Relativistic dissipative hydrodynamics from kinetic theory in the relaxation-time approximation

Amaresh Jaiswal

GSI Darmstadt, Germany

MITP Workshop on
Relativistic Hydrodynamics: Theory and Modern Applications

October 11, 2016

Introduction

- Hydrodynamics describes the long-wavelength, low-frequency limit of the microscopic dynamics of a system.
- Relativistic hydrodynamics has been used to study high-energy heavy-ion collisions with considerable success.
- As all fluids are non-ideal in nature, dissipation must be included in the formulation of hydrodynamic equations.
- Relativistic generalization of the Navier-Stokes theory (first-order in gradients) shows acausal behavior.
- Israel-Stewart theory (second-order corrections) restores causality.
- However there are several ways in which one can derive a second-order theory of relativistic dissipative hydrodynamics.
- Here I discuss formulations based on kinetic theory.

Ideal and dissipative hydrodynamics

- Hydrodynamic equations are conservation of energy-momentum and particle current, i.e., $\partial_\mu T^{\mu\nu} = 0$ and $\partial_\mu N^\mu = 0$.

Ideal	**Dissipative**
$T^{\mu\nu} = \epsilon u^\mu u^\nu - P\Delta^{\mu\nu}$	$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$
$N^\mu = n u^\mu$	$N^\mu = n u^\mu + n^\mu$
Unknowns: $\underbrace{\epsilon,\ P,\ n,\ u^\mu}_{1+1+\ 1+\ 3} = 6$	$\underbrace{\epsilon,\ P,\ n,\ u^\mu,\ \Pi,\ \pi^{\mu\nu},\ n^\mu}_{1+1+\ 1\ +\ 3\ +\ 1\ +\ 5\ +\ 3} = 15$
Equations: $\underbrace{\partial_\mu T^{\mu\nu} = 0,\quad \partial_\mu N^\mu = 0,\quad EOS}_{4\quad +\quad 1\quad +\quad 1} \quad = 6$	
Closed set of equations	9 more equations required

- Here $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$.
- Landau frame chosen: $T^{\mu\nu} u_\nu = \epsilon u^\mu$.

Relaivistic kinetic theory

- Kinetic theory: calculation of macroscopic quantities by means of statistical description in terms of distribution function.
- For large no. of particles, one can introduce a function $f(x,p)$ which gives a distribution of particle momenta at each space-time point.
- In terms of the distribution function, the energy momentum tensor and particle four-current can be written as:

$$T^{\mu\nu}(x) = \frac{g}{(2\pi)^3}\int\frac{d^3p}{p^0}p^\mu\, p^\nu\, f(x,p)\ ; \quad N^\mu(x) = \frac{g}{(2\pi)^3}\int\frac{d^3p}{p^0}p^\mu\, f(x,p)$$

- For a system which is not in equilibrium, $f = f_0 + \delta f$.
- With suitable projections, the dissipative quantities can be written in terms of δf as:

$$\Pi = -\frac{1}{3}\Delta_{\alpha\beta}\int dP\, p^\alpha p^\beta\, \delta f, \quad \pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta}\int dp\, p^\alpha p^\beta\, \delta f, \quad n^\mu = \Delta^\mu_\alpha\int dp\, p^\alpha\, \delta f.$$

Dissipative evolution equations

- In order to preserve causality, it is necessary to have relaxation-type equations for dissipative quantities rather than constituent equations.

- The best prescription [Denicol, Koide, Rischke, PRL 2010] is to consider

$$\dot{\Pi} = -\frac{1}{3}\Delta_{\alpha\beta}\int dP\, p^\alpha p^\beta\, \dot{\delta f}, \quad \dot{\pi}^{\langle\mu\nu\rangle} = \Delta^{\mu\nu}_{\alpha\beta}\int dp\, p^\alpha p^\beta\, \dot{\delta f}, \quad \dot{n}^{\langle\mu\rangle} = \Delta^\mu_\alpha\int dp\, p^\alpha\, \dot{\delta f},$$

where $\dot{\Pi} = u^\mu\partial_\mu\Pi$, $\dot{\pi}^{\langle\mu\nu\rangle} = \Delta^{\mu\nu}_{\alpha\beta}\dot{\pi}^{\alpha\beta}$ and $\dot{n}^{\langle\mu\rangle} = \Delta^\mu_\alpha\dot{n}^\alpha$.

