Simulations for FCC-ee beam self-polarization

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Contents:

- Sokolov-Ternov polarization in a 100 km ring
- Polarization in presence of wigglers; parametric studies
- Simulations at 45 and 80 GeV in presence of misalignments
- Some considerations on energy calibration
- Summary

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Introduction

- High precision beam energy measurement ($\ll$ 100 keV) is needed for Z pole physics at 90 GeV CM energy and W physics at 160 GeV CM energy.
- If not at cost of luminosity, longitudinal beam polarization improves Z peak measurements, but it is not essential.
- Self-polarization through Sokolov-Ternov effect strongly depends on bending radius and beam energy: not obvious for FCC.

Sokolov-Ternov polarization

Beam get vertically polarized in the vertical guiding field of the ring

$$P_\infty = 92.3\% \qquad \tau_p^{-1} = \frac{5\sqrt{3}}{8}\frac{r_e\gamma^5\hbar}{m_0 C}\oint\frac{ds}{|\rho|^3}$$

For FCC-e^+e^- with $\rho \simeq$ 10424 m, fixed by the maximum attainable dipole field for the hh case, it is

E (GeV)	U_0 (MeV)	σ_E/E (%)	τ_{pol} (h)
45	35	0.038	256
80	349	0.067	14

Effect of wigglers

$\boldsymbol{\tau_p}$ may be reduced by introducing wigglers:

$$\tau_p^{-1} = F\gamma^5 \Big[\int_{dip} \frac{ds}{|\rho_d|^3} + \int_{wig} \frac{ds}{|\rho_w|^3} \Big] \qquad F \equiv \frac{5\sqrt{3}}{8} \frac{r_e \hbar}{m_0 C}$$

Polarization

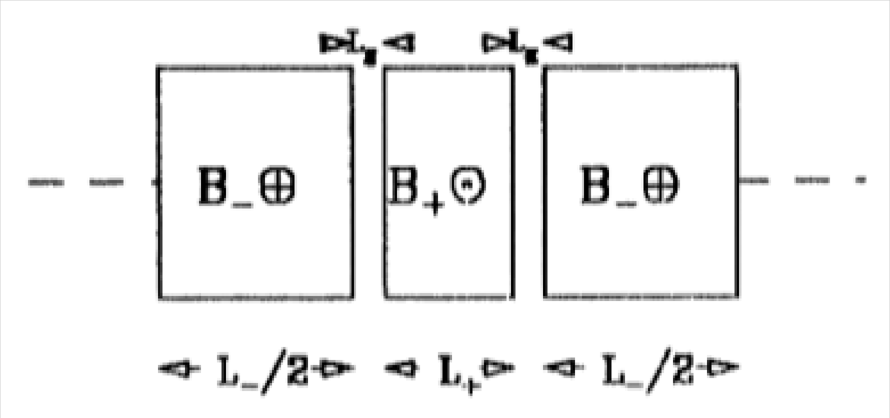
$$P_\infty = \frac{8}{5\sqrt{3}} \frac{\oint ds \frac{\hat{B}\cdot\hat{n}_0}{|\rho|^3}}{\oint ds \frac{1}{|\rho|^3}} \propto \tau_p \Big[\int_{dip} ds\, \frac{\hat{B}_d \cdot \hat{n}_0}{|\rho_d|^3} + \int_{wig} ds\, \frac{\hat{B}_w \cdot \hat{n}_0}{|\rho_w|^3} \Big]$$

$\hat{n}_0 \equiv \hat{y}$ in a perfectly planar ring.

Constraints:

- $x' = 0$ outside the wiggler $\Rightarrow \int_{wig} ds\, B_w = 0$ (vanishing field integral)
- $x = 0$ outside the wiggler $\Rightarrow \int_{wig} ds\, sB_w = 0$ (true for symmetric field)
- P large $\Rightarrow \int_{wig} ds\, B_w^3$ must be large

The LEP polarization wigglers have been considered



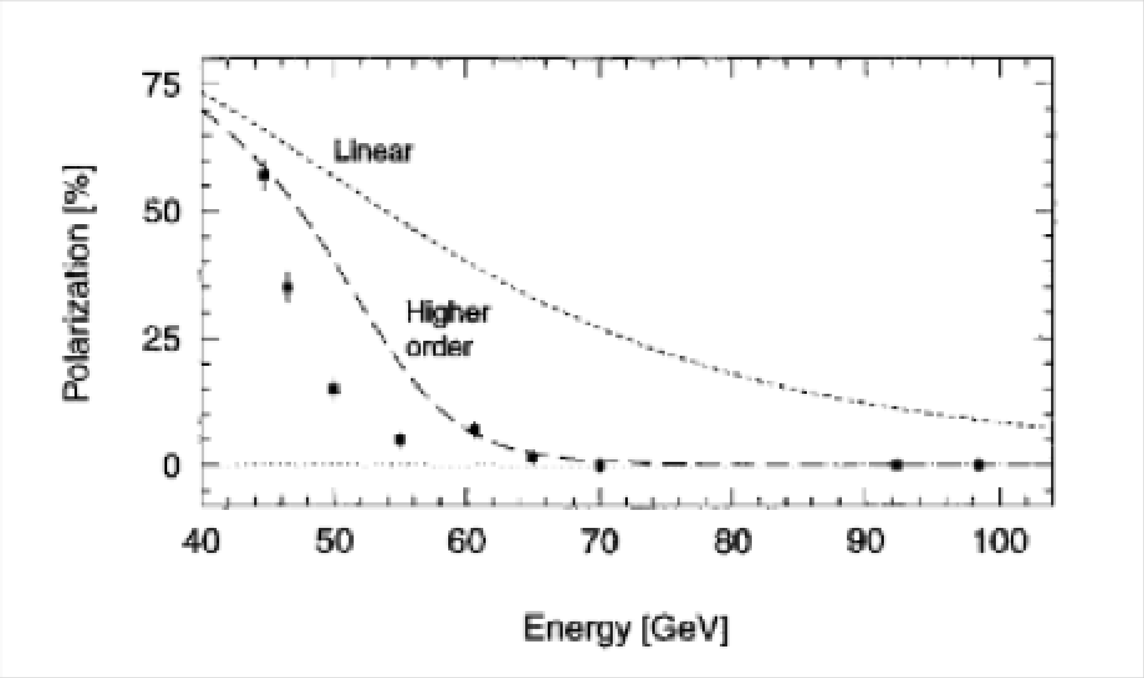
$$\int_{wig} ds \, \frac{1}{\rho_w^3} = \frac{L_+}{\rho_+^3}\left(1 - \frac{1}{N^2}\right) \qquad N \equiv L_-/L_+ = B_+/B_-$$

N should be large for keeping polarization high!

4 such wigglers with $\boldsymbol{N = 6}$ and $\boldsymbol{L_+}$=1.3 m have been introduced in dispersion free regions of a simplified FCC ring (*"toy ring"*). At 45 GeV:

$\boldsymbol{B_+}$ (T)	$\boldsymbol{U_0}$ (MeV)	$\boldsymbol{\Delta E/E}$ (%)	$\boldsymbol{\Delta E}$ (MeV)	$\boldsymbol{\epsilon_x}$ (μm)	$\boldsymbol{\tau_x}$ (s)	$\boldsymbol{P}$ (%)	$\boldsymbol{\tau_{pol}}$ (min)
0	37	.04	18	.8e-3	.82	92.4	14e3
1.3	64	.22	99	.5e-2	.48	87.6	247
2.6	144	.41	184	.070	.21	87.6	31
3.9	278	.55	247	.274	.11	87.6	9
5.2	466	.65	292	.691	.06	87.6	4

LEP measured polarization



(R. Assmann et al., SPIN2000, Osaka)

Polarization strongly depending on energy and no polarization observed above 65 GeV!

Sokolov-Ternov effect
in the guiding dipole field

↓

Polarisation

Perturbations
(v-bends, vertical orbit in quads etc.)

↓

Depolarisation

↘ ↙

Equilibrium polarisation $(< \boldsymbol{P_{ST}})$

Derbenev-Kondratenko expression for equilibrium polarization

$$P_{DK} = \frac{8}{5\sqrt{3}} \frac{\oint ds < \frac{1}{|\rho|^3} \hat{b} \cdot (\hat{n} - \frac{\partial \hat{n}}{\partial \delta}) >}{\oint ds < \frac{1}{|\rho|^3} \left[1 - \frac{2}{9} (\hat{n} \cdot \hat{s})^2 + \frac{11}{18} (\frac{\partial \hat{n}}{\partial \delta})^2 \right] >}$$

with

$$\hat{b} \equiv \vec{v} \times \dot{\vec{v}} / |\vec{v} \times \dot{\vec{v}}|$$

$\partial \hat{n} / \partial \delta$ ($\delta \equiv \delta E / E$) quantifies the depolarizing effects resulting from the trajectory perturbations consequent to photon emission.

Perfectly planar machine: $\partial \hat{n} / \partial \delta$=0.
In presence of radial fields: $\partial \hat{n} / \partial \delta \neq$0 and large when

$$\nu_{spin} \pm m Q_x \pm n Q_y \pm p Q_s = \text{integer} \qquad \nu_{spin} \simeq a\gamma$$

Usually the dominant higher order resonances are the *synchrotron sidebands* of the first order resonances.

LEP lack of polarization at high energy is understood as due to the *larger* beam energy spread. Wigglers increase the energy spread of FCC-e+e- beams!

Is it possible to improve the wiggler design to get lower energy spread at constant τ_{pol}?

