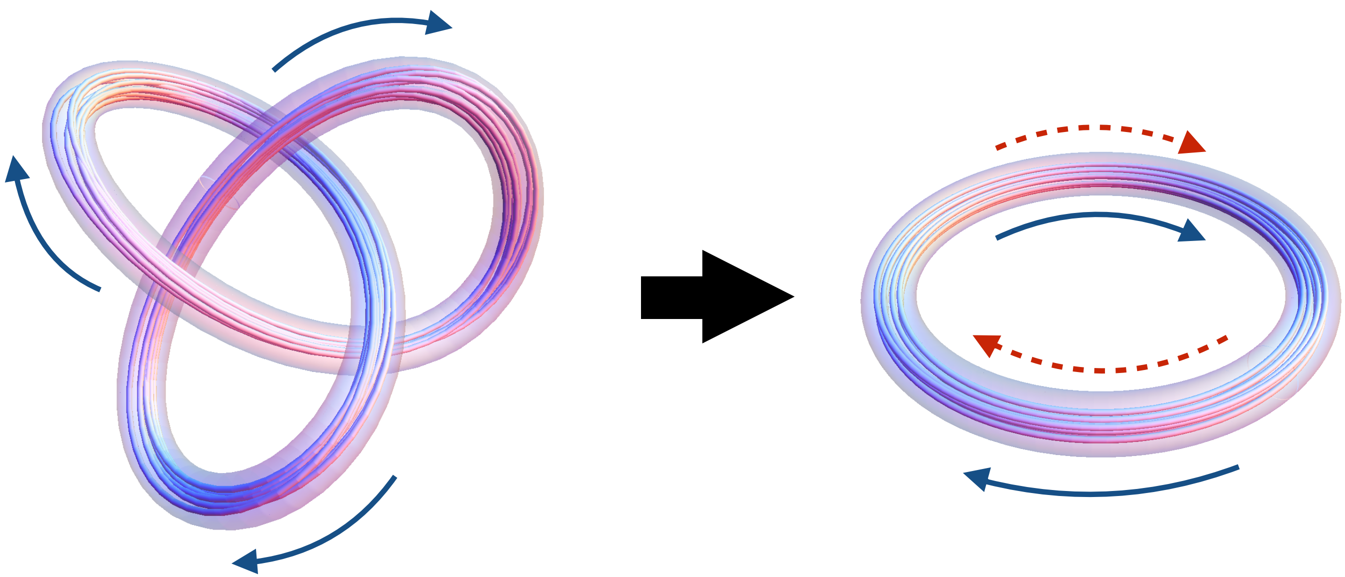
Topology of B fields and Anomalous chiral effects



Yuji Hirono

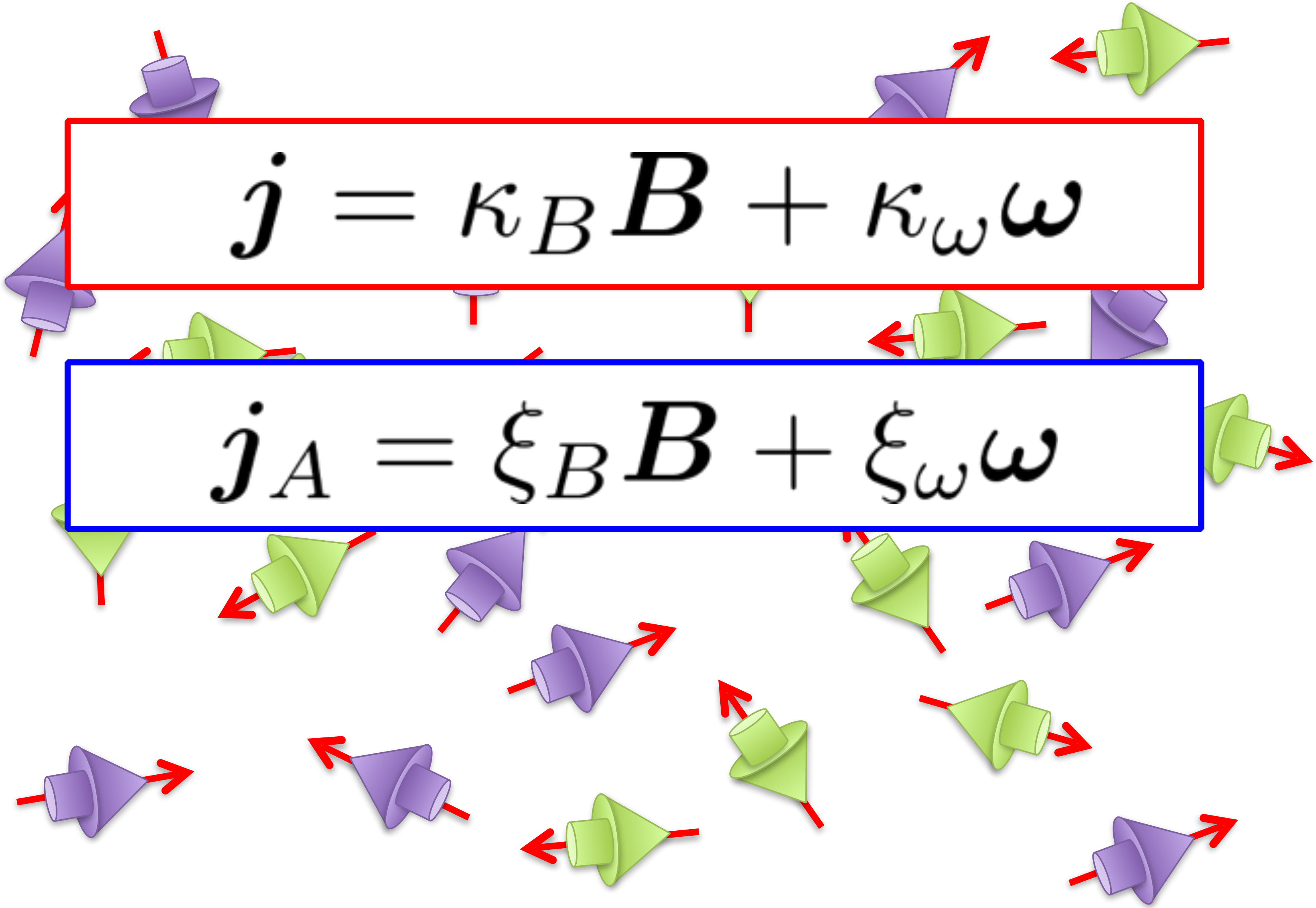
Brookhaven National Laboratory

Anomalous chiral effects

Anomalous chiral effects

$$\boldsymbol{j} = \kappa_B \boldsymbol{B} + \kappa_\omega \boldsymbol{\omega}$$

$$\boldsymbol{j}_A = \xi_B \boldsymbol{B} + \xi_\omega \boldsymbol{\omega}$$



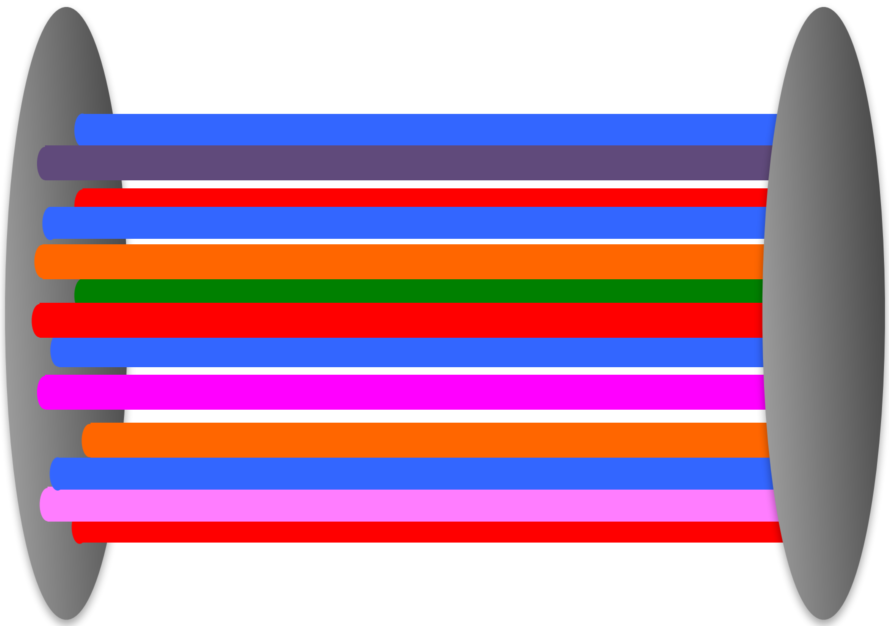
Anomalous chiral effects

- Charge dependent correlations from event-by-event anomalous hydrodynamics

  [Hirono-Hirano-Khazeev [1412.0311]]

- CME current from reconnections of magnetic flux

  [Hirono-Khazeev-Yi, PRL in press [1606.09611]]

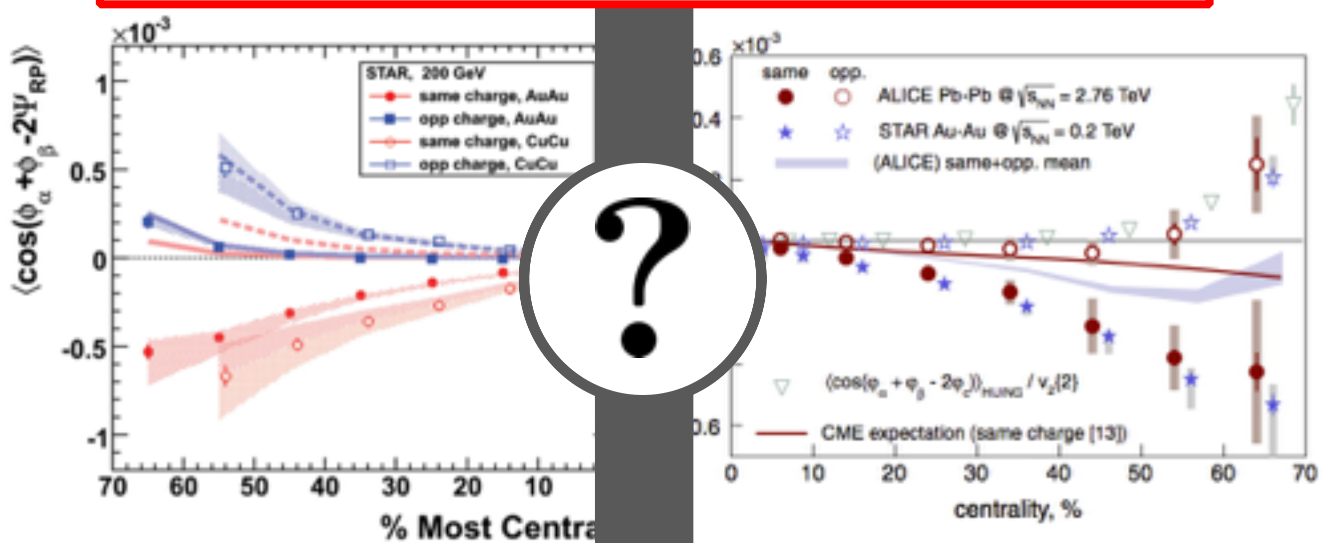


Charge dependent correlations from event-by-event anomalous hydrodynamics

[Hirono-Hirano-Khazeev [1412.0311]]

Anomalous transport in heavy-ion collisions?

$$j = \frac{e^2 \mu_5}{2\pi^2} B \qquad j_5 = \frac{e^2 \mu}{2\pi^2} B$$



[STAR, PRL2009, PRC2010]

[ALICE, PRL2013]

$$\langle \cos(\phi_1^\alpha + \phi_2^\beta - 2\Psi_{\rm RP}) \rangle$$

Anomalous transport in heavy-ion collisions?

$$\boldsymbol{j} = \frac{e^2 \mu_5}{2\pi^2}\boldsymbol{B} \qquad \boldsymbol{j}_5 = \frac{e^2 \mu}{2\pi^2}\boldsymbol{B}$$

1. Physical meaning of obs.
2. EbE anomalous hydro
3. Results

$$\langle \cos(\phi_1^\alpha + \phi_2^\beta - 2\Psi_{\rm RP}) \rangle$$

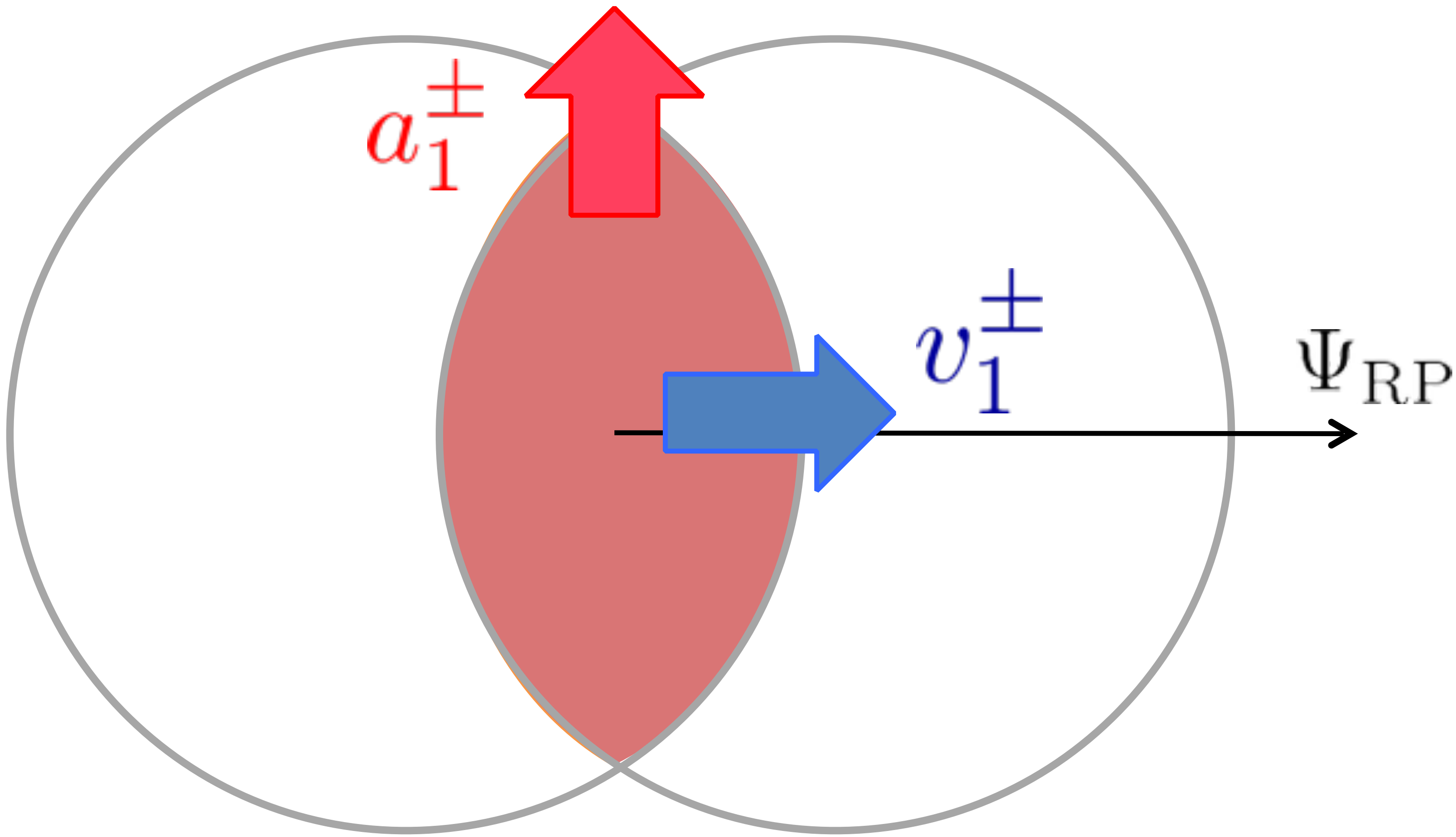
Charge dependent correlations [STAR]

$$\boxed{\langle \cos(\phi_1^\alpha + \phi_2^\beta - 2\Psi_{\mathrm{RP}}) \rangle}$$

$$\alpha, \beta \in \{+, -\}$$

$$\langle \cos(\phi_1^+ + \phi_2^+ - 2\Psi_{\mathrm{RP}}) \rangle$$
$$= \langle \cos(\phi_1^+ - \Psi_{\mathrm{RP}}) \cos(\phi_2^+ - \Psi_{\mathrm{RP}}) \rangle - \langle \sin(\phi_1^+ - \Psi_{\mathrm{RP}}) \sin(\phi_2^+ - \Psi_{\mathrm{RP}}) \rangle$$
$$= \langle (v_1^+)^2 \rangle - \langle (a_1^+)^2 \rangle$$

