(Progress towards) Initial Conditions for Anomalous Hydrodynamics

Mark Mace
Stony Brook U./BNL

Based on MM, S. Schlichting, R. Venugopalan
Phys.Rev. D93 (2016) 074036 (arXiv:1601.07342 [hep-ph]),
MM, N. Mueller, S. Schlichting, S. Sharma, in preparation

Hydrodynamics 2016- Mainz ITP
October 10, 2016

Stony Brook University BEST COLLABORATION BROOKHAVEN NATIONAL LABORATORY

Motivation

- Want to quantitatively study of anomalous transport (CME/CSE/CMW…) in heavy ion collisions
- Anomalous hydrodynamics needed for phenomenology
  (Son & Surowka PRL 103191601; Hirono, Hirano, Kharzeev arXiv:1412.0311)

$$\partial_\mu T^{\mu\nu} = F^{\nu\rho} j_\rho$$
$$\partial_\mu j^\mu = 0$$
$$\partial_\mu j_a^\mu = C E^\mu B_\mu$$

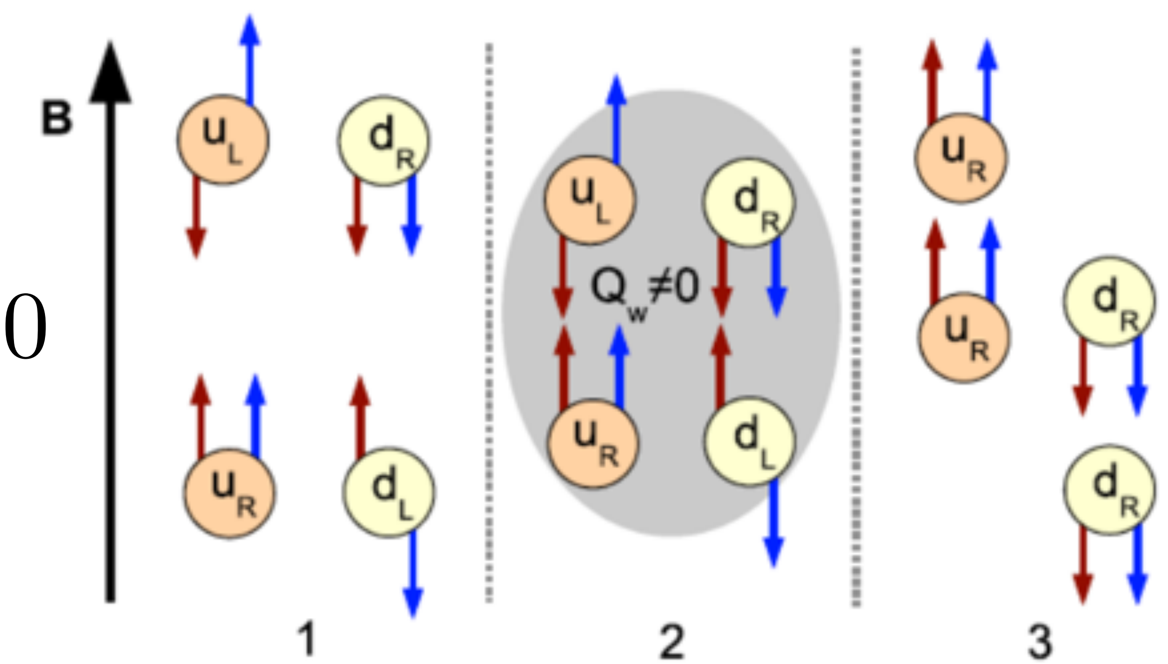
- What else is needed to do phenomenology?
  - Initial axial charge production, initial axial and vector currents,…
  - Formulate in terms of microscopic, field-theoretic description, study macroscopic effects to be able to interface with bulk evolution of anomalous hydrodynamics
- For this, I will discuss studies building this up, specifically axial charge production out of equilibrium at weak coupling

Chiral Magnetic Effect

- Chiral Magnetic Effect: $\mathbf{j}_V \sim n_5 \mathbf{B}$

  (Kharzeev, McLerran, Warringa NPA803, 227; Fukushima, Kharzeev, Warringa PRD78 074033)

$$\mathbf{B} \gg 0$$



Red=momentum
Blue=spin

(Kharzeev Prog. Part. Nucl. Phys. 75 (2014) 133-151)

- Chiral Separation Effect: $\mathbf{j}_a \sim n_V \mathbf{B}$

  (Son & Zhitnitsky, PRD 70, 074018; Metlitski & Zhitnitsky, PRD 72, 045011)

- Chiral Magnetic Wave: collective gapless excitation from the coupling between density waves of electric and chiral charges

  (Kharzeev & Yee, PRD 83, 085007)

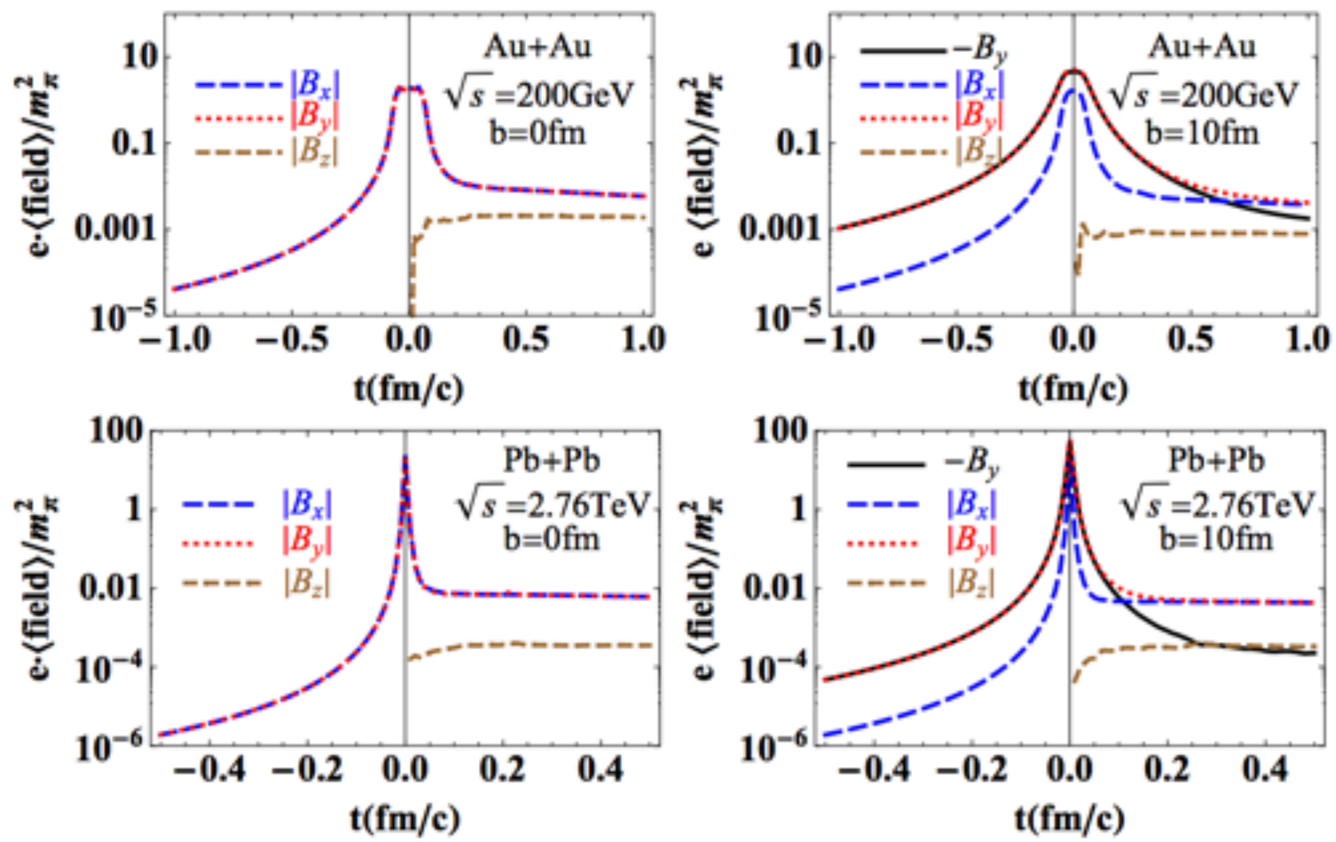
Magnetic Field in HIC

- Off-center collisions creates initially strong magnetic field

$$e\mathbf{B} \sim (m_\pi)^2 \sim 10^{12}\mathrm{T}$$

- Very short lifetime

$$t \sim 0.1 - 0.2\frac{\mathrm{fm}}{\mathrm{c}}$$



(Deng and Huang, PRC85, 044907)

Greatest fraction of chiral effects should be at very initial times when magnetic field is strongest

(Skokov,Illarionov,Toneev IJMP A24 5925-5932, Deng and Huang, PRC85, 044907; McLerran and Skokov NPA929 (2014) 184-190; Tuchin et al PRC91 (2015) 064902, arXiv:1604.04572)

Axial Charge in HIC $\mathbf{j}_V \sim n_5 \mathbf{B}$

- Axial anomaly: Axial charge sourced by fluctuations in non-Abelian field strength tensor

$$\partial_\mu j_a^\mu(x) = 2im\langle\hat{\bar{\psi}}(x)\gamma_5\hat{\psi}(x)\rangle - \frac{g^2}{8\pi^2}\mathrm{Tr}F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)$$

Axial current $j_a^\mu = (j_a^0, \mathbf{j}_5)$

Quark mass

Receives contributions from sphaleron transitions, color flux tubes, field strength fluctuations,…