The important interconnected parameters are

$$U_{loss} = \frac{C_\gamma E^4}{2\pi} \oint \frac{ds}{\rho^2} \qquad\qquad (\sigma_E/E)^2 = \frac{C_q}{J_\epsilon}\gamma^2 \oint \frac{ds}{|\rho|^3} / \oint \frac{ds}{\rho^2}$$

$$\tau_p^{-1} = F\gamma^5 \Big[\int_{dip} \frac{ds}{|\rho_d|^3} + \int_{wig} \frac{ds}{|\rho_w|^3}\Big] = F\gamma^5 \Big[\int_{dip} \frac{ds}{|\rho_d|^3} + \frac{L^+}{|\rho^+|^3}(1 + \frac{1}{N^2})\Big]$$

$$P_\infty = \frac{8F\gamma^5}{5\sqrt{3}}\, \tau_p \Big[\int_{dip} ds\, \frac{\hat{B}_d \cdot \hat{n}_0}{|\rho_d|^3} + \frac{L^+}{|\rho^+|^3}(1 - \frac{1}{N^2})\Big] \qquad \hat{n}_0 \equiv \hat{y} \text{ in a } \textit{planar} \text{ ring}$$

For energy calibration the actual important parameter is the time, $\tau_{10\%}$, needed to reach $P \simeq 10\%$ rather than τ_p

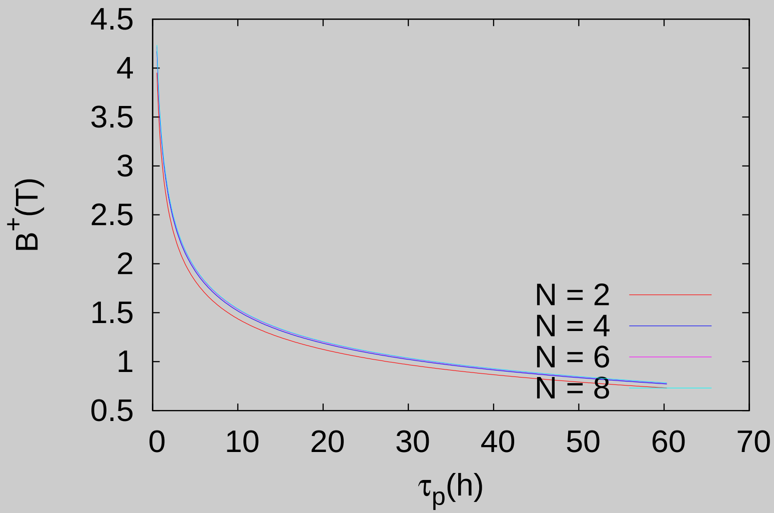
$$\tau_{10\%} = -\tau_p \times \ln(1 - 0.1/P_\infty) \qquad \text{depends upon } P_\infty$$

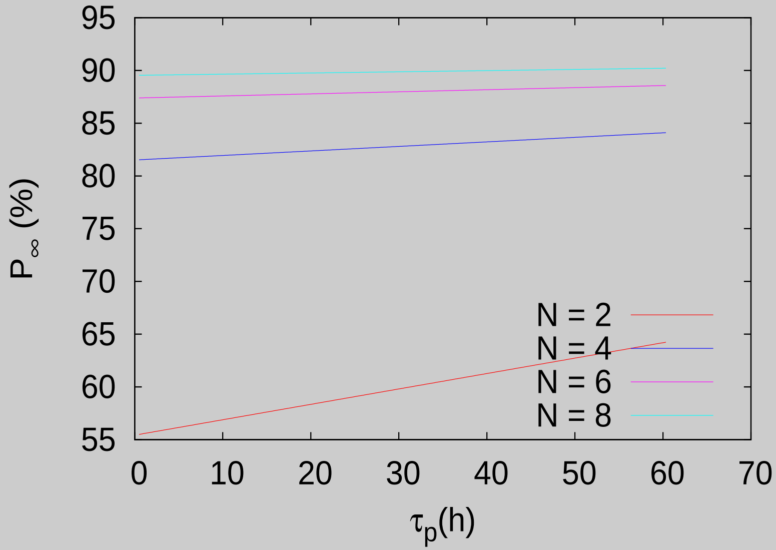
The energy spread may written as

$$(\sigma_E/E)^2 = \frac{C_q C_\gamma E^4}{2\pi J_\epsilon F \gamma^3} \frac{1}{\tau_p U_{loss}}$$

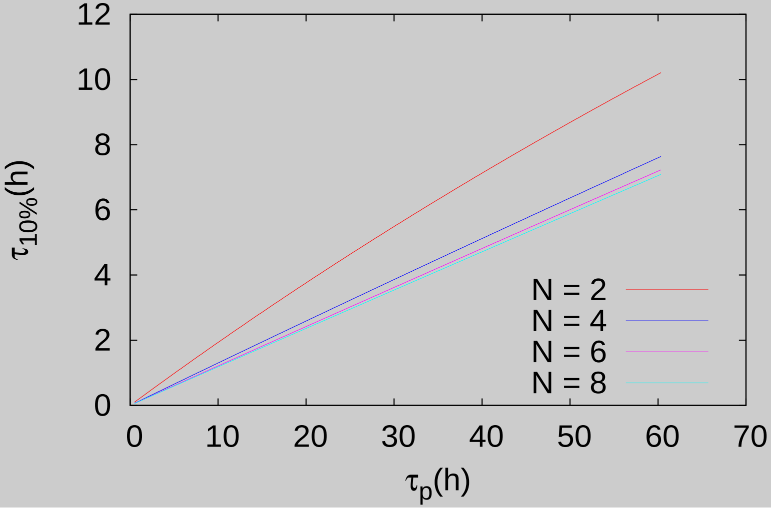
i.e. small σ_E and τ_p are at the price of higher U_{loss}.

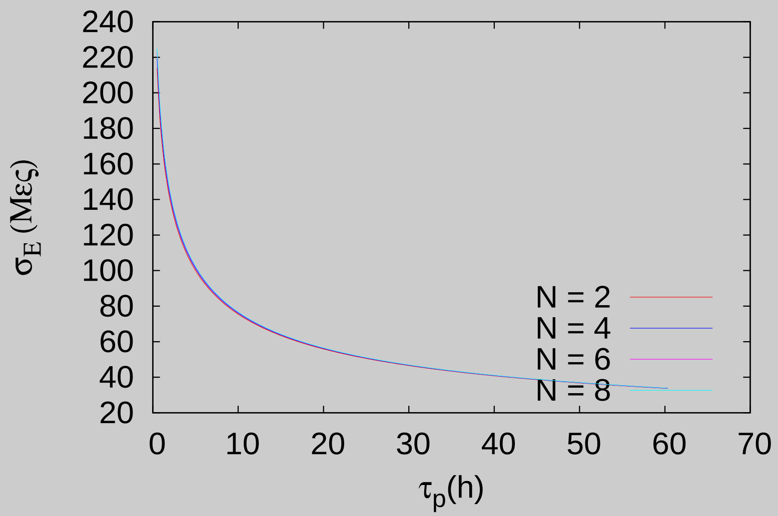
Effect of one wiggler - 45 GeV



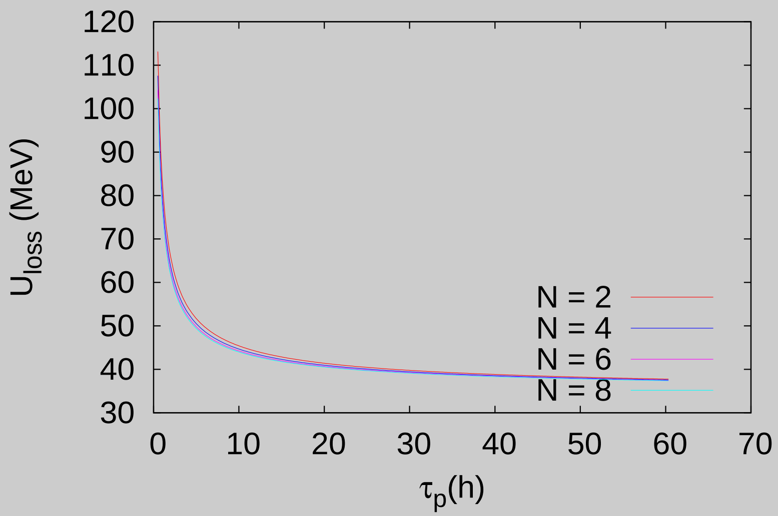


nb: $\boldsymbol{L_{-}} = \boldsymbol{N L_{+}}$, with $\boldsymbol{L_{+}}$=1.3 m





σE (Mες)
240
220
200
180
160
140
120
100
80
60
40
20
0
10
20
30
40
50
60
70
τp(h)
N = 2
N = 4
N = 6
N = 8



Uloss (MeV)
120
110
100
90
80
70
60
50
40
30
0
10
20
30
40
50
60
70
τp(h)
N = 2
N = 4
N = 6
N = 8

Fixing σ_E=50 MeV (LEP σ_E at 60 GeV) $\Rightarrow B^+ \simeq 1$ T for any value of N.

$$L_- = N L_+, \text{ with } L_+ = 1.3 \text{ m}$$

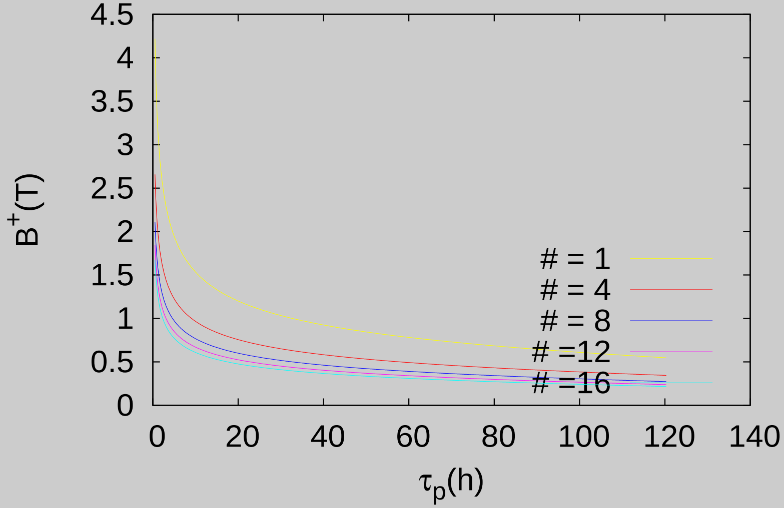
N	B^+ (T)	U_{loss} (MeV)	σ_E (MeV)	P (%)	τ_{pol} (h)	$\tau_{10\%}$ (h)
2	1.03	40.4	50.1	59.1	25.5	4.7
4	1.08	39.9	50.0	82.6	25.8	3.3
6	1.09	39.7	50.1	87.9	26.0	3.1
8	1.09	39.5	50.0	89.8	26.0	3.1

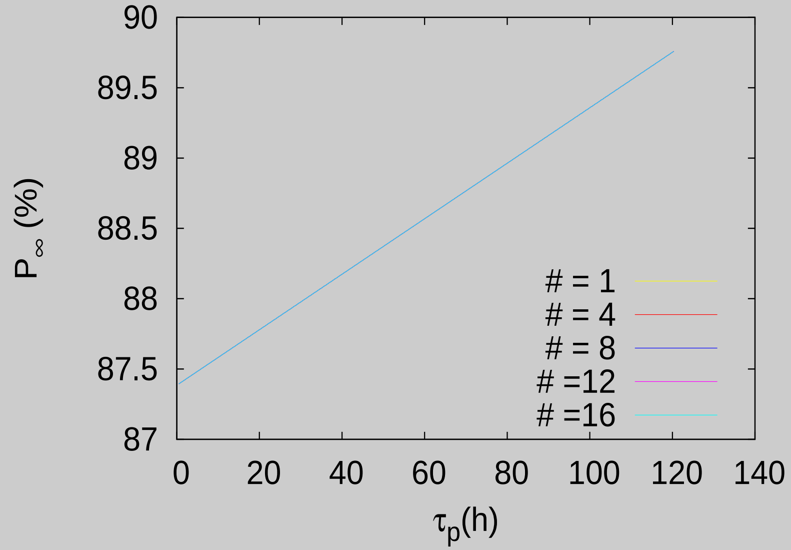
For such field only N=2 should be avoided because of the larger $\tau_{10\%}$.