Charge dependent correlations [STAR]



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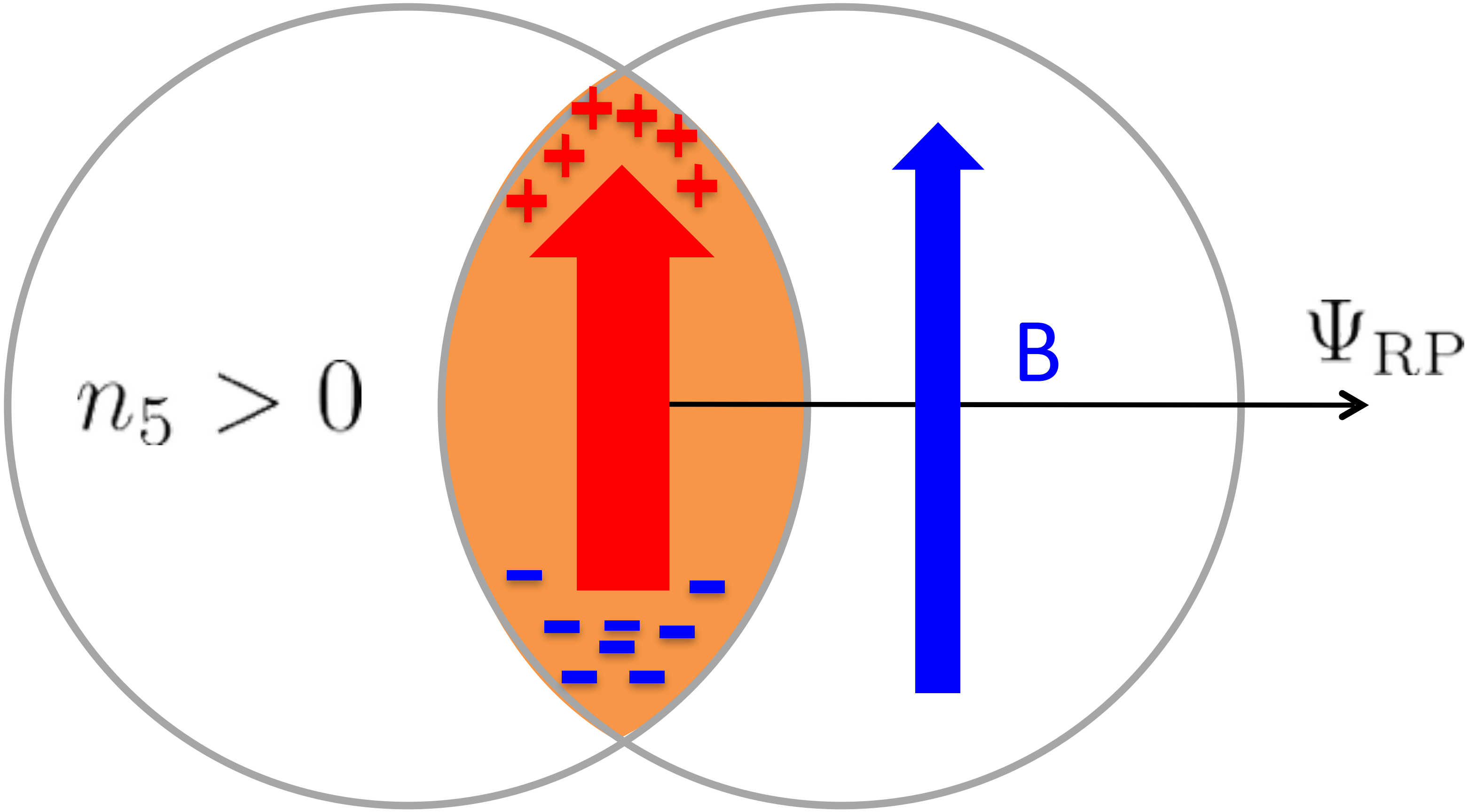
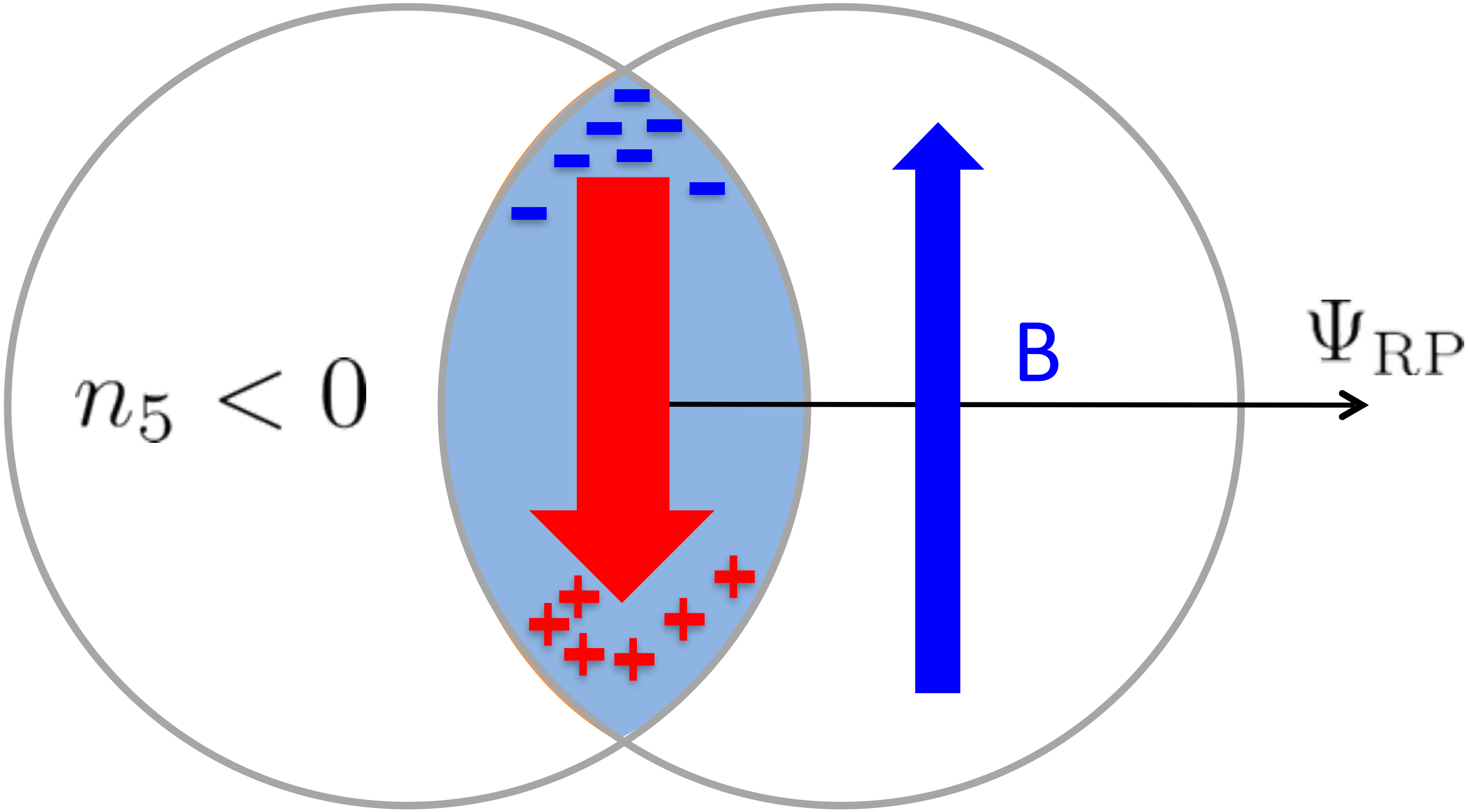
Charge dependent correlations [STAR]



$a_1^+ > 0 \quad a_1^- < 0$

Charge dependent correlations [STAR]



$a_1^+ < 0 \quad a_1^- > 0$

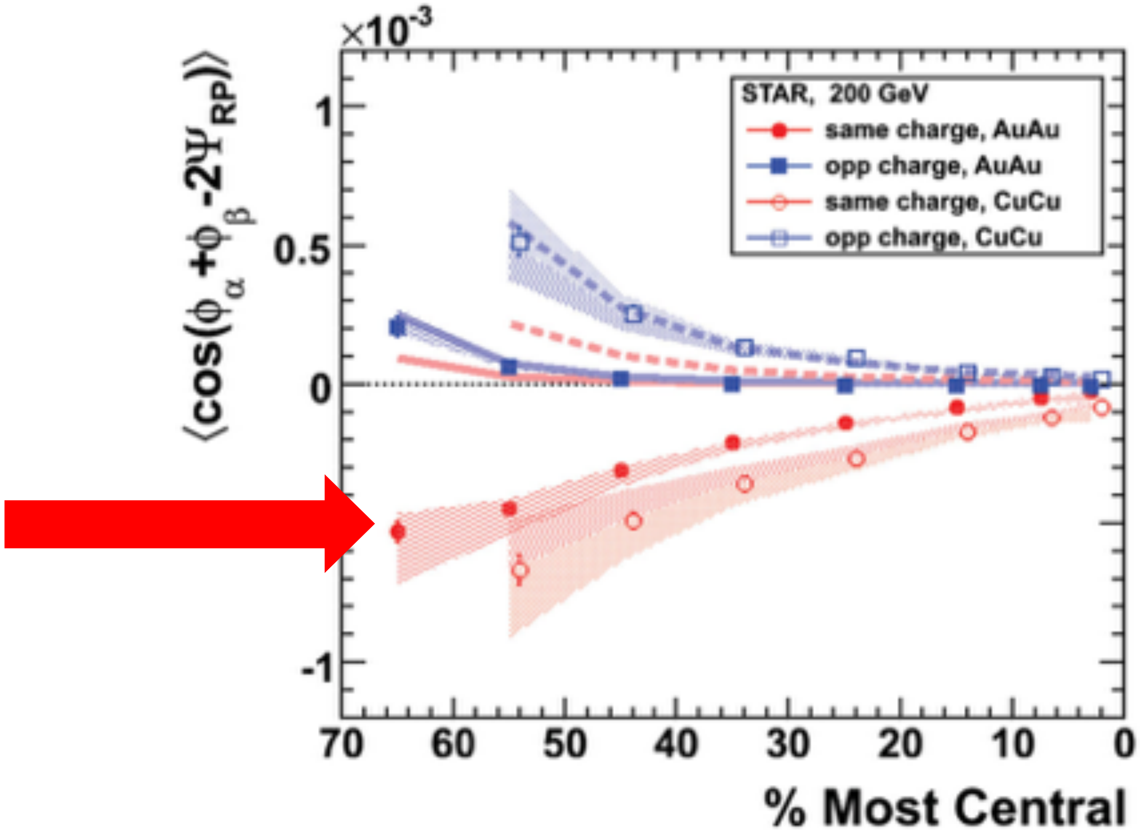
Charge dependent correlations [STAR]

$$\langle a_1^+ \rangle = \langle a_1^- \rangle = 0$$

$$\langle (a_1^+)^2 \rangle = \langle (a_1^-)^2 \rangle > 0$$

$$\langle a_1^+ a_1^- \rangle < 0$$

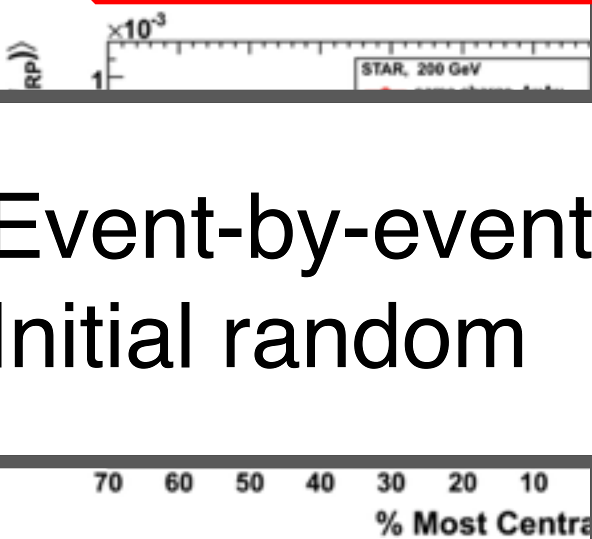
Charge dependent correlations [STAR]

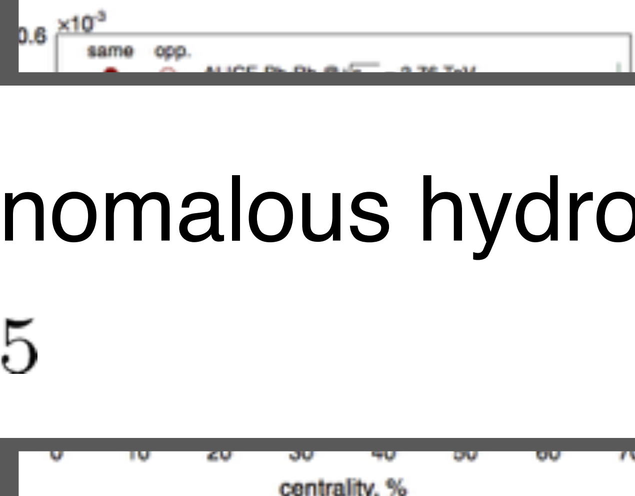


$$\langle \cos(\phi_1^+ + \phi_2^+ - 2\Psi_{\mathrm{RP}})\rangle = \langle (v_1^+)^2 \rangle - \langle (a_1^+)^2 \rangle$$

Anomalous transport in heavy-ion collisions?

$$j = \frac{e^2 \mu_5}{2\pi^2} B \qquad j_5 = \frac{e^2 \mu}{2\pi^2} B$$





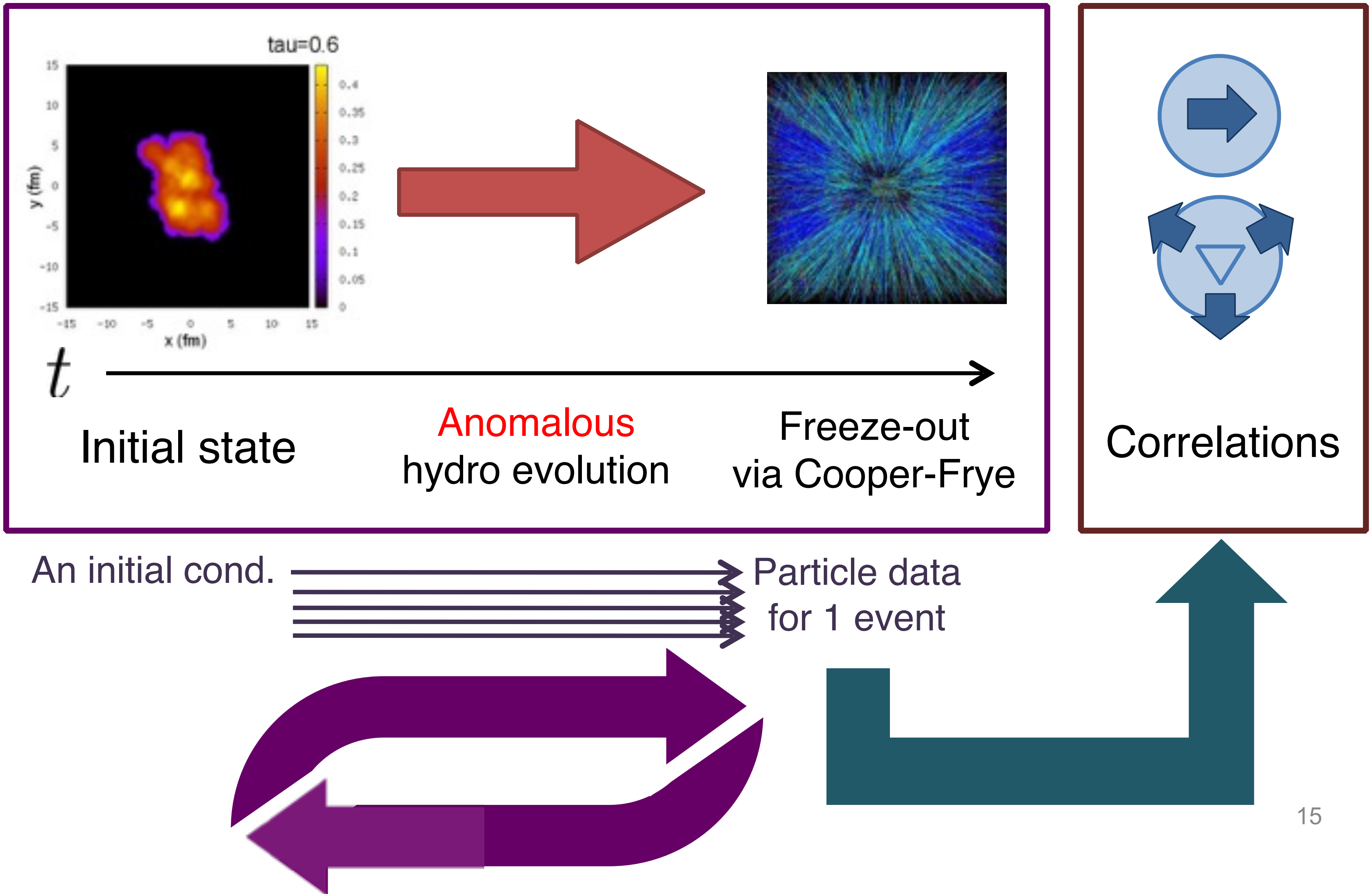
- Event-by-event anomalous hydro
- Initial random n_5

