Anisotropic matching principle for the hydrodynamic expansion

Outline

- Hydrodynamics basics and anisotropic background
- Improvements on the leading order
- Extended Landau Matching (and NLO)
- Latest results

Relativistic Hydrodynamics: Theory and modern applications
MITP, Oct. 14 2016

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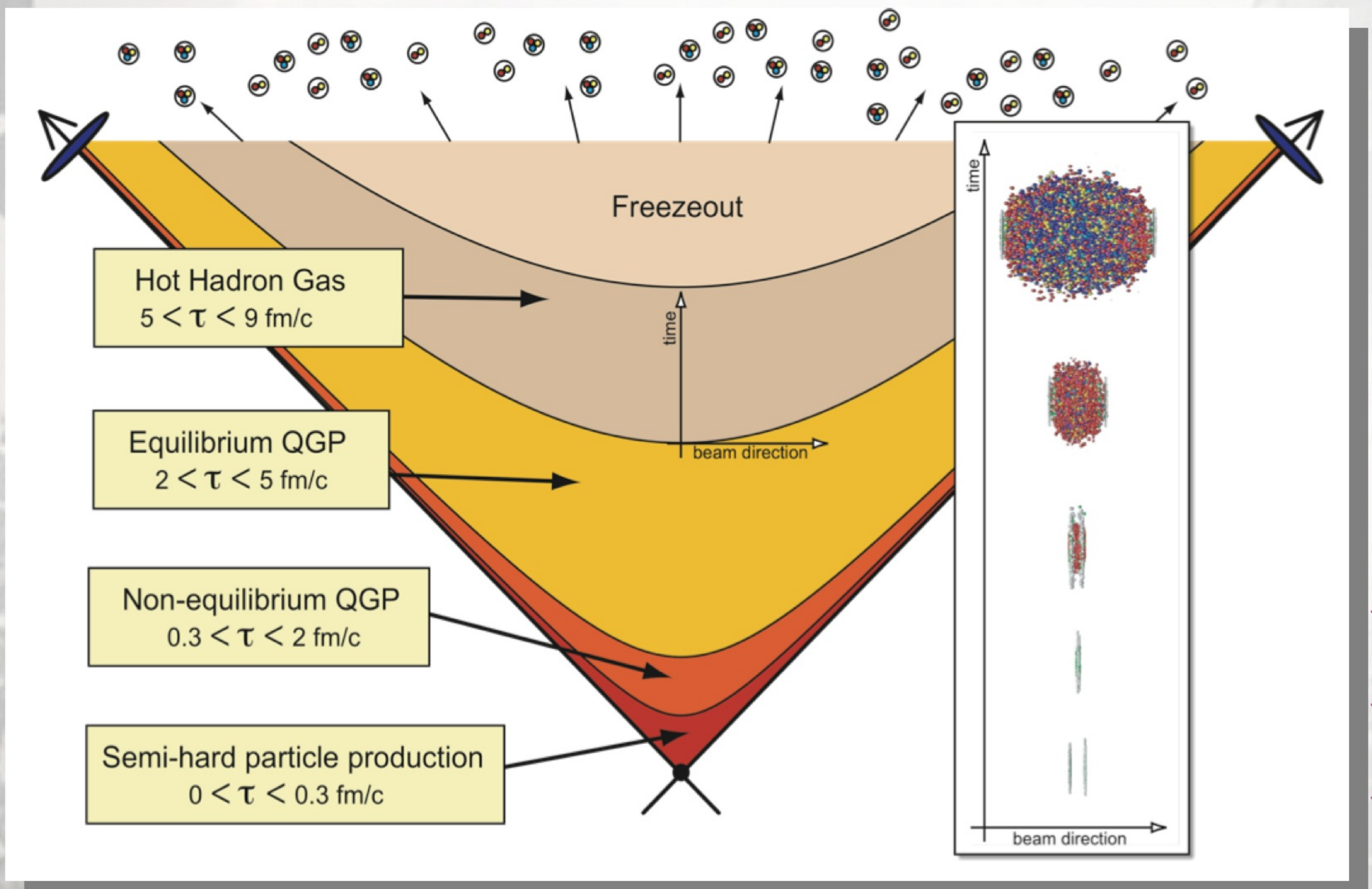
w Kielcach

Hydrodynamics

Hydrodynamic modeling of heavy ion collisions , “almost perfect liquid”.



Strong longitudinal expansion,

pressure anisotropy (also AdS/CFT)

Large momentum anisotropy from microscopic models (pQCCD, CGC)

Local equilibrium?

Equilibration?

Hydrodynamization?

Hydrodynamics

From quantum field theory

$$T^{\mu\nu} = \langle \widehat{T}^{\mu\nu} \rangle$$

$$\partial_\mu T^{\mu\nu} = \langle \partial_\mu \widehat{T}^{\mu\nu} \rangle$$

and translation invariance

$$\partial_\mu T^{\mu\nu} = 0$$

Very general but incomplete, approximations.

Hydrodynamics

Perfect fluid

$$T^{\mu\nu} = (\varepsilon + P)U^\mu U^\nu - g^{\mu\nu}P$$

From four-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Continuity and Euler equations

$$U^\mu \partial_\mu \varepsilon + (\varepsilon + P)\partial_\mu U^\mu = 0 \quad \rightarrow \quad \partial_t \rho + \nabla_{\mathbf{x}} \cdot (\rho \mathbf{v}) = 0$$

$$(\varepsilon + P)A^\mu - \nabla^\mu P = 0 \quad \rightarrow \quad \rho\,\mathbf{a} + \nabla_{\mathbf{x}} P = \rho\,\mathbf{a} + \nabla_{\mathbf{x}} \cdot T|_{\text{pf}} = 0$$

Hydrodynamics?

$$T^{\mu\nu} = (\varepsilon + P)U^\mu U^\nu - g^{\mu\nu} P + \delta T^{\mu\nu}$$

QFT

Equation of state

Kubo formulas

↓

Small deviations from background (equilibrium?)

How to connect them with transport coefficients?

Relativistic kinetic theory

Still embeds microscopic degrees of freedom

Well defined away from equilibrium

Four-momentum conservation (hydrodynamics)

Provides extra equations for the non ideal degrees of freedoms

From kinetic theory to hydrodynamics

Relativistic Boltzmann equation:

$$p^\mu \, \partial_\mu f(x,p) = -\mathcal{C}[f]$$

First moment:

$$\int dP \, p^\mu p^\nu \, \partial_\mu f = \partial_\mu T^{\mu\nu} =$$

$$= - \int dP \, p^\nu \mathcal{C}[f] \left[\, = 0 \right]$$

S. R. De Groot, W. A. van Leeuwen, Ch. G. van Weert, *Relativistic kinetic theory*, North Holland (1980).

Dissipative hydrodynamics

$$f = f_{\text{eq.}} + \delta f \quad \Rightarrow \quad T^{\mu\nu} = T^{\mu\nu}_{\text{eq.}} + \delta T^{\mu\nu}$$

$\delta f \Rightarrow \delta T^{\mu\nu}$ treated as, small, perturbations

Landau frame, massless particles

$$\delta T^{\mu\nu} = \pi^{\mu\nu}$$

$$U^{\mu}\pi_{\mu\nu} = 0 \qquad g_{\mu\nu}\pi^{\mu\nu} = 0$$

- ***Grad moments (inspired) treatment***
- ***Chapman-Enskog (inspired)***
- ***Entropy flux...***

Anisotropic hydrodynamics

Reorganization of the hydrodynamic expansion

$$f = f_{\text{eq.}} + \delta f$$

around an anisotropic background instead of the local equilibrium

$$f = f_{\text{aniso.}} + \delta \tilde{f}$$

"Romatschke-Strickland" form, extraequation from the zeroth moment of th Boltzmann equation

$$f_{\text{aniso.}} = k \exp\left[- \frac{\sqrt{\left(p \cdot U(x) \right)^2 + \xi(x) \left(p \cdot Z(x) \right)^2}}{\Lambda(x)} \right]$$