Keeping $L_+ + L_- = L_+(1+N) = 9.3$

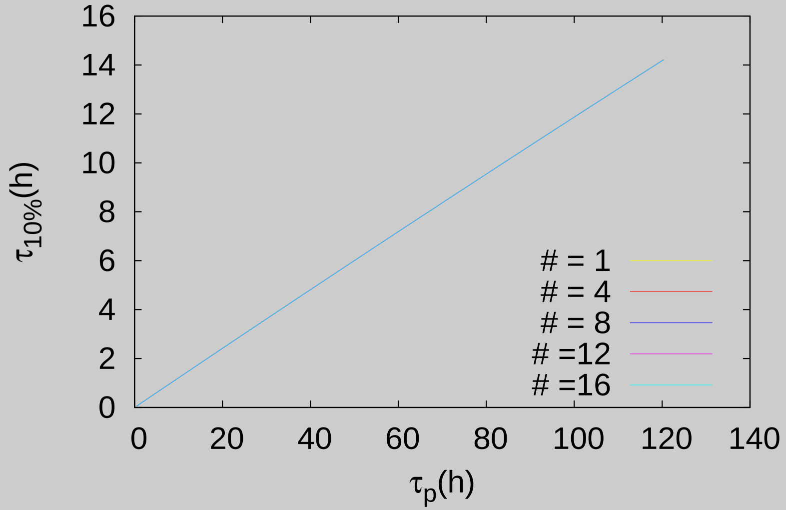
N	B^+ (T)	U_{loss} (MeV)	σ_E (MeV)	P (%)	τ_{pol} (h)	$\tau_{10\%}$ (h)
2	0.78	42.4	50.0	59.0	24.3	4.5
4	0.96	40.6	50.0	82.6	25.5	3.3
6	1.08	39.7	50.0	87.9	26.0	3.1
8	1.18	39.2	50.0	89.8	26.3	3.1

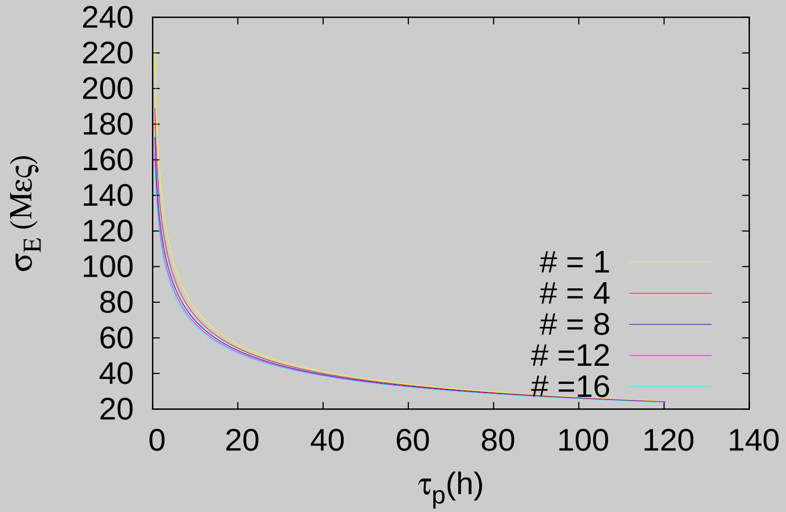
Effect of number of wigglers





nb: $L_- = N L_+$, with L_+=1.3 m and N=6





240
220
200
180
160
140
120
100
80
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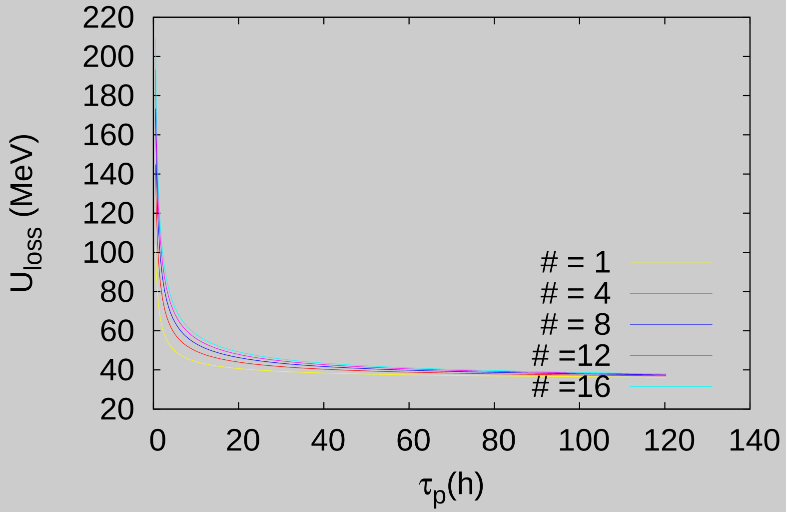
40
20
0
20
40
60
80
100
120
140
σ_E (Mες)
τ_p(h)
= 1
= 4
= 8
=12
=16



220
200
180
160
140
120
100
80
60
40
20
0
20
40
60
80
100
120
140
U_{loss} (MeV)
τ_p(h)
= 1
= 4
= 8
=12
=16

Fixing σ_E=50 MeV

#	B^+ (T)	U_{loss} (MeV)	σ_E (MeV)	P (%)	τ_{pol} (h)	$\tau_{10\%}$ (h)
1	1.09	39.7	50.1	87.9	26.0	3.1
4	0.71	42.8	50.0	87.9	24.2	2.9
8	0.57	45.3	50.0	87.8	22.8	2.8
12	0.51	47.1	50.0	87.8	22.0	2.7
16	0.47	48.6	50.0	87.8	21.3	2.6

No "miraculous" set of parameters, but larger number of wigglers is better:

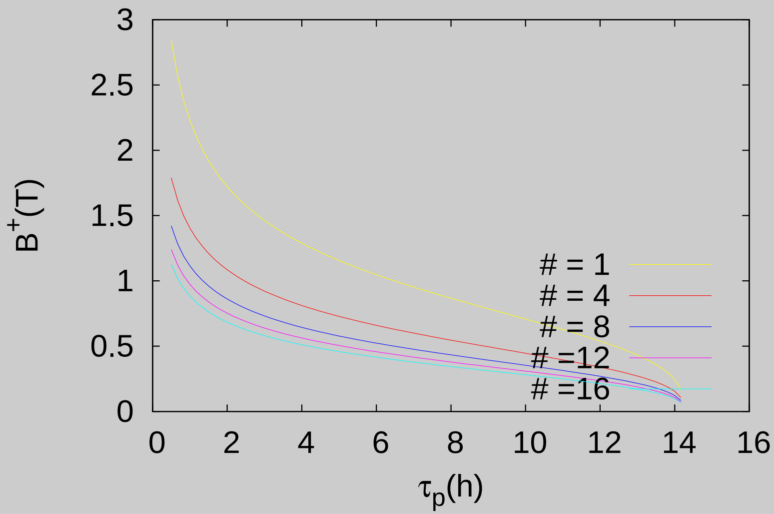
- polarization time decreases
- losses increase but they are better distributed; however with 16 wigglers P_{RF} increases from 51 to 70.5 MW for I=1450 mA (U_{loss}=35 MeV w/o wigglers)

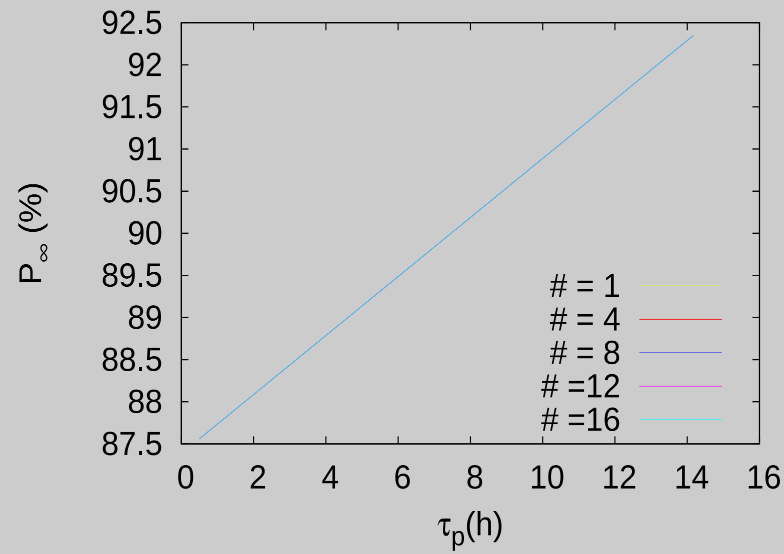
80 GeV case

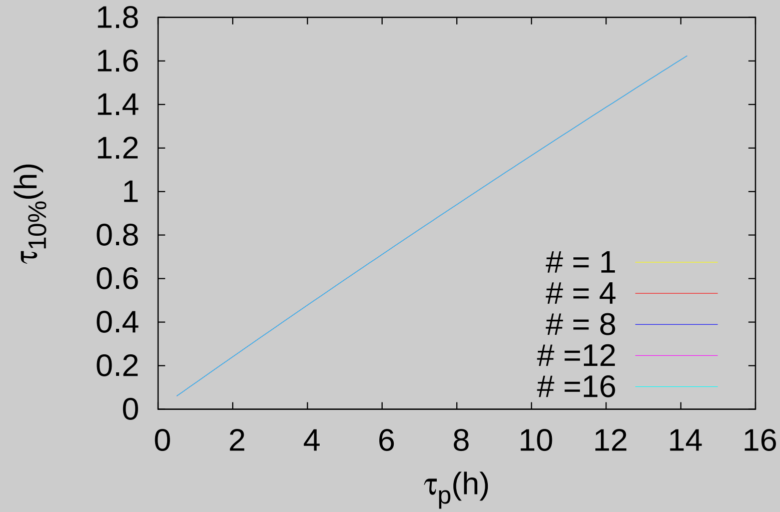
For curiosity...