[STAR, PRL2009, PRC2010]

[ALICE, PRL2013]

$$\langle \cos(\phi_1^\alpha + \phi_2^\beta - 2\Psi_{\rm RP}) \rangle$$

Event-by-event anomalous hydrodynamic model



Anomalous hydrodynamics equations

- Non-dissipative anomalous fluid in 3+1D
  - no viscosity/Ohmic conductivity
- Background electromagnetic fields

$$\partial_\mu T^{\mu\nu} = \boxed{F^{\nu\rho} j_\rho}$$

$$\partial_\mu j^\mu = 0 \qquad \partial_\mu j_5^\mu = \boxed{C E_\mu B^\mu}$$

$$C = \frac{N_\mathrm{c} N_\mathrm{f}}{2\pi^2}$$

Anomalous hydrodynamics equations

- Constitutive equations

$$j^\mu = n u^\mu + \boxed{\kappa_B B^\mu} \qquad j_5^\mu = n_5 u^\mu + \boxed{\xi_B B^\mu}$$

CME CSE

$$e\kappa_B = C\mu_5\left(1 - \frac{\mu n}{e+p}\right) \quad e\xi_B = C\mu\left(1 - \frac{\mu_5 n_5}{e+p}\right)$$

[Son & Surowka (2009)]
[Kalaydzhyan & Kirsch (2011)]

- Equation of state - conformal
  - massless quarks & gluons

$$p\left(T, \boxed{\mu, \mu_5}\right) = \frac{g_{\mathrm{qgp}}\pi^2}{90}T^4 + \frac{N_c N_f}{6}\left(\mu^2 + \mu_5^2\right)T^2 + \frac{N_c N_f}{12\pi^2}\left(\mu^4 + 6\mu^2\mu_5^2 + \mu_5^4\right)$$

MC Sampling of particles

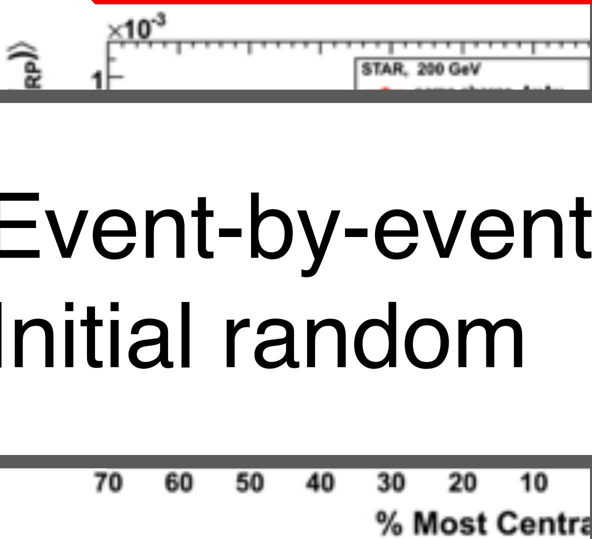
Isothermal hypersurface

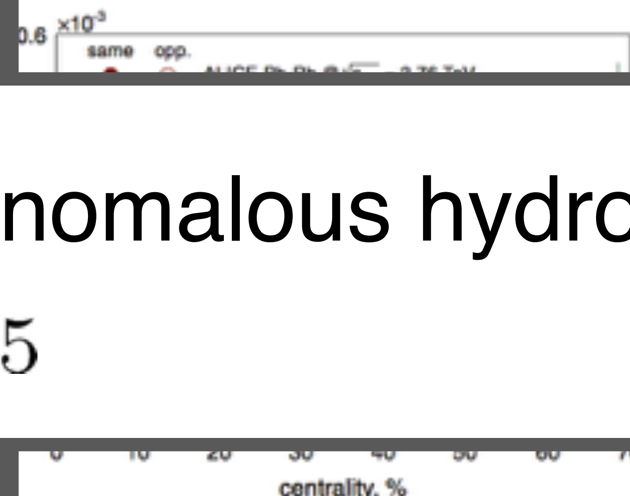
$$T_{\mathrm{fo}} = 160\ \mathrm{MeV}$$

$$dN = \int \frac{d^3p}{E} \frac{p_\mu d\sigma^\mu}{e^{\beta(p\cdot u - \mu)} \mp_{\mathrm{BF}} 1}$$

Anomalous transport in heavy-ion collisions?

$$j = \frac{e^2 \mu_5}{2\pi^2} B \qquad j_5 = \frac{e^2 \mu}{2\pi^2} B$$



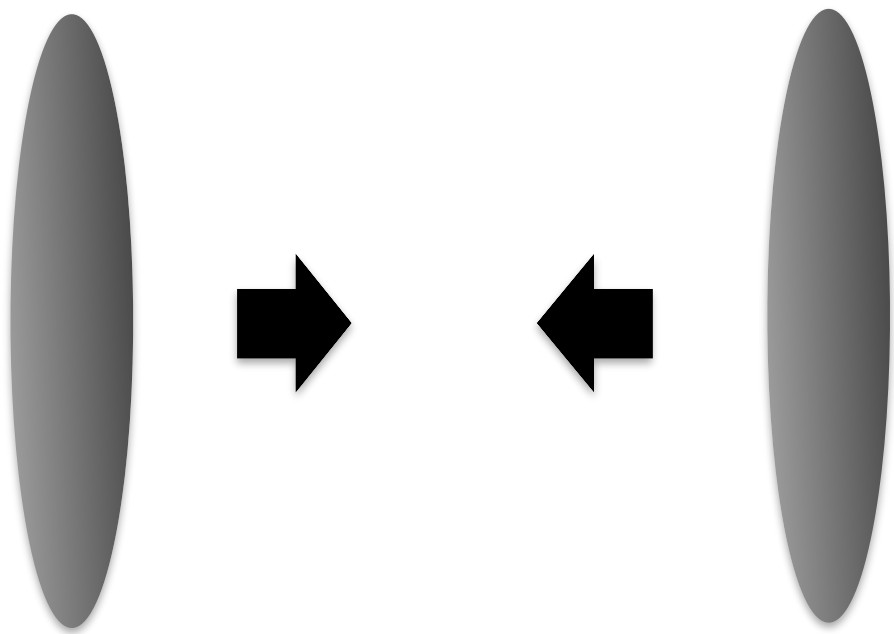


- ☑ Event-by-event anomalous hydro
- ☐ Initial random n_5

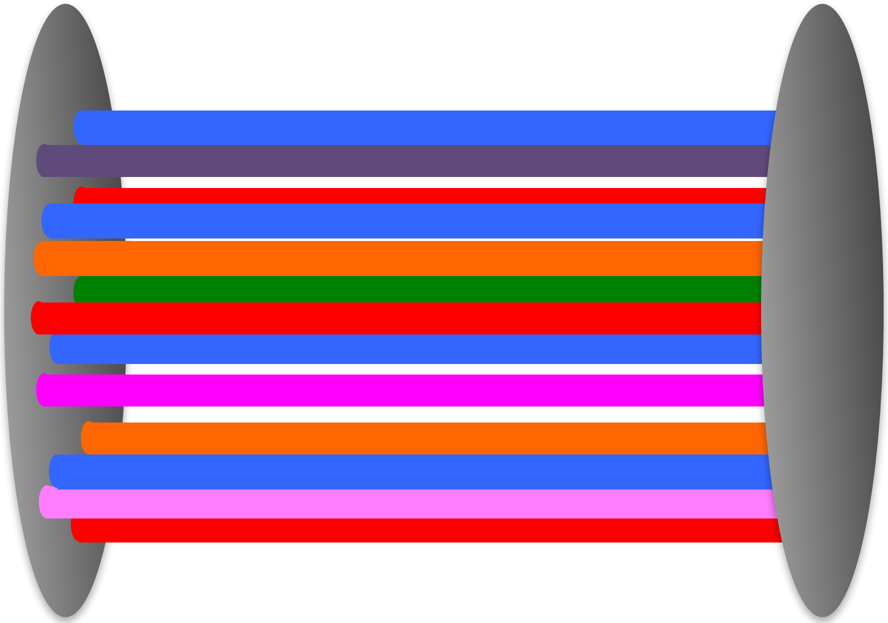
[STAR, PRL2009, PRC2010]

[ALICE, PRL2013]

$$\langle \cos(\phi_1^\alpha + \phi_2^\beta - 2\Psi_{\mathrm{RP}}) \rangle$$



$$\partial_\mu j_5^\mu = \frac{g^2}{16\pi^2} \boldsymbol{E}^a \cdot \boldsymbol{B}^a$$



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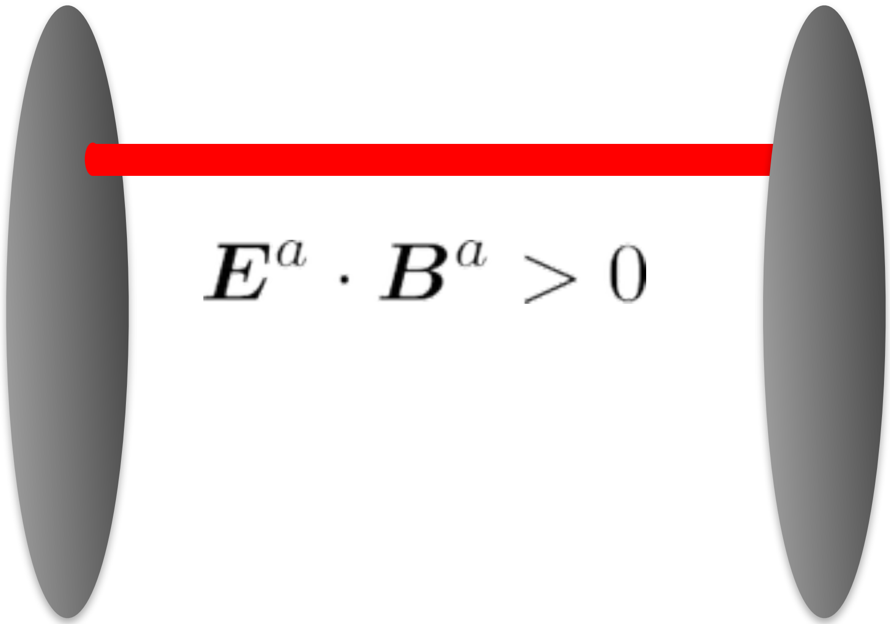

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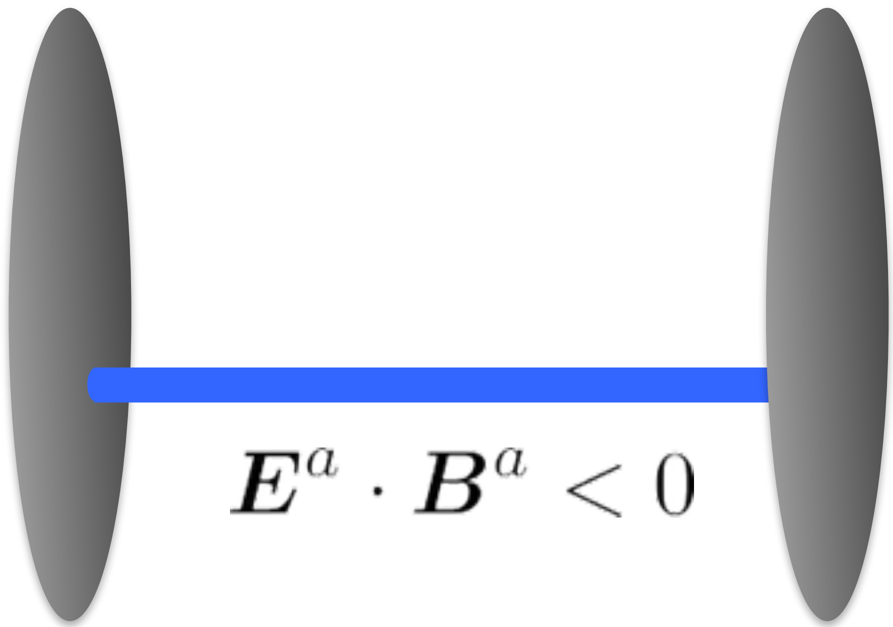
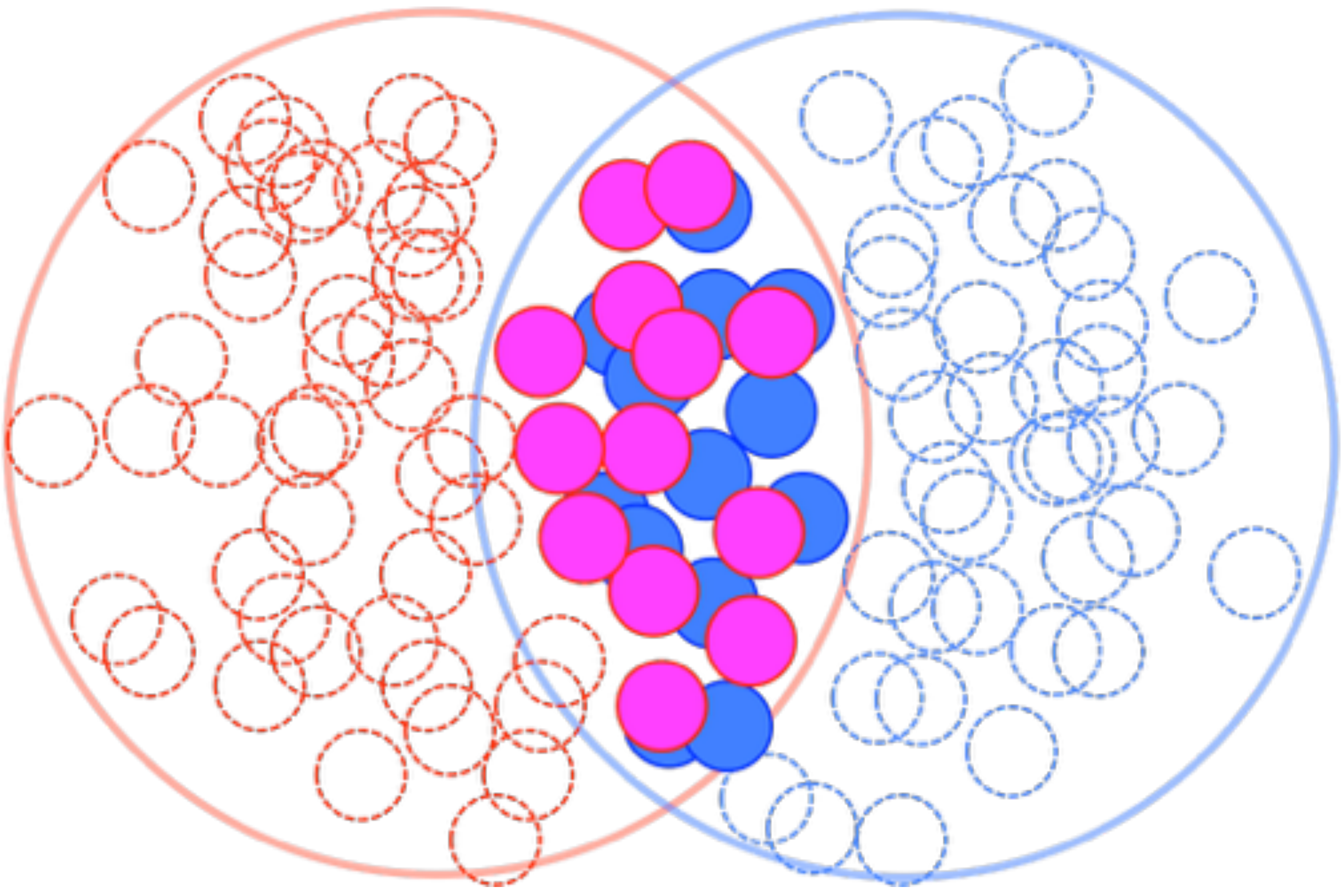


