3+1d Anisotropic Hydrodynamics - Phenomenological applications

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Relativistic Hydrodynamics: Theory and Modern Applications
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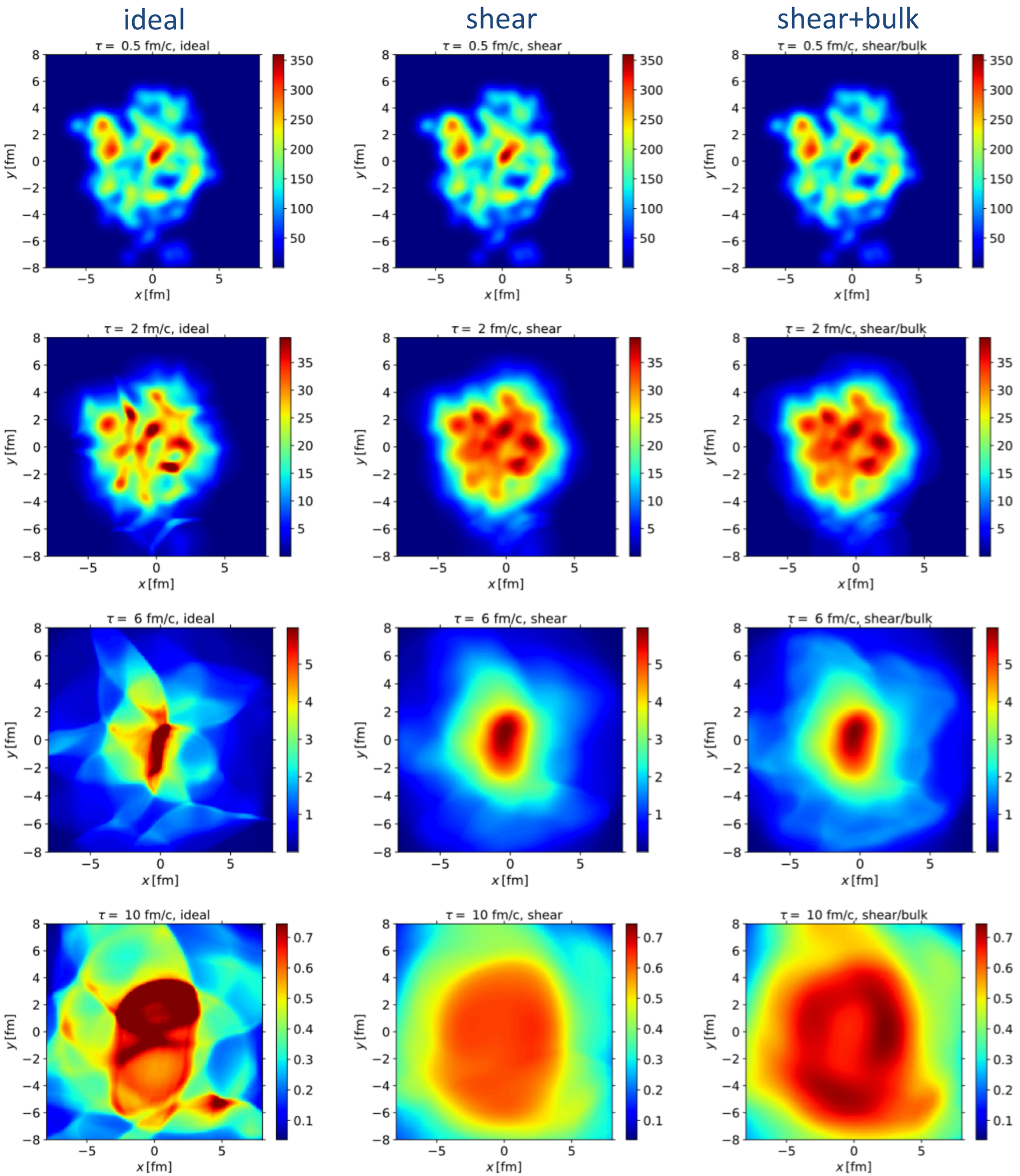
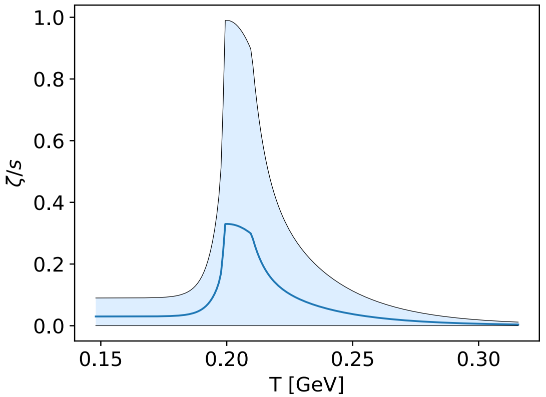
Some pretty pictures from 3d viscous hydro

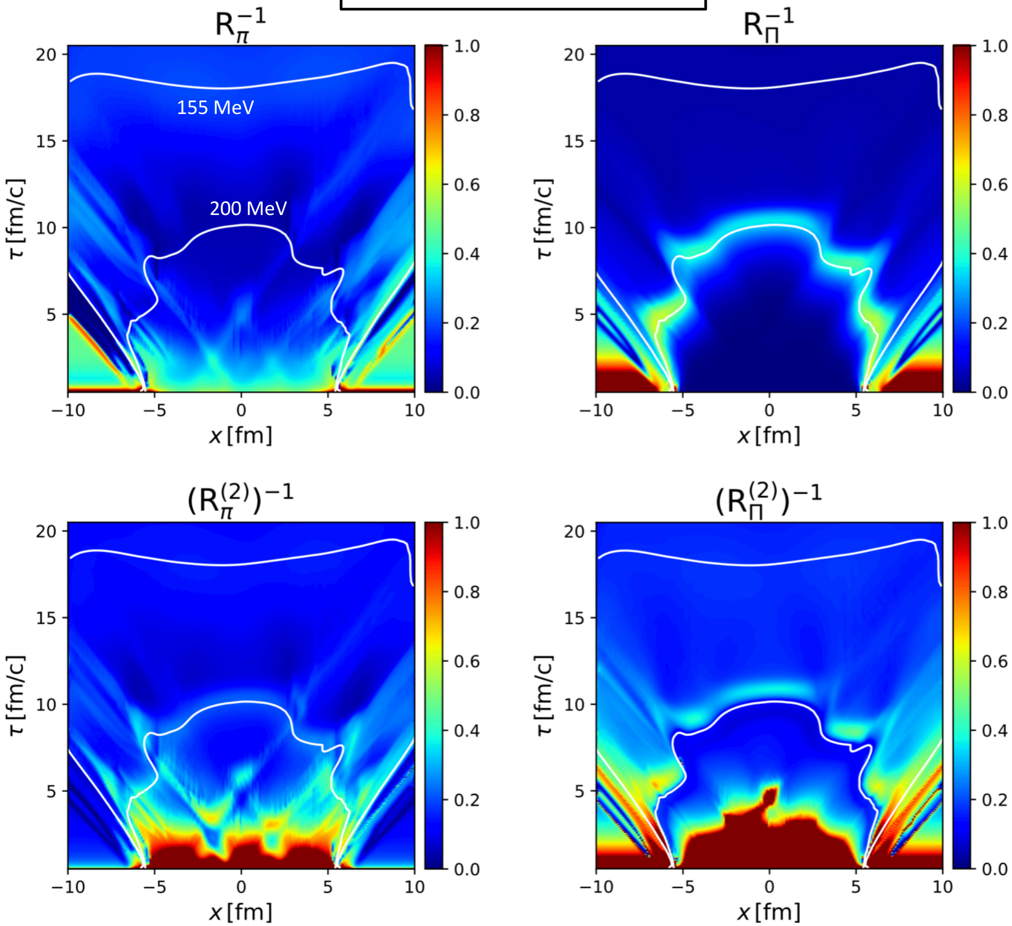


- Left panels show output from the Ohio State/Kent State GPU-based viscous hydro code
  [Bazow, Heinz, and MS, 1608.06577]
- Solves the non-conformal DNMR (Denicol, Niemi, Molnar, Rischke) equations with a realistic EoS
- Parameterized ζ/s (plot below)
- $\eta/s = 0.2$
- $T_0 = 600$ MeV @ $t_0 = 0.5$ fm/c



Pb-Pb @ 2.76 TeV - Don’t worry, be happy

$$R_\pi^{-1} \equiv \frac{\sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}}{\mathscr{P}_0}, \qquad R_\Pi^{-1} \equiv \frac{|\Pi|}{\mathscr{P}_0}.$$



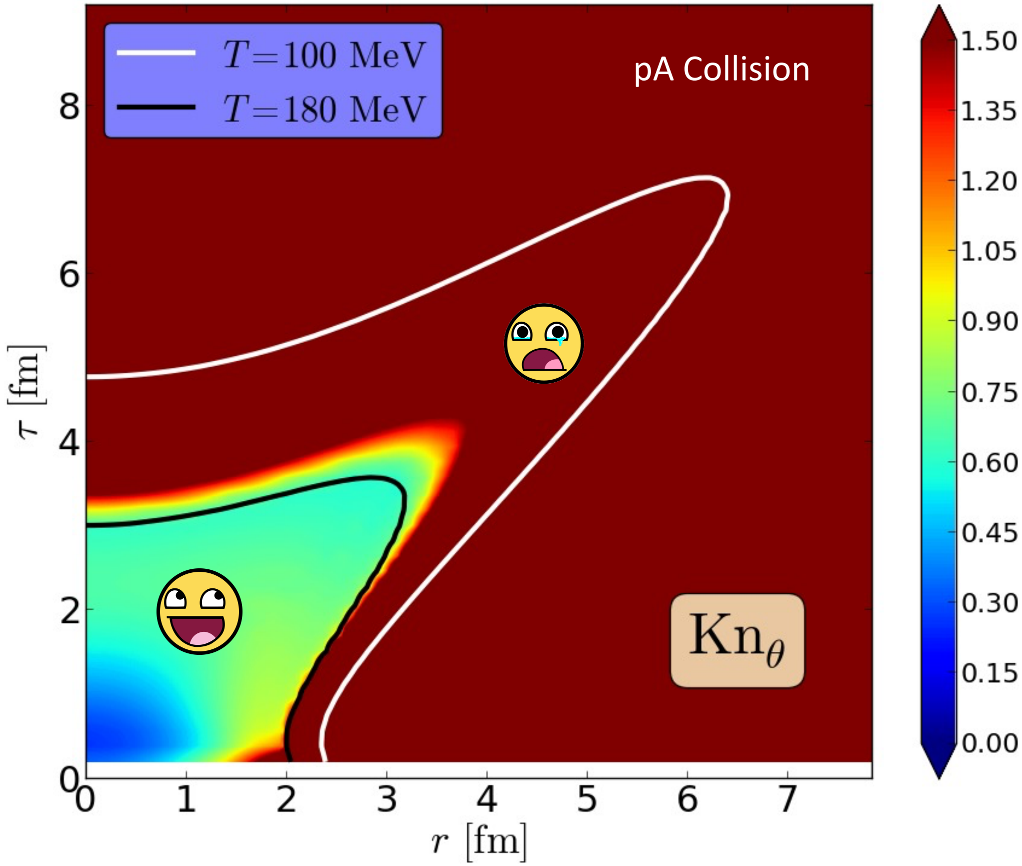
$$(R_\pi^{(2)})^{-1} \equiv \frac{\sqrt{\mathscr{J}^{\mu\nu}\mathscr{J}_{\mu\nu}}}{2\eta\sqrt{\sigma^{\mu\nu}\sigma_{\mu\nu}}}, \qquad (R_\Pi^{(2)})^{-1} \equiv \frac{|\mathscr{J}|}{\zeta|\theta|}.$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \mathscr{J} + \mathscr{K} + \mathscr{R}\,,$$
$$\tau_n \dot{n}^{\langle\mu\rangle} + n^\mu = \kappa I^\mu + \mathscr{J}^\mu + \mathscr{K}^\mu + \mathscr{R}^\mu\,,$$
$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \mathscr{J}^{\mu\nu} + \mathscr{K}^{\mu\nu} + \mathscr{R}^{\mu\nu}\,.$$