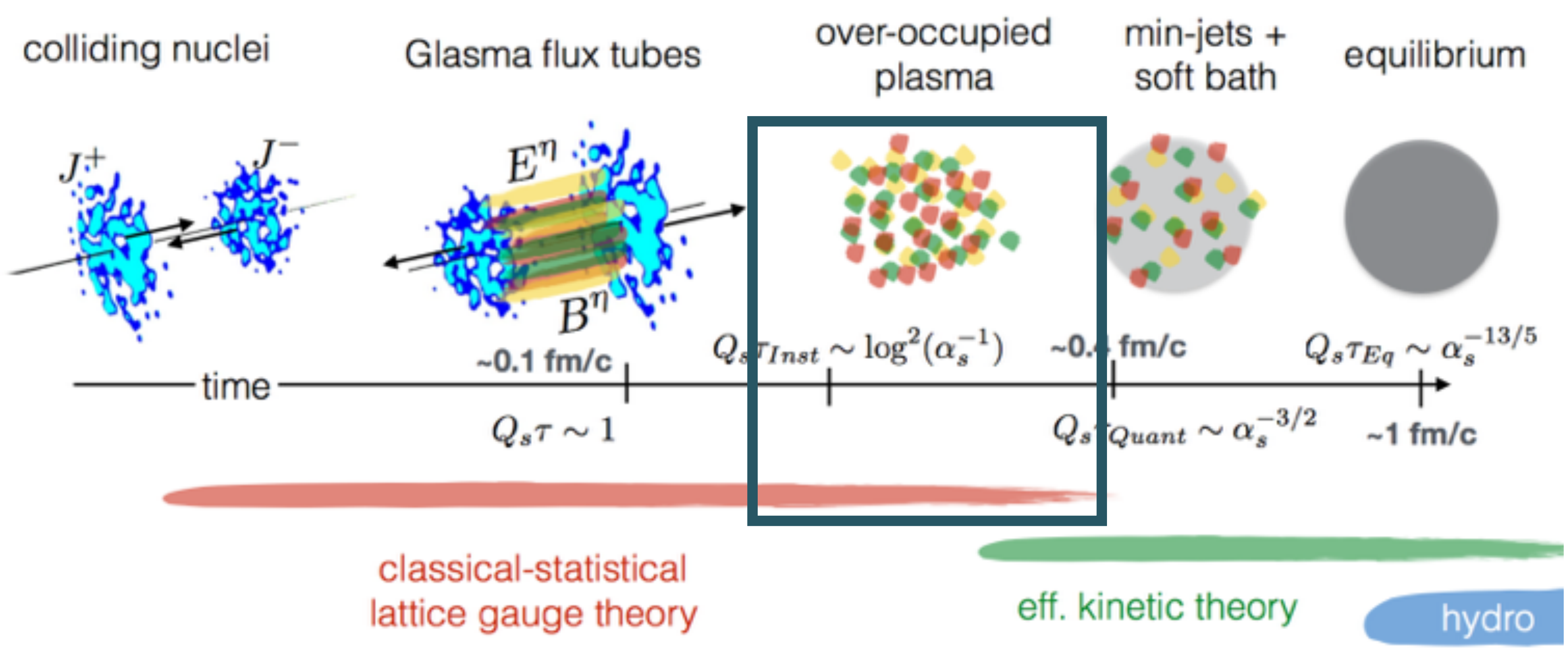
(Klinkhamer, Manton PRD30 (1984) 2212)

Sphaleron transition (real time topological transitions) leads to unit change of Chern-Simons number and induce axial charge imbalance

$$\Delta J_5^0 = -2\Delta N_{CS} + 2m_f \int d^4x \langle\bar{\psi} i\gamma_5\psi\rangle$$

Early stages of HIC

- System is far from equilibrium, gluon dominated
- Much progress in understanding equilibration of the bulk, but axial charge generation lacks understanding
  - Focus on earliest times where classical description is applicable



(Adapted from S.Schlichting 2016)

Topology on the lattice at early times after HIC

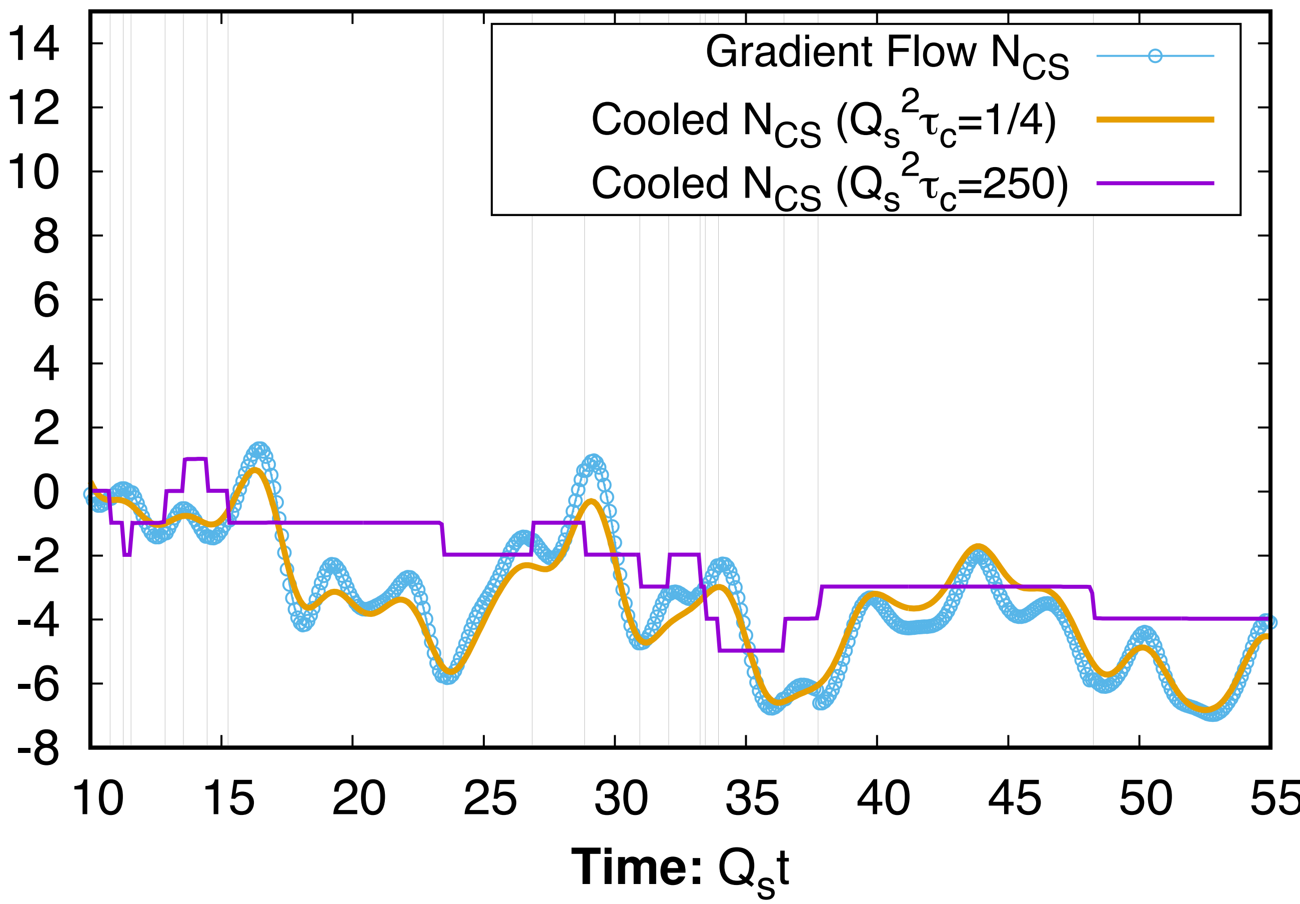
- Early time dynamics described by classical Yang-Mills
  (McLerran, Venugopalan PRD49 2233 (1994))
  - Non-perturbatively large gluon phase space density $f(p \sim Q_s) \sim \frac{1}{\alpha_s}$
  - Sphaleron production included in this framework
- Use real time, non-equilibrium lattice gauge theory
- Track right hand side of anomaly by measuring Chern-Simons number

$$\frac{dN_{CS}}{dt} = \frac{g^2}{8\pi^2} \int d^3x \, E_i^a(\mathbf{x}) B_i^a(\mathbf{x})$$

- Use cooling to remove short range fluctuation and isolate topological transitions
- Neglect longitudinal expansion and use SU(2) for simplicity