$$\boldsymbol{E}^a \cdot \boldsymbol{B}^a < 0$$

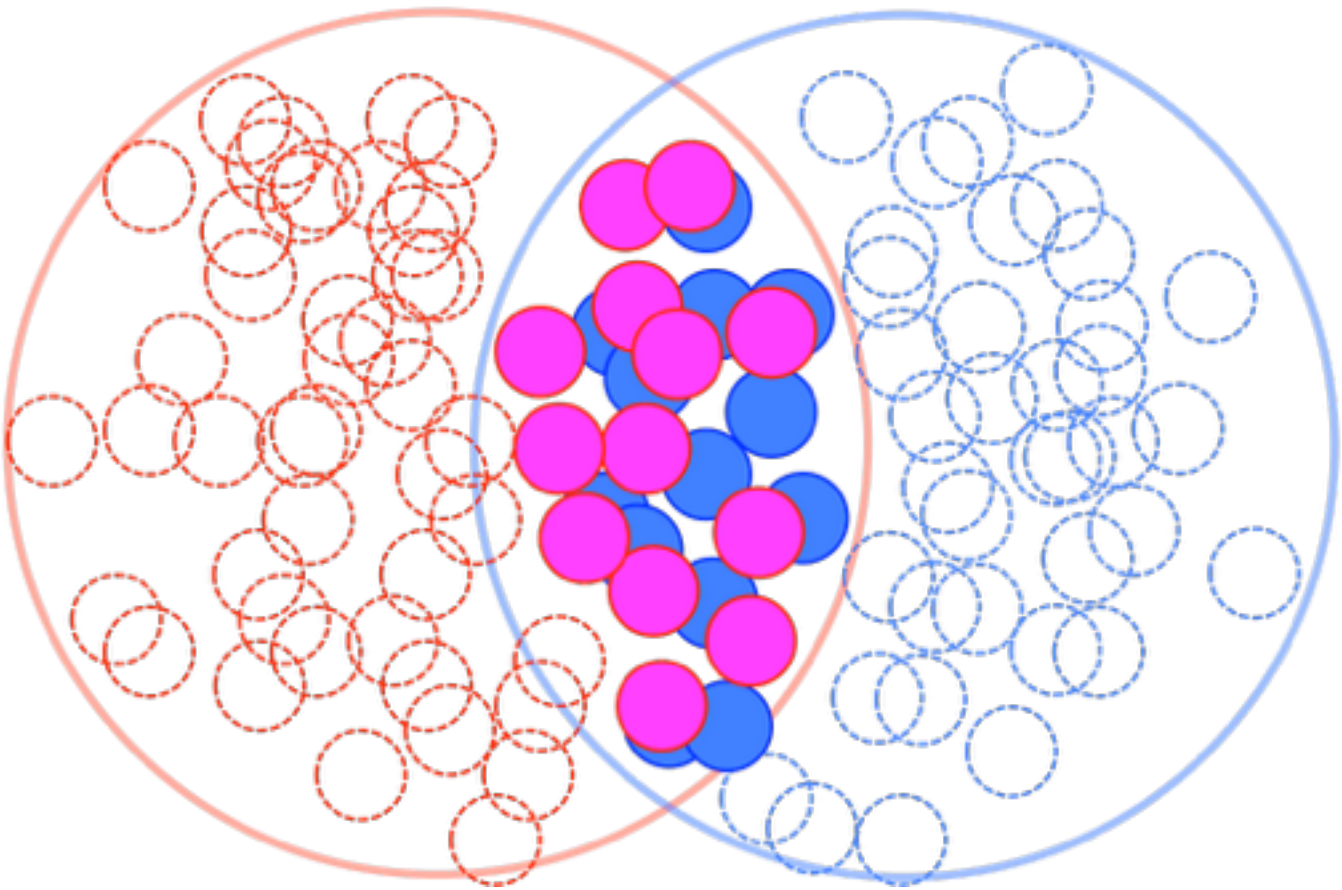
Axial charges from color flux tubes

$N_{\mathrm{part}}^{\mathrm{A(B)}}(\boldsymbol{x}_{\mathrm{T}})$: # of ( ) $N_{\mathrm{coll}}(\boldsymbol{x}_{\mathrm{T}})$: # of pairs



Axial charges from color flux tubes

$$s(\boldsymbol{x}_\mathrm{T}, \eta_s) = A f(\eta_s) \left[\frac{1-\alpha}{2} \frac{d^2 N_\mathrm{part}}{dx_\mathrm{T}^2} + \alpha \frac{d^2 N_\mathrm{coll}}{dx_\mathrm{T}^2} \right]$$

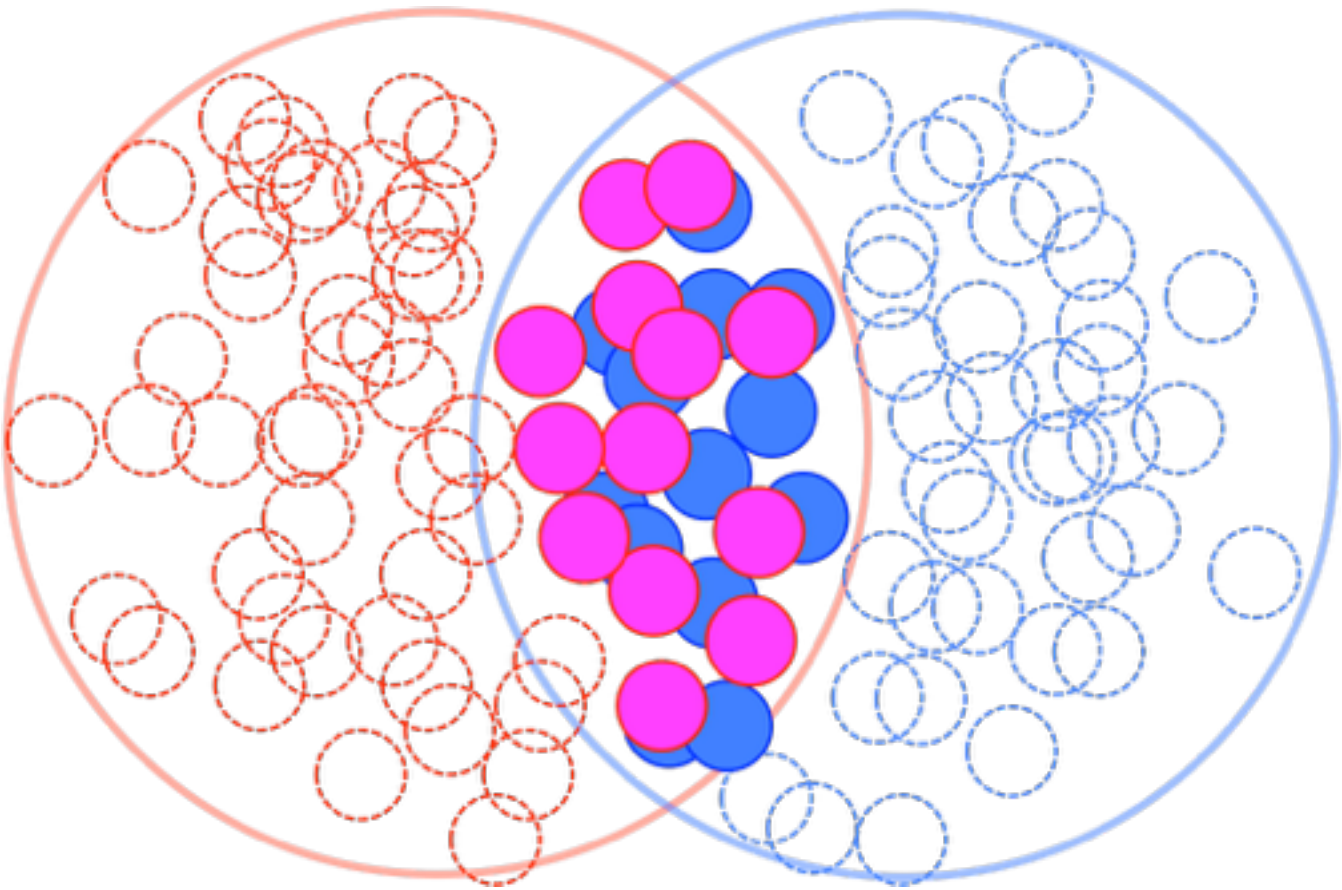


Axial charges from color flux tubes

$X_j \in \{+1, -1\}$

Sign of $\boldsymbol{E}^a \cdot \boldsymbol{B}^a$

$$\mu_5(\boldsymbol{x}_\mathrm{T}) = C_{\mu_5} \sum_{j=1}^{N_\mathrm{coll}(\boldsymbol{x}_\mathrm{T})} X_j$$



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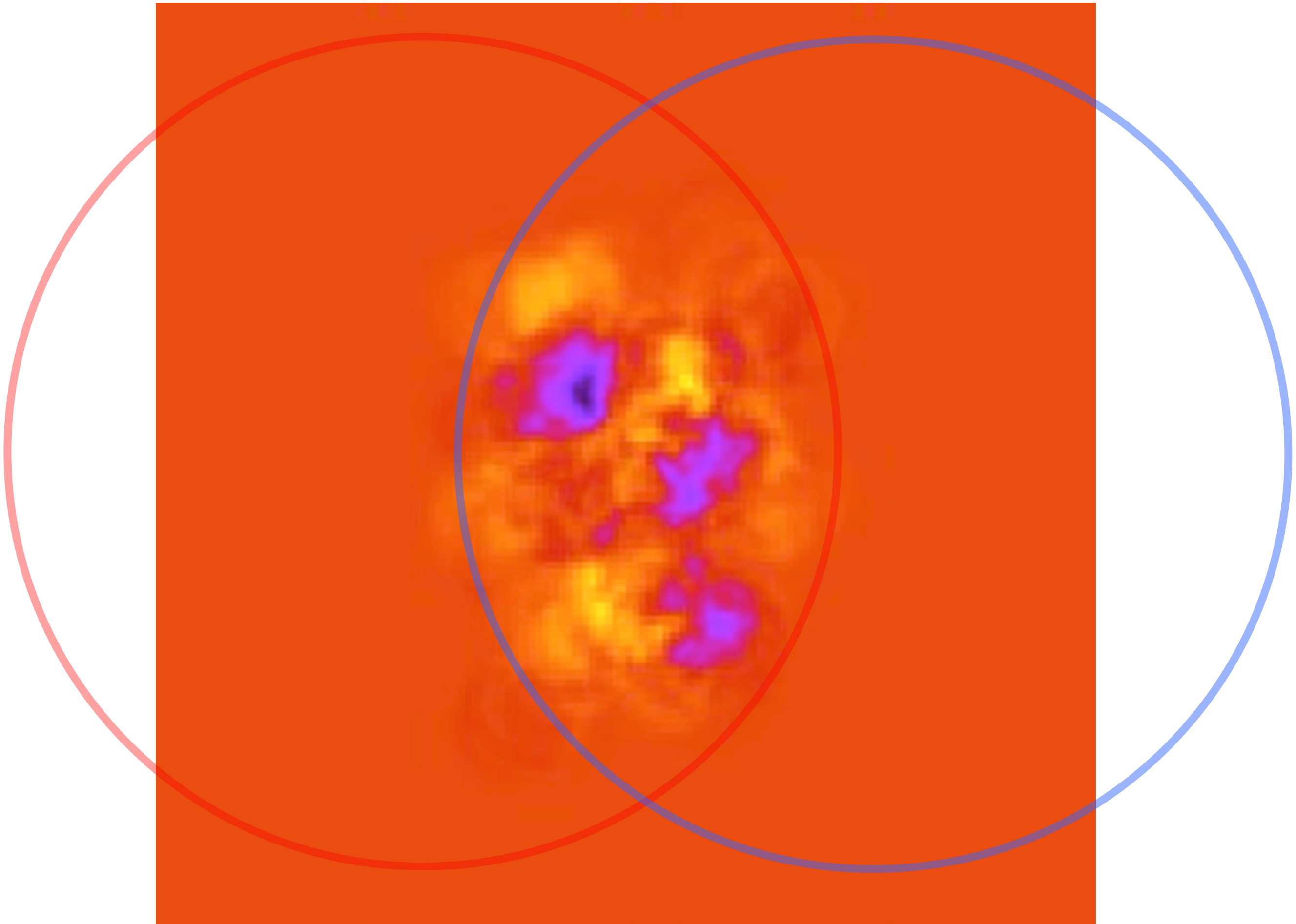
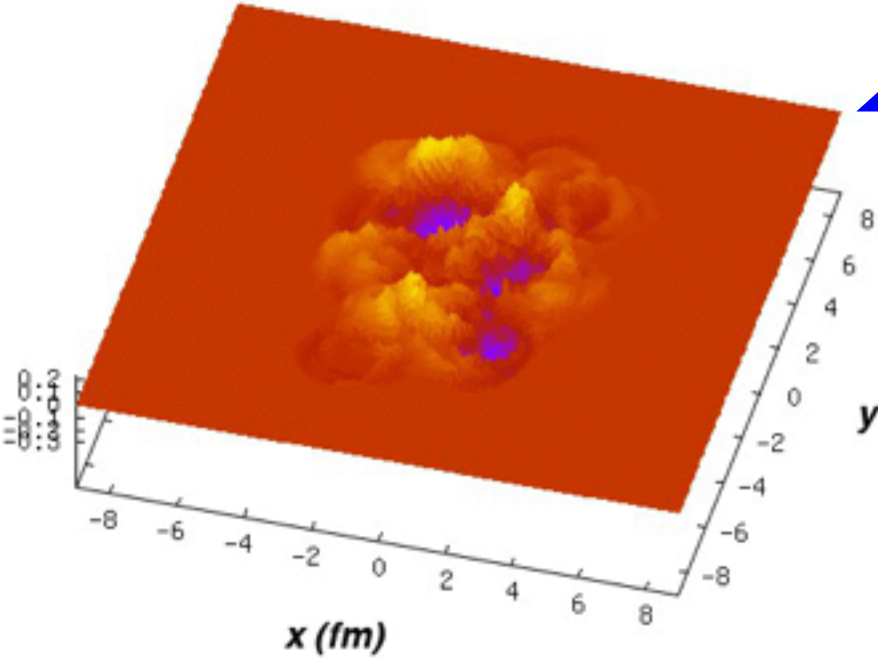
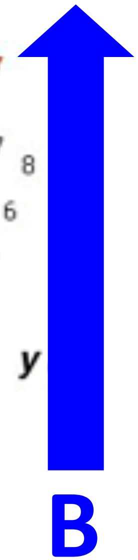
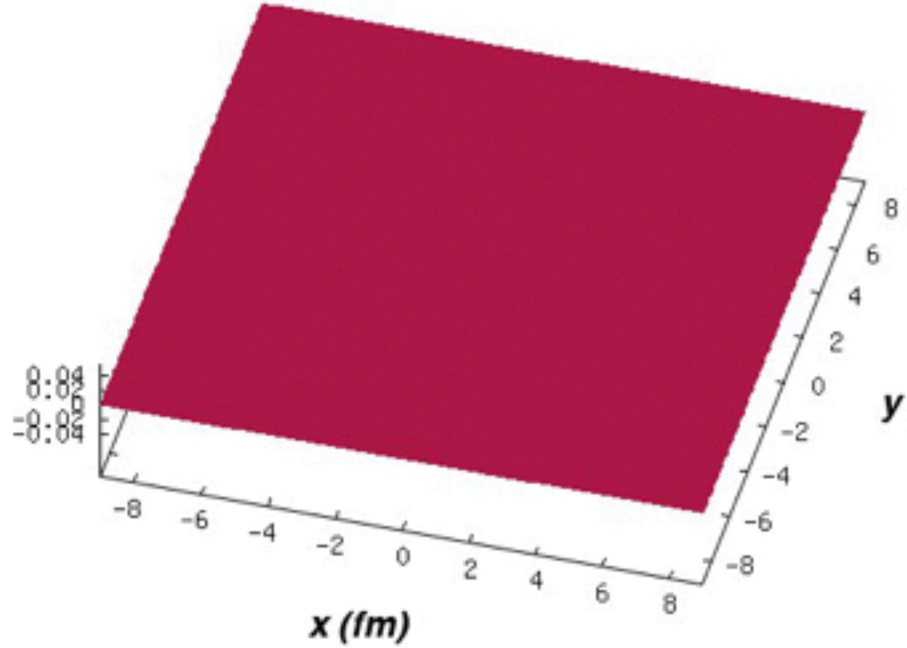