- To obtain $\dot{\delta f}$, one can use the Boltzmann equation in the relaxation time approximation:

$$p^\mu\partial_\mu f = -\frac{u\cdot p}{\tau_R}\delta f.$$

- Keeping in mind that $f = f_0 + \delta f$, and after some rearrangements,

$$\dot{\delta f} = -\dot{f}_0 - \frac{1}{u\cdot p}p^\gamma\nabla_\gamma f - \frac{\delta f}{\tau_R}.$$

Dissipative evolution equations contd.

- Substituting $\dot{\delta f}$ in expressions for $\dot{\Pi}$, $\dot{\pi}^{\langle\mu\nu\rangle}$ and $\dot{n}^{\langle\mu\rangle}$

$$\dot{\Pi} + \frac{\Pi}{\tau_R} = \frac{\Delta_{\alpha\beta}}{3} \int dp\, p^\alpha p^\beta \left[\dot{f}_0 + \frac{1}{u\cdot p} p^\gamma \nabla_\gamma \left(f_0 + \delta f\right)\right],$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_R} = -\Delta^{\mu\nu}_{\alpha\beta} \int dp\, p^\alpha p^\beta \left[\dot{f}_0 + \frac{1}{u\cdot p} p^\gamma \nabla_\gamma \left(f_0 + \delta f\right)\right],$$

$$\dot{n}^{\langle\mu\rangle} + \frac{n^\mu}{\tau_R} = -\Delta^\mu_\alpha \int dp\, p^\alpha \left[\dot{f}_0 + \frac{1}{u\cdot p} p^\gamma \nabla_\gamma \left(f_0 + \delta f\right)\right].$$

- The above equations are relaxation-type equations with a single relaxation time-scale τ_R.
- Now the form of δf needs to be specified.
    - Grad's 14-moment method.
    - Chapman-Enskog like iterative solution of the Boltzmann equation.

Grad's 14-moment method

- The equilibrium distribution functions can be written as

$$f_0 = [\exp\{y_0(x,p)\} + r]^{-1}, \quad y_0 = -\beta(u \cdot p) + \alpha, \quad r = 0, \pm 1$$

- Away from equilibrium, $f = [\exp\{y(x,p)\} + r]^{-1}$, where

$$\phi(x,p) \equiv y(x,p) - y_0(x,p) = \varepsilon(x) - \varepsilon_\mu(x) p^\mu + \varepsilon_{\mu\nu}(x) p^\mu p^\nu + \cdots$$

- Approximations: Taylor expansion around equilibrium up to linear in ϕ and truncated up to quadratic in p^μ

$$f = f_0 + \delta f, \quad \delta f = f_0 \tilde{f}_0 \phi, \quad \text{where, } \tilde{f}_0 = 1 - r f_0$$

- Assumption: ε, ε_μ and $\varepsilon_{\mu\nu}$ are linear in Π, n_μ and $\pi_{\mu\nu}$

$$\varepsilon = A_0 \Pi, \quad \varepsilon_\mu = A_1 \Pi u_\mu + B_0 n_\mu, \quad \varepsilon_{\mu\nu} = A_2 (3 u_\mu u_\nu - \Delta_{\mu\nu}) \Pi - B_1 u_{(\mu} n_{\nu)} + C_0 \pi_{\mu\nu}$$

- A_0, A_1, A_2, B_0, B_1 and C_0 are determined from the definitions of dissipative quantities, matching conditions and frame definition.