- $\mathcal{J}, \mathcal{J}^\mu$, and $\mathcal{J}^{\mu\nu}$ are O(Kn R^{-1})
- $\mathcal{K}, \mathcal{K}^\mu$, and $\mathcal{K}^{\mu\nu}$ are O(Kn^2)
- $\mathcal{R}, \mathcal{R}^\mu$, and $\mathcal{R}^{\mu\nu}$ are O(R^{-2})
- DNMR derivation assumes that Kn ~ R^{-1}
- For this to be a reasonable approx, the 2nd order terms should be smaller than the O(Kn) Navier-Stokes terms
- **In order for code to run stably, it is necessary to “dynamically regulate” the viscous corrections**

Pb-Pb @ 2.76 TeV - Don't worry, be happy

Figure (sans emoticons): H. Niemi and G. Denicol, 1404.7327



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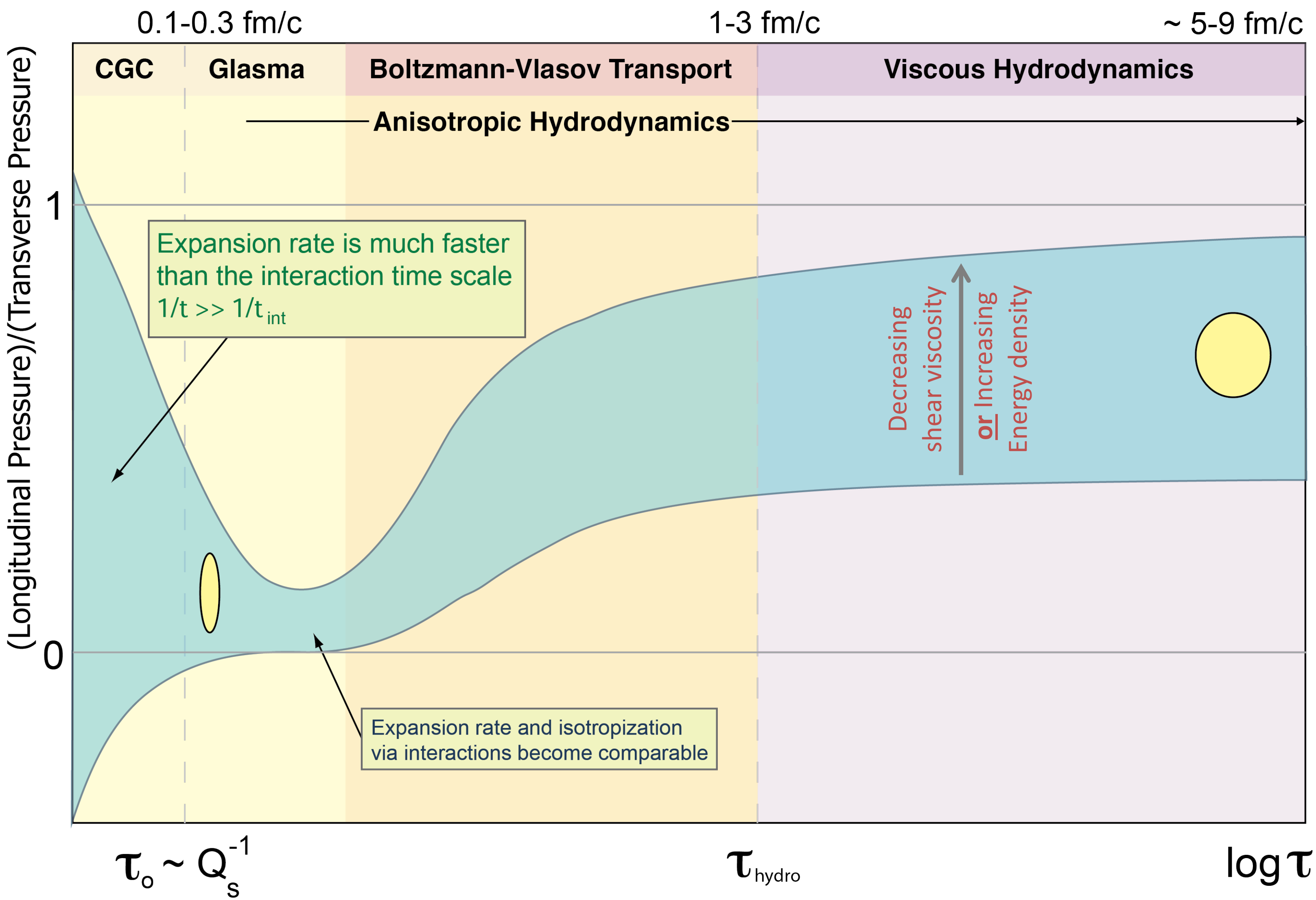
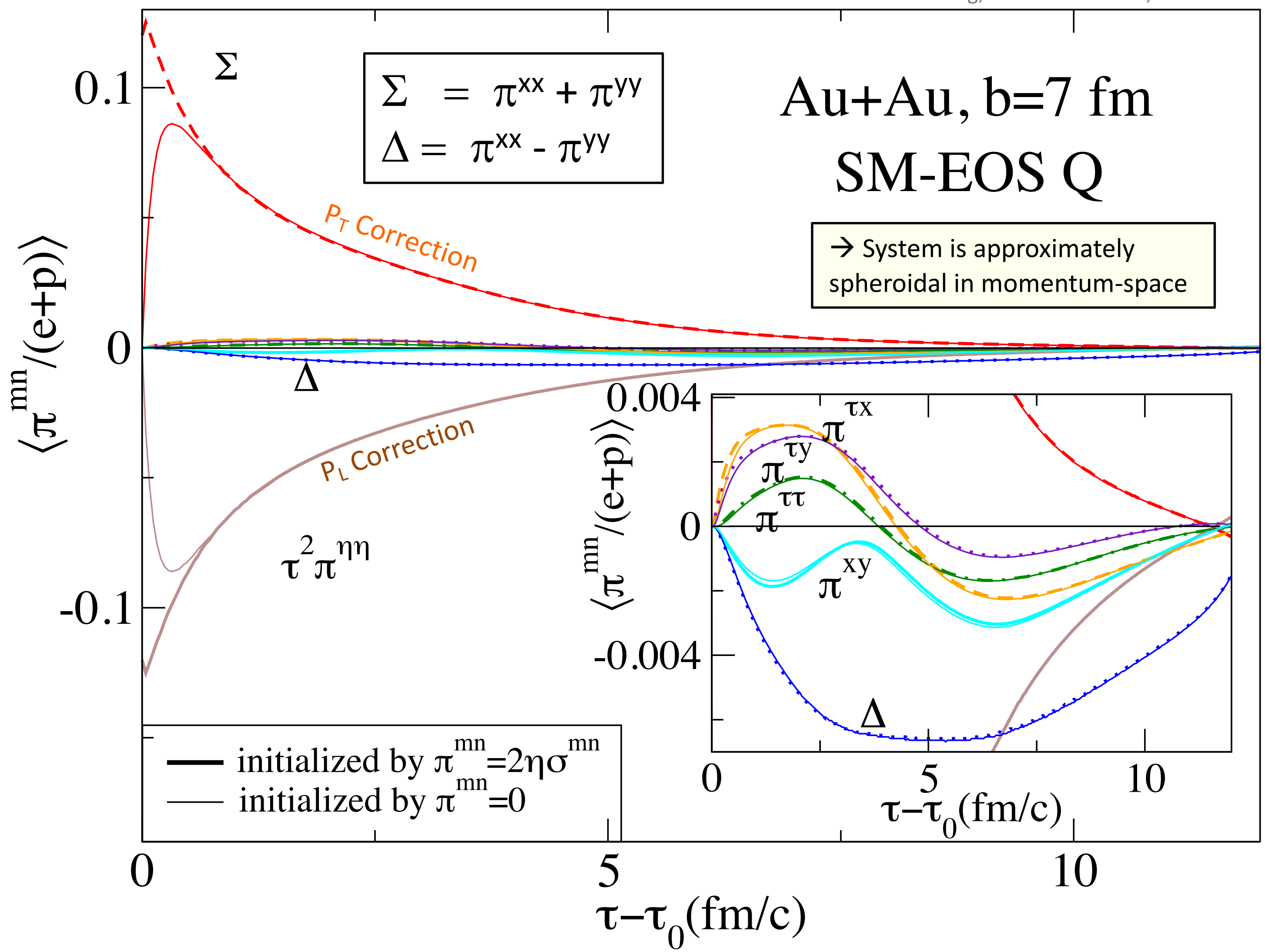
QGP momentum anisotropy cartoon



What are the largest viscous corrections?

H. Song, PhD Dissertation, 0908.3656



Spheroidal expansion method

Viscous Hydrodynamics Expansion

$$f(\tau,\mathbf{x},\mathbf{p}) = \underbrace{f_{\rm eq}(\mathbf{p},T(\tau,\mathbf{x}))}_{\text{Isotropic in momentum space}} + \delta f$$

See e.g.
- W. Florkowski and R. Ryblewski, 1007.0130
- M. Martinez and MS, 1007.0889
- D. Bazow, U. Heinz, and MS, 1311.6720
- D. Bazow, U. Heinz, and M. Martinez, 1503.07443
- E. Molnar, H. Niemi, and D. Rischke, 1602.00573; 1606.09019

Anisotropic Hydrodynamics (aHydro) Expansion

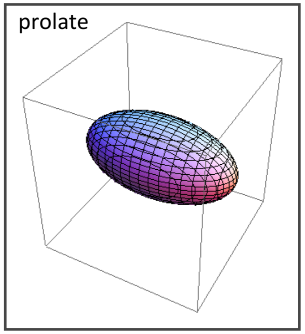
$$f(\tau,\mathbf{x},\mathbf{p}) = f_{\rm aniso}(\mathbf{p},\underbrace{\Lambda(\tau,\mathbf{x})}_{T_\perp},\underbrace{\xi(\tau,\mathbf{x})}_{\rm anisotropy}) + \delta\tilde{f}$$

Treat this term perturbatively
→ "NLO aHydro"

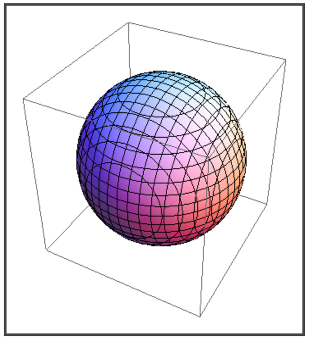
→ "Romatschke-Strickland" form in LRF

$$\boxed{f^{LRF}_{\rm aniso} = f_{\rm iso}\left(\frac{\sqrt{\mathbf{p}^2+\xi(\mathbf{x},\tau)p_z^2}}{\Lambda(\mathbf{x},\tau)}\right)}$$

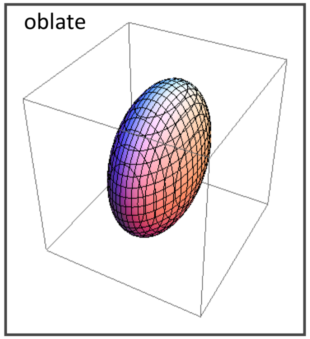
$$\boxed{\xi = \frac{\langle p_T^2\rangle}{2\langle p_L^2\rangle} - 1}$$



$-1 < \xi < 0$



$\xi = 0$



$\xi > 0$

Why spheroidal form at LO?

