Non-boost-invariant dissipative hydrodynamics

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details in:
W. Florkowski, R. R., M. Strickland, L. Tinti, arXiv: 1609.06293

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Motivation (I)

- the ultimate goal of the URHIC experiments is to understand/extract the properties of the produced matter ($\eta/s, \zeta/s, ...$)
- predictions of standard effective dissipative approaches agree with data surprisingly well
- for their derivation small deviations from LE are usually assumed
- URHIC dynamics leads inevitably to rapid longitudinal expansion and large pressure anisotropies
- their presence triggered intense development of the effective fluid dynamical models
- in addition to the improvements of the standard perturbative frameworks the anisotropic hydrodynamics was invented ...
  W. Florkowski and R. R., Phys. Rev. C83, 034907 (2011), 1007.0130
  M. Martinez and M. Strickland, Nucl. Phys. A848, 183 (2010), 1007.0889
- ... and further improved
  D. Bazow, U. W. Heinz, and M. Strickland, Phys. Rev. C90, 054910 (2014)
  D. Bazow, U. W. Heinz, and M. Martinez, Phys. Rev. C91, 064903 (2015)
  E. Molnar, H. Niemi, and D. H. Rischke, Phys. Rev. D93, 114025 (2016)
  L. Tinti, Phys. Rev. C 94, 044902 (2016)

→ see W. Florkowski talk

- within the underlying RTA kinetic theory good performance of *aHydro was shown for certain highly-symmetric flows (Bjorken, Gubser) **→ see U. Heinz talk**
  W. Florkowski, R. R., and M. Strickland, Nucl. Phys. A916, 249 (2013)
  G. S. Denicol, U. W. Heinz, M. Martinez, J. Noronha, and M. Strickland, Phys. Rev. Lett. 113, 202301 (2014)
  M. Nopoush, R. R., and M. Strickland, Phys. Rev. D91, 045007 (2015)
- the ability to capture both close-to-equilibrium and free-streaming limits makes it a unique tool

Motivation (II)

- due to limited knowledge of longitudinal dynamics in URHIC boost-invariance is employed usually in simulations
- if relaxed, the gradients in the rapidity direction provide a new source of pressure corrections that affect the hydrodynamical evolution
- since aHydro is devoted to handle them more precisely we expect some quantitative differences as compared to standard dissipative approaches
- in order to make the problem numerically and analytically well controlled we assume conformal symmetry and (1+1)D transversely homogenous geometry
- often fluid dynamical approaches are derived from kinetic theory
- we use RTA collisional kernel as the common denominator for the study (classical statistics)

General principles

- neglect conserved charges
- conservation of energy and momentum yields $\partial_\mu T^{\mu\nu} = 0$
- use Landau definition of four-velocity $U_\mu T^{\mu\nu} = \mathcal{E} U^\nu$
- the most general decomposition of $T^{\mu\nu}$ in this frame is

$$T^{\mu\nu} = \mathcal{E}\, U^\mu U^\nu - \left(\mathcal{P} + \Pi\right) \Delta^{\mu\nu} + \pi^{\mu\nu}$$

- temperature is defined via Landau matching $\mathcal{E}(x) = \mathcal{E}_{\rm eq}\left(T(x)\right)$
- consider conformal fluid $\mathcal{P} = \mathcal{E}/3$, $\Pi = 0$

$$U_\nu \partial_\mu T^{\mu\nu} = 0 \qquad \Longrightarrow \qquad D\mathcal{E} = -\left(\mathcal{E} + \mathcal{P}\right)\theta + \pi_{\mu\nu}\sigma^{\mu\nu}$$

$$\Delta^\alpha_\nu \partial_\mu T^{\mu\nu} = 0 \qquad \Longrightarrow \qquad \left(\mathcal{E} + \mathcal{P}\right) DU^\alpha = \nabla^\alpha \mathcal{P} - \Delta^\alpha_\mu \partial_\nu \pi^{\mu\nu}$$