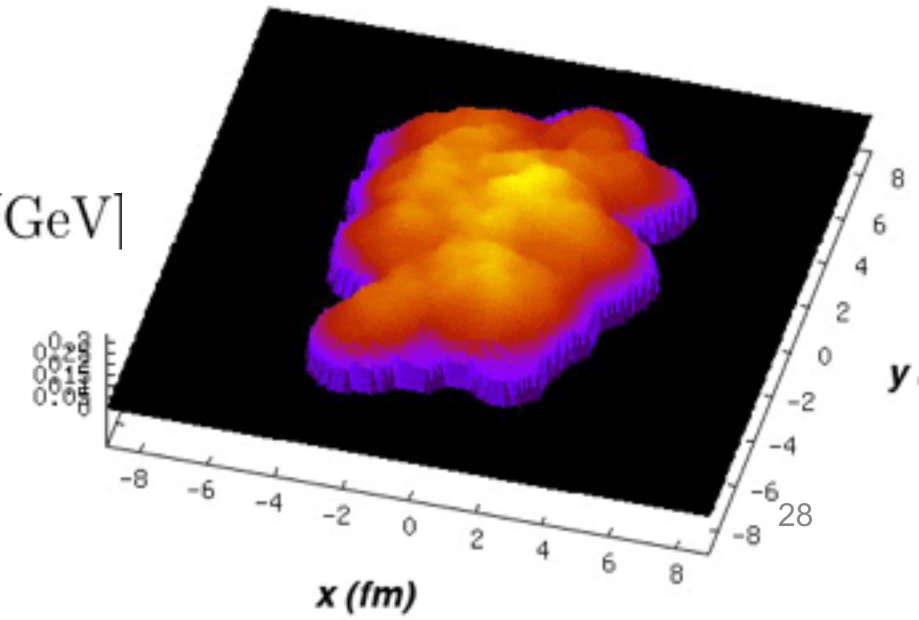
μ_5 [GeV]

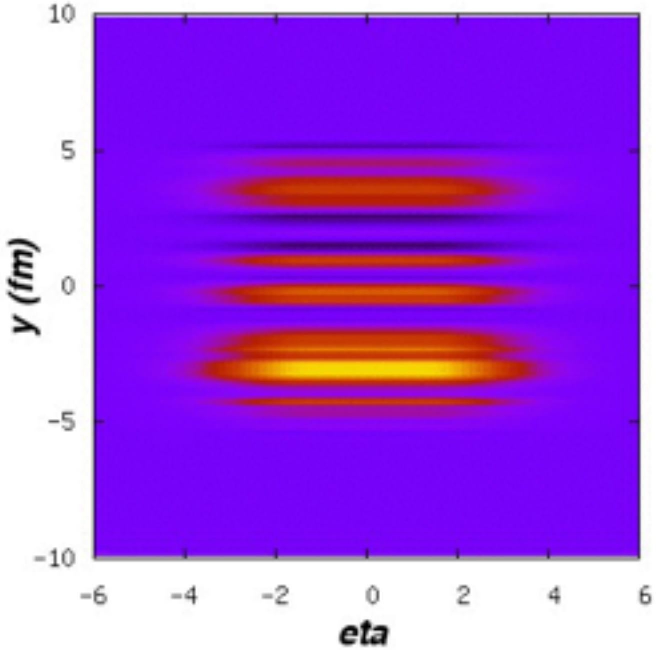


μ [GeV]

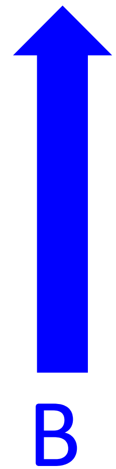
- RHIC energy
- b=7.2fm(20-30%)
- $eB_{\max} \simeq (2m_\pi)^2$
- $\tau_B = 3$ [fm]
- $C_{\mu_5} = 0.1$ [GeV]

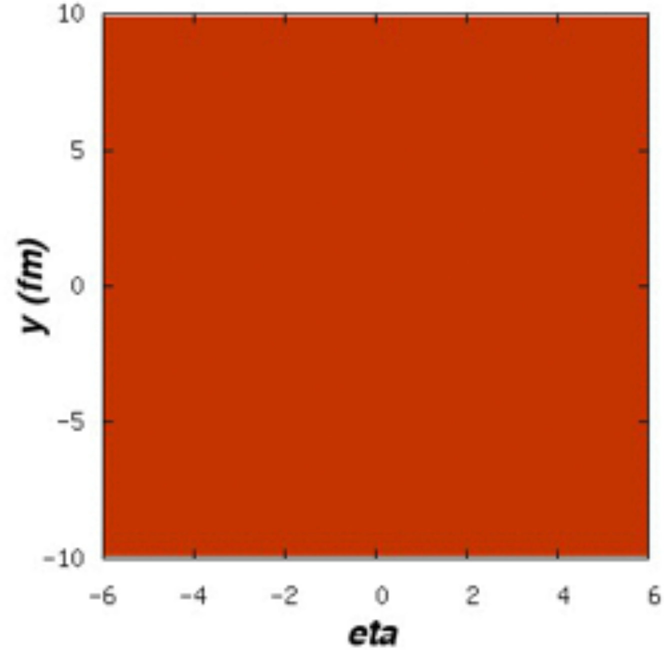
T [GeV]



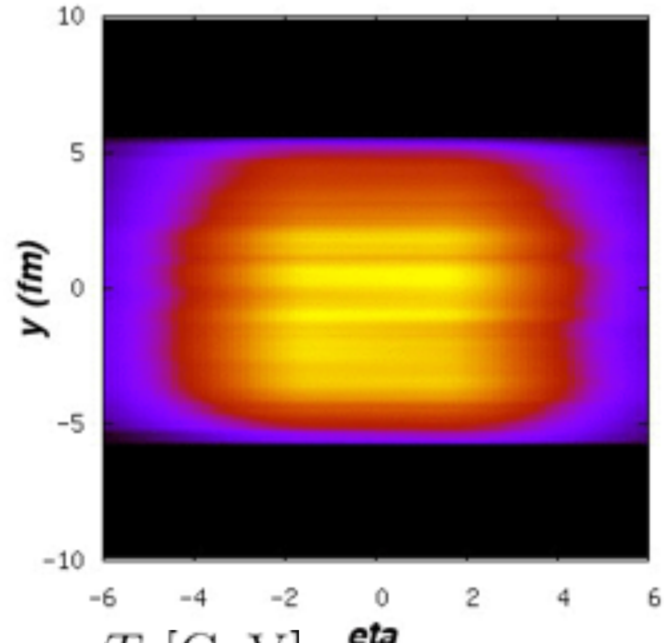


μ_5 [GeV]





μ [GeV]



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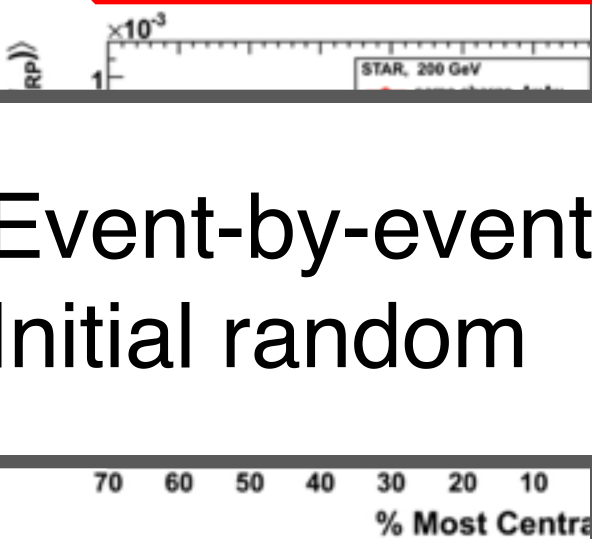
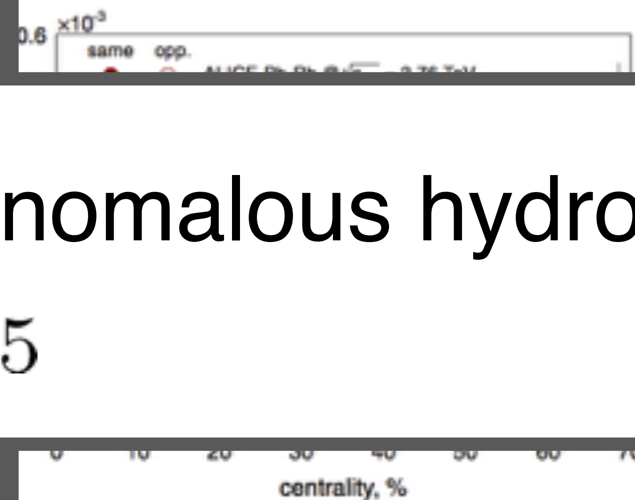




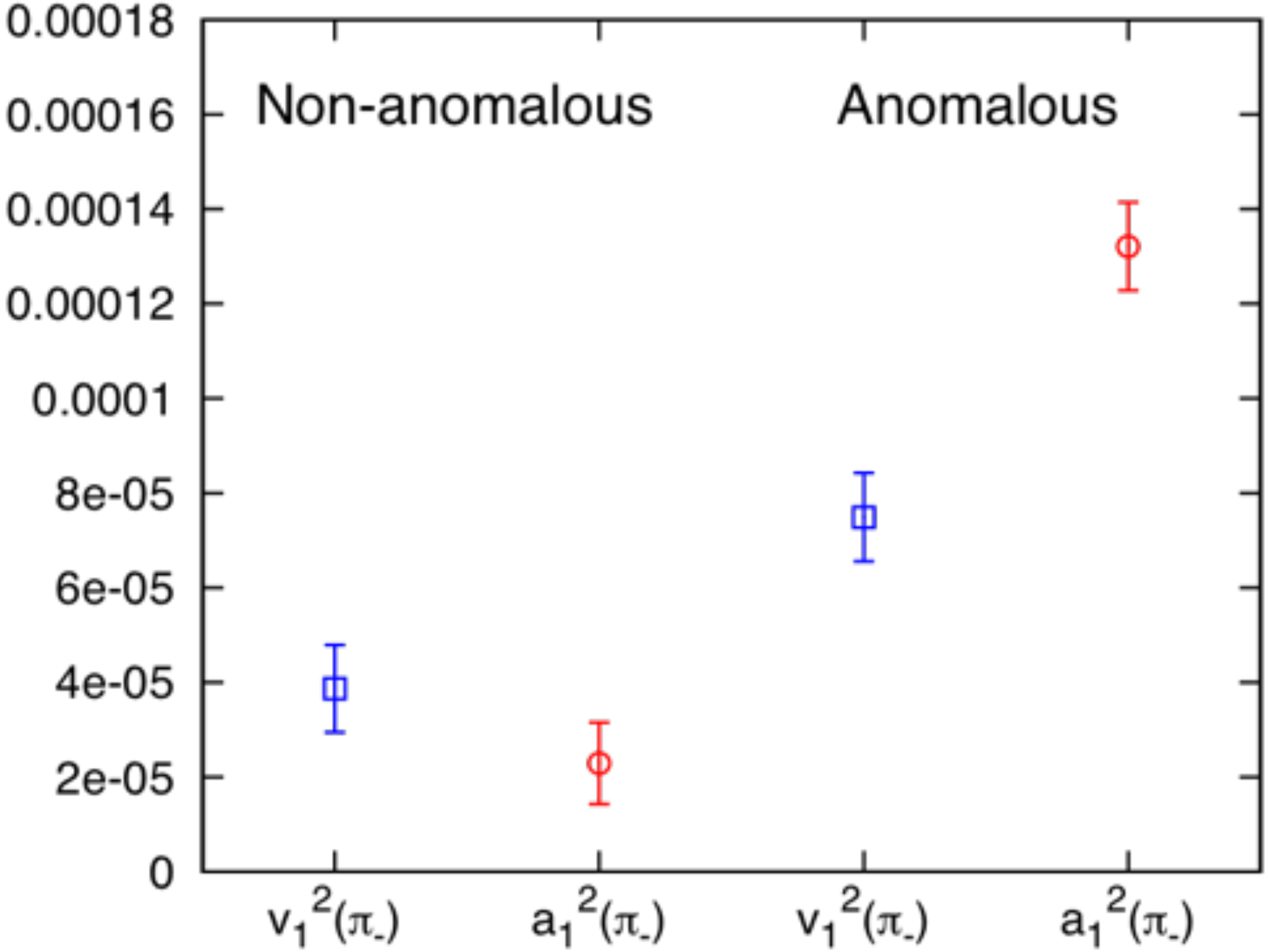
- ☑ Event-by-event anomalous hydro
- ☑ Initial random n_5

[STAR, PRL2009, PRC2010]

