Consideration for polarization in FCChh

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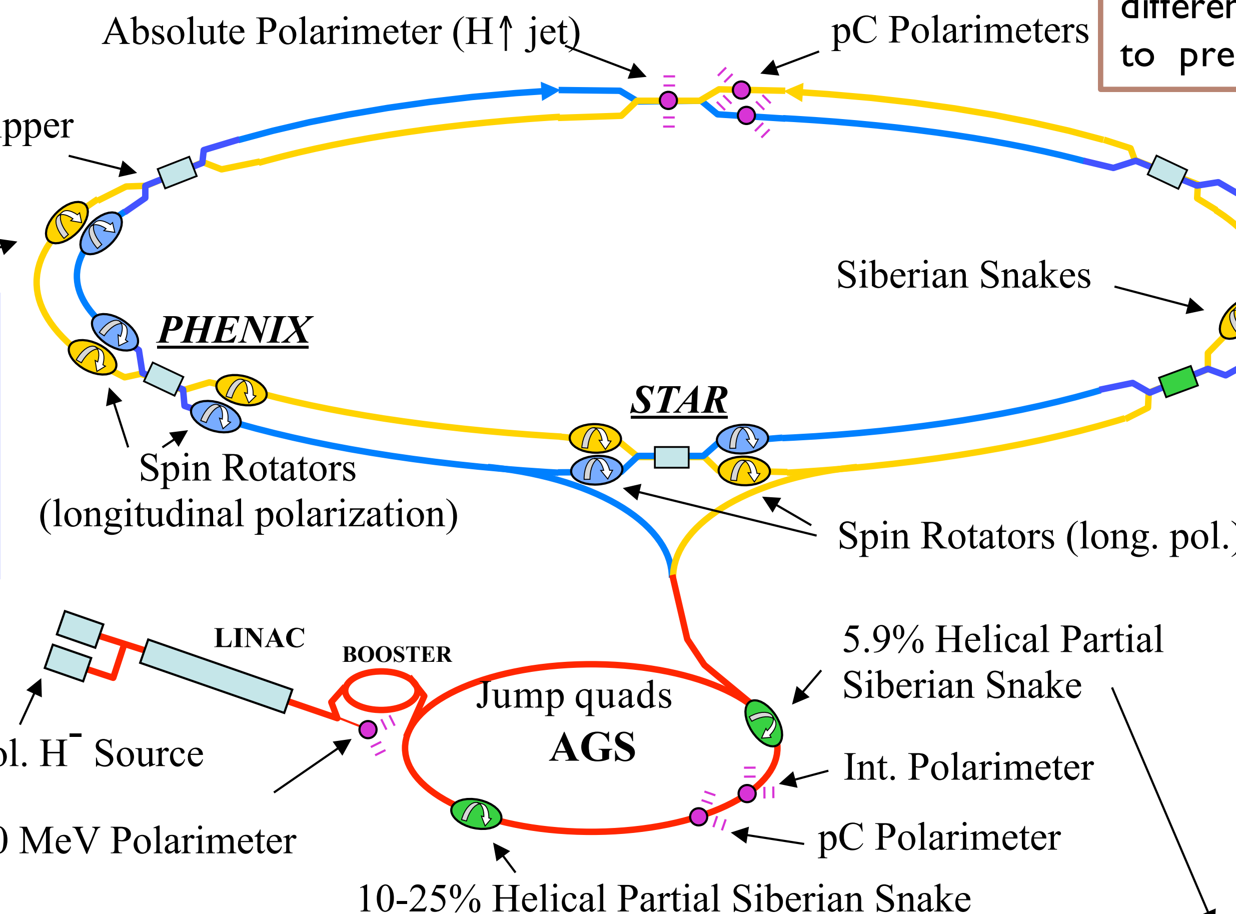
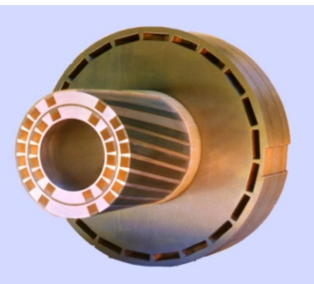
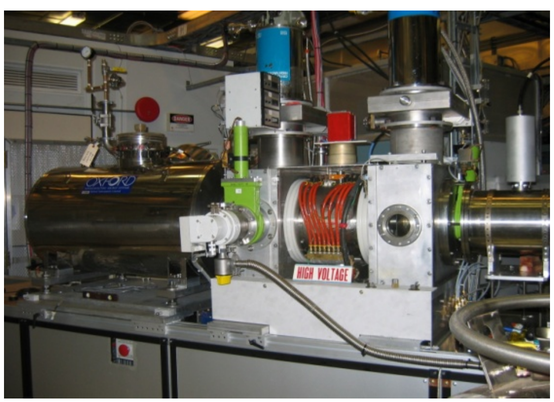
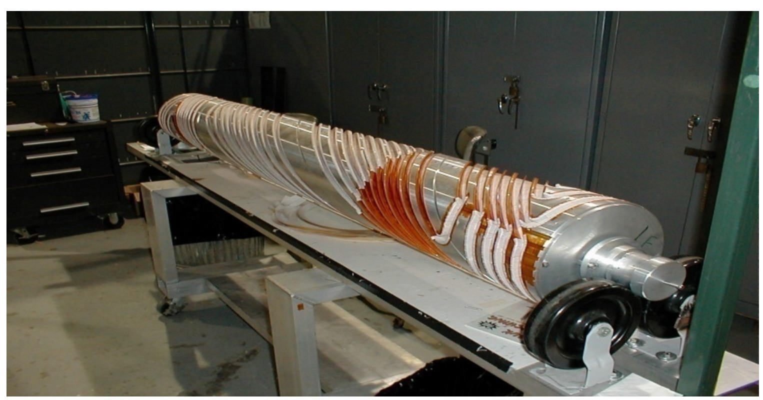
RHIC – First Polarized Hadron Collider

Diverse technologies are used on different stages of accelerator chain to preserve and measure polarization