(0+1)d: It recovers the free streaming limit, and it is consistent with viscous hydrodynamics close to equilibrium

0+1 dimensions

Boost invariant in the longitudinal direction, homogeneous in the transverse plane

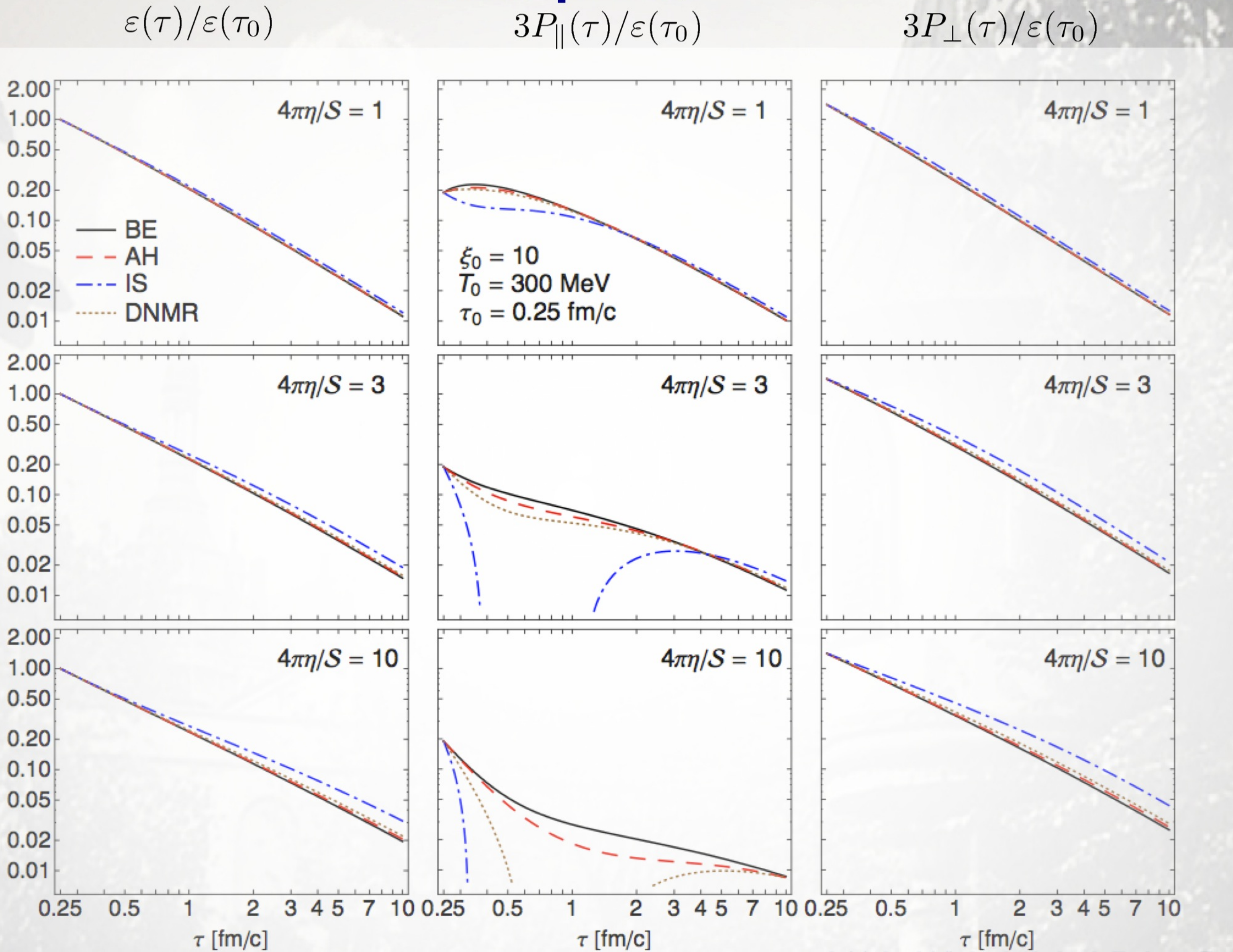
Exact solutions for the Boltzmann equation with the collisional kernel treated in relaxation time approximation

$$\mathcal{C}[f] = (p \cdot U) \, \frac{f - f_{\rm eq.}}{\tau_{\rm eq.}}$$

Test for viscous and anisotropic hydrodynamics!

- **W Florkowski, R Ryblewski and M Strickland, Phys. Rev. C88, 024903 (2013)**

Some plots



$\varepsilon(\tau)/\varepsilon(\tau_0)$
$3P_{\parallel}(\tau)/\varepsilon(\tau_0)$
$3P_{\perp}(\tau)/\varepsilon(\tau_0)$
$4\pi\eta/\mathcal{S} = 1$
$4\pi\eta/\mathcal{S} = 3$
$4\pi\eta/\mathcal{S} = 10$
BE
AH
IS
DNMR
$\xi_0 = 10$
$T_0 = 300$ MeV
$\tau_0 = 0.25$ fm/c
2.00
1.00
0.50
0.20
0.10
0.05
0.02
0.01
0.25
0.5
1
2
3
4
5
7
10
τ [fm/c]

Higher dimensions

Radial flow, pressure asymmetries in the transverse plane

Non trivial transverse dynamics is important to explain collective behavior like the elliptic flow