- What is special about this form at leading order?

$$f_{\rm aniso}^{LRF} = f_{\rm iso}\left(\frac{\sqrt{\mathbf{p}^2+\xi(\mathbf{x},\tau)p_z^2}}{\Lambda(\mathbf{x},\tau)}\right)$$

- Gives the ideal hydro limit when $\xi=0$ ($\Lambda \rightarrow T$)
- For longitudinal (0+1d) free streaming, the LRF distribution function is of spheroidal form; limit emerges automatically in 0+1d aHydro

$$\xi_{\rm FS}(\tau) = (1+\xi_0)\left(\frac{\tau}{\tau_0}\right)^2 - 1$$

- Since $f_{iso} \geq 0$, the one-particle distribution function and pressures are ≥ 0 (not guaranteed in standard 2nd-order viscous hydro)
- Reduces to 2nd-order viscous hydrodynamics in limit of small anisotropies

M. Martinez and MS, 1007.0889

$$\frac{\Pi}{\mathcal{E}_{\rm eq}} = \frac{8}{45}\xi + \mathcal{O}(\xi^2)$$

For 3+1d proof of equivalence to second-order viscous hydrodynamics in the near-equilibrium limit see Tinti 1411.7268.

The growing anisotropic hydrodynamics family

- There are two approaches being actively followed in the literature to address this problem

  A. Linearize around a spheroidal distribution function and treat the perturbations using standard kinetic vHydro methods [“vaHydro”]
  Bazow, Martinez, Molnar, Niemi, Rischke, Heinz, MS

  B. Introduce a generalized anisotropy tensor which replaces the entire viscous stress tensor at LO and then linearize around that instead
  Tinti, Ryblewski, Martinez, Nopoush, Alqahtani, Florkowski, Molnar, Niemi, Rischke Schaefer, Bluhm, MS

- Each of these methods has its own advantages.

- In what I will show today, I will use the generalized method (B) at leading order.

Generalized aHydro formalism

In generalized aHydro, so far one assumes that the distribution function is of the form

$$f(x,p) = f_{\rm eq}\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu}(x) p^\nu}}{\lambda(x)}, \frac{\mu(x)}{\lambda(x)}\right) + \delta\tilde{f}(x,p)$$

$$\Xi^{\mu\nu} = \underbrace{u^\mu u^\nu}_{\text{LRF four velocity}} + \underbrace{\xi^{\mu\nu}}_{\text{Traceless symmetric anisotropy tensor}} - \underbrace{\Delta^{\mu\nu}}_{\text{Transverse projector}} \underbrace{\Phi}_{\text{"Bulk"}}$$

$$u^\mu u_\mu = 1$$
$$\xi^\mu{}_\mu = 0$$
$$\Delta^\mu{}_\mu = 3$$
$$u_\mu \xi^{\mu\nu} = u_\mu \Delta^{\mu\nu} = 0$$

- 3 degrees of freedom in u^μ
- 5 degrees of freedom in $\xi^{\mu\nu}$
- 1 degree of freedom in Φ
- 1 degree of freedom in λ
- 1 degree of freedom in μ

→ 11 DOFs

See e.g.
- M. Martinez, R. Ryblewski, and MS, 1204.1473
- L. Tinti and W. Florkowski, 1312.6614
- M. Nopoush, R. Ryblewski, and MS, 1405.1355

Equations of Motion

- Herein the EOM are obtained from moments of the Boltzmann equation in the relaxation time approximation (RTA)

$$p^\mu \partial_\mu f = -\mathcal{C}[f] \qquad\qquad \mathcal{C}[f] = \frac{p^\mu u_\mu}{\tau_{\rm eq}}(f - f_{\rm eq})$$

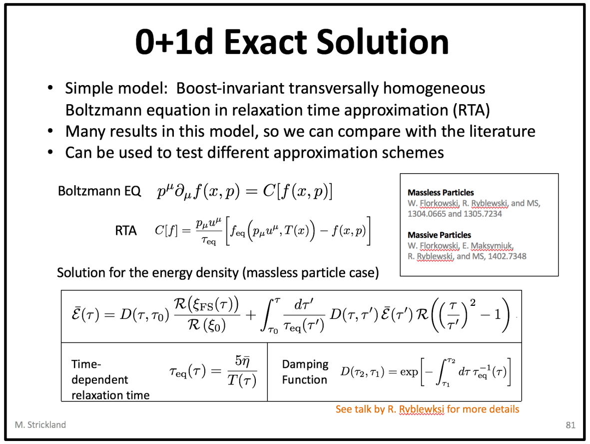
- **It is relatively straightforward to use other collisional kernels (forthcoming)**

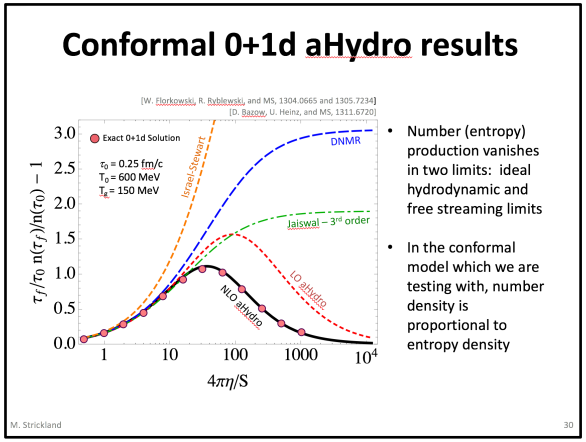
- 1 equation from the 0$^{\rm th}$ moment [number (non-conservation)]
- 4 equations from the 1$^{\rm st}$ moment [energy-momentum conservation]
- 6 equations from the 2$^{\rm nd}$ moment [dissipative dynamics]
- We must also specify the relation between the equilibrium (isotropic) pressure and energy density. More on this later.

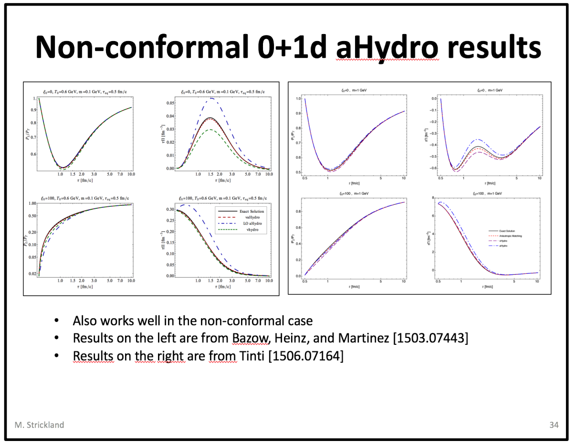
$$\begin{aligned}
D_u n + n\theta_u &= \frac{1}{\tau_{\rm eq}}(n_{\rm eq} - n) \\
\partial_\mu T^{\mu\nu} &= 0 \\
\partial_\mu \mathcal{I}^{\mu\nu\lambda} &= \frac{1}{\tau_{\rm eq}}(u_\mu \mathcal{I}^{\mu\nu\lambda}_{\rm eq} - u_\mu \mathcal{I}^{\mu\nu\lambda})
\end{aligned}$$

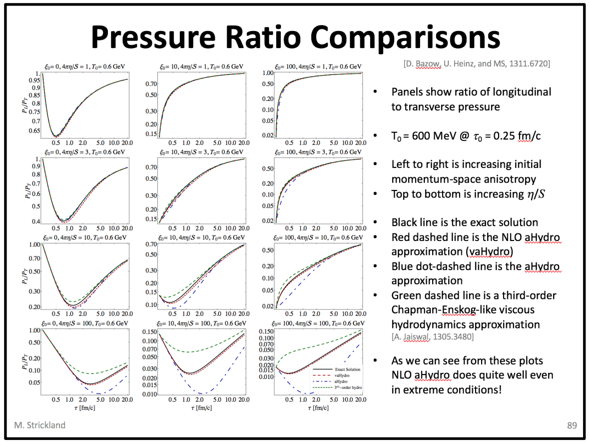
Is it really better?

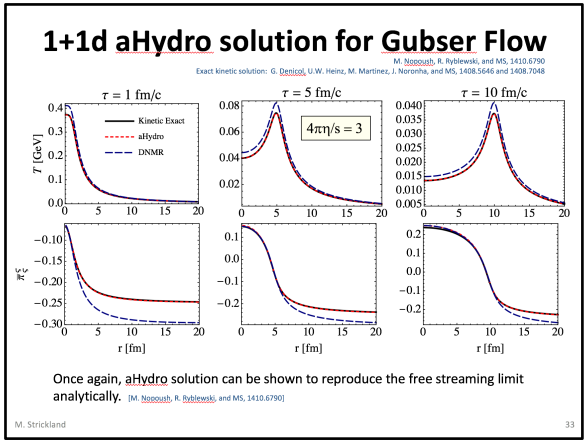
aHydro better reproduces exact solutions to the Boltzmann equation in a variety of expanding backgrounds better than standard viscous Hydro. [See L. Tinti's talk for more details]

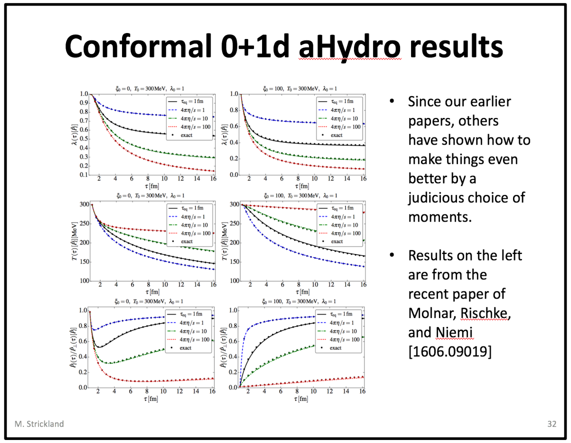












Towards phenomenology

3+1d aHydro Equations of Motion

- Assuming an ellipsoidal form for the anisotropy tensor (ignoring off-diagonal components for now), one has seven degrees of freedom: ξ_x, ξ_y, ξ_z, u_x, u_y, u_z, and λ.
- For the EoS we use a lattice-based EoS with the effective temperature T determined via Landau matching.

$$
\boxed{
\begin{aligned}
D_u\mathcal{E} + \mathcal{E}\theta_u + \mathcal{P}_x u_\mu D_x X^\mu + \mathcal{P}_y u_\mu D_y Y^\mu + \mathcal{P}_z u_\mu D_z Z^\mu &= 0\,,\\
D_x\mathcal{P}_x + \mathcal{P}_x\theta_x - \mathcal{E} X_\mu D_u u^\mu - \mathcal{P}_y X_\mu D_y Y^\mu - \mathcal{P}_z X_\mu D_z Z^\mu &= 0\,,\\
D_y\mathcal{P}_y + \mathcal{P}_y\theta_y - \mathcal{E} Y_\mu D_u u^\mu - \mathcal{P}_x Y_\mu D_x X^\mu - \mathcal{P}_z Y_\mu D_z Z^\mu &= 0\,,\\
D_z\mathcal{P}_z + \mathcal{P}_z\theta_z - \mathcal{E} Z_\mu D_u u^\mu - \mathcal{P}_x Z_\mu D_x X^\mu - \mathcal{P}_y Z_\mu D_y Y^\mu &= 0\,.
\end{aligned}
}
$$

First Moment

$$
\boxed{
\begin{aligned}
D_u\mathcal{I}_x + \mathcal{I}_x(\theta_u + 2u_\mu D_x X^\mu) &= \frac{1}{\tau_{\rm eq}}(\mathcal{I}_{\rm eq} - \mathcal{I}_x)\,,\\
D_u\mathcal{I}_y + \mathcal{I}_y(\theta_u + 2u_\mu D_y Y^\mu) &= \frac{1}{\tau_{\rm eq}}(\mathcal{I}_{\rm eq} - \mathcal{I}_y)\,,\\
D_u\mathcal{I}_z + \mathcal{I}_z(\theta_u + 2u_\mu D_z Z^\mu) &= \frac{1}{\tau_{\rm eq}}(\mathcal{I}_{\rm eq} - \mathcal{I}_z)\,.
\end{aligned}
}
$$

Second Moment

Florkowski, Haque, Nopoush, Ryblewski, MS, forthcoming

Implementing the equation of state

R Ryblewski and F. Florkowski, 1204.2624
M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101

