Weak x strong coupling non-equilibrium dynamics in the early universe

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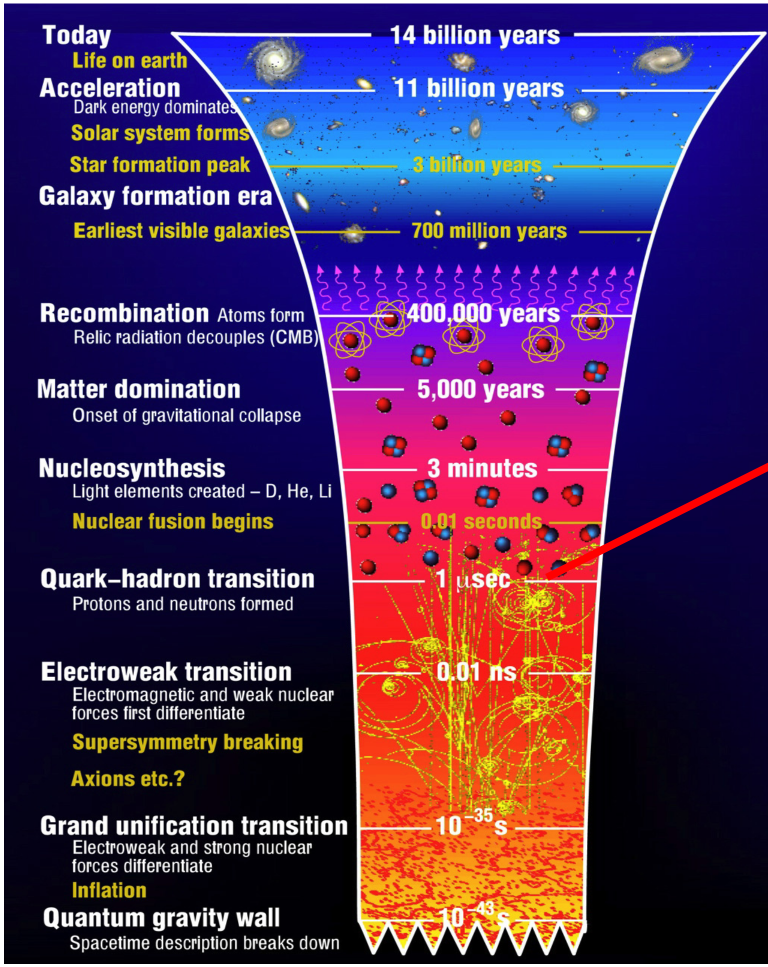
Based on BDHMN, Phys. Rev. Lett. 116 (2016) 2, 022301, arXiv:1507.07834 [hep-ph]

arXiv:1607.05245

and Buchel, Heller, JN, arXiv:1603.05344 [hep-th] + other new stuff

Relativistic hydrodynamics: theory and modern applications, MITP, Oct. 2016

How do Standard Model fields thermalize in the early Universe?



A thermal history of the Universe

Event	time t	redshift z	temperature T
Inflation	10^{-34} s (?)	–	–
Baryogenesis	?	?	?
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μs	10^{12}	150 MeV
Dark matter freeze-out	?	?	?
Neutrino decoupling	1 s	6×10^9	1 MeV
Electron-positron annihilation	6 s	2×10^9	500 keV
Big Bang nucleosynthesis	3 min	4×10^8	100 keV
Matter-radiation equality	60 kyr	3400	0.75 eV
Recombination	260–380 kyr	1100–1400	0.26–0.33 eV
Photon decoupling	380 kyr	1000–1200	0.23–0.28 eV
Reionization	100–400 Myr	11–30	2.6–7.0 meV
Dark energy-matter equality	9 Gyr	0.4	0.33 meV
Present	13.8 Gyr	0	0.24 meV

How do Standard Model fields thermalize?

- 20 ps < time < 20 microsecs after Big Bang

- Temperature of the Universe dropped from ~ 100 GeV to 150 MeV

If you focus on the QCD fields: $\psi_f, \bar{\psi}_f, A^a_\mu$

1) When T >> 1 GeV $\rightarrow$ QCD is a gas of quasiparticles

2) For T ~ 200 MeV $\rightarrow$ QCD is a non-conformal ($\varepsilon \neq 3P$)

strongly interacting plasma

OUTLINE

I) Expanding universe as the simplest setup to study thermalization of rapidly evolving systems

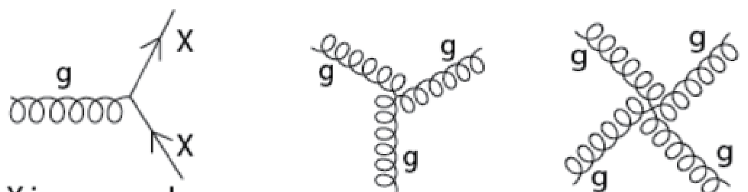
II) Toy model at weak coupling: Boltzmann equation

III) Toy model at strong coupling: N=2* plasma

IV) Conclusions

Quark-gluon plasma: The smallest fluid ever made

QCD = confinement + asymptotic freedom

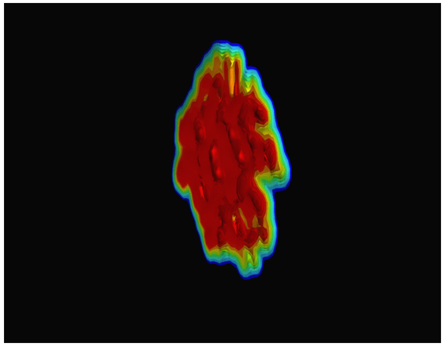


X is any quark.

gluon self-interactions

?