The Romatschke-Strickland form has only one anisotropy parameter

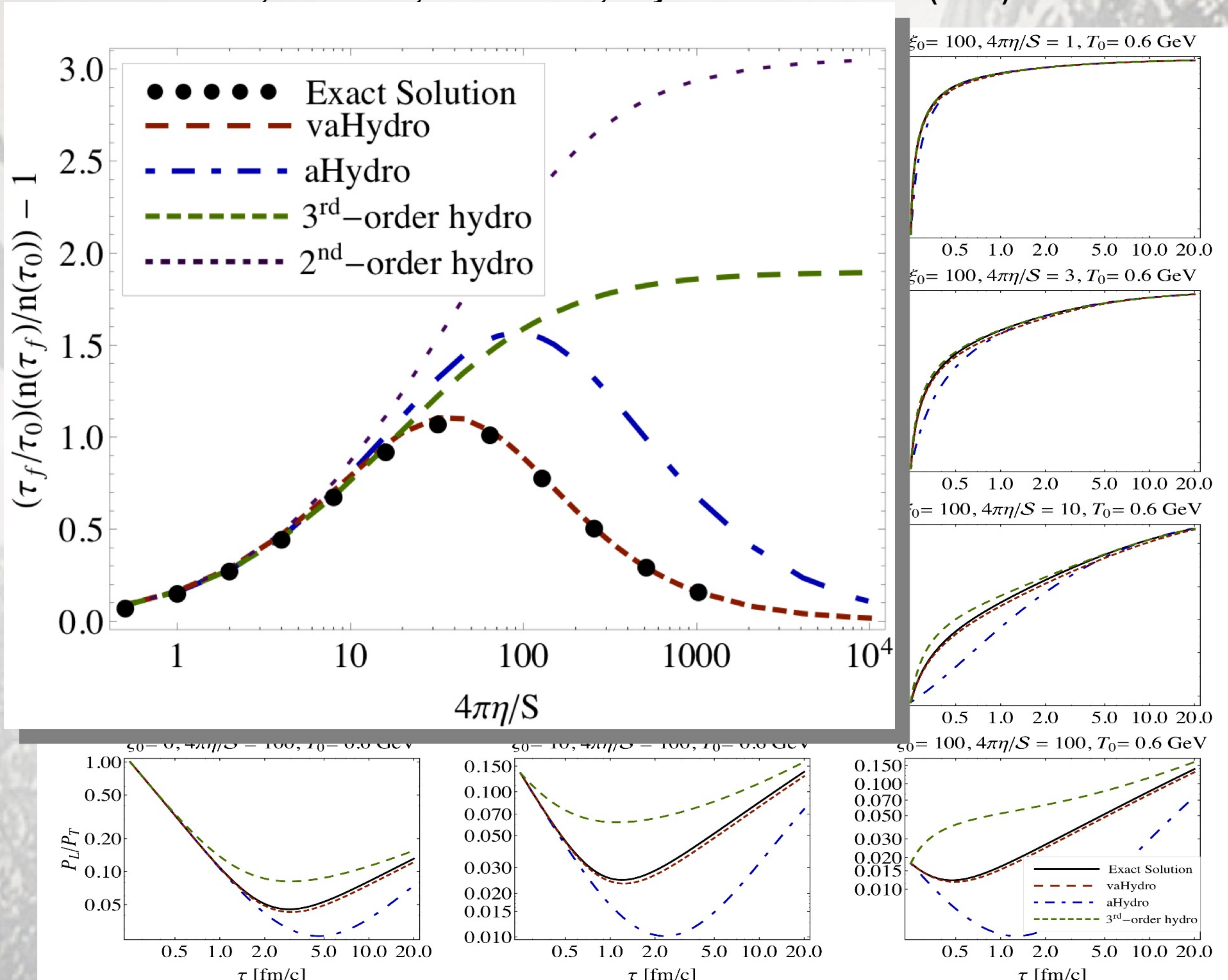
Possible solutions

Next to leading order

- **D Bazow, U W Heinz, M Strickland, Phys.Rev. C 90 054910 (2014)**
- **D Bazow, U W Heinz, M Martinez, Phys.Rev. C 91 064903 (2015)**
- **E. Molnar, H. Niemi, D.H. Rischke, Phys.Rev. D93 114025 (2016)**

or **Improve the leading order**

D Bazow, U W Heinz, M Strickland, Phys.Rev. C 90 054910 (2014)



First step, cylindrically symmetric radial flow

For a conformal system

$$f_{\mathrm{aniso.}} = k \exp\left[-\frac{\sqrt{(1+\xi_X)\,(p\cdot X)^2 + (1+\xi_Y)\,(p\cdot Y)^2 + (1+\xi_Z)\,(p\cdot Z)^2}}{\lambda(x)}\right]$$

Pressure not isotropic in the transverse plane

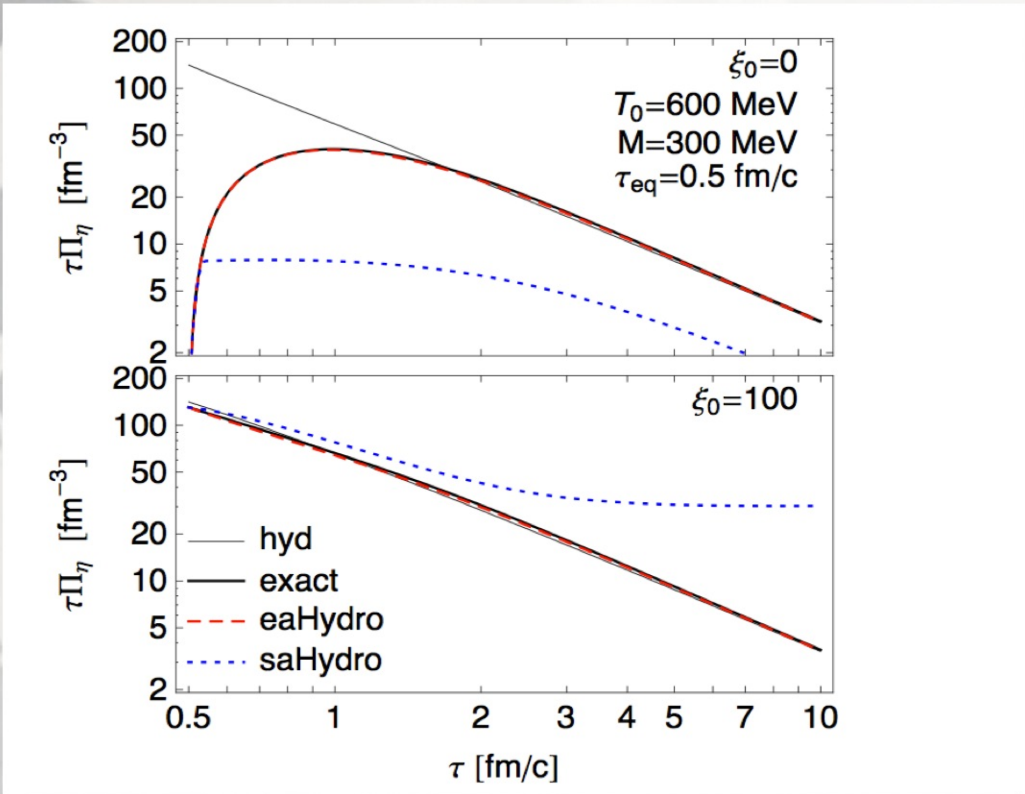
$$\sum_I \xi_I = 0$$

Dynamical equations from the second moment of the Boltzmann equation and four-momentum conservation

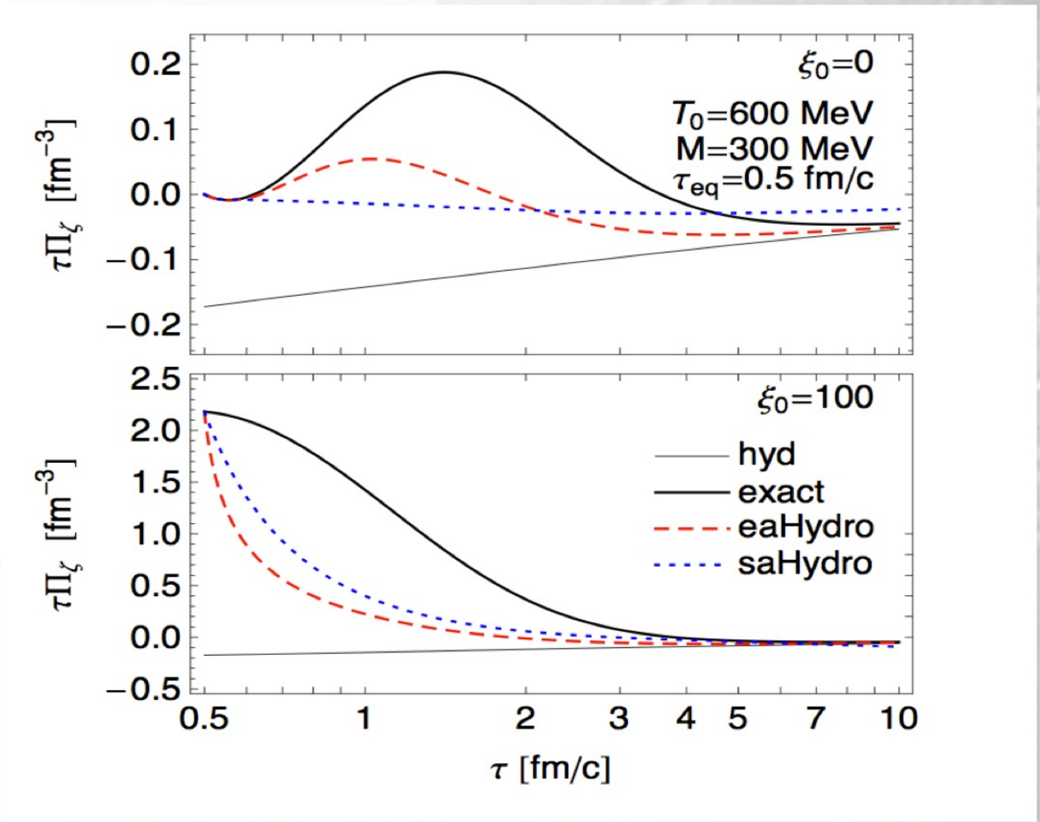
- L Tinti and W Florkowski , Phys. Rev. C 89, 034907 (2014)

