

# Initial state with shear and vorticity in streak by streak Bjorken coordinates

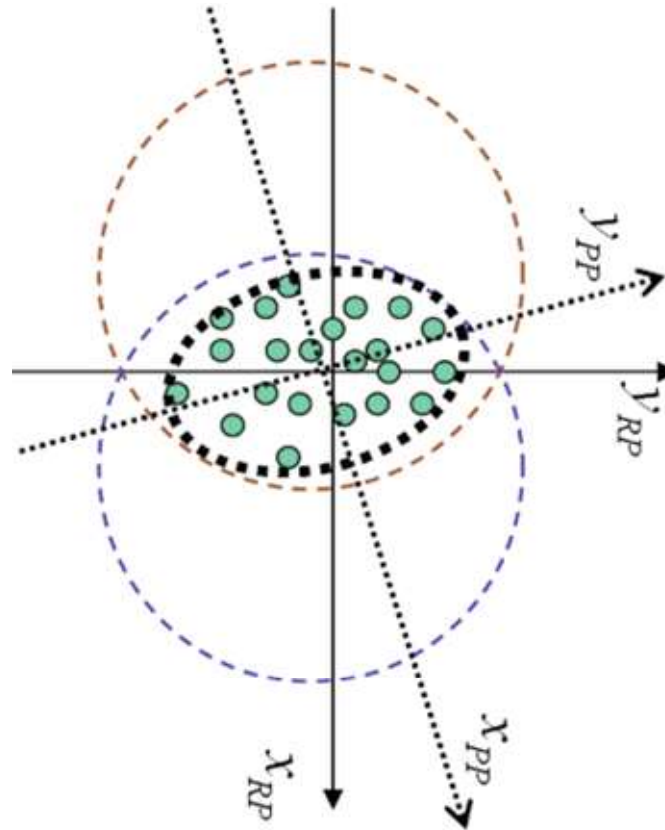
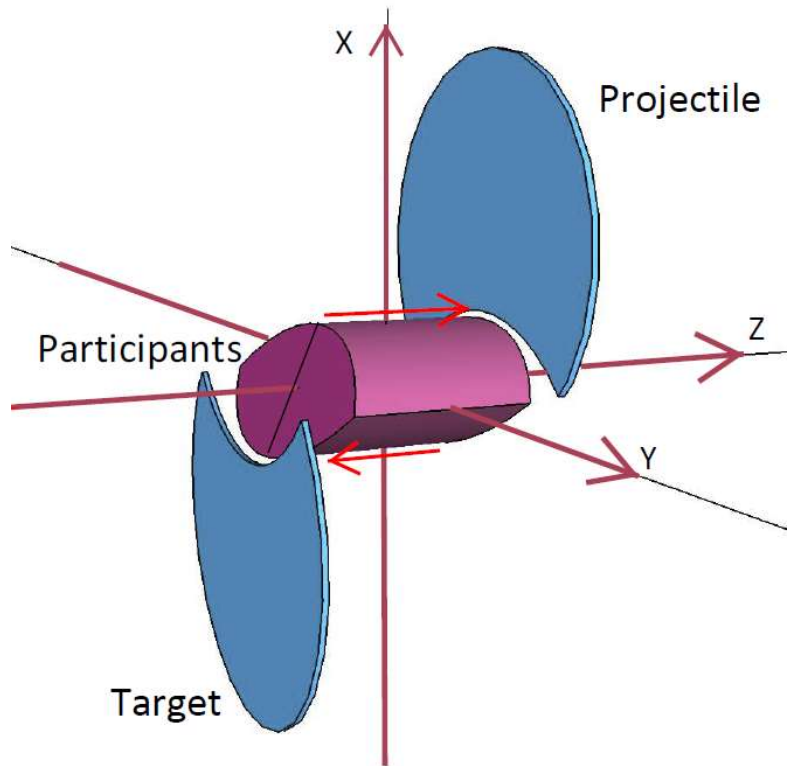
**mitp**  
Mainz Institute for  
Theoretical Physics

L. P. Csernai,  
V. K. Magas,  
D. Strottman,  
S. Velle,  
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Uo Bergen

Relativistic  
Hydrodynamics: Theory and  
Modern Applications, 10-14 October 2016, Mainz Institute  
for Theoretical Physics, J. Gutenberg University, Mainz, Germany

Recent experimental and theoretical developments indicate that angular momentum, local vorticity exist and the arising particle polarization is observable. Parton kinetic models show less longitudinal extent than earlier assumed. An initial state construction is presented, in streak by streak,  $\tau$ ,  $x$ ,  $y$ ,  $\eta$  coordinates, which reflects these features. This might be applicable for some theoretical models using these coordinates, although numerical fluid dynamical models with changing anisotropy and cell-size may be problematic. The expected consequences of this initial state are presented on the basis of recent calculations with the PICR code, which used initial states with shear and vorticity using  $\tau$ ,  $x$ ,  $y$ ,  $\eta$  coordinates.

# Peripheral Collisions (A+A) → $v_2$ flow



$$\frac{d^3N}{dydp_t d\phi} = \frac{1}{2\pi} \frac{d^2N}{dydp_t} [1 + 2v_1(y, p_t) \cos(\phi) + \underline{2v_2(y, p_t) \cos(2\phi)} + \dots]$$

# Fluctuations and polarization, CMB

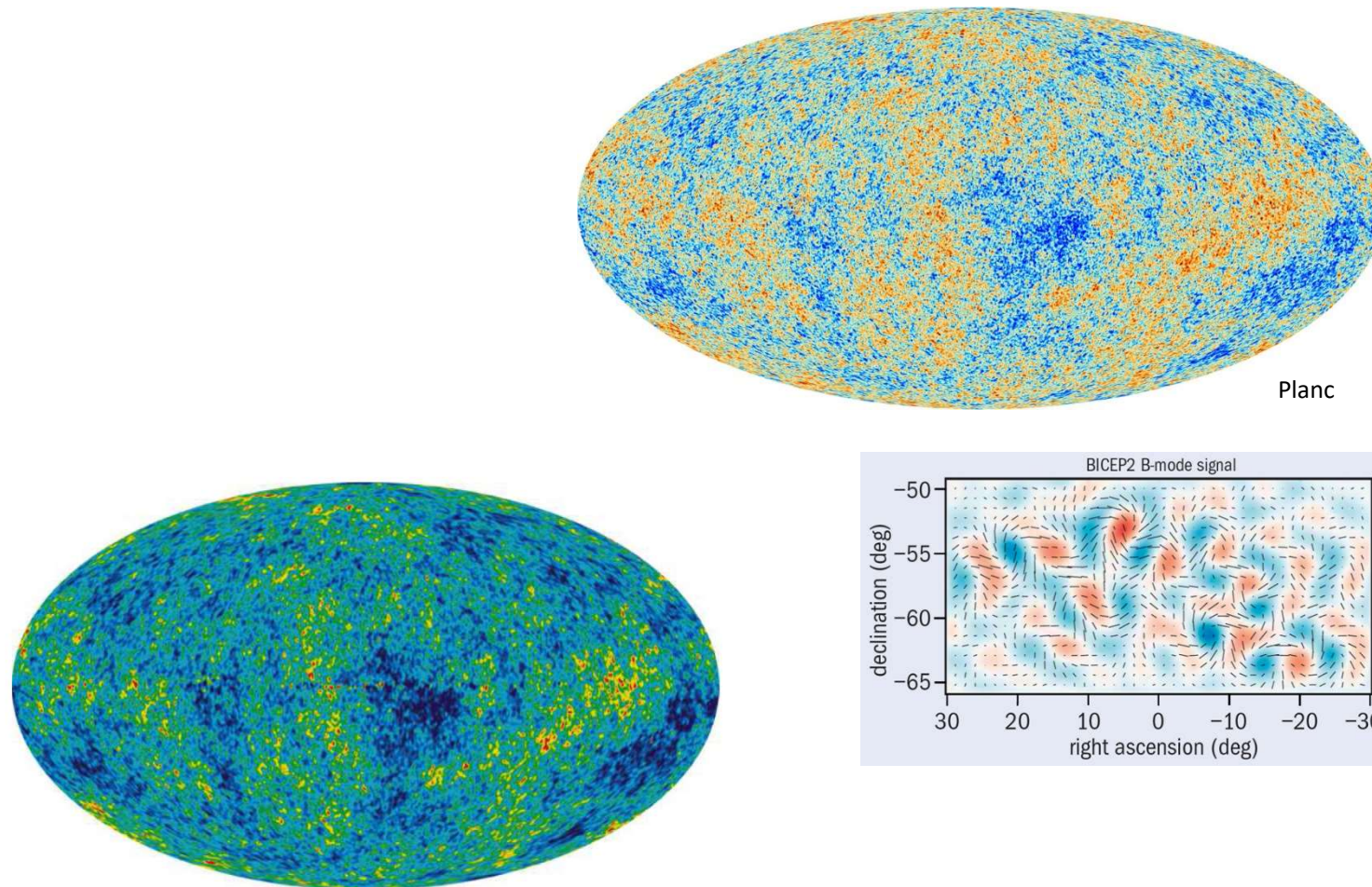
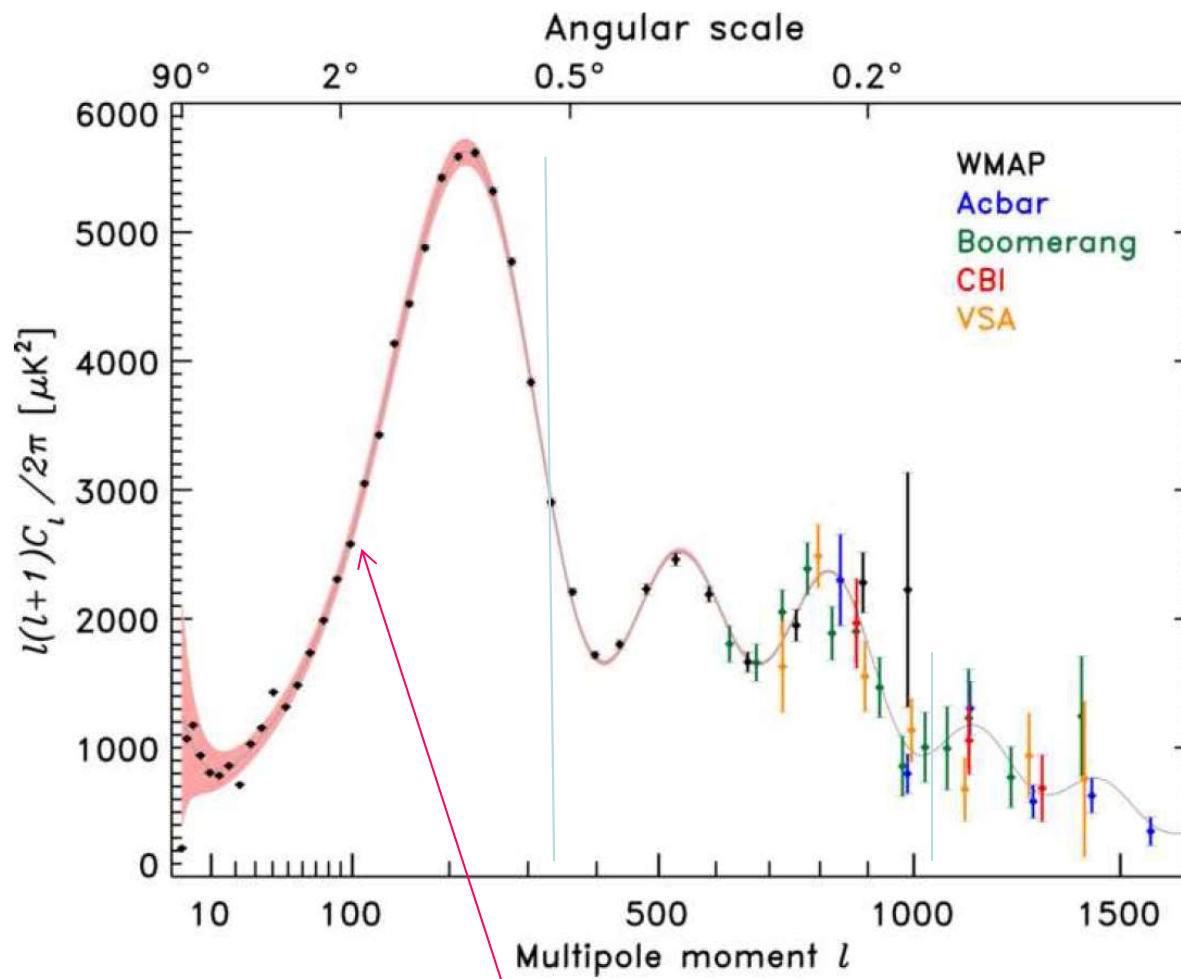


