From strong to weak coupling in holographic models of quark-gluon plasma

Andrei Starinets

Rudolf Peierls Centre for Theoretical Physics
Oxford University

Based on:
S.Grozdanov and A.Starinets:
Teor. Mat. Fiz. 182, no. 1, 76 (2014)
arXiv: 1412.5685 [hep-th]
arXiv: 1610.XXXX [hep-th]
S.Grozdanov, N.Kaplis and A.Starinets
arXiv: 1605.02173 [hep-th]

Relativistic hydrodynamics: theory and modern applications
MITP, Mainz, FRG
11 October 2016

Plan

Transport properties and analytic structure of correlation functions
in weakly interacting many-body quantum system (particles or quasiparticles)

Transport properties and analytic structure of correlation functions
in strongly interacting many-body quantum systems (via holography)

Real systems are at intermediate coupling (e.g. QGP)

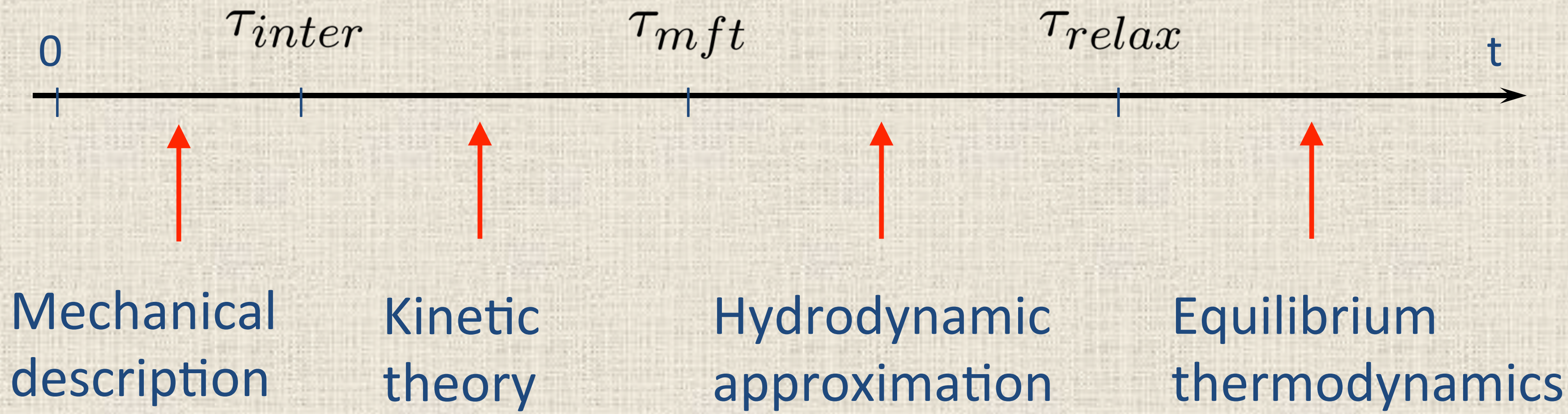
The problem of interpolation

Computing corrections to infinite coupling results from higher-derivative gravity

Hydrodynamic regime in kinetic theory

Hierarchy of times (e.g. in Bogolyubov's kinetic theory)

$$\tau_{mft} \ll t \ll T_{existence}$$



Hierarchy of scales

$$l_{mfp} \ll l \ll L$$

(L is a macroscopic size of a system)

The hydrodynamic regime (continued)

Degrees of freedom

0 — τ_{inter} — τ_{mft} — τ_{relax} — t

Mechanical description	Kinetic theory	Hydrodynamic approximation	Equilibrium thermodynamics
Coordinates, momenta of individual particles	Coordinate- and time-dependent distribution functions	Local densities of conserved charges $\epsilon(x,t)$	Globally conserved charges E

Hydro regime: $\boxed{\tau_{\text{micro}} \ll \tau \ll t_{\text{global}}}$ $\boxed{l_{\text{micro}} \ll l \ll L_{\text{global}}}$

Relaxation time in kinetic theory

Kinetic equation
$$\frac{\partial F}{\partial t} + \frac{p_i}{m}\frac{\partial F}{\partial r^i} - \frac{\partial U(r)}{\partial r^i}\frac{\partial F}{\partial p_i} = C[F]$$

Linearized by
$$F(t,\mathbf{r},\mathbf{p}) = F_0(\mathbf{r},\mathbf{p})\left[1+\varphi(t,\mathbf{r},\mathbf{p})\right]$$

Leads to
$$\frac{\partial \varphi}{\partial t} = -\frac{p_i}{m}\frac{\partial \varphi}{\partial r^i} + \frac{\partial U(r)}{\partial r^i}\frac{\partial \varphi}{\partial p_i} + L_0[\varphi]$$

For spatially homogeneous distributions:
$$\varphi(t,\mathbf{p}) = e^{-\nu t}h(\mathbf{p})$$

Eigenvalue problem:
$$-\nu h = L_0[h]$$

Solution:
$$\varphi(t,\mathbf{p}) = \sum_n C_n e^{-\nu_n t} h_n(\mathbf{p})$$

Relaxation time in kinetic theory

Kinetic equation
$$\frac{\partial F}{\partial t} + \frac{p_i}{m}\frac{\partial F}{\partial r^i} - \frac{\partial U(r)}{\partial r^i}\frac{\partial F}{\partial p_i} = C[F]$$

Linearized by
$$F(t,\mathbf{r},\mathbf{p}) = F_0(\mathbf{r},\mathbf{p})\left[1+\varphi(t,\mathbf{r},\mathbf{p})\right]$$

Leads to
$$\frac{\partial \varphi}{\partial t} = -\frac{p_i}{m}\frac{\partial \varphi}{\partial r^i} + \frac{\partial U(r)}{\partial r^i}\frac{\partial \varphi}{\partial p_i} + L_0[\varphi]$$

