

Charm Physics at BESIII

Lei Li

For BESIII Collaboration

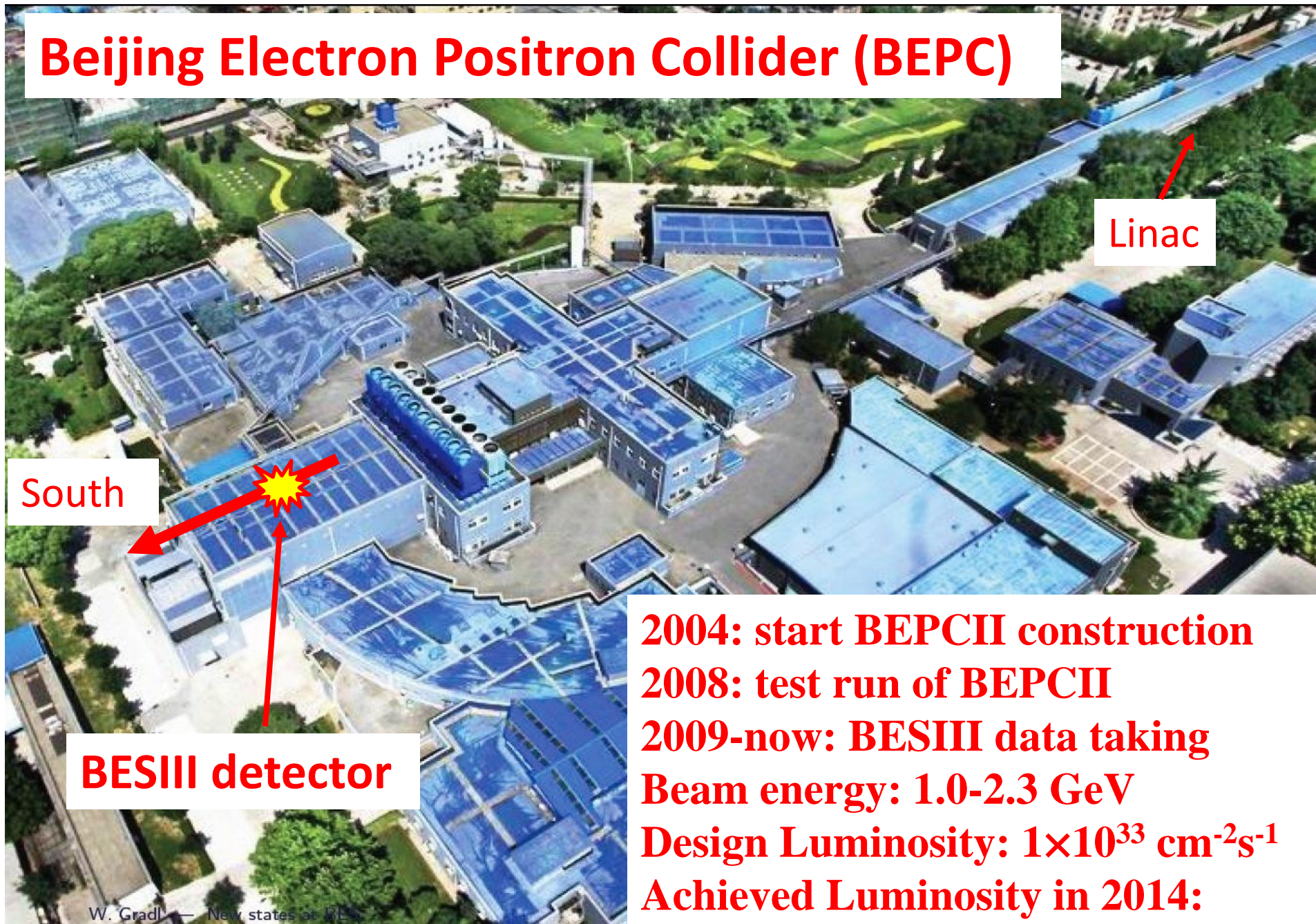
Beijing Institute of Petro-chemical Technology (BIPT)

**54th International Winter Meeting On Nuclear Physics
Jan. 25-29, 2016, Bormio, Italy**

Outline

- **Introduction**
- **D decays**
 - **D leptonic and semi-leptonic decays**
 - **D hadronic decays**
- **Λ_c^+ decays**
 - **Λ_c^+ semi-leptonic decays**
 - **Λ_c^+ hadronic decays**
- **Summary**

Beijing Electron Positron Collider (BEPC)



2004: start BEPCII construction
2008: test run of BEPCII
2009-now: BESIII data taking
Beam energy: 1.0-2.3 GeV
Design Luminosity: $1 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$
Achieved Luminosity in 2014:
 $L_{\text{peak}} = 0.85 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$

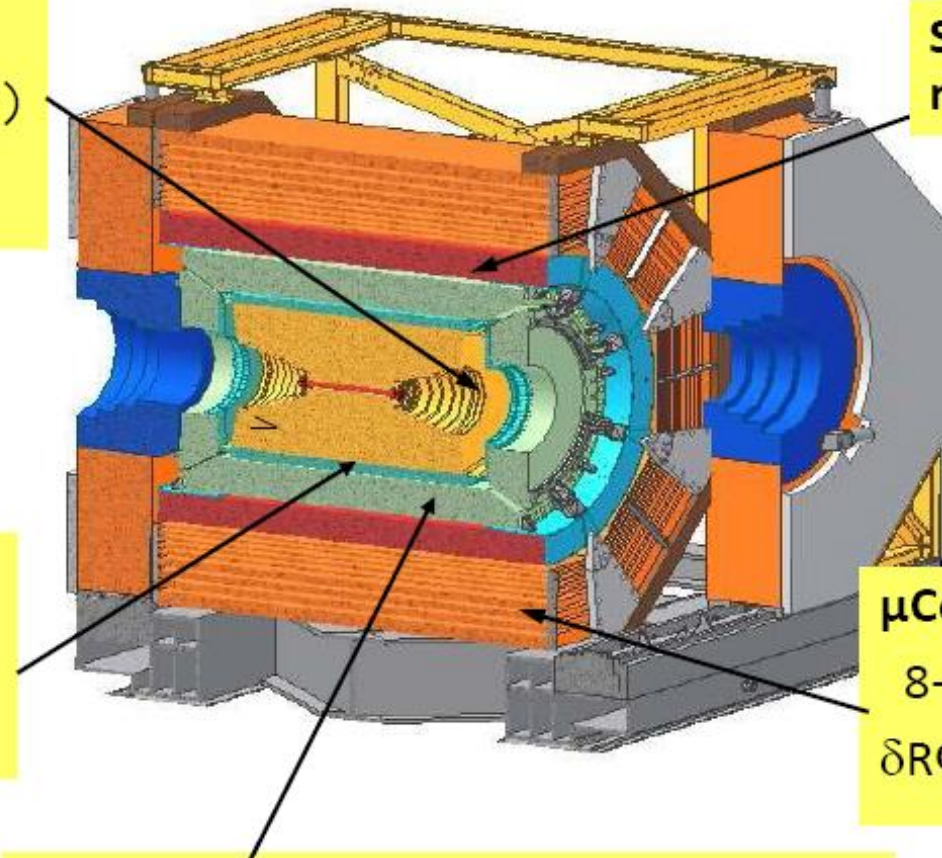
BESIII Detector

Drift Chamber (MDC)

$$\sigma_{P/P} (^{\circ}/_0) = 0.5\% (1\text{GeV})$$

$$\sigma_{dE/dx} (^{\circ}/_0) = 6\%$$

Super-conducting
magnet (1.0 tesla)



μ Counter

8- 9 layers RPC

$$\delta R\Phi = 1.4 \text{ cm} \sim 1.7 \text{ cm}$$

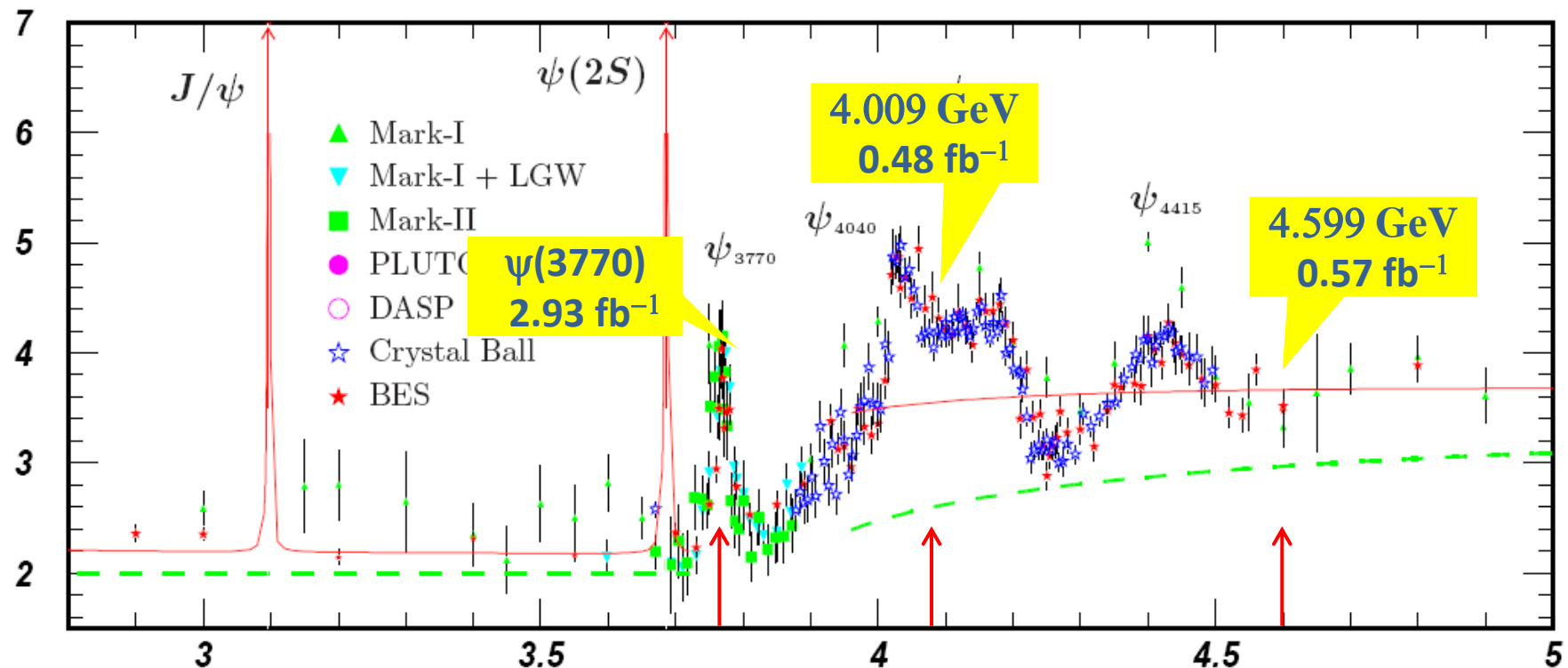
Time Of Flight (TOF)

σ_T : 90 ps Barrel
110 ps endcap

EMC: $\sigma_{E/\sqrt{E}} (^{\circ}/_0) = 2.5 \% (1 \text{ GeV})$
(CsI) $\sigma_{z,\phi} (\text{cm}) = 0.5 - 0.7 \text{ cm}/\sqrt{E}$