Improvements even for the 0+1-dimensional expansion

- W Florkowski, R Ryblewski, M Strickland, L Tinti, Phys. Rev. C 89, 054909 (2014)



Much better agreement for the shear pressure corrections

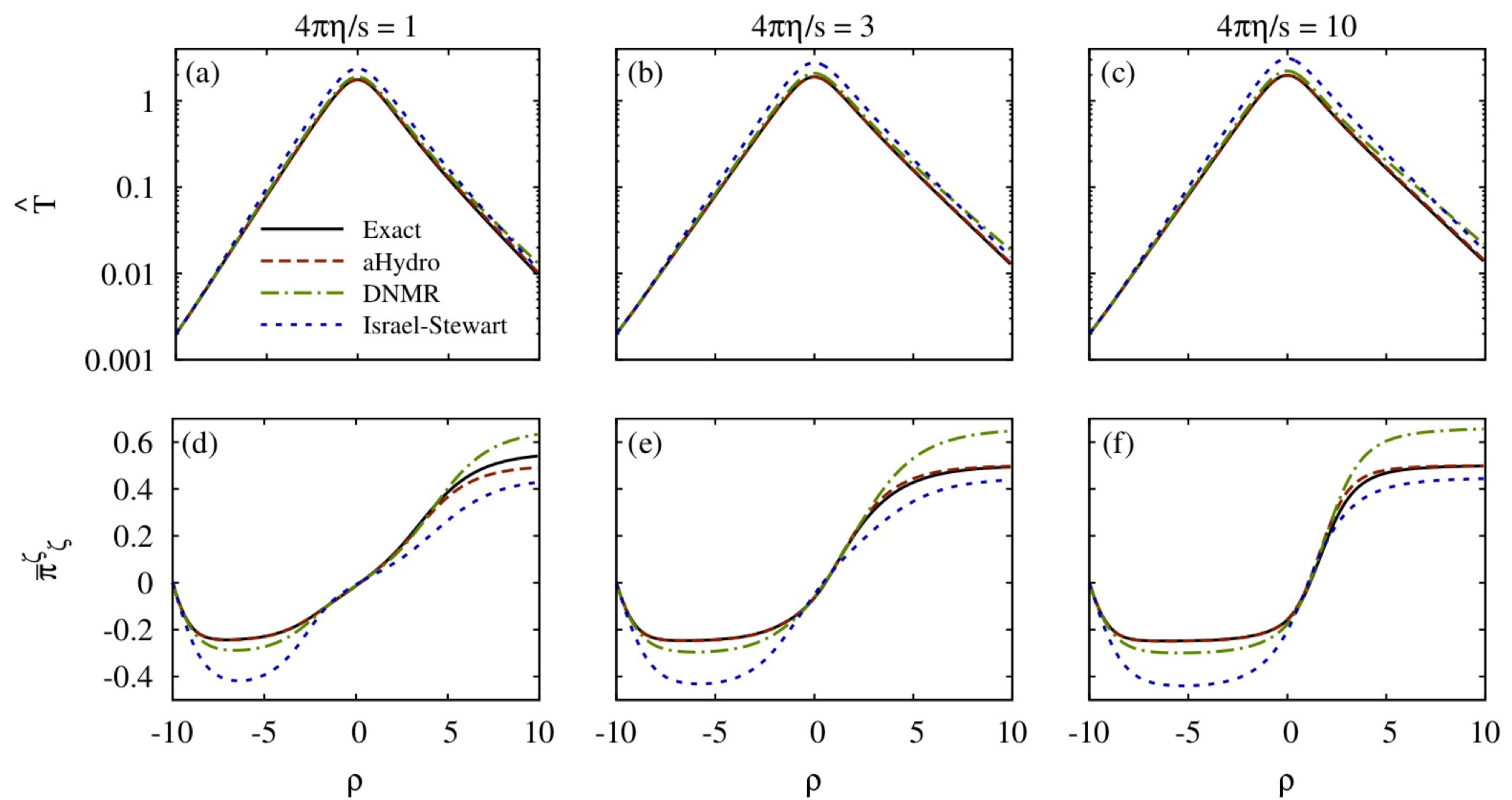


but not for bulk viscosity...

Bulk degree of freedom (M Nopoush, R Ryblewski, M Strickland, Phys. Rev. C 90, 014908 (2014)

Another exact solutions of the Botlzmann equation

Gubser Flow • M Nopoush, R Ryblewski, M Strickland, Phys. Rev. D 91, 045007 (2015)



Another exact solutions of the Botlzmann equation

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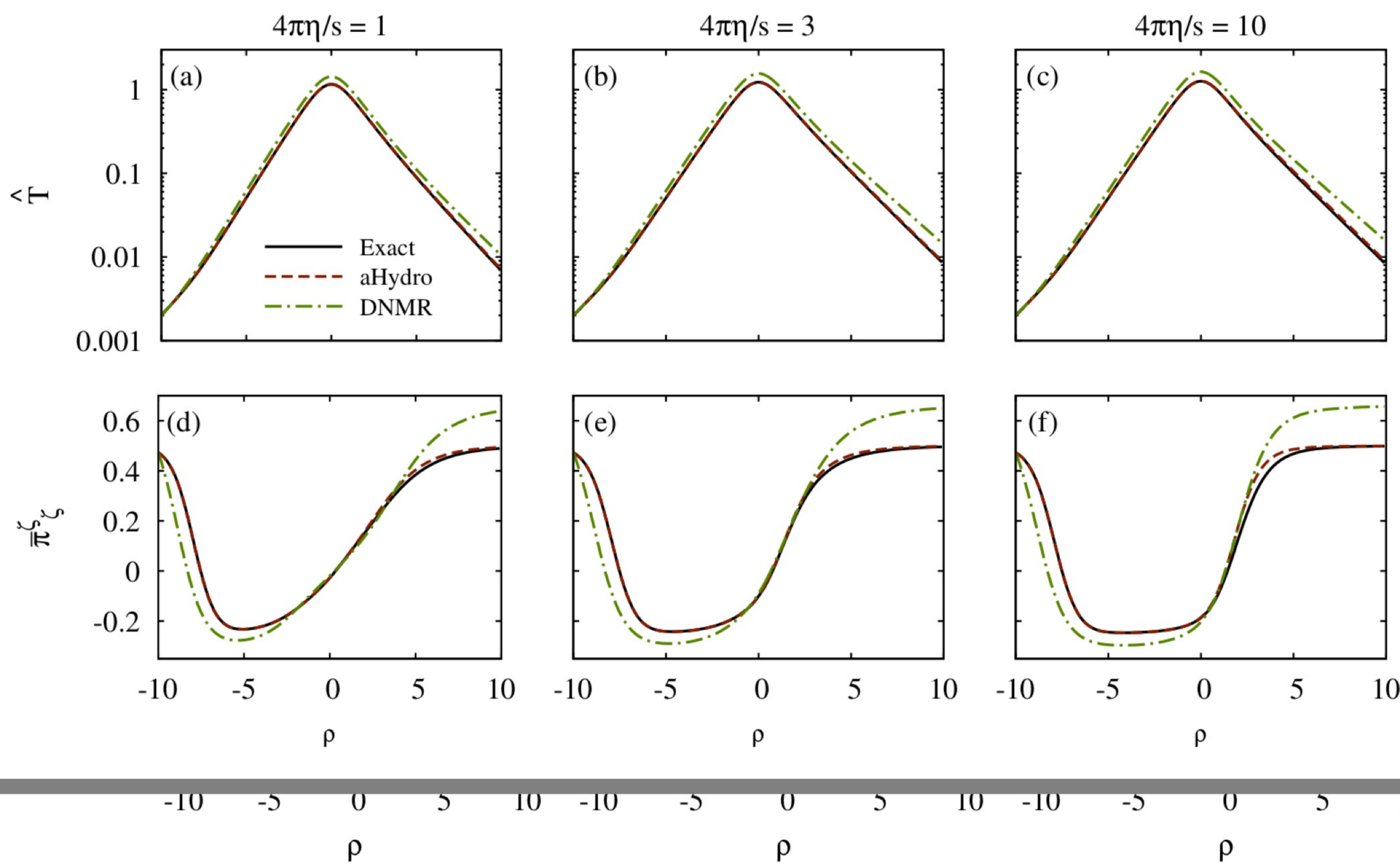