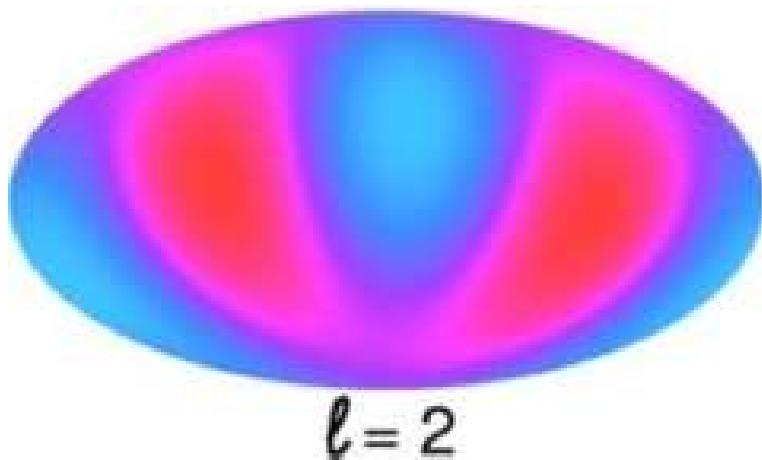
Figure 32: The CMB radiation temperature fluctuations from the 5-year WMAP data seen over the full sky. The average temperature is 2.725K, and the colors represent small temperature fluctuations. Red regions are warmer, and blue colder by about 0.0002 K.



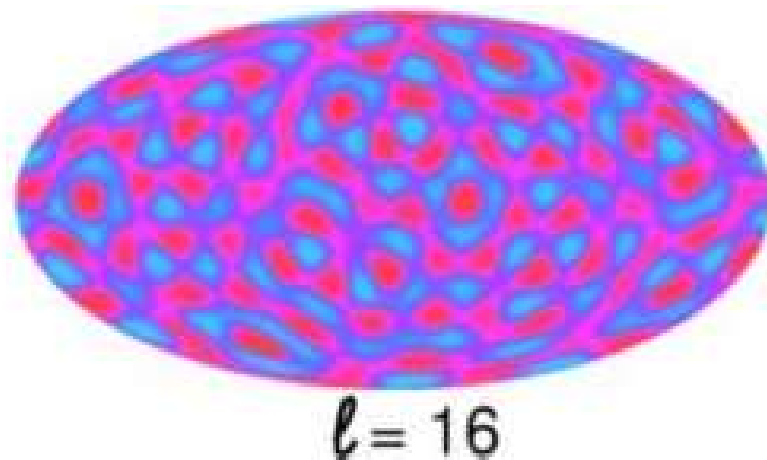
Longer tail on the negative ( low  $l$  ) side !