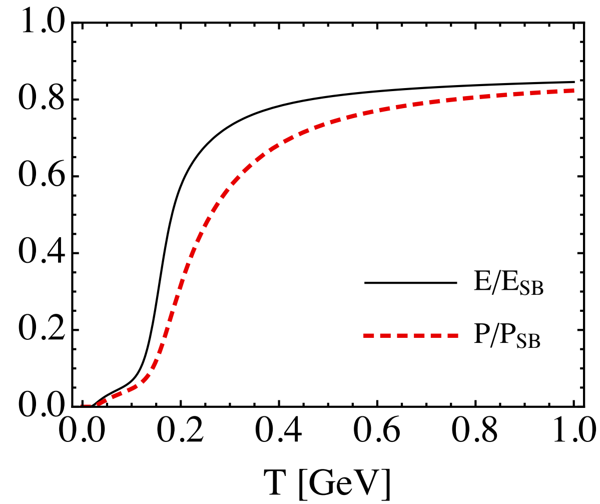
Standard Method

$$n(\Lambda,\xi) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} f_{\rm aniso} = \frac{n_{\rm iso}(\Lambda)}{\sqrt{1+\xi}}$$

$$\mathcal{E}(\Lambda,\xi) = T^{\tau\tau} = \mathcal{R}(\xi)\,\mathcal{E}_{\rm iso}(\Lambda)$$

$$\mathcal{P}_\perp(\Lambda,\xi) = \frac{1}{2}\left(T^{xx}+T^{yy}\right) = \mathcal{R}_\perp(\xi)\mathcal{P}_{\rm iso}(\Lambda)$$

$$\mathcal{P}_L(\Lambda,\xi) = -T^\varsigma_\varsigma = \mathcal{R}_{\rm L}(\xi)\mathcal{P}_{\rm iso}(\Lambda)$$

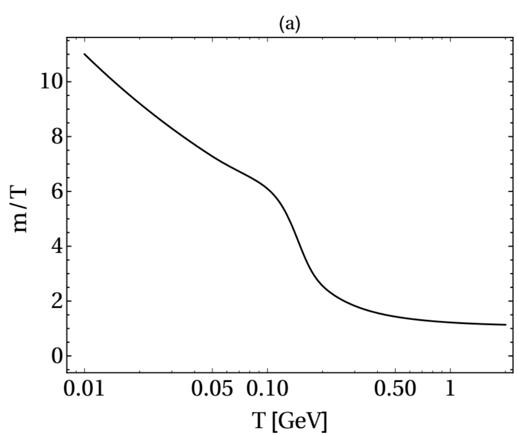


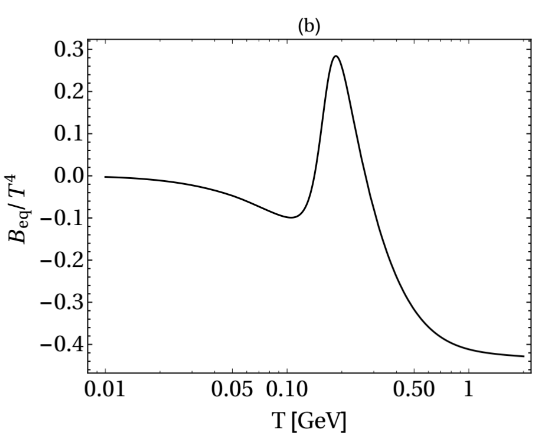
Quasiparticle Method

$$T^{\mu\nu}_{\rm eq} = T^{\mu\nu}_{\rm kinetic,eq} + g^{\mu\nu}B_{\rm eq}$$

$$p^\mu\partial_\mu f + \frac{1}{2}\partial_i m^2 \partial^i_{(p)} f = -\mathcal{C}[f]$$

$$\partial_\mu B = -\frac{1}{2}\partial_\mu m^2 \int dP\, f(x,p)$$





Anisotropic "Cooper-Frye" Freezeout

Bazow, Heinz, Martinez, Nopoush, Ryblewski, MS, 1506.05278
Florkowski, Haque, Nopoush, Ryblewski, MS, forthcoming

- Use same ellipsoidal form for "anisotropic freeze-out" at LO.
- Form includes both shear and bulk corrections to to the distribution function.
- Use energy density (scalar) to determine the freeze-out hyper-surface Σ → e.g. $T_{\rm eff,FO}$ = 150 MeV

$$f(x,p) = f_{\rm iso}\left(\frac{1}{\lambda}\sqrt{p_\mu \Xi^{\mu\nu} p_\nu}\right)$$

$$\Xi^{\mu\nu} = \underbrace{u^\mu u^\nu}_{\text{isotropic}} + \underbrace{\xi^{\mu\nu}}_{\text{anisotropy tensor}} - \underbrace{\Phi\Delta^{\mu\nu}}_{\text{bulk correction}}$$

$$\xi^{\mu\nu}_{\rm LRF} \equiv {\rm diag}(0,\xi_x,\xi_y,\xi_z)$$

$$\xi^\mu{}_\mu = 0 \qquad u_\mu \xi^\mu{}_\nu = 0$$

$$\left(p^0\frac{dN}{dp^3}\right)_i = \frac{\mathcal{N}_i}{(2\pi)^3}\int f_i(x,p)\, p^\mu d\Sigma_\mu \,,$$

NOTE: Usual 2nd-order viscous hydro form

$$f(p,x) = f_{\rm eq}\left[1 + (1 - a f_{\rm eq})\frac{p_\mu p_\nu \Pi^{\mu\nu}}{2(\epsilon+P)T^2}\right]$$

$$f_{\rm eq} = 1/[\exp(p\cdot u/T) + a] \qquad a = \text{-1, +1, or 0}$$

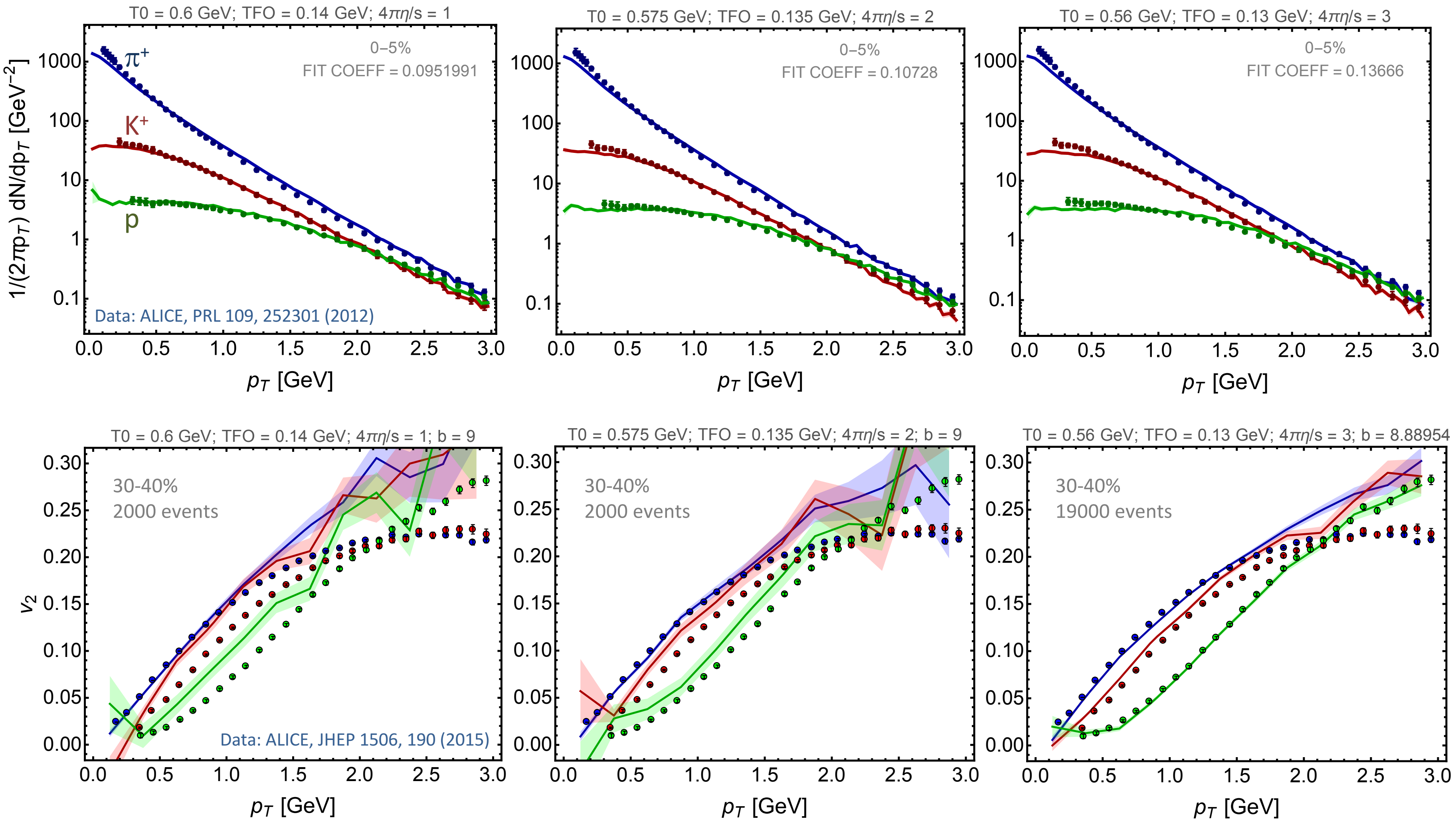
- This form suffers from the problem that the distribution function can be negative in some regions of phase space → unphysical
- **Problem becomes worse when including the bulk viscous correction** (see forthcoming slides).

The phenomenological setup

- As a first pass let's see if we can get close to the data with this simple model using smooth optical Glauber initial conditions.
- For initial conditions we use a mixture of wounded nucleon and binary collision profiles with a binary mixing fraction of 0.15 (empirically suggested).
- In the rapidity direction, we use a rapidity profile with a "tilted" central plateau and Gaussian "wings".
- We take all anisotropy parameters to be 1 initially (isotropic IC).
- We then run the code and extract the freeze-out hypersurface.
- The primordial particle production is then Monte-Carlo sampled using the Therminator 2. [Chojnacki, Kisiel, Florkowski, and Broniowski, arXiv:1102.0273]
- Therminator also takes care of all resonance feed down.
- We do not have a kinetic "afterburner" e.g. URQMD, yet.
- All data shown are from the ALICE collaboration. Error bars are statistical only. <u>Systematic errors are (unfortunately) not provided in the ALICE tables.</u>

Preliminary results – Optical Glauber ICs

LHC 2.76 TeV Pb+Pb collisions; top row shows spectra, bottom row shows differential v_2