Quark-Gluon Plasma



Ex: Schenke, Jeon, Gale, PRL 2011

QGP perfect fluidity: $\frac{\eta}{s} \sim \frac{1}{4\pi}$ $\rightarrow$ emergent property of QCD.

This seems to appear even in elementary proton+proton collisions.

Fluid dynamics at length scales of the size of a proton.

The expanding universe provides a much simpler case to study.

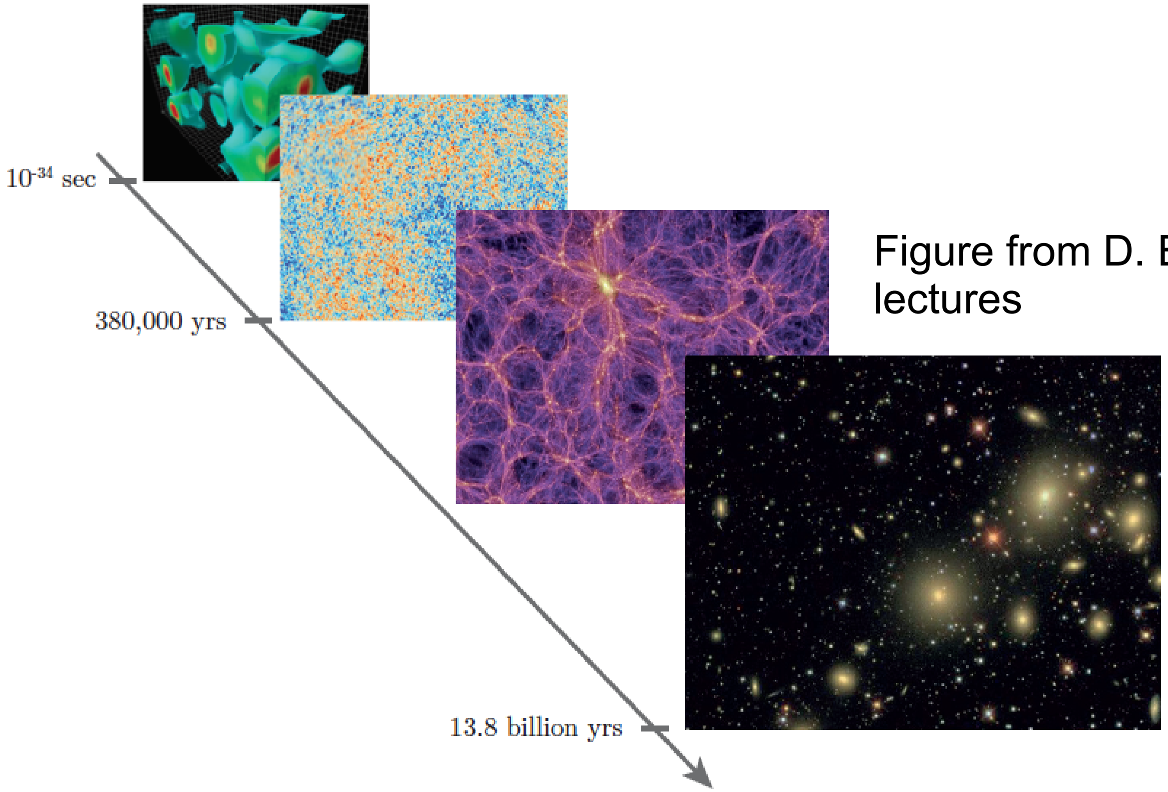


Figure from D. Baumann's lectures

More symmetries than HIC, though there is only one event to analyze …

Friedmann-Robertson-Lemaitre-Walker (FRLW) spacetime

Maximally (spatially) symmetric spacetime

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1 - K\,r^2} + r^2\,d\Omega^2\,.\right]$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G\,T_{\mu\nu}$$

Einstein's equations

K ~ 0 (spatially flat –> our universe)

K = 1, -1

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\varepsilon - \frac{K}{a^2}$$

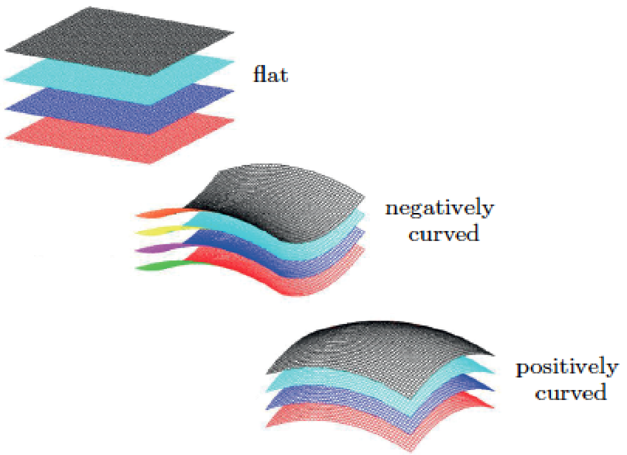
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\varepsilon + 3P)$$

$$\varepsilon \propto \begin{cases} a^{-3} & \text{matter} \\ a^{-4} & \text{radiation} \\ a^{0} & \text{vacuum} \end{cases}$$