Iterative solution of the Boltzmann equation

- Botzmann equation in the relaxation-time approximation:

$$p^\mu\partial_\mu f = -\frac{u\cdot p}{\tau_R}(f-f_0) \;\Rightarrow\; f = f_0 - \frac{\tau_R}{u\cdot p}\,p^\mu\partial_\mu f$$

- Writing $f = f_0 + \delta f^{(1)} + \delta f^{(2)} + \cdots$ and solving iteratively,

$$\delta f^{(1)} = -\frac{\tau_R}{u\cdot p}\,p^\mu\partial_\mu f_0, \quad \delta f^{(2)} = \frac{\tau_R}{u\cdot p}p^\mu p^\nu\partial_\mu\left(\frac{\tau_R}{u\cdot p}\partial_\nu f_0\right), \quad \cdots$$

- Using $\delta f^{(1)}$ in the definition of dissipative quantities, one gets

$$\Pi = -\beta_\Pi\,\tau_R\,\theta, \quad \pi^{\mu\nu} = 2\beta_\pi\,\tau_R\,\sigma^{\mu\nu}, \quad n^\mu = \beta_n\,\tau_R\,\nabla^\mu\alpha.$$

- Using these first-order relations in $\delta f^{(1)}$,

$$\delta f_1 = \left(\lambda_\Pi\Pi + \lambda_n n_\alpha p^\alpha + \lambda_\pi\pi_{\alpha\beta}p^\alpha p^\beta\right) f_0\tilde{f}_0.$$

- Identical to small anisotropy expansion in aHydro; see talk by L. Tinti.

Second-order viscous corrections

- For a system of massless particles at vanishing chemical potential, $\delta f = \delta f^{(1)} + \delta f^{(2)}$ can be written as

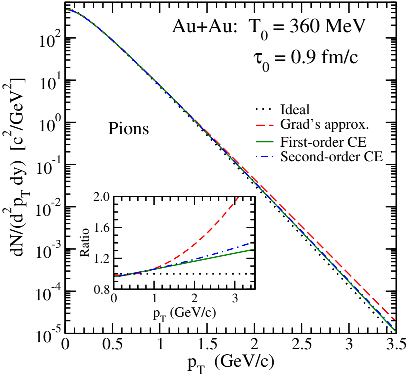
$$
\begin{aligned}
\delta f = & \frac{f_0\beta}{2\beta_\pi(u\cdot p)}\, p^\alpha p^\beta \pi_{\alpha\beta} - \frac{f_0\beta}{\beta_\pi}\Bigg[\frac{\tau_\pi}{u\cdot p}\, p^\alpha p^\beta \pi^\gamma_\alpha\, \omega_{\beta\gamma} - \frac{5}{14\beta_\pi(u\cdot p)}\, p^\alpha p^\beta \pi^\gamma_\alpha\, \pi_{\beta\gamma} \\
& + \frac{\tau_\pi}{3(u\cdot p)}\, p^\alpha p^\beta \pi_{\alpha\beta}\theta - \frac{6\tau_\pi}{5}\, p^\alpha \dot{u}^\beta \pi_{\alpha\beta} + \frac{(u\cdot p)}{70\beta_\pi}\, \pi^{\alpha\beta}\pi_{\alpha\beta} + \frac{\tau_\pi}{5}\, p^\alpha \left(\nabla^\beta \pi_{\alpha\beta}\right) \\
& - \frac{3\tau_\pi}{(u\cdot p)^2}\, p^\alpha p^\beta p^\gamma \pi_{\alpha\beta}\dot{u}_\gamma + \frac{\tau_\pi}{2(u\cdot p)^2}\, p^\alpha p^\beta p^\gamma \left(\nabla_\gamma \pi_{\alpha\beta}\right) \\
& - \frac{\beta + (u\cdot p)^{-1}}{4(u\cdot p)^2\beta_\pi}\, p^\alpha p^\beta p^\gamma p^\delta \pi_{\alpha\beta}\pi_{\gamma\delta}\Bigg] + \mathcal{O}(\delta^3).
\end{aligned}
$$

[R. S. Bhalerao, AJ, S. Pal, and V. Sreekanth, PRC 89, 054903 (2014)]

- The first-order correction can be compared to that due to Grad's 14-moment approximation

$$
\delta f_{CE} = \frac{5 f_0}{2(u\cdot p)(\epsilon + P)T}\, p^\alpha p^\beta \pi_{\alpha\beta}, \qquad \delta f_G = \frac{f_0}{2(\epsilon + P)T^2}\, p^\alpha p^\beta \pi_{\alpha\beta}
$$

