Testing the validity of fluid dynamics in (2+1)-dimensional boost-invariant expansion

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with
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Conservation laws & tensor decompositions

$$\partial_\mu N^\mu = 0$$
$$\partial_\mu T^{\mu\nu} = 0$$
$$N^\mu = n u^\mu + n^\mu$$
$$T^{\mu\nu} = e u^\mu u^\nu - (p + \Pi)\Delta^{\mu\nu} + 2W^{(\mu}u^{\nu)} + \pi^{\mu\nu}$$

$n = u_\mu N^\mu$	LRF particle density
$n^\mu = \Delta^\mu_\alpha N^\alpha$	particle diffusion current
$e = u_\mu T^{\mu\nu} u_\nu$	LRF energy density
$W^\mu = \Delta^{\mu\alpha} T_{\alpha\beta} u^\beta$	energy diffusion current
$p(e,n) + \Pi = -\frac{1}{3}\Delta_{\mu\nu} T^{\mu\nu}$	isotropic pressure (p_{eq} + *bulk*)
$\pi^{\mu\nu} = T^{\langle\mu\nu\rangle}$	shear stress tensor

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$
$$T^{\langle\mu\nu\rangle} = \left[\frac{1}{2}\left(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\nu_\alpha \Delta^\mu_\beta\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}\right] T^{\alpha\beta}$$

Fluid dynamics can be derived from the Boltzmann equation

- Close to thermal equilibrium: inverse Reynolds number $\frac{|\pi^{\mu\nu}|}{p} \lesssim 1$
- Separation of microscopic and macroscopic scales: Knudsen number $\lambda_{\mathrm{mfp}}\theta \lesssim 1$

Denicol, Niemi, Molnar, Rischke, Phys. Rev. D **85**, 114047 (2012)

$$k^\mu \partial_\mu f_{\mathbf{k}} = C[f]$$

$$\mathrm{Kn} \lesssim 1 \text{ and } R^{-1} \lesssim 1$$
$$\Downarrow$$

$$\dot{n}^{\langle\mu\rangle} + \frac{n^\mu}{\tau_n} = \frac{\kappa_n}{\tau_n}\nabla^\mu \alpha_0 + \mathcal{J}^\mu + \mathcal{R}^\mu + \mathcal{K}^\mu ,$$
$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\frac{\eta}{\tau_\pi}\sigma^{\mu\nu} + \mathcal{J}^{\mu\nu} + \mathcal{R}^{\mu\nu} + \mathcal{K}^{\mu\nu}$$

- How small/large is the small Knudsen/Reynold number
- Conditions for validity of fluid dynamics
- A+A, p+A collisions: How good is the mapping from v_2 etc. to matter properties η, ζ, etc.

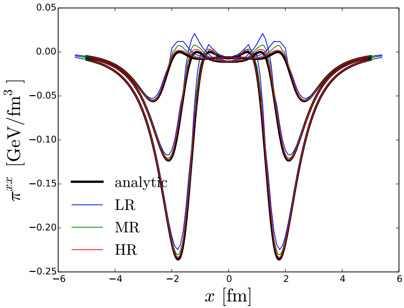
We test fluid dynamics by comparing to the direct solutions of the Boltzmann equation.

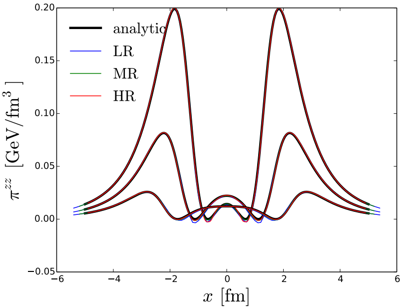
- Vary system size and cross section
- Gaussian number density profiles with $R = 1$ and 3 fm.
- Glauber binary collision profile, with $b = 7.5$ fm.
- Constant cross section $\sigma = 100$, 50, 20, 5, 1 mb
- Binary collisions
- Boost-invariant (2+1)-dimensional expansion