FLRW spacetime

Spatial isotropy + homogeneity

Universe



Isotropic and homogeneous expanding FLRW spacetime

(zero spatial curvature)

Ex: metric

$$ds^2 = dt^2 - a^2(t)\left(dx^2 + dy^2 + dz^2\right)$$

Determined from Einstein's equations

Friedmann-Lemaitre-Robertson-Walker spacetime

We consider an isotropic and homogeneous expanding FRW spacetime

(zero spatial curvature)

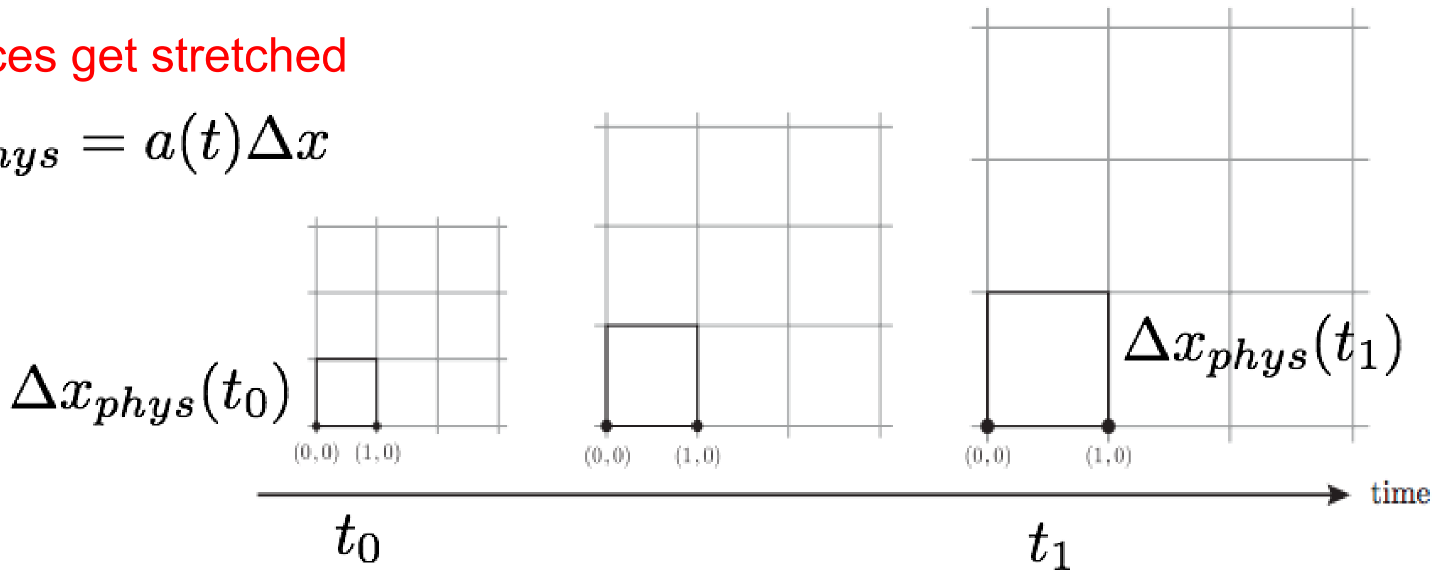
metric
$$ds^2 = dt^2 - a^2(t)\left(dx^2 + dy^2 + dz^2\right)$$

Cosmological scale factor (e.g., radiation) $a(t) \sim t^{1/2}$

Hubble parameter $H = \dot{a}/a > 0$

Distances get stretched

$$\Delta x_{phys} = a(t)\Delta x$$



Thermalization in an expanding Universe

Pros:

- Applications in cosmology

- Spatial isotropy + homogeneity = strong constraining symmetries

- Underlying expansion of the Universe is "simple"
(in comparison to HIC)

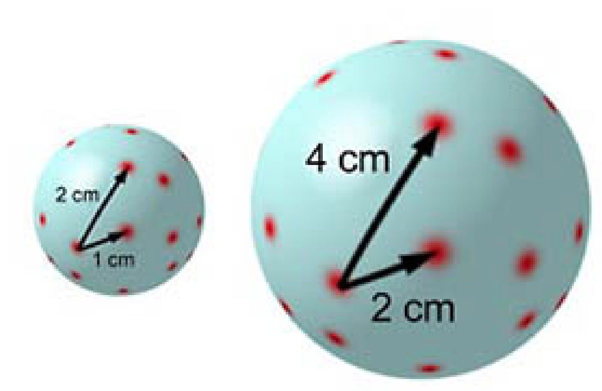
Cons:

- Inclusion of general relativistic effects: numerics more involved

- Why would the inclusion of something "difficult" (GR) help anybody here???

If your goal is to understand the thermalization process of SM fields in an expanding case:

Nothing is easier than studying how a locally static system thermalizes (or not) in an expanding Universe.



Universe expands

Flow locally static

I will show you in the following two toy models to make you think about this problem …

First toy model: General relativistic Boltzmann equation

- Dilute gases display complex non-equilibrium dynamics.

- The Boltzmann equation has been instrumental in physics and mathematics (e.g., 2010 Fields Medal).

General Relativistic Boltzmann equation

$$k^{\mu}(u_{\mu}D+\nabla_{\mu})f(x,k)+k_{\lambda}k^{\mu}\Gamma^{\lambda}_{\mu i}\frac{\partial f(x,k)}{\partial k_i}=\mathcal{C}[f],$$

Space-time variation

Collision term

- It describes how the particle distribution function $f_k(x,k)$ varies in time and space due to the effects of collisions (and external fields).