[ALICE, PRL2013]

$$\langle \cos(\phi_1^{\alpha} + \phi_2^{\beta} - 2\Psi_{\mathrm{RP}}) \rangle$$

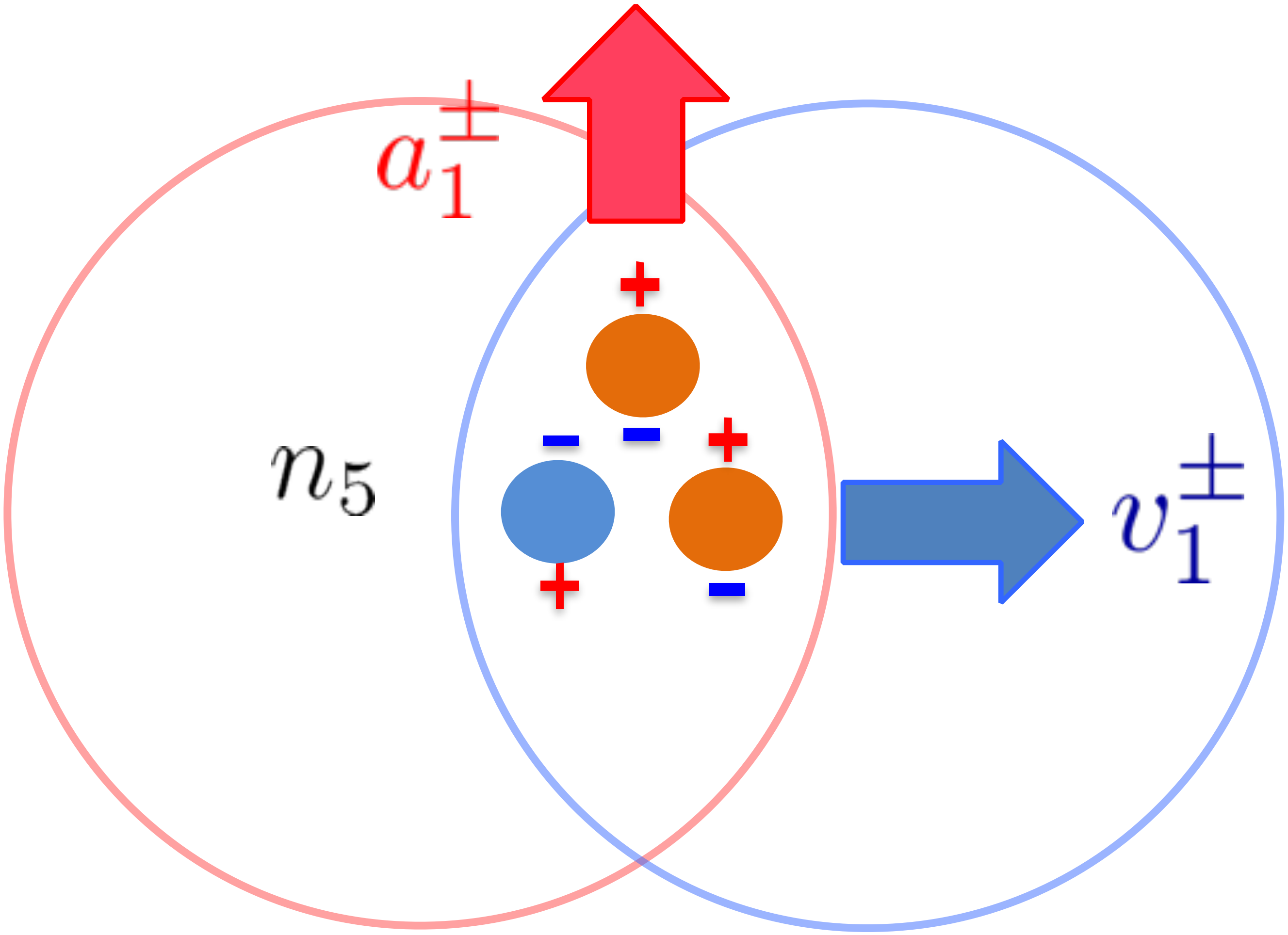
Correlations: $\langle (v_1^-)^2 \rangle$, $\langle (a_1^-)^2 \rangle$



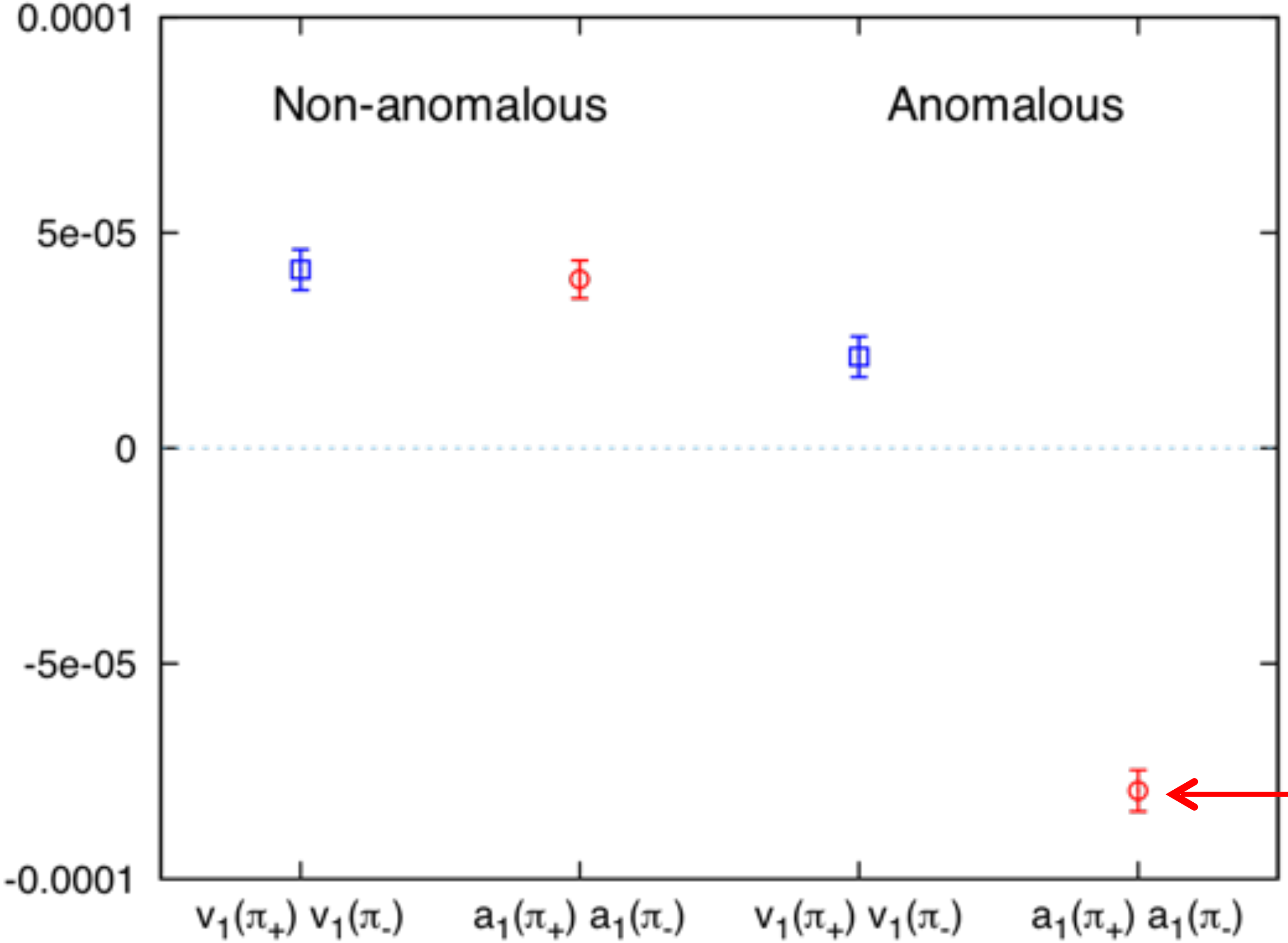
- b=7.2(20-30%)
- RHIC energy
- 100k events
- $eB_{\max} \simeq (3m_\pi)^2$

$$\langle \cos(\phi_1^- + \phi_2^- - 2\Psi_{RP}) \rangle = \langle (v_1^-)^2 \rangle - \langle (a_1^-)^2 \rangle$$

Why v_1 fluctuation also become larger?



Correlations: $\langle v_1^+ v_1^- \rangle$, $\langle a_1^+ a_1^- \rangle$



- b=7.2(20-30%)
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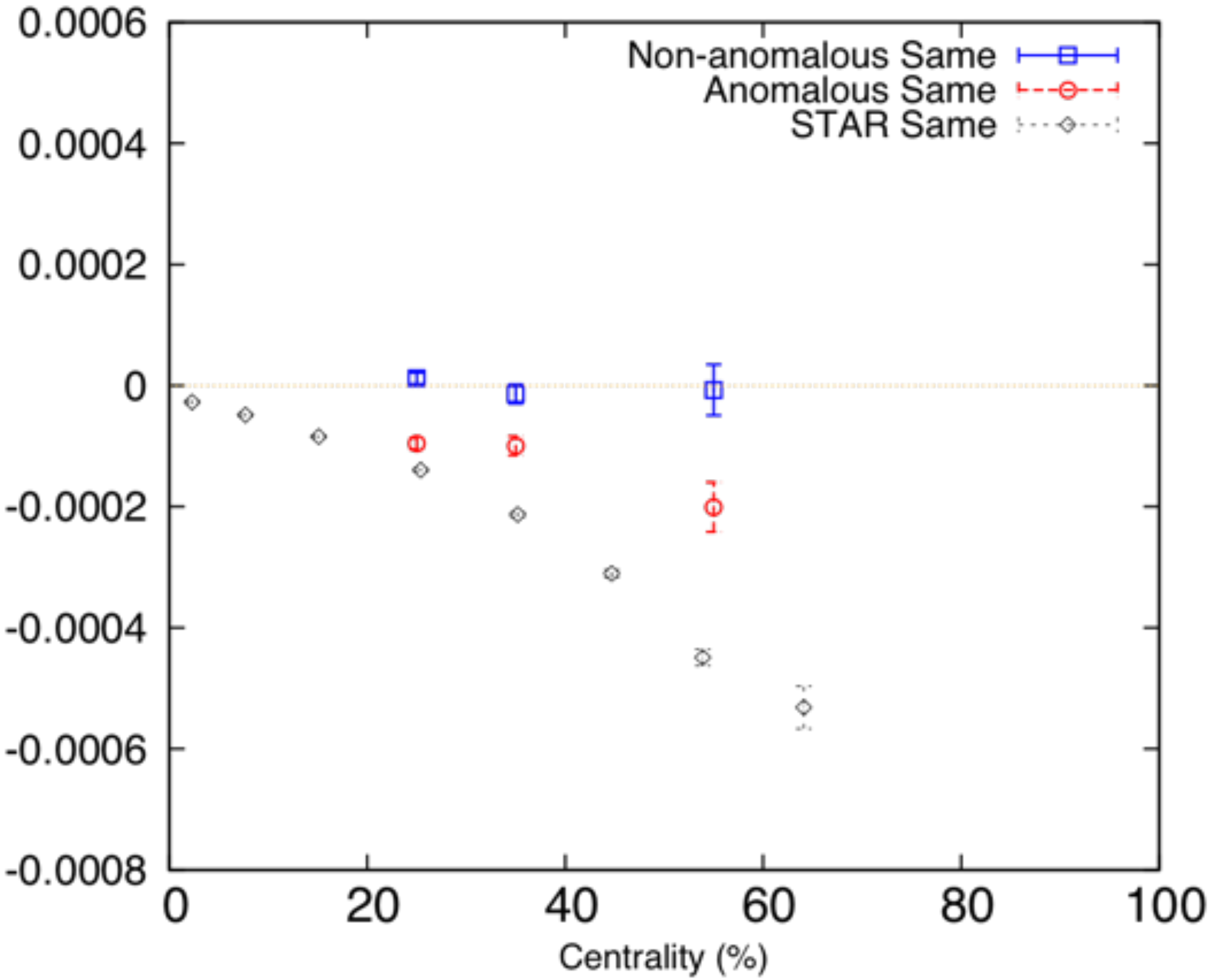
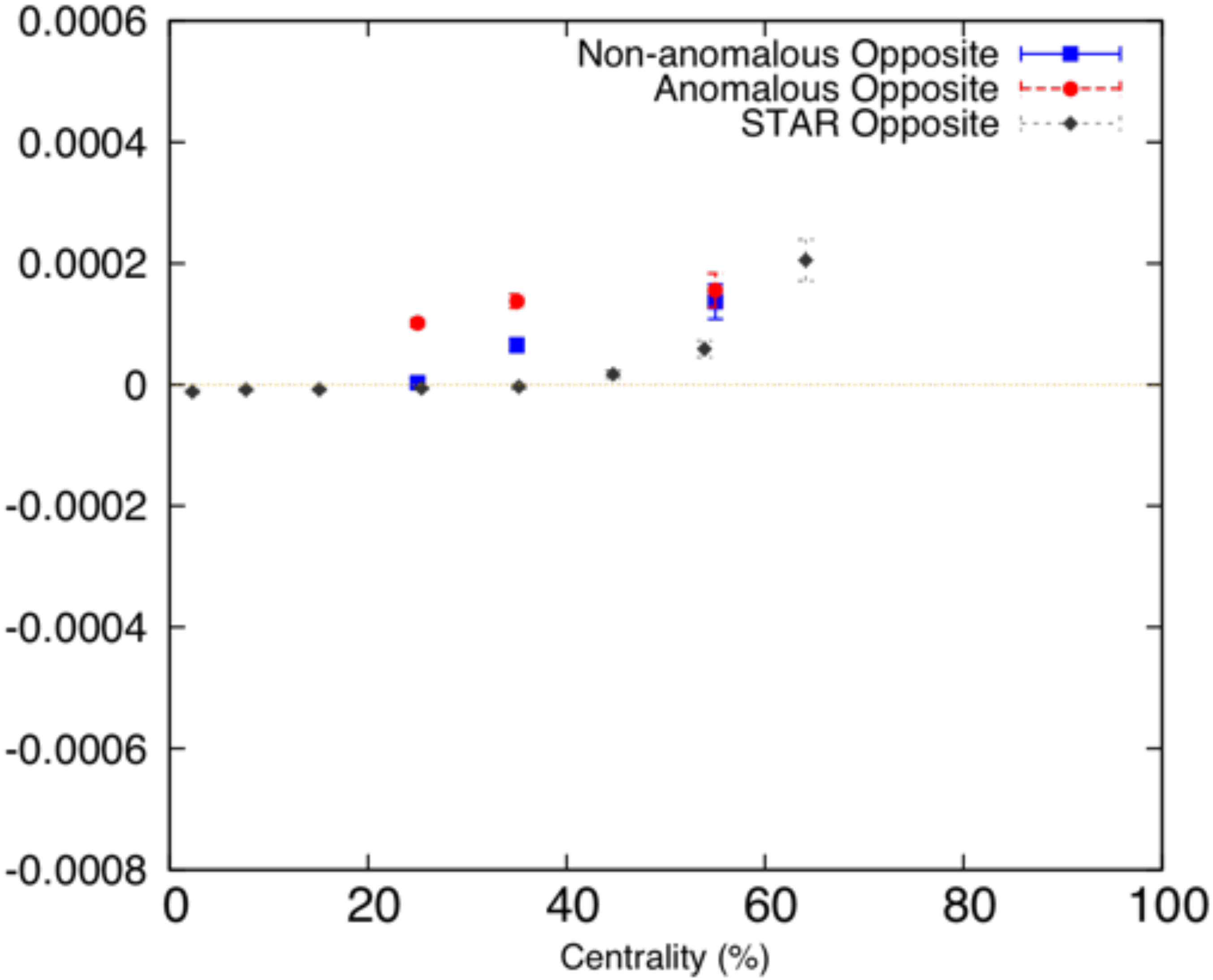
Anti-correlation btw. a_1^+ & a_1^-

$\langle\cos(\phi_1^\alpha + \phi_2^\beta - 2\Psi_{\mathrm{RP}})\rangle$ (same) vs centrality



$\langle\cos(\phi_1^\alpha + \phi_2^\beta - 2\Psi_{\mathrm{RP}})\rangle$ (opposite) vs centrality



Background effects

- Transverse momentum conservation [S. Pratt, S. Schlichting, S. Gavin, PRC(2011)] [A. Bzdak, V. Koch, J. Liao, PRC(2011)]
- Local charge conservation [S. Schlichting and S. Pratt, PRC(2011)] [Y. Hori, S. Schlichting, et al [1208.0603]]
- Cluster particle correlations [F. Wang, PRC(2010)]

In our current simulations,
multi-particle correlations are not imposed

$$E\frac{d^3N}{dp^3}(\boldsymbol{p}) = \int \frac{p^\mu d\sigma_\mu}{e^{\beta(p\cdot u-\mu)} \mp_{\rm BF} 1}$$

Cooper-Frye formula → single-particle distributions

Estimate of TMC contributions in STAR data [Y. Yin, J. Liao, PLB(2016)]

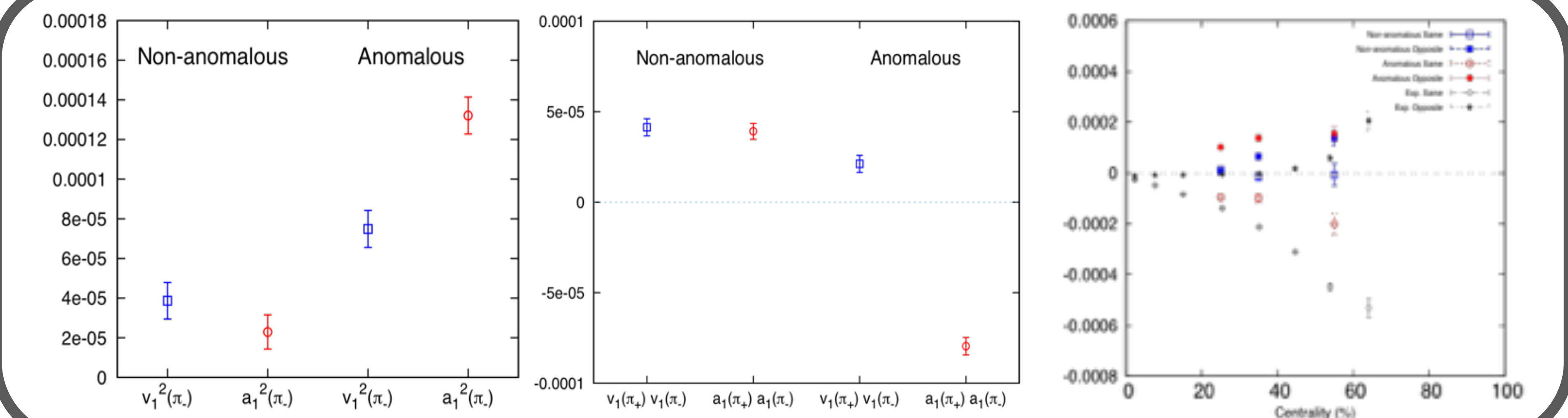
Anomalous transport in heavy-ion collisions?