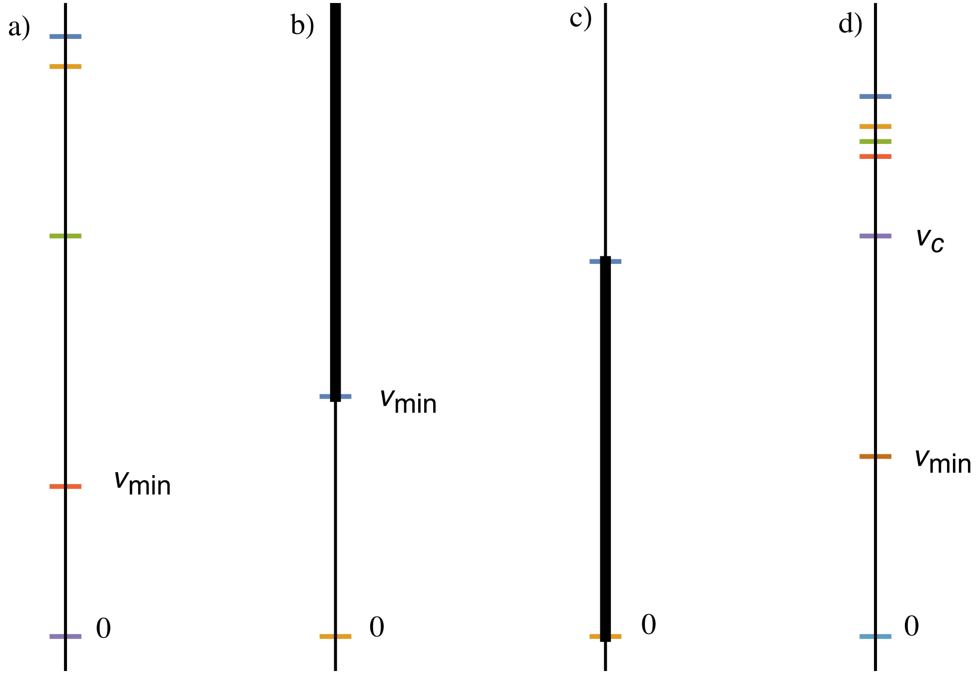
For spatially homogeneous distributions:
$$\varphi(t,\mathbf{p}) = e^{-\nu t}h(\mathbf{p})$$

Eigenvalue problem:
$$-\nu h = L_0[h]$$

Solution:
$$\varphi(t,\mathbf{p}) = \sum_n C_n e^{-\nu_n t}h_n(\mathbf{p})$$

Spectrum of linearized kinetic operator

Wang Chang & Uhlenbeck (1952), Grad (1963)



a) Discrete spectrum, $U = \alpha/r^4$

b) Continuous spectrum with a gap, $U = \alpha/r^n$, $n > 4$

c) Continuous gapless spectrum, $U = \alpha/r^n$, $n < 4$

d) Hod spectrum

Relaxation time in kinetic theory (continued)

$$\varphi(t,\mathbf{p}) = \sum_n C_n e^{-\nu_n t} h_n(\mathbf{p})$$

$$-\nu h = L_0[h]$$

The hierarchy of relaxation times is determined by the spectrum of linearized kinetic operator

$$\tau_R = 1/\nu_{min}$$

For weakly inhomogeneous systems:

$$\frac{\partial F}{\partial t} + \frac{p_i}{m}\frac{\partial F}{\partial r^i} - \frac{\partial U(r)}{\partial r^i}\frac{\partial F}{\partial p_i} = -\frac{F - F_0}{\tau_R}$$

Krook-Gross-Bhatnagar (KGB) equation (1959)

Transport is then essentially determined by the relaxation time, e.g. shear viscosity is

$$\eta = \tau_R\, s\, T$$

How to compute shear viscosity in strongly interacting systems?

D.T.Son next to Maxwell's viscometer at Cavendish Laboratory, Cambridge

Fluid dynamics is an effective theory valid in the long-wavelength, long-time limit

Fundamental degrees of freedom = densities of conserved charges

Equations of motion = conservation laws + constitutive relations^*

Example I

$$\left.\begin{aligned} &\partial_a J^a = 0 \\ &J^i = -D\,\nabla^i\, J^0 + \cdots \end{aligned}\right\} \quad \partial_t J^0 = D\,\nabla^2 J^0 + \cdots$$

Example II

$$\left.\begin{aligned} &\partial_a T^{ab} = 0 \\ &T^{ab} = \varepsilon u^a u^b + P(\varepsilon)\left(g^{ab} + u^a u^b\right) + \Pi^{ab} + \cdots \end{aligned}\right\}$$

Navier-Stokes eqs
Burnett eqs
.....

* Modulo assumptions e.g. analyticity

** E.o.m. universal, transport coefficients depend on underlying microscopic theory

Consider relativistic neutral conformal fluid in a d-dimensional (curved) space-time

$$T^{ab} = \varepsilon u^a u^b + P(\varepsilon)\left(g^{ab} + u^a u^b\right) + \Pi^{ab} + \cdots$$

Including only terms with first and second derivatives of fluid velocity:

$$\begin{aligned}\Pi^{ab} = &-\eta\sigma^{ab} \\ &+ \eta\tau_\Pi\left[{}^{\langle}D\sigma^{ab\rangle} + \frac{1}{d-1}\sigma^{ab}\left(\nabla\cdot u\right)\right] \\ &+ \kappa\left[R^{\langle ab\rangle} - (d-2)u_c R^{c\langle ab\rangle d}u_d\right] \\ &+ \lambda_1 \sigma^{\langle a}{}_c\sigma^{b\rangle c} + \lambda_2 \sigma^{\langle a}{}_c\Omega^{b\rangle c} + \lambda_3 \Omega^{\langle a}{}_c\Omega^{b\rangle c}\end{aligned}$$

Transport coefficients (in conformal case): $\eta\,,\tau_\Pi\,,\kappa\,,\lambda_1\,,\lambda_2\,,\lambda_3$

Non-conformal case: 2 first order coefficients, 15 (10) second order coefficients
(see S.Bhattacharyya, 1201.4654 [hep-th])

Beyond second order hydrodynamics

Tensors structures appearing in the derivative expansion have been analyzed using computer algebra in 1507.02461 [hep-th] by Grozdanov & Kaplis.

At third order, there are 20 relevant structures in the conformal case and 68 in the non-conformal one.

This still needs an entropy current analysis similar to the one in S.Bhattacharyya, 1201.4654 [hep-th]

Example: dispersion relations in conformal case

$$\omega = -i\frac{\eta}{\varepsilon+P}k^2 - i\left[\frac{\eta^2\tau_\Pi}{(\varepsilon+P)^2} - \frac{\theta_1}{2(\varepsilon+P)}\right]k^4 + \cdots$$

$$\omega = \pm c_s k - i\Gamma k^2 \mp \frac{\Gamma}{2c_s}\left(\Gamma - 2c_s^2\tau_\Pi\right)k^3 - i\left[\frac{8\eta^2\tau_\Pi}{9(\varepsilon+P)^2} - \frac{\theta_1+\theta_2}{3(\varepsilon+P)}\right]k^4 + \cdots$$

Here $c_s = 1/\sqrt{3}$ $\qquad \Gamma = \eta/(\varepsilon + P)$

Notations used in the derivative expansion

$$D \equiv u^a \nabla_a$$

$$\Delta^{ab} \equiv g^{ab} + u^a u^b$$

$$A^{\langle ab \rangle} \equiv \frac{1}{2}\Delta^{ac}\Delta^{bd}\left(A_{cd} + A_{dc}\right) - \frac{1}{d-1}\Delta^{ab}\Delta^{cd}A_{cd} \equiv \langle A^{ab} \rangle$$

$$\sigma^{ab} = 2^{\langle}\nabla^a u^{b\rangle}$$

$$\Omega^{ab} = \frac{1}{2}\Delta^{ac}\Delta^{bd}\left(\nabla_c u_d - \nabla_d u_c\right)$$