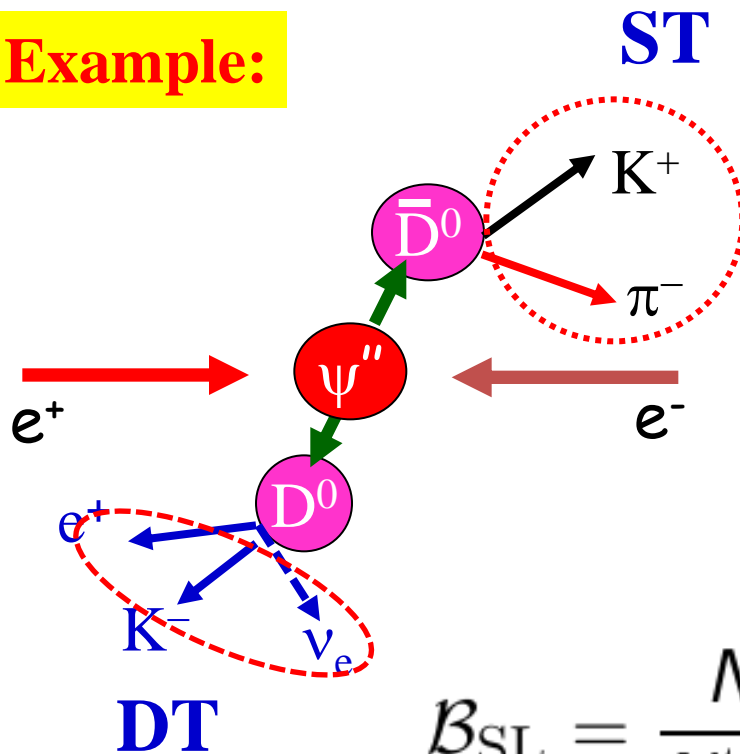
Data samples in this talk

- 2.93 fb⁻¹ data@3.773 GeV for D⁰ \bar{D}^0 , D⁺D⁻ production;
- 0.48 fb⁻¹ data@4.009 GeV for D_s⁺D_s⁻ production;
- 0.57 fb⁻¹ data@4.599 GeV for Λ_C^+ $\bar{\Lambda}_C^-$ production;



Analysis Technique

Example:



Single Tags (ST)

$$M_{bc} = \sqrt{E_{\text{beam}}^2 - |\vec{p}_{\bar{D}^0}|^2}$$

Double Tags (ST)

$$U_{\text{miss}} = E_{\text{miss}} - c|\vec{p}_{\text{miss}}|$$

[For Semi-Leptonic decays]

$$\mathcal{B}_{\text{SL}} = \frac{N^{\text{semi}}}{N^{\text{tag}} \times \epsilon}$$

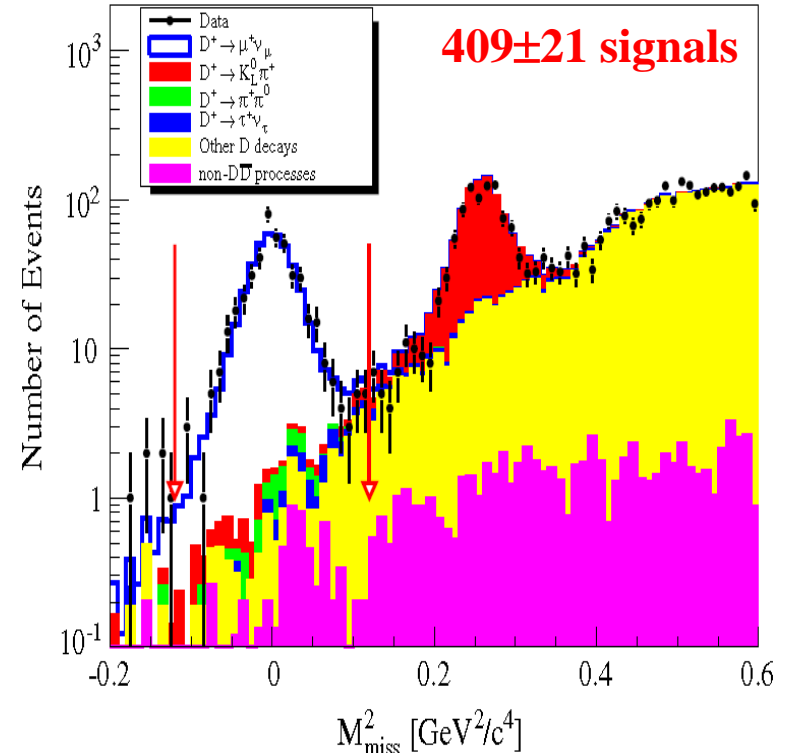
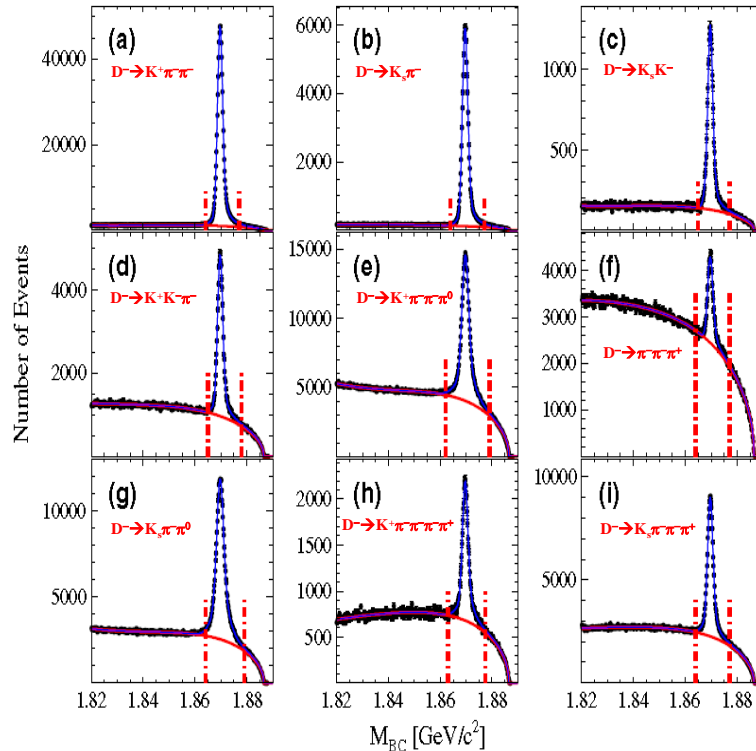
Clean sample of ST charmed hadrons can be fully reconstructed by hadronic decays with large BF's. Based on this, one can access to absolute BF's and dynamics in the decays.

Measurement of $B[D^+ \rightarrow \mu^+ \nu]$, f_{D^+} and $|V_{cd}|$

$$e^+e^- \rightarrow \psi(3770) \rightarrow D^+ D^-$$

2.93 fb⁻¹ data@ 3.773 GeV

PRD89(2014)051104R



$$N_{D_{tag}} = (170.31 \pm 0.34) \times 10^4$$

$$B[D^+ \rightarrow \mu^+ \nu] = (3.71 \pm 0.19 \pm 0.06) \times 10^{-4}$$

Input t_{D^+} , m_{D^+} , m_{μ^+} on PDG
and $|V_{cd}|$ of CKM-Fitter

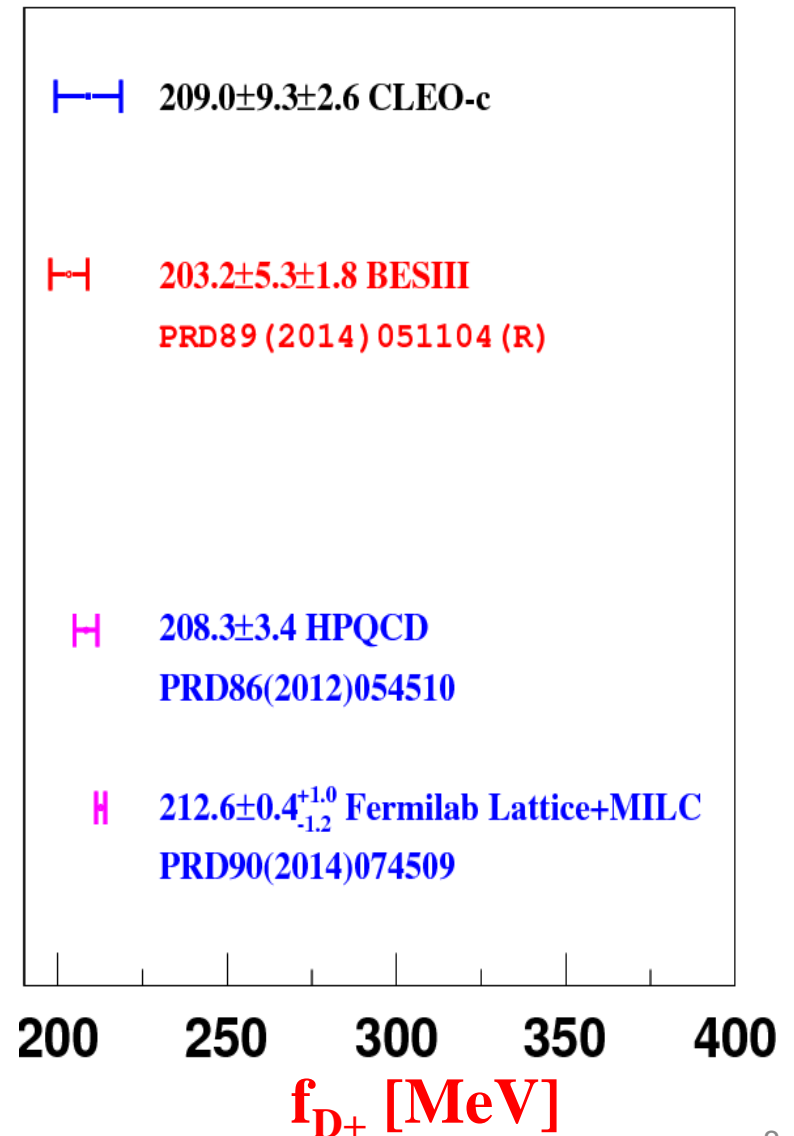
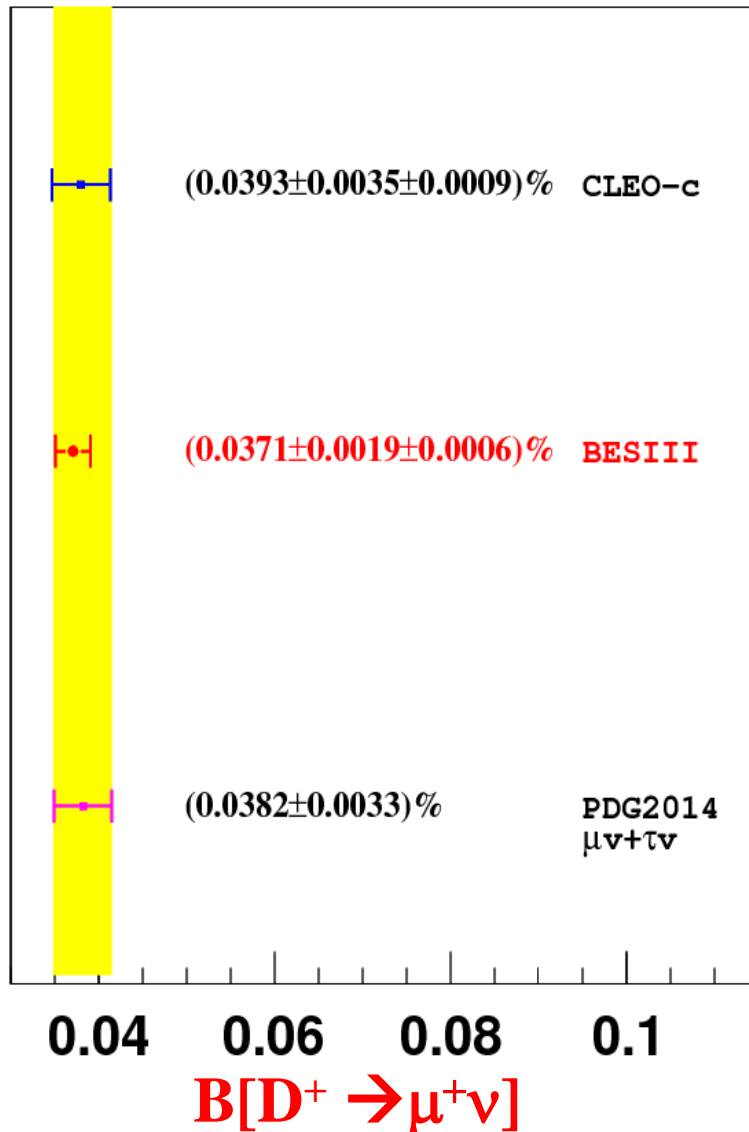
BES III