Here: 14-moment approximation (Denicol, Koide, Rischke, PRL **105**, 162501 (2010))

- spacetime evolution of $T^{\mu\nu}$
- Freeze-out: freeze-out condition, δf-correction

Testing the numerics: Gubser flow

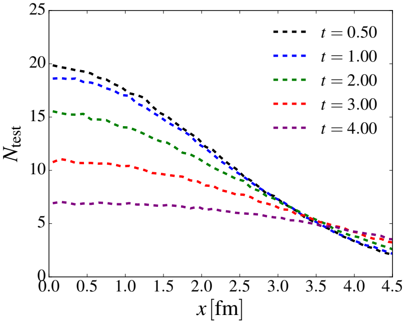


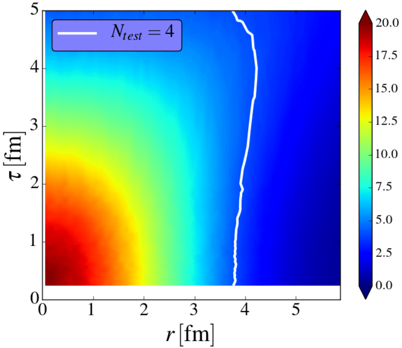


- fluid dynamics: SHASTA
- Test against exact solution (Gubser flow): Marrochio, Noronha, Denicol, Luzum, Jeon and Gale, Phys. Rev. C **91**, no. 1, 014903 (2015)
- Need to resolve 2 (short) timescales: longitudinal expansion $1/\tau$ and relaxation times τ_π
- $\longrightarrow$ adaptive time-step.

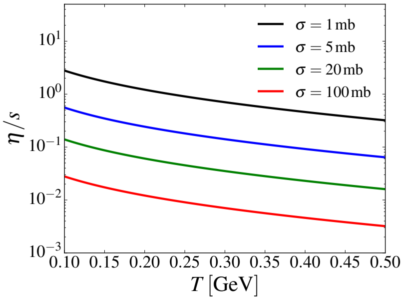
Solving the Boltzmann equation: BAMPS

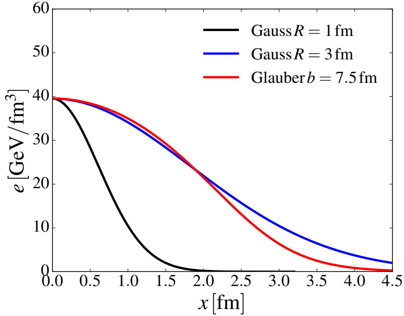
- Boltzmann solver: BAMPS (Xu, Greiner, Phys. Rev. C **71** (2005) 064901)
- test particles
- (test) particles can interact within the computational cell $\Delta^3 x$
- If the number of test particles in the cell $< 4 \longrightarrow$ free gas