*Hydro definitions differ in the literature – see footnote 91 on page 128 of M.Haehl, R.Loganayagam, M.Rangamani, 1502.00636 [hep-th]

Computing transport coefficients from "first principles"

Fluctuation-dissipation theory
(Callen, Welton, Green, Kubo)

Kubo formulae allows one to calculate transport coefficients from microscopic models

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d^3x e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

In the regime described by a gravity dual the correlator can be computed using gauge theory/gravity duality

Kubo formulas for second order transport coefficients

First order transport coefficients can be computed from two-point functions of the corresponding operators using Kubo formulas

$$\eta = \lim_{\omega\to 0}\frac{1}{2\omega}\int dt\, d^3x e^{i\omega t}\langle\left[T_{xy}(t,x), T_{xy}(0,0)\right]\rangle$$

Similarly, second transport coefficients can be computed from three-point functions

$$\lambda_2 = 2\eta\tau_\Pi - 4\lim_{p,q\to 0}\frac{\partial^2}{\partial p^0\partial q^z}G^{xy,ty,xz}_{RAA}(p,q)$$

Moore, Sohrabi, Saremi, 2010, 2011; Arnold, Vaman, Wu, Xiao, 2011

Schwinger-Keldysh generating functional

$$G^{ab,cd,\dots}_{RA\dots}(0,x,\dots) = \left.\frac{(-i)^{n-1}(-2i)^n\delta^n W}{\delta h_{A\;ab}(0)\delta h_{R\;cd}(x)\dots}\right|_{h=0} = (-i)^{n-1}\left\langle T^{ab}_R(0)T^{cd}_A(x)\dots\right\rangle$$

$$W\left[h^+,h^-\right] = \ln\int\mathcal{D}\phi^+\mathcal{D}\phi^-\mathcal{D}\varphi\exp\left\{i\int d^4x^+\sqrt{-g^+}\mathcal{L}\left[\phi^+(x^+),h^+\right] - \int_0^\beta d^4y\mathcal{L}_E\left[\varphi(y)\right] - i\int d^4x^-\sqrt{-g^-}\mathcal{L}\right.$$

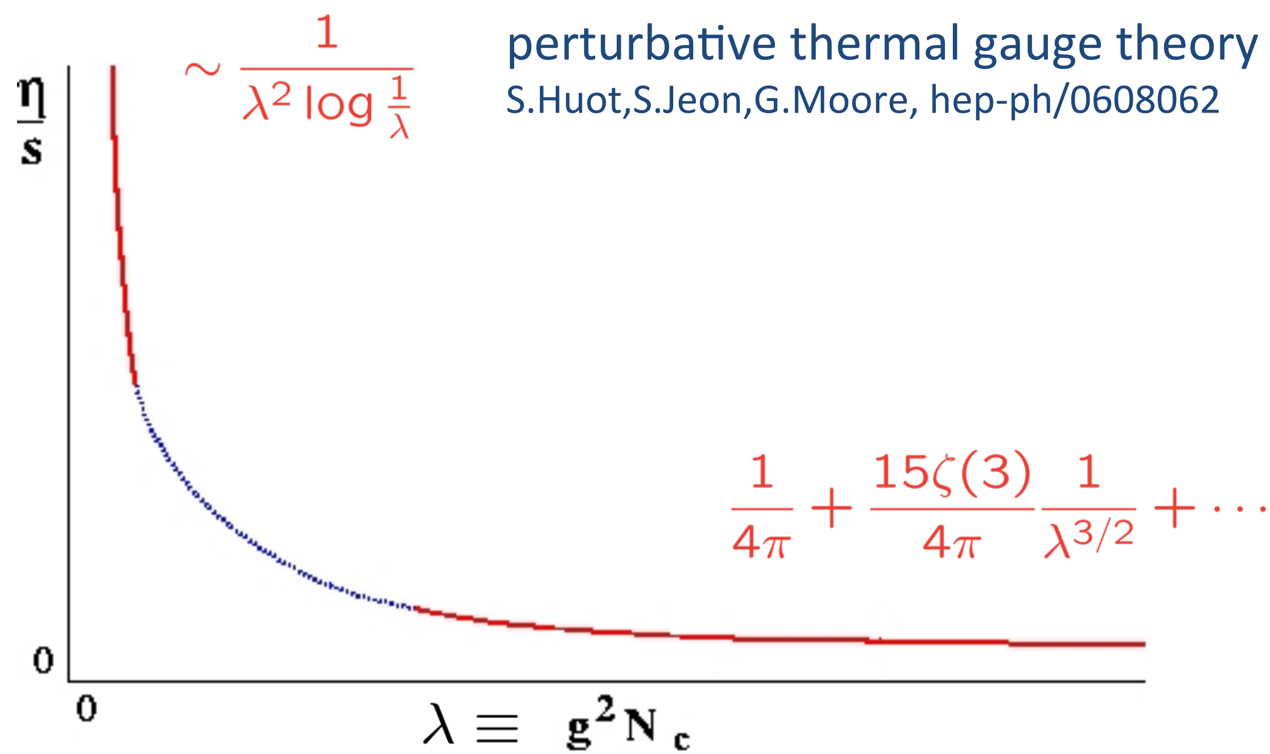
How to compute second order transport coefficients?

Fluid-gravity correspondence [Bhattacharyya et al, 2007]

Quasinormal spectrum [Baier et al, 2007]

Kubo formulas & three-point functions
[Moore, Sohrabi, Saremi, 2010, 2011; Arnold, Vaman, Wu, Xiao, 2011]

Shear viscosity in $\mathcal{N}=4$ SYM



Correction to $1/4\pi$: Buchel, Liu, A.S., hep-th/0406264

Buchel, 0805.2683 [hep-th]; Myers, Paulos, Sinha, 0806.2156 [hep-th]

First and second order transport coefficients of *conformal* holographic fluids *to leading order* in supergravity approximation

$$\eta = s/4\pi\,,$$
$$\tau_\Pi = \frac{d}{4\pi T}\left(1 + \frac{1}{d}\left[\gamma_E + \psi\left(\frac{2}{d}\right)\right]\right)\,,$$
$$\kappa = \frac{d}{d-2}\,\frac{\eta}{2\pi T}\,,$$
$$\lambda_1 = \frac{d\eta}{8\pi T}\,,$$
$$\lambda_2 = \left[\gamma_E + \psi\left(\frac{2}{d}\right)\right]\frac{\eta}{2\pi T}\,,$$
$$\lambda_3 = 0$$

Bhattacharyya et al, 2008

Note: $2\eta\tau_\Pi - 4\lambda_1 - \lambda_2 = 0$

Transport coefficients $\eta\,, \tau_{\Pi}\,, \kappa\,, \lambda_1\,, \lambda_2\,, \lambda_3$ are non-trivial functions of the parameters of the underlying microscopic theory, even in the simplest case of conformal liquids

$$\frac{\eta}{s} = F\left(N_c, N_f, \frac{m}{T}, \frac{\Lambda}{T}, \ldots\right)$$

Some information can be obtained from kinetic theory at weak coupling and from gauge-string duality at strong coupling (for theories with string or gravity duals). In the latter case, it is natural to look for "universal" (independent of the specific string construction) results.