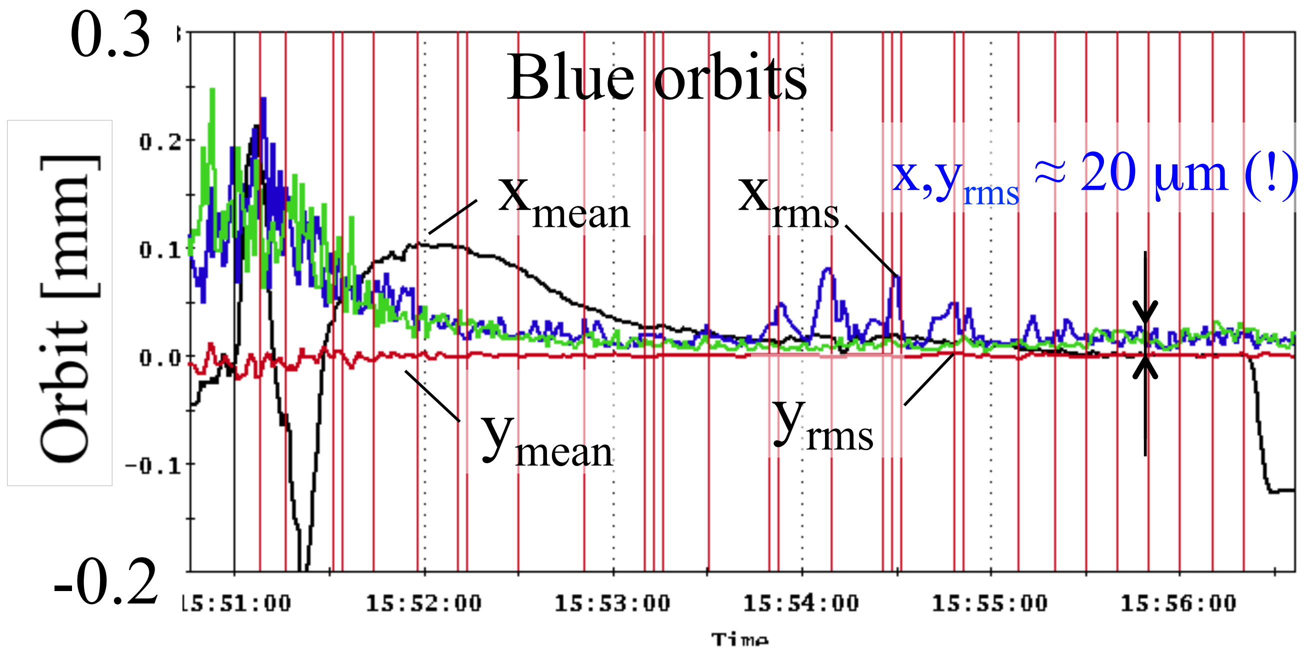


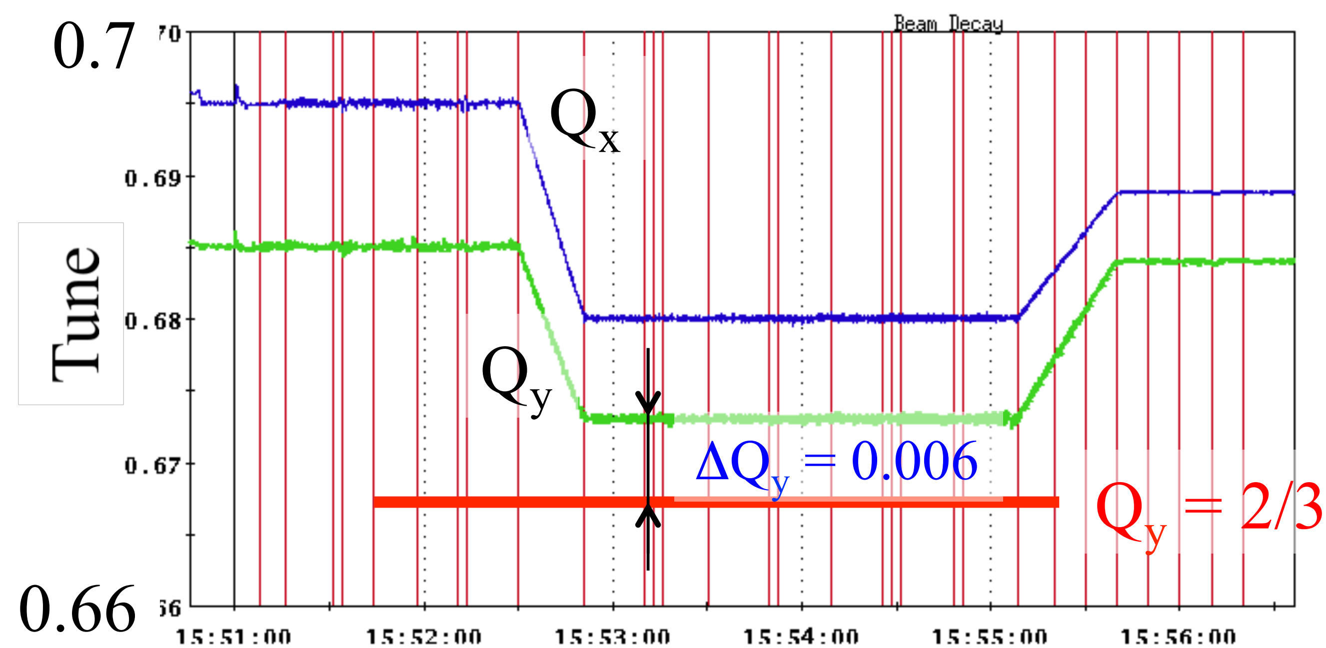






Beam control improvement – feedbacks on ramp

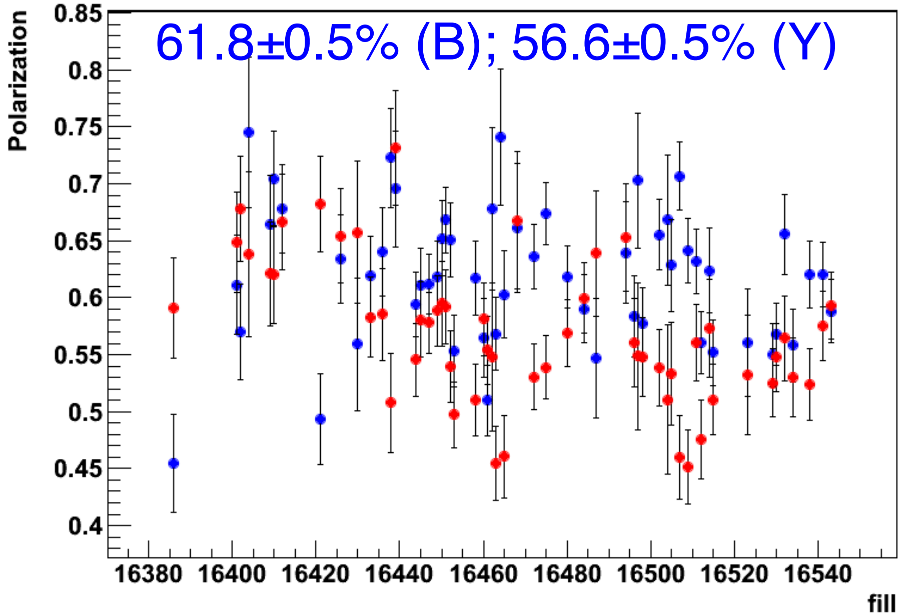




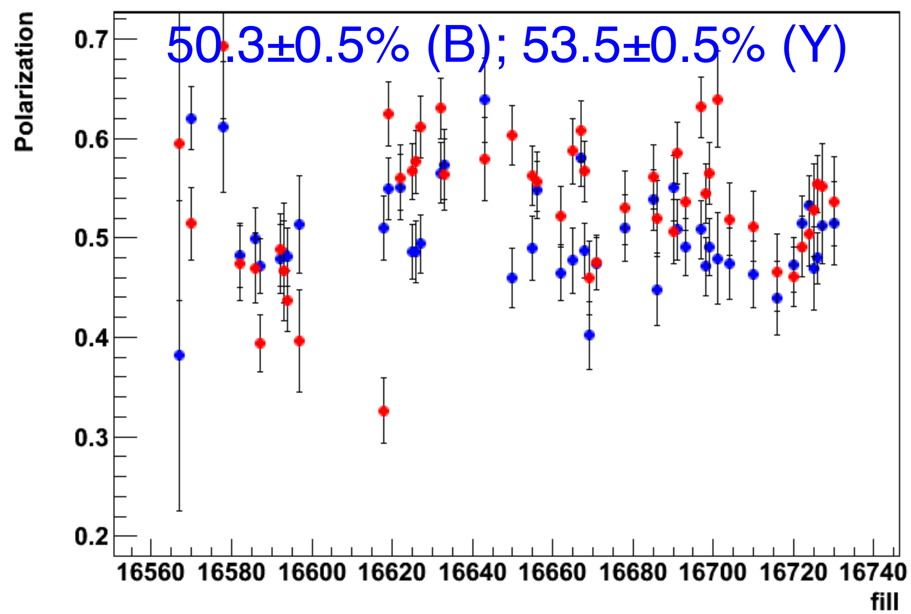
- Slow orbit feedback on every ramp allows for
  - Smaller y_{rms} (smaller imperfection resonance strength)
  - Ramp reproducibility (have 24 h orbit variation)
- Continues fast 10 Hz orbit feedback eliminates effect of vibrating triplets
- Tune/coupling feedback on every ramp allows for
  - Acceleration near $Q_y = 2/3$ with better polarization transmission

Polarization results (from Run12)