Input t_{D^+} , m_{D^+} , m_{μ^+} on PDG and
LQCD calculated $f_{D^+} = 207 \pm 4$
MeV [PRL100(2008)062002]

$$f_{D^+} = (203.2 \pm 5.3 \pm 1.8) \text{ MeV}$$

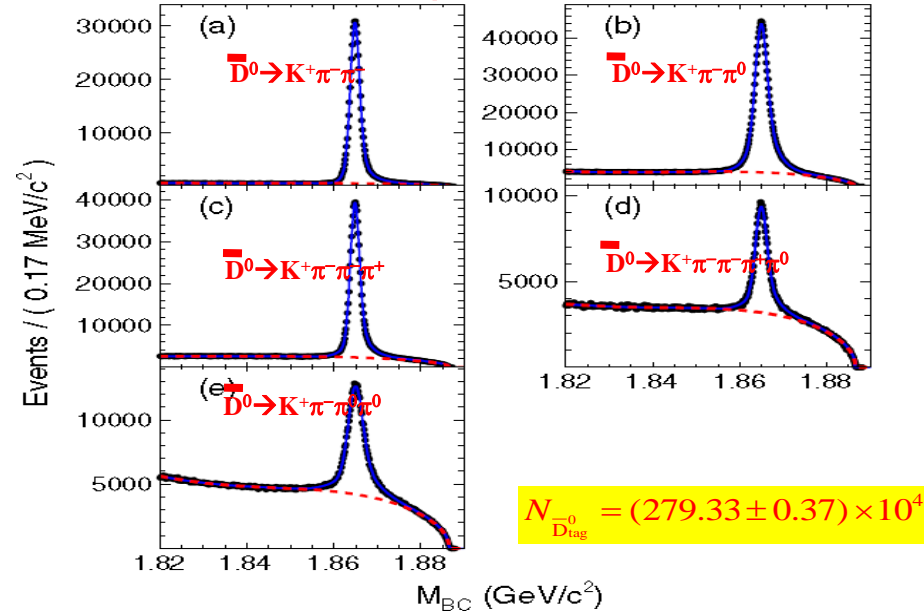
$$|V_{cd}| = 0.2210 \pm 0.0058 \pm 0.0047$$

Comparisons of $B[D^+ \rightarrow \mu^+ \nu_\mu]$ and f_{D^+}

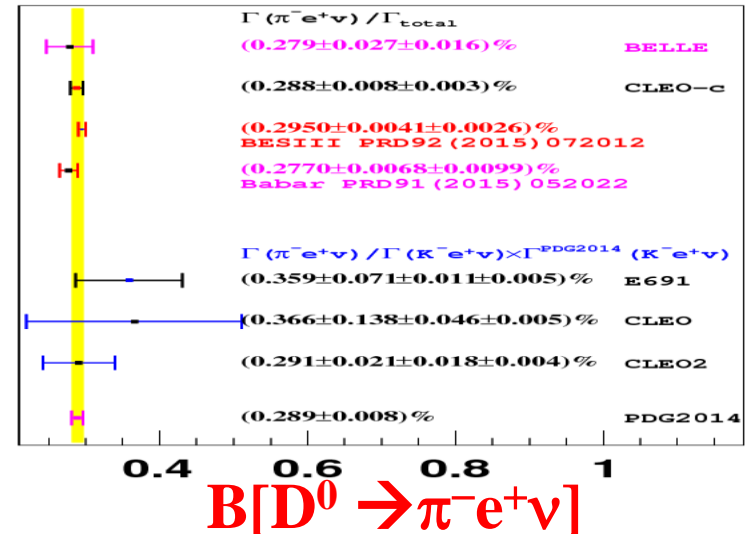
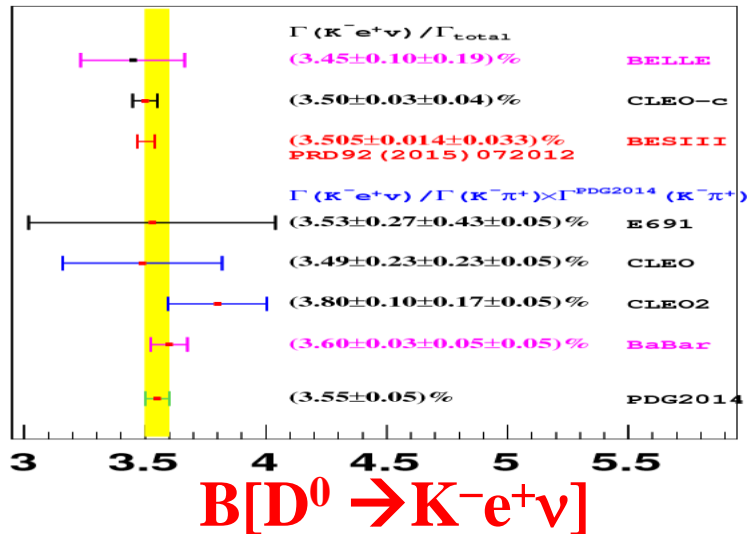
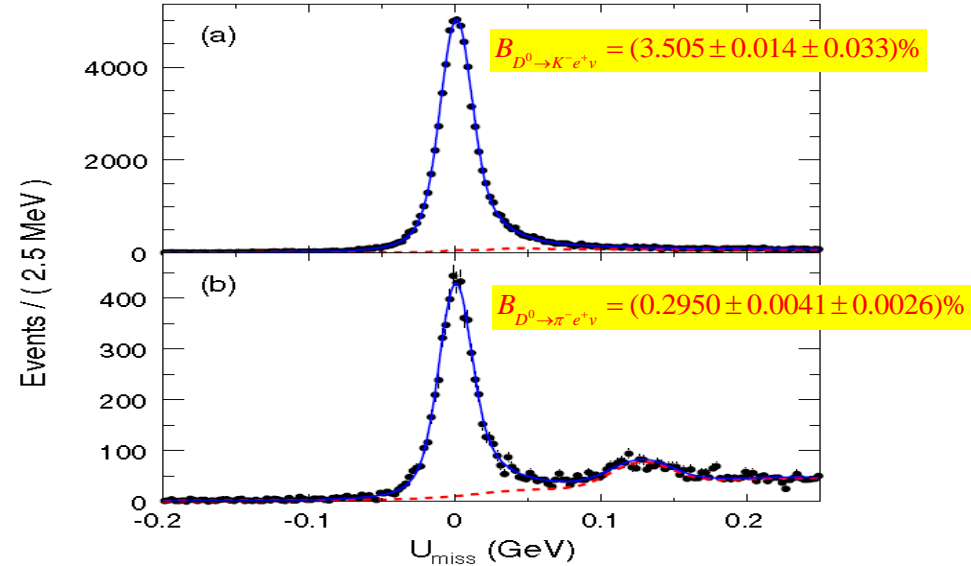


Measurement of $B[D^0 \rightarrow K(\pi)^- e^+ \nu]$

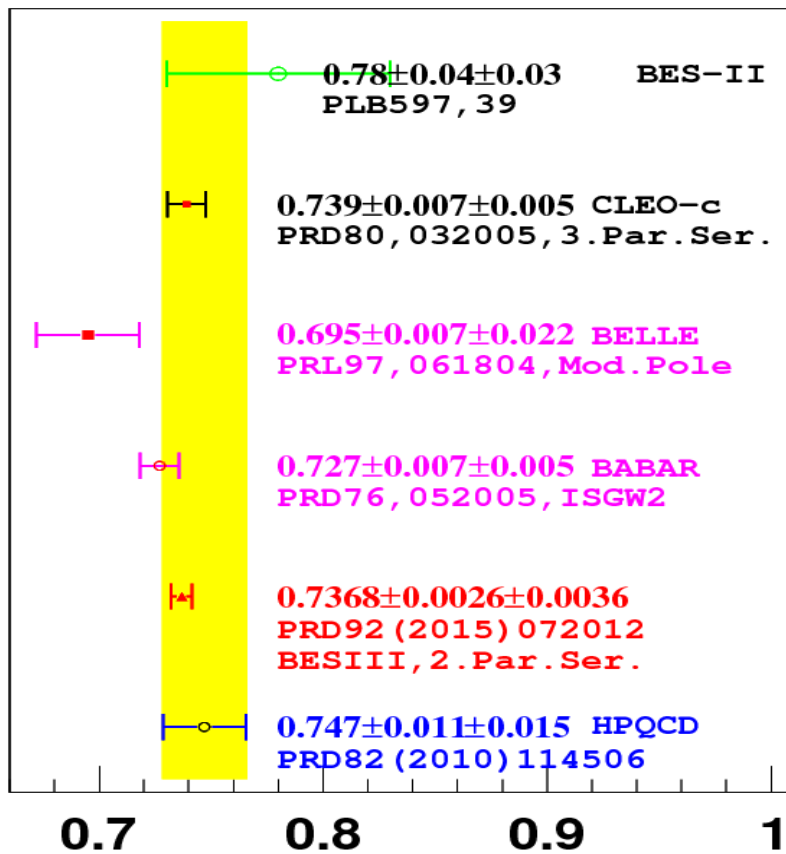
$e^+e^- \rightarrow \psi(3770) \rightarrow D^0 \bar{D}^0$



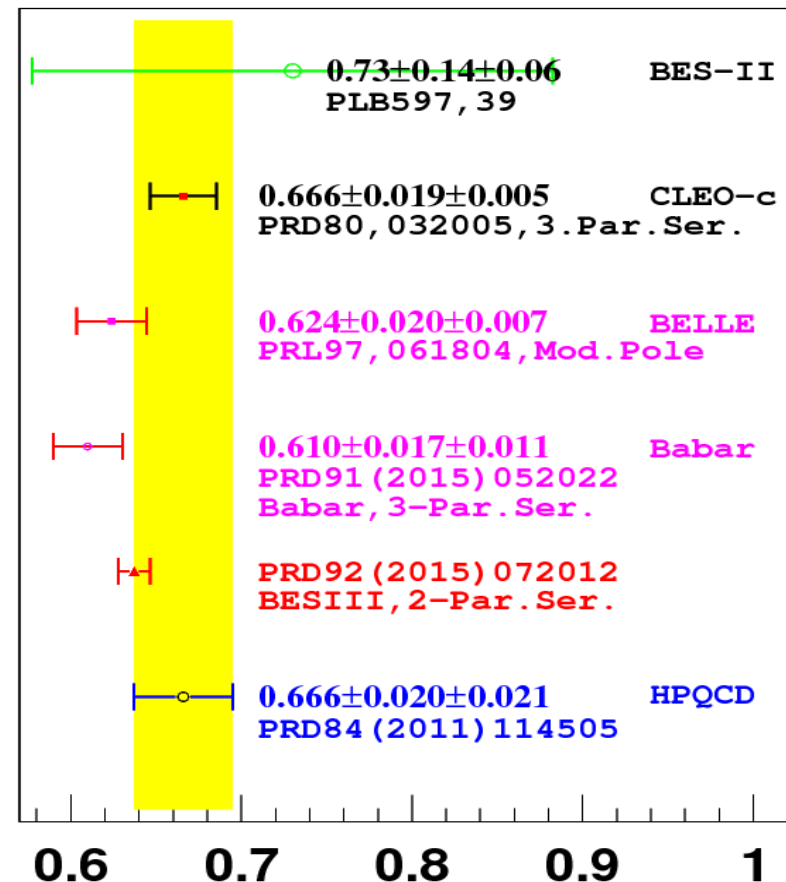
PRD92(2015)072012



Measurement of $f_+^{K(\pi)}(0)$



$f_+^{K(0)}$



$f_+^{\pi(0)}$

Analysis of $D^+ \rightarrow K_L e^+ \nu$

[PRD92(2015) 112008]