$$D = U^\mu \partial_\mu, \qquad \theta = \partial_\mu U^\mu \qquad \nabla^\mu = \Delta^{\mu\nu}\partial_\nu, \qquad \sigma^{\mu\nu} = \partial^{\langle\mu} U^{\nu\rangle}$$

$$A^{\langle\mu\nu\rangle} = \Delta^{\mu\nu}_{\alpha\beta} \qquad A^{\alpha\beta}\Delta^{\mu\nu}_{\alpha\beta} = \left(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\nu_\alpha \Delta^\mu_\beta\right)/2 - \Delta^{\mu\nu}\Delta_{\alpha\beta}/3$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - U^\mu U^\nu$$

Dissipative hydrodynamics formulations (I)

- the conservation equations are supplemented with dynamic equations for $\pi^{\mu\nu}$
- later on we restrict ourselves to (1+1)D evolution thus we put $\omega^{\mu\nu} = 0$
- as the underlying microscopic theory we consider RTA Boltzmann equation
- Müller-Israel-Stewart (MIS) approach
  I. Muller, Z. Phys. 198, 329 (1967)
  W. Israel, Annals Phys. 100, 310 (1976)
  J. M. Stewart, Proc. R. Soc. London A 357 , 59 (1977)

  a common version of it has the following form
  A. Muronga, Phys. Rev. Lett. 88, 062302 (2002)

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}\right) = -\pi^{\mu\nu}\frac{T\beta_\pi}{2}\partial_\rho\left(\frac{1}{T\beta_\pi}U^\rho\right) \approx -\frac{4}{3}\pi^{\mu\nu}\theta$$

- Baier-Romatschke-Son-Starinets-Stephanov (BRSSS) approach
  R. Baier, P. Romatschke, D. T. Son, A. O. Starinets, and M. A. Stephanov, JHEP 04, 100 (2008)

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}\right) = -\frac{d}{d-1}\pi^{\mu\nu}\theta + \frac{\lambda_1}{\tau_\pi\eta^2}\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha}$$

  matching its gradient expansion to the one of RTA BE yields: $\lambda_1 = \frac{5}{7}\tau_\pi\eta$
  W. Florkowski, R. R. and M. Spaliński, arXiv:1608.07558
  M. P. Heller, A. Kurkela and M. Spaliński, arXiv:1609.04803

→ see W. Florkowski talk

  for RTA BE one also has $\beta_\pi = 4\mathcal{P}/5$

Dissipative hydrodynamics formulations (II)

- Denicol-Niemi-Molnar-Rischke (DNMR) approach
  G. S. Denicol, H. Niemi, E. Molnar, and D. H. Rischke, Phys. Rev. D85, 114047 (2012)

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}\right) = -\frac{\delta_{\pi\pi}}{\tau_\pi}\pi^{\mu\nu}\theta - \frac{\tau_{\pi\pi}}{\tau_\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \frac{\varphi_7}{\tau_\pi}\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha}$$

  for RTA BE: $\frac{\delta_{\pi\pi}}{\tau_\pi} = \frac{4}{3}, \quad \frac{\tau_{\pi\pi}}{\tau_\pi} = \frac{10}{7}, \quad \phi_7 = 0$

  same result obtained by Jaiswal with modified Chapmann-Enskog method
  A. Jaiswal, Phys. Rev. C87, 051901 (2013)