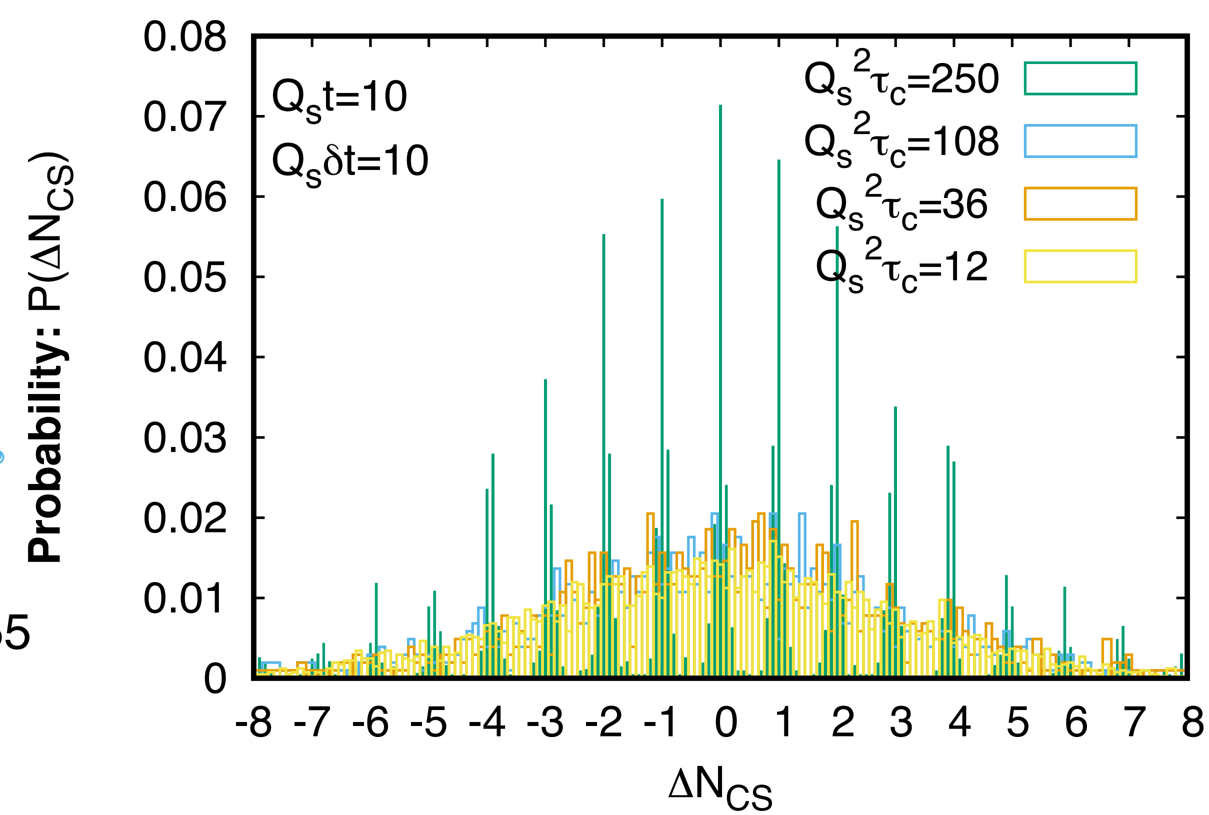
Sphalerons in the glasma

Detect integer changes in Chern-Simons number for single configuration



(MM, Schlichting, Venugopalan PRD 074036)

Histogram Chern-Simons diffusion of many configurations shows transitions between different topological sectors occurs

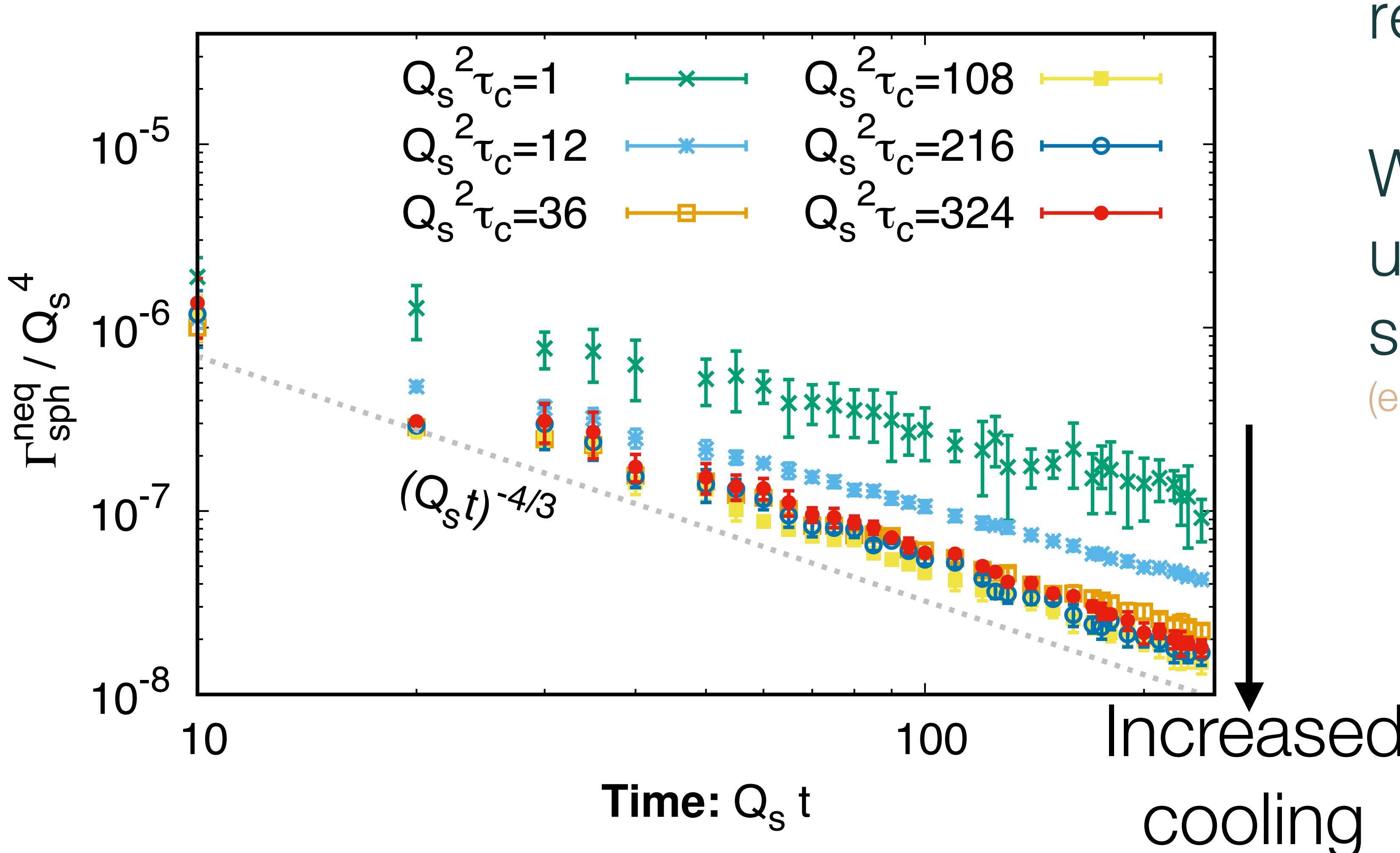


Can isolate transitions from background

Significant number on order of few $1/Q_s$

Non-equilibrium sphaleron transition rate

- Strongly time dependent, largest are early times
- Significant contributions from field strength fluctuations
- Non-markovian, not random walk like thermal equilibrium



(MM, Schlichting, Venugopalan PRD 074036)

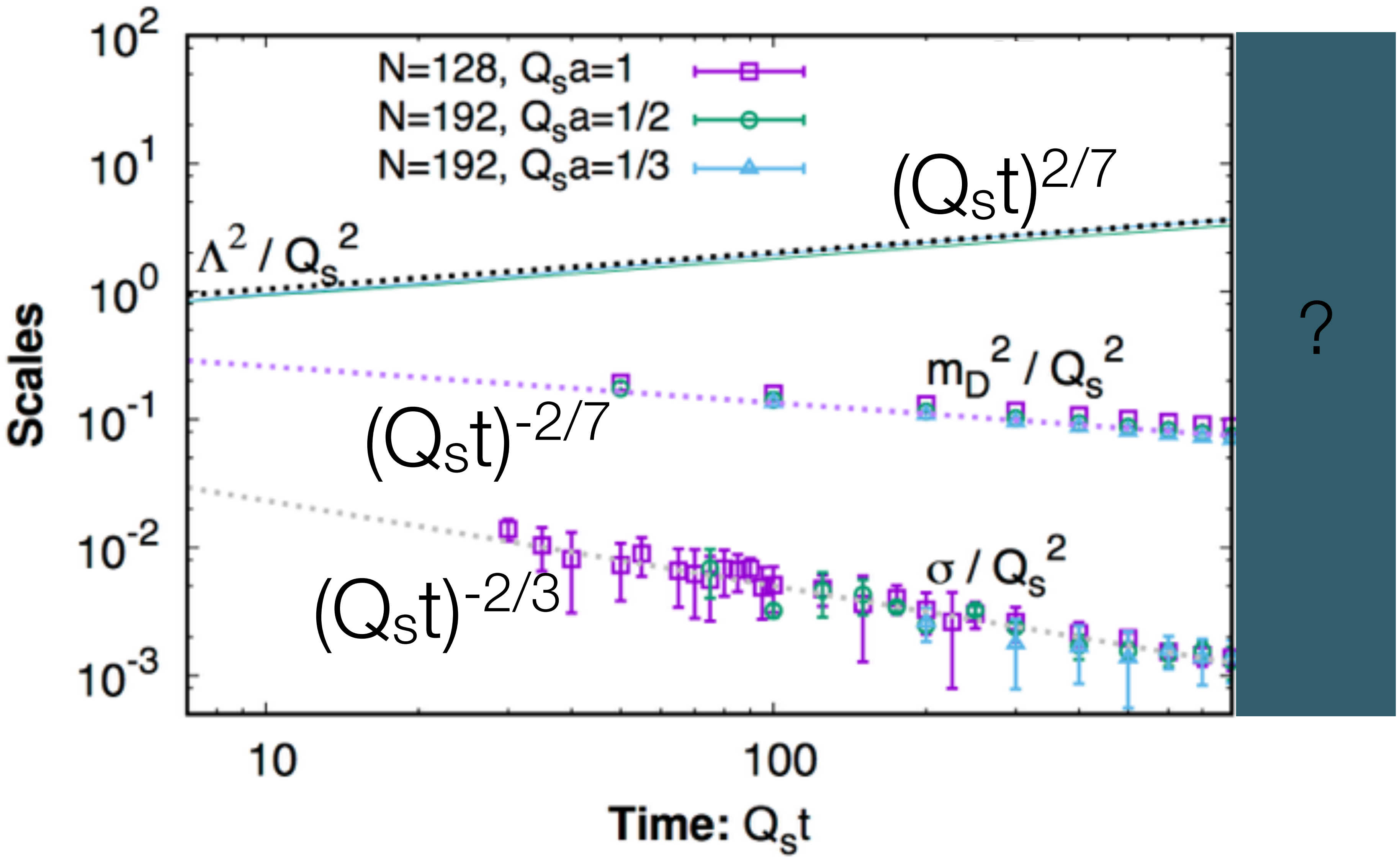
Expect qualitatively similar features when extending to longitudinal expanding, relevant to HIC