# In Central Heavy Ion Collisions

~ like Elliptic flow,  $v_2$

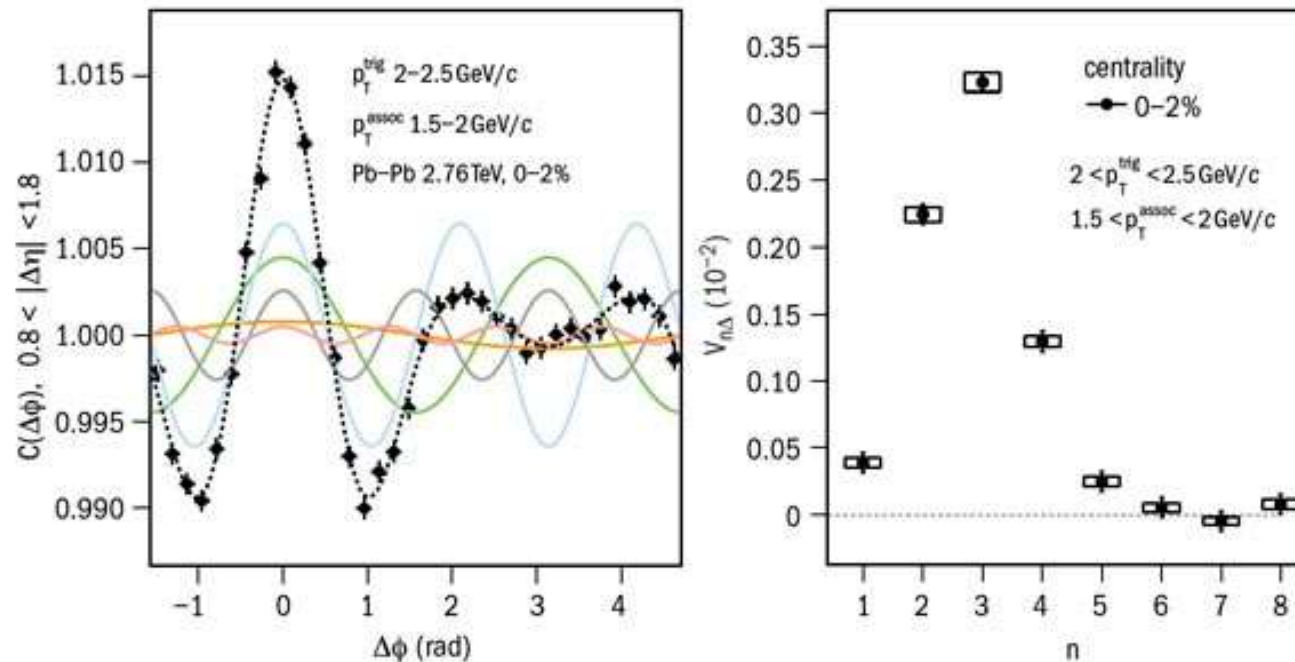


~ spherical with many (16) nearly equal perturbations



Sep 23, 2011

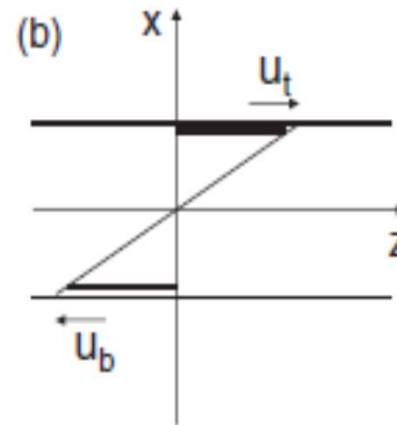
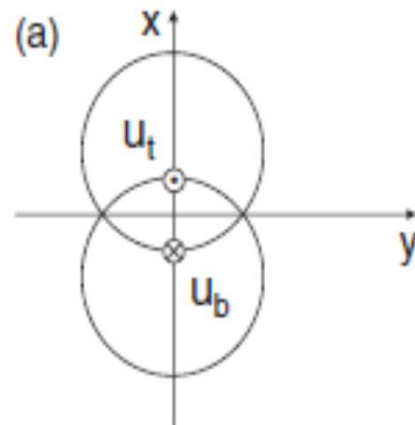
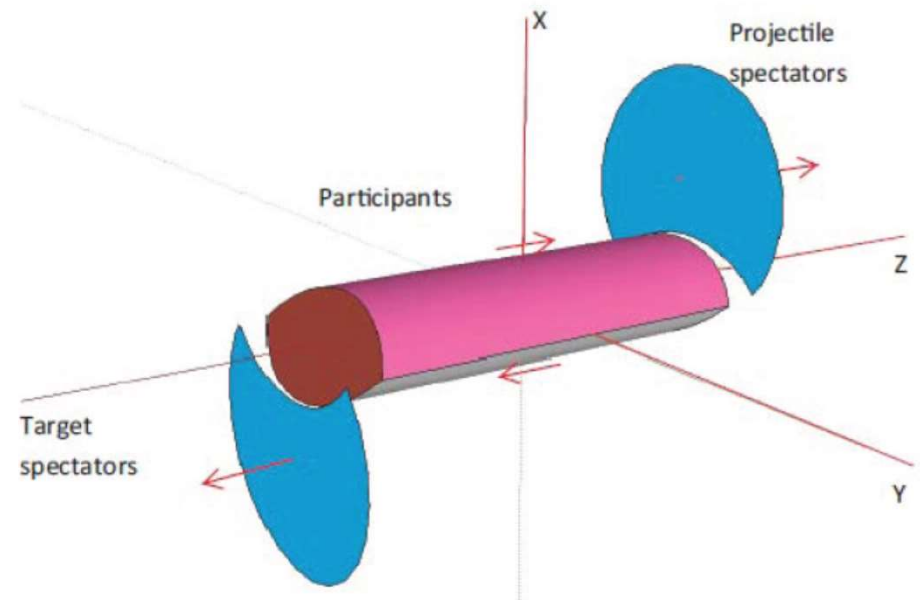
## ALICE measures the shape of head-on lead-lead collisions



Flow originating from initial state fluctuations is significant and dominant in central and semi-central collisions (where from global symmetry no azimuthal asymmetry could occur, all Collective  $v_n = 0$ ) !

# Peripheral Collisions - Initial State

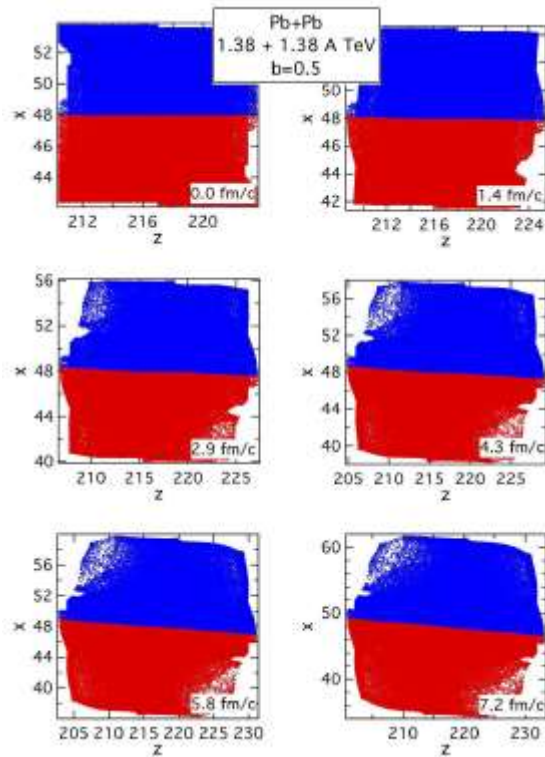
- Peripheral reactions
- $\Xi$  Shear & vorticity
- $L$ : in  $-y$  direction



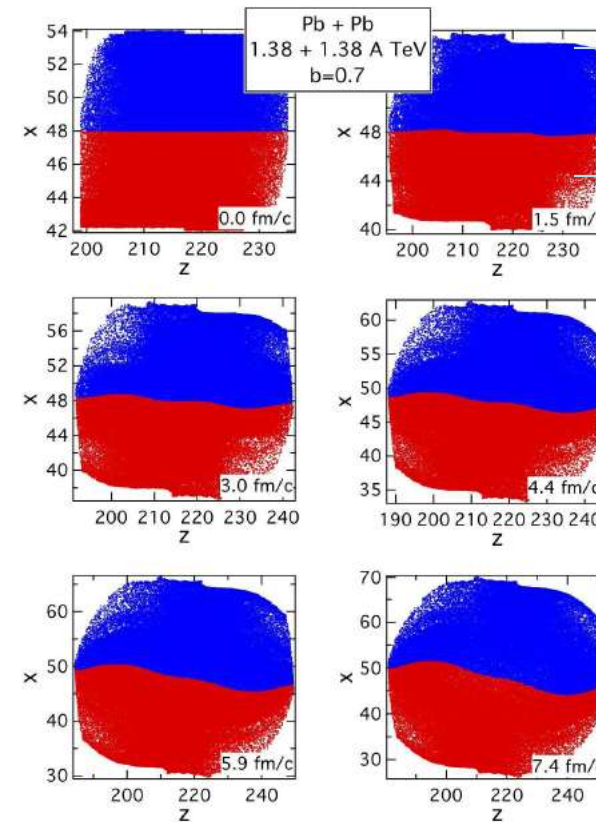
# Shear & Turbulence → KHI

L.P. Csernai<sup>1,2,3</sup>, D.D. Strottman<sup>2,3</sup>, and Cs. Anderlik<sup>4</sup>  
 PHYSICAL REVIEW C **85**, 054901 (2012)

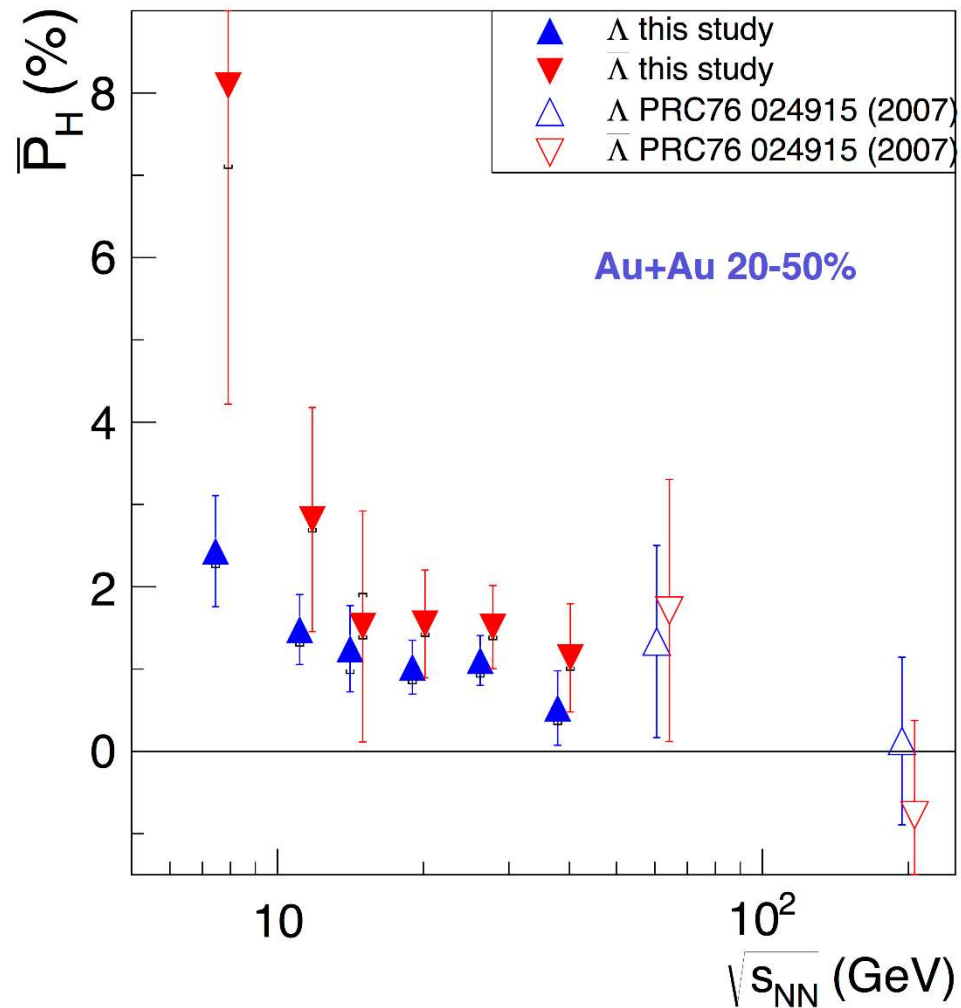
ROTATION – high  $\eta$



KHI – low  $\eta$



## Rotation and Turbulence - (2015-16)



### ● STAR results

[M.A. Lisa, et al. (STAR Collaboration), Invited talk, QCD Chirality Workshop - UCLA, February 23-26, 2016, Los Angeles, USA.]