In the limit of infinite N (gauge group rank) and infinite coupling (i.e. in the supergravity approximation from dual string theory point of view)

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Policastro, Kovtun, Son, AOS, 2001-2008
Buchel, J.Liu, 2003; Buchel, 2004
Iqbal, H.Liu, 2008

$$\frac{\sigma}{\chi} = \frac{1}{4\pi T}\frac{d}{d-2}$$

[for conformal relativistic fluids]: Kovtun, Ritz, 2008

$$2\eta\tau_{\Pi} - 4\lambda_1 - \lambda_2 = 0$$

[for conformal relativistic fluids]: Haack, Yarom, 2008

More speculative statements inspired by holography

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$$

Kovtun, Son, AOS, 2004 (violated by some models with Higher derivative gravity, Kats and Petrov, 2007; Brigante, Myers, H.Liu, Myers, Shenker, Yaida, 2008)

$$\frac{\zeta}{\eta} \geq 2\left(\frac{1}{d-1} - c_s^2\right)$$

Buchel, 2008 (counterexample: Buchel, 2012)

See also Kanitscheider and Skenderis, 2009

$$c_s^2 \leq c_{s,conf}^2 = \frac{1}{d-1}$$

Hohler and Stephanov, 2009;
Cherman, Cohen, Nellore, 2009

$$\frac{\sigma}{\chi} \geq \frac{\hbar v^2}{4\pi T}\frac{d}{d-2}$$

Kovtun and Ritz, 2008

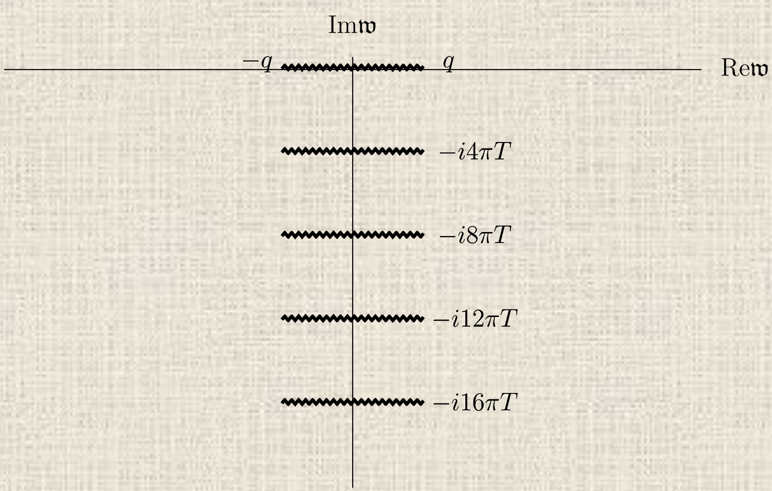
$$D \gtrsim \frac{\hbar v_F}{k_B T}$$

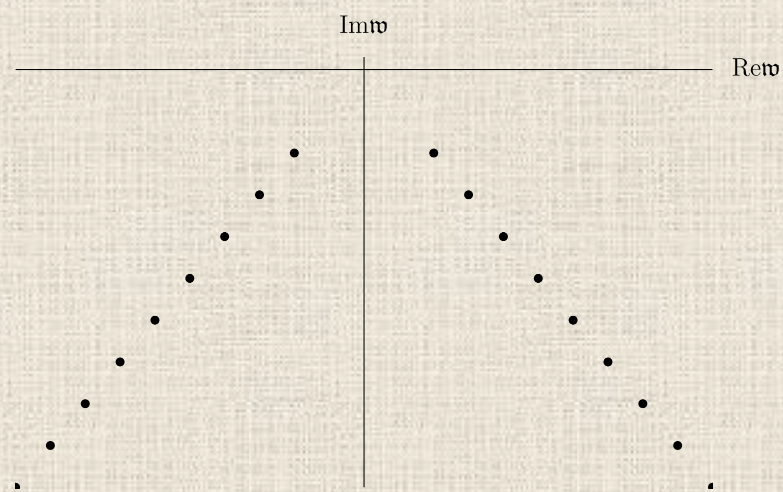
Hartnoll, 2014

See also "Fluctuation bounds…" by Kovtun 1407.0690 [hep-th]

Cuts versus poles: a mystery

Singularities of a Green's function in the complex frequency plane





Weak (vanishing) coupling

Hartnoll, Kumar (2005)

Strong (infinite) coupling

Starinets (2002)

We should be able to interpolate between the two limits…

Coupling constant corrections to N=4 SYM transport coefficients

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left(R + \frac{12}{L^2} + \gamma\mathcal{W}\right) \qquad \gamma = \lambda^{-3/2}\zeta(3)/8$$

$$\eta = \frac{\pi}{8}N_c^2T^3\left(1 + 135\gamma + \dots\right)$$

$$\tau_\Pi = \frac{(2-\ln 2)}{2\pi T} + \frac{375\gamma}{4\pi T} + \dots$$

$$\kappa = \frac{N_c^2T^2}{8}\left(1 - 10\gamma + \dots\right)$$

$$\lambda_1 = \frac{N_c^2T^2}{16}\left(1 + 350\gamma + \dots\right)$$

$$\lambda_2 = -\frac{N_c^2T^2}{16}\left(2\ln 2 + 5\left(97 + 54\ln 2\right)\gamma + \dots\right)$$

NEW!

$$\lambda_3 = \frac{25N_c^2T^2}{2}\gamma + \dots$$

Note: $2\eta\tau_\Pi - 4\lambda_1 - \lambda_2 = 0$

Curvature squared corrections to transport coefficients of a (hypothetical) strongly coupled liquid

$$S_{R^2} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left[R - 2\Lambda + L^2\left(\alpha_1 R^2 + \alpha_2 R_{\mu\nu}R^{\mu\nu} + \alpha_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\right]$$

$$\eta = \frac{r_+^3}{2\kappa_5^2}\left(1 - 8\left(5\alpha_1 + \alpha_2\right)\right) + \ldots$$

$$\eta\tau_\Pi = \frac{r_+^2\left(2 - \ln 2\right)}{4\kappa_5^2}\left(1 - \frac{26}{3}\left(5\alpha_1 + \alpha_2\right)\right) - \frac{r_+^2\left(23 + 5\ln 2\right)}{12\kappa_5^2}\alpha_3 + \ldots$$

$$\kappa = \frac{r_+^2}{2\kappa_5^2}\left(1 - \frac{26}{3}\left(5\alpha_1 + \alpha_2\right)\right) - \frac{25 r_+^2}{6\kappa_5^2}\alpha_3 + \ldots$$