Work still ongoing in understanding longitudinal structure of initial state

(e.g. Schenke, Schlichting arXiv:1605.07158)

Dynamical separation of scales

In glasma, initially one scale Q_s, dynamically generate scales



In equilibrium, hierarchy of scales

Hard Scale
$\Lambda^{eq} \sim T$

Electric Screening
$m_D^{eq} \sim gT$

Magnetic Screening
$\Lambda_s^{eq} \sim \sqrt{\sigma} \sim g^2 T$

(MM, Schlichting, Venugopalan PRD 074036)

Non-equilibrium sphaleron transition rate

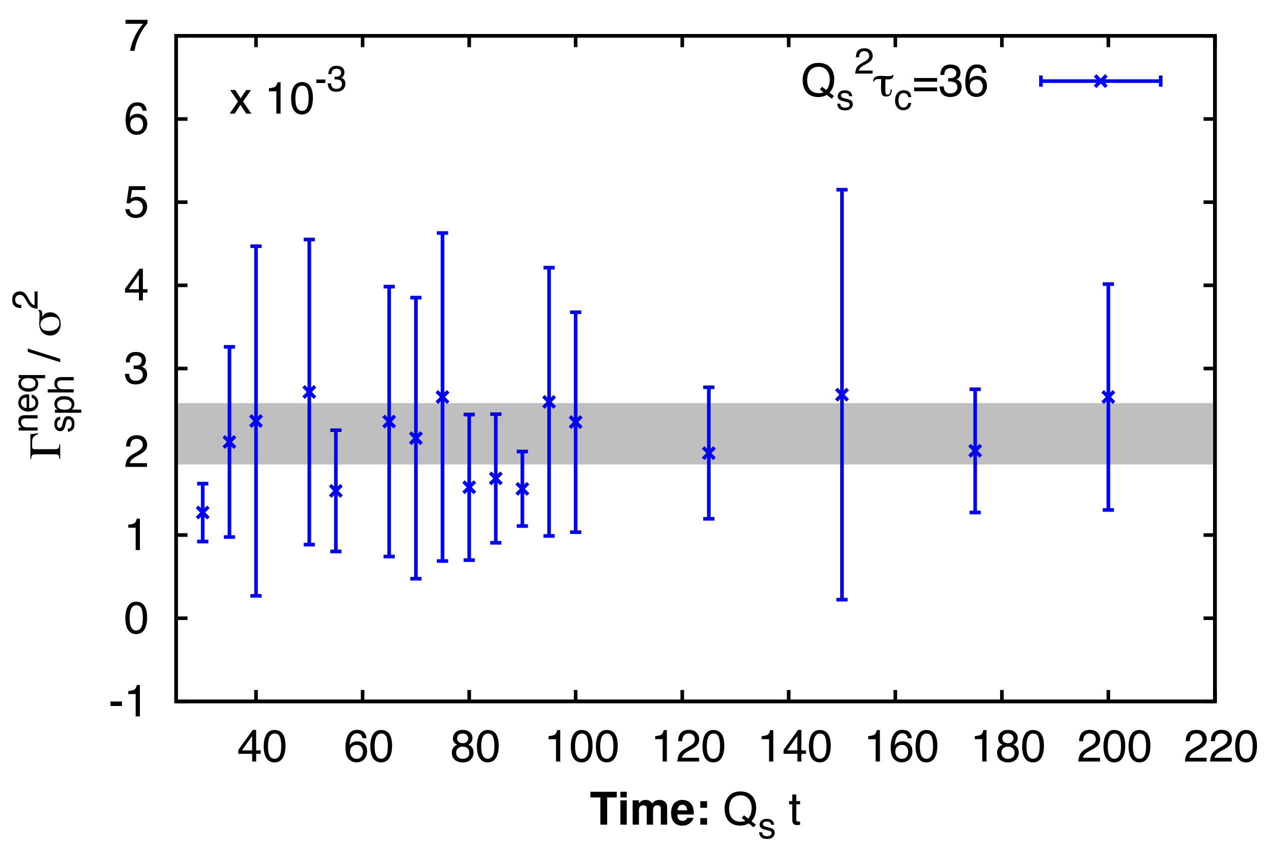
- In equilibrium, sphaleron rate controlled by magnetic modes

- From dynamical separation of scales, non-equilibrium sphaleron rate controlled by modes of order magnetic screening

$$\Gamma_{sph}^{neq}(t) = 2 \times 10^{-2} \sigma^2(t)$$

$$\sigma^2(t) \approx Q_s^2 (Q_s t)^{-2/3}$$

(MM, Schlichting, Venugopalan PRD 074036)



Sphalerons in non-Abelian plasmas

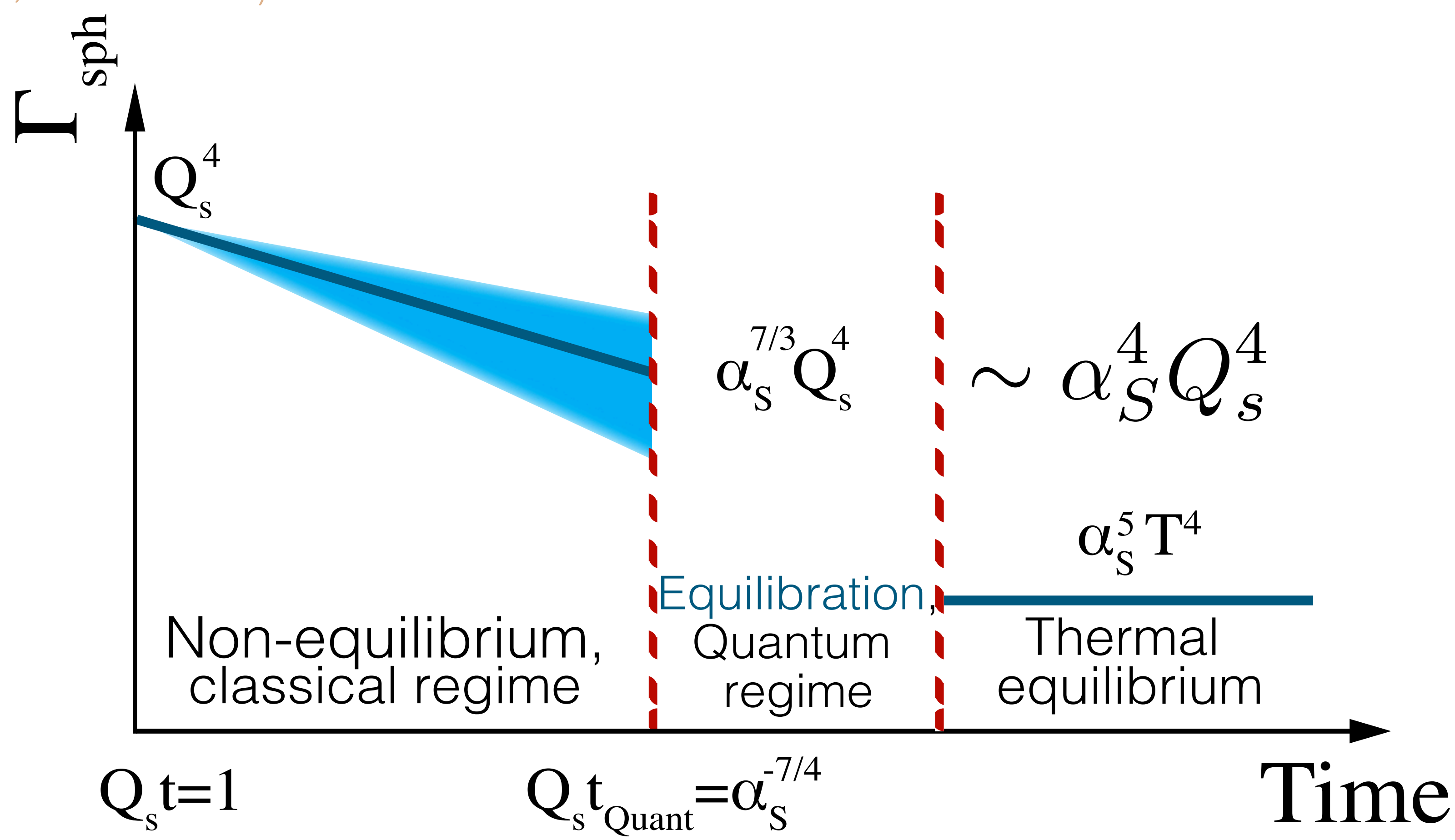
From dynamical separation of scales, non-equilibrium sphaleron rate controlled by modes of order magnetic screening $\sigma^2(t) \approx Q_s^2 (Q_s t)^{-2/3}$

Glasma (early times): $\Gamma_{sph}^{neq} \approx Q_s^4 \rightarrow \Gamma_{sph}^{neq}(t) \sim Q_s^4 (Q_s t)^{-\frac{4}{3}} \sim \sigma(t)^2$

(MM, Schlichting, Venugopalan)

Equilibrium (late times): $\Gamma_{sph}^{eq} \approx \alpha_S^5 T^4$

(Arnold, Son, Yaffee; Moore et al.)