- 100 GeV
  - Average store pol. from Hjet

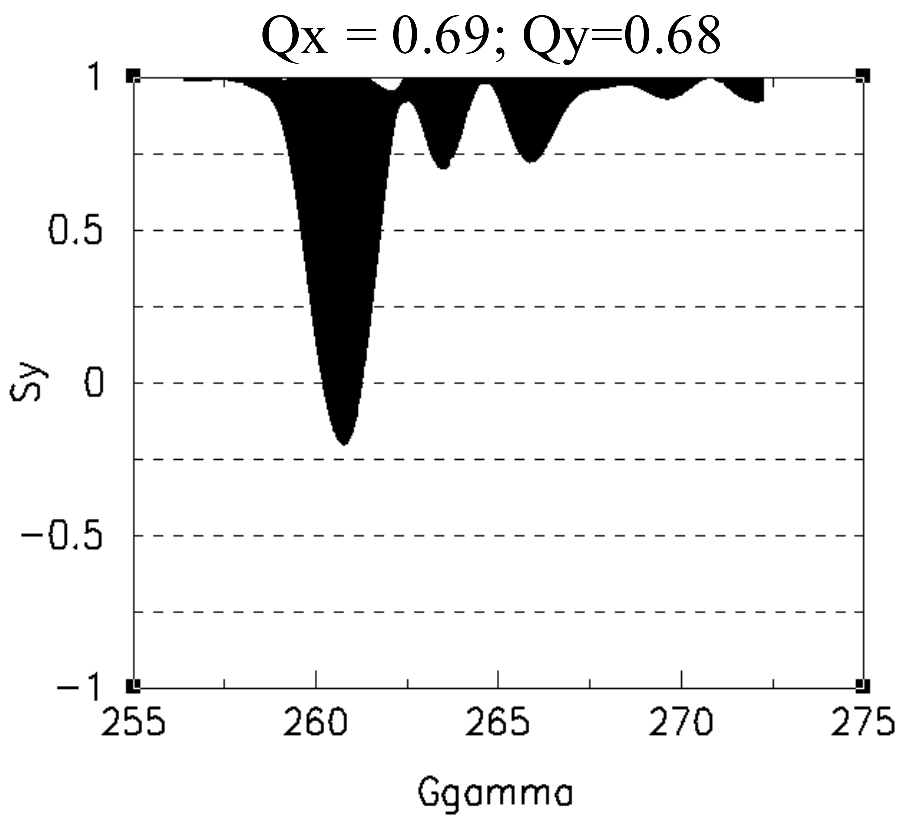


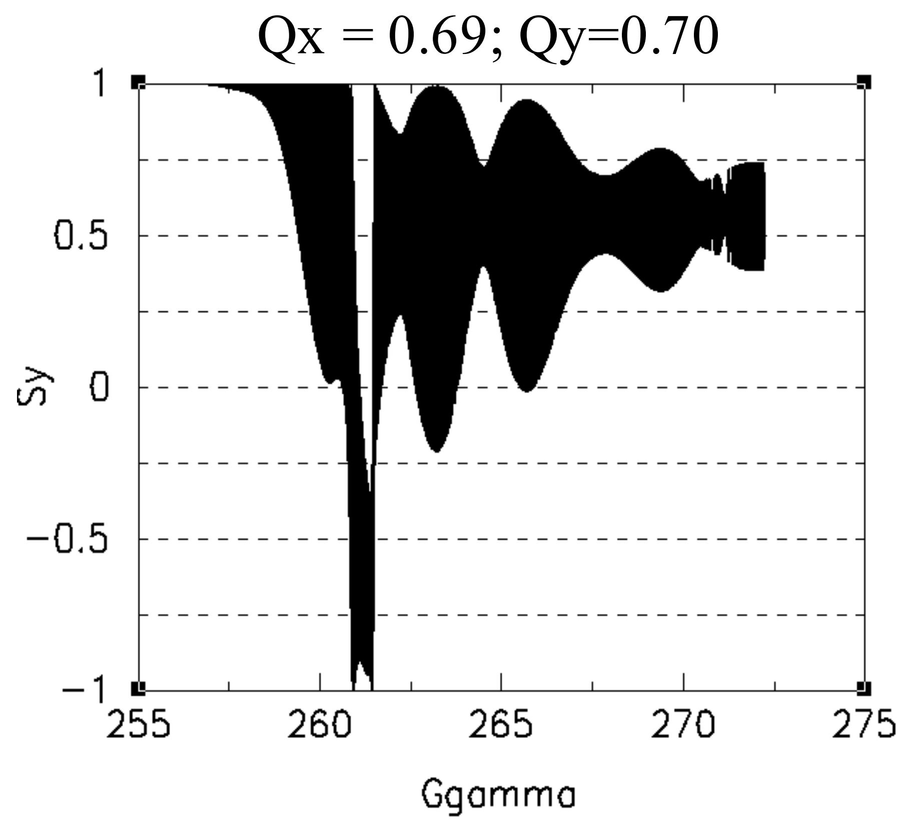
- 255 GeV beam
  - Average store pol. from HJet



Smaller polarization at 255 GeV is due to both polarization losses during acceleration ramp and slow polarization decay at the store.

Example of RHIC simulations of resonance crossing with Snakes





Crossing is adiabatical. In resonance zone spin may deviate significantly from vertical, but it comes back to vertical after passing the resonance.

Adiabaticity of resonance crossing can be broken if a Snake resonance happens. Then, the polarization loss occurs.