E (GeV)	U_0 (MeV)	σ_E/E (%)	σ_E (MeV)	τ_{pol} (h)	τ_{10} (h)
45	35	0.038	17.1	256	29.0
80	349	0.067	53.6	14	1.6

Do we need wigglers? No, as polarization is not needed for physics.

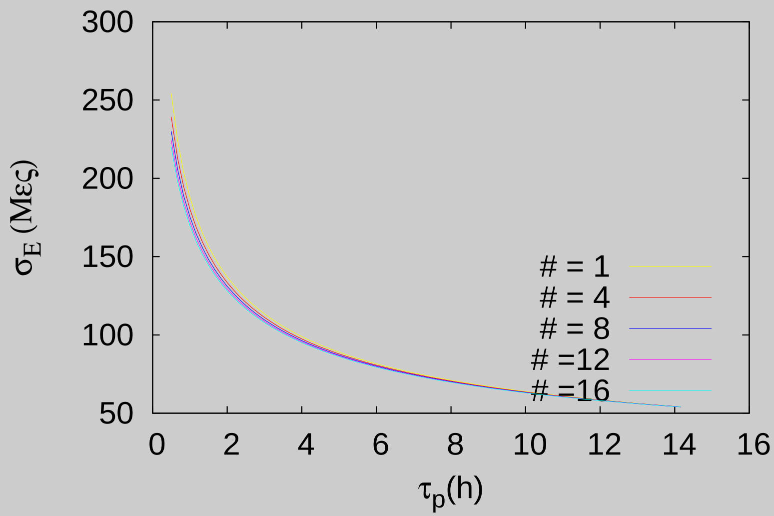


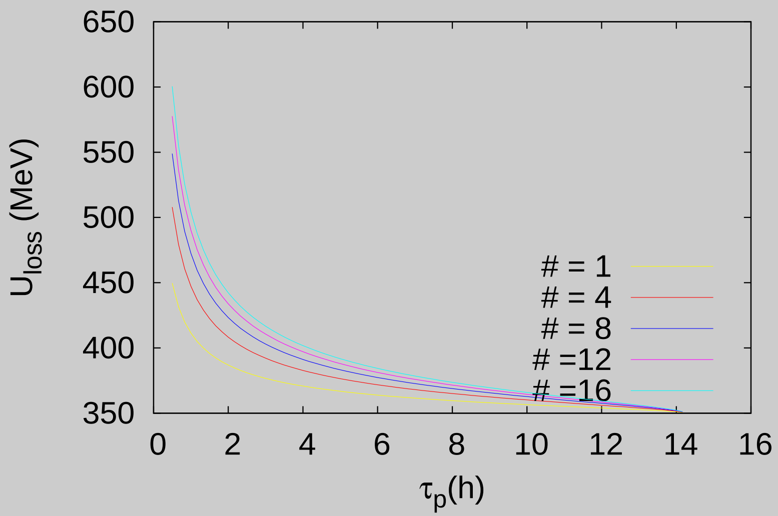




$\boldsymbol{L_{-}}=\boldsymbol{N}\boldsymbol{L_{+}}$, $\boldsymbol{L_{+}}$=1.3 m

80 GeV beam energy


σE (Mες)
τp(h)
= 1
= 4
= 8
=12
=16


Uloss (MeV)
τp(h)
= 1
= 4
= 8
=12
=16

Parameter values for halving $\tau_{10\%}$

#	B^+ (T)	U_{loss} (MeV)	σ_E (MeV)	P (%)	τ_{pol} (h)	$\tau_{10\%}$ (h)
0	-	350.4	53.9	92.3	14.2	1.6
1	0.94	361.2	74.9	89.9	7.2	0.8
4	0.60	368.1	75.1	89.8	7.0	0.8
8	0.48	372.7	74.6	89.8	7.0	0.8
12	0.42	376.7	75.1	89.8	6.8	0.8
16	0.38	379.4	74.9	89.8	6.8	0.8

No advantage from large number of wigglers, a part from better distributed losses.

Resonances are awakened by imperfections!

Question: how *perfect* the ring must be for keeping resonances "sleeping"?

Simulations in presence of realistic errors and corrections are needed.

- MAD-X used for simulating quadrupole misalignments and orbit correction
- SITROS (by J. Kewish) used for computing the resulting polarization. It is a tracking code with 2th order orbit description and non-linear spin motion. It has been used for HERA-e in the version improved by M. Böge and M. Berglund.
  - HERA-e like *Harmonic Bumps* optimization for $\delta\hat{\boldsymbol{n}}_0$ correction in the FCC-e+e- ring implemented.

SLIM by A. Chao is used for linear calculations.

SLICKTRACK by D. Barber is available too, but it needs extra work to avoid using the costly NAG library.

Washington week:

- 45 GeV case with 4 wigglers
  - effect of quadrupole vertical mis-alignment for various wiggler field strength was considered
  - in absence of BPMs errors polarization was not a mission impossible

In this talk:

- 45 GeV
  - limit ΔE=50 MeV (extrapolating from LEP)
  - 4 wigglers with B^{+}= 0.7 T
  - 10% polarization in 2.9 h for energy calibration
- 80 GeV
  - no wigglers
  - 10% polarization in 1.6 h for energy calibration
- BPMs errors added to quadrupole misalignments

Simulations at 45 GeV

"Toy" ring, 4 wigglers with B_+=0.7 T

- Q_x=0.1278
  Q_y=0.2085
  Q_s=0.1174 (U_{rf}=900 MV, f_{RF}=400 MHz)

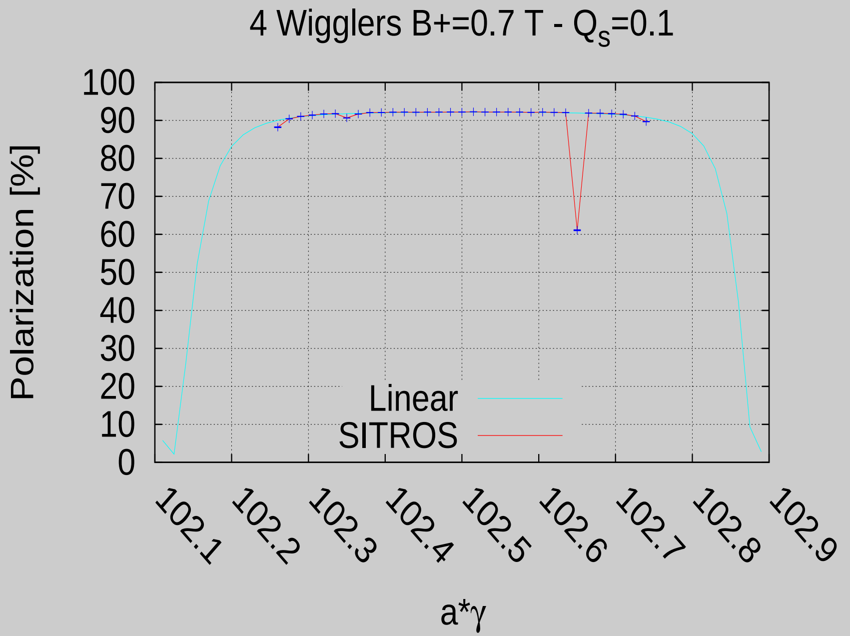
Closed orbit correction scheme:

- BPM introduced close to each quadrupole
- one vertical corrector introduced close to each vertical focusing quadrupole
- orbit corrected either by
  - SVD using all 1096 correctors
    or
  - 110 correctors (MICADO algorithm)
- polarization axis $\hat{n}_0(s)$ distortion corrected by 8 "Harmonic Bumps" à la HERA-e