(3+1)-dimensional framework

No symmetry constraints from boost invariance or cylindrical symmetry

Generalized "Romatschke-Strickland" form

$$f = k \exp\left(-\frac{1}{\lambda}\sqrt{p_\mu \Xi^{\mu\nu} p_\nu}\right)$$

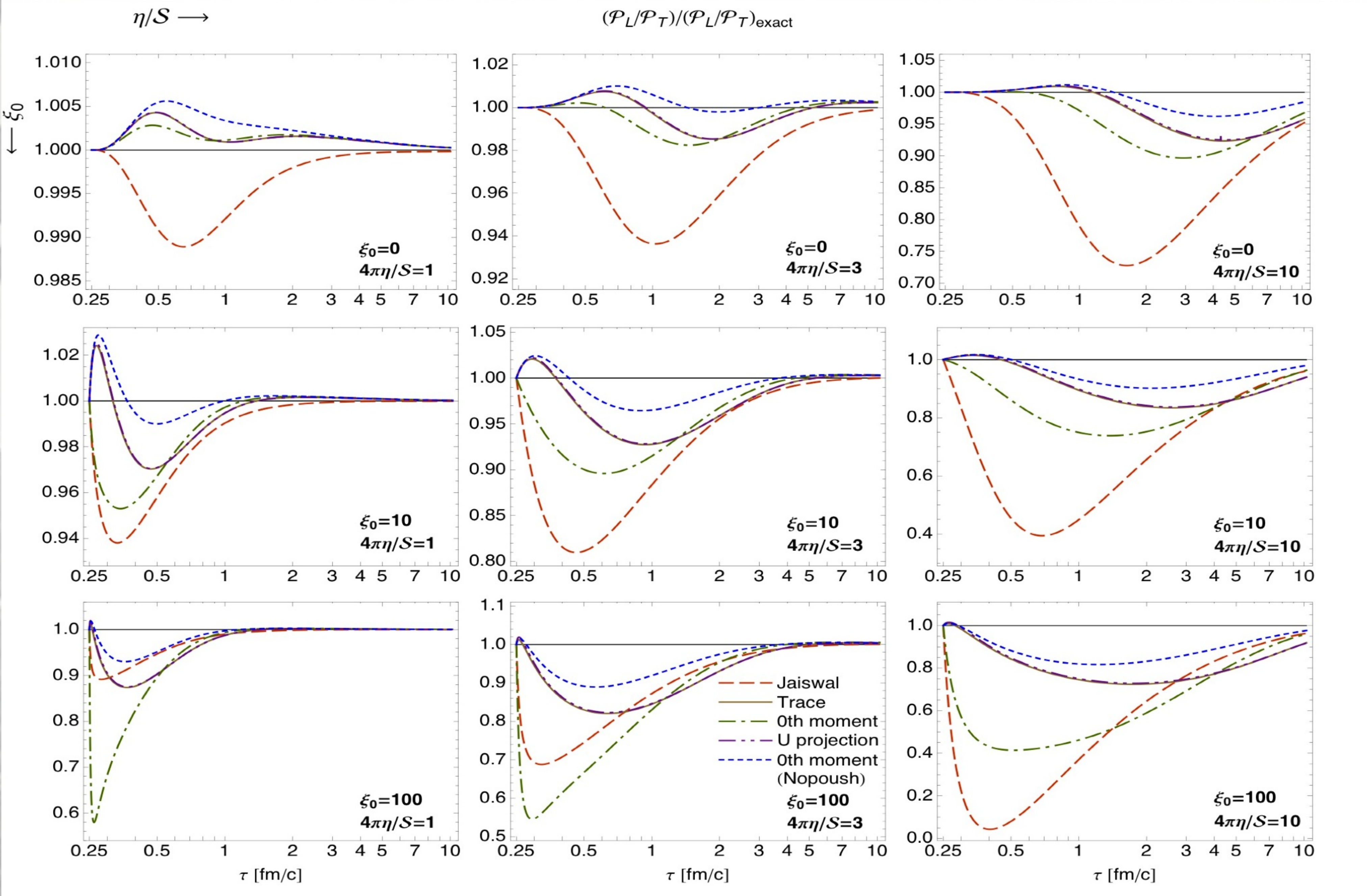
Dynamical equations from the second moment of the Boltzmann equation, the zeroth moment, and four-momentum conservation

- **L Tinti, arXiv:1411.7268**

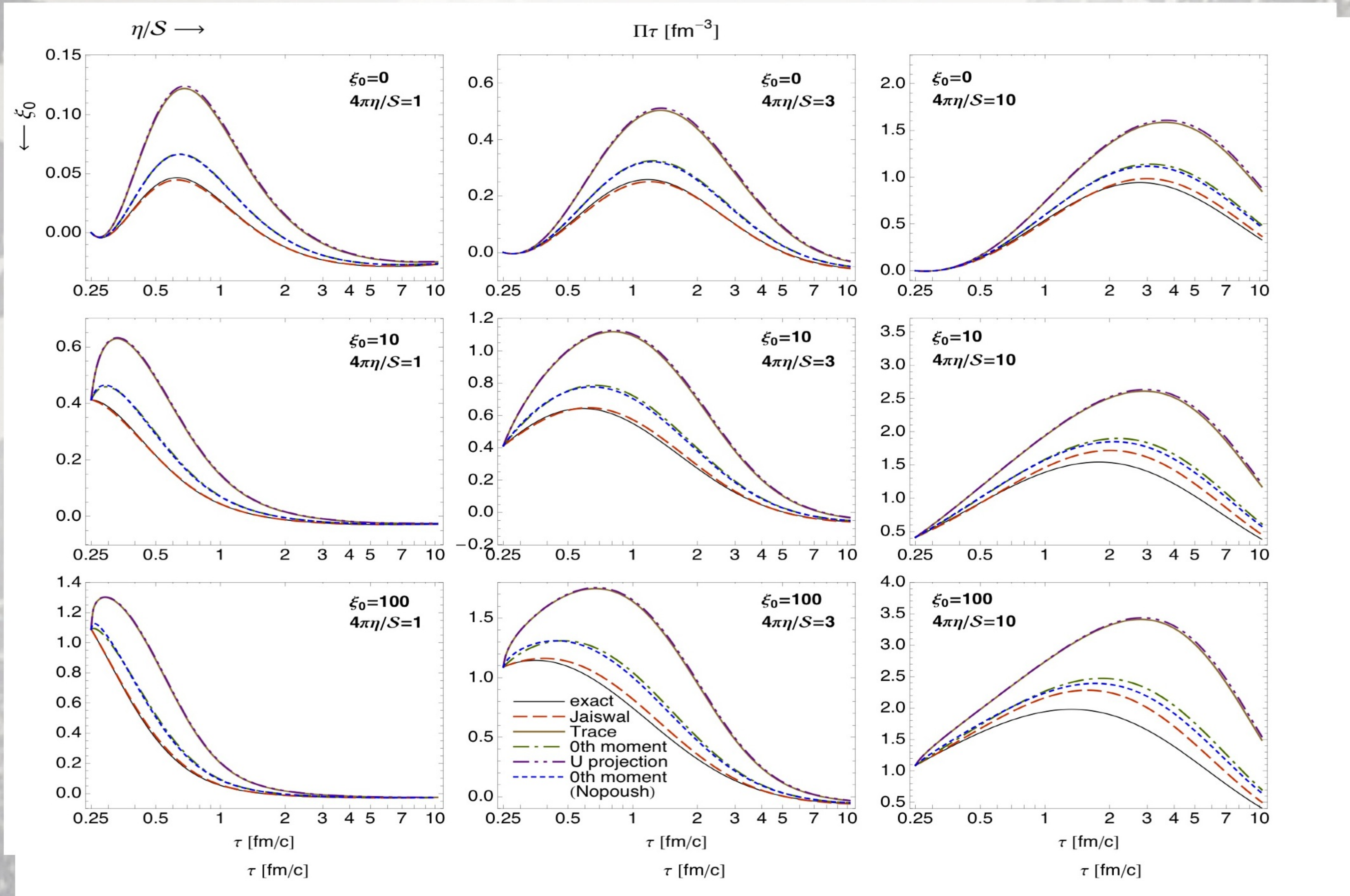
Unfortunately this generalization does not improve the agreement with the Boltzmann equation...