# Observable consequences

Mike Lisa &  
STAR  
 $\Lambda$  & anti- $\Lambda$   
polarization

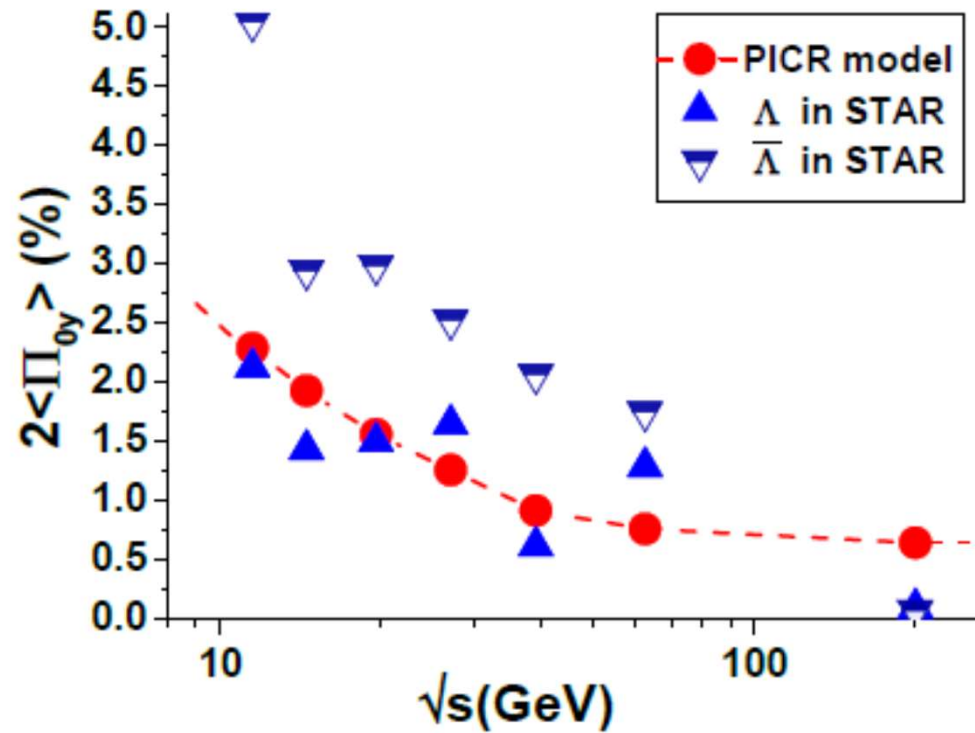
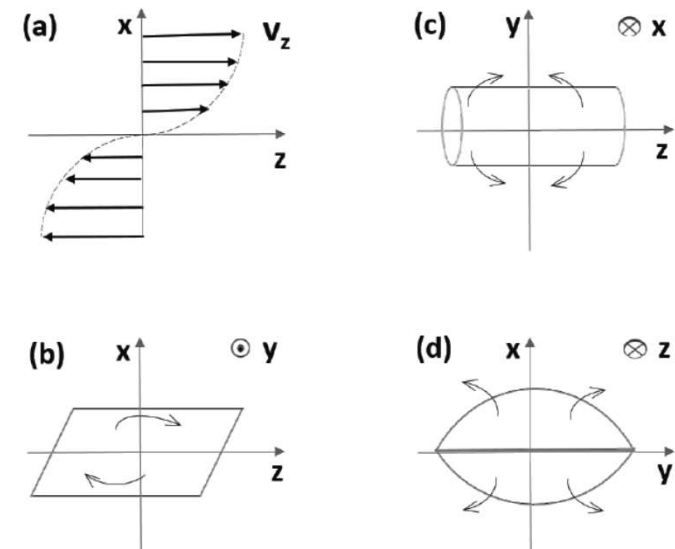
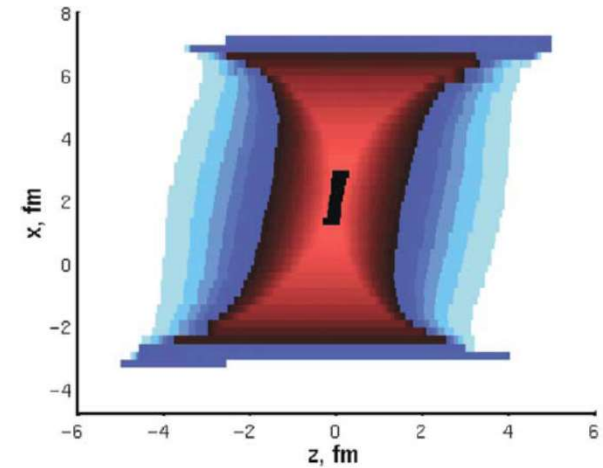
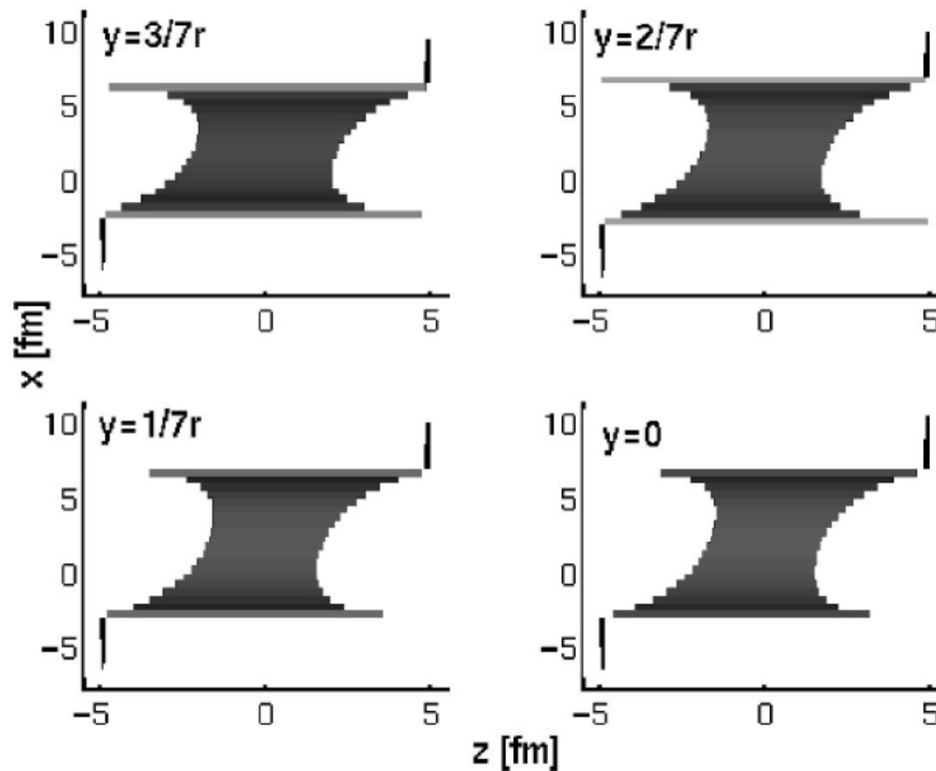


FIG. 4. (Color online) The global polarization,  $2\langle \Pi_{0y} \rangle_p$ , in our PICR hydro-model (red circle) and STAR BES experiments (green triangle), at energies  $\sqrt{s}$  of 11.5GeV, 14.5GeV, 19.6GeV, 27GeV, 39GeV, 62.4GeV, and 200GeV. The red The experimental data were extracted from Ref[Mike Lisa], dropping the error bars.

# Initial State – Peripheral reactions

Magas, Csernai, Strottman (2001), (2002)

- Yang-Mills flux tube model for longitudinal streaks
- String tension is decreasing at the periphery
- Initial shear & vorticity is present