  → see A. Jaiswal talk

- anisotropic hydrodynamics (aHydro) employing Tinti *anisotropic matching principle*
  L. Tinti, Phys.Rev. C94 (2016) 044902

  for the special case of RTA BE EOM read

  → see L. Tinti talk

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{1}{\tau_{\rm eq}}\pi^{\mu\nu} = -\left(\sigma_{\rho\sigma} + \frac{1}{3}\,\theta\,\Delta_{\rho\sigma}\right)\int dP\,\frac{p^{\langle\mu}p^{\nu\rangle}p^\rho p^\sigma f_a}{(p\cdot U)^2} - 2\,\pi_\alpha^{<\mu}\sigma^{\nu>\alpha} + 2\mathcal{P}\,\sigma^{\mu\nu} - \frac{5}{3}\,\theta\,\pi^{\mu\nu}$$

$$f_a(x,p) = k\exp\left[-\frac{1}{\Lambda(x)}\sqrt{p_\mu\Xi^{\mu\nu}(x)p_\nu}\right]$$

$$\Xi^{\mu\nu} = \phi_U\,U^\mu U^\nu + \xi^{\langle\mu\nu\rangle}$$

Implementation of (1+1)D non-boost-invariant expansion

- consider the transversally homogenous system
- in hyperbolic coordinates choose the complete orthonormal basis

$$U^\mu = \Big(\cosh(\eta+\theta_\|(\tau,\eta)), 0, 0, \sinh(\eta+\theta_\|(\tau,\eta))\Big)$$
$$X^\mu = (0,1,0,0)$$
$$Y^\mu = (0,0,1,0)$$
$$Z^\mu = \Big(\sinh(\eta+\theta_\|(\tau,\eta)), 0, 0, \cosh(\eta+\theta_\|(\tau,\eta))\Big)$$

$$\Delta^{\mu\nu} = -\sum_I I^\mu I^\nu \qquad I \in \{X, Y, Z\}$$

- the symmetry implies significant simplifications

$$\pi^{\mu\nu} = \sum_I \pi_I I^\mu I^\nu \qquad \pi_X = \pi_Y = \pi_s(\tau,\eta)/2 \qquad \pi_Z = -\pi_s(\tau,\eta)$$
$$\mathcal{P}_I = I_\mu I_\nu T^{\mu\nu} \qquad \mathcal{P}_T = \mathcal{P} + \frac{\pi_s}{2} \qquad \mathcal{P}_L = \mathcal{P} - \pi_s$$

- energy-momentum conservation equations read

$$D\mathcal{E} = -\Big(\mathcal{E}+\mathcal{P}_L\Big)\theta, \qquad D_L\mathcal{P}_L = -\Big(\mathcal{E}+\mathcal{P}_L\Big)\theta_L$$

$$D_L = Z^\mu\partial_\mu, \qquad \theta_L = \partial_\mu Z^\mu$$

Dissipative hydrodynamics equations in (1+1)D setup (I)

- Müller-Israel-Stewart (MIS) approach

$$D\pi_s + \frac{1}{\tau_\pi}\left(\pi_s - \frac{4\eta}{3}\theta\right) = -\frac{4}{3}\theta\pi_s$$

- Baier-Romatschke-Son-Starinets-Stephanov (BRSSS) approach

$$D\pi_s + \frac{1}{\tau_\pi}\left(\pi_s - \frac{4\eta}{3}\theta\right) = -\frac{4}{3}\,\theta\,\pi_s - \frac{5}{14\eta}\,\pi_s^2$$

- Denicol-Niemi-Molnar-Rischke (DNMR) approach

$$D\pi_s + \frac{1}{\tau_\pi}\left(\pi_s - \frac{4\eta}{3}\theta\right) = -\frac{38}{21}\,\theta\,\pi_s$$

Dissipative hydrodynamics equations in (1+1)D setup (II)

- anisotropic hydrodynamics (aHydro) approach

$$f_a(x,p) = k\, f_{iso}\left[-\frac{1}{\Lambda(x)}\sqrt{(p_\mu U^\mu)^2 + \xi (p_\mu Z^\mu)^2}\right]$$