$$\lambda_1 = \frac{r_+^2}{4\kappa_5^2}\left(1 - \frac{26}{3}\left(5\alpha_1 + \alpha_2\right)\right) - \frac{r_+^2}{12\kappa_5^2}\alpha_3 + \ldots$$

$$\lambda_2 = -\frac{r_+^2\ln 2}{2\kappa_5^2}\left(1 - \frac{26}{3}\left(5\alpha_1 + \alpha_2\right)\right) - \frac{r_+^2\left(21 + 5\ln 2\right)}{6\kappa_5^2}\alpha_3 + \ldots$$

$$\lambda_3 = -\frac{28 r_+^2}{\kappa_5^2}\alpha_3 + \ldots$$

Note: $2\eta\tau_\Pi - 4\lambda_1 - \lambda_2 = 0$

Gauss-Bonnet corrections to transport coefficients of a (hypothetical) strongly coupled liquid

$$S_{GB} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left[R + \frac{12}{L^2} + \frac{\lambda_{GB}}{2}L^2\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\right]$$

$$\eta = \frac{r_+^2}{2\kappa_5^2}\left(1 - 4\lambda_{GB}\right)$$

$$\eta\tau_\Pi = \frac{r_+^2\left(2 - \ln 2\right)}{4\kappa_5^2} - \frac{r_+^2\left(25 - 7\ln 2\right)}{8\kappa_5^2}\lambda_{GB} + \ldots$$

$$\kappa = \frac{r_+^2}{2\kappa_5^2} - \frac{17r_+^2}{4\kappa_5^2}\lambda_{GB} + \ldots$$

$$\lambda_1 = \frac{r_+^2}{4\kappa_5^2} - \frac{9r_+^2}{8\kappa_5^2}\lambda_{GB} + \ldots$$

$$\lambda_2 = -\frac{r_+^2\ln 2}{2\kappa_5^2} - \frac{7r_+^2\left(1 - \ln 2\right)}{4\kappa_5^2}\lambda_{GB} + \ldots$$

$$\lambda_3 = -\frac{14r_+^2}{\kappa_5^2}\lambda_{GB} + \ldots$$

Brigante, Myers, H.Liu, Myers, Shenker, Yaida, 2008 Shaverin, Yarom, 2012

Note: $2\eta\tau_\Pi - 4\lambda_1 - \lambda_2 = 0$

Non-perturbative Gauss-Bonnet corrections to transport coefficients of a (hypothetical) strongly coupled liquid

$$S_{GB} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left[R + \frac{12}{L^2} + \frac{\lambda_{GB}}{2}L^2\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\right]$$

$$\eta = \frac{s}{4\pi}\gamma^2 = \frac{s}{4\pi}(1-4\lambda_{GB})$$

Brigante et al, 2008

$$\tau_\Pi = \frac{1}{2\pi T}\left(\frac{1}{4}(1+\gamma)\left(5+\gamma-\frac{2}{\gamma}\right) - \frac{1}{2}\log\left[\frac{2(1+\gamma)}{\gamma}\right]\right)$$

Banerjee and Dutta, 2011

$$\lambda_1 = \frac{\eta}{2\pi T}\left(\frac{(1+\gamma)(3-4\gamma+2\gamma^3)}{2\gamma^2}\right)$$

Grozdanov and AOS, 2014

$$\lambda_2 = -\frac{\eta}{\pi T}\left(-\frac{1}{4}(1+\gamma)\left(1+\gamma-\frac{2}{\gamma}\right) + \frac{1}{2}\log\left[\frac{2(1+\gamma)}{\gamma}\right]\right)$$

Grozdanov and AOS, 2014

$$\lambda_3 = -\frac{\eta}{\pi T}\left(\frac{(1+\gamma)(3+\gamma-4\gamma^2)}{\gamma^2}\right)$$

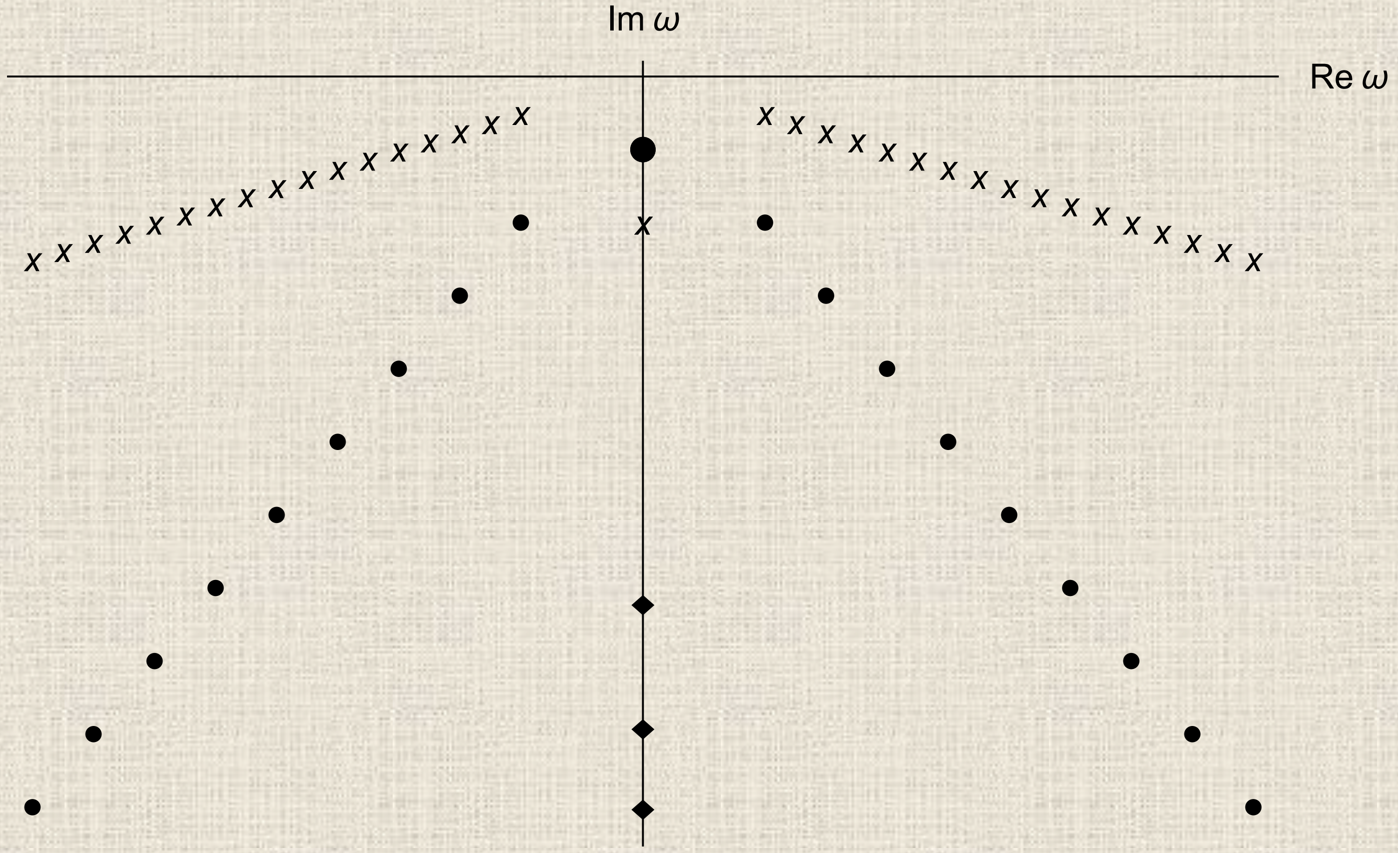
Grozdanov and AOS, 2014

$$\kappa = \frac{\eta}{\pi T}\left(\frac{(1+\gamma)(2\gamma^2-1)}{2\gamma^2}\right)$$