Quadrupole vertical misalignments

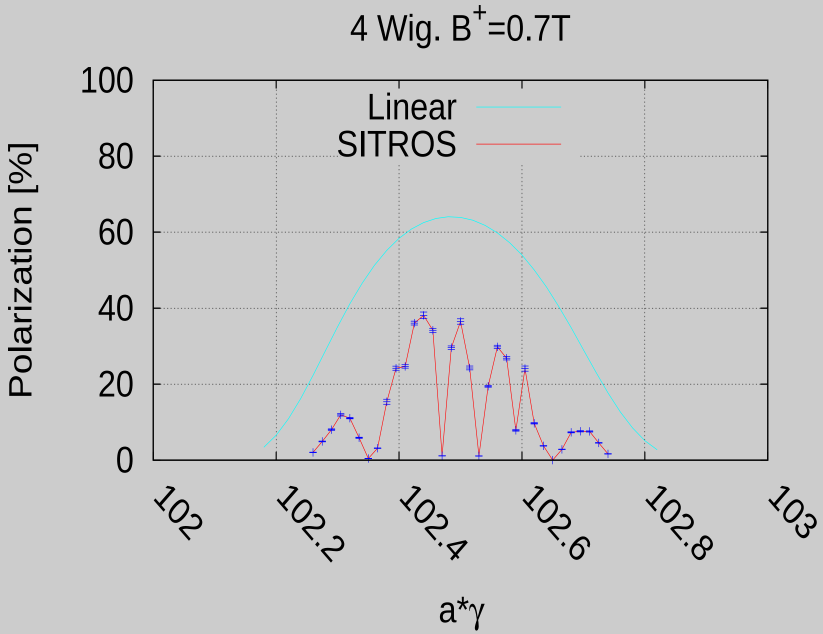
- $\delta_y^Q = 200\ \mu$m

	y_{rms} (mm)	$\delta\hat{n}_{0,rms}$ (mrad)
	8.	26.4
SVD	0.05	0.3

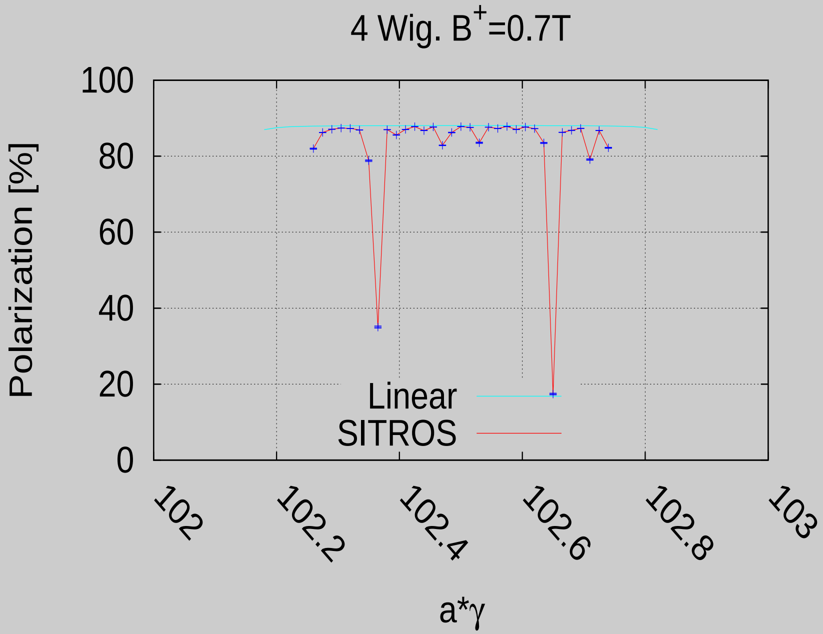


- $\delta_y^Q = 200\ \mu$m
- BPMs errors
  - $\delta_y^M = 200\ \mu$m
  - 10% calibration errors

	y_{rms} (mm)	$\delta\hat{n}_{0,rms}$ (mrad)
	8.	26.4
SVD	0.8	3.9
+bumps	0.9	2.0



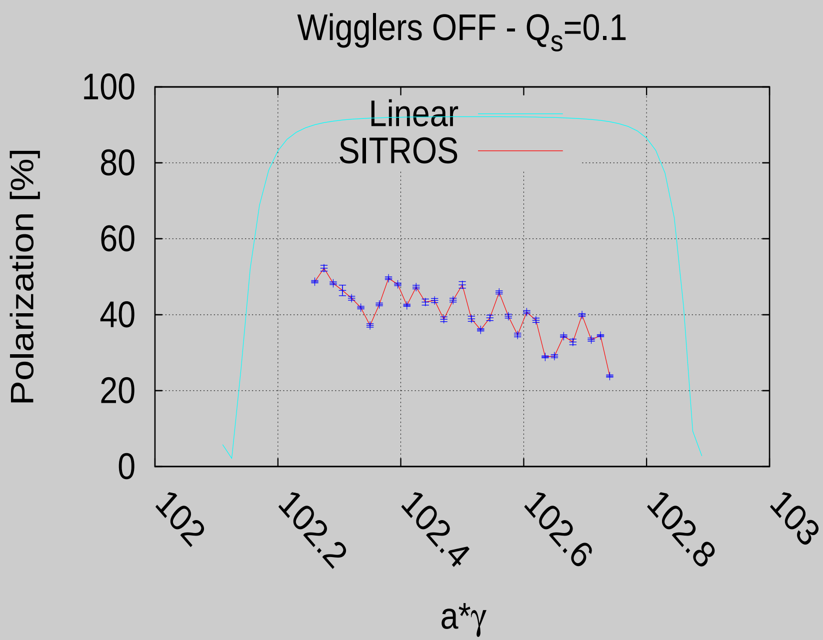
SVD



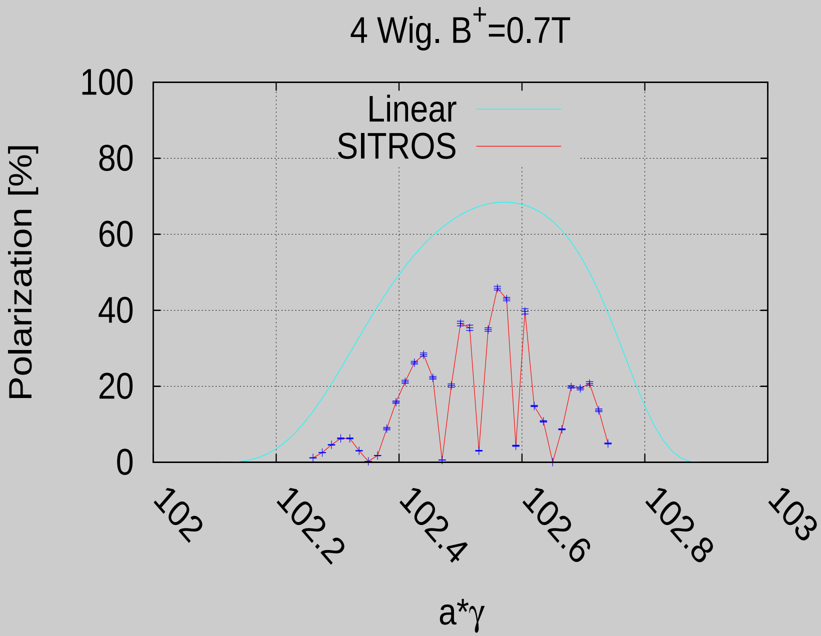
SVD + harmonic bumps

Increasing wiggler strength and keeping errors/correctors (orbit and $\delta\hat{n}_0$ are unchanged), ie

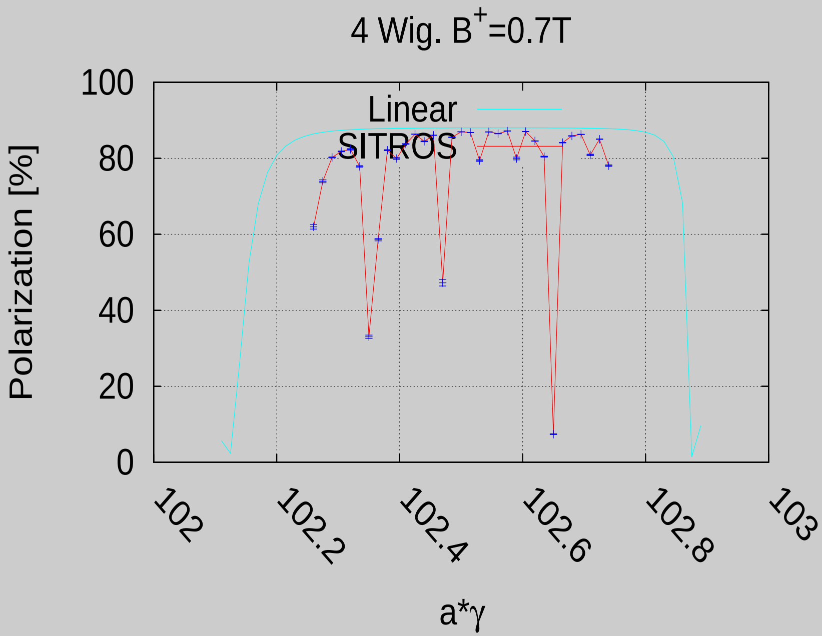
- 4 wigglers with B^+=3.9 T (ΔE=247 MeV at 45 GeV !)
- δ_y^Q= 200 μm
- BPMs errors
  - δ_y^M= 200 μm
  - 10% calibration errors
- SVD correction + hb



	y_{rms} (mm)	$\delta\hat{n}_{0,rms}$ (mrad)
	8.	26.4
MICADO	0.6	3.9
+bumps	0.7	2.2

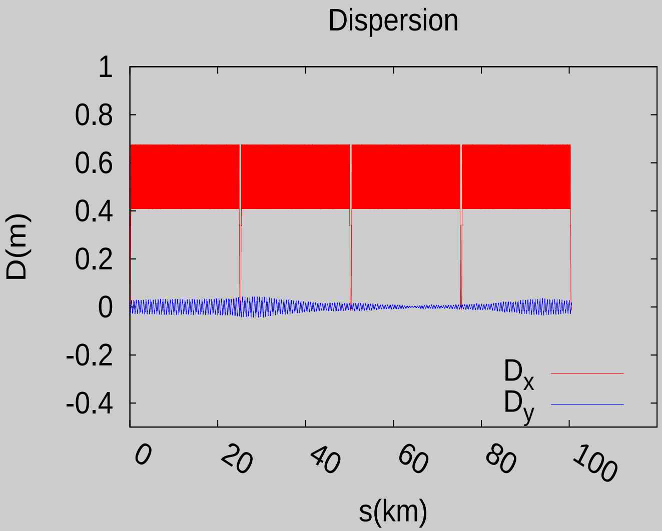


MICADO



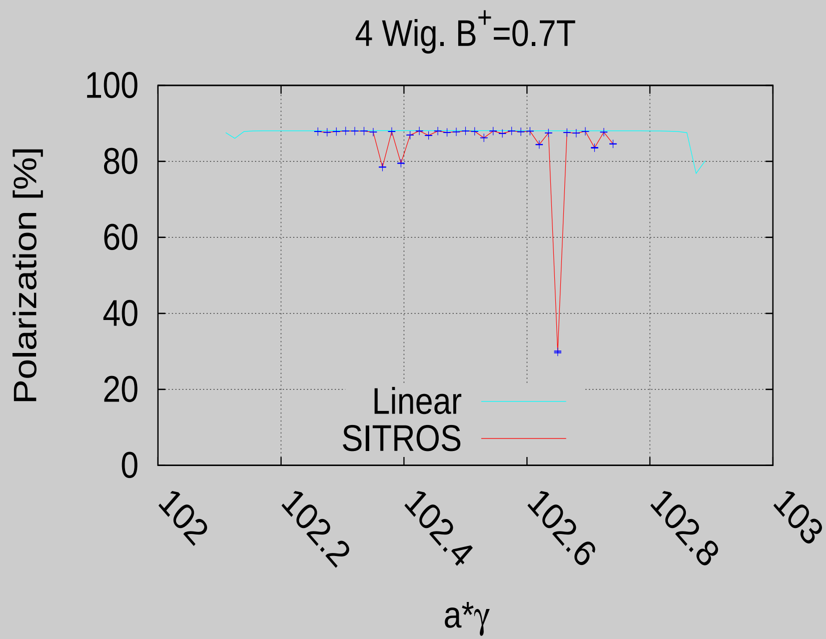
MICADO + harmonic bumps

Effect of quadrupole roll angle.