- thermodynamic variables, $f_{iso}(x) = e^{-x}$

$$\begin{aligned}
\mathcal{E} &= \mathcal{E}_{\rm eq}(\Lambda)\mathcal{R}(\xi) = 3\mathcal{P}_{\rm eq}(\Lambda)\mathcal{R}(\xi)\\
\mathcal{P}_L &= \mathcal{P}_{\rm eq}(\Lambda)\mathcal{R}_L(\xi)\\
\mathcal{P}_T &= \mathcal{P}_{\rm eq}(\Lambda)\mathcal{R}_T(\xi)
\end{aligned}$$

- energy-momentum conservation

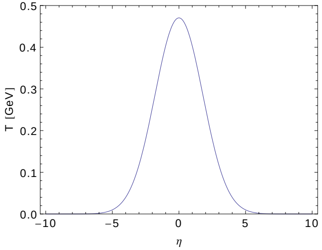
$$\mathcal{R}(\xi) D \ln \mathcal{P}_{\rm eq}(\Lambda) + \mathcal{R}'(\xi) D\xi + \frac{2}{3}\Big[\mathcal{R}_T(\xi) + \mathcal{R}_L(\xi)\Big]\theta = 0$$

$$\mathcal{R}_L(\xi) D_L \ln \mathcal{P}_{\rm eq}(\Lambda) + \mathcal{R}'_L(\xi) D_L\xi + 2\Big[\mathcal{R}_T(\xi) + \mathcal{R}_L(\xi)\Big]\theta_L = 0$$

- anisotropy evolution

$$\Big[\mathcal{R}_T(\xi) - \mathcal{R}_L(\xi)\Big]\left[D \ln \mathcal{P}_{\rm eq}(\Lambda) + \frac{1}{\tau_{\rm eq}}\right] + \Big[\mathcal{R}'_T(\xi) - \mathcal{R}'_L(\xi)\Big]\Big[D\xi - 2(1+\xi)\theta\Big] = 0$$

Setup



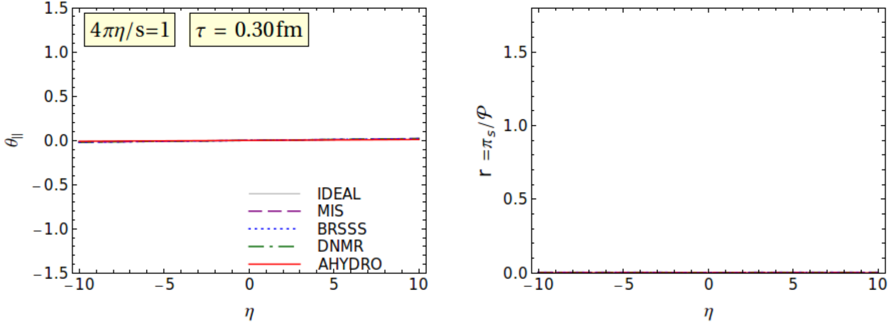
- we have to solve 3 coupled PDE for:
  $T(\tau,\eta), \theta_{\|}(\tau,\eta), \pi_s(\tau,\eta)$ (viscous) or
  $\Lambda(\tau,\eta), \theta_{\|}(\tau,\eta), \xi(\tau,\eta)$ (anisotropic)

- initial setup:
  $\mathcal{E}(\tau_0,\eta) = \mathcal{E}_0 \exp\left(-\eta^2/(2a^2)\right)$ with $a = 0.9$, $\mathcal{E}_0 = 100\ \mathrm{GeV/fm^3}$ at $\tau_0 = 0.3$ fm
  $\alpha T^4 = \mathcal{E} \qquad \alpha = 15.63$
  $\theta_{\|}(\tau_0,\eta) = b\,\eta \qquad b = 10^{-10}$
  $\pi_s(\tau_0,\eta) = c\,\mathcal{P}(\tau_0,\eta) \qquad c = 10^{-10}$
  $\frac{\pi_s(\tau_0,\eta)}{\mathcal{P}(\tau_0,\eta)} = 1 - \frac{\mathcal{R}_L(\xi(\tau_0,\eta))}{\mathcal{R}(\xi(\tau_0,\eta))} \quad \rightarrow \xi(\tau_0,\eta)$