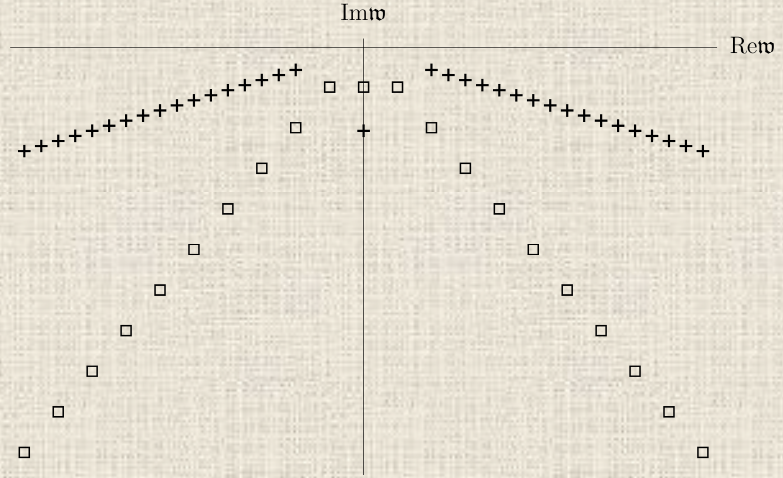
Banerjee and Dutta, 2011

$$H(\lambda_{GB}) = 2\eta\tau_\Pi - 4\lambda_1 - \lambda_2 = -\frac{\eta}{\pi T}\frac{(1-\gamma_{GB})(1-\gamma_{GB}^2)(3+2\gamma_{GB})}{\gamma_{GB}^2} = -\frac{40\lambda_{GB}^2\eta}{\pi T} + \dots$$

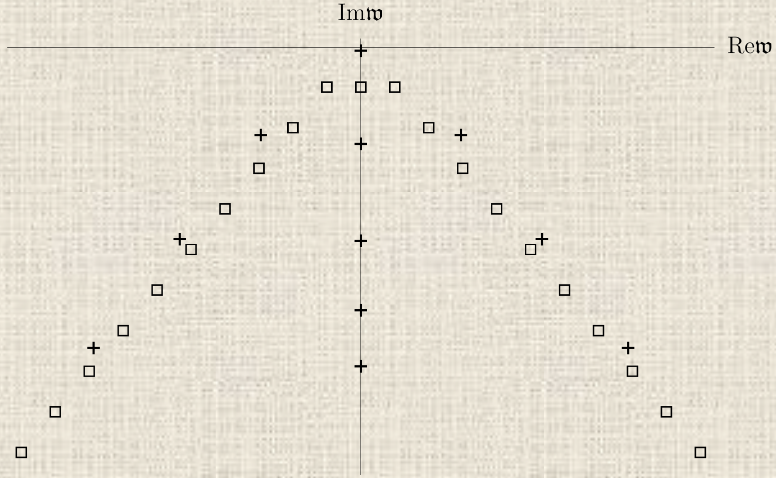
Singularities of stress-energy tensor Green's function at infinite (black dots) and finite (black crosses and diamonds) coupling



Singularities of stress-energy tensor Green's function in different regimes of viscosity-entropy ratio