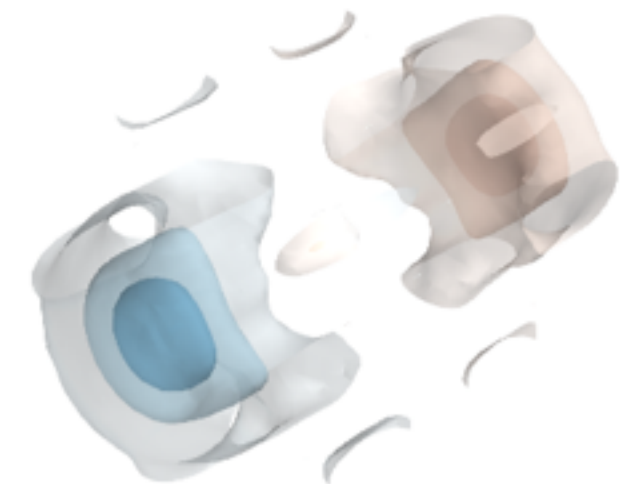


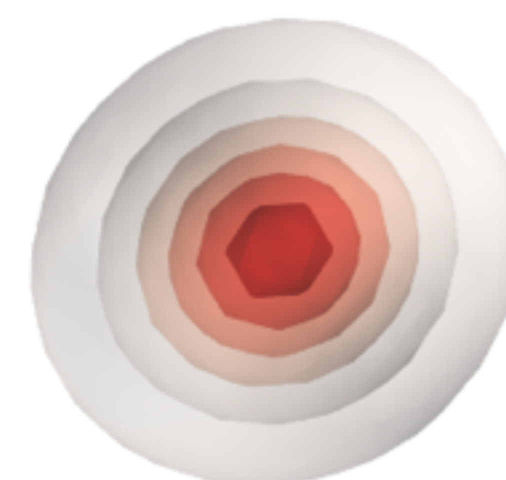
(MM, Schlichting, Venugopalan PRD 074036)

Implications for the CME

- Generalizing for longitudinal expansion in the glasma
  - Initial times appear to be generally dominant
    - Greatest magnetic field
    - Dominant amount of axial charge (should be) generated
      - Glasma: $\Gamma^{neq}_{sph} \sim {Q_s}^4$ Equilibrium: $\Gamma^{eq}_{sph} \sim \alpha_S^5 T^4 \sim \alpha_S^4 Q_s^4$
- First order business: study sphaleron generated axial charge
  - Pilot study with real-time dynamical lattice fermions

Chiral magnetic effect and anomaly induced transport from real time lattice simulations





Real Time Lattice QCD

- To study full CME dynamics, need to extend non-equilibrium gauge field studies to include fermions
- Cannot resort to classical approximation for fermions because of Pauli principle
- Expand initial fermion field in operator basis, evolve wave function by solving Dirac equation $(i\not{D} - m)\hat{\psi} = 0$

SU(2)+U(1)

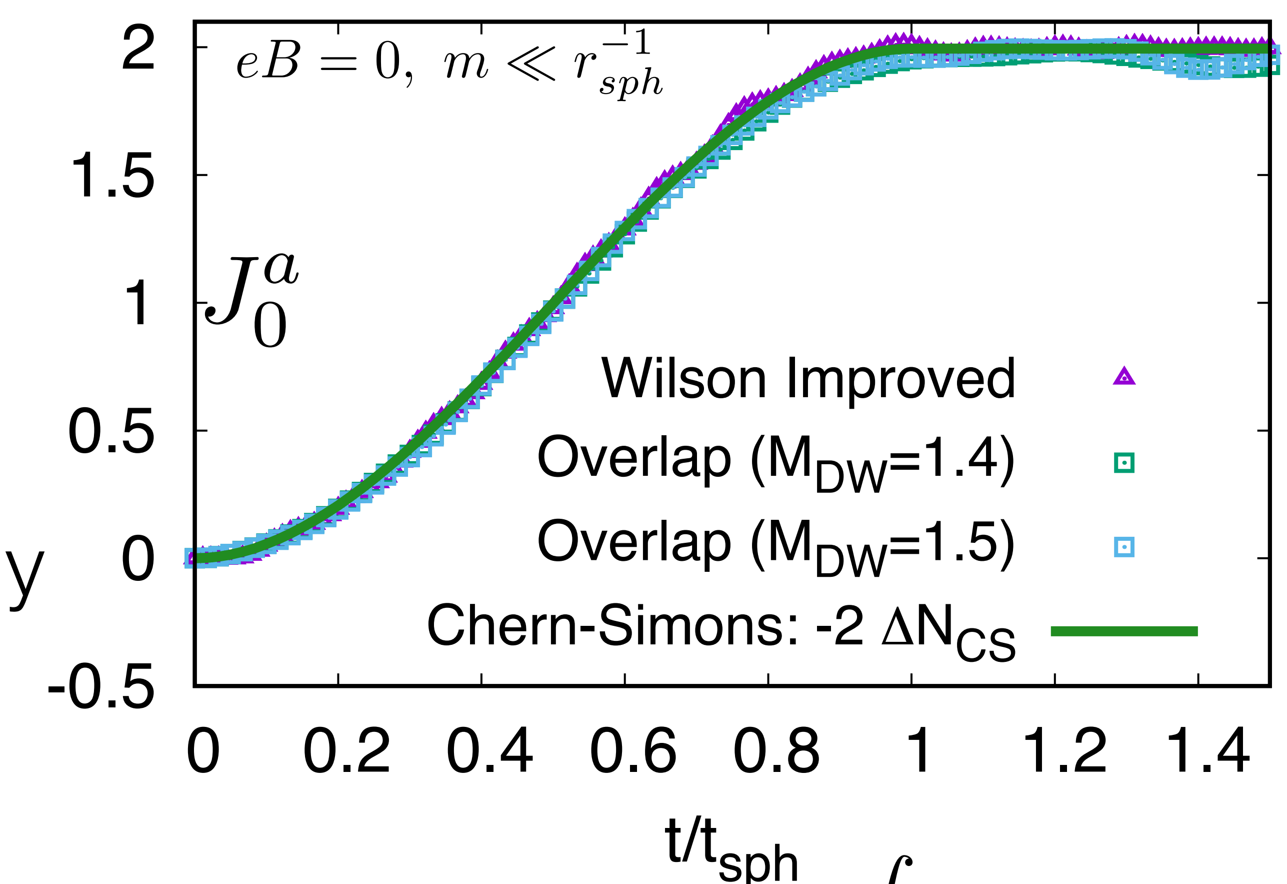
$$\hat{\psi}_x(t) = \frac{1}{\sqrt{V}} \sum_\lambda \left(\hat{b}_\lambda(t=0)\phi^u_\lambda(t,x) + \hat{d}^\dagger_\lambda(t=0)\phi^v_\lambda(t,x) \right)$$

- Measure operator expectation values, i.e $j^\mu_V = q\langle \hat{\bar{\psi}}(x)\gamma^\mu \bar{\psi}(x)\rangle$
- Lattice fermions have well known doubling problem
- Neglect back coupling for now $D_\mu F^{\mu\nu} = j^\nu$

Fermions on the lattice

Wilson (tree level-improved):

Add term to action to remove doublers. Explicitly breaks chiral symmetry, only recovered in continuum limit. Has own anomaly relation
(Karsten & Smit NPB 183)



Overlap Fermions:
(Narayanan & Neuberger PRL71 3251, Neuberger PLB417 141)

$$\Delta J_0^a = -2\Delta N_{CS} + 2m_f \int d^4x \langle \bar{\psi} i \gamma_5 \psi \rangle$$

Constructed to satisfy Ginsparg-Wilson relation. Exact Chiral symmetry. Separate chiral and continuum limit. Can be massless. Extremely numerically expensive. First Minkowski study to our knowledge.