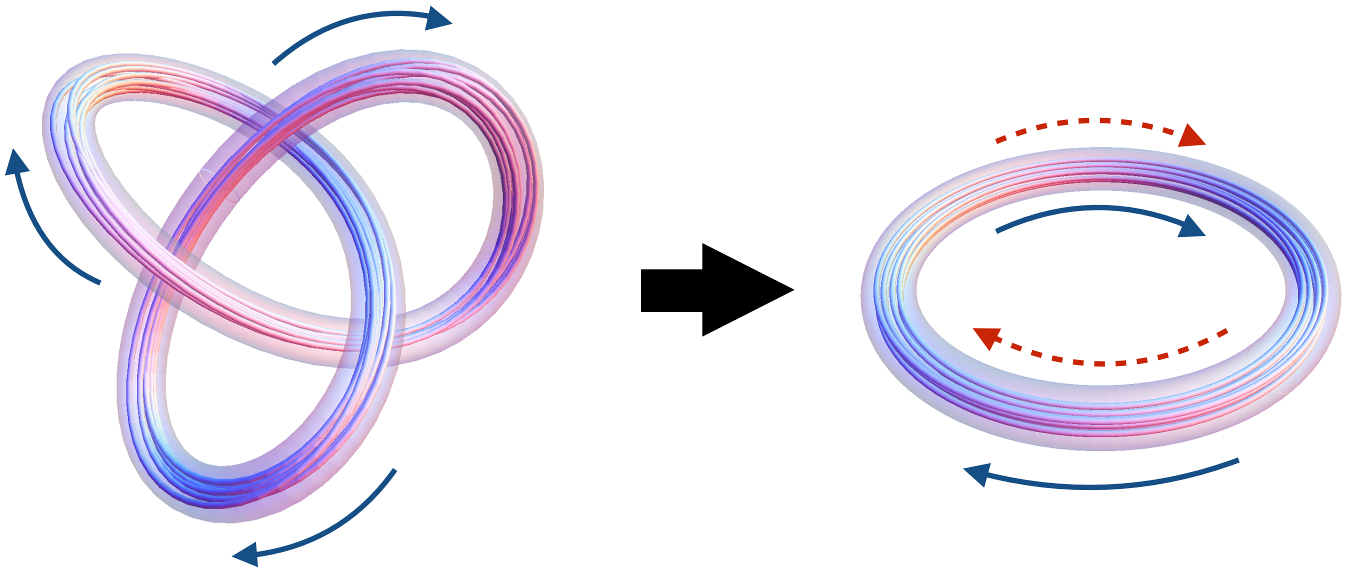
Outlook

- Quantification of the background contribution
- Contribution from the pre-hydro stage
- Dissipations/Dynamical EM fields/…

☑Event-by-event anomalous hydro

☑Initial random n_5



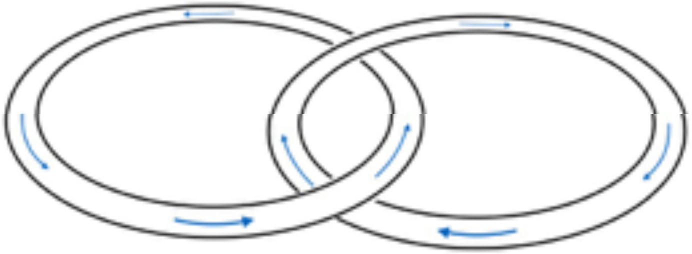


CME current from reconnections of magnetic flux

[Hirono-Khazeev-Yi, PRL in press [1606.09611]]

Magnetic helicity knows topology

$$\mathcal{H} = \int d^3x\, \boldsymbol{A} \cdot \boldsymbol{B}$$



$$= \sum_i \mathcal{S}_i \varphi_i^2 + 2 \sum_{i,j} \mathcal{L}_{ij} \varphi_i \varphi_j$$

Self-linking number

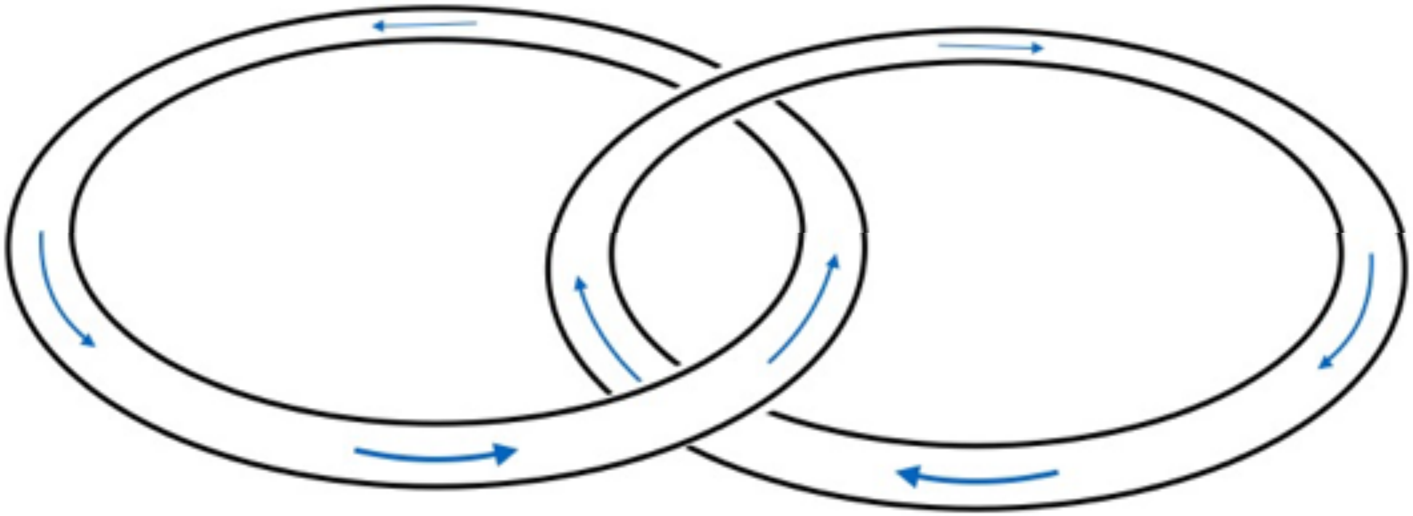
Linking number

Magnetic helicity knows topology

- In ideal MHD, the topology of magnetic fields does not change (no reconnection).
- Reconnections can occur for resistive MHD.
- Magnetic reconnections lead to **CME currents**

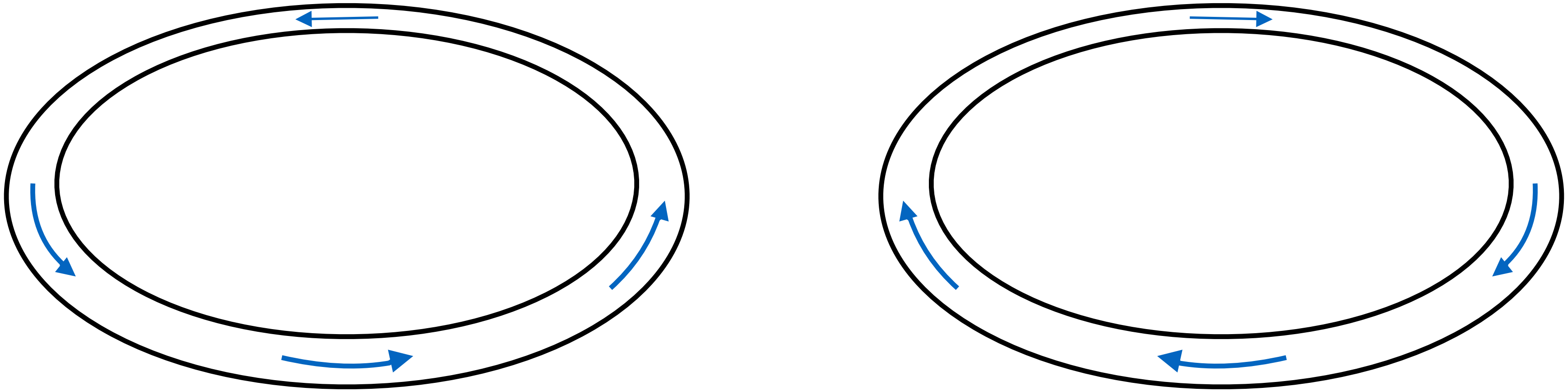
Change of integrated CME currents is given by change in helicity

$$\sum_i \oint_{C_i} \Delta \boldsymbol{J} \cdot d\boldsymbol{x} = -\frac{e^3}{2\pi^2} \Delta \mathcal{H}$$



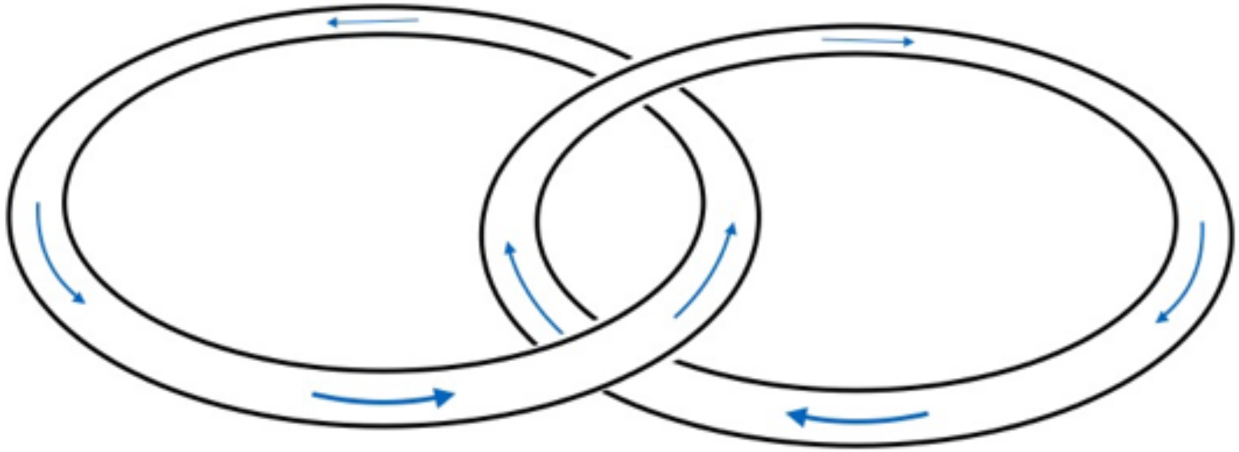
Parity odd

$$\mathcal{H} = 2\varphi^2$$

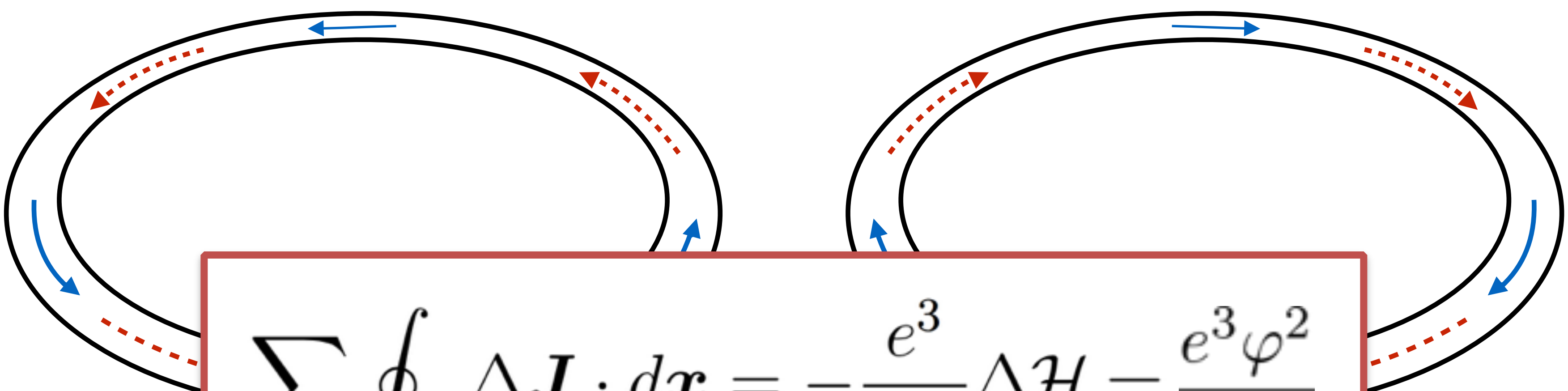


Parity even

$$\mathcal{H} = 0$$

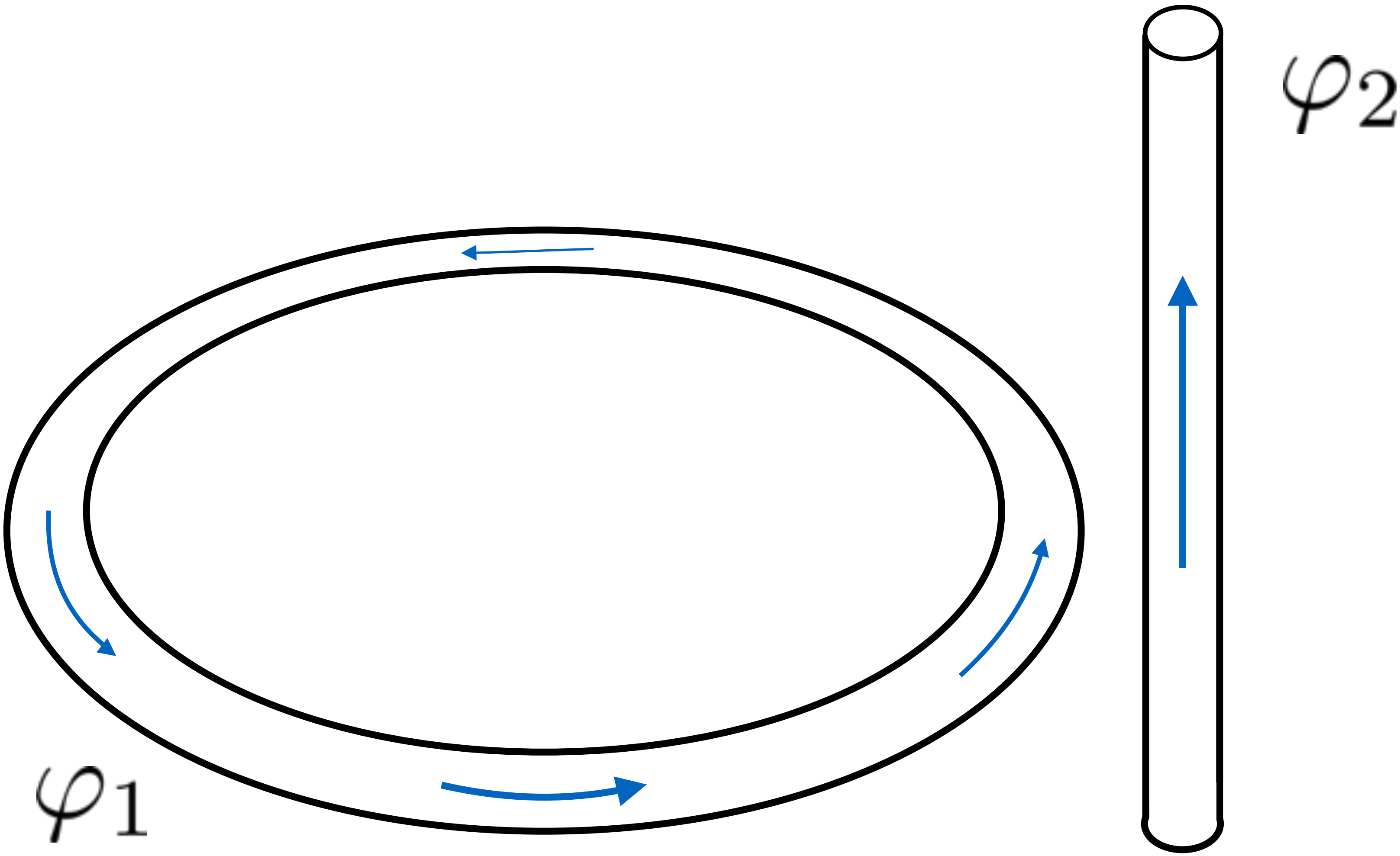


$$\Delta \mathcal{H} = -2\varphi^2$$



$$\sum_i \oint_{C_i} \Delta \boldsymbol{J} \cdot d\boldsymbol{x} = -\frac{e^3}{2\pi^2}\Delta \mathcal{H} = \frac{e^3 \varphi^2}{\pi^2}$$