Shear viscosity over entropy ratio and initial energy density profiles

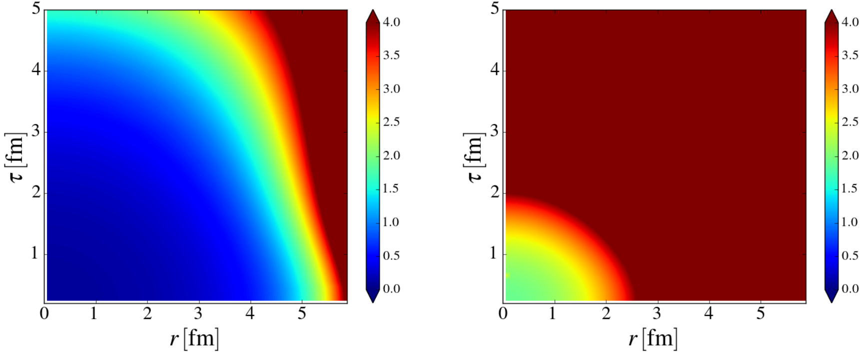




Gaussian n profile $R = 3$ fm

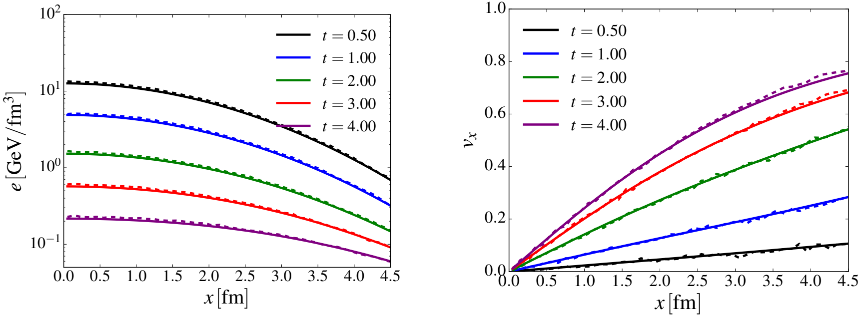
spacetime-evolution of Knudsen number $\lambda_{\rm mfp}\theta$

$\sigma = 20$ mb $\qquad\qquad$ $\sigma = 1$ mb



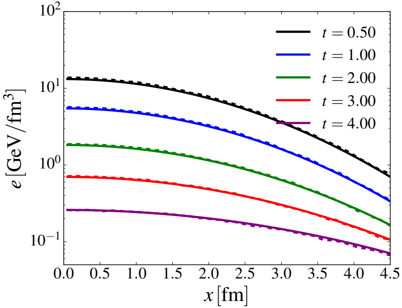
Gaussian profile $R = 3$ fm, $\sigma = 20$ mb

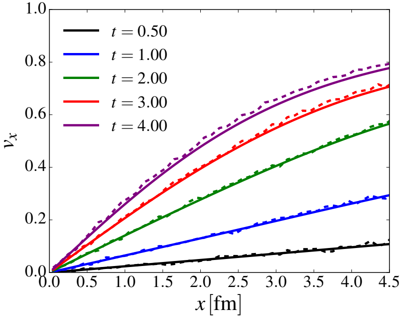
Energy density and velocity profiles, $\sigma = 20$ mb



Gaussian profile $R = 3$ fm, $\sigma = 5$ mb

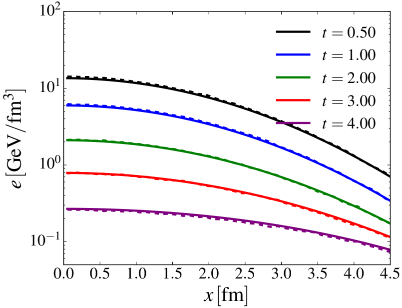
Energy density and velocity profiles, $\sigma = 5$ mb

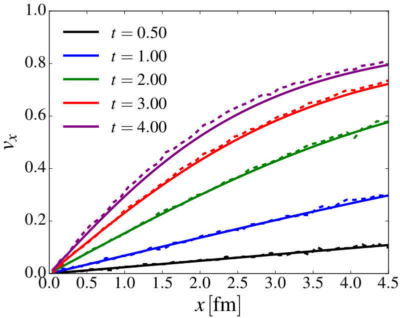




Gaussian profile $R = 3$ fm, $\sigma = 1$ mb

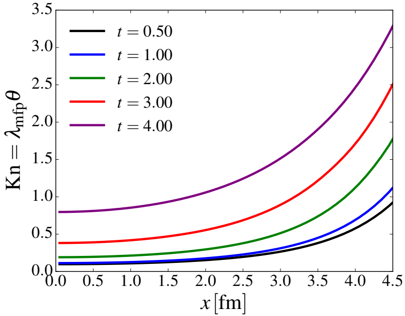
Energy density and velocity profiles, $\sigma = 1$ mb