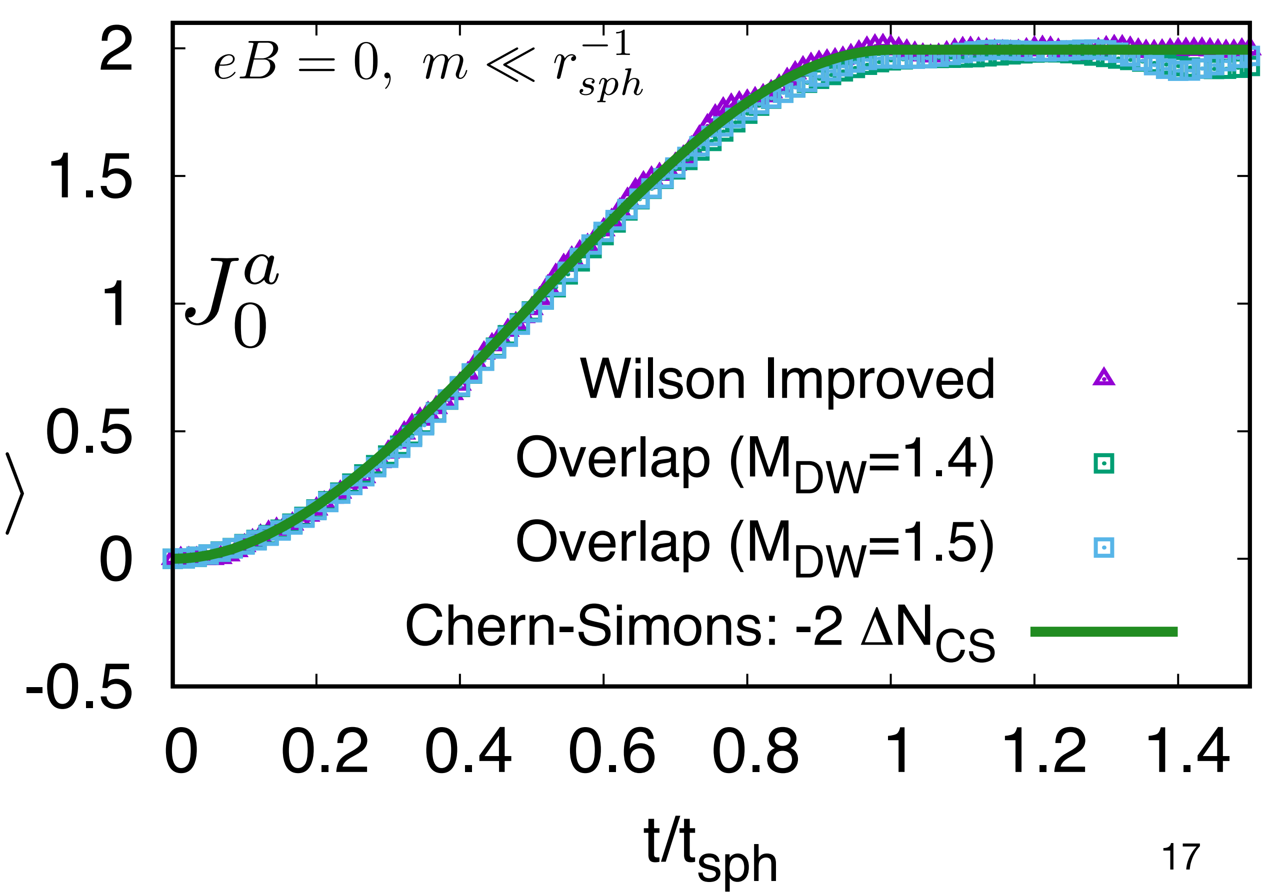
Simulate with both as a cross check

Initial study

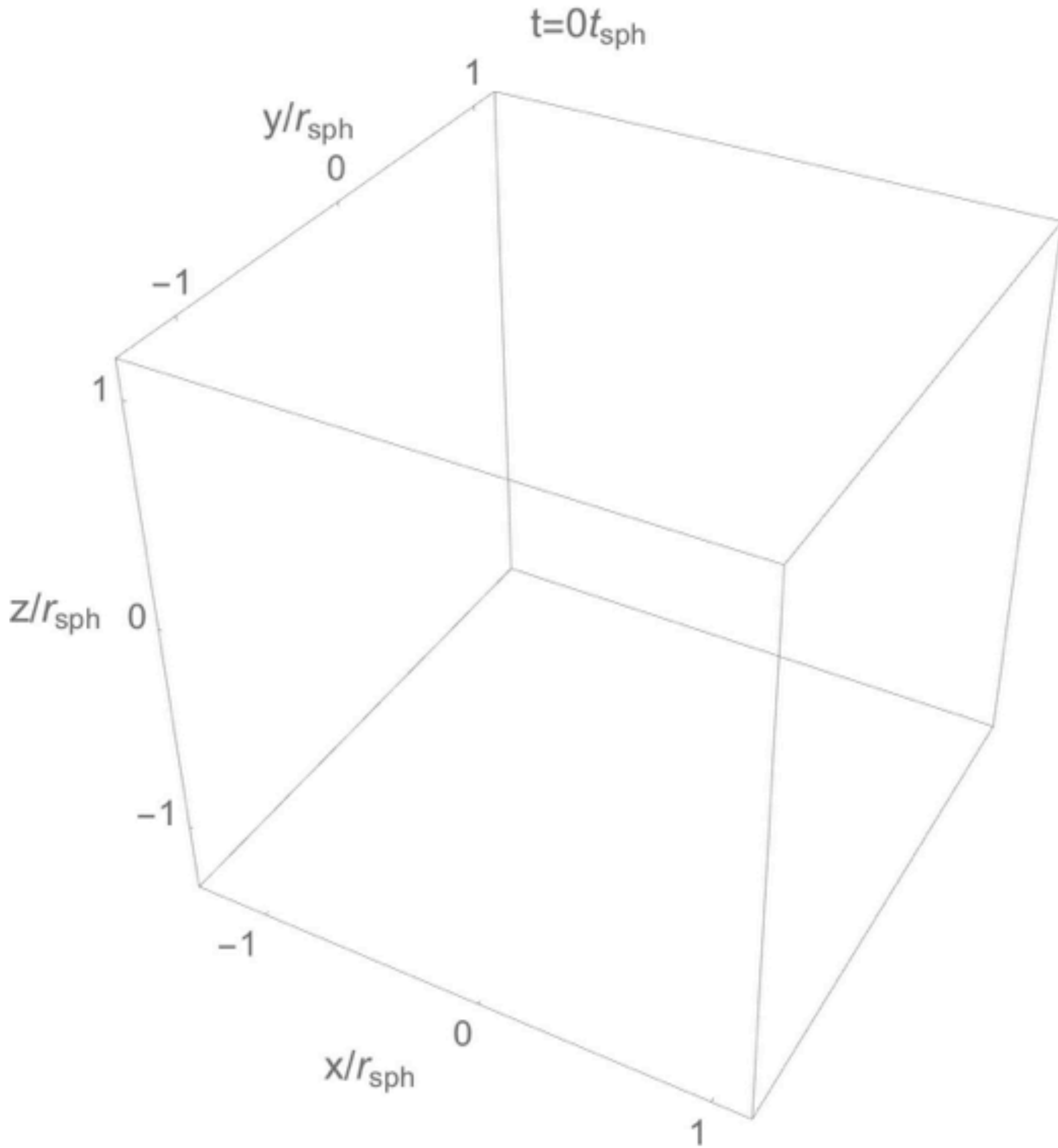
- Want to simulate in clean, controlled environment to establish theoretical techniques
- Consider single sphaleron
    - Unit change in Chern-Simons number induces axial charge imbalance

$$\Delta N_{CS} = \frac{g^2}{8\pi} \int d^4x \mathbf{E}^a \cdot \mathbf{B}^a$$

$$\Delta J_5^0 = -2\Delta N_{CS} + 2m_f \int d^4x \langle \bar{\psi} i \gamma_5 \psi \rangle$$



Isolated sphaleron



Local anomaly relation

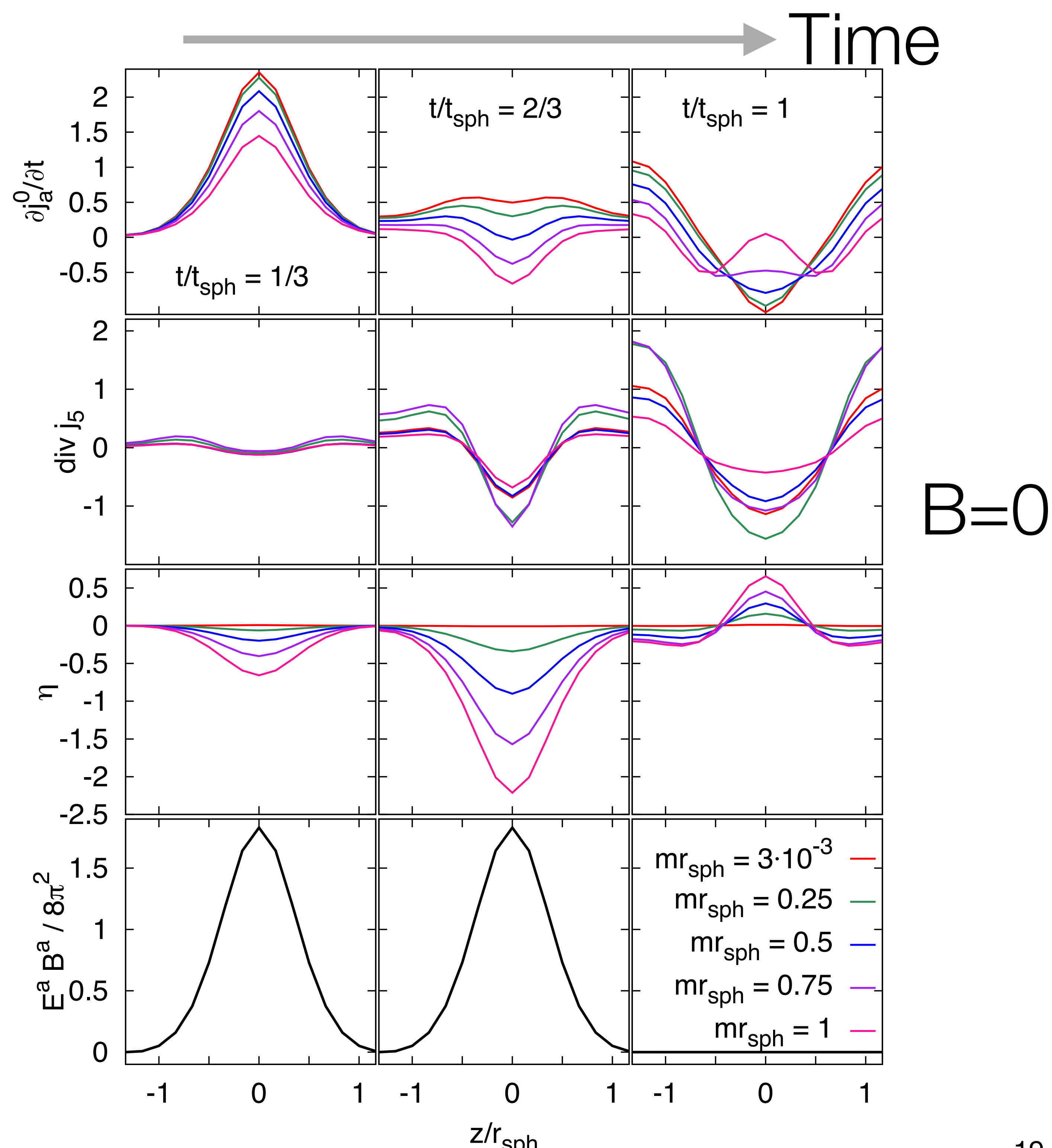
Study local anomaly as a function of fermion mass

$$\partial_\mu j^\mu_a = 2m_f \bar{q} i \gamma_5 q - \frac{g^2}{16\pi^2} F^a_{\mu\nu}(x)\tilde{F}^{\mu\nu}_a(x)$$

$$= 2m_f \eta_a - \frac{g^2}{4\pi^2} E^a_i(x) B^a_i(x)$$

For larger masses, reduction of axial charge

Chirality flipping term takes over

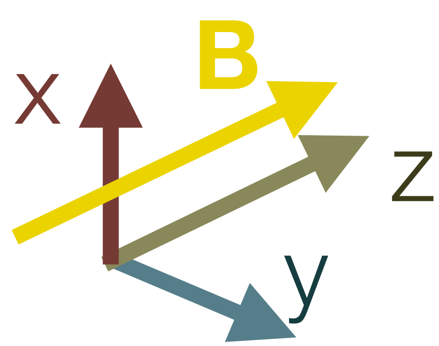


(MM, Mueller, Schlichting, Sharma, in preparation)