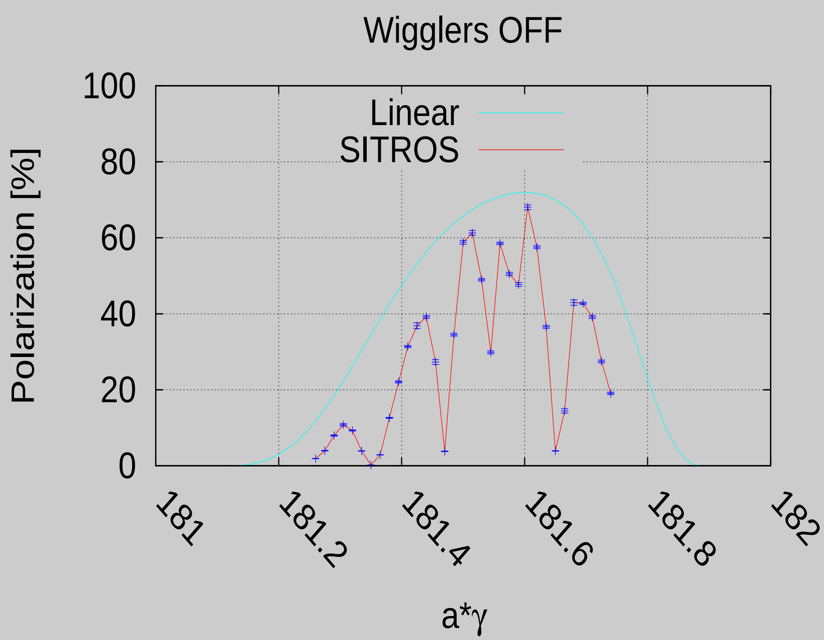
$\Delta\theta^{Q}_{rms}$=0.25 mrad

ϵ_x (μ)	ϵ_y (μ)	ratio (%)
0.1048e-2	0.27e-4	2.6

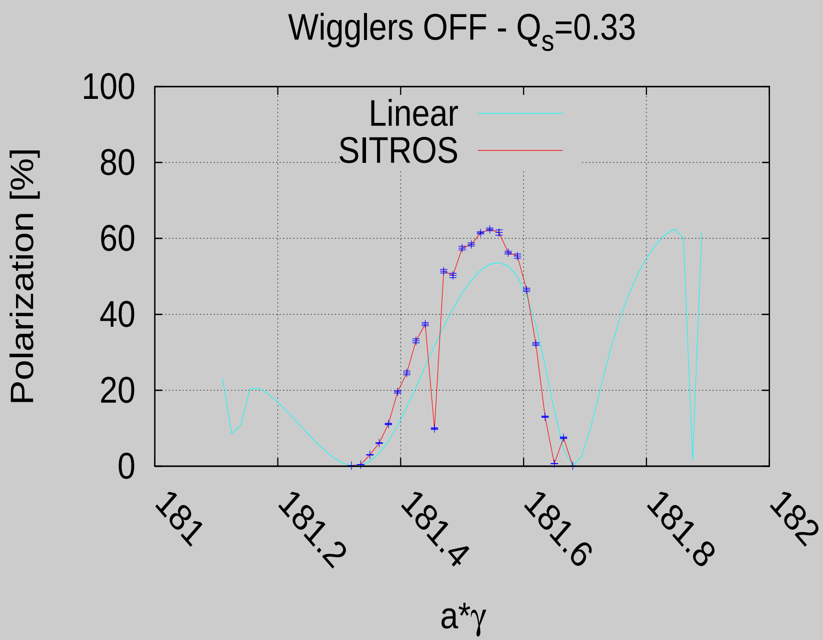


Simulations at 80 GeV

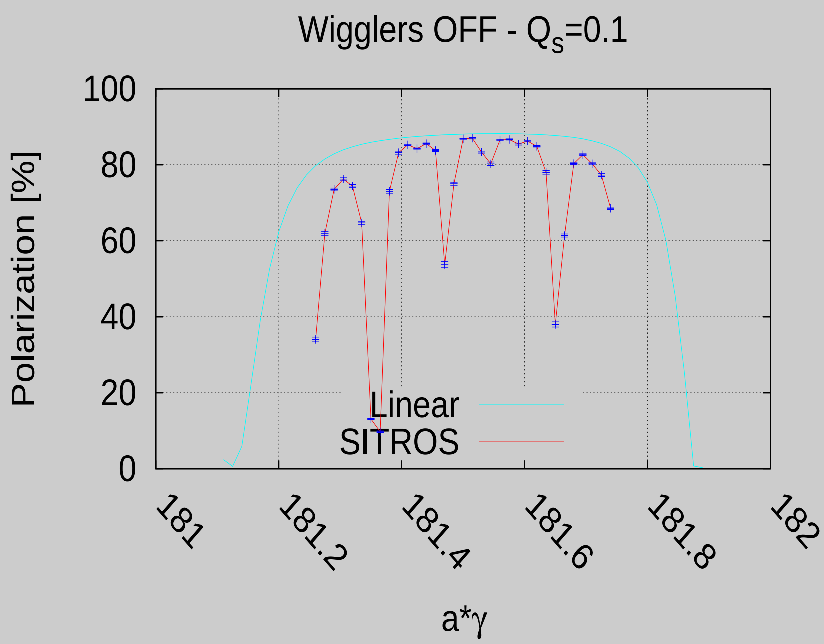
- no wigglers
- $\delta_y^Q = 200\ \mu$m
- no BPMs errors
- orbit correction by SVD
  - y_{rms}=0.05 mm
  - $\epsilon_y/\epsilon_x \simeq 0$
  - $\delta\hat{n}_{0,rms}$=3 mrad at 79.98 GeV



Increasing Q_s to 0.3[a]



Correcting $\delta\hat{n}_{0,rms}$=2.5 mrad

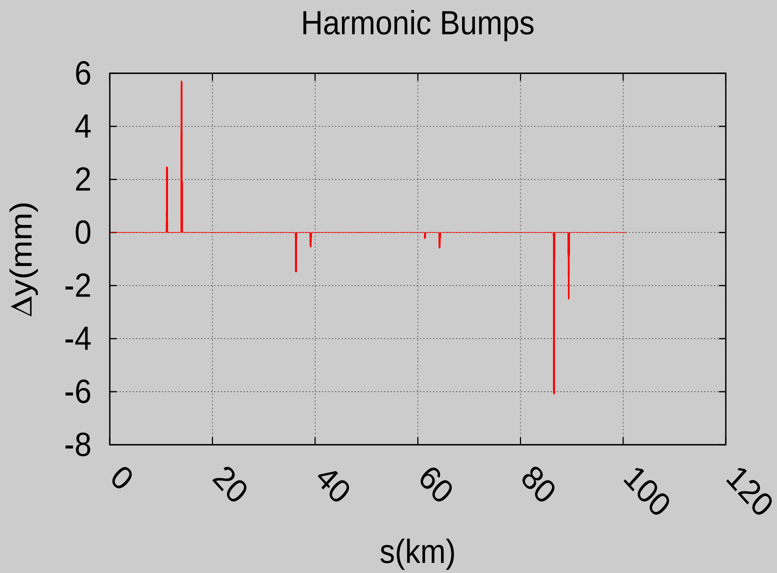


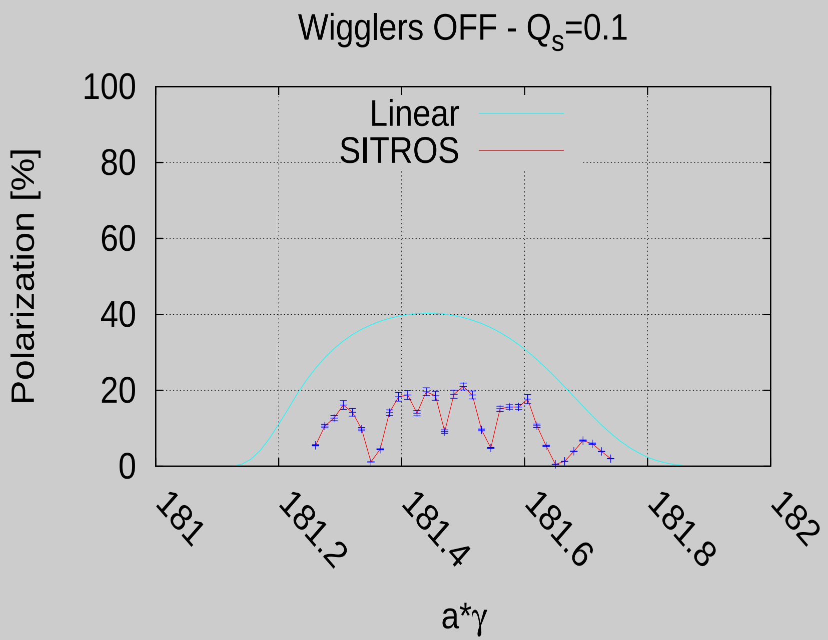
[a] Enhancement factor

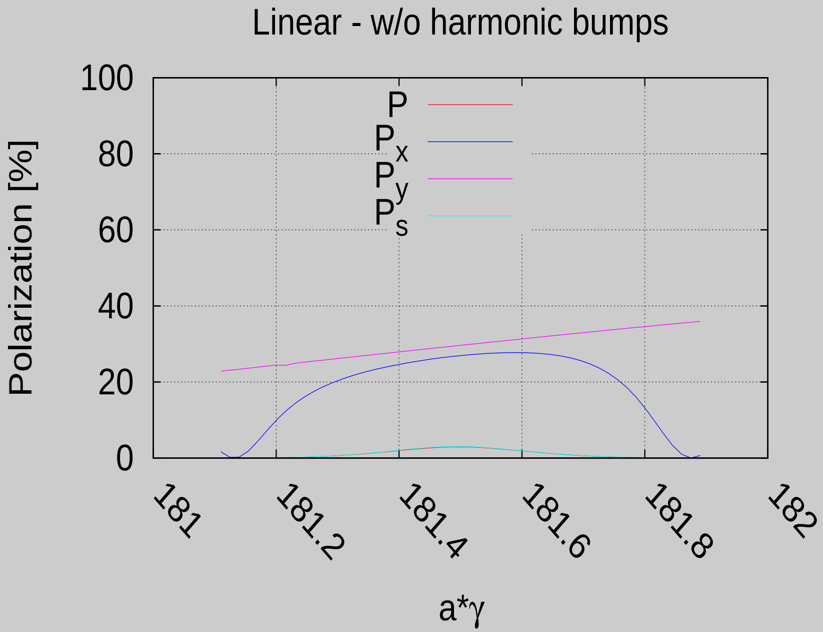
$$\xi = \left(\frac{a\gamma}{Q_s}\frac{\Delta E}{E}\right)^2$$