Effect of viscous corrections on observables



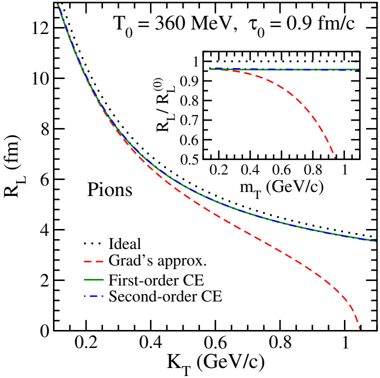


Figure: Effect of viscous corrections on pion spectra and longitudinal HBT radii.

[R. S. Bhalerao, AJ, S. Pal, and V. Sreekanth, PRC 89, 054903 (2014)]

- Grad's approximation for δf violate $1/\sqrt{m_T}$ scaling of the longitudinal HBT radii.

Shear evolution for low density fluids of massless particles

- For $\mu_b = m = 0$,

$$\cancel{n^\mu = \Delta^\mu_\alpha \int dp\, p^\alpha\, \delta f}\,, \quad \cancel{\Pi = -\frac{1}{3}\Delta_{\alpha\beta}\int dP\, p^\alpha p^\beta\, \delta f}\,, \quad \pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta}\int dp\, p^\alpha p^\beta\, \delta f\,.$$

- Using 14-moment approximation [G. S. Denicol, T. Koide and D. H. Rischke, PRL 105, 162501 (2010)],

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi\sigma^{\mu\nu} - \frac{4}{3}\pi^{\mu\nu}\theta + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \frac{10}{7}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma}, \qquad \beta_\pi = \frac{4P}{5}.$$

Shear evolution for low density fluids of massless particles

- For $\mu_b = m = 0$,

$$\cancel{n^\mu = \Delta^\mu_\alpha \int dp\, p^\alpha\, \delta f}\,, \quad \cancel{\Pi = -\frac{1}{3}\Delta_{\alpha\beta}\int dP\, p^\alpha p^\beta\, \delta f}\,, \quad \pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta}\int dp\, p^\alpha p^\beta\, \delta f\,.$$

- Using 14-moment approximation [G. S. Denicol, T. Koide and D. H. Rischke, PRL 105, 162501 (2010)],

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi\sigma^{\mu\nu} - \frac{4}{3}\pi^{\mu\nu}\theta + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \frac{10}{7}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma}, \qquad \beta_\pi = \frac{4P}{5}.$$

- Using Champan-Enskog like iterative solution of the Boltzmann equation [AJ, PRC 87, 051901 (2013)],

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi\sigma^{\mu\nu} - \frac{4}{3}\pi^{\mu\nu}\theta + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \frac{10}{7}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma}, \qquad \beta_\pi = \frac{4P}{5}.$$

- Identical in the conformal case, not in general!

Shear evolution for low density fluids of massless particles

- For $\mu_b = m = 0$,

$$\cancel{n^\mu = \Delta^\mu_\alpha \int dp\, p^\alpha\, \delta f}\,, \quad \cancel{\Pi = -\frac{1}{3}\Delta_{\alpha\beta}\int dP\, p^\alpha p^\beta\, \delta f}\,, \quad \pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta}\int dp\, p^\alpha p^\beta\, \delta f\,.$$

- Using 14-moment approximation [G. S. Denicol, T. Koide and D. H. Rischke, PRL 105, 162501 (2010)],

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi\sigma^{\mu\nu} - \frac{4}{3}\pi^{\mu\nu}\theta + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \frac{10}{7}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma}, \qquad \beta_\pi = \frac{4P}{5}.$$

- Using Champan-Enskog like iterative solution of the Boltzmann equation [AJ, PRC 87, 051901 (2013)],

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi\sigma^{\mu\nu} - \frac{4}{3}\pi^{\mu\nu}\theta + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \frac{10}{7}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma}, \qquad \beta_\pi = \frac{4P}{5}.$$

- Identical in the conformal case, not in general!