➤ Regardless of long flight distance, K_L interact with EMC and deposit part of energy, thus giving position information

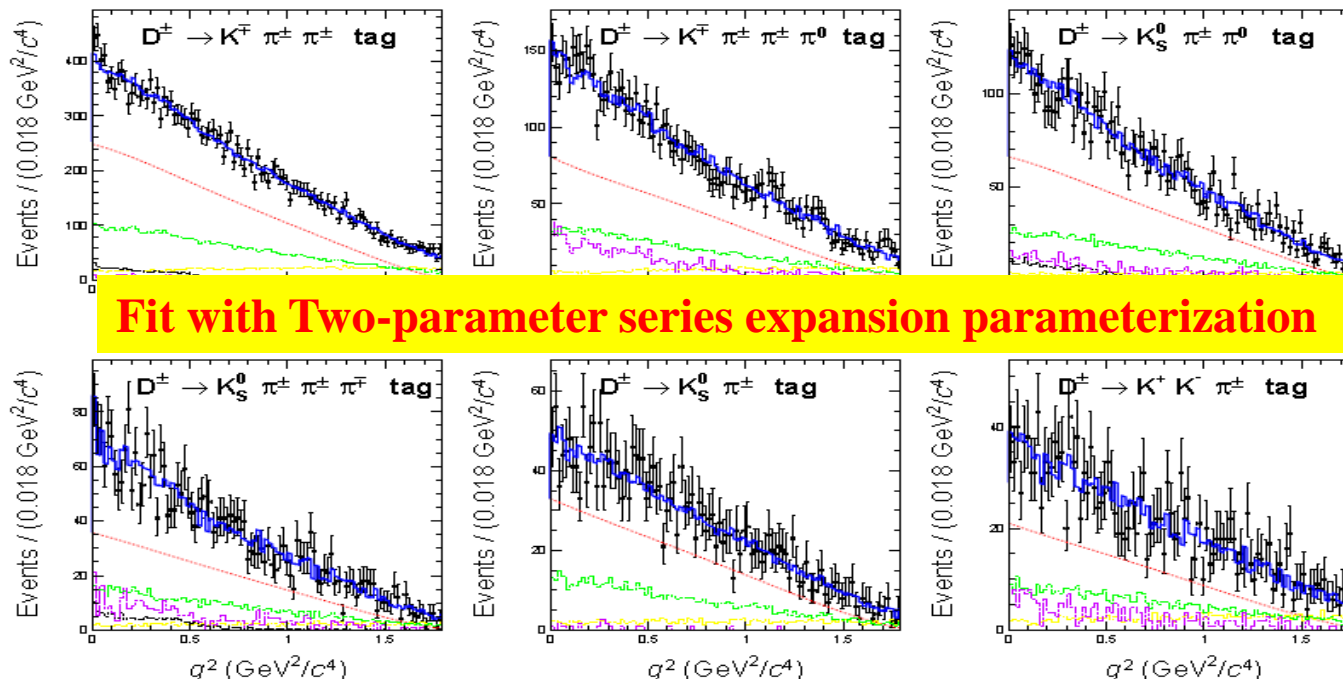
➤ After reconstructing all other particles, K_L can be inferred with position information and constraint $U_{\text{miss}} \rightarrow 0$

$$B(D^+ \rightarrow K_L e^+ \nu) = (4.482 \pm 0.027 \pm 0.103)\%$$

$$A_{CP} \equiv \frac{B(D^+ \rightarrow K_L^0 e^+ \nu_e) - B(D^- \rightarrow K_L^0 e^- \bar{\nu}_e)}{B(D^+ \rightarrow K_L^0 e^+ \nu_e) + B(D^- \rightarrow K_L^0 e^- \bar{\nu}_e)}$$

$$A_{CP}^{D^+ \rightarrow K_L e^+ \nu} = (-0.59 \pm 0.60 \pm 1.50)\%$$

Simultaneous fit to observed numbers of DT candidates



The first measurement of the BR for $D^+ \rightarrow K_L e^+ \nu_e$

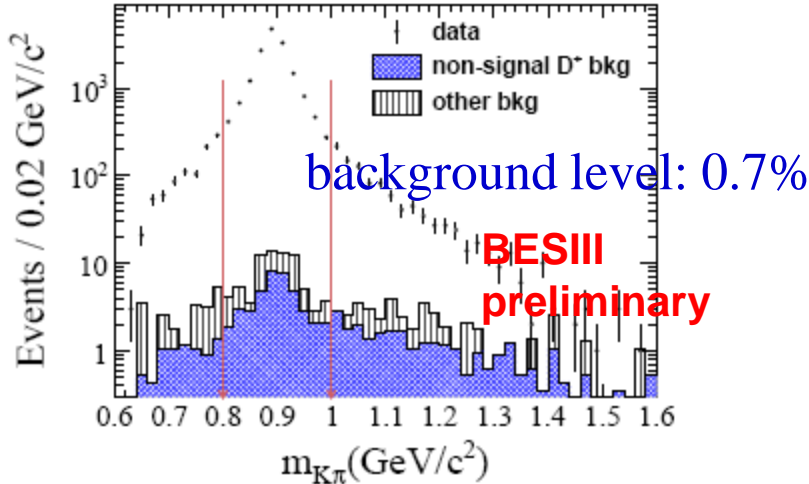
Fit with Two-parameter series expansion parameterization

$$f_K^+(0)|V_{cs}| = 0.728 \pm 0.006 \pm 0.011$$

Analysis of $D^+ \rightarrow K^- \pi^+ e^+ \nu$

arXiv:1512.08627

$M_{K\pi}$ distribution



$$B(D^+ \rightarrow K^- \pi^+ e^+ \nu_e) = (3.71 \pm 0.03 \pm 0.08)\%$$

$$B(D^+ \rightarrow K^- \pi^+ e^+ \nu_e)_{[0.8,1]} = (3.33 \pm 0.03 \pm 0.07)\%$$

Fit Results (Preliminary)

■ Fitted fractions of the component

$$f(D^+ \rightarrow (K^- \pi^+)_{K^{*0}(892)} e^+ \nu_e) = (93.93 \pm 0.22 \pm 0.18)\%$$

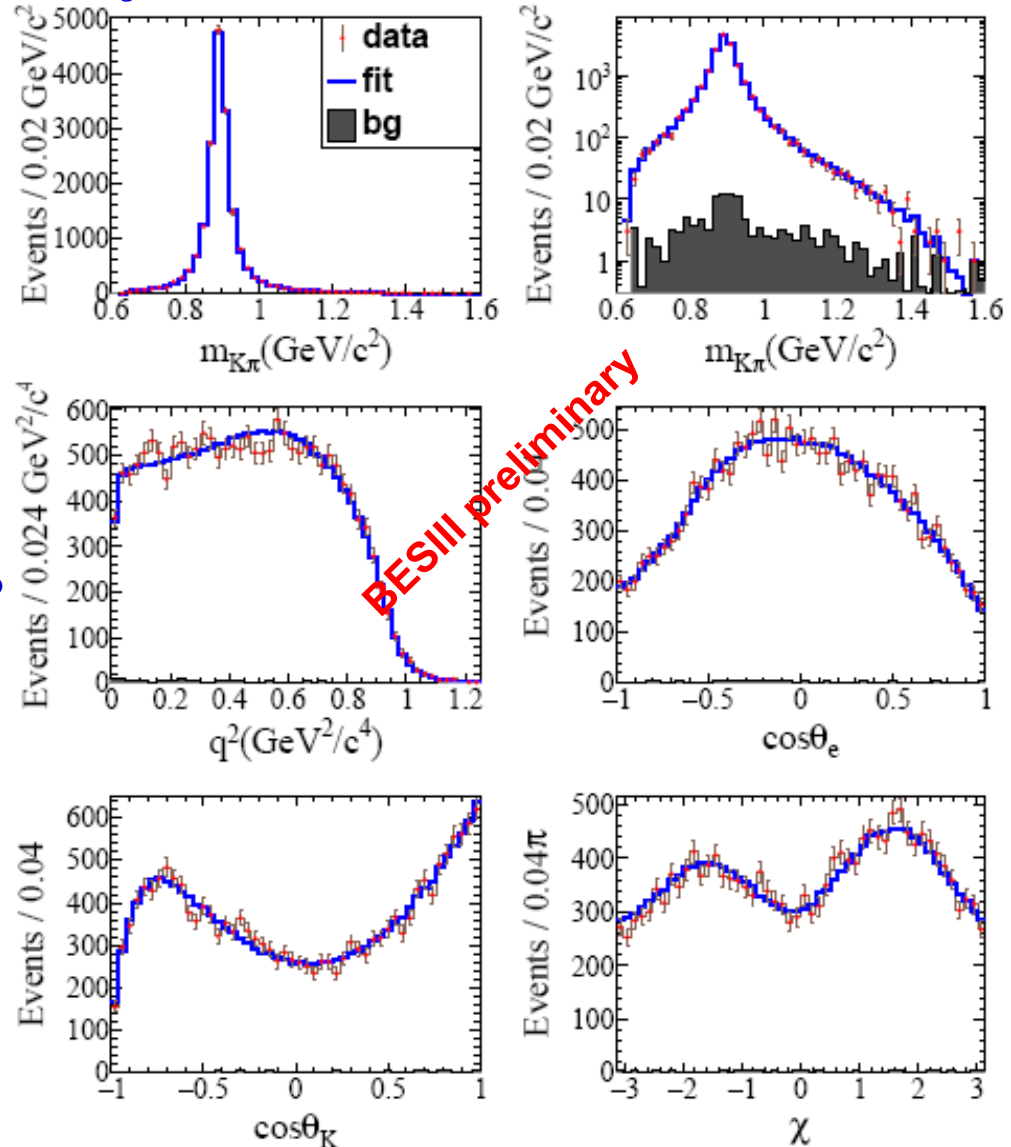
$$f(D^+ \rightarrow (K^- \pi^+)_{S\text{-wave}} e^+ \nu_e) = (6.05 \pm 0.22 \pm 0.18)\%$$

■ Parameters of $K^{*0}(892)$

$$m_{K^{*0}(892)} = (894.60 \pm 0.25 \pm 0.08) \text{ MeV}/c^2$$

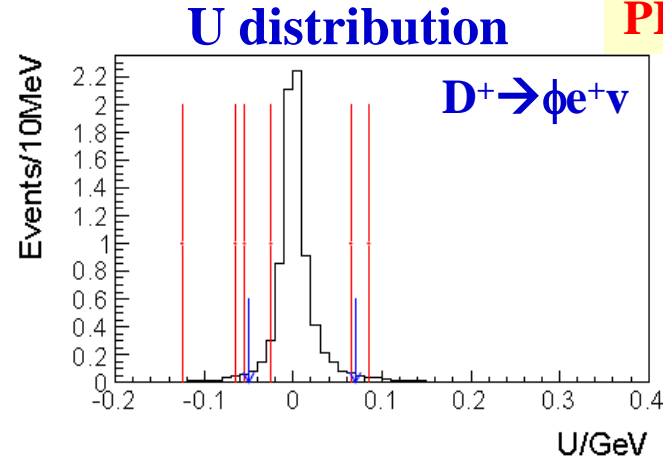
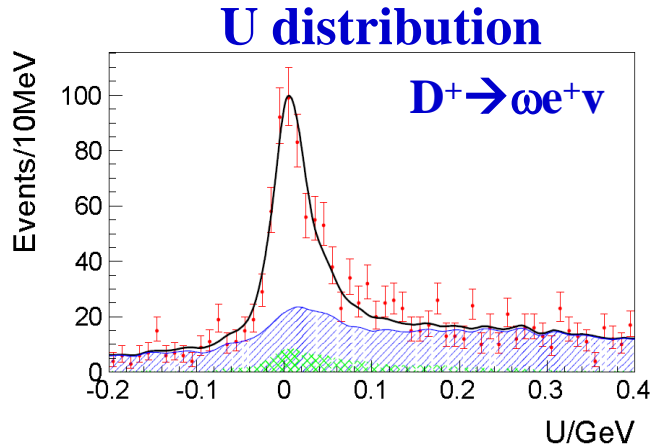
$$\Gamma_{K^{*0}(892)} = (46.42 \pm 0.56 \pm 0.15) \text{ MeV}/c^2$$