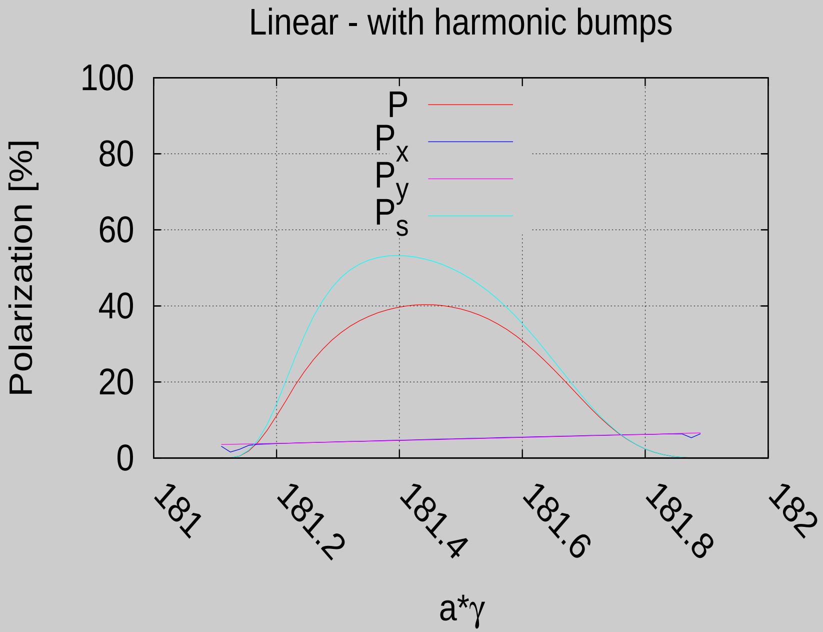
Adding BPMs errors

- no wigglers
- $\delta_y^Q = 200\ \mu$m
- BPMs errors
  - $\delta_y^M = 200\ \mu$m
  - 10% calibration errors
- orbit correction by SVD
  - y_{rms}=0.8 mm
  - ϵ_y/ϵ_x=0.2%
  - $\delta\hat{n}_{0,rms}$=19.8 mrad at 79.98 GeV
- with harmonic bumps
  - $\delta\hat{n}_{0,rms}$=8.6 mrad
  - ϵ_y/ϵ_x=2%

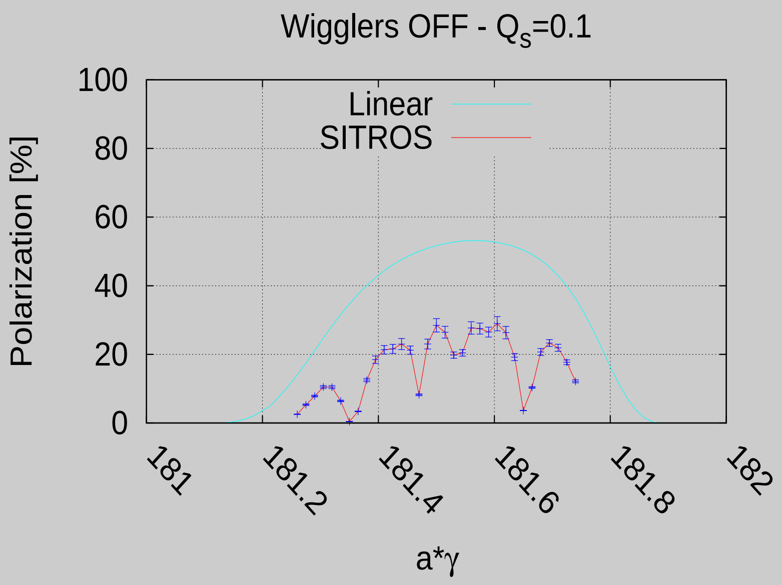


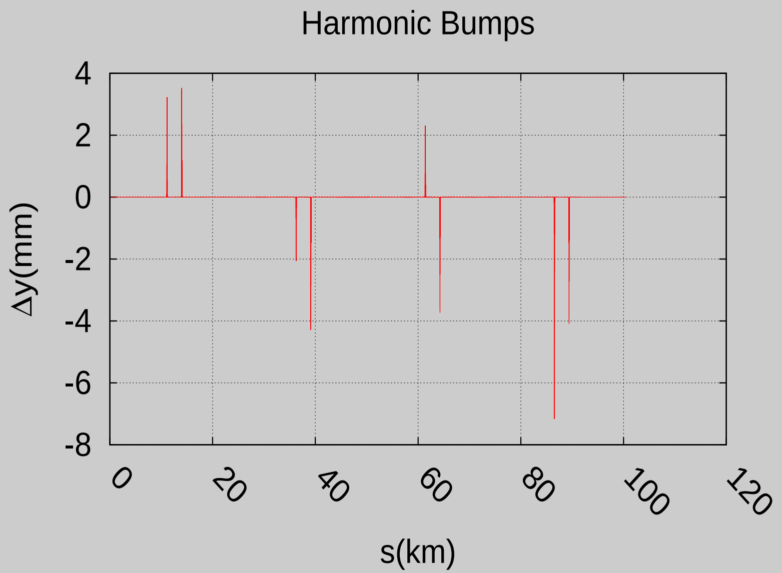


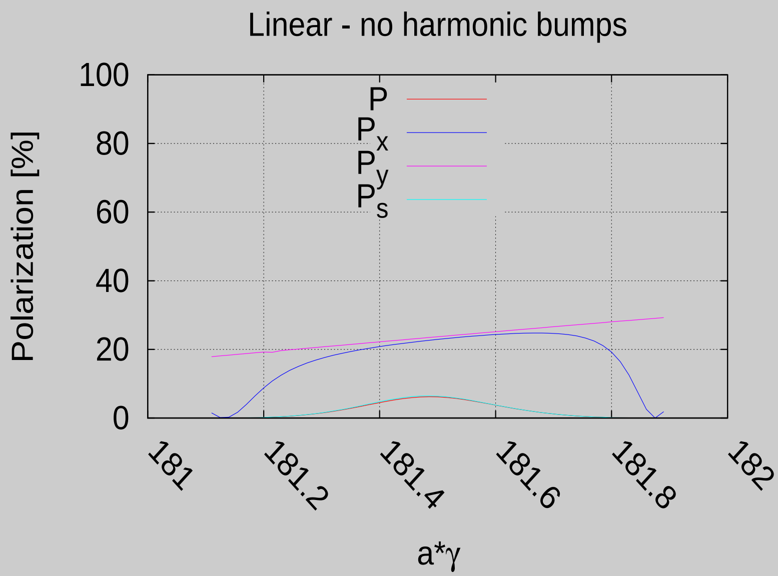

Linear - w/o harmonic bumps
Polarization [%]
a*γ
P
Px
Py
Ps
0
20
40
60
80
100
181
181.2
181.4
181.6
181.8
182

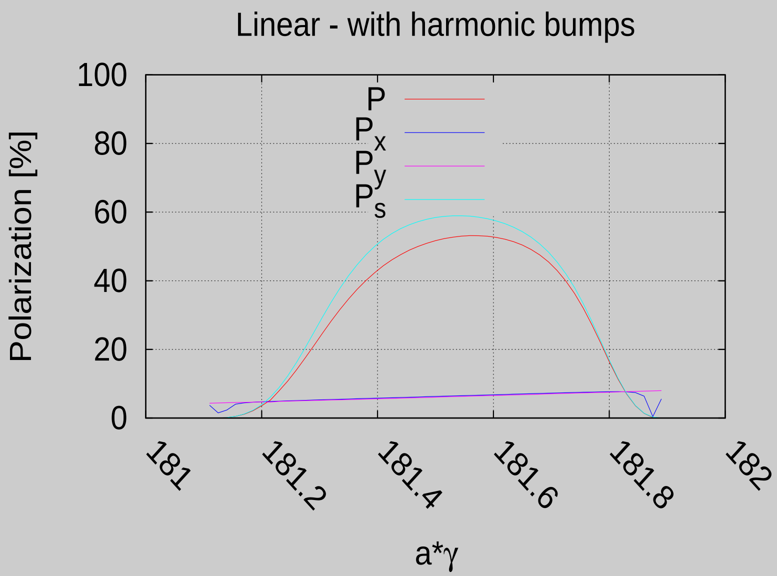

Linear - with harmonic bumps
Polarization [%]
a*γ
P
Px
Py
Ps
0
20
40
60
80
100
181
181.2
181.4
181.6
181.8
182

- no wigglers
- δ_y^Q= 200 μm
- BPMs errors
  - δ_y^M= 200 μm
  - 5% calibration errors
- orbit correction by SVD
  - y_{rms}=0.6 mm
  - ϵ_y/ϵ_x=0.3%
  - $\delta\hat{n}_{0,rms}$=14.4 mrad at 79.98 GeV
- $\delta\hat{n}_{0,rms}$=6.9 mrad with harmonic bumps
- ϵ_y/ϵ_x=2.5%