Third-order hydrodynamics

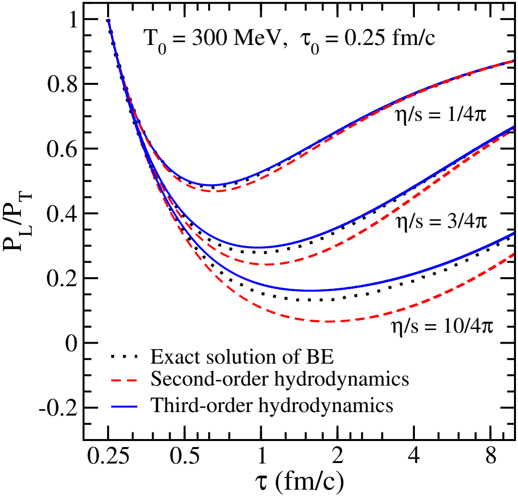
- Third-order equation for shear stress tensor [AJ, PRC 88, 021903 (2013)]:

$$
\begin{aligned}
\dot{\pi}^{\langle\mu\nu\rangle} = & -\frac{\pi^{\mu\nu}}{\tau_\pi} + 2\beta_\pi\sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \frac{10}{7}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma} - \frac{4}{3}\pi^{\mu\nu}\theta - \frac{10}{63}\pi^{\mu\nu}\theta^2 \\
& + \tau_\pi\left[\frac{50}{7}\pi^{\rho\langle\mu}\omega^{\nu\rangle\gamma}\sigma_{\rho\gamma} - \frac{76}{245}\pi^{\mu\nu}\sigma^{\rho\gamma}\sigma_{\rho\gamma} - \frac{44}{49}\pi^{\rho\langle\mu}\sigma^{\nu\rangle\gamma}\sigma_{\rho\gamma}\right. \\
& \left. - \frac{2}{7}\pi^{\rho\langle\mu}\omega^{\nu\rangle\gamma}\omega_{\rho\gamma} - \frac{2}{7}\omega^{\rho\langle\mu}\omega^{\nu\rangle\gamma}\pi_{\rho\gamma} + \frac{26}{21}\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma}\theta - \frac{2}{3}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma}\theta\right] \\
& - \frac{24}{35}\nabla^{\langle\mu}\left(\pi^{\nu\rangle\gamma}\dot{u}_\gamma\tau_\pi\right) + \frac{6}{7}\nabla_\gamma\left(\tau_\pi\dot{u}^\gamma\pi^{\langle\mu\nu\rangle}\right) + \frac{4}{35}\nabla^{\langle\mu}\left(\tau_\pi\nabla_\gamma\pi^{\nu\rangle\gamma}\right) \\
& - \frac{2}{7}\nabla_\gamma\left(\tau_\pi\nabla^{\langle\mu}\pi^{\nu\rangle\gamma}\right) - \frac{1}{7}\nabla_\gamma\left(\tau_\pi\nabla^\gamma\pi^{\langle\mu\nu\rangle}\right) + \frac{12}{7}\nabla_\gamma\left(\tau_\pi\dot{u}^{\langle\mu}\pi^{\nu\rangle\gamma}\right).
\end{aligned}
$$

- 14 new transport coefficients obtained; 15 predicted from conformal analysis [S. Grozdanov and N. Kaplis, PRD 93, 066012 (2016)].

- Misses $\omega^{\rho\langle\mu}\omega^{\nu\rangle\gamma}\omega_{\rho\gamma}$ similar to $\omega^{\rho\langle\mu}\omega^{\nu\rangle}{}_\rho$ at second-order.

One dimensional Bjorken evolution of pressure anisotropy



[AJ, PRC 88, 021903 (2013)]

Exact solution of the BE:

[W. Florkowski, R. Ryblewski and M. Strickland, PRC 88, 024903 (2013); NPA 916, 249 (2013); W. Florkowski, E. Maksymiuk, R. Ryblewski and M. Strickland, PRC 89,054908 (2014); W. Florkowski and E. Maksymiuk, JPG 42, 045106 (2015)]