Projections of data and fitted MC distributions



Study of $D^+ \rightarrow \omega e^+ \nu$ and search for $D^+ \rightarrow \phi e^+ \nu$

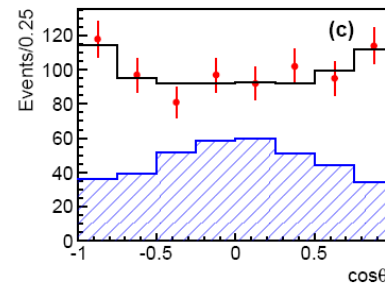
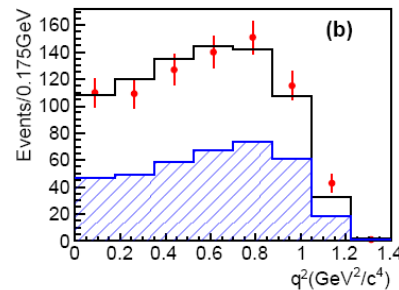
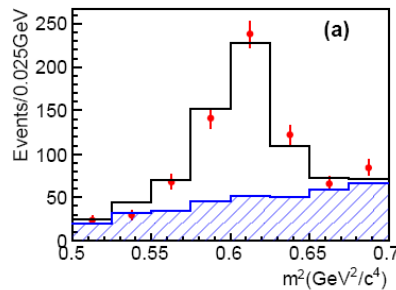
PRD92(2015) 071101R



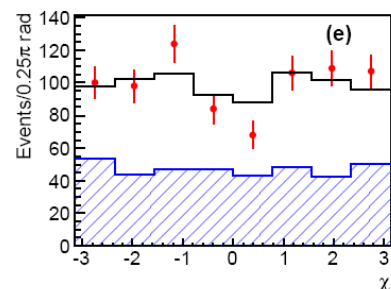
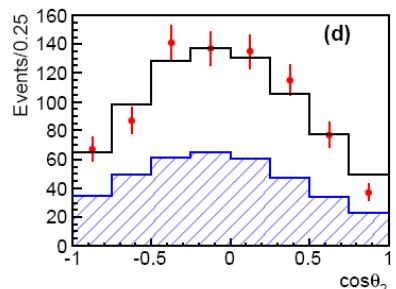
Red dots: data;
Arrows: signal
region.

Mode	This work	Previous
$\omega e^+ \nu_e$	$(1.63 \pm 0.11 \pm 0.08) \times 10^{-3}$	$(1.82 \pm 0.18 \pm 0.07) \times 10^{-3}$
$\phi e^+ \nu_e$	$< 1.3 \times 10^{-5}$ (90%C.L.)	$< 9.0 \times 10^{-5}$ (90%C.L.)

**Better precision
or sensitivity**



Amplitude analysis of
 $D^+ \rightarrow \omega e^+ \nu$ is performed
for the first time



Results of form factor ratios:

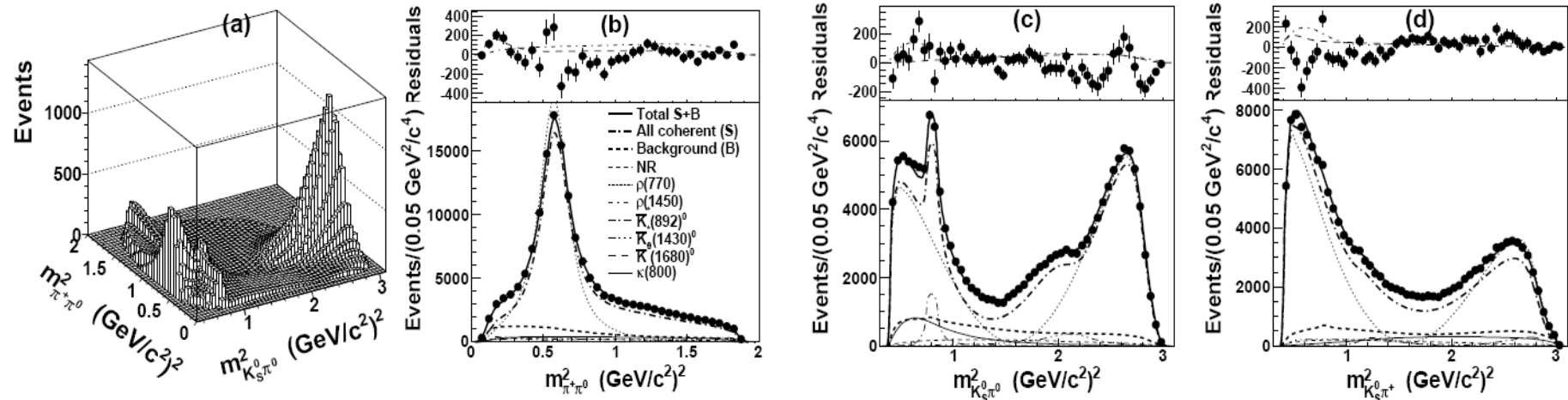
$$r_V = V(0)/A_1(0) = 1.24 \pm 0.09 \pm 0.06$$

$$r_2 = A_2(0)/A_1(0) = 1.06 \pm 0.15 \pm 0.05$$

Dalitz Plot Analysis of $D^+ \rightarrow K_S^0 \pi^+ \pi^0$

Distribution of (a) fitted p.d.f, and projections on (b) $M^2_{\pi^+\pi^0}$, (c) $M^2_{K_S^0\pi^0}$, and (d) $M^2_{K_S^0\pi^+}$.

PRD89(2014) 052001



Partial BFs calculated by combining fitted fractions with PDG's $B[D^+ \rightarrow K_S^0 \pi^+ \pi^0]$.

Mode	Partial Branching Fraction (%)
$D^+ \rightarrow K_S^0 \pi^+ \pi^0$ Non Resonant	$0.32 \pm 0.05 \pm 0.25^{+0.28}_{-0.25}$
$D^+ \rightarrow \rho^+ K_S^0, \rho^+ \rightarrow \pi^+ \pi^0$	$5.83 \pm 0.16 \pm 0.30^{+0.45}_{-0.15}$
$D^+ \rightarrow \rho(1450)^+ K_S^0, \rho(1450)^+ \rightarrow \pi^+ \pi^0$	$0.15 \pm 0.02 \pm 0.09^{+0.07}_{-0.11}$
$D^+ \rightarrow \bar{K}^{*0}(892) \pi^+, \bar{K}^{*0}(892) \rightarrow K_S^0 \pi^0$	$0.250 \pm 0.012 \pm 0.015^{+0.025}_{-0.024}$
$D^+ \rightarrow \bar{K}^{*0}(1430) \pi^+, \bar{K}^{*0}(1430) \rightarrow K_S^0 \pi^0$	$0.26 \pm 0.04 \pm 0.05 \pm 0.06$
$D^+ \rightarrow \bar{K}^{*0}(1680) \pi^+, \bar{K}^{*0}(1680) \rightarrow K_S^0 \pi^0$	$0.09 \pm 0.01 \pm 0.05^{+0.04}_{-0.08}$
$D^+ \rightarrow \bar{K}^0 \pi^+, \bar{K}^0 \rightarrow K_S^0 \pi^0$	$0.54 \pm 0.09 \pm 0.28^{+0.36}_{-0.19}$
$NR + \bar{K}^0 \pi^+$	$1.30 \pm 0.12 \pm 0.12^{+0.12}_{-0.30}$
$K_S^0 \pi^0$ S-wave	$1.21 \pm 0.10 \pm 0.16^{+0.19}_{-0.27}$

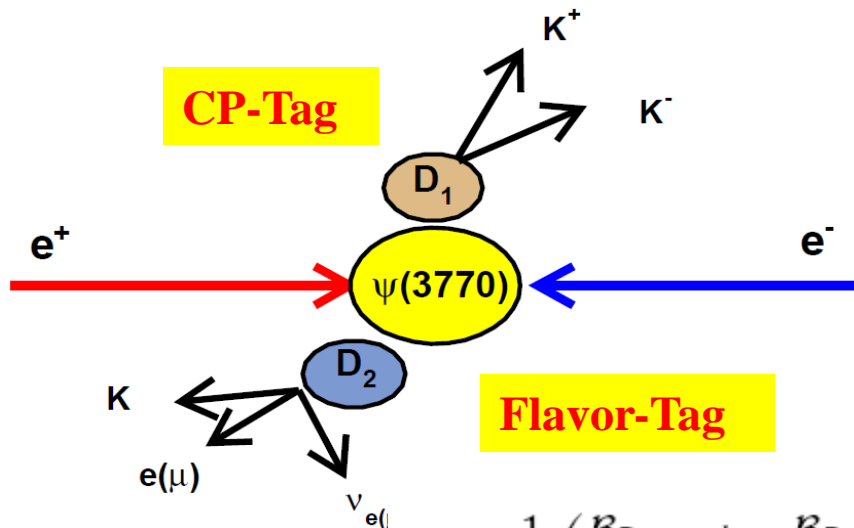
Partial branching ratios are measured with higher precision than previous measurements.

$D\bar{D}$ mixing parameter y_{CP}

PLB 744, 339 (2015)

We measure the y_{CP} using CP-tagged semi-leptonic D decays, which allows to access CP asymmetry in mixing and decays.

Compatible with the previous measurements.