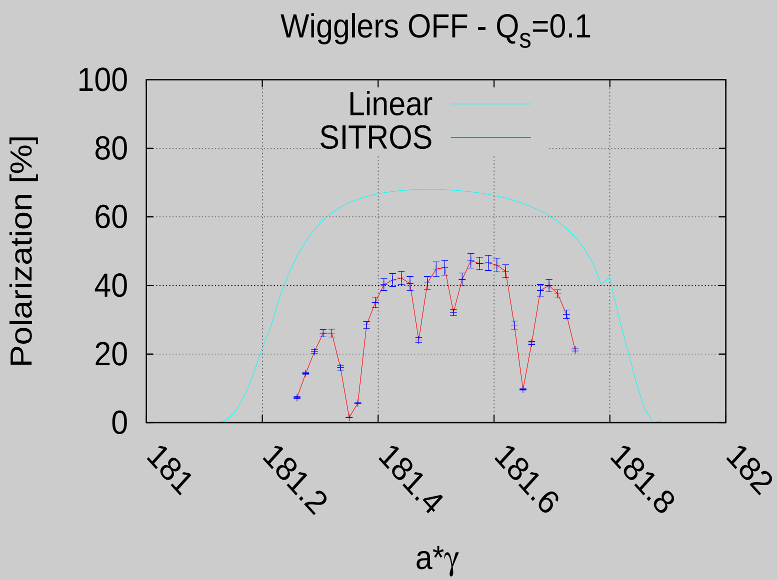


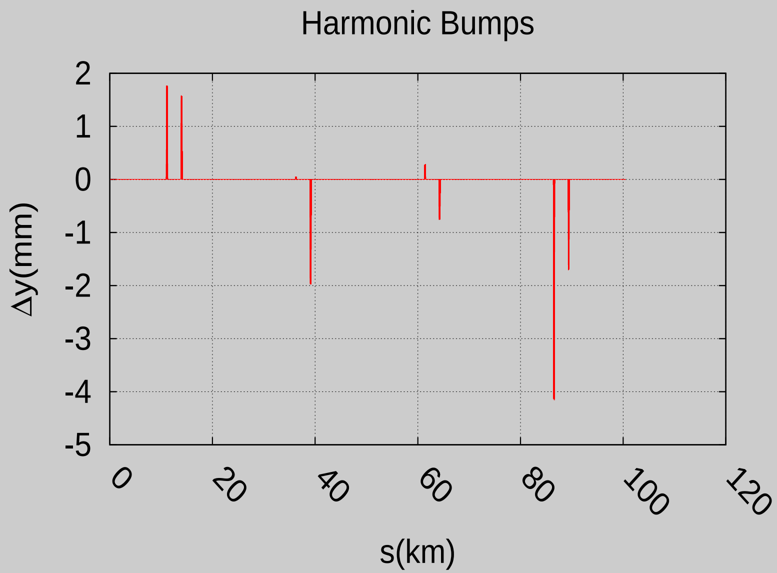




The large vertical bumps increase the vertical emittance!

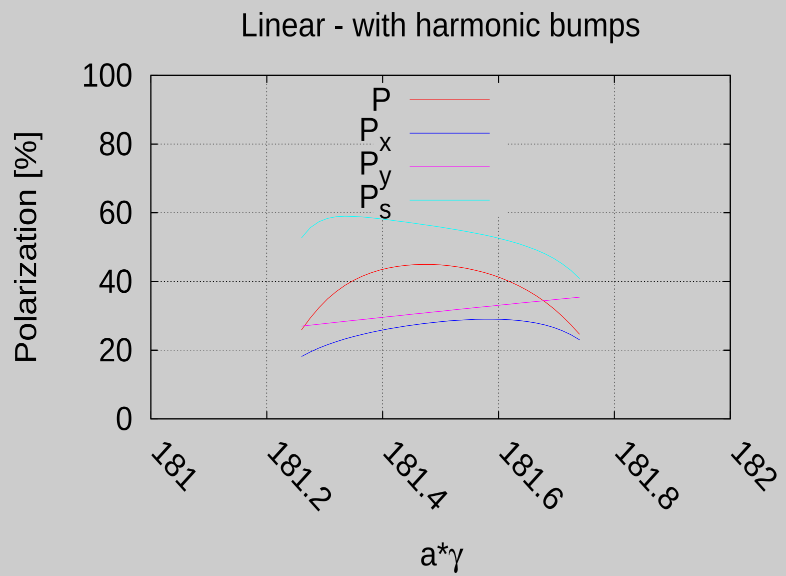
- no wigglers
- $\delta_y^Q = 200\ \mu$m
- BPMs errors
  - $\delta_y^M = 0\ \mu$m
  - 5% calibration errors
- orbit correction by SVD
  - y_{rms}=0.4 mm
  - $\delta\hat{n}_{0,rms}$=11.5 mrad at 79.98 GeV
- $\delta\hat{n}_{0,rms}$=5 mrad with harmonic bumps
- ϵ_y/ϵ_x=1.2%

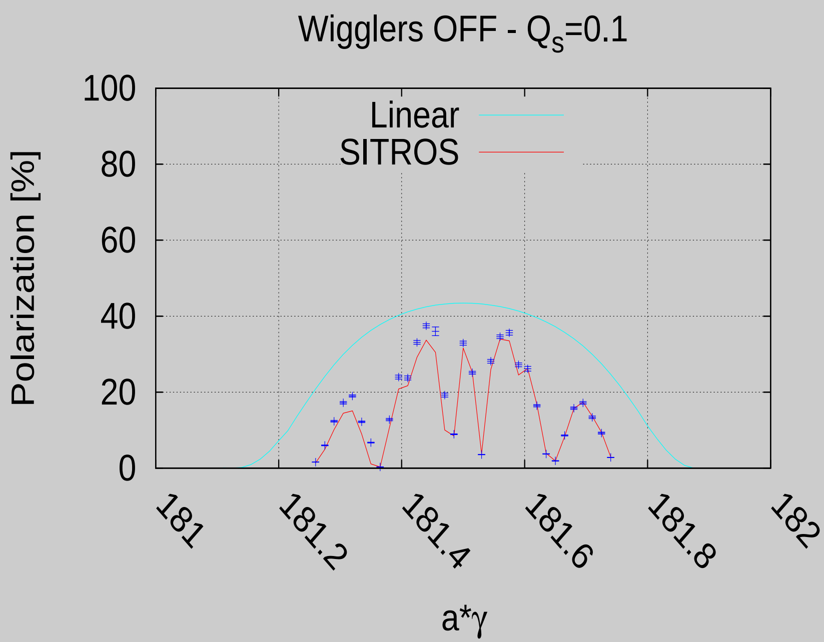




Idea: use 5 coils to get dispersion-free bumps.

- no wigglers
- $\delta_y^Q = 200\ \mu$m
- BPMs errors
  - $\delta_y^M = 200\ \mu$m
  - 10% calibration errors
- orbit correction by SVD
  - y_{rms}=0.8 mm
  - ϵ_y/ϵ_x=0.2%
  - $\delta\hat{n}_{0,rms}$=19.9 mrad at 79.98 GeV
- with harmonic bumps
  - $\delta\hat{n}_{0,rms}$=9.7 mrad
  - ϵ_y/ϵ_x=0.2%





Some considerations on energy calibration through resonant depolarization

It is based on the relationships

$$\nu_{spin} = a\gamma \qquad\qquad a \equiv \text{gyromagnetic anomaly}$$

Required precision: better than 100 KeV.

To be taken into account

- beam energy dependence upon
  - orbit length → "continuous" monitoring
  - position along the ring
- short luminosity lifetime (1-3 hours) calls for top-up injection → use of non-colliding bunches for polarization
  - non-colliding bunches may have a different energy

One more basic problem

- is it always $\nu_{spin} = a\gamma$?

The relationships $\nu_{spin} = a\gamma$ holds for a purely planar ring

- Effect of radial fields depends upon energy and unperturbed spin tune. For the toy ring, averaging over 10 seeds[a]

	ΔE (KeV)
45 GeV	6.3 $\pm$ 3.0
80 GeV	20.0 $\pm$ 9.4

- Effect of RF electric field (term $\vec{\beta} \times \vec{\mathcal{E}}_{RF}$ in BMT-equation)[b]

	ΔE (KeV)
45 GeV	$\alpha_{rms} \times$ 43
80 GeV	$\alpha_{rms} \times$ 76

$\alpha \equiv$ angle between orbit and electric field (mrad).

[a] Using formulas from R. Assmann thesis

[b] From Yu. I. Eidelman et al. formulas

The spin tune changes as computed by SITF (linear) for the actual cases presented here (with BPMs errors) give

	ΔE (KeV)	
	svd	+hb
45 GeV	36	52
80 GeV	162	135

The effect seems to be larger than expected; it should be better investigated!

Summary and outlook.

Studies for the 45 GeV and 80 GeV case have been presented.

- The large bending radius requires wigglers for reducing the polarization time at low energy keeping a high asymptotic polarization level in *absence* of errors.
- In presence of errors, in particular the vertical misalignment of quadrupoles, depolarizing resonances appear. Synchrotron side-bands become more dangerous with increasing energy spread. Their importance can be quantified only by non-linear calculations, like in SITROS.
- Maintaining acceptable level of polarization calls for well planned correction schemes, in particular at 80 GeV.
- With the proposed scheme it seems that maintaining polarization for energy calibration at 45 GeV is not a *mission impossible*, but space must be provided in the FODO cells for BPMs and correctors!

- At larger energy ϵ_x increases
  - larger effect of coupling and $\delta\hat{n}_0$
- At 80 GeV, $\delta\hat{n}_0$ due to the *same* misalignments increases and although the energy spread is the same as at 45 GeV with wigglers, the polarization is lower! The "LEP limit" shouldn't be applied to lower energies.
- The large bumps required for the correction cause a even larger vertical emittance increase.
  - A more efficient $\delta\hat{n}_0$ correction has been considered, likely there is still space for improvements.
  - The reach of beam-based alignments techniques should be investigated.
- Effect of solenoids ($\delta\hat{n}_0$ and coupling) must be compensated, better with anti-solenoids at proper locations. The planned solution is highly recommended!

End of the 3th Episode

Thanks!