Chiral magnetic effect

Sphaleron transition induces local imbalance
of axial charge density j_5^0

Axial charge j_5^0 Vector current j_V^z Vector charge j_V^0



Non-zero magnetic field B_z -> vector current j_V^z is generated

Vector current j_V^z leads to separation of electric charges j_V^0
along the B-field direction

(Slide adapted from S. Schlichting CERN 2016) (Mueller, Schlichting, Sharma arXiv:1606.00342;
MM, Mueller, Schlichting, Sharma, in preparatior

Chiral magnetic effect

Sphaleron transition induces local imbalance
of axial charge density j_5^0

Axial charge j_5^0 | Vector current j_V^z | Vector charge j_V^0

Non-zero magnetic field B_z -> vector current j_V^z is generated

Vector current j_V^z leads to separation of electric charges j_V^0
along the B-field direction

(Slide adapted from S. Schlichting CERN 2016)

(Mueller, Schlichting, Sharma arXiv:1606.00342;
MM, Mueller, Schlichting, Sharma, in preparation

Chiral magnetic wave

Vector charge imbalance j_V^0 generates an axial current j_5^z
so that axial charge also flows along the B-field direction

Axial charge j_5^0 Vector current j_V^z Vector charge j_V^0

Shock-wave of vector charge and axial charge
propagating along B-field direction

(Slide adapted from S. Schlichting CERN 2016)

(Mueller, Schlichting, Sharma arXiv:1606.00342;
MM, Mueller, Schlichting, Sharma, in preparation

Chiral magnetic wave

Vector charge imbalance j_V^0 generates an axial current j_5^z
so that axial charge also flows along the B-field direction

Axial charge j_5^0 | Vector current j_V^z | Vector charge j_V^0

Shock-wave of vector charge and axial charge
propagating along B-field direction

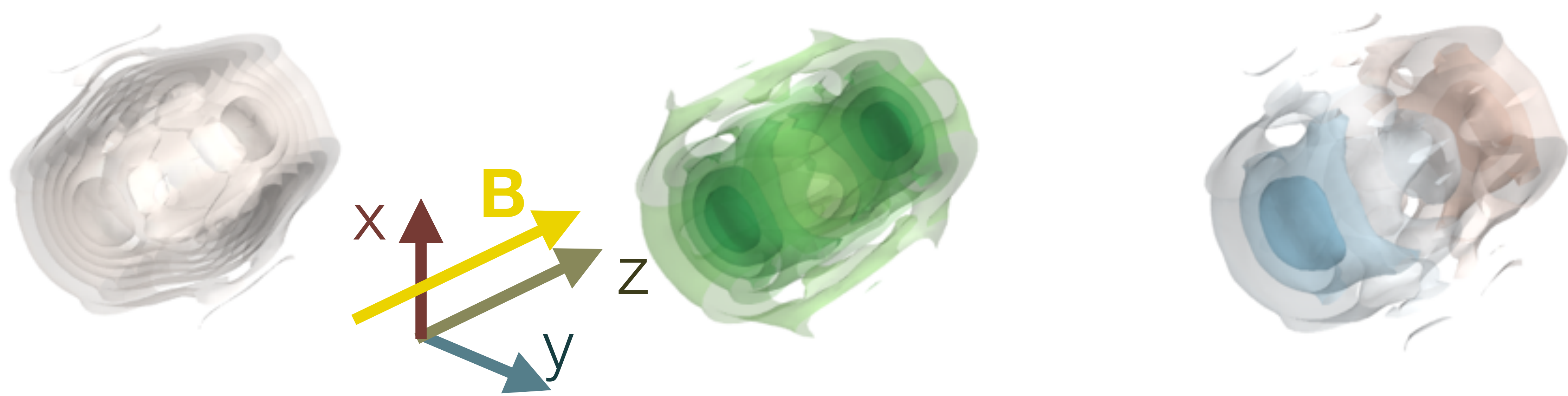
(Slide adapted from S. Schlichting CERN 2016)

(Mueller, Schlichting, Sharma arXiv:1606.00342;
MM, Mueller, Schlichting, Sharma, in preparation

Chiral magnetic wave

Vector charge imbalance j_V^0 generates an axial current j_5^z
so that axial charge also flows along the B-field direction

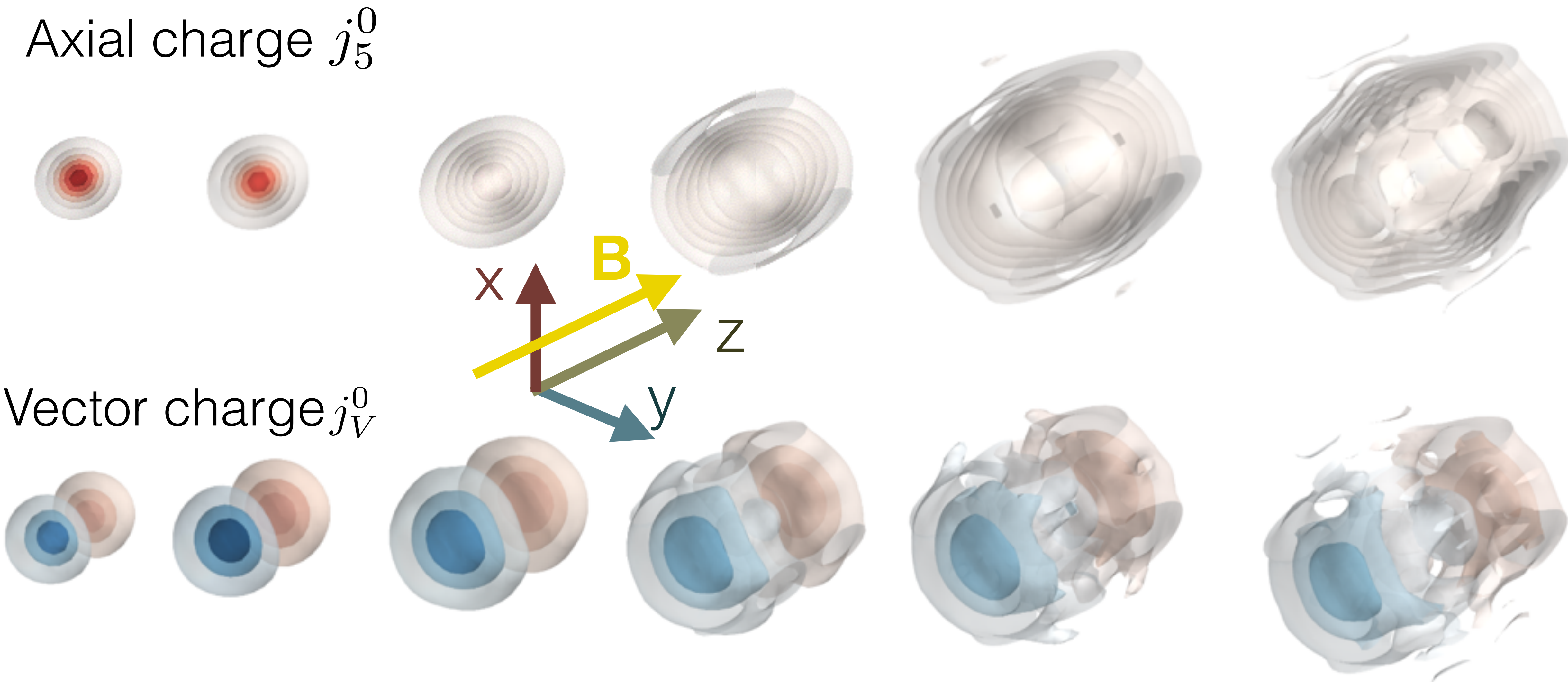
Axial charge j_5^0 Vector current j_V^z Vector charge j_V^0



Shock-wave of vector charge and axial charge
propagating along B-field direction

(Slide adapted from S. Schlichting CERN 2016) (Mueller, Schlichting, Sharma arXiv:1606.00342;
MM, Mueller, Schlichting, Sharma, in preparation

CME/CMW Dynamically



Clear separation of electric charge j_V^0 along the B-field direction

Proof of principle of a microscopic description of anomalous transport phenomena out-of-equilibrium

(Slide adapted from S. Schlichting CERN 2016)

(Mueller, Schlichting, Sharma arXiv:1606.00342; MM, Mueller, Schlichting, Sharma, in preparation)

Magnetic Field Dependence

- Compare with anomalous hydrodynamics
  (Son & Surowka PRL 103 (2009) 191601)