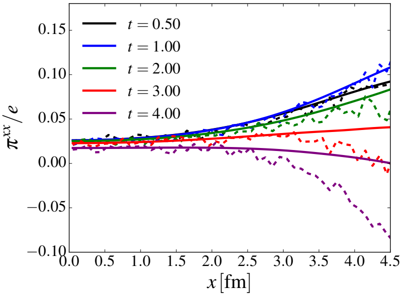




Gaussian profile $R = 3$ fm, $\sigma = 20$ mb

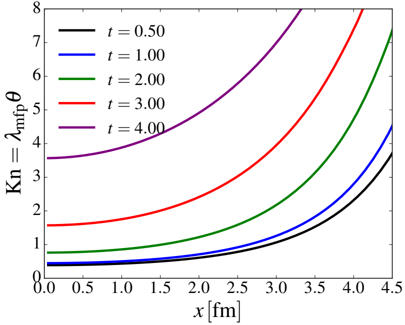
Knudsen number and π^{xx}/e, $\sigma = 20$ mb

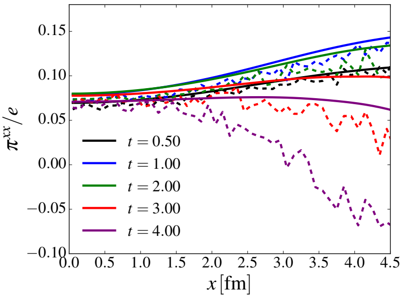




Gaussian profile $R = 3$ fm, $\sigma = 5$ mb

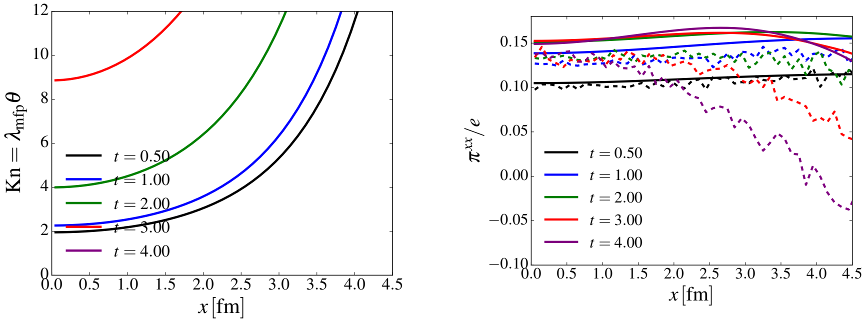
Knudsen number and π^{xx}/e, $\sigma = 5$ mb





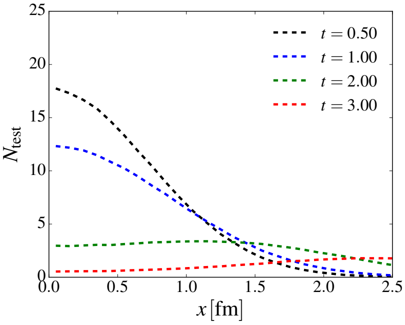
Gaussian profile $R = 3$ fm, $\sigma = 1$ mb

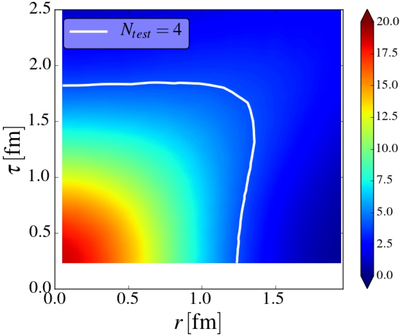
Knudsen number and π^{xx}/e, $\sigma = 1$ mb



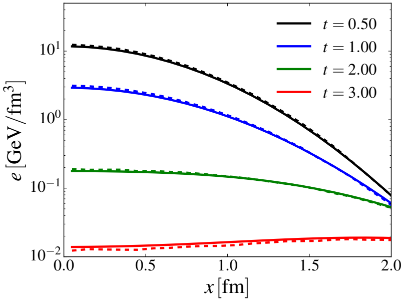
Gaussian n profile, $R = 1$ fm

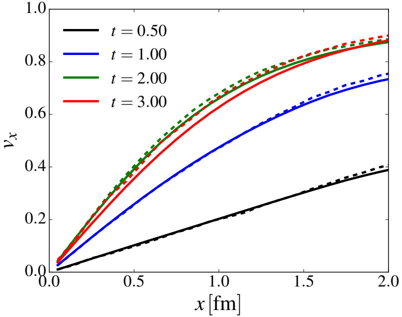
Test particles per cell:



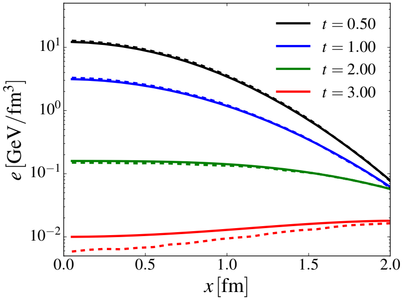


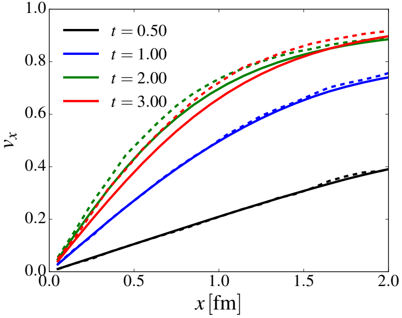
Gaussian profile $R = 1$ fm, $\sigma = 20$ mb



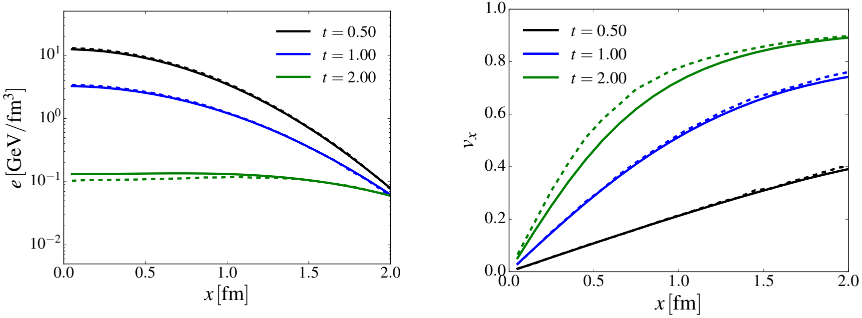


Gaussian profile $R = 1$ fm, $\sigma = 5$ mb



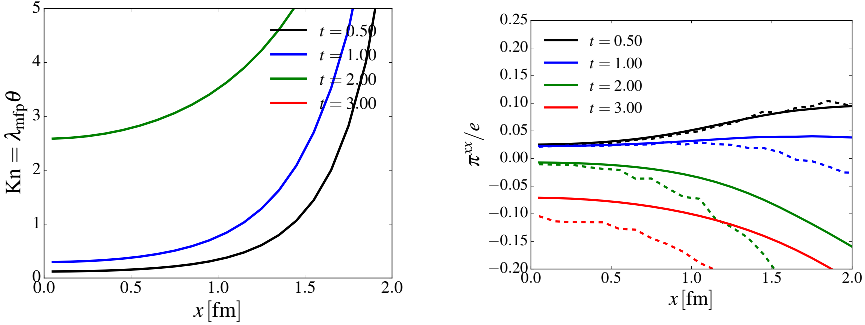


Gaussian profile $R = 1$ fm, $\sigma = 1$ mb



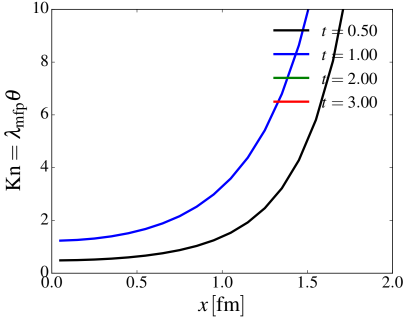
Gaussian profile $R = 1$ fm, $\sigma = 20$ mb