- technicalities:
  large spatial gradients → weighted LAX (wLAX) algorithm with $\lambda_{\mathrm{LAX}} = 0.01$
  M. Martinez, R. R., and M. Strickland, Phys. Rev. C85, 064913 (2012)
  standard fourth-order Runge-Kutta for temporal update

Results

- $\eta/s = 1/(4\pi)$ [1]



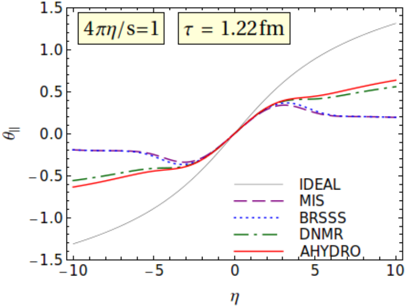
$$\mathrm{R}^{-1} = \sqrt{\pi_{\mu\nu}\pi^{\mu\nu}}/\mathcal{P} = \sqrt{3/2}\pi_s/\mathcal{P} = \sqrt{3/2}r$$

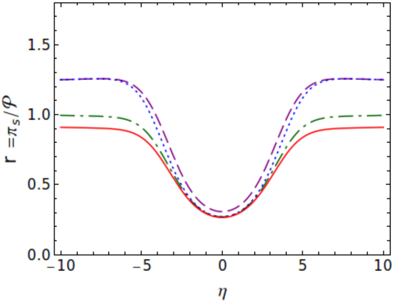
$$\tau^{\mathrm{BRSSS}}_{\mathrm{shock}} < \tau^{\mathrm{MIS}}_{\mathrm{shock}} < \tau^{\mathrm{DNMR}}_{\mathrm{shock}} \ll \tau^{\mathrm{AHYDRO}}_{\mathrm{shock}}$$

all formulations crash eventually unless shock-dealing proceedures are employed

[1] Animation: open in Adobe Reader.

Results



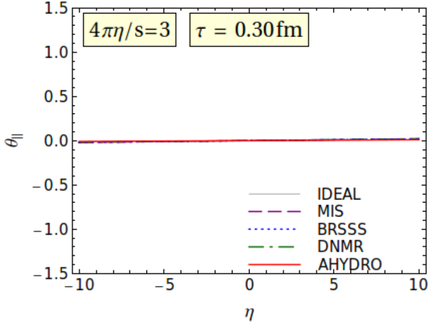


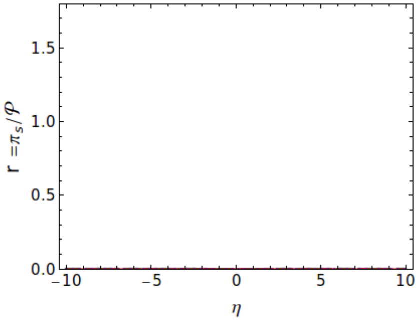
BRSSS: $\sigma^{\langle\mu\alpha}\sigma_{\alpha}{}^{\nu\rangle} \rightarrow \pi^{\langle\mu\alpha}\pi_{\alpha}{}^{\nu\rangle}$ through $\pi^{\mu\alpha} = 2\eta\sigma^{\mu\alpha}$