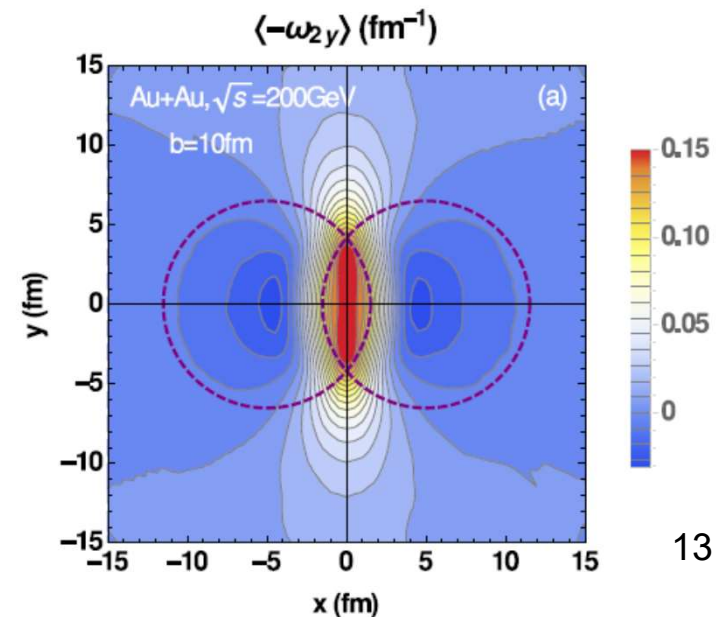
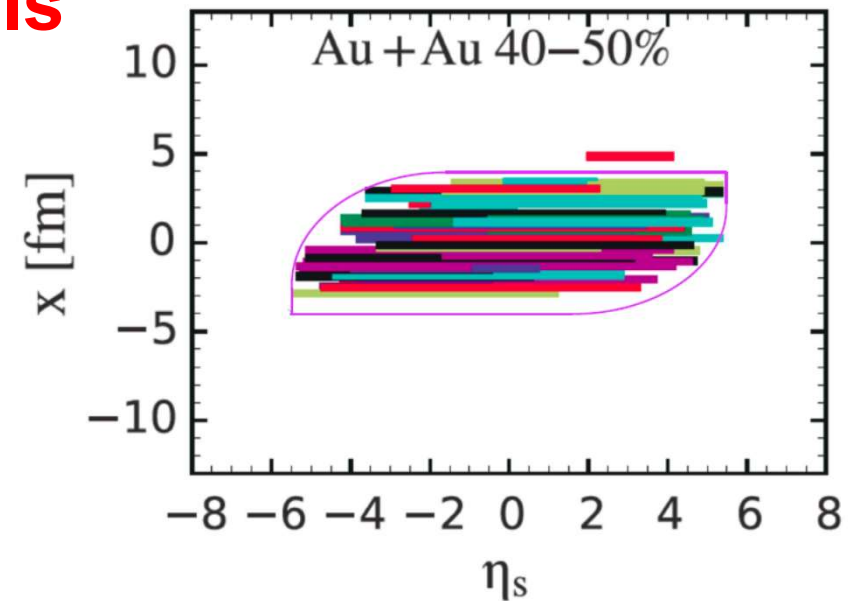
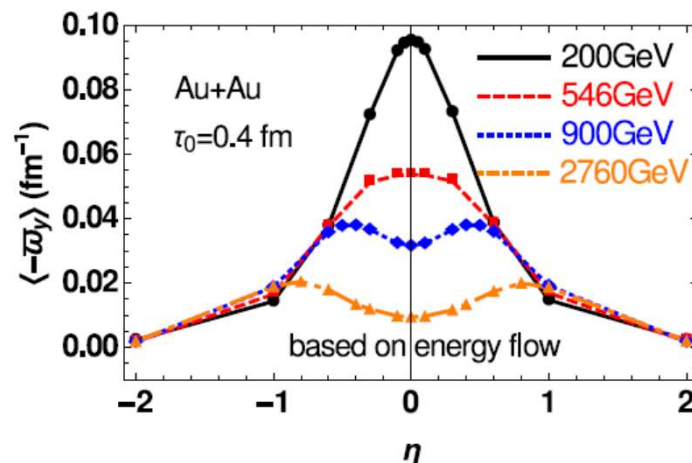
# Present parton kinetic models

## - HIJING, AMPT, PACIAE

Different space-time configurations

[Long-Gang Pang, Hannah Petersen, Guang-You Qin, Victor Roy and Xin-Nian Wang, 27 September - 3 October 2015, Kobe, Japan; and Long-Gang Pang, Hannah Petersen, Guang-You Qin, Victor Roy, Xin-Nian Wang, arXiv: 1511.04131 ]

[Wei-Tian Deng, and Xu-Guang Huang, arXiv: 1609.01801]

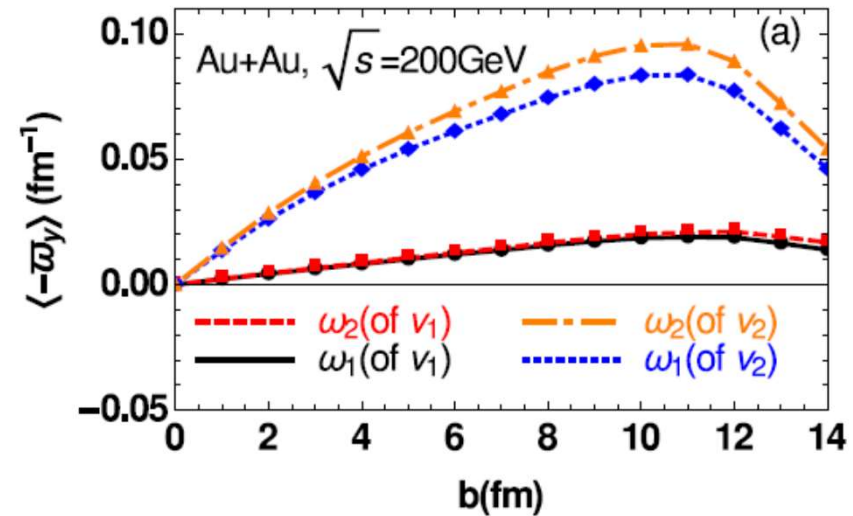
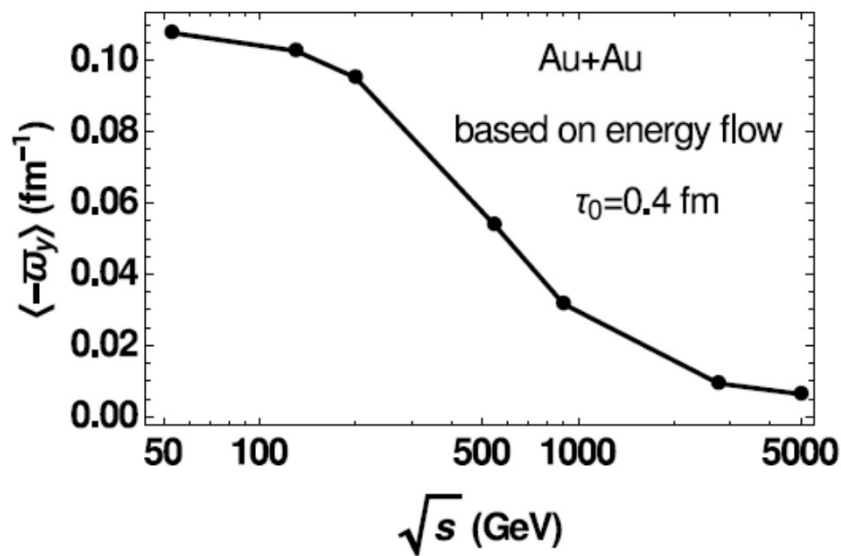
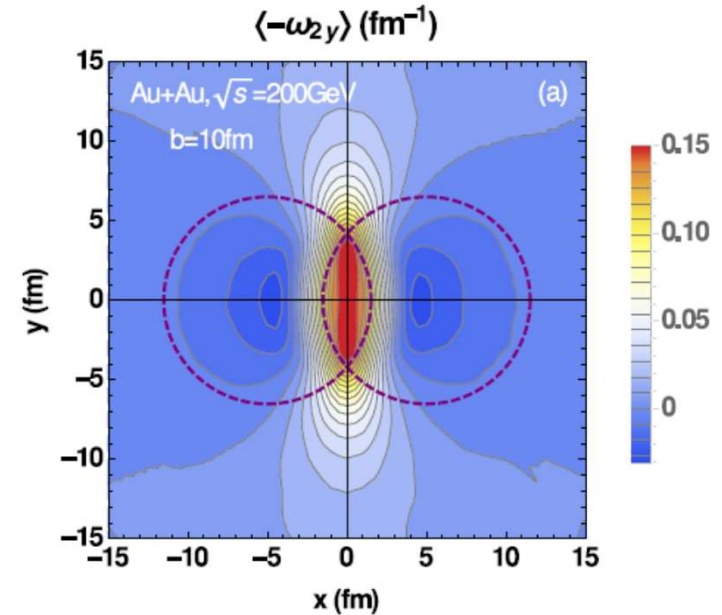


# Present parton kinetic models

- **HIJING, AMPT, PATHIA**

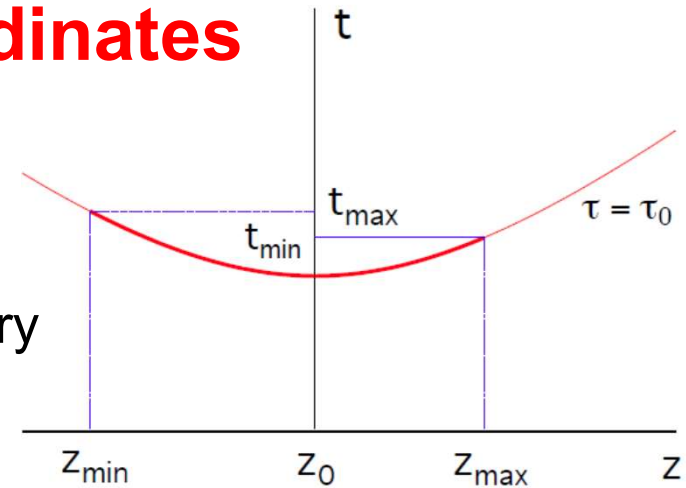
Different space-time configurations

[Wei-Tian Deng, and Xu-Guang Huang,  
arXiv: 1609.01801]



## Initiative: new I.S. in $\tau, \eta$ coordinates

- Separately for each longitudinal streak
- String tension is not decreasing at the periphery
- Initial shear & vorticity is present !