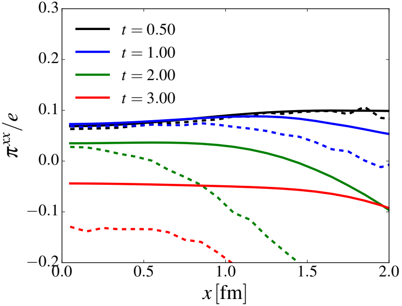
Knudsen number and π^{xx}/e, $\sigma = 20$ mb



Gaussian profile $R = 1$ fm, $\sigma = 5$ mb

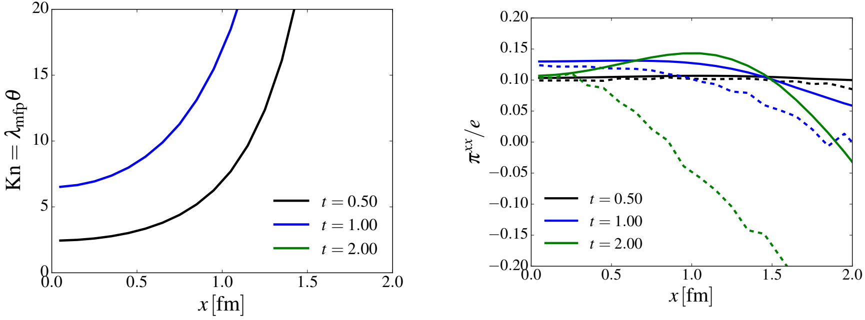
Knudsen number and π^{xx}/e, $\sigma = 5$ mb





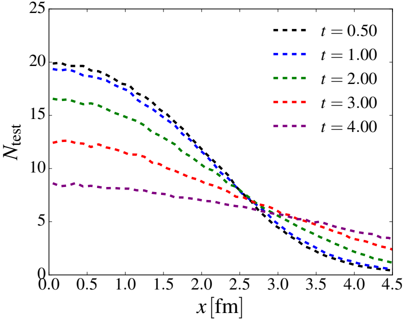
Gaussian profile $R = 1$ fm, $\sigma = 1$ mb

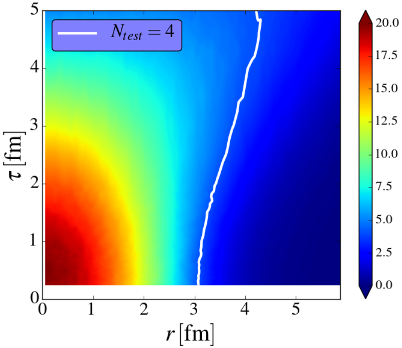
Knudsen number and π^{xx}/e, $\sigma = 1$ mb



Binary profile $b = 7.5$ fm

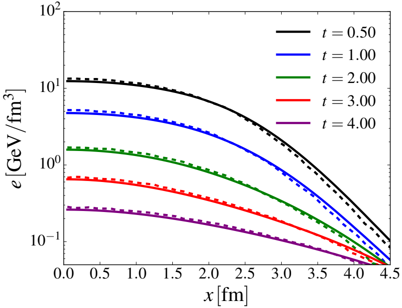
Test particles per cell:

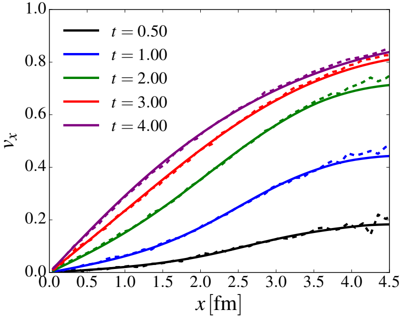




Binary profile $b = 7.5$ fm, $\sigma = 100$ mb

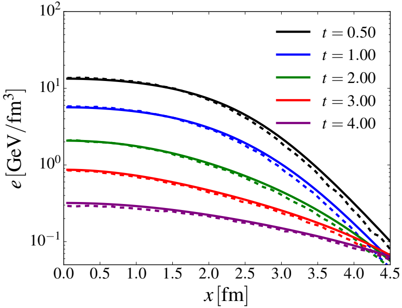
Energy density and velocity profiles, $\sigma = 100$ mb