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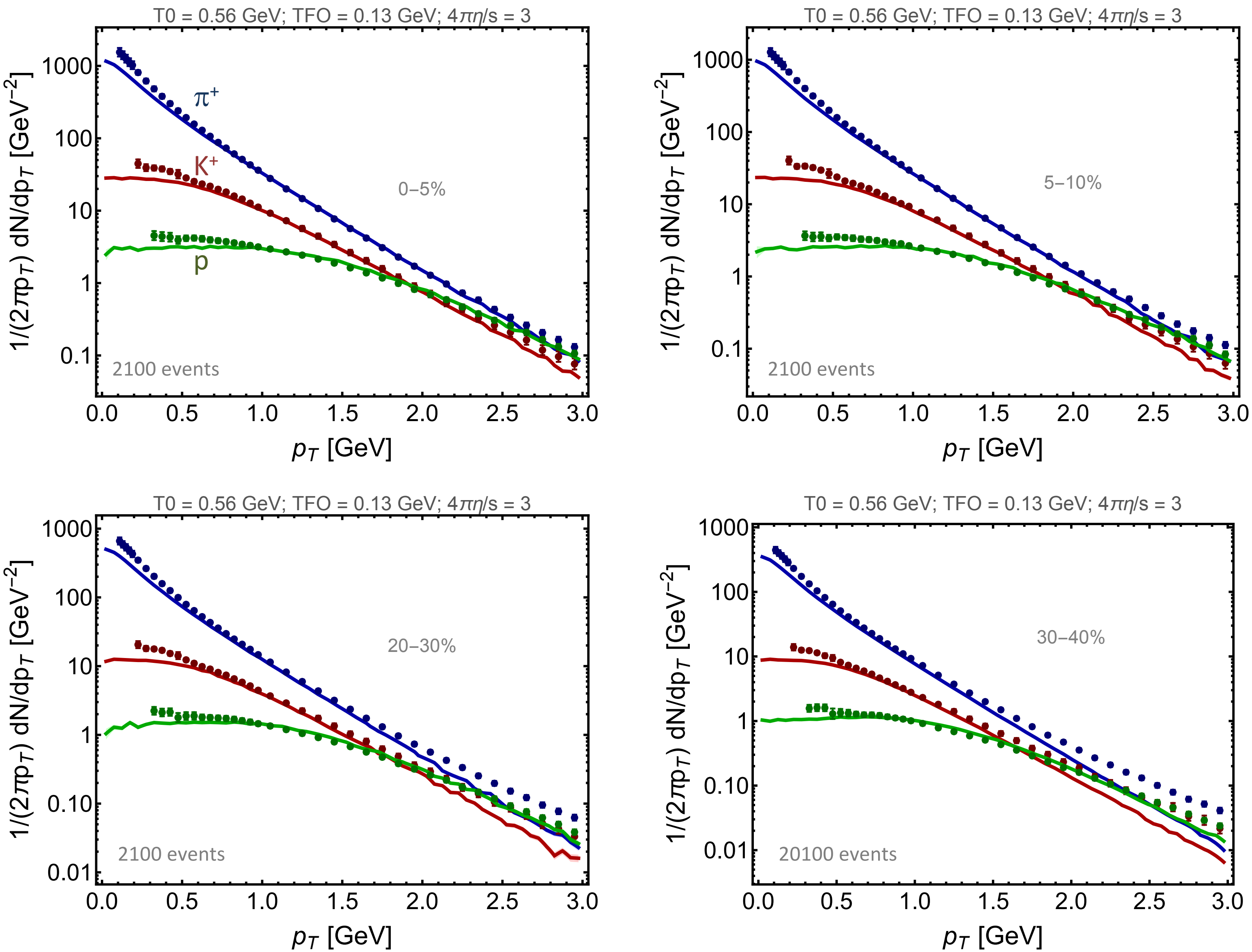

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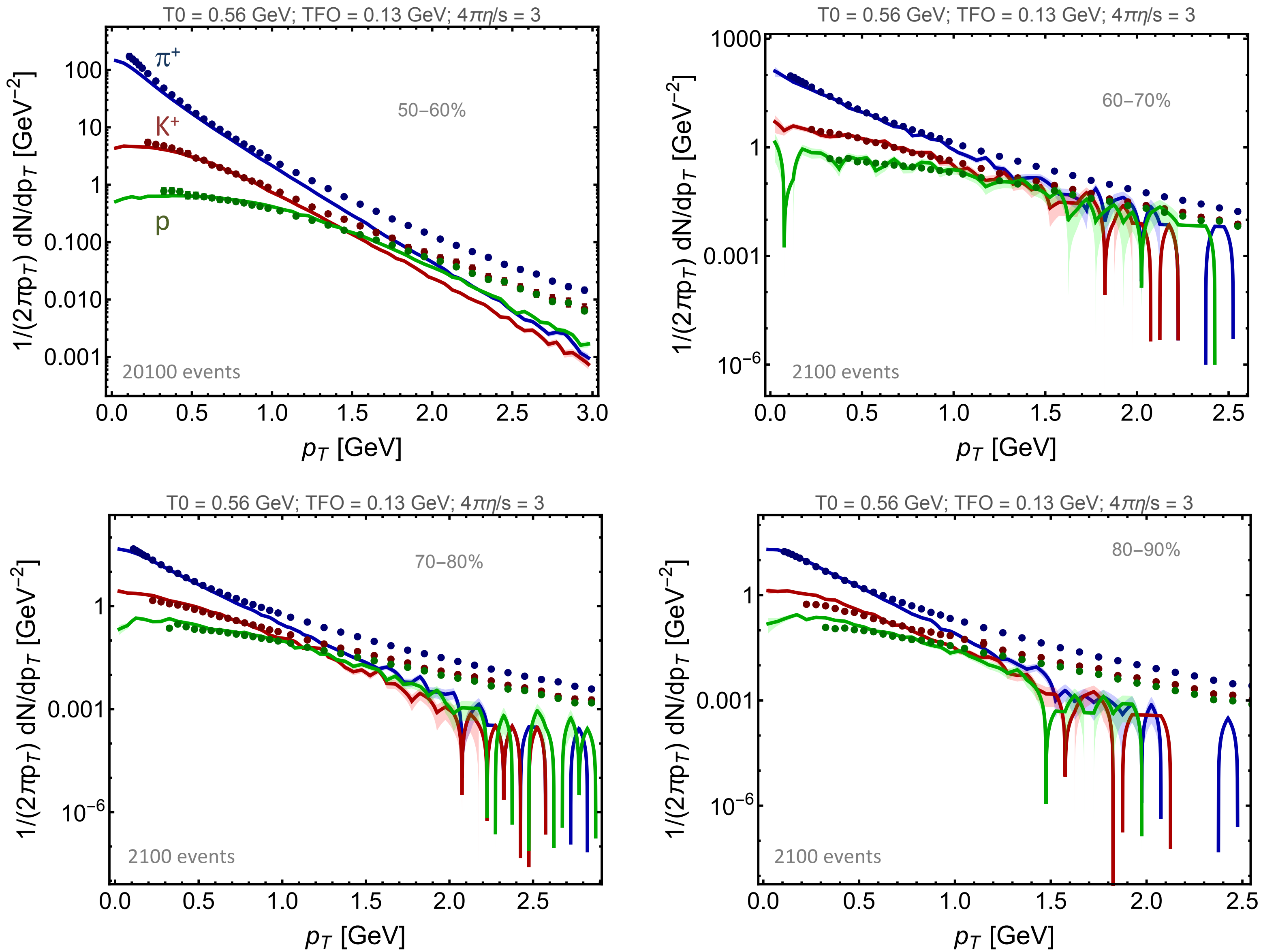

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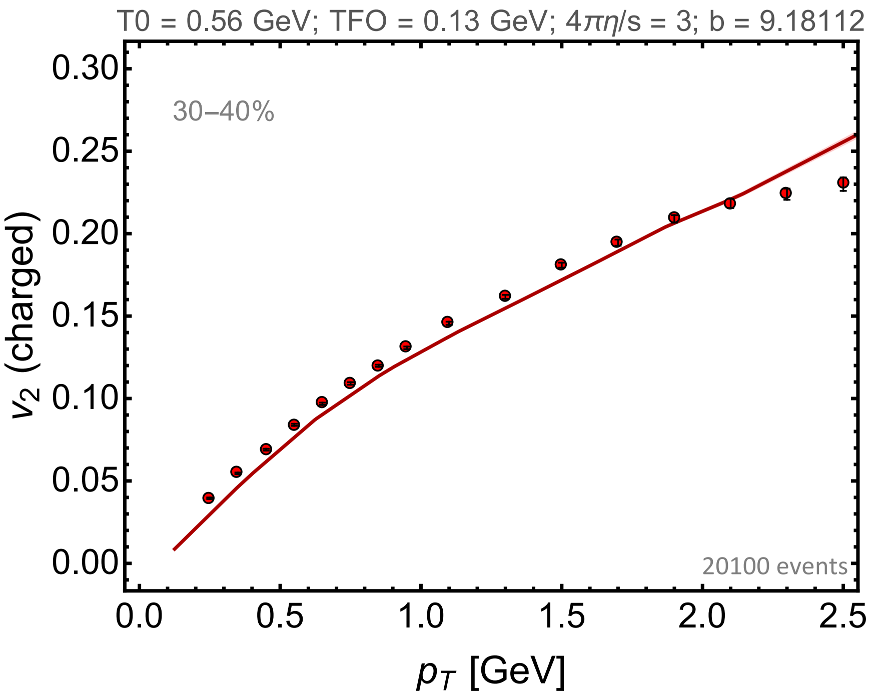