The normal four vector of a hypersurface at  $\tau = \text{const.}$  is

$$d\Sigma^\mu = A \tau u^\mu, \quad (3)$$

- Conservation laws:  $dN = d\Sigma_\mu N^\mu = \tau A n u_\mu u^\mu d\eta$

$$N_i = N_1 + N_2 = \tau_0 n(\tau_0) A (\eta_{max} - \eta_{min})$$

$$E_i = E_1 + E_2 = \tau_0 e(\tau_0) A (\sinh \eta_{max} - \sinh \eta_{min})$$

$$P_{iz} = P_{1z} - P_{2z} = \tau_0 A e (\cosh \eta_{max} - \cosh \eta_{min}) \quad 15$$

# Initiative: new I.S. in $\tau, \eta$ coordinates

- For the i-th streak  $(t, z)$  and  $(\tau, \eta)$  coordinates are connected as

$$t - t_0 = \tau / \sqrt{\coth^2 \eta + 1} ,$$

$$z - z_0 = \tau / \sqrt{\tanh^2 \eta + 1} ,$$

$$\tau = \sqrt{(t - t_0)^2 + (z - z_0)^2} ,$$

$$\eta = \frac{1}{2} \ln \left( \frac{t - t_0 + z - z_0}{t - t_0 - (z - z_0)} \right)$$

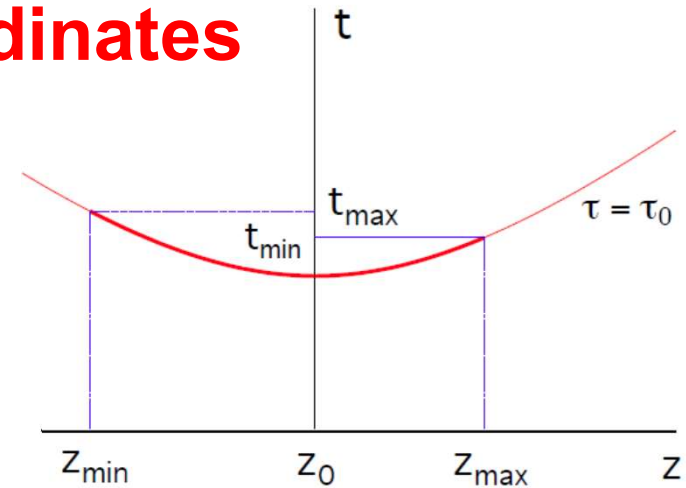
$$= \operatorname{arctanh} \frac{z - z_0}{t - t_0} ,$$

- For the central streak:

$$\frac{1}{2} \Delta \eta_c = \operatorname{arcsinh} \left( \frac{E_c}{2\tau_0 e(\tau_0) A} \right) \quad \text{and}$$

$$z_{c-max} = \tau_0 / \sqrt{\tanh^2 \Delta \eta_c + 1} ,$$

$$t_{c-max} = \tau_0 / \sqrt{\tanh^{-2} \Delta \eta_c + 1}$$



# Initiative: new I.S. in $\tau, \eta$ coordinates

- For the i-th streak:  $e(\tau_0) = E_c / \left[ \tau_0 A 2 \sinh \left( \frac{1}{2} \Delta \eta_c \right) \right]$

$$\frac{1}{2} \Delta \eta_i = \operatorname{arcsinh} \left( \frac{E_i}{2 \tau_0 e(\tau_0) A} \right)$$

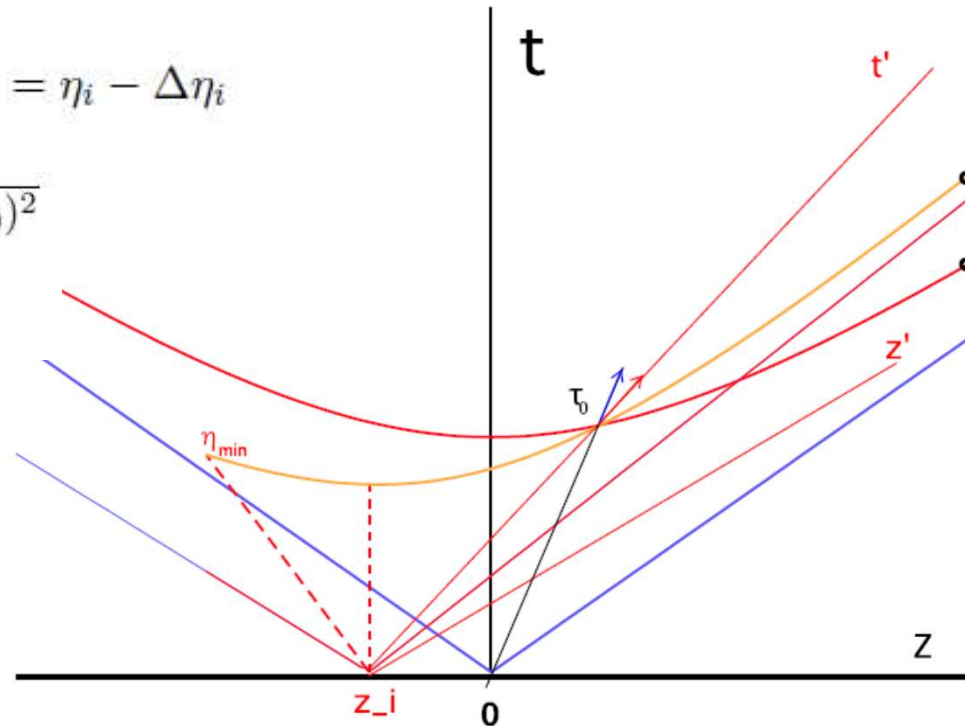
$$\eta_i = \operatorname{arctanh} \frac{P_{iz}}{E_i}.$$

$$\eta_{i-\max_P} = \eta_i + \Delta \eta_i, \quad \eta_{i-\min_T} = \eta_i - \Delta \eta_i$$

$$\tau_0 = \sqrt{(t_{\max} - t_{i0})^2 + (z_{\max} - z_{i0})^2}$$

$$\eta_{i-\max_P} = \operatorname{arctanh} \left( \frac{z_{\max} - z_{i0}}{t_{\max} - t_{i0}} \right).$$

- $\rightarrow$  The origin,  $t_{0i}, z_{0i}$ , will be different for each streak.



## Initiative: new I.S. in $\tau, \eta$ coordinates

Thus for each streak,  $i$ , we can get the origin of the  $\tau=\tau_0$  hyperbola,  $t_{i0}$  &  $z_{i0}$ .

from:

$$z_{i0} = z_{max} - \frac{\tau_0 (\tanh \eta_{i-max_P})}{\sqrt{1 + (\tanh \eta_{i-max_P})^2}}$$

$$\tau_0^2 = \left( t_{i0} + \frac{\tau_0}{\sqrt{\coth^2 \eta_i + 1}} \right)^2 + \left( z_{i0} + \frac{\tau_0}{\sqrt{\tanh^2 \eta_i + 1}} \right)^2.$$

## Matching I.S. to hydro

PHYSICAL REVIEW C **81**, 064910 (2010)

### **Matching stages of heavy-ion collision models,**

Yun Cheng, L. P. Csernai, V. K. Magas, B. R. Schlei, and D. Strottman

There are hydro options: Cartesian / Bjorken coordinates

Transition surface,  $\tau = \text{const.}$ ,  $t = \text{const.}$ , curved  $h.s.$

In all cases I.S.  $T^{\mu\nu} \rightarrow$  Conservation laws due to EoS.

# Consequences:

- Will be similar to the 2001-2 I.S. in (t,z) coordinates
- More compact  $\rightarrow$  vorticity may survive better
- The earlier results will remain qualitatively similar:

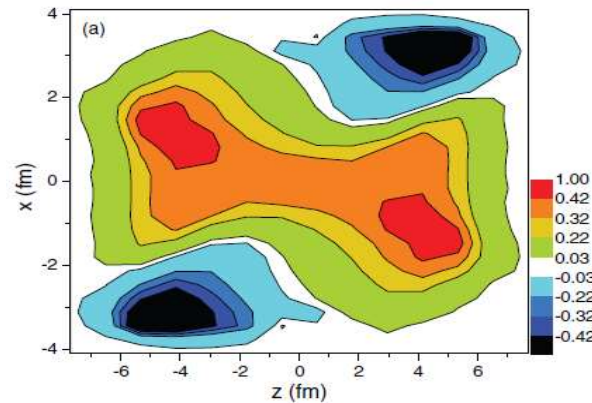
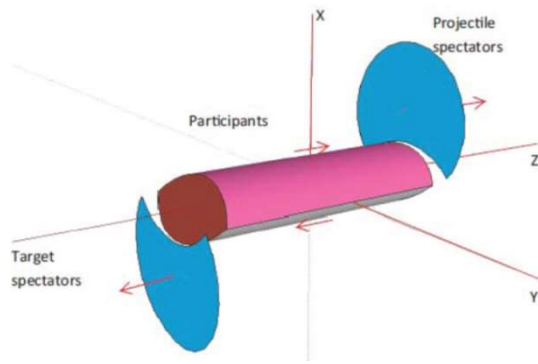


Fig. 3 The vorticity calculated in the reaction (xz) plane at  $t = 0.17$  fm/c after the start of fluid dynamical evolution.

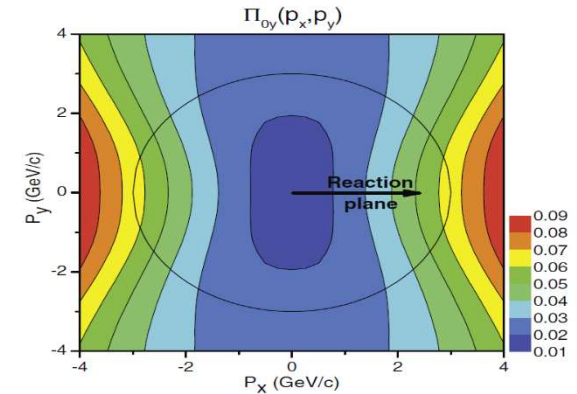


Fig. 4. The dominant y component of the observable polarization,  $\Pi_0(\mathbf{p})$  in the  $\Lambda$ 's rest frame.

The initial rotation can lead to observable vorticity (Fig. 3), and polarization (Fig. 4): Leading vorticity term.  
The initial angular momentum can be transferred to the polarization at final state, via spin-orbit coupling or equipartition.