DNMR: $\sigma^{\langle\mu\alpha}\pi_{\alpha}{}^{\nu\rangle}$

Results

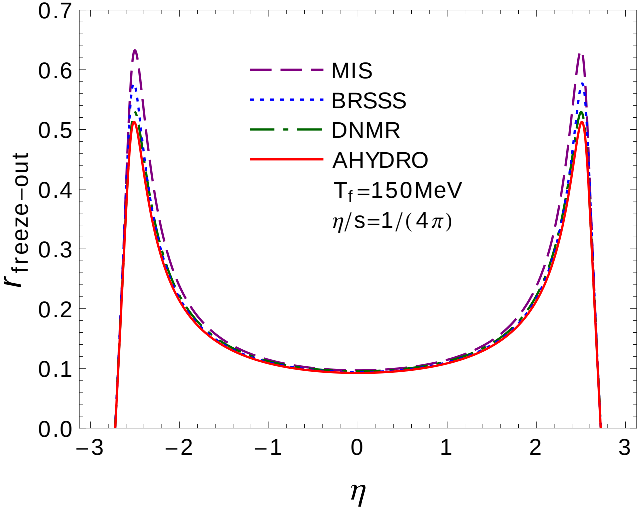
- $\eta/s = 3/(4\pi)$ [2]





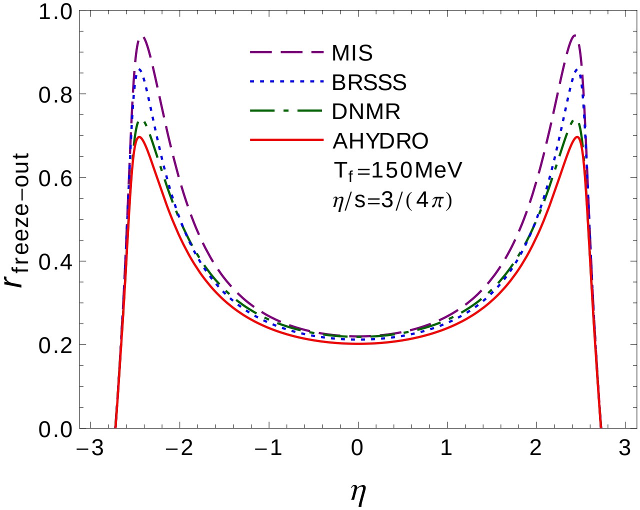
[2] Animation: open in Adobe Reader.

Results



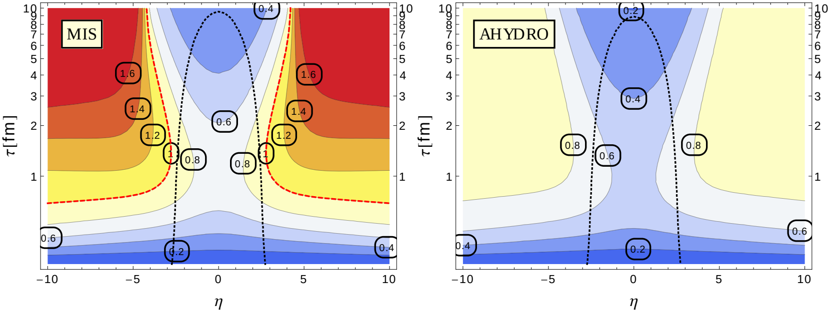
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$r_{\text{freeze-out}}$
MIS
BRSSS
DNMR
AHYDRO
T_f=150MeV
$\eta/s=1/(4\pi)$
−3
−2
−1
0
1
2
3
η

Results



1.0
0.8
0.6
0.4
0.2
0.0
−3
−2
−1
0
1
2
3
$r_{\text{freeze-out}}$
η
MIS
BRSSS
DNMR
AHYDRO
T_f=150MeV
$\eta/s=3/(4\pi)$

Results



Summary

- negative pressure is present in standard hydrodynamic approaches - no such situation allowed in underlying kinetic theory (if fields are neglected)
- pressure in aHydro is always positive thus the dynamics is self-regularized at large spacetime rapidity
- negative pressure at large spacetime rapidity results in matter pile-up and faster development of a shock front
- aHydro substantially delays the formation of a shock
- different treatment of the shear-to-shear coupling terms in the EOM of the shear-stress tensor produces, contrary to common assumptions, a significant difference in the evolution of the system away from equilibrium

Thank you for your attention!