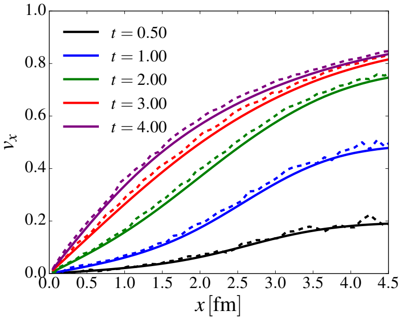




Binary profile $b = 7.5$ fm, $\sigma = 5$ mb

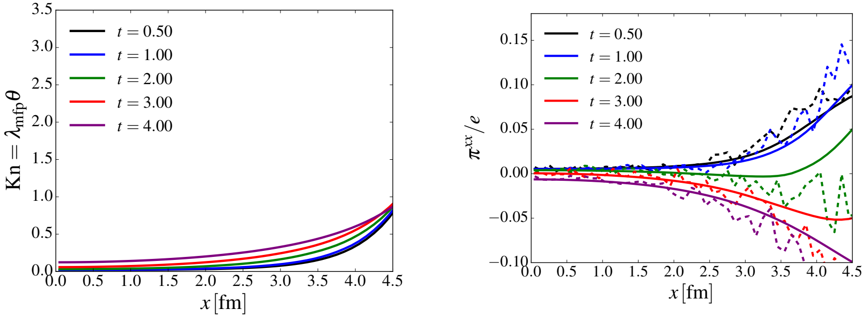
Energy density and velocity profiles, $\sigma = 5$ mb





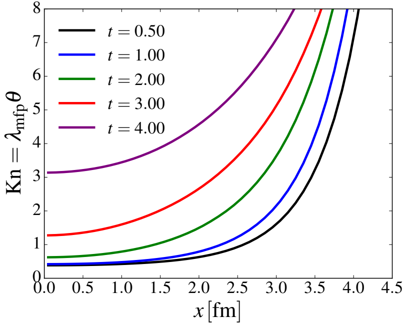
Binary profile $b = 7.5$ fm, $\sigma = 100$ mb

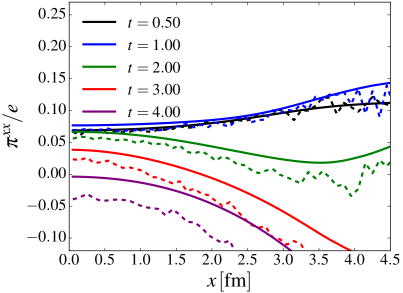
Knudsen number and π^{xx}/e, $\sigma = 100$ mb



Binary profile $b = 7.5$ fm, $\sigma = 5$ mb

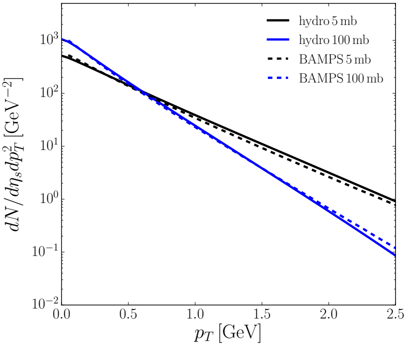
Knudsen number and π^{xx}/e, $\sigma = 5$ mb

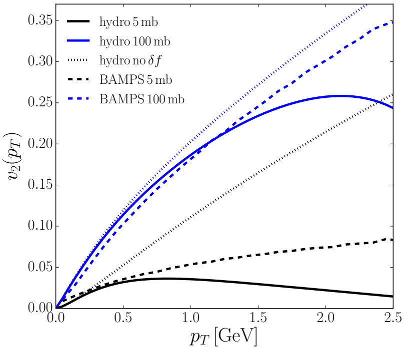




Binary profile $b = 7.5$ fm, $\mathrm{Kn}_{fr} = 2$

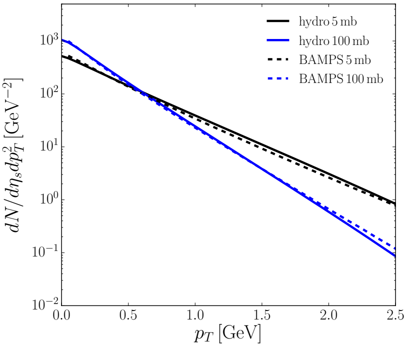
- Cooper-Frye freeze-out
- Decoupling condition: $\mathrm{Kn} = 2$ and $N_{\mathrm{test}} = 4$