[L. P. Csernai, et al, PRC **87**, 034906 (2013)]

[F. Becattini, et al. PRC **88**, 034905 (2013)]

## Consequences:

Based on Ref. [Becattini, 2013],  $\Lambda$  polarization can be calculated as:

$$\Pi(p) = \frac{\hbar\epsilon}{8m} \frac{\int dV n_F(x, p) (\nabla \times \beta)}{\int dV n_F(x, p)} \quad \leftarrow \text{Vorticity, 1st}$$

$$+ \frac{\hbar p}{8m} \times \frac{\int dV n_F(x, p) (\partial_t \beta + \nabla \beta^0)}{\int dV n_F(x, p)} \quad \leftarrow \text{Expansion, 2nd}$$

where  $\beta^\mu(x) = [1/T(x)]u^\mu(x)$  is the inverse temperature four-vector field. Then thermal vorticity is  $\omega = \nabla \times \beta$ .

The polarization 3-vector in the rest frame of particle can be found by Lorentz-boosting the above four-vector:

$$\Pi_0(p) = \Pi(p) - \frac{p}{p^0(p^0 + m)} \Pi(p) \cdot p ,$$

[F. Becattini, L.P. Csernai, and D.J. Wang, Phys. Rev. C **88**, 034905 (2013)]

# Consequences:

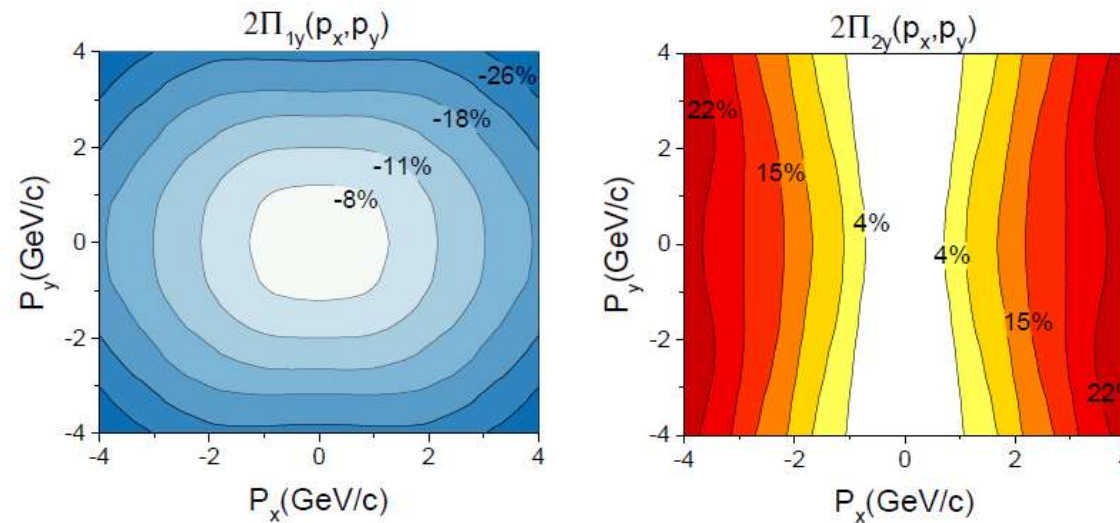
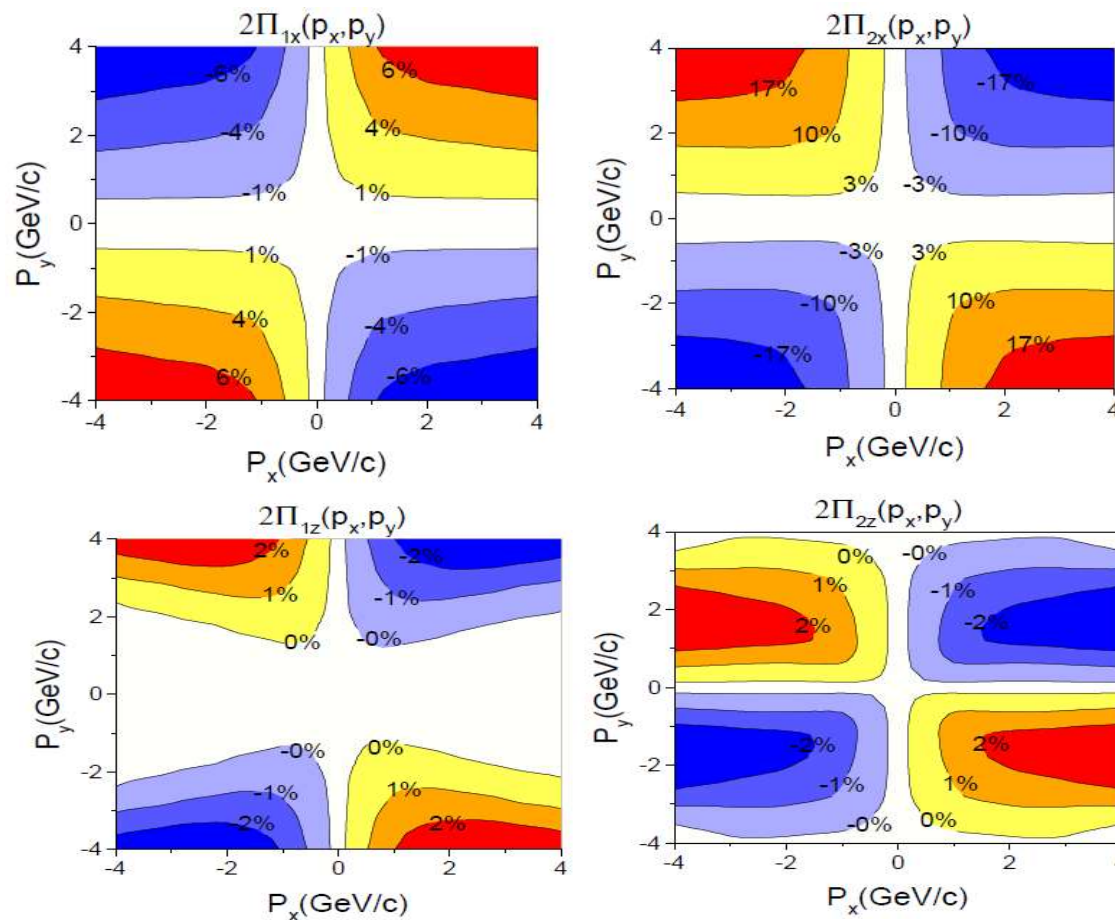


Fig. 6 The first (left) and second (right) term of the dominant  $y$  component of the  $\Lambda$  polarization for momentum vectors in the transverse plane at  $p_z = 0$ , for the FAIR U+U reaction at 8.0 GeV

- The  $y$  component is dominant, is up to  $\sim 20\%$ , as we can compare it with  $x$  and  $z$  components later.
- 1<sup>st</sup> & 2<sup>nd</sup> terms are opposite direction. Result into a relatively smaller value of global polarization.

# Consequences

/ c.m. !