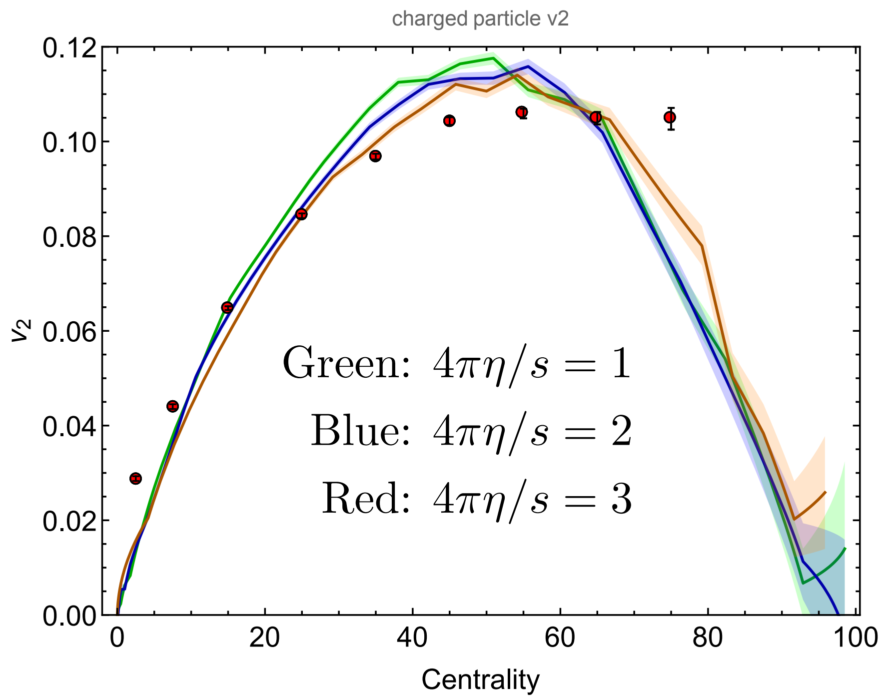

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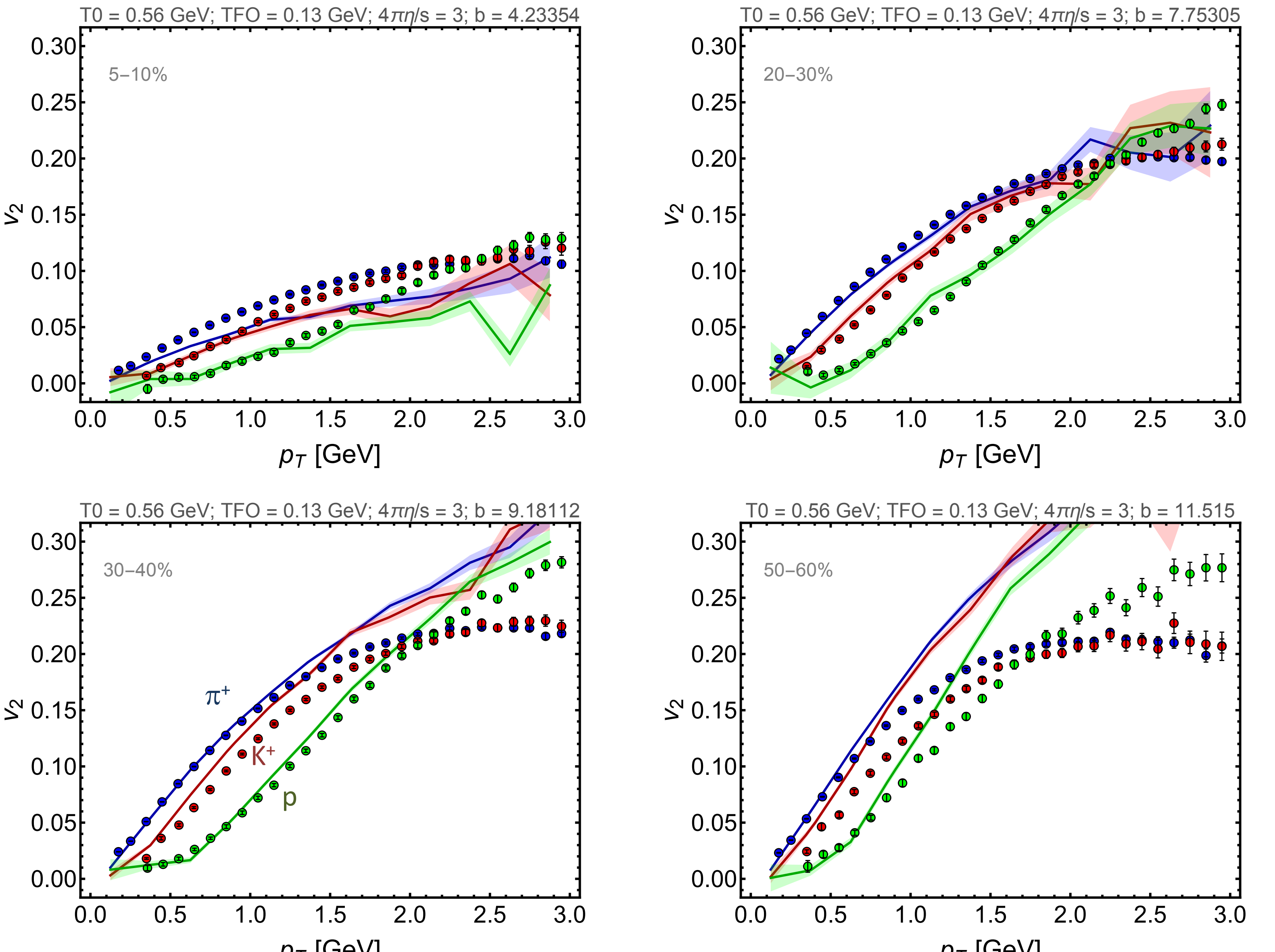
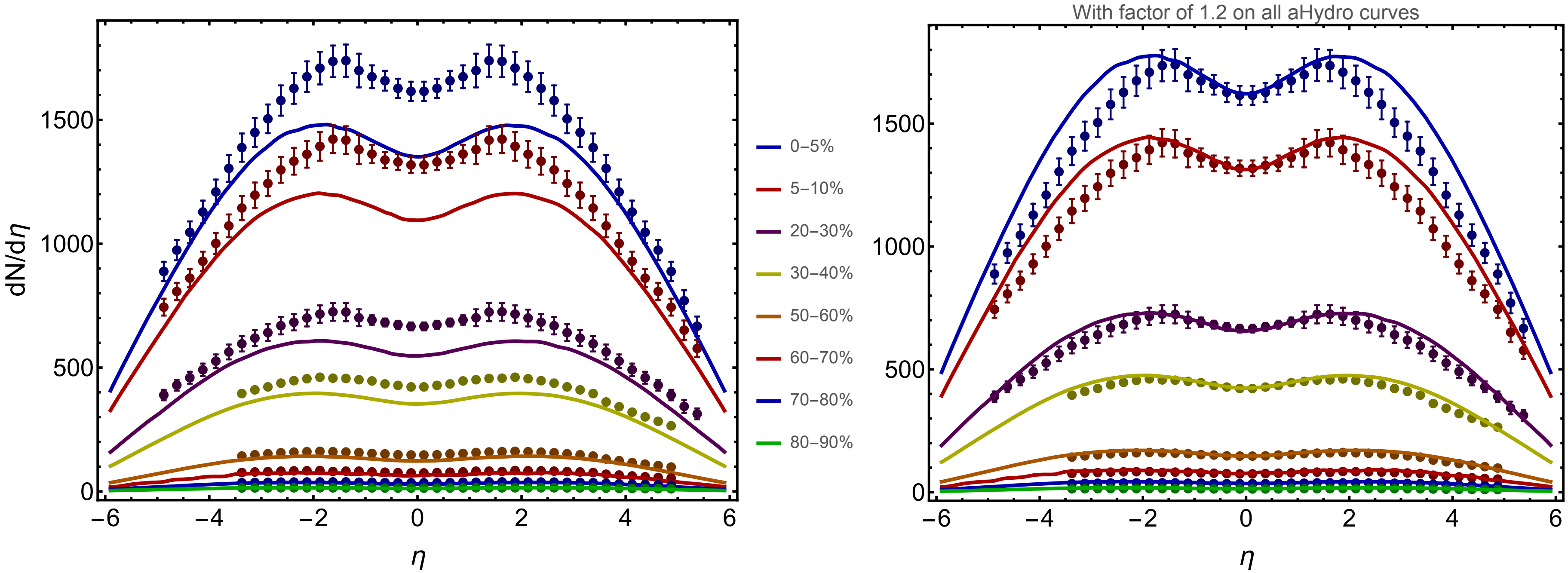


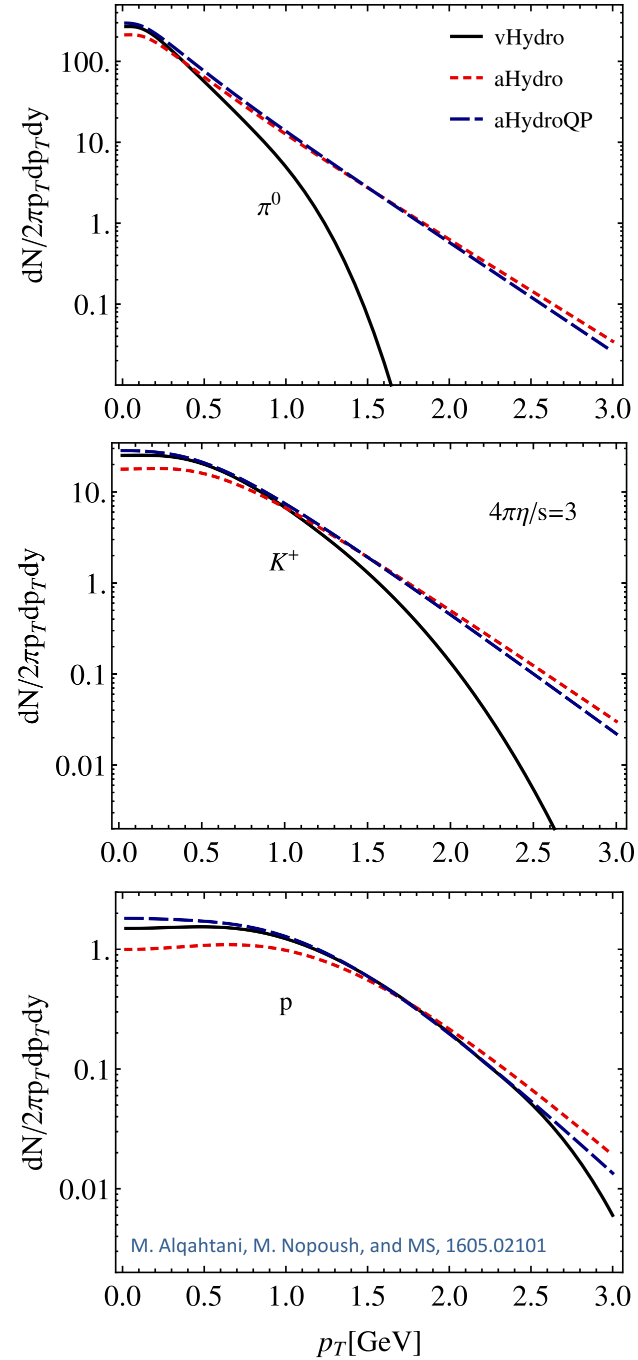
Preliminary results – Optical Glauber ICs

- We currently have a problem get the total multiplicities right
- This is related to the under prediction of the particle spectra at low transverse momenta



What's wrong?

- Not entirely sure why the low-p_T spectra and hence multiplicities are off.
- I think that the problem is related to the way we have implemented the realistic equation of state.
- In the standard approach I took the massless limit of the 1st and 2nd moment equations and simply enforced a realistic EoS by hand.
- I think we have to include the non-conformality in the 2nd moment equation or use aHydroQP (numerically too demanding at the moment).



Conclusions and Outlook

- Anisotropic hydrodynamics builds upon prior advances in relativistic hydrodynamics in an attempt to create a (hopefully) more quantitatively reliable model of QGP evolution.
- It incorporates some “facts of life” specific to the conditions generated in relativistic heavy ion collisions and, in doing so, optimizes the dissipative hydrodynamics approach.
- We now have a running 3+1d aHydro code with realistic EoS, anisotropic freeze-out, and fluctuating initial conditions.
- Our preliminary fits to experimental data using optical Glauber look “reasonable”; however, we have a problem at the moment with the low-momentum part of the spectra and multiplicities
- Also need to add the off-diagonal anisotropies and turn on the fluctuating initial conditions . . . Lots of work yet to do.

Connection to Viscous Hydro

For small departures from equilibrium we can linearize

$$\boxed{f(x,p) = f_{\rm eq}\left(\frac{p^\mu u_\mu}{T}\right)(1+\delta f(x,p))}$$

$$\begin{aligned} T^{\mu\nu} &= T^{\mu\nu}_{\rm ideal} + \int dP\; p^\mu p^\nu f_{\rm eq}\,\delta f \\ &\equiv T^{\mu\nu}_{\rm ideal} + \Pi^{\mu\nu} \end{aligned} \quad \longrightarrow \quad \boxed{\Pi^{\mu\nu} = \int dP\; p^\mu p^\nu f_{\rm eq}\,\delta f}$$

For viscous hydro one expands δf in a gradient expansion: n^{th} order in gradients → n^{th}-order viscous Hydro

- 1^{st} order Hydro : Relativistic Navier-Stokes (parabolic diff eqs → acausal) [e.g. Eckart and Landau-Lifshitz]
- 2^{nd} order Hydro : Including quadratic gradients fixes causality problem; hyperbolic diff eqs [e.g. Israel-Stewart]
- …

1^{st} Order Hydro

- Expand kinetic equations to first order in gradients.