$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

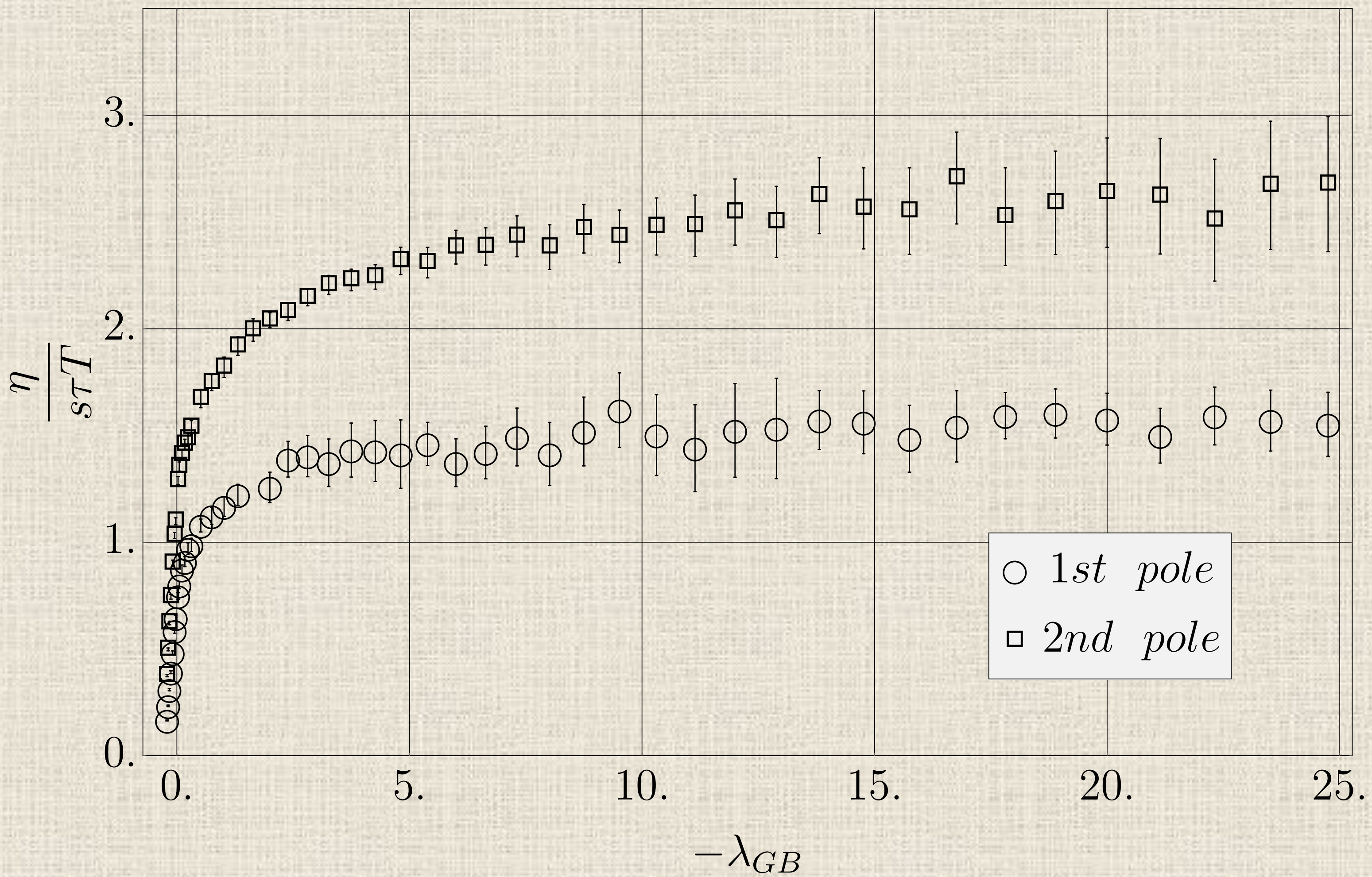


$$\frac{\eta}{s} < \frac{1}{4\pi}$$

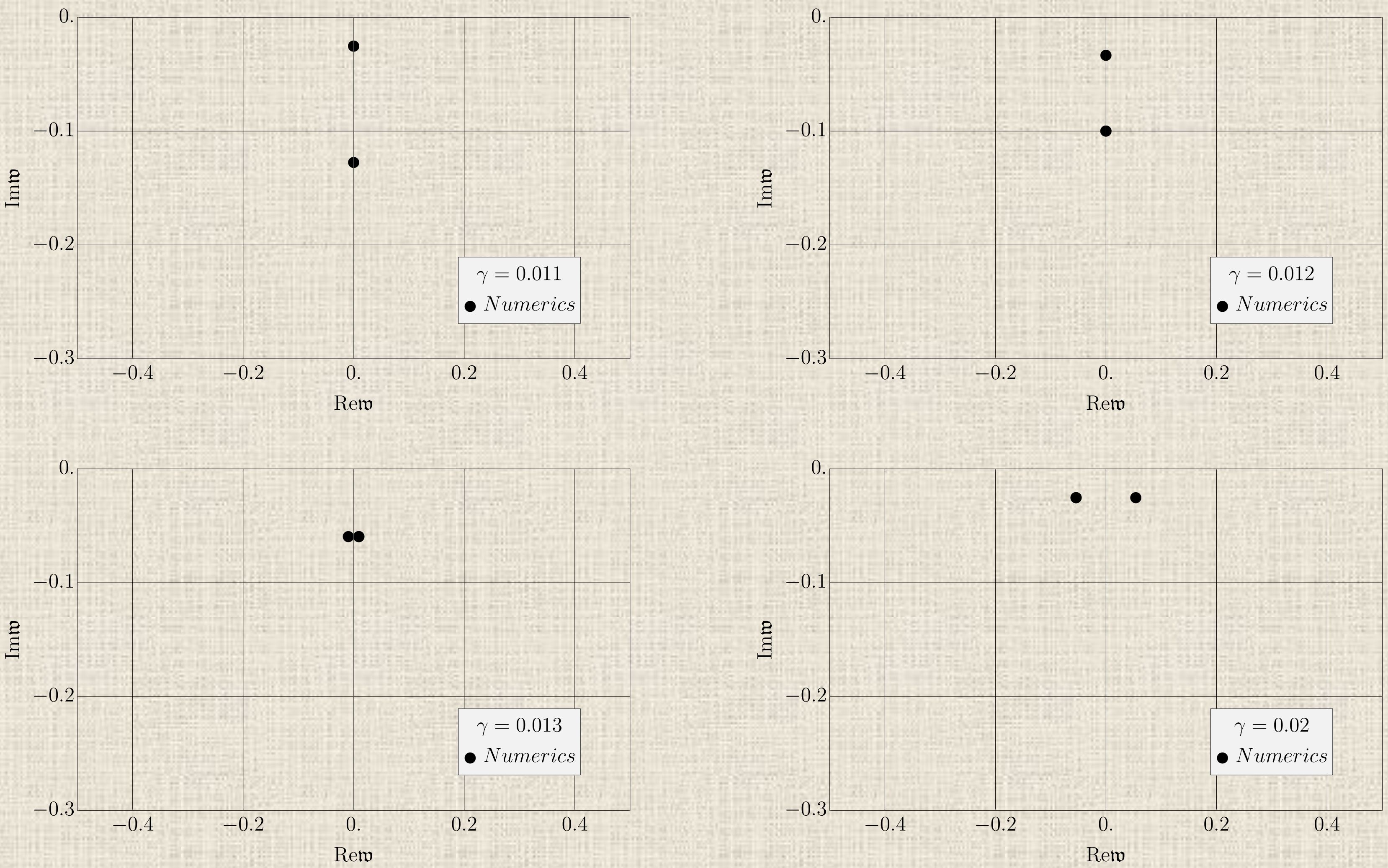
White squares: poles at infinite coupling
Crosses: poles at finite coupling