Low density fluids of massive particles

- Massive particles $m \neq 0$ and low net Baryon number density $\mu_b = 0$

$$\cancel{n^\mu = \Delta^\mu_\alpha \int dp\, p^\alpha\, \delta f}\,, \quad \Pi = -\frac{1}{3}\Delta_{\alpha\beta}\int dP\, p^\alpha p^\beta\, \delta f\,, \quad \pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta}\int dp\, p^\alpha p^\beta\, \delta f\,.$$

- Second-order evolution equations are obtained as,

$$\dot{\Pi} = -\frac{\Pi}{\tau_\Pi} - \beta_\Pi\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu},$$

$$\dot{\pi}^{\langle\mu\nu\rangle} = -\frac{\pi^{\mu\nu}}{\tau_\pi} + 2\beta_\pi\sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \tau_{\pi\pi}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}.$$

- In relaxation-time approximation, $\tau_\Pi = \tau_\pi = \tau_R \;\Rightarrow\; \zeta/\eta = \beta_\Pi/\beta_\pi$.

- For $m/T \ll 1$,

$$\frac{\zeta}{\eta} = \Lambda\left(\frac{1}{3} - c_s^2\right)^2, \qquad \Lambda = \begin{cases} 75 & \text{for MB} \\ 48 & \text{for FD} \\ \infty & \text{for BE} \\ 15 & \text{Weinberg} \end{cases}$$

[AJ, R. Ryblewski, M. Strickland, PRC 90, 044908 (2014); W. Florkowski, AJ, E. Maksymiuk, R. Ryblewski, M. Strickland, PRC 91, 054907 (2015)]

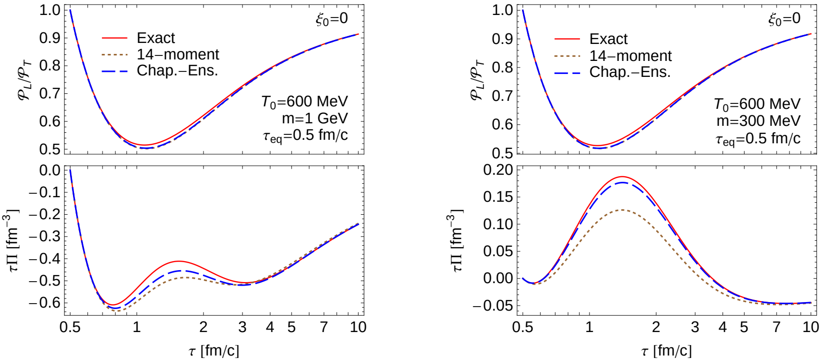


Figure: [AJ, R. Ryblewski, M. Strickland, PRC 90, 044908 (2014); W. Florkowski, AJ, E. Maksymiuk, R. Ryblewski, M. Strickland, PRC 91, 054907 (2015)].

- Chapman-Enskog method performs better than moment method.
- Results valid for all distributions.

High density fluids of massless particles

- Massless particles $m = 0$ and net Baryon number density $\mu_b \neq 0$,

$$n^\mu = \Delta^\mu_\alpha \int dp\, p^\alpha\, \delta f\,, \quad \cancel{\Pi = -\frac{1}{3}\Delta_{\alpha\beta}\int dP\, p^\alpha p^\beta\, \delta f}\,, \quad \pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta}\int dp\, p^\alpha p^\beta\, \delta f\,.$$

- Second-order evolution equations are obtained as,

$$\dot{n}^{\langle\mu\rangle} + \frac{n^\mu}{\tau_n} = \beta_n \nabla^\mu \alpha - n_\nu \omega^{\nu\mu} - n^\mu \theta - \frac{9}{5} n_\nu \sigma^{\nu\mu},$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \frac{4}{3}\pi^{\mu\nu}\theta - \frac{10}{7}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma}.$$