Boltzmann Equation in FLRW spacetime

Simplest (unrealistic) toy model of an out-of-equilibrium Universe:

- Massless particles, classical statistics, constant cross section: σ

- Weakly coupled QCD at high T is much more complicated than this

- However, the toy model captures the physics I need for this talk

- You will see that such a system flows as a perfect fluid though it is dissipative (entropy is always being produced)

$$k^0 = k/a(t) \quad \text{with} \quad k = |\boldsymbol{k}|$$

$$\int_k \equiv \int d^3k / \left[(2\pi)^3 \sqrt{-g}\, k^0\right]$$

<u>Our Boltzmann equation:</u>

$$k^0 \partial_t f_k = \frac{(2\pi)^5}{2} \sqrt{-g}\, \sigma \int_{k'pp'} s\, \delta^4(k+k'-p-p')(f_p f_{p'} - f_k f_{k'})$$

$$s = (k^\mu + k'^\mu)(k_\mu + k'_\mu)$$

This equation includes general relativistic effects + full nonlinear collision dynamics

<u>We want to find solutions for the distribution function</u>

Given an initial condition: $f(t_0, k)$ and $n(t_0)$, $\varepsilon(t_0)$

How does one solve this type of nonlinear integro-differential equation?

The moments method

- Originally introduced by Grad (1949) and used by Israel and Stewart (1979) in the relativistic regime.

- Perfected for applications in HIC by DNMR, Phys. Rev. D 85 (2012) 114047

- Used more recently in Phys. Rev. Lett. 116 (2016) 2, 022301

The idea is simple

Instead of solving for the distribution function itself directly, one uses the Boltzmann eq. to find exact equations of motion for the moments of the distribution function.

Ex: The particle density $n(t) = \int_k (u.k) f_k(t)$ is a scalar moment

with equation $\partial_t n + 3n\, H(t) = 0$

Defining the scaled time: $\hat{t} = t/\ell_0$ where $\ell_0 = 1/(\sigma\, n(t_0))$

(constant mean free path)

And the normalized moments $M_m(\hat{t}) = \dfrac{\rho_m(\hat{t})}{\rho_m^{eq}(\hat{t})}$ which obey the **exact** set of eqs:

See PRL 2016, arXiv:1507.07834 [hep-ph]

ALL THE NONLINEAR BOLTZMANN DYNAMICS IS ENCODED HERE

$$a^3(\hat{t})\,\frac{\partial}{\partial \hat{t}} M_m(\hat{t}) + M_m(\hat{t}) = \frac{1}{m+1}\sum_{j=0}^{m} M_j(\hat{t})M_{m-j}(\hat{t})$$

GR effect

Simple recursive nonlinearity

Conservation laws require $M_0 = M_1 = 1$

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Ex: The energy density $\varepsilon(t) = \int_k (u.k)^2 f_k(t)$ is a scalar moment

with equation $\partial_t \varepsilon + 4\varepsilon\, H(t) = 0$

Clearly, due to the symmetries, here only scalar moments can be nonzero.

Thus, if we can find the time dependence of the scalar moments

$$\rho_m(t) = \int_{\boldsymbol{k}} (u \cdot k)^{m+1} f_k(t) \quad (m \in \mathbb{N}_0) \qquad \rho_0 = n,\ \rho_1 = \varepsilon$$

via solving their exact equations of motion, one should be able to recover $f_k(t)$

“Fourier” transforming the Boltzmann equation

G. Denicol and JN, to appear soon

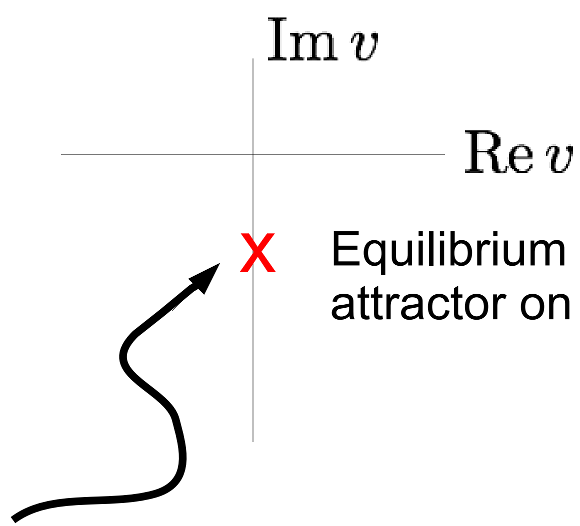
If the moments are what we want, it makes sense to define the generating function

$$\Phi(t,v) \equiv \int_k e^{iv(u\cdot k)}(u\cdot k) f_k(t) \Longrightarrow \rho_n(t) = \frac{\partial^n \Phi(t,v)}{i^n \partial v^n}\Big|_{v=0}$$

where v is a <u>complex number</u>

Thermalization → development of a pole at $v = -1/T$

$$\Phi(t,v) \to \Phi^{eq}(v) = \frac{\rho_0}{(1-iTv)^3}$$



Equilibrium = global attractor on the plane

Thermalization process is mapped onto how the analytical structure of this function changes with time.