FCChh lattice for spin resonance calculation

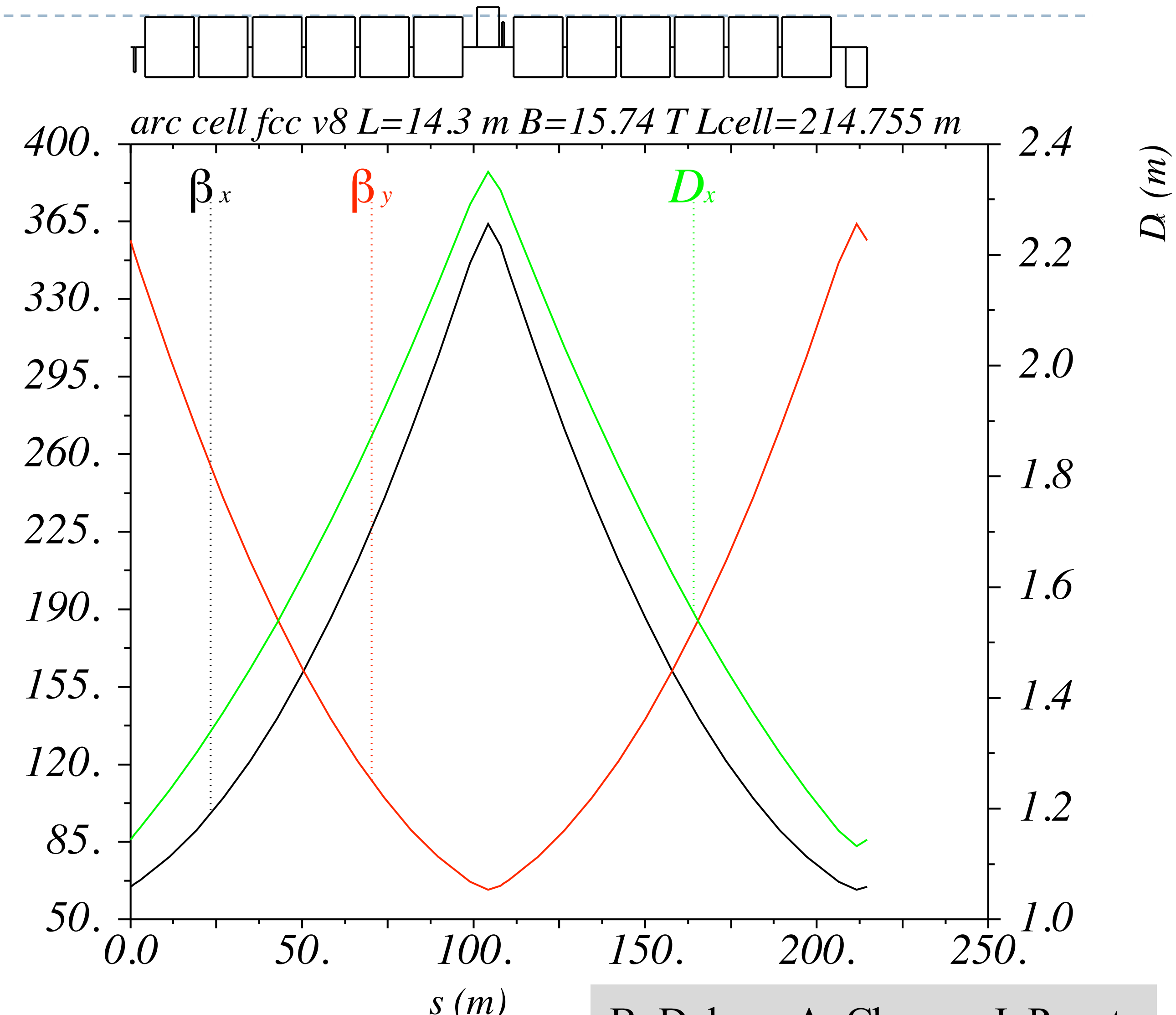
12 dipoles with L=14.3m

Dipole field = 16 T

L_{cell}=214.755m

Circumference ~ 100 km

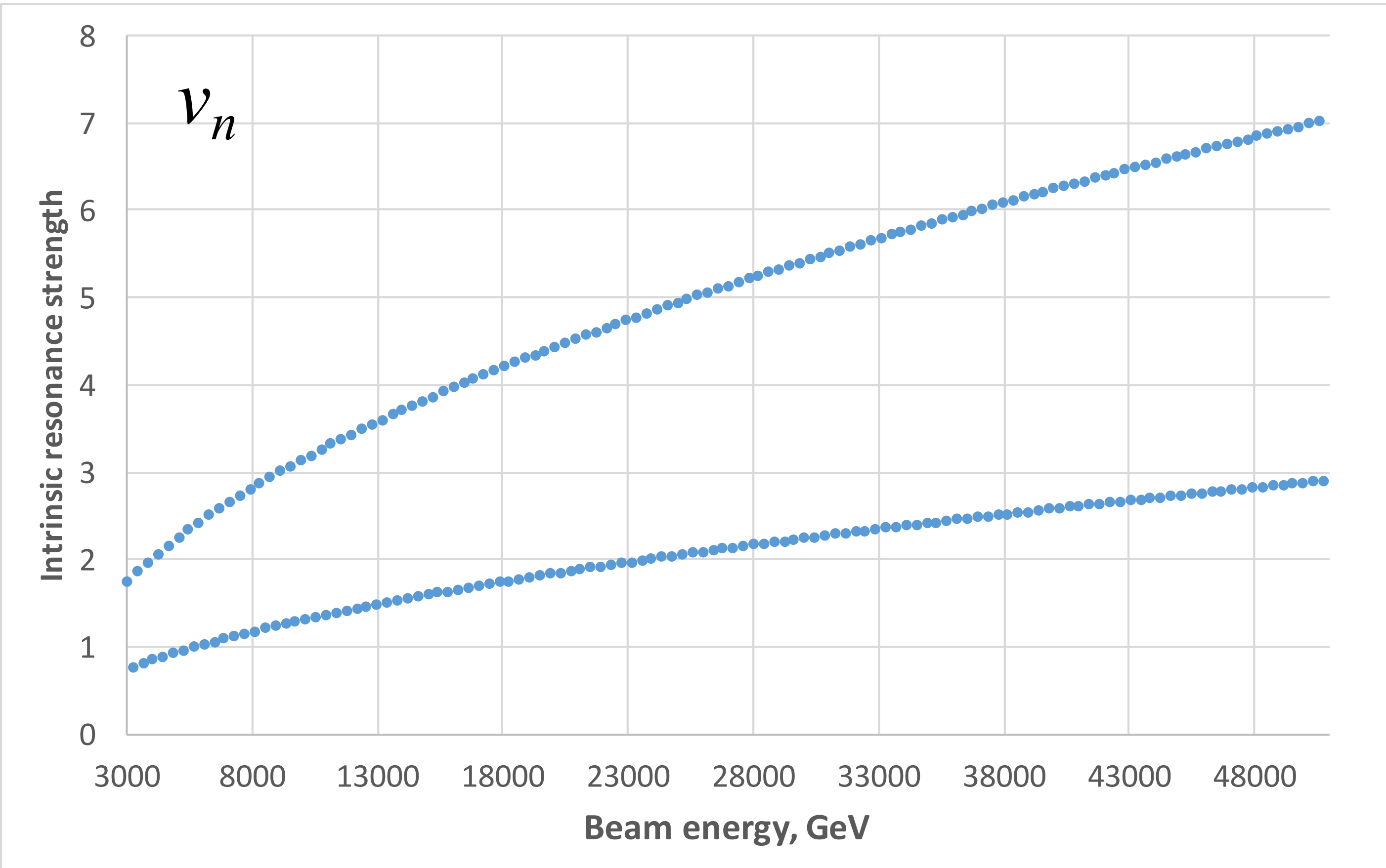
Normal. emittance =2.2 mm*mrad



B. Dalena, A. Chance, J. Payet

FCChh model resonance calculation

Intrinsic resonances calculated without Snakes



Uses 388 cells
to make full turn

Calculation done for
1σ vertical betatron
amplitude

SRM - single resonance model

The important case is the "single resonance model", or SRM, where single resonance (circular) harmonic υ_n, defined in the accelerator without the Snakes, is considered together with the Snake spin transformations.

Single resonance (circular) harmonic υ_n is transformed in the accelerator with the Snakes into the orthogonal series of linear harmonics (so, the "single" resonance is not really single in the proper frame) :

$$\upsilon_n e^{i\nu_n\theta} \rightarrow -e^{i\tilde{\alpha}} \sum_k i^k w_{nk} \sin(\delta_k \theta + \phi_n - \xi)$$

$$w_{nk} = |\upsilon_n| \frac{\sin(\chi_k \pi/2)}{\chi_k \pi/2}, \quad \delta_k = \nu_n - (k+1/2),$$