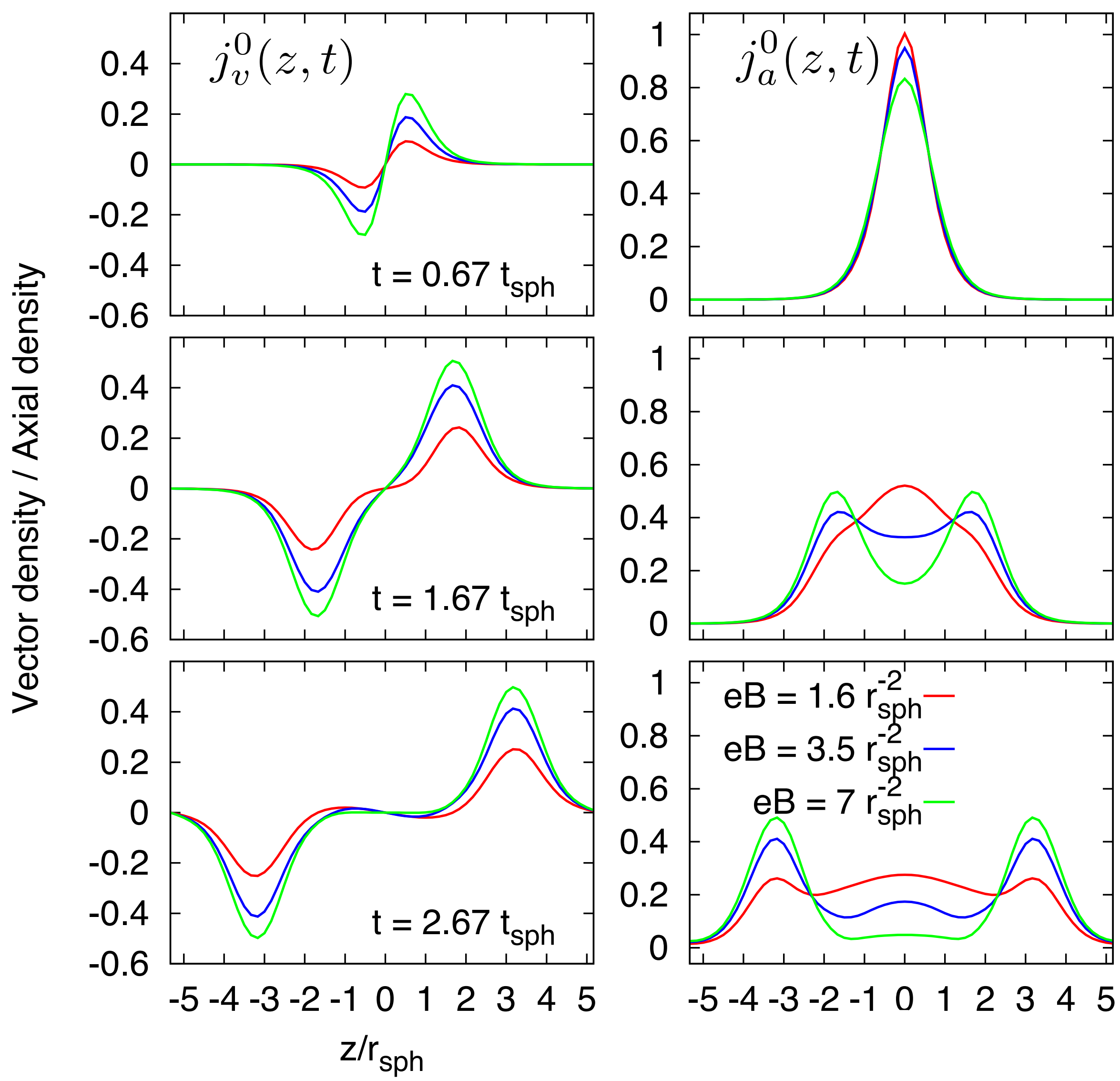
$$j^{\mu}_{v,a} = n_{v,a} u^{\mu} + D_{v,a} \Delta^{\mu} n_{v,a} + \sigma^{B}_{v,a} B^{\mu}$$

- Strong field limit $eB \gg r_{sph}^{-2}, m_f$

$$\sigma^{B}_{v,a} = n_{a,v}/B$$

$$\partial_t \begin{pmatrix} j^0_v(t,z) \\ j^0_a(t,z) \end{pmatrix} = -\partial_z \begin{pmatrix} j^0_a(t,z) \\ j^0_v(t,z) \end{pmatrix} + \begin{pmatrix} 0 \\ S(t,z) \end{pmatrix}$$

$m_f \ll r_{sph}^{-1}$



Shock-wave solution

$$j^0_{v,a}(t > t_{\rm sph}, z) = \frac{1}{2} \int_0^{t_{\rm sph}} dt' \Big(S\big(t', z - c(t-t')\big) \mp S\big(t', z + c(t-t')\big) \Big)$$

What happens away from this limit?

$$S(t,z) = -\frac{g^2}{8\pi^2} \int d^2x_{\perp} \mathrm{Tr}\, F^{\mu\nu} \tilde{F}_{\mu\nu}$$

(MM, Mueller, Schlichting, Sharma, in preparation)

Quark mass dependence

Light quarks $m_f \ll r_{sph}^{-1}$

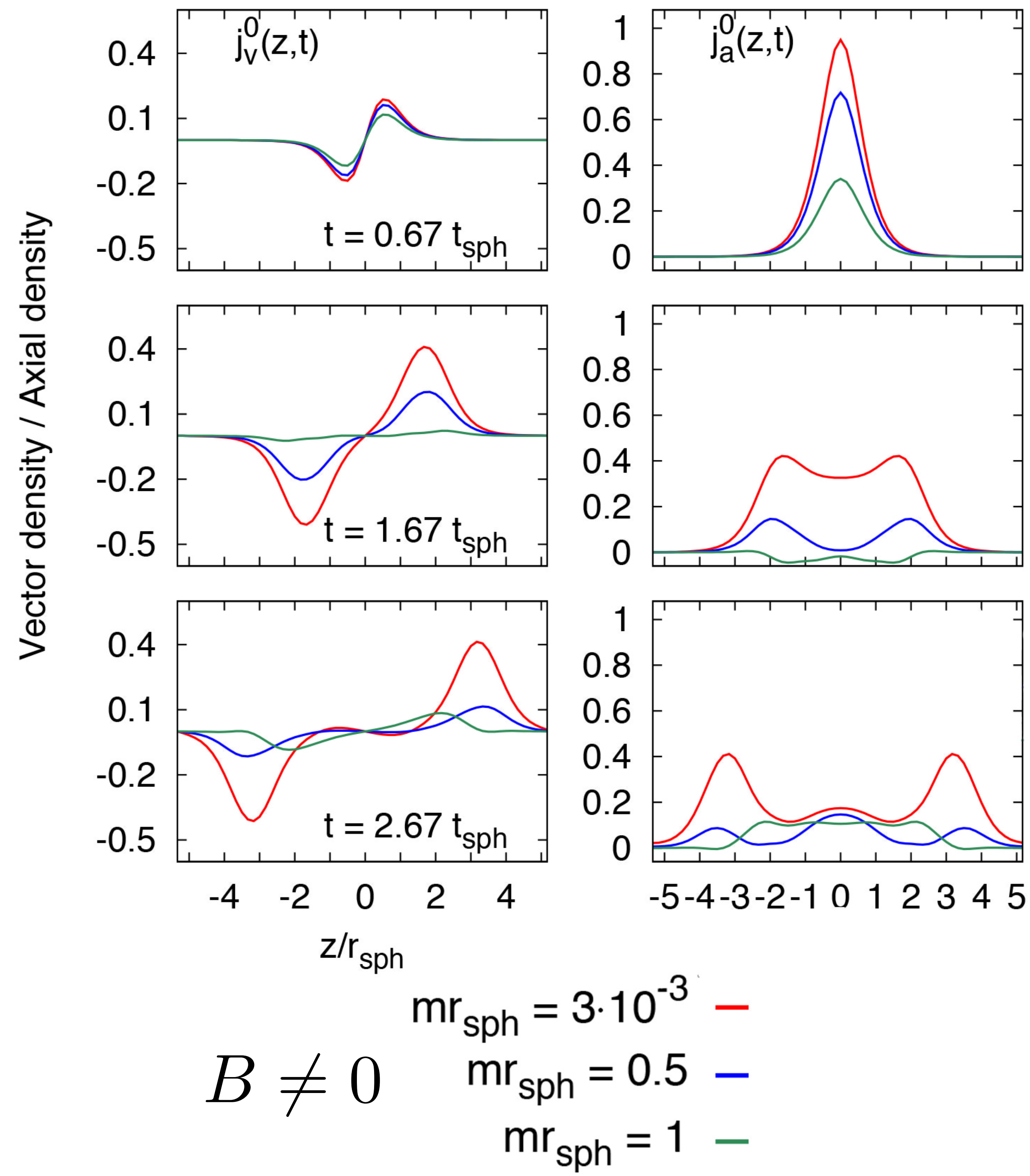
Non-dissipative transport from Chiral Magnetic Wave

Well described by anomalous hydrodynamic "shock-wave"

Heavy quarks $m_f \sim r_{sph}^{-1}$

Dissipation of axial charge leads to reduced axial charge density

How to include in macroscopic picture?



(MM, Mueller, Schlichting, Sharma, in preparation)

Next steps

First calculation of axial charge production from weakly coupled pre-equilibrium dynamics

Sphaleron rate greatest at early times, axial charge from sphalerons studied

Techniques of real-time lattice fermions established

Successful microscopic description of simple system

Quark production and axial charge production from more realistic gauge configurations next

Goal to provide first principles initial conditions for anomalous hydrodynamics

Thanks!