“Fourier” transforming the Boltzmann equation

G. Denicol and JN, to appear soon

This way to see the thermalization process is valid for any type of cross section (does not depend on the mass, quantum statistics changes the pole)

It is easy to show that this →

$$\partial_t(a^3\Phi) = \frac{\sigma}{2}\int_{kk'} f_k f_{k'} s(2\pi)^5 \int_{pp'} \delta^{(4)}(k+k'-p-p')e^{iv(u\cdot p)}$$
$$-\frac{\sigma}{2}\int_{kk'} f_k f_{k'} e^{iv(u\cdot k)} s(2\pi)^5 \int_{pp'} \delta^{(4)}(k+k'-p-p')$$

Becomes this:

$$\partial_t(a^3\Phi) + \sigma\rho_0\Phi = \sigma\left(\int_0^1 d\alpha\,\Phi(t,\alpha v)\right)^2 - \sigma\left(\int_0^1 d\alpha\,(2\alpha-1)\Phi(t,\alpha v)\right)^2$$

Taking derivatives w.r.t. v one can easily find the equation for the moments

Full Analytical Solution

Redefining time

Using the moments equations in this form

$$\tau = \int_{\hat{t}_0}^{\hat{t}} dt'/a^3(t')$$

$$\partial_\tau M_m(\tau) + M_m(\tau) = \frac{1}{m+1} \sum_{j=0}^{m} M_j(\tau) M_{m-j}(\tau).$$

One can show that

$$M_m(\tau) = \mathcal{K}(\tau)^{m-1} \left[m - (m-1)\mathcal{K}(\tau) \right] \quad (m \geq 0)$$

is an analytical solution of the moments equations !

$$\mathcal{K}(\tau) = 1 - \frac{e^{-\tau/6}}{4} \sim e^{-1/K_N}$$

Non-perturbative in $K_N = \ell(t)H(t)$

Full Analytical Solution

1st analytical solution of the Boltzmann equation for an expanding gas (since 1872)

BDHMN, PRL (2016) arXiv:1507.07834 [hep-ph]

λ = fugacity

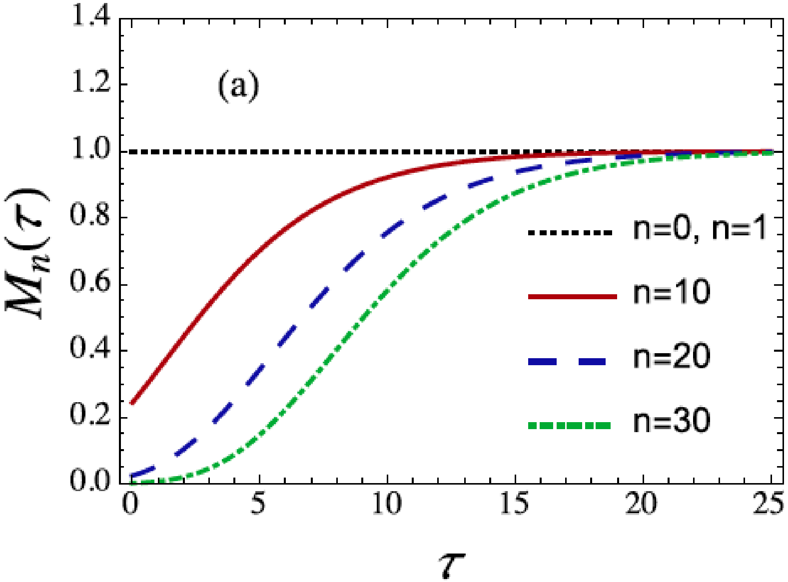
$$f_k(\tau) = \lambda \exp\left(-\frac{u\cdot k}{\mathcal{K}(\tau)T(\tau)}\right) \times \left[\frac{4\mathcal{K}(\tau)-3}{\mathcal{K}^4(\tau)} + \frac{u\cdot k}{T(\tau)}\left(\frac{1-\mathcal{K}(\tau)}{\mathcal{K}^5(\tau)}\right)\right]$$

Initial condition $f_k(0) = \frac{256}{243}(k/T_0)\,\lambda\,\exp[-4k/(3T_0)] > 0$

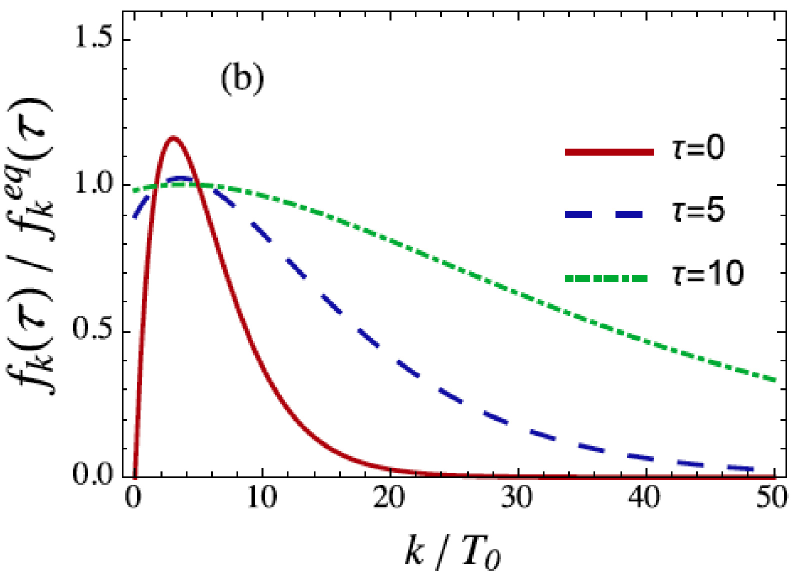
See arXiv:1607.05245 for many more details about this and other solutions

Full Analytical Solution

Time evolution



Momentum dependence



For radiation dominated universe higher order moments will certainly not erase the info about initial conditions → system never equilibrate due to expansion.