$$\chi_k = \nu_0 - (k+1/2), \quad \tilde{\alpha} = \xi_1 - \nu_0 \tfrac{\pi}{2} - \tfrac{\pi}{4}$$

•The analytical solution for the SRM with the Snakes was obtained by S. Mane.

Mane's solution for Snake resonances for SRM

S. Mane derived the analytical solutions for the SRM with the Snakes. For instance, for the case of two Snakes the solution for the periodical spin vector field n is:

$$n_z = \mathscr{A}_0(\eta, \pi\delta) + 2 \sum_{m=2,4,6,...} \mathscr{A}_m(\eta, \pi\delta)\cos(m(\phi - \xi))$$

$$n_x + in_y = -2ig \sum_{m=1,3,5,...} \mathscr{B}_m(\eta, \pi\delta)\sin(m(\phi - \xi))$$

betatron phase (ϕ)   Snake axis angle (ξ)

$$g_2 = \left[\cos\frac{\pi\Omega}{2} + \mathrm{i}\,\frac{\nu_0 - Q}{\Omega}\sin\frac{\pi\Omega}{2}\right] e^{\mathrm{i}\xi}\, e^{\mathrm{i}\pi\delta/2}.$$

$$\eta = \frac{|\nu_k|}{\Omega}\sin\left(\frac{\pi\Omega}{N_s}\right)$$

$$\Omega = \sqrt{(G\gamma - Q_k)^2 + |\nu_k|^2}$$

$$\delta = Q_k - \frac{1}{2}$$

Sine-Bessel functions:

$$\mathscr{A}_m(z,\alpha) := \cos\left(\frac{1}{2}m\alpha\right)\sum_{k=0}^{\infty}(-1)^k \frac{C^2_{m/2+k-1}(\alpha)}{S_k(\alpha)S_{k+m}(\alpha)} z^{m+2k}$$