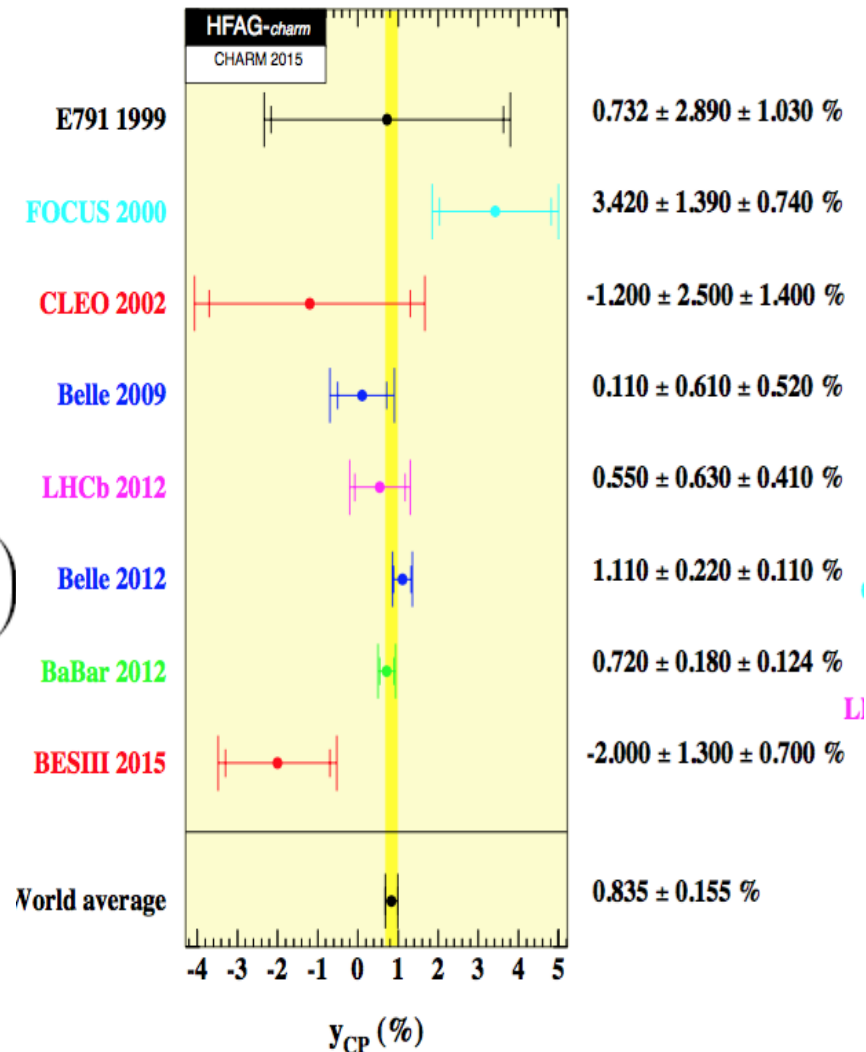


$$y_{CP} \approx \frac{1}{4} \left(\frac{\mathcal{B}_{D_{CP-} \rightarrow l}}{\mathcal{B}_{D_{CP+} \rightarrow l}} - \frac{\mathcal{B}_{D_{CP+} \rightarrow l}}{\mathcal{B}_{D_{CP-} \rightarrow l}} \right)$$

Reconstructed Modes:

Type	Mode
CP+	$K^+K^-, \pi^+\pi^-, K_S^0\pi^0\pi^0$
CP-	$K_S^0\pi^0, K_S^0\omega, K_S^0\eta$
Semileptonic	$K^\mp e^\pm \nu, K^\mp \mu^\pm \nu$

$$y_{CP} = (-2.1 \pm 1.3 \pm 0.7)\%$$

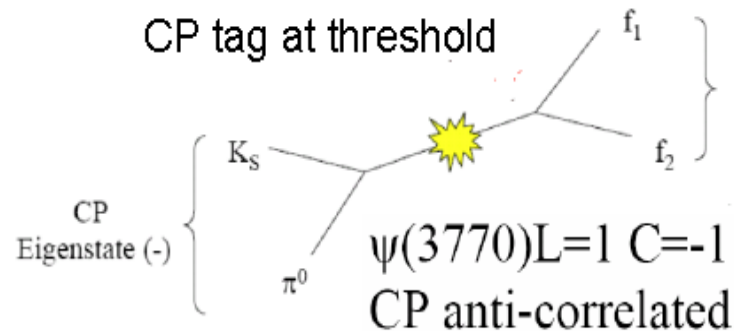


Strong phase difference $\delta_{K\pi}$

PLB 734, 227 (2014)

Quantum correlation \rightarrow Interference \rightarrow access strong phase!

If CP violation in charm is neglected: mass eigenstates = CP eigenstates



$$\mathcal{A}_{K\pi}^{CP} \equiv \frac{\mathcal{B}_{D^{S-} \rightarrow K^- \pi^+} - \mathcal{B}_{D^{S+} \rightarrow K^- \pi^+}}{\mathcal{B}_{D^{S-} \rightarrow K^- \pi^+} + \mathcal{B}_{D^{S+} \rightarrow K^- \pi^+}}$$

$$2r \cos \delta_{K\pi} + y = (1 + R_{WS}) \cdot \mathcal{A}_{CP \rightarrow K\pi},$$

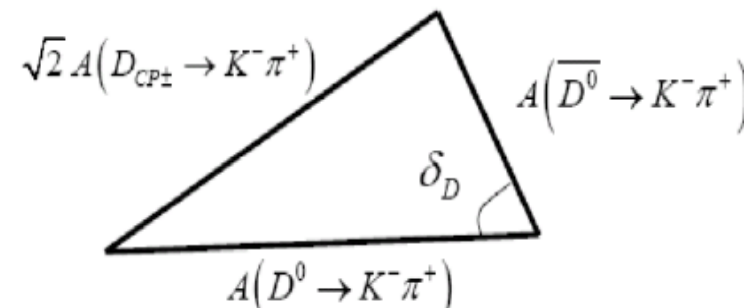
$$|D_1\rangle \equiv \frac{|D^0\rangle + |\bar{D}^0\rangle}{\sqrt{2}} \quad |D_2\rangle \equiv \frac{|D^0\rangle - |\bar{D}^0\rangle}{\sqrt{2}}$$

$$\mathcal{A}_{CP}^{K\pi} = (12.7 \pm 1.3 \pm 0.7) \times 10^{-2}$$

With external inputs of the parameters in HFAG2013 and PDG

$$R_D = 3.47 \pm 0.06\%, \quad y = 6.6 \pm 0.9\% \quad R_{WS} = 3.80 \pm 0.05\%$$

$$\cos \delta_{K\pi} = 1.02 \pm 0.11 \pm 0.06 \pm 0.01$$



$\delta_{K\pi}$ is important to relate to mixing parameters x and y from x' and y'

Reconstructed Modes:

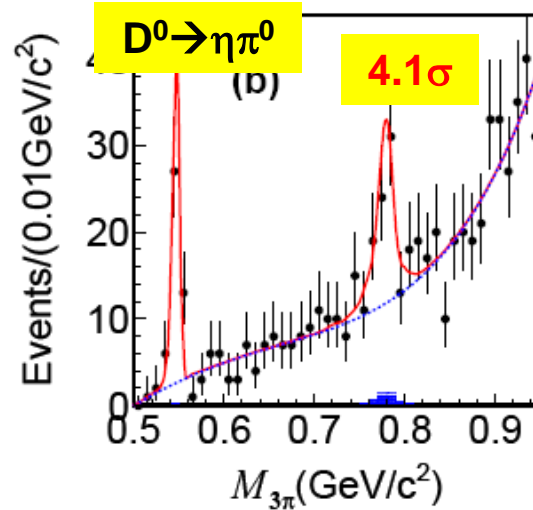
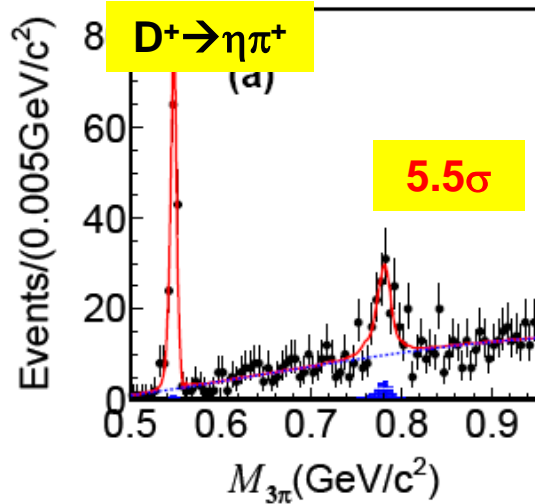
Type	Mode
Flavored	$K^- \pi^+, K^+ \pi^-$
S+	$K^+ K^-, \pi^+ \pi^-, K_S^0 \pi^0 \pi^0, \pi^0 \pi^0, \rho^0 \pi^0$
S-	$K_S^0 \pi^0, K_S^0 \eta, K_S^0 \omega$

most precise to date

Observation/Evidence for SCS decay $D^{+ (0)} \rightarrow \omega \pi^{+ (0)}$

Suppress background via DT method

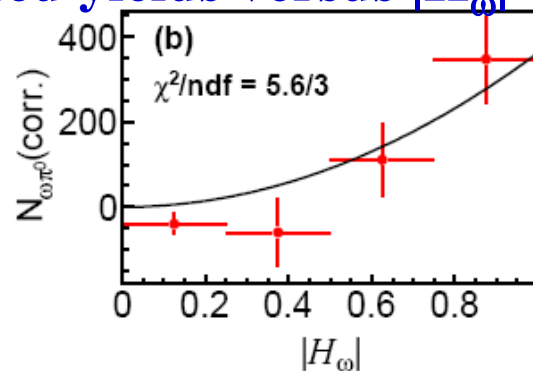
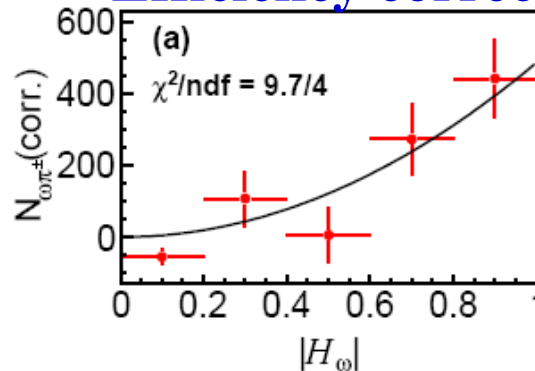
arXiv:1512.06998



➤ Predications of $B[D \rightarrow \omega \pi]$ at 1.0×10^{-4} level.

➤ $D \rightarrow \omega \pi$ were studied at CLEO-c with ST method, but only set BF upper limits

Efficiency corrected yields versus $|H_\omega|$



Summary of BFs measurements

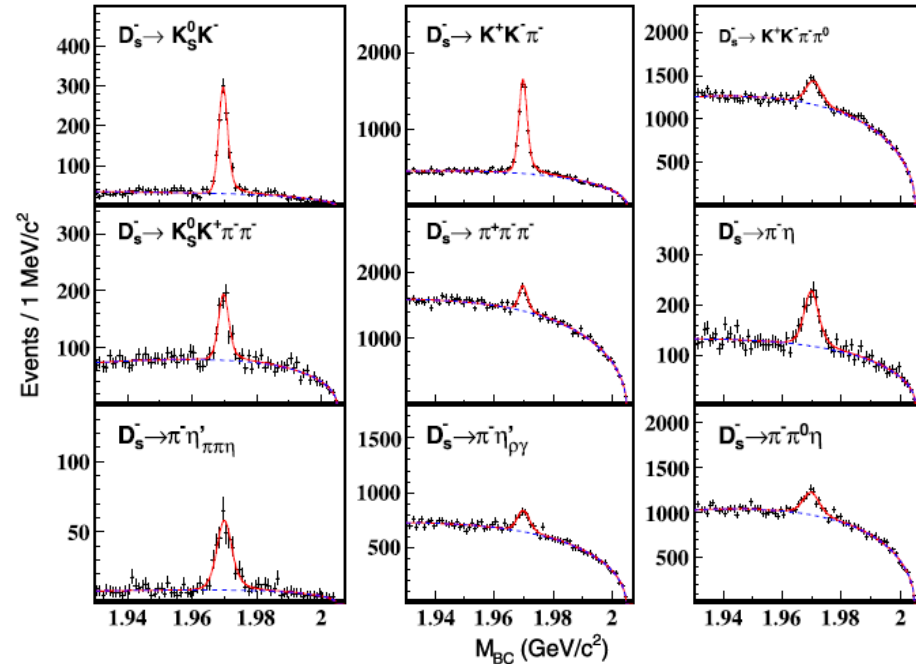
Mode	This work	Previous measurements
$D^+ \rightarrow \omega \pi^+$	$(2.79 \pm 0.57 \pm 0.16) \times 10^{-4}$	$< 3.4 \times 10^{-4}$ at 90% C.L.
$D^0 \rightarrow \omega \pi^0$	$(1.17 \pm 0.34 \pm 0.07) \times 10^{-4}$	$< 2.6 \times 10^{-4}$ at 90% C.L.
$D^+ \rightarrow \eta \pi^+$	$(3.07 \pm 0.22 \pm 0.13) \times 10^{-3}$	$(3.53 \pm 0.21) \times 10^{-3}$
$D^0 \rightarrow \eta \pi^0$	$(0.65 \pm 0.09 \pm 0.04) \times 10^{-3}$	$(0.68 \pm 0.07) \times 10^{-3}$