The approach to equilibrium here depends on the occupancy of each moment.

Full Analytical Solution – Generating function

For the analytical solution

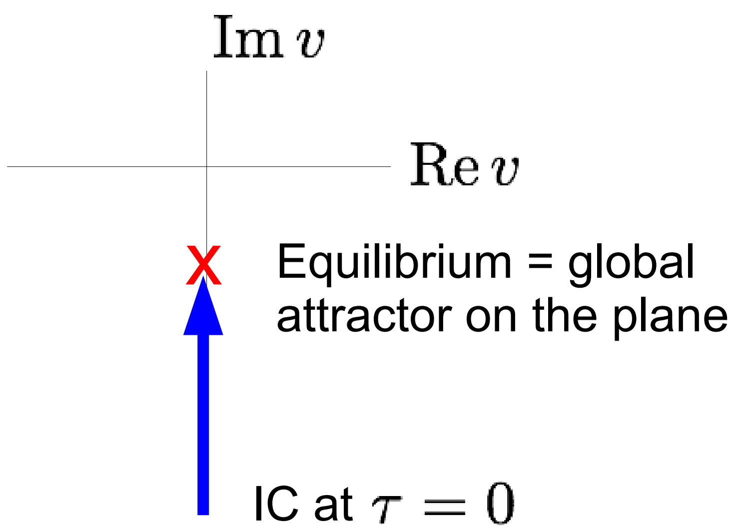
$$M_m(\tau) = \mathcal{K}(\tau)^{m-1}\left[m - (m-1)\mathcal{K}(\tau)\right] \quad (m \geq 0)$$

one finds

$$\Phi(t,v) = \rho_0 \frac{[1 + iTv(3 - 4\mathcal{K}(\tau))]}{[1 - iTv\mathcal{K}(\tau)]^4}$$

Time dependent pole at

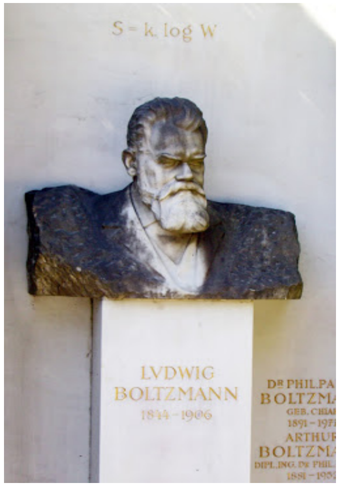
$$v = -i/(T\mathcal{K}(\tau))$$



- Thermalization process of different initial conditions correspond to other trajectories on the plane.

- Non-thermal fixed points???? Universality?

Non-equilibrium entropy



See arXiv:1607.05245

$$\mathcal{S}^\mu = -\int_k k^\mu f_k (\ln f_k - 1)$$

One can prove that H-theorem is valid here. Entropy production solely from non-hydrodynamic modes (hydro modes have decoupled).

$$\nabla_\mu \mathcal{S}^\mu = \frac{1}{4}\int_{kk'pp'} W_{kk\leftrightarrow pp'} \left[\frac{f_p f_{p'}}{f_k f_{k'}} - \ln\left(\frac{f_p f_{p'}}{f_k f_{k'}}\right) - 1\right] f_k f_{k'} \geq 0$$

Even though energy-momentum tensor always the same as in equilibrium.

Expansion is never truly adiabatic in this toy Universe.

This is all very nice but QCD is a non-Abelian gauge theory.

Close to the QCD phase transition, QCD is likely strongly coupled.

Boltzmann description not applicable.

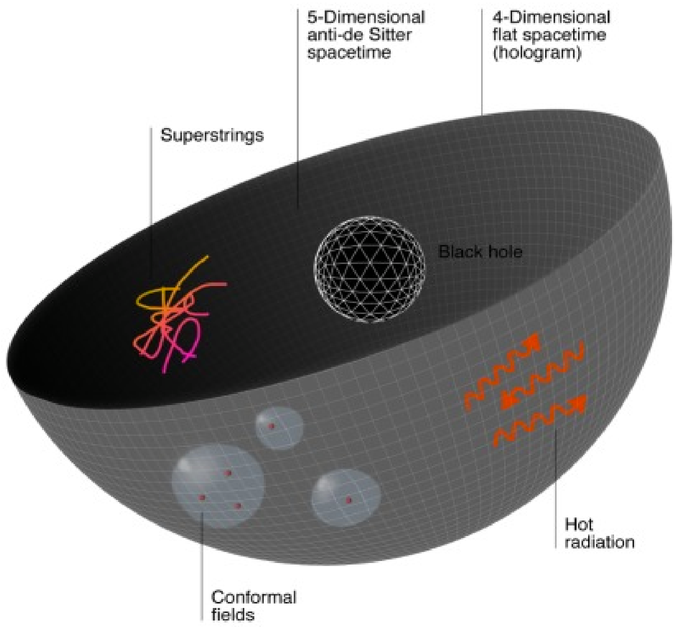
How do we study thermalization for T ~ QCD phase transition in the early universe?