$$\mathscr{B}_m(z,\alpha) := \sum_{k=0}^{\infty}(-1)^k \frac{C^2_{(m-1)/2+k}(\alpha)}{S_k(\alpha)S_{k+m}(\alpha)} (ze^{\mathrm{i}\alpha/2})^{m+2k}.$$

$$S_n(\alpha) := \sin\alpha\,\sin(2\alpha)\cdots\sin(n\alpha)$$

$$C_n(\alpha) := \cos\alpha\,\cos(2\alpha)\cdots\cos(n\alpha)$$

The functions $\mathscr{A}_m$ and $\mathscr{B}_m$ has the resonance behavior at the snake resonance conditions:

$$Q_z = \frac{2m+1}{2(2n+1)}$$

Limits of SRM solution

- The SRM solution does not provide answer for:
  - The resonances due to the interaction of imperfection and intrinsic resonance harmonics (that is integer and Q_z related harmonics)
  - The resonances due to the interaction of the horizontal and vertical resonance harmonics
  - Due to interference of several strong resonance harmonics.

The general Snake resonance condition is:

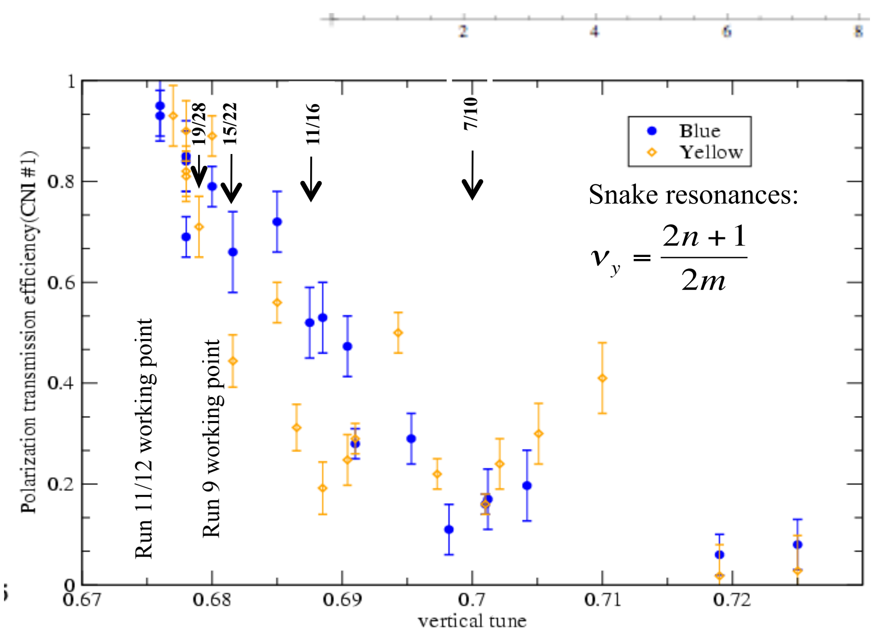
$$\nu_k \pm 2pQ_y = \frac{1-2p}{2}$$

$$\nu_k = \pm Q_y + k$$
$$Q_y = \pm \frac{1-2(p+k)}{2(2p\pm 1)}$$

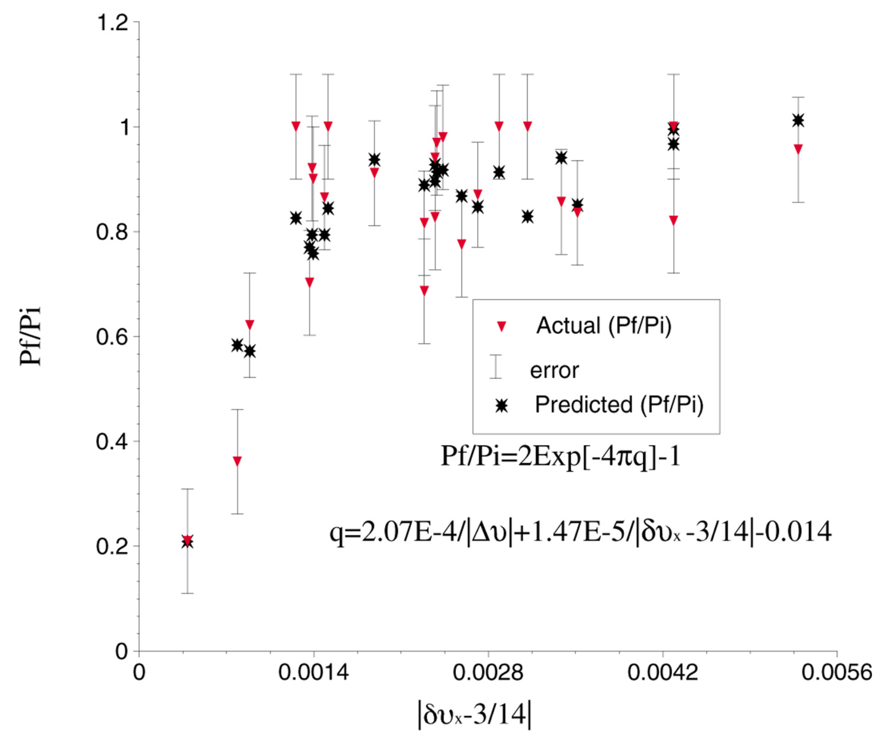
$$\nu_k = k$$
$$Q_y = \pm \frac{1-2(p+k)}{4p}$$

$$\nu_k = \pm Q_x + k$$
$$Q_x \pm 2pQ_y = \pm \frac{1-2(p+k)}{2}$$

Snake resonances observation in RHIC



Experimental data. (M. Bai)



Resonance caused by betatron coupling.
Operational data. (V. Ranjbar, V. Ptitsyn)