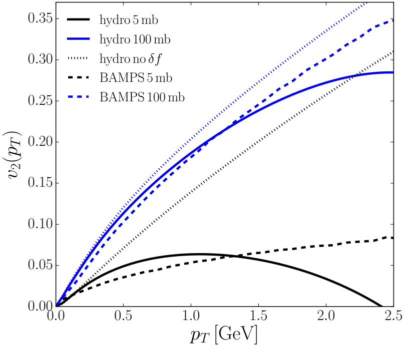




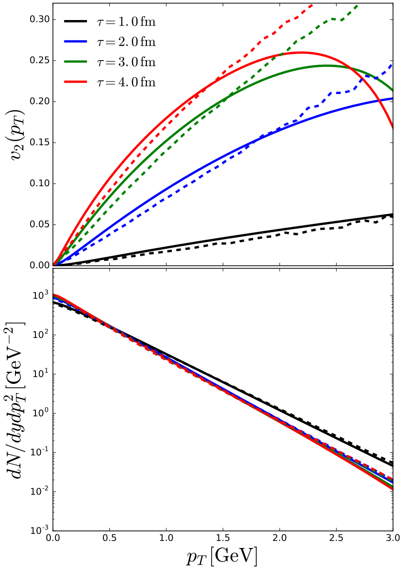
Binary profile $b = 7.5$ fm, $\mathrm{Kn}_{fr} = 4$

- Decoupling condition: $\mathrm{Kn} = 4$ and $N_{\mathrm{test}} = 4$
- 100 mb: Knudsen decoupling doesn't matter: $N_{\mathrm{test}} = 4$ happens first
- 5mb: Not possible to get low p_T $v_2(p_T)$ and p_T-integrated v_2 simultaneously



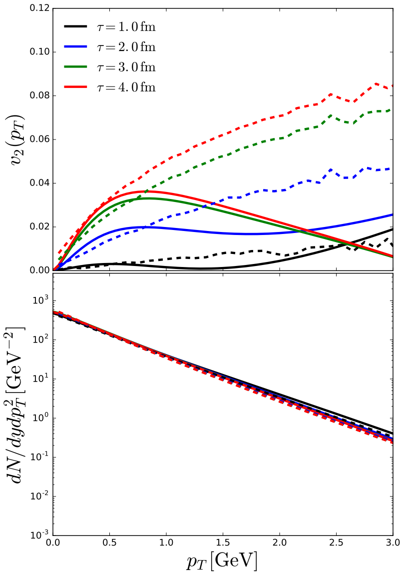


Binary profile $b = 7.5$ fm, $\mathrm{Kn}_{fr} = 2$, $\sigma = 100$ mb



- $\sigma = 100$ mb
- Decoupling at constant times + $\mathrm{Kn} = 2$ at the edge of the fireball
- Quite similar agreement at all times
- BAMPS: constant t decoupling, HYDRO: contant proper time τ (these are not exactly the same)

Binary profile $b = 7.5$ fm, $\mathrm{Kn}_{fr} = 2$, $\sigma = 5$ mb



- $\sigma = 5$ mb
- Decoupling at constant times + $\mathrm{Kn} = 2$ at the edge of the fireball
- Similar (dis)-agreement at all times. Even at early times where $\mathrm{Kn} \sim 0.5$.

Summary

- BAMPS + hydro: framework to test the validity of fluid dynamics in different situations
- spacetime evolution: good agreement up to $\mathrm{Kn} \sim 1-2$
- Elliptic flow: $\mathrm{Kn} \sim 0.5$ too large?