Lattice QCD cannot be used here (need real time dynamics)

A reasonable thing to do is to “jump” into a black hole (brane)

Holography (gauge/string duality)

Maldacena 1997; Witten 1998; Gubser, Polyakov, Klebanov 1998



Strong coupling limit of QFT in 4 dimensions

⇕

String Theory/Classical gravity in d>4 dimensions

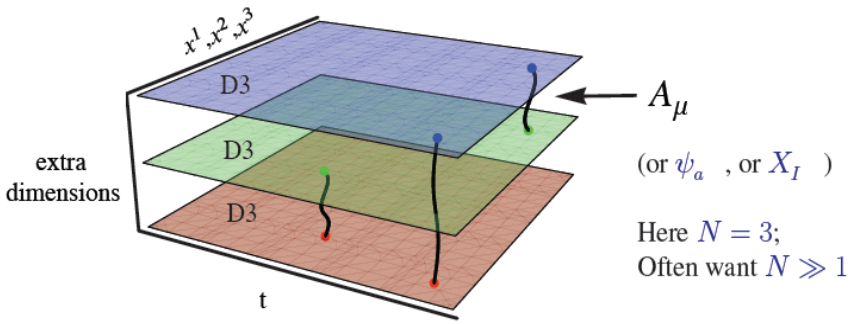
Universality of nearly perfect fluids

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Kovtun, Son, Starinets, PRL 2005

STANDARD EXAMPLE

$\mathcal{N}=4$ SU(Nc) Supersymmetric Yang-Mills in d=4



Fields in the adjoint rep. of SU(Nc)

Gluons, Fermions, Scalars

$\beta = 0$ **CFT !!!!**

Maldacena, 1997: This gauge theory is dual to Type IIB string theory on AdS_5 x S_5

Strongly-coupled, large Nc gauge theory	Weakly-coupled, low energy string theory
$N_c \to \infty$	$g_s \to 0$
$\lambda = R^4/\ell_s^4 \to \infty$	$\ell_s/R \to 0$
t'Hooft coupling in the gauge theory	

Universality and nearly perfect fluidity

$\lambda \gg 1$ in QFT $\rightarrow$ string theory in weakly curved backgrounds

d.o.f. / vol. $\rightarrow \infty$ in QFT $\rightarrow$ vanishing string coupling

T, μ in QFT $\rightarrow$ spatially isotropic black brane

For anisotropic models
there is violation
see, e.g, arXiv:1406.6019

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Universality of shear viscosity

Kovtun,Son,Starinets, PRL 2005

Universality of black hole horizons → **HOLOGRAPHY** → Universality of transport coefficient in QFT

Given that heavy ion data indicates that T ~ QCD transition the QGP is a nearly perfect fluid …

There must have been nearly perfect fluidity in the early universe

Experimental consequences of that are not yet known (are there any??)

Given that around those temperatures QCD is not conformal, we would like to use a nonconformal gravity dual in a FLRW spacetime

This was done by A. Buchel, M. Heller, JN in arXiv:1603.05344 [hep-th]

2^{nd} toy model: N=2* gauge theory

Pilch, Warner, Buchel, Peet, Polchinski, 2000
A. Buchel, S. Deakin, P. Kerner and J. T. Liu, NPB 784 (2007) 72

A relevant deformation of SYM: Breaking of SUSY

N = 4 SYM theory + $\delta\mathcal{L} = -2\int d^4x\,\left[\,m_b^2\,\mathcal{O}_b + m_f\,\mathcal{O}_f\,\right]$

$m_b^2\,\mathcal{O}_b$ → Bosonic mass

$m_f\,\mathcal{O}_f$ → Fermionic mass

$$\mathcal{O}_b = \frac{1}{3}\mathrm{Tr}\left(|\phi_1|^2 + |\phi_2|^2 - 2\,|\phi_3|^2\right),$$

$$\mathcal{O}_f = -\mathrm{Tr}\Big(i\,\psi_1\psi_2 - \sqrt{2}g_{\rm YM}\,\phi_3[\phi_1,\phi_1^\dagger] + \sqrt{2}g_{\rm YM}\,\phi_3[\phi_2^\dagger,\phi_2]$$

$$+\mathrm{h.c.}\Big) + \frac{2}{3}m_f\,\mathrm{Tr}\left(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2\right)$$

C. Hoyos, S. Paik, and L. G. Yaffe, JHEP 10, 062 (2011)

2nd toy model: N=2* gauge theory

Pilch, Warner, Buchel, Peet, Polchinski, 2000

Classical gravity dual action:

$$S = \frac{1}{16\pi G_5} \int \mathrm{d}^5 x \sqrt{-g} \left(R - 12(\partial\alpha)^2 - 4(\partial\chi)^2 - V \right),$$

Scalar potential

$$V = -e^{-4\alpha} - 2e^{2\alpha} \cosh 2\chi + \frac{1}{4} e^{8\alpha} \sinh^2 2\chi.$$

Bulk viscosity

$$\frac{\zeta}{\eta} \sim \mathcal{O}(1) \left(\frac{1}{3} - c_s^2 \right)$$

- Well defined stringy origin

- Non-conformal strongly interacting plasma: $\varepsilon \neq 3p$

- Used in tests of holography in non-conformal setting

N=2* gauge theory in a FLRW Universe

Buchel, Heller, JN, arXiv:1603.05344 [hep-th]

Characteristic formulation of gravitational dynamics in asymptoticaly AdS5 spacetimes

Chesler,Yaffe, 2013

Assuming spatial isotropy and homogeneity $\boldsymbol{x} = \{x, y, z\}$ leads to

$$ds_5^2 = 2dt\,(dr - Adt) + \Sigma^2\, d\boldsymbol{x}^2,$$

$$\Sigma = \frac{a}{r} + \mathcal{O}(r^{-1}), \; A = \frac{r^2}{8} - \frac{\dot{a}r}{a} + \mathcal{O}(r^0)$$

$$\alpha = -\frac{8m_b^2 \ln r}{3r^2} + \mathcal{O}(r^{-2}), \; \chi = \frac{2m_f}{r} + \mathcal{O}(r^{-2}).$$

→ Encode non-equilibrium dynamics in an expanding Universe !!!