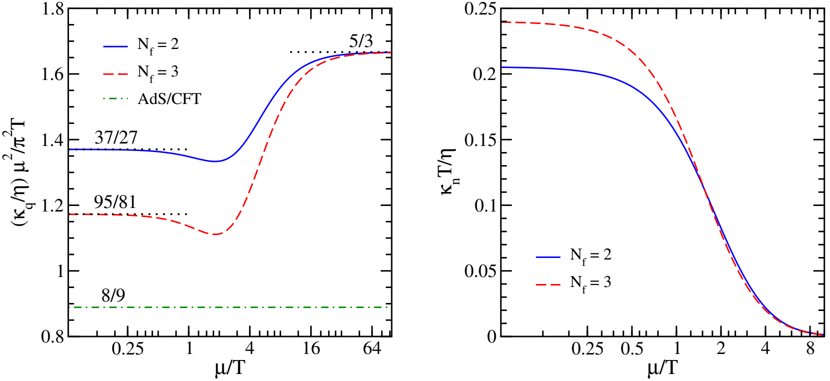
- Charge: $\kappa_n/\eta = \beta_n/\beta_\pi$; heat: $\kappa_q/\eta = (\beta_n/\beta_\pi)[(\epsilon + P)/nT]^2$.

- Wiedemann-Franz law [AJ, B. Friman, K. Redlich, PLB 751, 548 (2015)]:

$$\frac{\kappa_q}{\eta} = C\frac{\pi^2 T}{\mu^2}, \quad C = \begin{cases} 37/27 & \text{for 2 flavor QGP}, \ \mu/T \ll 1 \\ 95/81 & \text{for 3 flavor QGP}, \ \mu/T \ll 1 \\ 5/3 & \text{for } \mu/T \gg 1 \\ 8/9 & \text{AdS/CFT [Son \& Starinets, JHEP 0603, 052 (2006)]} \end{cases}$$

(μ: quark chemical potential)

Heat and charge conductivity



[AJ, B. Friman, K. Redlich, PLB 751, 548 (2015)].

- Intriguing similarity with AdS/CFT results for heat conductivity.
- At high densities, charge conductivity of QGP is small compared to shear viscosity.

Summary

- One can derive causal hydrodynamics from Boltzmann equation without resorting to moment method for δf.
- The simplest case with relaxation-time approximation is presented here.
- It is consistent with aHydro in the limit of small anisotropy [L. Tinti, PRC 94, 044902 (2016)].
- The method presented here seems to work better than 14-moment approximation.
- Interesting features for the ratio of transport coefficients observed within relaxation-time approximation.
- RG method can be used to solve the Boltzmann equation with $2 \leftrightarrow 2$ collision kernel and derive second-order hydro equations [K. Tsumura, Y. Kikuchi, T. Kunihiro, PRD 92, 085048 (2015); PRC 92, 064909 (2015); 1604.07458].

Thank you for your attention!

Backup slide 1: Non-local Collision term

- Collision term generalised to include non-local effects by including gradients of $f(x,p)$

$$C[f]_{\text{gen}} = C[f] + \partial_\mu \left(A^\mu f\right) + \partial_\mu \partial_\nu \left(B^{\mu\nu} f\right)$$

Where A^μ and $B^{\mu\nu}$ are tensor coefficients of the non-local terms.

- This form of collision term explicitly derived for $2 \leftrightarrow 2$ elastic collision:

$$C[f] = \frac{1}{2} \int dp' dk \; dk' \; W_{pp' \to kk'} \left(f_k f_{k'} \tilde{f}_p \tilde{f}_{p'} - f_p f_{p'} \tilde{f}_k \tilde{f}_{k'} \right)$$

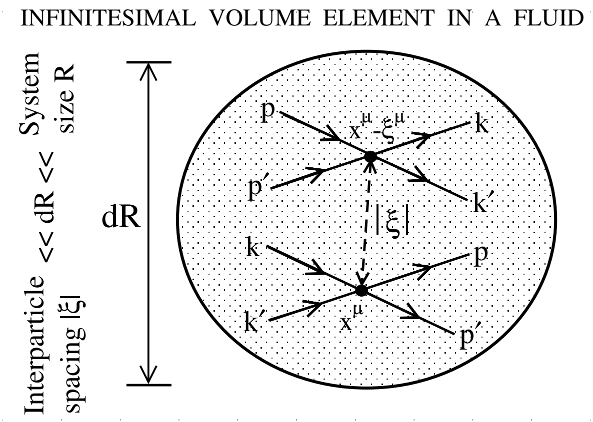
Where, $W_{pp' \to kk'}$ is the transition matrix element and $f_k = f(k,x)$.