1. Anti-symmetry

2. Trivial.

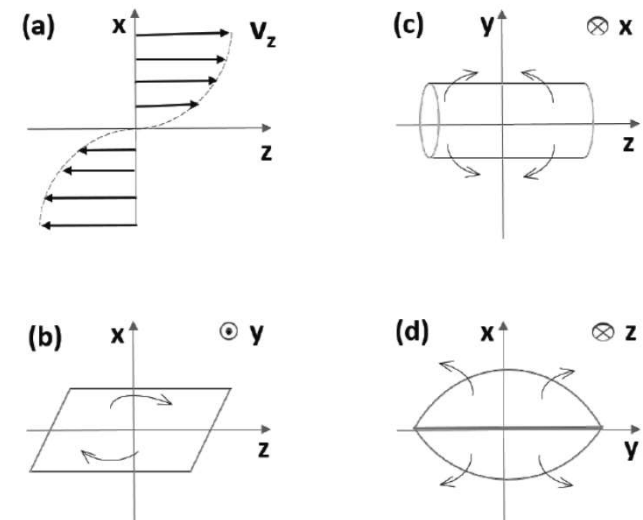
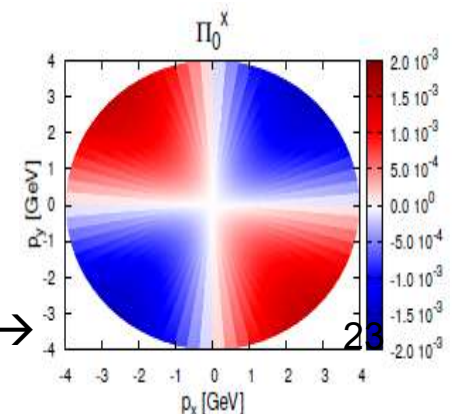
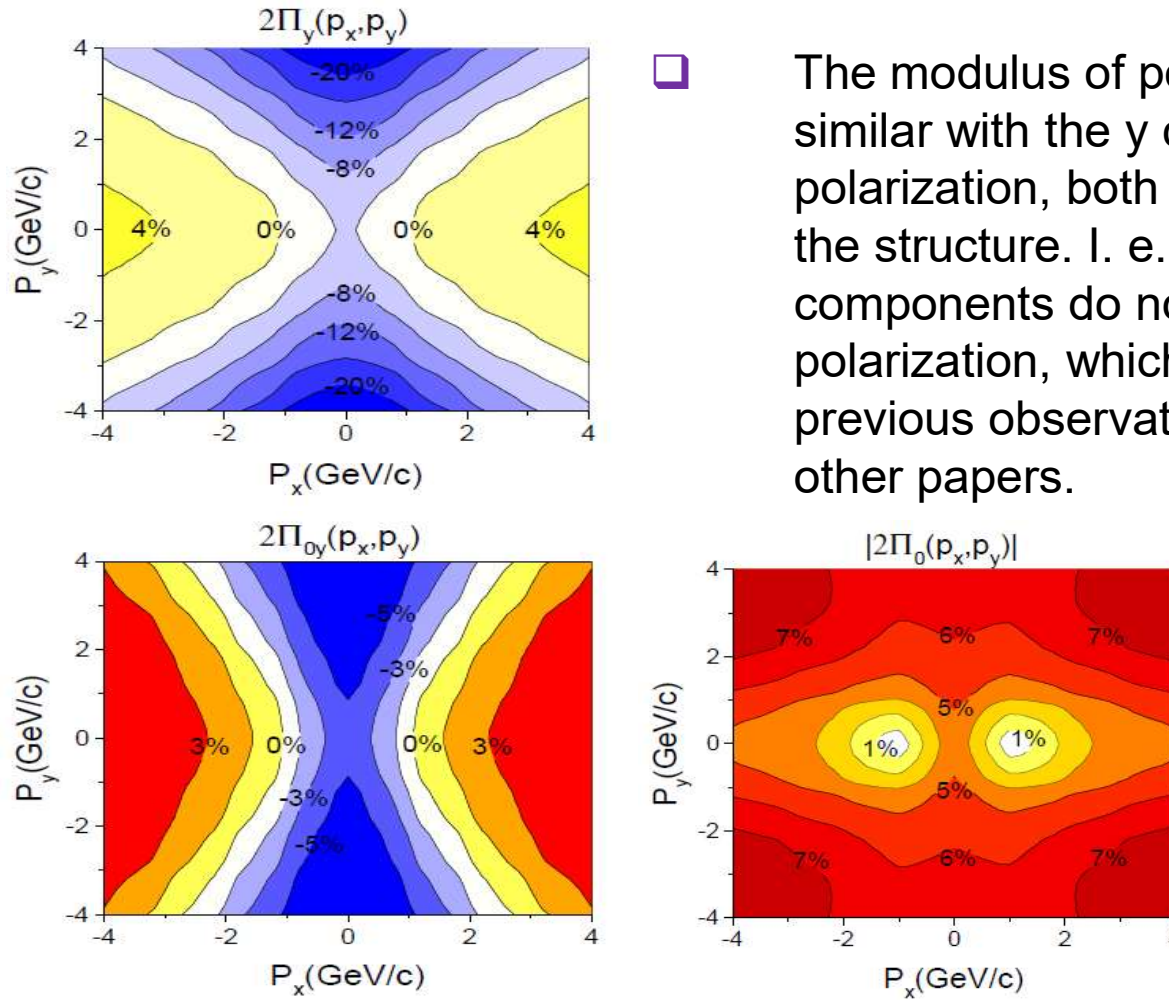


Fig. 7 The first (left) and second (right) terms of the  $\Lambda$  polarization for momentum vectors in the transverse plane at  $p_z = 0$ , for the FAIR U+U reaction at 8.0 GeV

[Becattini, et al., Eur. Phys. J. C 75, 406 (2015).] →



# Consequences FAIR



The modulus of polarization is very similar with the y component of polarization, both in magnitude and the structure. I. e. the other x and z components do not contribute to the polarization, which is in line with previous observations in this work and other papers.

Fig. 8 The y component (left) of polarization vector in center of mass frame and  $\Lambda$ 's rest frame. The right sub-figure are the modulus of the polarization in  $\Lambda$ 's rest frame. At FAIR, 8.0 GeV at time 2.5+4.75 fm/c.

# Consequences NICA

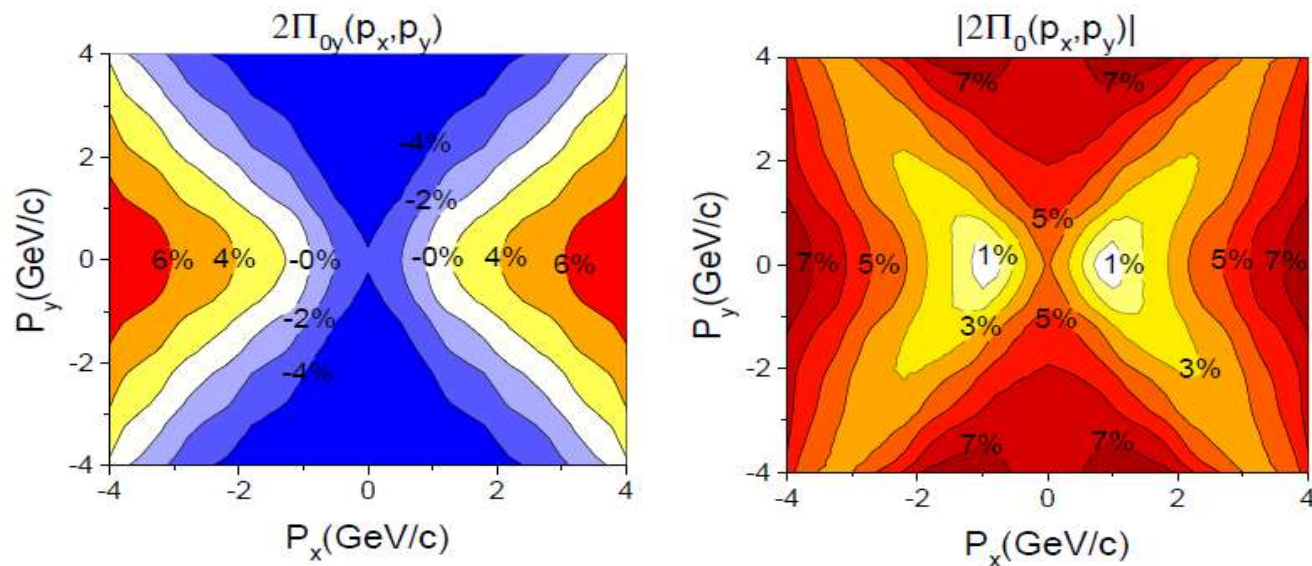


Fig. 9 The y component (left) and the modulus (right) of the polarization for momentum vectors in the transverse plane at  $p_z = 0$ , for the NICA Au+Au reaction at 9.3 GeV. The figure is in the  $\Lambda$ 's rest frame.

- Similarity between y component and modulus of Polarization, in magnitude and structure.
- Similarity between NICA and FAIR's polarization results.
- The net polarization is still negative, which means the first term is larger than the second term, at this time.

# Consequences FAIR

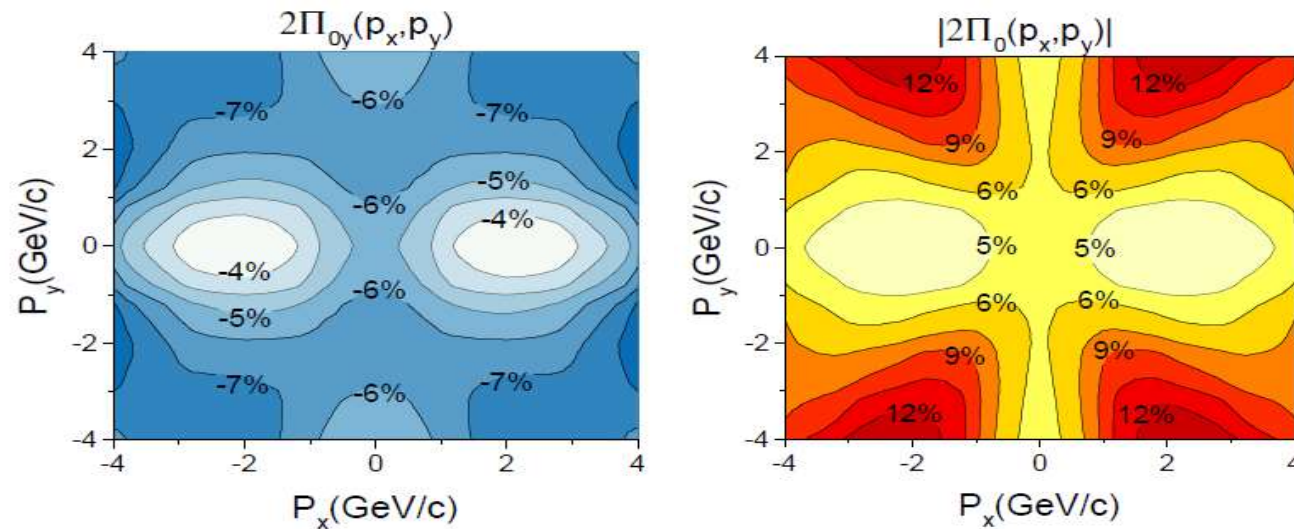


Fig. 9 The y component (left) and the modulus (right) of the polarization for momentum vectors in the transverse plane at  $p_z = 0$ , for the FAIR U+U reaction at 8.0 GeV, but at an earlier time  $t = 2.5 + 1.7$  fm/c. The figure is in the  $\Lambda$ 's rest frame.

- Initially, the first term is very dominant

## Conclusions

Peripheral reactions show shear, vorticity (turbulence)

I.S. can be implemented in  $(t, z)$  and  $(\tau, \eta)$  hydro codes

Different components,  $-y, x, z$ , and momentum dependence do show the weight of different dynamical flow patterns.