N=2* gauge theory in a FLRW Universe

Buchel, Heller, JN, arXiv:1603.05344 [hep-th]

Conformal limit When $m_b = m_f = 0$,

Analytical solution for SYM in FLRW spacetime

$$\alpha = \chi = 0\,,\ \Sigma = \frac{ar}{2}\,,\ A = \frac{r^2}{8}\left(1 - \frac{\mu^4}{r^4 a^4}\right) - \frac{\dot{a}}{a}\,r,$$

First studied by P. S. Apostolopoulos, G. Siopsis, and N. Tetradis, PRL, (2009)

Temperature

$$T = \frac{\mu}{4\pi\, a}.$$

Energy density

$$\epsilon = \frac{3}{8}\pi^2 N^2 T^4 + \frac{3N^2(\dot{a})^4}{32\pi^2 a^4}$$

Pressure

$$P = \frac{1}{3}\epsilon - \frac{N^2(\dot{a})^2\ddot{a}}{8\pi^2 a^3}$$

Conformal anomaly!!!! $-\epsilon + 3P = \frac{N^2}{32\pi^2}\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right)$

Divergence of the hydrodynamic series

Buchel, Heller, JN, arXiv:1603.05344 [hep-th]

Viscous hydrodynamics → Knudsen series expansion

Separation of scales → macroscopic: L microscopic: ℓ

Knudsen number gradient expansion:

$$K_N \sim \frac{\ell}{L} \ll 1$$

→ **FLUID**

- Used in kinetic theory (Chappman-Enskog)

- Within the fluid/gravity duality (Minwalla, Hubeny, Rangamani, etc)

Divergence of the hydrodynamic series

Buchel, Heller, JN, arXiv:1603.05344 [hep-th]

In our FLRW case, such a Knudsen gradient series gives

Energy-momentum tensor

$$T_{\mu\nu} = T_{\mu\nu}^{eq} + \Pi_{\mu\nu}(\dot{a}, \{\dot{a}^2, \ddot{a}\}, \cdots),$$

$T_{\mu\nu}^{eq}$ → equilibrium

$\Pi_{\mu\nu}$ → dissipation

In terms of the energy density and pressure out-of-equilibrium

$$\epsilon = \epsilon^{eq} + \mathcal{O}\left(\dot{a}^2, \ddot{a}\right) \ , \ P = P^{eq} - \zeta(\nabla \cdot u) + \mathcal{O}\left(\dot{a}^2, \ddot{a}\right) ,$$

$\zeta(\nabla \cdot u)$ → Bulk viscosity

Divergence of the hydrodynamic series

Buchel, Heller, JN, arXiv:1603.05344 [hep-th]

Entropy production

$$\frac{d(a^3 s)}{dt} = \frac{N^2}{16\pi} a^{7-2\Delta} \mu^2 \delta_\Delta^2 (4-\Delta)^2 s_\Delta \times \Omega_\Delta^2,$$

Apparent horizon: $a^3 s = N^2 \Sigma^3/(16\pi)|_{r=r_h}$

$$\Omega_\Delta \equiv \sum_{n=0}^{\infty} \mathcal{T}_{\Delta,n+1}[a] \frac{F_{\Delta,n}(1)}{\mu^n}.$$

For single component cosmologies

$$a(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3+3\omega}} \quad \text{with constant } \omega \text{ and } H \equiv \dot{a}/a$$

$$\mathcal{T}_{\Delta,n}[a] = \left(-\frac{1}{2} - \frac{3\omega}{2}\right)^n \Gamma\left(n + \frac{2(\Delta-4)}{1+3\omega}\right) a^n H^n.$$ → **Factorial growth!!!**

Divergence of the hydrodynamic series

Buchel, Heller, JN, arXiv:1603.05344 [hep-th]

1st analytical proof of the divergence of gradient expansion:

→ Knudsen gradient series has zero radius of convergence

→ Knudsen series leads to acausal and unstable dynamics

→ There must be a new way to define hydrodynamics
beyond the gradient expansion

→ A recent way to understand that involves resurgence (see, e.g. Heller's talk). For a different approach, see Denicol's talk.

Conclusions

- The early Universe may be the simplest “way” to study how Standard Model quantum fields thermalize.

- Exactly solvable nonlinear kinetic models in a FLRW can be studied (led to the 1^{st} analytical solution of the Boltzmann equation for expanding gas).

- Due to strong coupling near the QCD phase transition in the early Universe, non-equilibrium dynamics may be studied using the gauge/gravity duality.

- Toy model of QCD, N=2* gauge theory, behaves as a nearly perfect fluid but the hydrodynamic expansion has zero radius of convergence.

- New ideas are needed to make further progress in this field ...