- **L Tinti, R Ryblewski, W Florkowski, M Strickland,arXiv:1505.06456**

(3+1)-dimensional framework



(3+1)-dimensional framework



Apparently, generalizing the background distribution function is not enough ...

Of course, there's the next to leading order, but...

The Structure of the Boltzmann equation is very rich, many options to close the system of equations

The choice of the dynamical equations can be as important as the form of the anisotropic background

On top of the requirements of recovering viscous hydrodynamics close to equilibrium...

- **Is there any unambiguous choice for the dynamical equations?**
- **Is there any physical interpretation of the anisotropy parameters?**

It is not necessary to take the equations from the moments of the Boltzmann equation!

Kinetic theory already provides exact equations for the pressure corrections

$$D\pi^{\langle\mu\nu\rangle} + \mathcal{C}_{-1}^{\langle\mu\nu\rangle} = -\Delta^{\mu\nu}_{\rho\sigma}\nabla_\alpha \int dP\, \frac{p^\rho p^\sigma p^\alpha f}{(p\cdot U)} - \left(\sigma_{\rho\sigma} + \frac{1}{3}\,\theta\,\Delta_{\rho\sigma}\right)\int dP\, \frac{p^{\langle\mu}p^{\nu\rangle}p^\rho p^\sigma f}{(p\cdot U)^2}$$

$$D\Pi - \frac{1}{3}\Delta_{\mu\nu}\mathcal{C}_{-1}^{\mu\nu} = -D\mathcal{P}_{\rm eq.} + \frac{1}{3}\Delta_{\mu\nu}\nabla_\rho \int dP\, \frac{p^\mu p^\nu p^\rho f}{(p\cdot U)} + \frac{1}{3}\left(\sigma_{\rho\sigma} + \frac{1}{3}\,\theta\,\Delta_{\rho\sigma}\right)\int dP\, \frac{(p\cdot\Delta\cdot p)p^\rho p^\sigma f}{(p\cdot U)^2}$$

Together with the four-momentum conservation equations they provide the exact time evolution of the stress-energy tensor

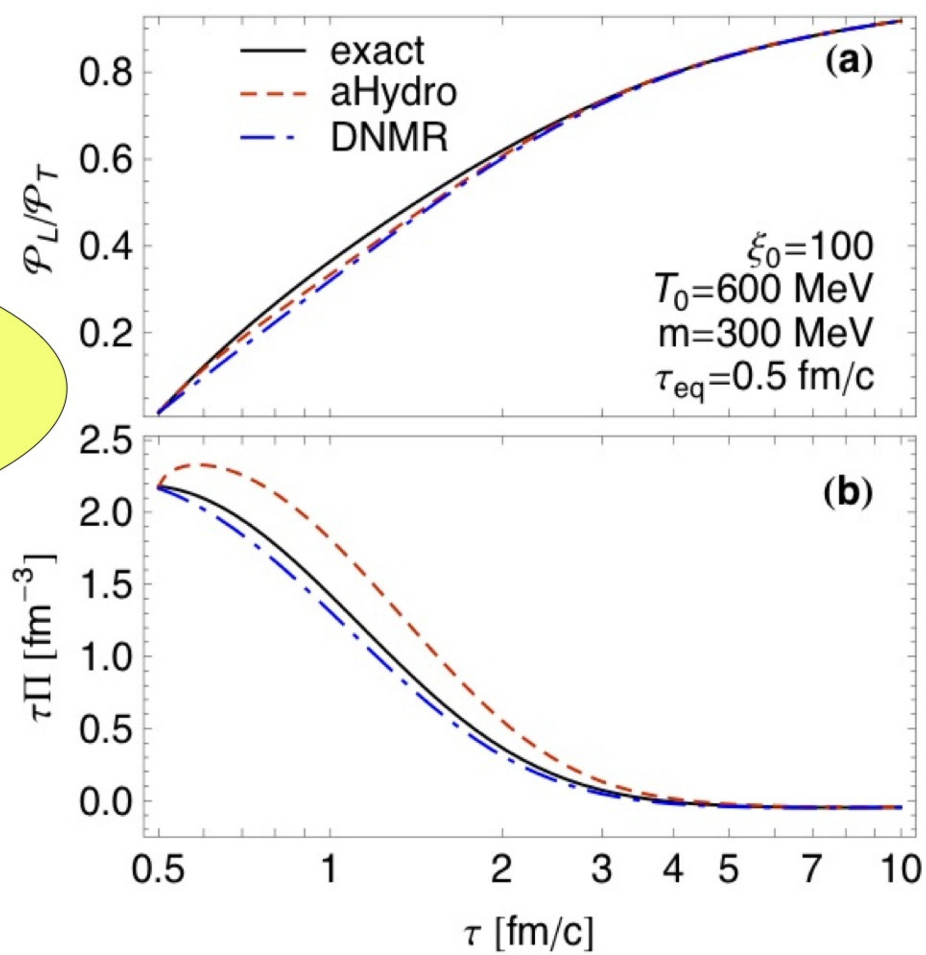
It is not necessary to take the equations from the moments of the Boltzmann equation!

Kinetic theory already pro...
for the pressure...

$$D\pi^{\langle\mu\nu\rangle} + \mathcal{C}_{-1}^{\langle\mu\nu\rangle} = -\Delta^{\mu\nu}_{\rho\sigma}\nabla_\alpha \int dP \ldots$$

$$D\Pi - \frac{1}{3}\Delta_{\mu\nu}\mathcal{C}^{\mu\nu}_{-1} = -D\mathcal{P}_{eq} \ldots$$

Very successful application to viscous hydrodynamics



Together with the four-mome... provide the exact time evolu...

- G S Denicol, W Florkowski, R Ryblewski, M Strickland,Phys.Rev. C 90 044905 (2014)

Physical meaning of the anisotropy parameter(s)

When we define local equilibrium in kinetic theory, the parameters that we use are chosen to reproduce the exact values of the stress energy tensor

- The four-velocity is the time-like eigenvector of the stress energy tensor (Landau frame)
- The effective temperature (chemical potential) fixes the proper energy density (number density)
- All of the other component are left to the next to leading order corrections.