- Probability of the process $(kk' \to pp') \propto f_k f_{k'} \tilde{f}_p \tilde{f}_{p'} \leftarrow$ occurs at x

Probability of the process $(pp' \to kk') \propto f_p f_{p'} \tilde{f}_k \tilde{f}_{k'} \leftarrow$ also occurs at x

[**AJ**, R. S. Bhalerao and S. Pal, PLB 720, 347 (2013)]

Backup slide 2: Non-local effects



Assumption that the two proccesses $(kk' \rightarrow pp')$ and $(pp' \rightarrow kk')$ occur at the same space-time point relaxed to include a seperation ξ.

[**AJ**, R. S. Bhalerao and S. Pal, PLB 720, 347 (2013)]

Backup slide 3: Dissipative equations with non-locality

- Final evolution equations for the dissipative fluxes:

$$\Pi = \tilde{a}\Pi_{\rm NS} - \beta_{\dot{\Pi}}\tau_{\Pi}\dot{\Pi} + \tau_{\Pi n} n\cdot\dot{u} - l_{\Pi n}\partial\cdot n - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi n} n\cdot\nabla\alpha$$
$$+ \lambda_{\Pi\pi}\pi_{\mu\nu}\sigma^{\mu\nu} + \Lambda_{\Pi\dot{u}}\dot{u}\cdot\dot{u} + \Lambda_{\Pi\omega}\omega_{\mu\nu}\omega^{\nu\mu} + (8\ \text{terms}),$$

$$n^{\mu} = \tilde{a}n^{\mu}_{\rm NS} - \beta_{\dot{n}}\tau_{n}\dot{n}^{\langle\mu\rangle} + \lambda_{nn}n_{\nu}\omega^{\nu\mu} - \delta_{nn}n^{\mu}\theta + l_{n\Pi}\nabla^{\mu}\Pi - l_{n\pi}\Delta^{\mu\nu}\partial_{\gamma}\pi^{\gamma}_{\nu}$$
$$- \tau_{n\Pi}\Pi\dot{u}^{\mu} - \tau_{n\pi}\pi^{\mu\nu}\dot{u}_{\nu} + \lambda_{n\pi}n_{\nu}\pi^{\mu\nu} + \lambda_{n\Pi}\Pi n^{\mu} + \Lambda_{n\dot{u}}\omega^{\mu\nu}\dot{u}_{\nu}$$
$$+ \Lambda_{n\omega}\Delta^{\mu}_{\nu}\partial_{\gamma}\omega^{\gamma\nu} + (9\ \text{terms}),$$

$$\pi^{\mu\nu} = \tilde{a}\pi^{\mu\nu}_{\rm NS} - \beta_{\dot{\pi}}\tau_{\pi}\dot{\pi}^{\langle\mu\nu\rangle} + \tau_{\pi n}n^{\langle\mu}\dot{u}^{\nu\rangle} + l_{\pi n}\nabla^{\langle\mu}n^{\nu\rangle} + \lambda_{\pi\pi}\pi^{\langle\mu}_{\rho}\omega^{\nu\rangle\rho}$$
$$- \lambda_{\pi n}n^{\langle\mu}\nabla^{\nu\rangle}\alpha - \tau_{\pi\pi}\pi^{\langle\mu}_{\rho}\sigma^{\nu\rangle\rho} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \Lambda_{\pi\dot{u}}\dot{u}^{\langle\mu}\dot{u}^{\nu\rangle}$$
$$+ \Lambda_{\pi\omega}\omega^{\langle\mu}_{\rho}\omega^{\nu\rangle\rho} + \chi_1\dot{b}_2\pi^{\mu\nu} + \chi_2\dot{u}^{\langle\mu}\nabla^{\nu\rangle}b_2 + \chi_3\nabla^{\langle\mu}\nabla^{\nu\rangle}b_2.$$

- Where $\tilde{a} = (1-a)$, $\dot{X} = u^{\mu}\partial_{\mu}X$ and "8 terms" ("9 terms") involve second-order, scalar (vector) combinations of derivatives of b_1, b_2.