Approximation: 1^{st} order in gradients of u^ν → Relativistic Navier-Stokes

$$\Pi^{\mu\nu} = \pi^{\mu\nu} + \Delta^{\mu\nu}\Phi \qquad \pi^\mu{}_\mu = 0$$

$$\pi^{\mu\nu} = \eta\nabla^{\langle\mu}u^{\nu\rangle} \qquad \Phi = \zeta\nabla_\alpha u^\alpha$$

η = Shear Viscosity

ζ = Bulk Viscosity

$$\nabla^{\langle\mu}u^{\nu\rangle} \equiv 2\nabla^{(\mu}u^{\nu)} - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha$$

- For now simplicity, I will ignore the bulk viscosity
- If $f_{\rm eq}$ is a Boltzmann distribution one finds

$$\boxed{f(x,p) = f_{\rm eq}\left(\frac{p^\mu u_\mu}{T}\right)\left[1 + \frac{p^\alpha p^\beta \pi_{\alpha\beta}}{2(\mathcal{E}+\mathcal{P})T^2}\right]}$$

1^{st} Order Hydro – 0+1d

$$u^\mu = (\cosh\varsigma, 0, 0, \sinh\varsigma) = \left(\frac{t}{\tau}, 0, 0, \frac{z}{\tau}\right)$$

$$\rightarrow \pi^{\mu\nu} = \eta\nabla^{\langle\mu}u^{\nu\rangle}$$

$$\nabla^{\langle\mu}u^{\nu\rangle} \equiv 2\nabla^{(\mu}u^{\nu)} - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha$$

$$\pi^{xx} = \eta\left(2\underbrace{\nabla^{(x}u^{x)}}_{0} - \frac{2}{3}\underbrace{\Delta^{xx}}_{-1}\underbrace{\partial_\mu u^\mu}_{1/\tau}\right) = \frac{2\eta}{3\tau} = \pi^{yy}$$

$$\pi^{zz} = -(\pi^{xx} + \pi^{yy}) = -\frac{4\eta}{3\tau}$$

$$\mathcal{P}_T \equiv \mathcal{P}_{\rm eq} + \pi^{xx} = \mathcal{P}_{\rm eq} + \frac{2\eta}{3\tau}$$

$$\mathcal{P}_L \equiv \mathcal{P}_{\rm eq} + \pi^{zz} = \mathcal{P}_{\rm eq} - \frac{4\eta}{3\tau}$$

$$\boxed{\mathcal{P}_T \neq \mathcal{P}_L}$$

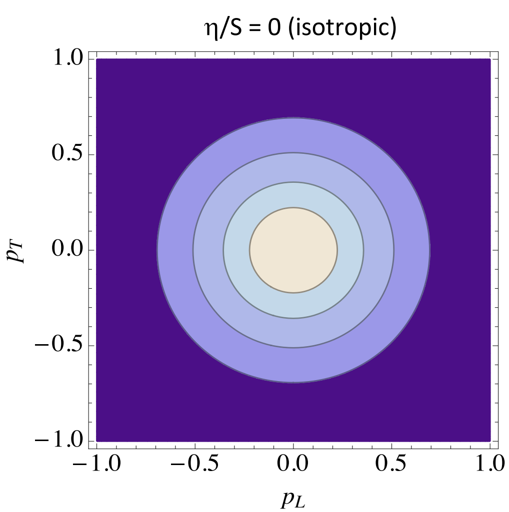
Longitudinal pressure can become negative!

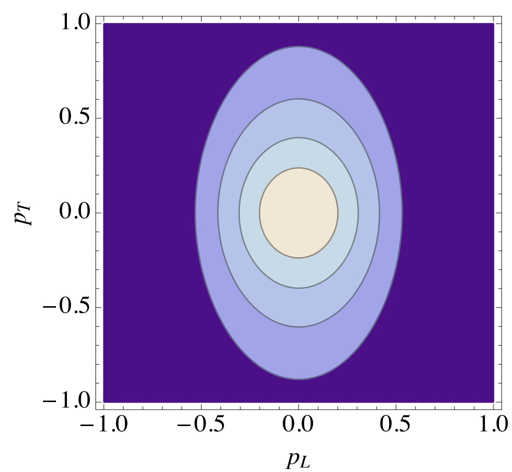
1st Order Hydro – 0+1d

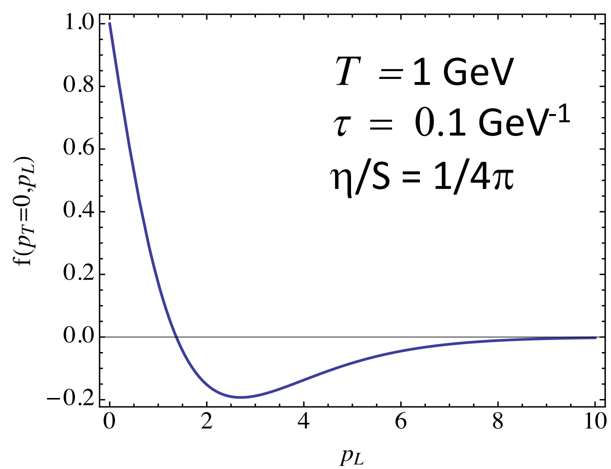
Additionally one finds for the first order distribution function

$$f(x,p) = f_{\rm eq}\left(\frac{p^\mu u_\mu}{T}\right)\left[1+\frac{p^\alpha p^\beta \pi_{\alpha\beta}}{2(\mathcal{E}+\mathcal{P})T^2}\right] \longrightarrow f_{\rm eq}\left(\frac{E}{T}\right)\left[1+\frac{\eta}{\mathcal{S}}\frac{p_x^2+p_y^2-2p_z^2}{3\tau T^3}\right]$$

- Distribution function becomes anisotropic in momentum space
- There are also regions where $f(x,p) < 0$
- Anisotropy and regions of negativity increase as τ or T decrease OR η/S increases







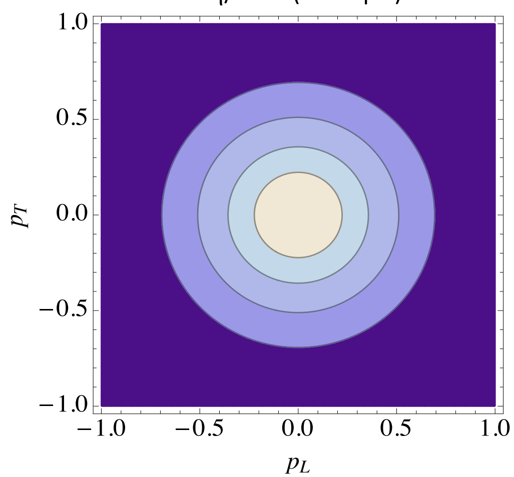
1st Order Hydro – 0+1d

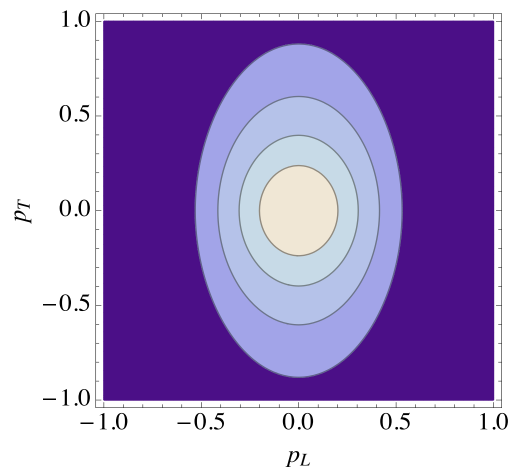
Additionally one finds for the first order distribution function

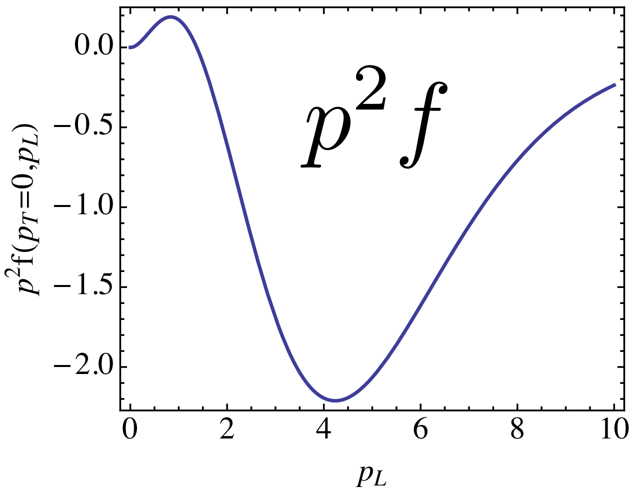
$$f(x,p) = f_{\rm eq}\left(\frac{p^\mu u_\mu}{T}\right)\left[1+\frac{p^\alpha p^\beta \pi_{\alpha\beta}}{2(\mathcal{E}+\mathcal{P})T^2}\right] \longrightarrow f_{\rm eq}\left(\frac{E}{T}\right)\left[1+\frac{\eta}{\mathcal{S}}\frac{p_x^2+p_y^2-2p_z^2}{3\tau T^3}\right]$$

- Distribution function becomes anisotropic in momentum space
- There are also regions where $f(x,p) < 0$
- Anisotropy and regions of negativity increase as τ or T decrease OR η/S increases

η/S = 0 (isotropic)