Best guess for number of Snakes in FCChh

$$\eta = \frac{|v_k|}{\Omega} \sin\left(\frac{\pi\Omega}{N_s}\right)$$

Effective spin resonance strength with Snakes.
It has maximum value exactly at the resonance ($G\gamma = Q_k$):

$$\Omega = \sqrt{(G\gamma - Q_k)^2 + |v_k|^2}$$

$$\eta_{max} = \sin\left(\frac{\pi|v_k|}{N_s}\right)$$

$$\delta = Q_k - \frac{1}{2}$$

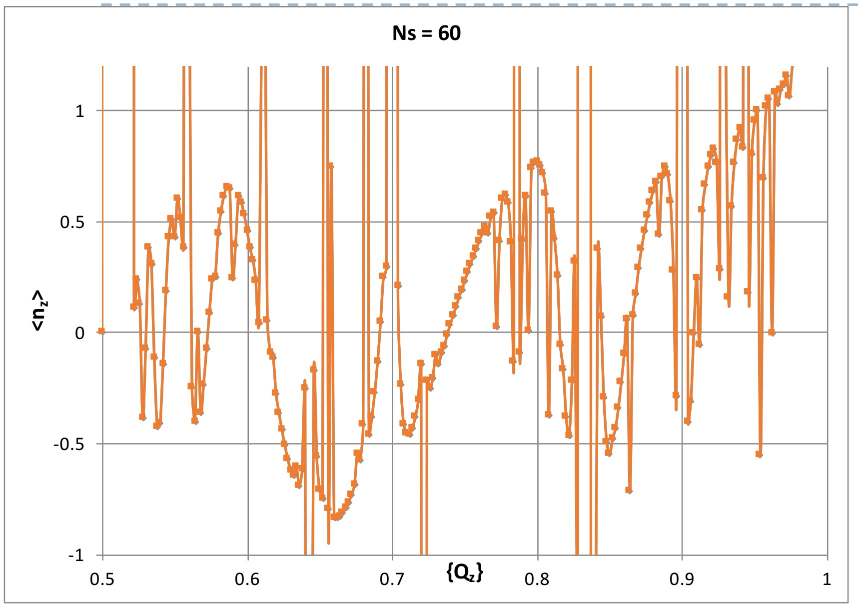
According to this approach the number of Snakes must grow proportionally to the resonance strength

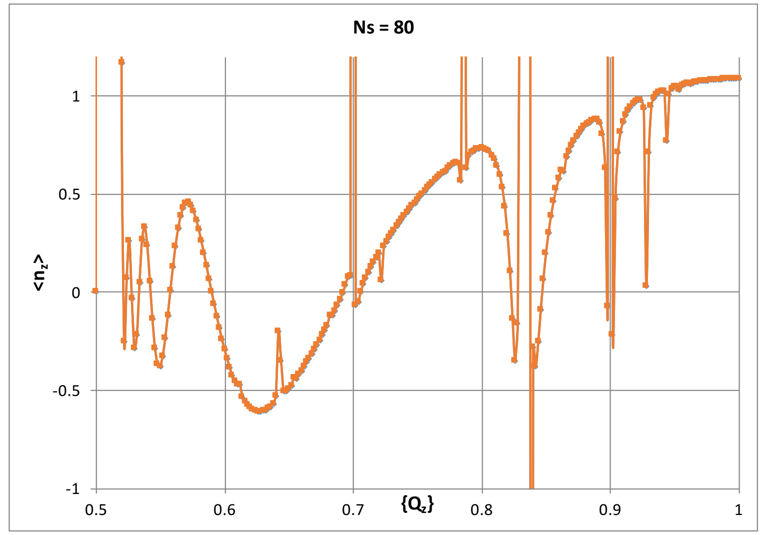
Max intrinsic resonance strength in RHIC ~ 0.25; 2 Snakes

Then, if max resonance strength ~7, one would need at least 60 Snakes.

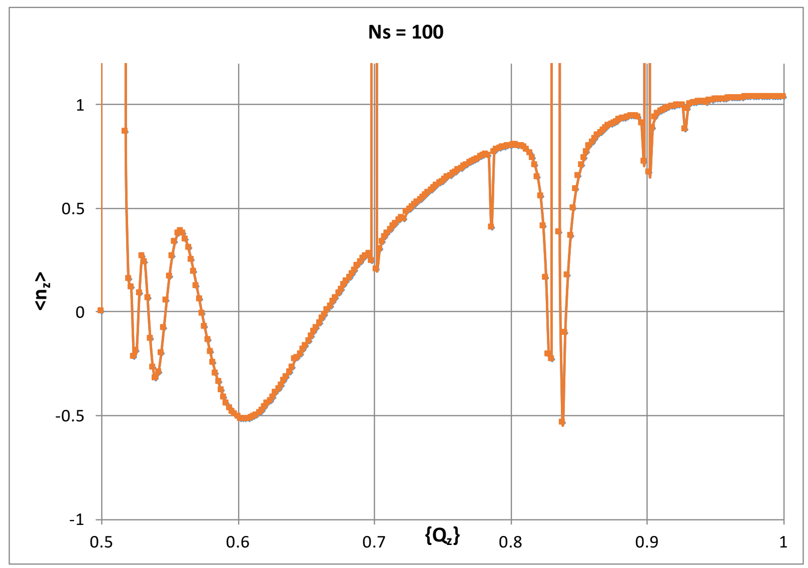
But, RHIC with 2 Snakes still suffer moderate polarization loss between 100 GeV and 250 GeV.

Snake resonances for FCChh for different number of Snakes





$$\langle n_z \rangle = \mathscr{A}_0(\eta, \pi\delta)$$



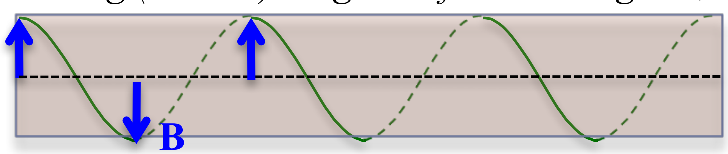
Plots show vertical polarization, averaged over betatron phase, for protons with 3σ vertical betatron amplitude versus fractional part of vertical betatron tune.

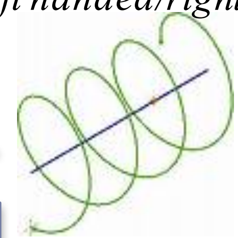
Irregular areas characterize the snake resonance width.