On the "unreasonable effectiveness" of kinetic theory at strong coupling

Recall that in kinetic theory $\eta = const\ s\,\tau_R\,T$

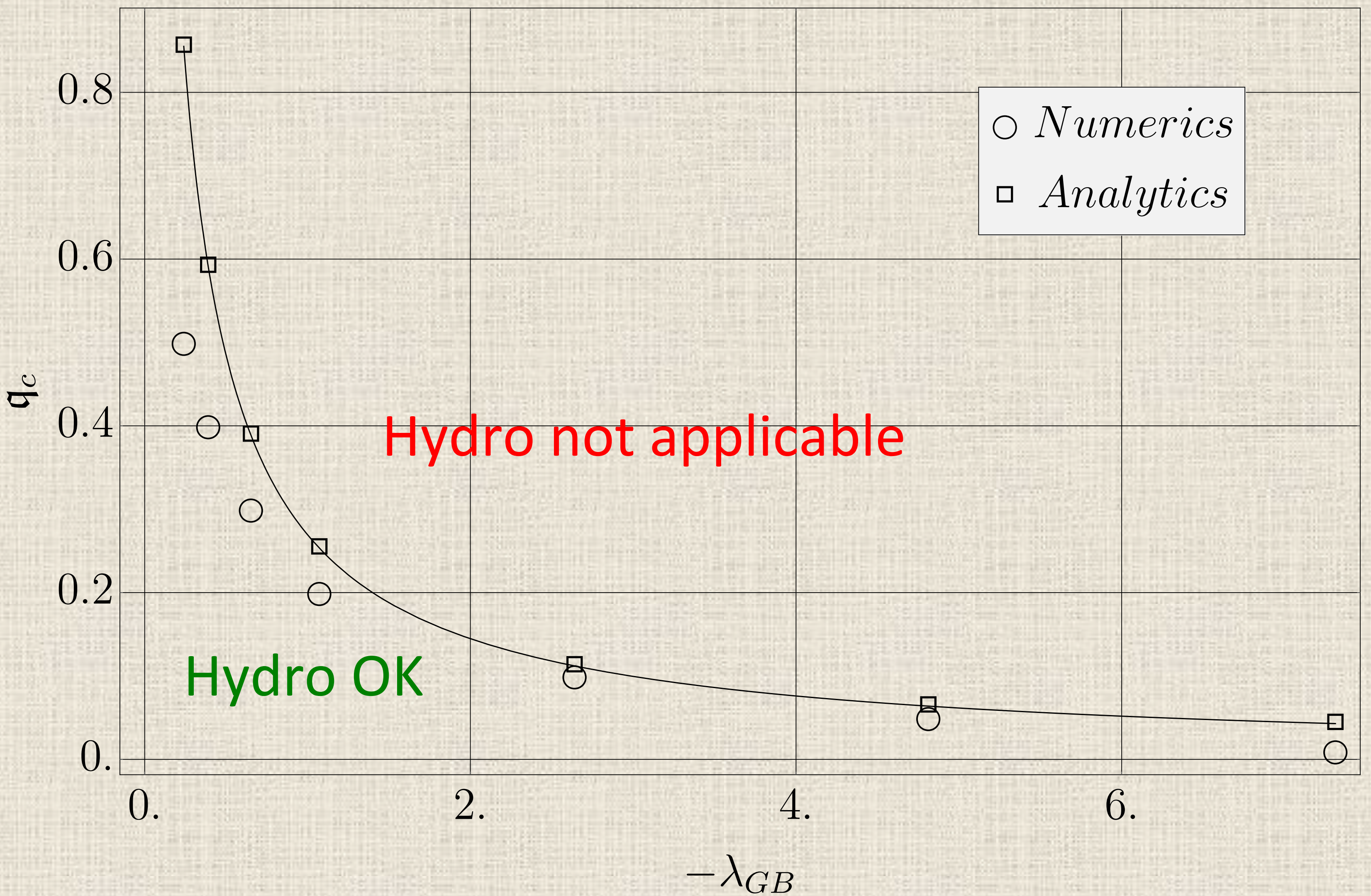


Breakdown of hydrodynamics at (large) finite coupling

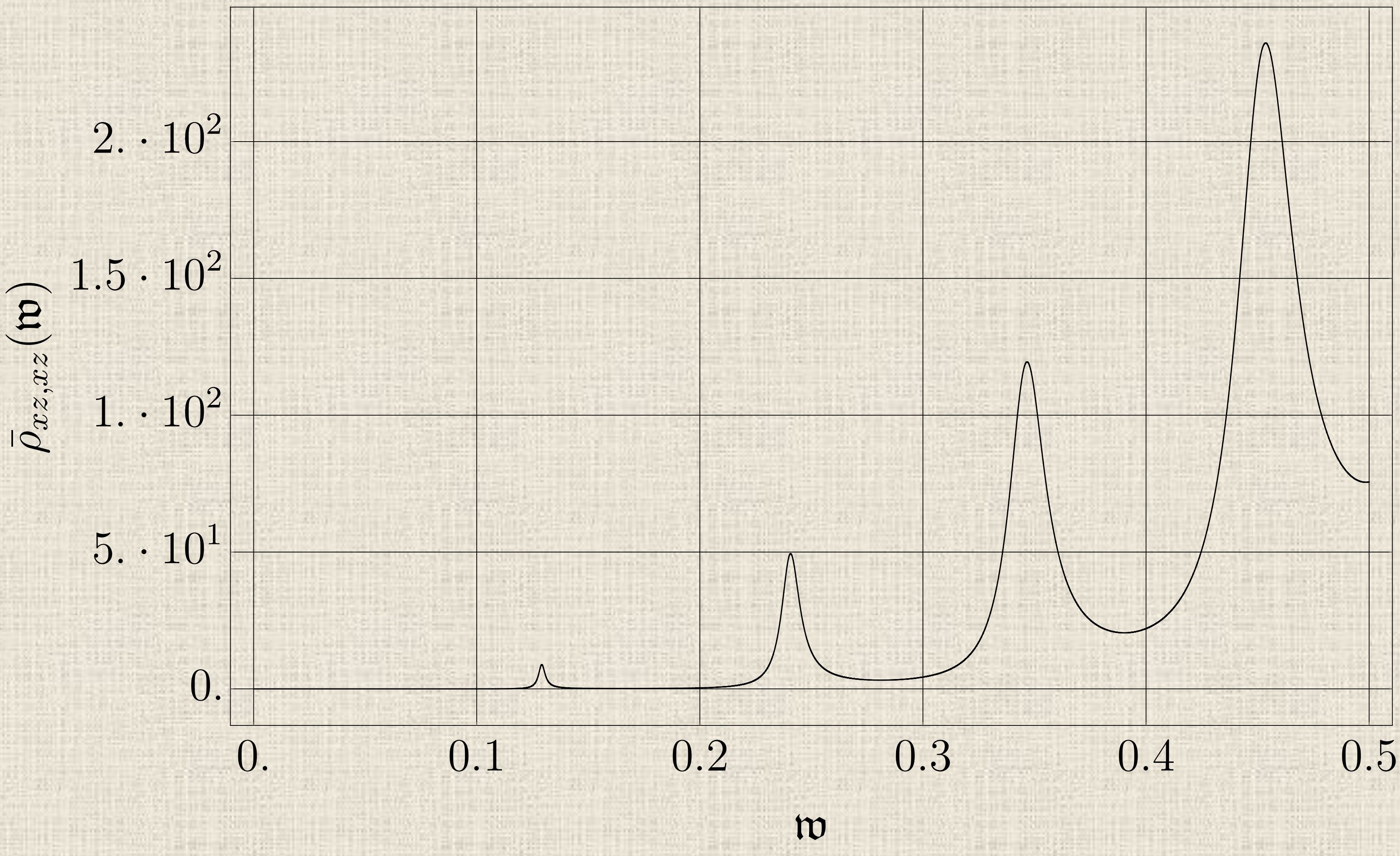


$$q_c = q_c(\lambda)$$

“Applicability of hydrodynamics” as a function of coupling



Quasiparticles from holography?



Conclusions & open questions

Finite coupling corrections seem to show qualitatively similar behavior irrespective of the precise structure of higher derivative terms in dual gravity (we did R^2 and R^4)

How robust are the results (structure of higher derivative expansion)?

We observe breakdown of hydrodynamics at coupling-dependent value of a wave-vector. The dependence on coupling suggests that hydrodynamics has a wider applicability range at stronger coupling

Our results suggest that kinetic theory results may be formally still applicable in the intermediate and strong coupling regime where the use of kinetic theory itself cannot be justified

We observe qualitatively different analytic structure of correlators depending on whether

$$\eta/s > 1/4\pi \text{ or } \eta/s < 1/4\pi$$

Is this of any significance?

THANK YOU!