Improve understanding of U-spin and SU(3) flavor symmetry breaking effects in D decays and benefitting theoretical prediction of CP violation in D decays

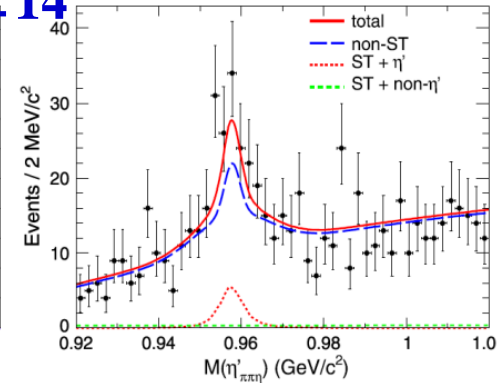
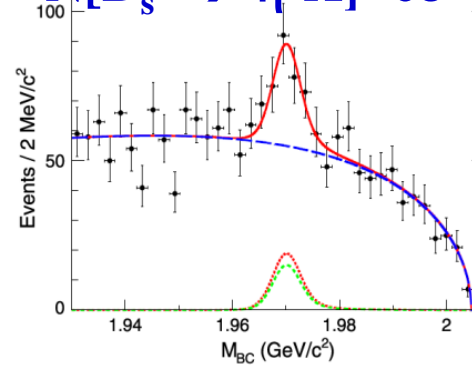
$D_s^+ \rightarrow \eta' X$ and $\eta' \rho^+$

About 15.6 K ST D_s^- events by using 9 ST modes

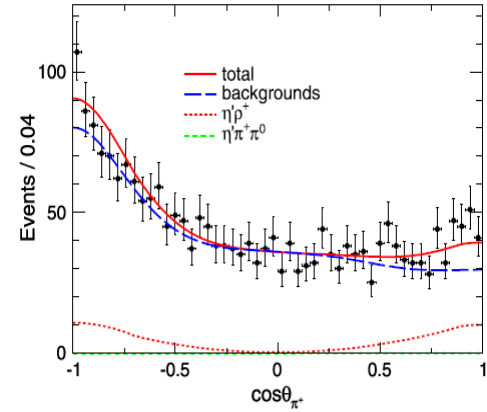
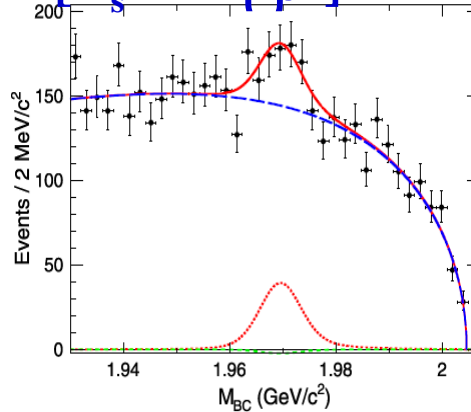
PLB750 466(2015)



$N[D_s^+ \rightarrow \eta' X] = 68 \pm 14$



$N[D_s^+ \rightarrow \eta' \rho^+] = 210 \pm 50$



$B[D_s^+ \rightarrow \eta' X] = (8.8 \pm 1.8 \pm 0.5)\%$

Consistent with CLEO measurements $B[D_s^+ \rightarrow \eta' X] = (11.7 \pm 1.8)\%$ [PRD79 112008(2009)]

$B[D_s^+ \rightarrow \eta' \rho^+] = (5.8 \pm 1.4 \pm 0.4)\%$ $B^{\text{exp}}[D_s^+ \rightarrow \eta' \rho^+] = (3.0 \pm 0.5)\%$ [PRD84 074019(2011)]

Resolve the disagreement between theoretical predication and CLEO-c's previous measurement. $B[D_s^+ \rightarrow \eta' \rho^+] = (12.5 \pm 2.2)\%$ [PRD58 052002(1998)]

Absolute BF for $\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e$

Theoretical calculations on the BF ranges from 1.4% to 9.2%

PDG2014: $(2.1 \pm 0.6)\%$

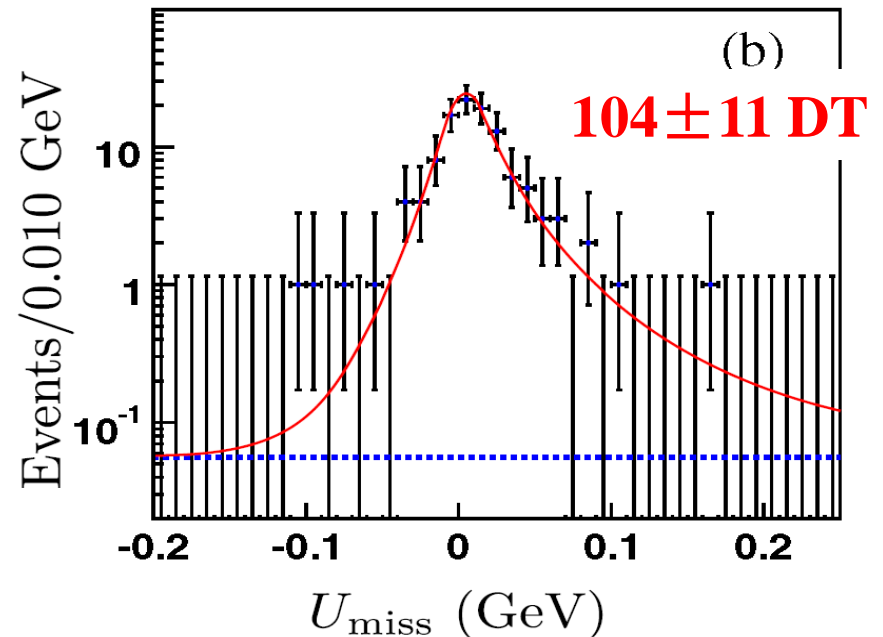
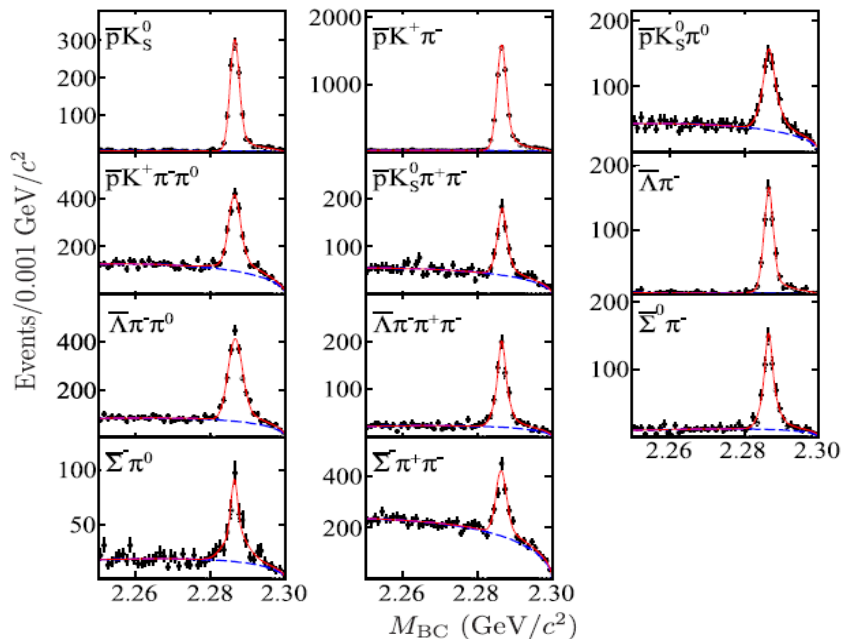
PDG2015: $(2.9 \pm 0.5)\%$



Input $B[\Lambda_c^+ \rightarrow p K^- \pi^+] = (6.84^{+0.32}_{-0.40})\%$
by BELLE [PRL113,042002(2014)]

14415 \pm 159 events with 11 ST modes

PRL115(2015)221805



$B[\Lambda_c^+ \rightarrow \Lambda e^+ \nu] = (3.63 \pm 0.38 \pm 0.20)\%$ First absolute measurement

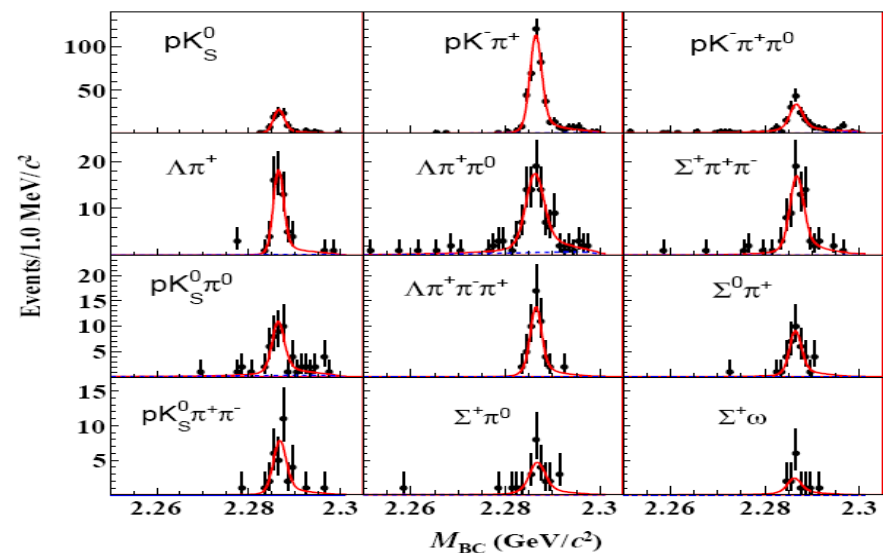
Important for test and calibrate the LQCD calculations.

Absolute BFs for Λ_c^+ hadron decays

Measurement using the threshold pair-productions via e^+e^- annihilation is unique: the most simple and straightforward

arXiv:1511.08380

Accepted by PRL



A global least-square fitter is utilized to improve the measured precision for 12 Λ_c^+ hadronic decay channels.

$$N_{-j}^{DT} = \sum_{i^+ \neq i} N_{i^+j^-}^{DT} + \sum_{i^- \neq i} N_{i^-j^+}^{DT} + N_{jj}^{DT}$$

✓ Absolute BFs are improved significantly.