Helical Siberian Snakes

Longitudinal field magnets -> unacceptable large (and energy dependent) field
Common dipole field magnets -> large beam orbit excursion

Rotating (helical) magnetic field: strength B, helicity H (left handed/right handed)

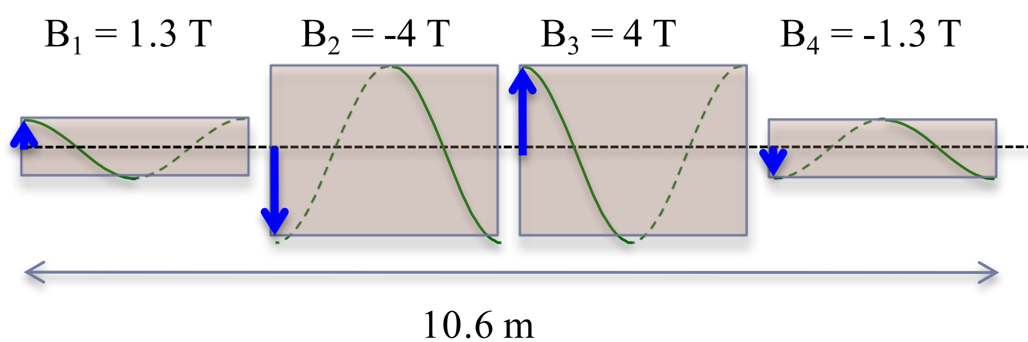




The solution: the design of "continuous" axis snake using four helical magnets (V. Ptitsyn, Yu.M. Shatunov, 1994)

Symmetry conditions:
$B_1 = -B_4$; $B_2 = -B_3$
$H_1 = H_4$; $H_2 = H_3$
$N_1 = N_4$; $N_2 = N_3$
(N-number of helical periods)

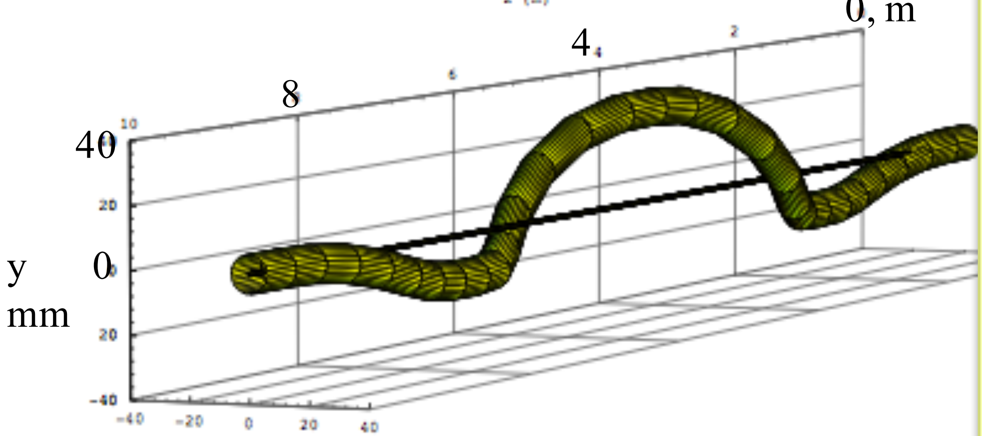
✓ The beam orbit restoration at the snake exit
✓ The rotation axis at the horizontal plane
✓ Orientation of the axis and the spin rotation angle are defined by B_1 and B_2

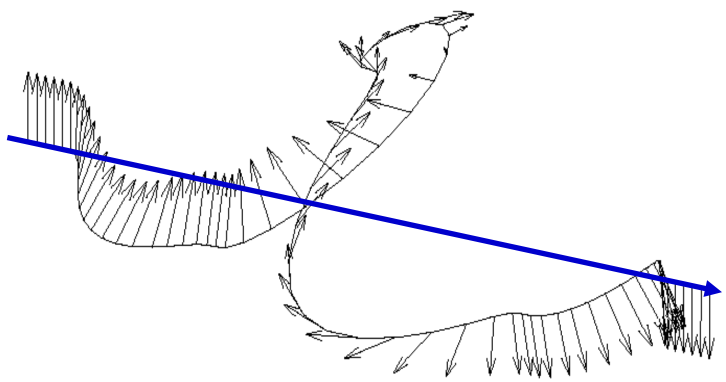


On similar principles the helical magnet design of spin rotators was also developed

Siberian Snakes in RHIC

Orbital excursion inside the Snake





Spin variation inside the Snake

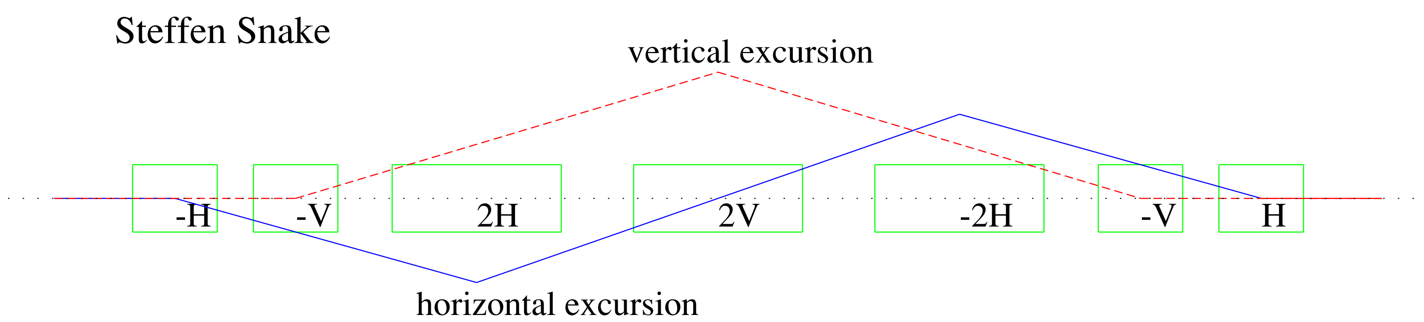
Helical magnets were designed and built at BNL with considerable support from RIKEN (Japan)

2002: Snakes were installed and commissioned
2003: The rotators were installed and commissioned

The Snakes in their cryostats

Dipole magnet Snake

- Snake with similar symmetry conditions can be constructed using common dipole magnets



Although the dipole magnet Snake produces orbit excursion large than helical one, for FCChh energies the excursion will be small (< 5mm)

Conclusion notes

- Scaling from RHIC polarization preservation experience and using S.Mane's model the number of Snakes for FCChh polarization preservation would be at least 60 (better 100). Choice of number of Snakes may depend on the choice of betatron tunes.

- Snake design: helical dipole (RHIC-like) or common dipole (Steffen snake) will work well.

- Difference from RHIC: fast ramping speed
  - Possible advantage: faster resonance crossing would reduce effect of Snake resonances
  - Possible disadvantage: the adiabaticity of resonance crossing has to be verified