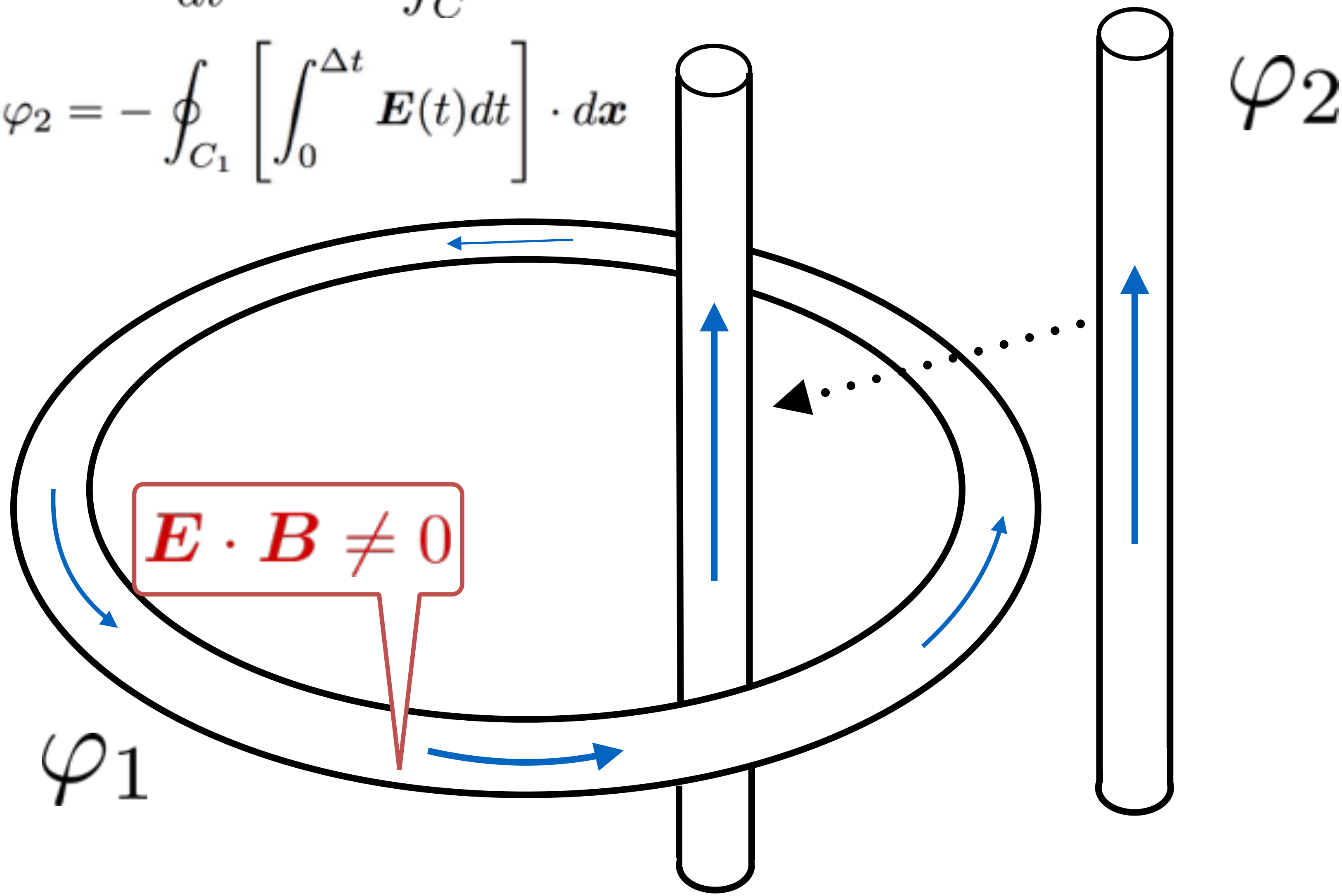
Derivation



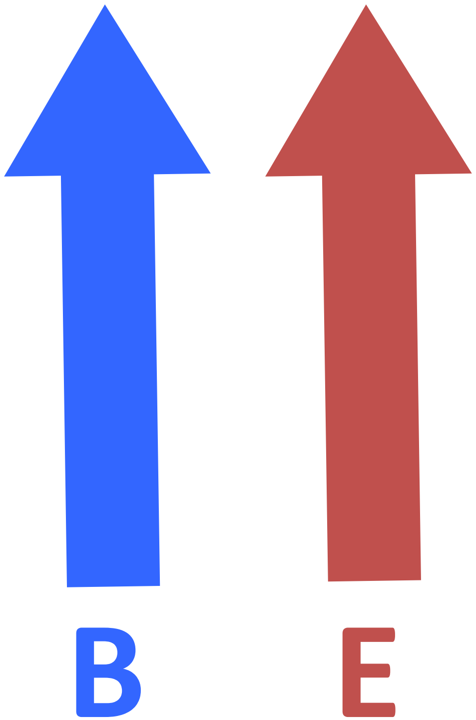
φ_2
φ_1

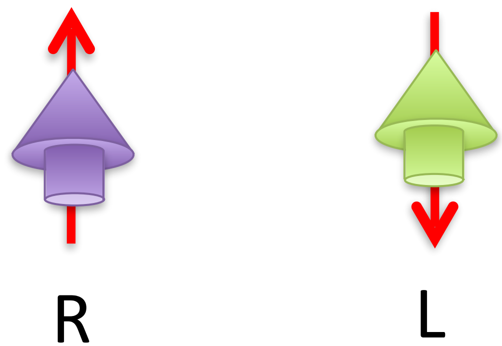
Faraday's law $\frac{d}{dt}\Phi = -\int_C \boldsymbol{E} \cdot d\boldsymbol{x}$

$$\Delta\Phi = \varphi_2 = -\oint_{C_1} \left[\int_0^{\Delta t} \boldsymbol{E}(t)dt\right] \cdot d\boldsymbol{x}$$



Lowest Landau Levels





$$\dot{\boldsymbol{p}}_F = \pm e \boldsymbol{E}$$

Particle generations

- Density of states in (1+1)D is given by $\frac{p_F}{2\pi}$

$$\oint \Delta \boldsymbol{p}_F(s) \cdot d\boldsymbol{x} = \pm e \oint_{C_1} \left[\int_0^{\Delta t} \boldsymbol{E}(t) dt \right] \cdot d\boldsymbol{x} = \mp e \varphi_2$$

$$\Delta \bar{N}_R = \frac{-\oint_{C_1} \Delta \boldsymbol{p}_F \cdot d\boldsymbol{x}}{2\pi} \times \frac{e\varphi_1}{2\pi} = \frac{e^2 \varphi_1 \varphi_2}{4\pi^2}$$

$$\Delta N_L = \frac{e^2 \varphi_1 \varphi_2}{4\pi^2}$$

Landau degeneracy factor

Generated currents

$$\oint_{C_1} \Delta \boldsymbol{J}_R \cdot d\boldsymbol{x} = (-e) \times \Delta \bar{N}_R = -\frac{e^3 \varphi_1 \varphi_2}{4\pi^2}$$

$$\oint_{C_1} \Delta \boldsymbol{J}_L \cdot d\boldsymbol{x} = (-) \times e \times \Delta N_L = -\frac{e^3 \varphi_1 \varphi_2}{4\pi^2}$$

$$\sum_{i \in \{1,2\}} \oint_{C_i} \Delta \boldsymbol{J} \cdot d\boldsymbol{x} = \sum_{i \in \{1,2\}} \oint_{C_i} \Delta \left[\boldsymbol{J}_R + \boldsymbol{J}_L \right] \cdot d\boldsymbol{x}$$

$$= -\frac{e^3}{2\pi^2} \times \boxed{2\varphi_1 \varphi_2}$$

Helicity change in the flux switching

- **Summary**
  - CME current is generated associated with the change of topology of magnetic fields.
  - We derived a relation that connects the amount of generated CME current and the change in magnetic helicity.
- **Outlook**
  - Alternative derivations based on, for example, the chiral kinetic theory.
  - Observables effects
    - Magnetars / heavy-ion collisions?

Backup slides

Anomalous magnetohydrodynamics

- In order to see anomalous effects, resistive MHD is necessary

$$\rho \frac{D}{Dt}\boldsymbol{v} = -\nabla p + \boldsymbol{J} \times \boldsymbol{B}$$

$$\boldsymbol{J} = \nabla \times \boldsymbol{B}$$

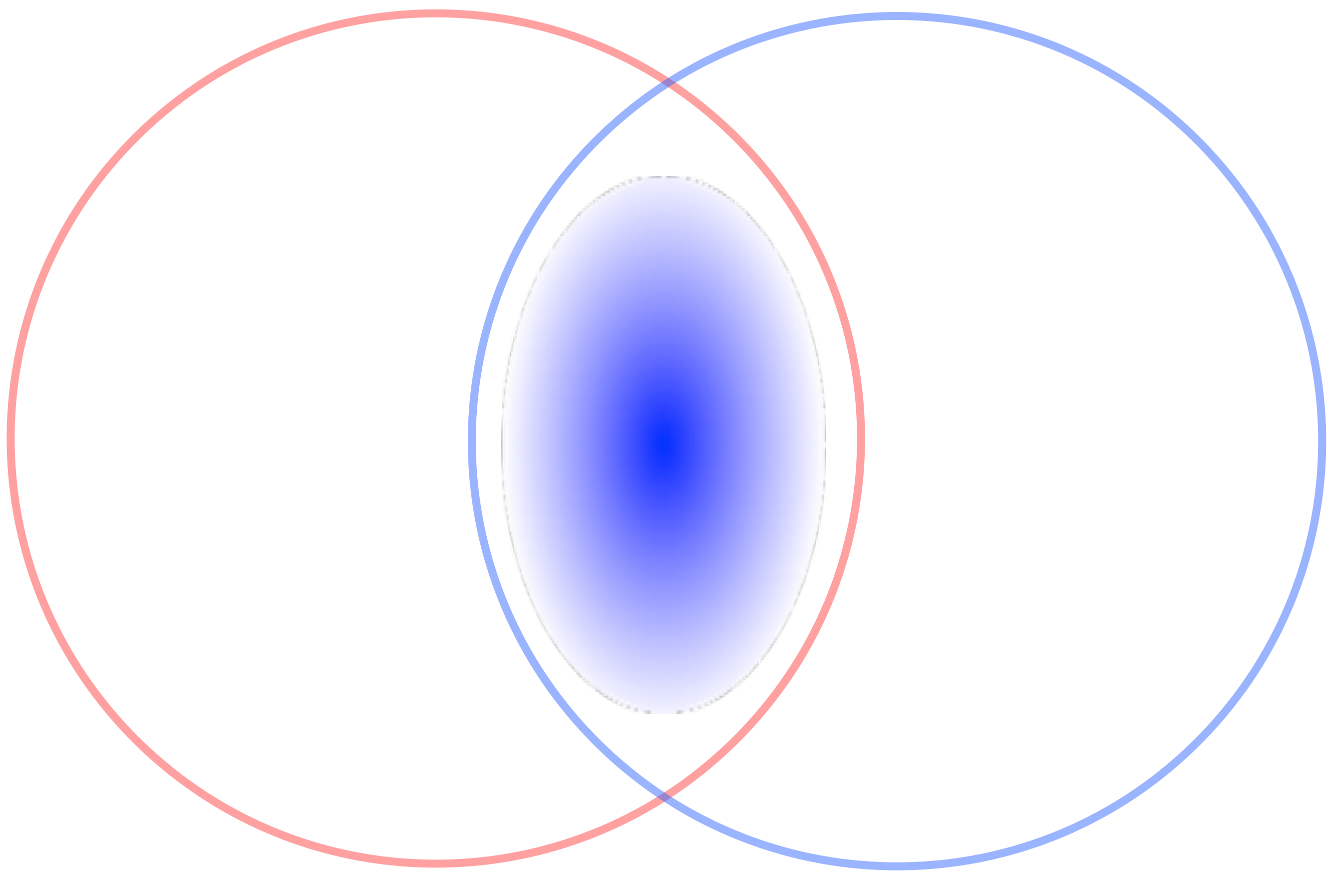
$$\boldsymbol{E} = -\boldsymbol{v} \times \boldsymbol{B} \qquad (\sigma \to \infty)$$

$$\partial_t \boldsymbol{B} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B})$$

Background magnetic fields

$$B_y(\tau, \eta_s, \boldsymbol{x}_\perp) = B_0 \frac{b}{2R} \exp\left[-\frac{x^2}{\sigma_x^2} - \frac{y^2}{\sigma_y^2} - \frac{\eta^2}{\sigma_{\eta_s}^2} - \frac{\tau}{\tau_{\rm B}} \right]$$

$$B_y(\tau_{\rm in}, 0, \mathbf{0}) \sim (3m_\pi)^2$$



C_{μ_5} estimated by anomaly equation

$$\partial_\mu j_5^\mu = \frac{g^2 N_\mathrm{f}}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

$$n_5(t_\mathrm{in}) \sim \frac{g^2 N_\mathrm{f}}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} \times t_\mathrm{in}$$

$$t_\mathrm{in} \sim 0.6 \ [\mathrm{fm}] \qquad g G^a_{\mu\nu} \sim 1 \ [\mathrm{GeV}^2]$$

$$n_5(t_\mathrm{in}) \sim (0.3 \ [\mathrm{GeV}])^3$$

$$\Rightarrow C_{\mu_5} = 0.1 \ [\mathrm{GeV}]$$

Correlations

same-charge

$$\langle (v_1^\alpha)^2 \rangle \equiv \left\langle \frac{1}{{}_{M_\alpha}\mathrm{P}_2} \sum_{<i,j>\in S_\alpha} \cos\Delta\phi_i^\alpha \cos\Delta\phi_j^\alpha \right\rangle$$

$$\langle (a_1^\alpha)^2 \rangle \equiv \left\langle \frac{1}{{}_{M_\alpha}\mathrm{P}_2} \sum_{<i,j>\in S_\alpha} \sin\Delta\phi_i^\alpha \sin\Delta\phi_j^\alpha \right\rangle$$

opposite-charge

$$\left\langle v_1^\alpha v_1^\beta \right\rangle \equiv \left\langle \frac{1}{M_\alpha M_\beta} \sum_{i\in S_\alpha, j\in S_\beta} \cos\Delta\phi_i^\alpha \cos\Delta\phi_j^\beta \right\rangle$$

$$\left\langle a_1^\alpha a_1^\beta \right\rangle \equiv \left\langle \frac{1}{M_\alpha M_\beta} \sum_{i\in S_\alpha, j\in S_\beta} \sin\Delta\phi_i^\alpha \sin\Delta\phi_j^\beta \right\rangle$$