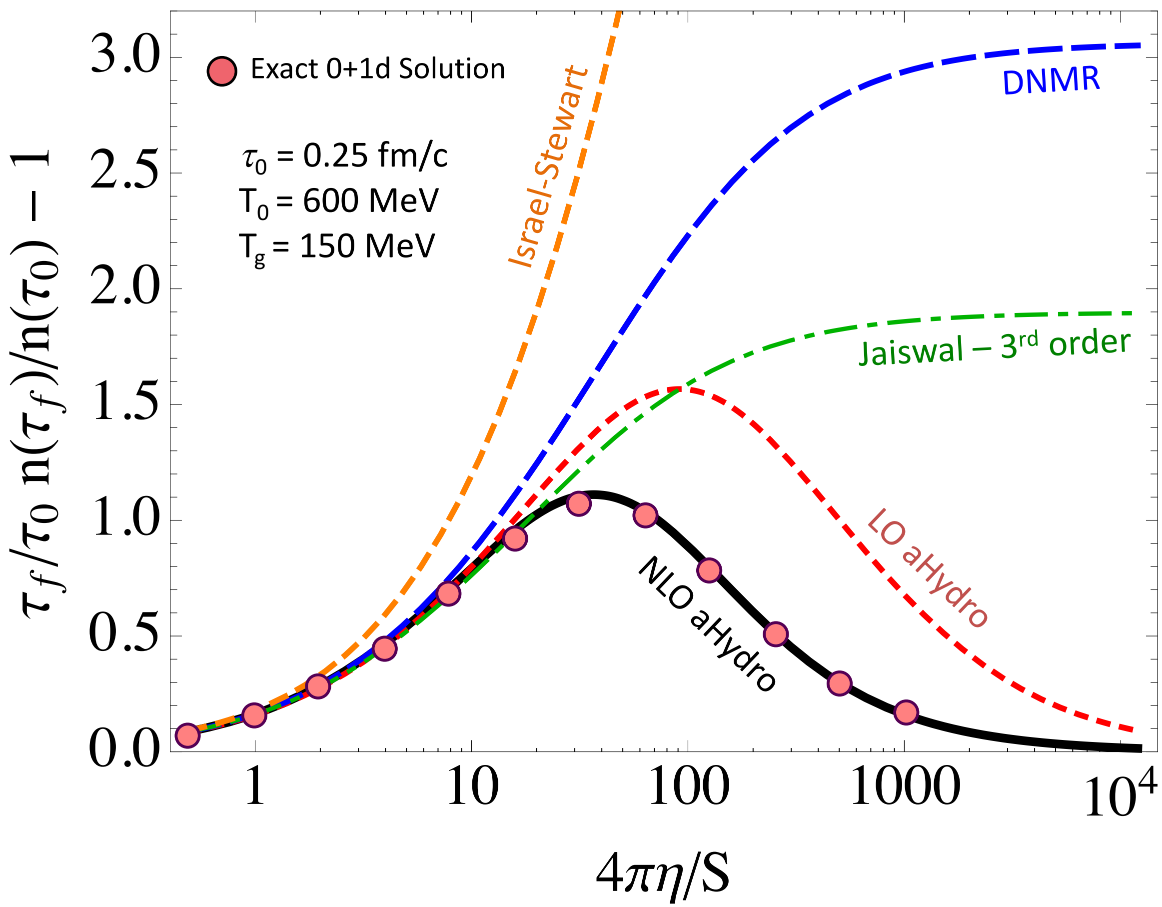






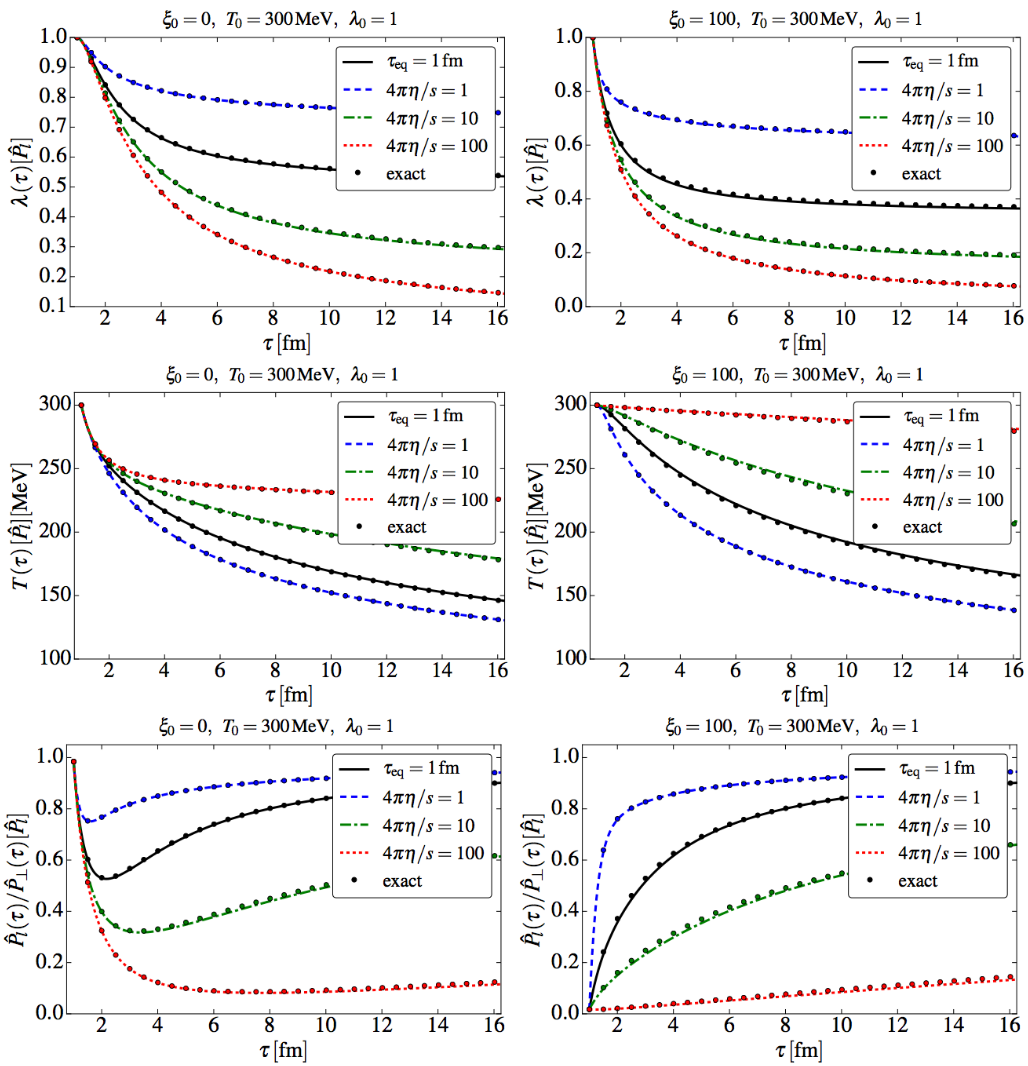
Conformal 0+1d aHydro results

[W. Florkowski, R. Ryblewski, and MS, 1304.0665 and 1305.7234]
[D. Bazow, U. Heinz, and MS, 1311.6720]



- Number (entropy) production vanishes in two limits: ideal hydrodynamic and free streaming limits
- In the conformal model which we are testing with, number density is proportional to entropy density

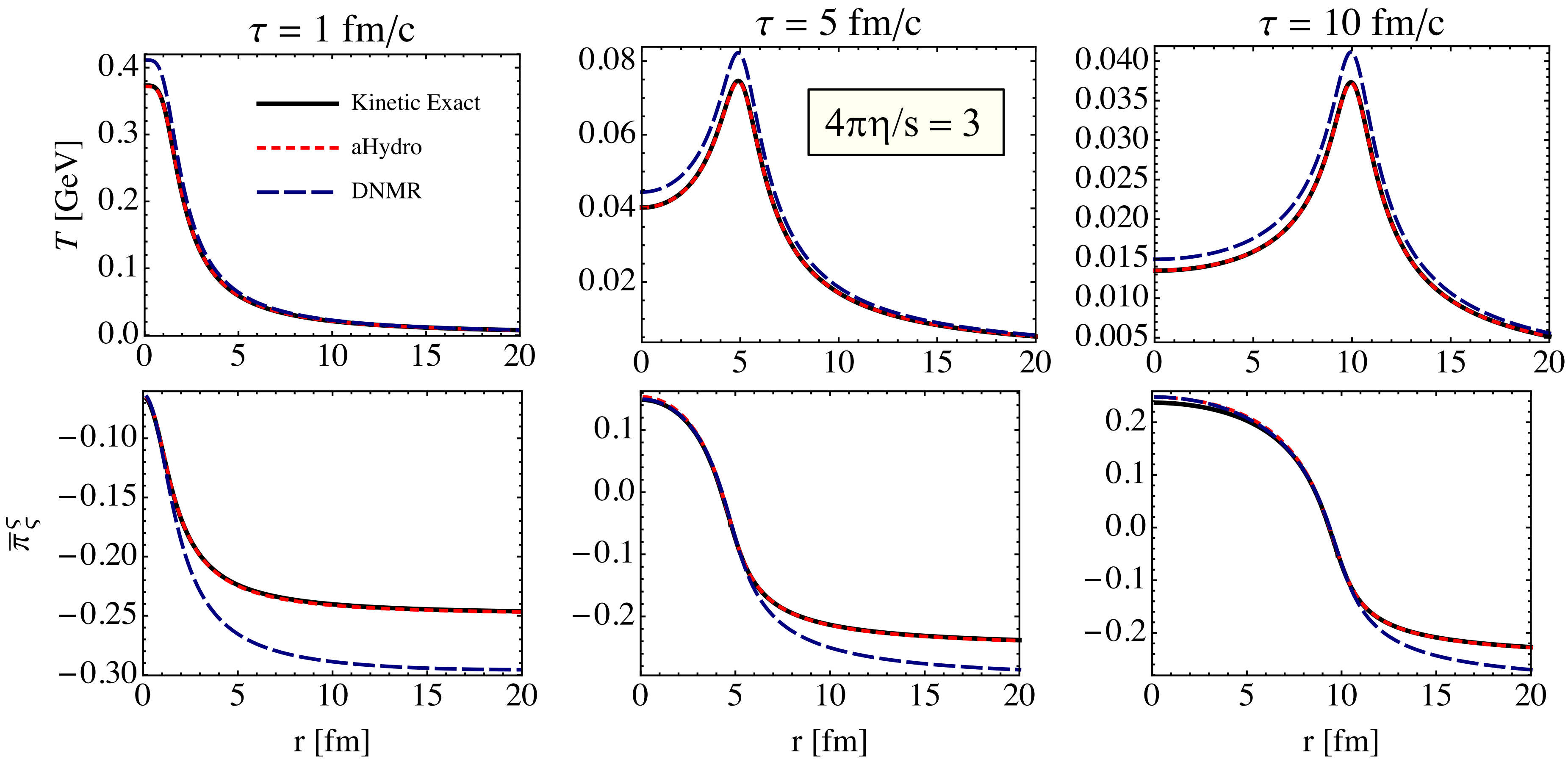
Conformal 0+1d aHydro results



- Since our earlier papers, others have shown how to make things even better by a judicious choice of moments.
- Results on the left are from the recent paper of Molnar, Rischke, and Niemi [1606.09019]

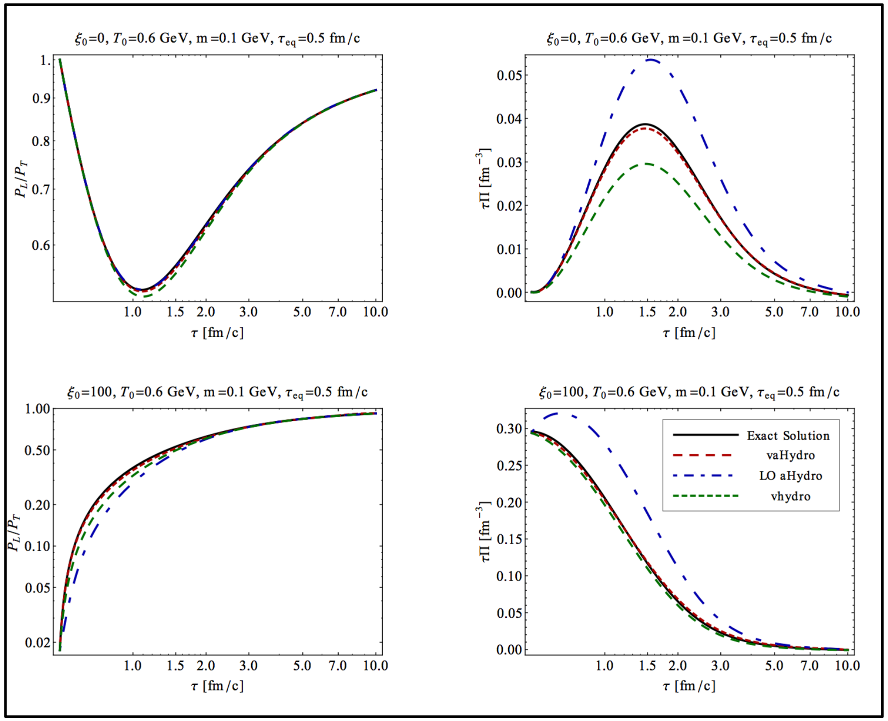
1+1d aHydro solution for Gubser Flow

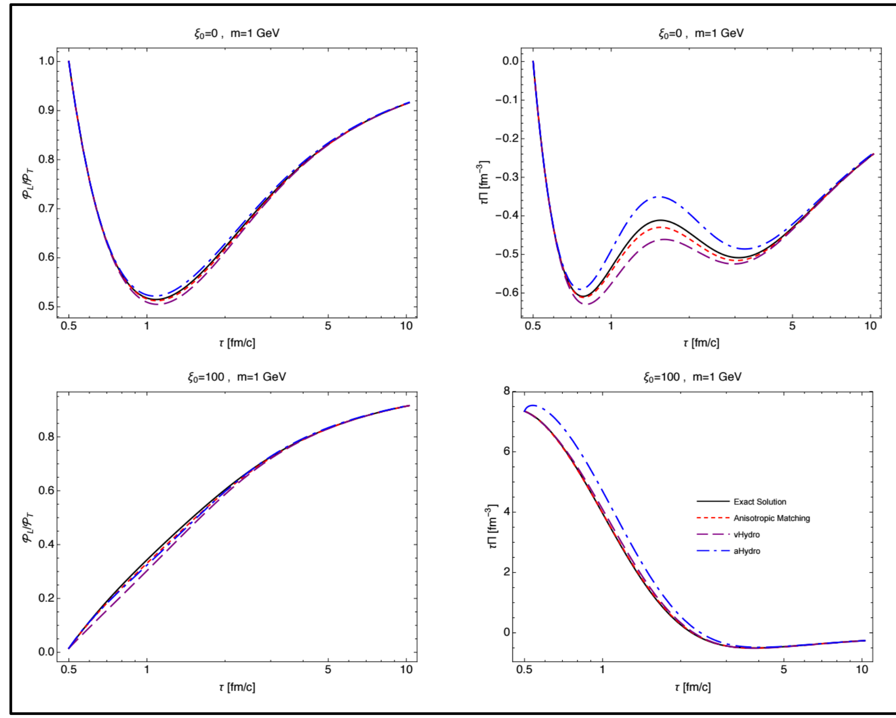
M. Nopoush, R. Ryblewski, and MS, 1410.6790
Exact kinetic solution: G. Denicol, U.W. Heinz, M. Martinez, J. Noronha, and MS, 1408.5646 and 1408.7048



Once again, aHydro solution can be shown to reproduce the free streaming limit analytically. [M. Nopoush, R. Ryblewski, and MS, 1410.6790]

Non-conformal 0+1d aHydro results





- Also works well in the non-conformal case
- Results on the left are from Bazow, Heinz, and Martinez [1503.07443]
- Results on the right are from Tinti [1506.07164]