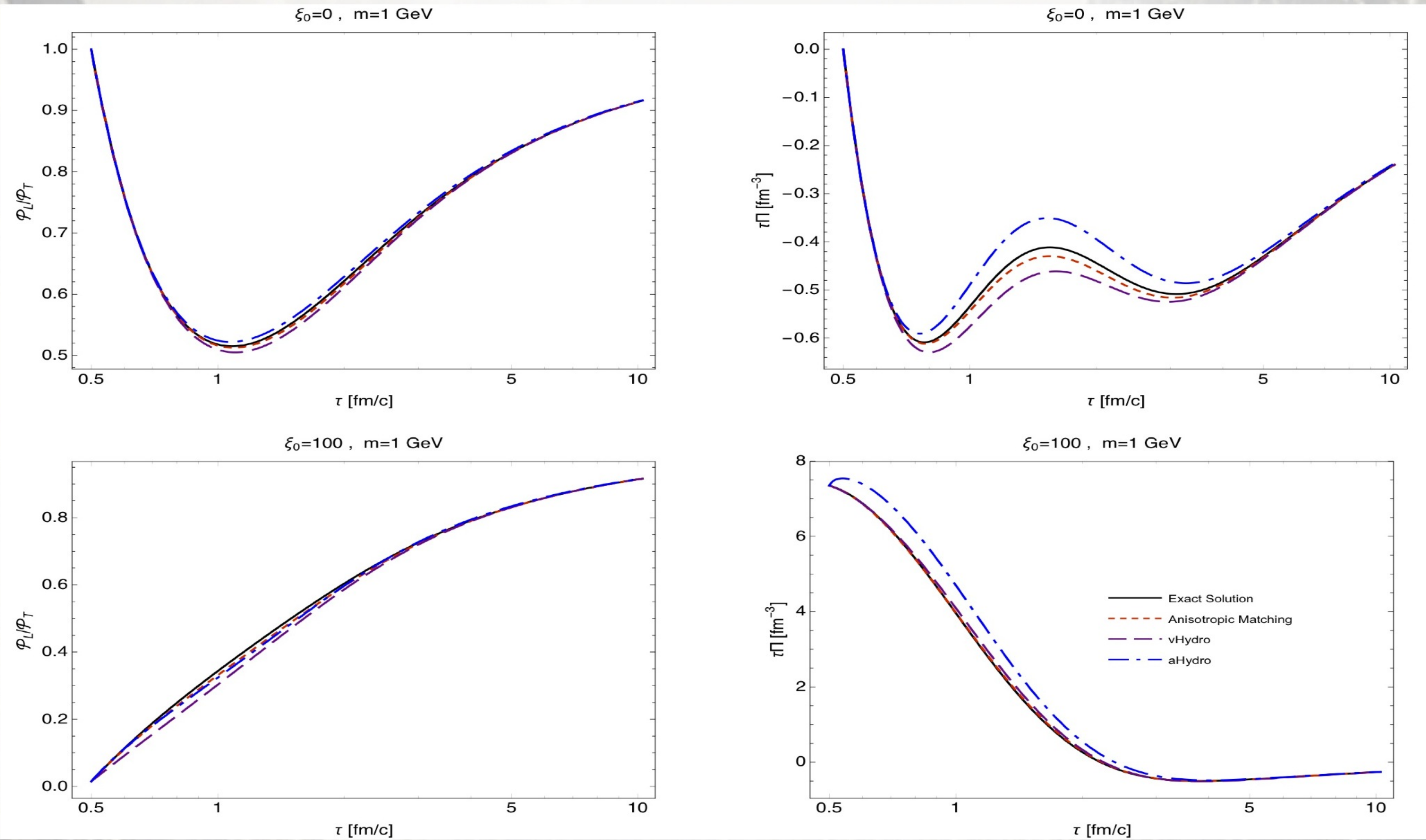
We can extend this Landau matching procedure to the anisotropic background

- Four-velocity still the eigenvector of the stress-energy tensor
- The momentum scale and the anisotropy parameter(s) fix the energy density and the longitudinal pressure (and the other components if one uses more than one extra parameter)
- NLO required for the other moments of the Boltzmann distribution

RTA => It recovers Jaiswal's prescription for second order viscous hydrodynamics in the close-to-equilibrium limit.

- **L Tinti, Phys.Rev. C94, 044902 (2016)**

Very successful application to anisotropic hydrodynamics too!

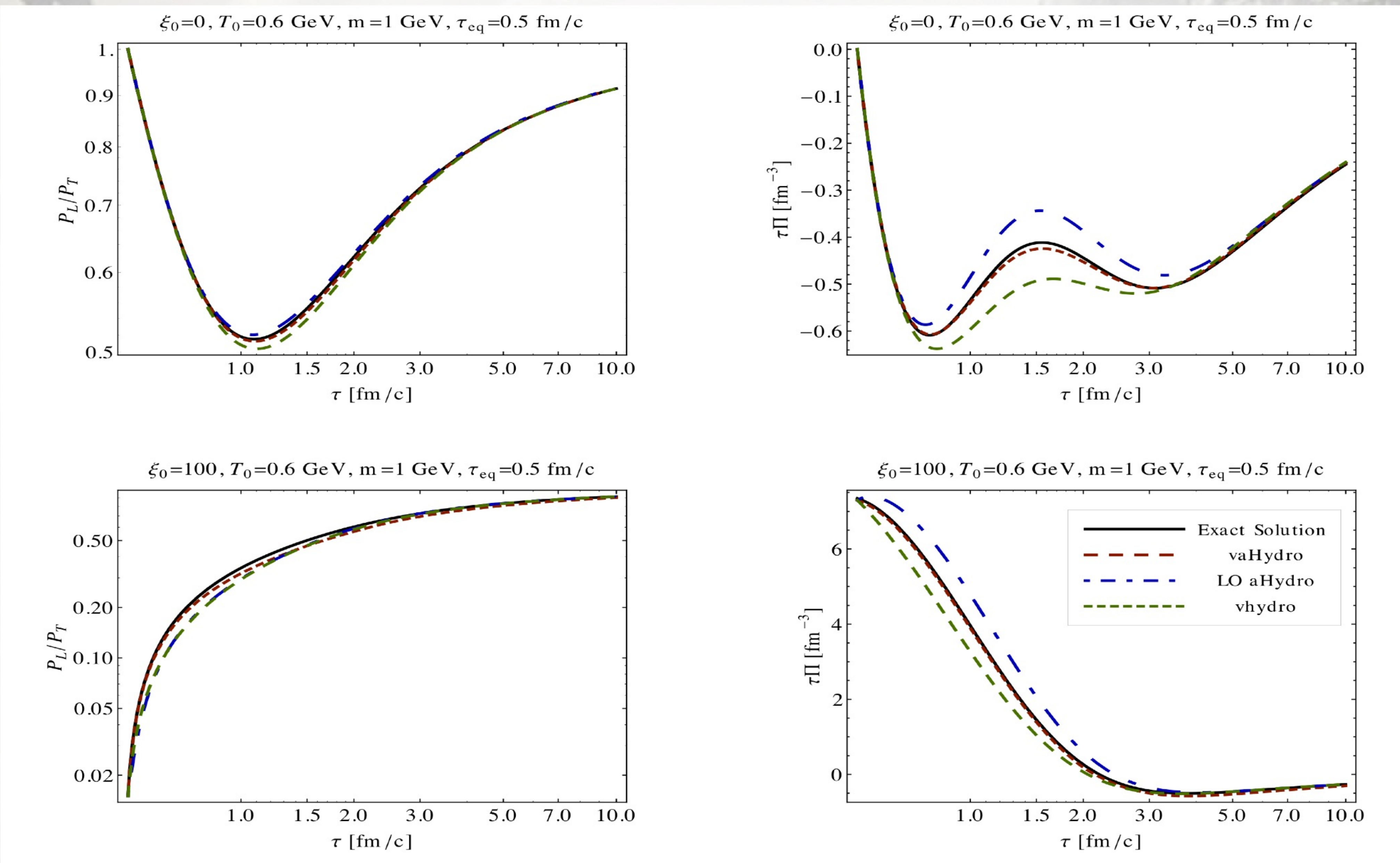


- **L Tintì, Phys.Rev. C94, 044902 (2016)**

See also: E. Molnar, H. Niemi, D. H. Rischke, Phys.Rev. D93 114025 (2016) for a systematic treatment of the next to leading order correction

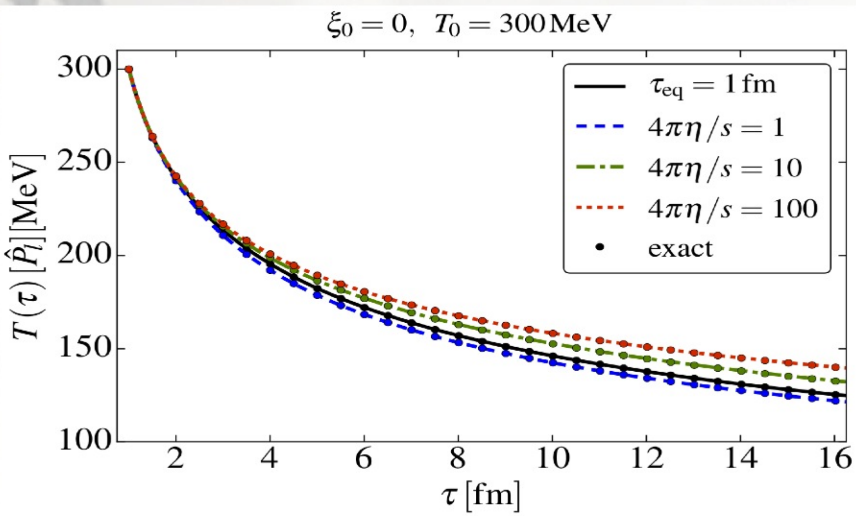
The results are already close to the next to leading order prescription

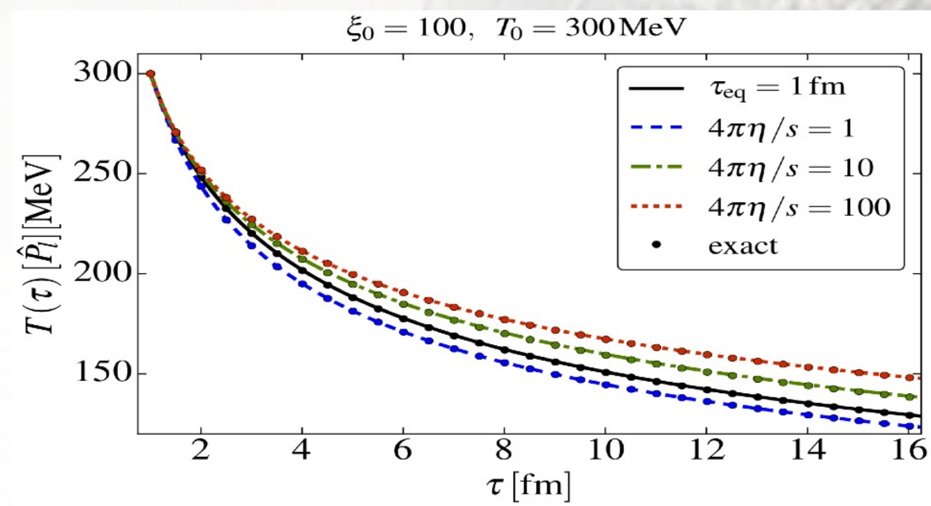
- D Bazow, U W Heinz, M Martinez, Phys.Rev. C 91 064903 (2015)

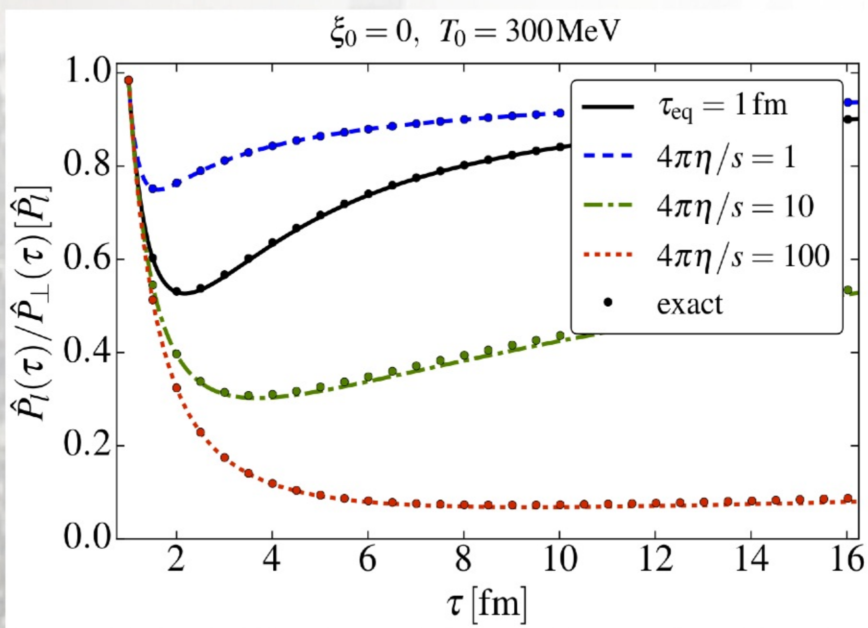


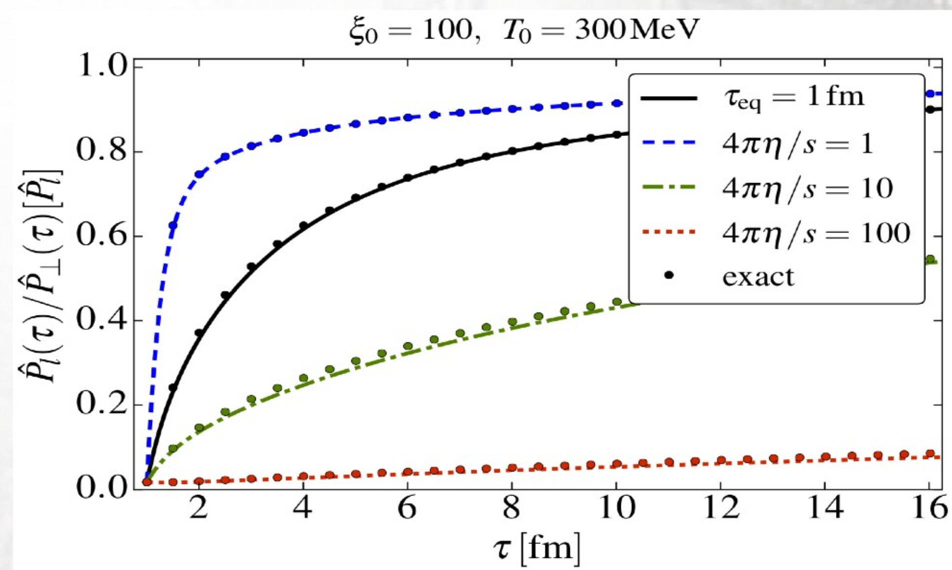
If you still have doubts you may have a look at

- E. Molnar, H. Niemi, D. H. Rischke, arXiv:1606.09019 (2016)









Summary & outlook

- Anisotropic hydrodynamics is a reorganization of the hydrodynamic expansion, around an anisotropic distribution.

- Pressure anisotropies already at the leading order, treated in a non-perturbative manner.

- Generalized ansatz for the leading order, consistent with second order viscous hydrodynamics close to equilibrium (full 3+1 expansion).

- Striking agreement with the exact solutions of the Boltzmann equation in the one-dimensional expansion

- … study of the non-hydro modes, higher rank moments, implementation of a non-ideal EOS..