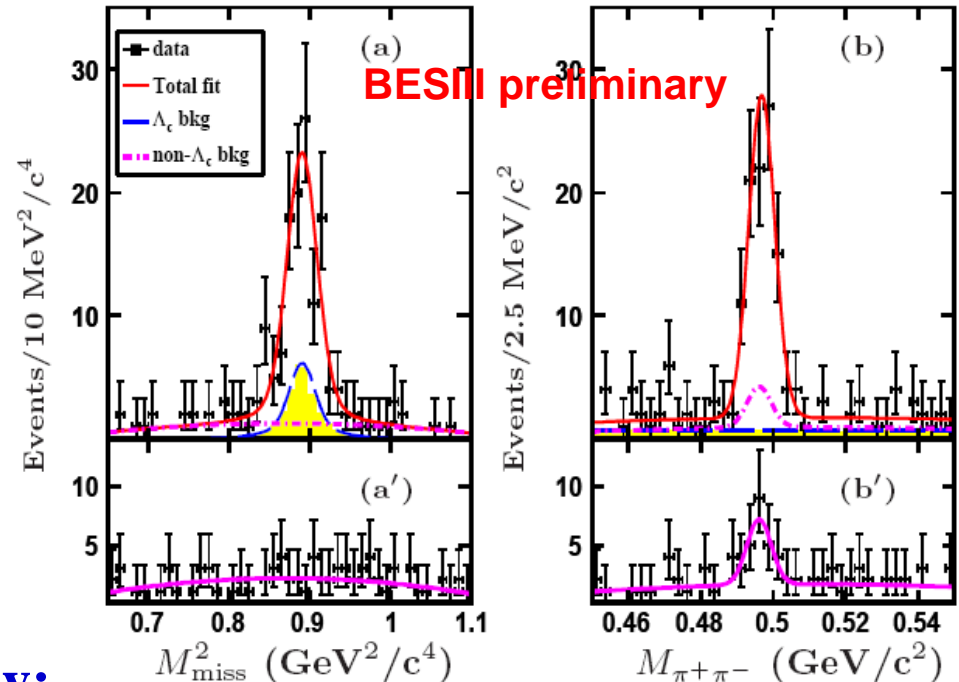
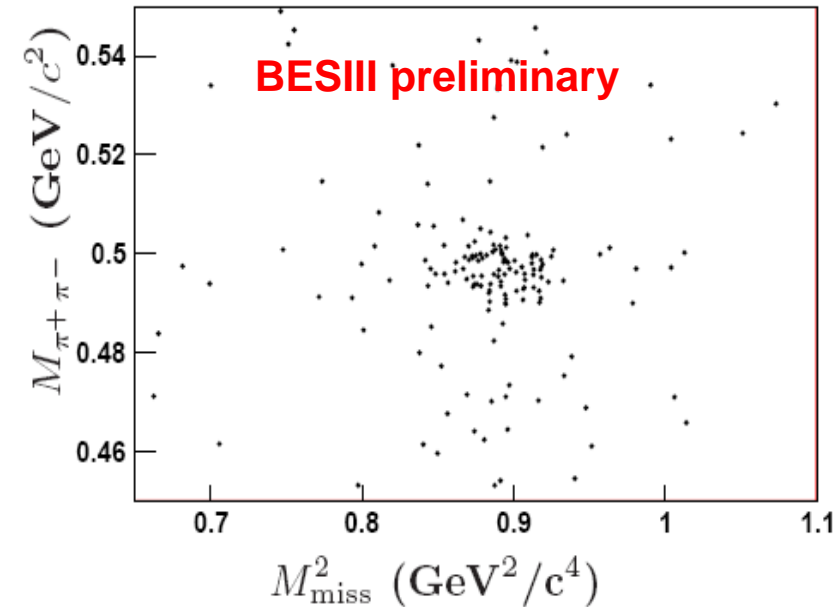
✓ BESIII BF for $\Lambda_c^+ \rightarrow pK^- \pi^+$ is smaller.

✓ Improved absolute BF of $pK^- \pi^+$ together with BELLE's result are key to calibrate other decays.

Mode	This work (%)	PDG (%)	Belle \mathcal{B}
pK_S^0	$1.52 \pm 0.08 \pm 0.03$	1.15 ± 0.30	
$pK^- \pi^+$	$5.84 \pm 0.27 \pm 0.23$	5.0 ± 1.3	$6.84 \pm 0.24^{+0.21}_{-0.27}$
$pK_S^0 \pi^0$	$1.87 \pm 0.13 \pm 0.05$	1.65 ± 0.50	
$pK_S^0 \pi^+ \pi^-$	$1.53 \pm 0.11 \pm 0.09$	1.30 ± 0.35	
$pK^- \pi^+ \pi^0$	$4.53 \pm 0.23 \pm 0.30$	3.4 ± 1.0	
$\Lambda \pi^+$	$1.24 \pm 0.07 \pm 0.03$	1.07 ± 0.28	
$\Lambda \pi^+ \pi^0$	$7.01 \pm 0.37 \pm 0.19$	3.6 ± 1.3	
$\Lambda \pi^+ \pi^- \pi^+$	$3.81 \pm 0.24 \pm 0.18$	2.6 ± 0.7	
$\Sigma^0 \pi^+$	$1.27 \pm 0.08 \pm 0.03$	1.05 ± 0.28	
$\Sigma^+ \pi^0$	$1.18 \pm 0.10 \pm 0.03$	1.00 ± 0.34	
$\Sigma^+ \pi^+ \pi^-$	$4.25 \pm 0.24 \pm 0.20$	3.6 ± 1.0	
$\Sigma^+ \omega$	$1.56 \pm 0.20 \pm 0.07$	2.7 ± 1.0	

Observation of $\Lambda_c^+ \rightarrow n K_S^0 \pi^+$

First observation of Λ_c^+ decays to final states involving the neutron.



The missing neutron is detected by:

$$M_{\text{miss}}^2 = (p_{\Lambda_c^+} - p_{K_S^0} - p_{\pi^+})^2 = E_{\text{miss}}^2 - c^2 |\vec{p}_{\text{miss}}|^2$$

83 ± 11 net signal events

BESIII Preliminary results:

$$\mathcal{B}[\Lambda_c^+ \rightarrow n K_S^0 \pi^+] = (1.82 \pm 0.23 \pm 0.11)\%$$

Fit to M_{miss}^2 and $M_{\pi^+\pi^-}$ spectra in (a,b) $\bar{\Lambda}_c^-$ signal region and (a',b') $\bar{\Lambda}_c^-$ sideband region simultaneously.

The relative BF of neutron-involved mode to proton-involved mode is essential to test the isospin symmetry for Λ_c^+ decays.
[arXiv:1601.04241]

Summary

▣ **BESIII provides important results on charm decays**

➤ **leptonic and semi-leptonic decays**

➤ **D hadronic decays**

➤ **D^0 mixing and strong phase**

➤ **Λ_c^+ hadronic and semi-leptonic decays**

important to test LQCD calculations, CKM matrix UT, search for NP beyond SM

▣ **More fruitful results will come out!**

Thanks!

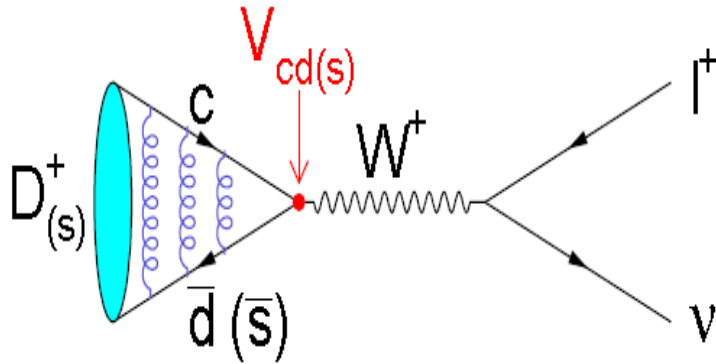
Introduction

Precision measurement of charm decays provide rich information to probe for strong and weak effects

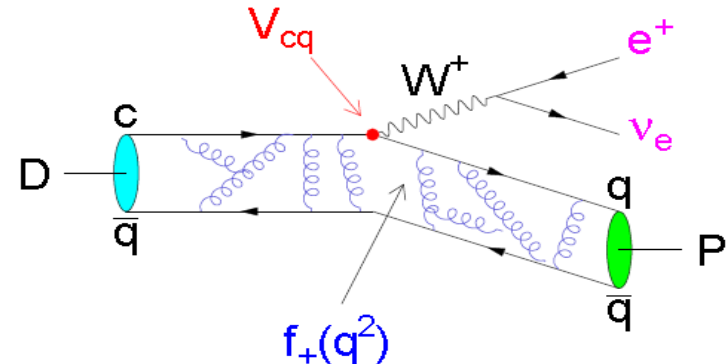
- **Unitarity test of CKM matrix:** direct access quark mixing matrix element $|V_{cs(d)}|$ or strong phase constrained γ/ϕ_3
- **LQCD calibration:** precise decay constant $f_{D(s)+}$, form factors $f_{D \rightarrow K(\pi)}(q^2)$ and others
- **New physics BSM:** evidence of rare decay/CP violation, or significant deviation of CKM unitarity/LQCD calculation
- **Important inputs for beauty physics:** Significantly improved decay rates or dynamics

D leptonic and semi-leptonic decays

Bridge to extract $D_{(s)}^+$ decay constant(s) $f_{D(s)+}$, form factors $f_+^{D \rightarrow K(\pi)}(q^2)$ and quark mixing matrix elements $|V_{cs(d)}|$



$$\Gamma(D_{(s)}^+ \rightarrow \ell^+ \nu_\ell) = \frac{G_F^2 f_{D_{(s)}^+}^2}{8\pi} |V_{cd(s)}|^2 m_\ell^2 m_{D_{(s)}^+}^2 \left(1 - \frac{m_\ell^2}{m_{D_{(s)}^+}^2}\right)^2$$



$$\frac{d\Gamma}{dq^2} = X \frac{G_F^2 |V_{cd(s)}|^2}{24\pi^3} p^3 |f_+(q^2)|^2$$

■ Improved $f_{D(s)+}$, $f_+^{D \rightarrow K(\pi)}(q^2)$ of D semi-leptonic decays calibrate LQCD calculations at higher accuracy. Once they pass experimental test, the precise LQCD calculations of f_D/f_B , f_{D_s}/f_{B_s} and form factor ratios are helpful for measurements in B decays

■ Recent LQCD calculations on $f_{D(s)+}$ [0.5(0.5)%], $f_+^{D \rightarrow K(\pi)}(0)$ [1.7(4.4)%] provide good chance to precisely measure $|V_{cs(d)}|$

$$\begin{aligned}
\frac{d\Gamma}{dq^2 d\cos\theta_1 d\cos\theta_2 d\chi dm_{\pi\pi\pi}} &= \frac{3}{8(4\pi)^4} G_F^2 |V_{cd}|^2 \frac{p_\omega q^2}{M_D^2} \mathcal{B}(\omega \rightarrow \pi\pi\pi) |\mathcal{BW}(m_{\pi\pi\pi})|^2 \\
&[(1 + \cos\theta_2)^2 \sin^2\theta_1 |H_+(q^2, m_{\pi\pi\pi})|^2 \\
&+ (1 - \cos\theta_2)^2 \sin^2\theta_1 |H_-(q^2, m_{\pi\pi\pi})|^2 + 4 \sin^2\theta_2 \cos^2\theta_1 |H_0(q^2, m_{\pi\pi\pi})|^2 \\
&+ 4 \sin\theta_2 (1 + \cos\theta_2) \sin\theta_1 \cos\theta_1 \cos\chi H_+(q^2, m_{\pi\pi\pi}) H_0(q^2, m_{\pi\pi\pi}) \\
&- 4 \sin\theta_2 (1 - \cos\theta_2) \sin\theta_1 \cos\theta_1 \cos\chi H_-(q^2, m_{\pi\pi\pi}) H_0(q^2, m_{\pi\pi\pi}) \\
&- 2 \sin^2\theta_2 \sin^2\theta_1 \cos 2\chi H_+(q^2, m_{\pi\pi\pi}) H_-(q^2, m_{\pi\pi\pi})].
\end{aligned}$$

$$\begin{aligned}
H_\pm(q^2) &= M A_1(q^2) \mp 2 \frac{M_D p_\omega}{M} V(q^2) \\
H_0(q^2) &= \frac{1}{2m_{\pi\pi\pi} \sqrt{q^2}} [(M_D^2 - m_{\pi\pi\pi}^2 - q^2) M A_1(q^2) \\
&\quad - 4 \frac{M_D^2 p_\omega^2}{M} A_2(q^2)]
\end{aligned} \quad (3)$$

where $M = M_D + m_{\pi\pi\pi}$. For the q^2 dependence, a single

pole parameterization [24] is applied:

$$V(q^2) = \frac{V(0)}{1 - q^2/m_V^2}, \quad A_{1,2}(q^2) = \frac{A_{1,2}(0)}{1 - q^2/m_A^2}, \quad (4)$$

where the pole masses m_V and m_A are expected to be close to $M_{D^*(1-)} = 2.01 \text{ GeV}/c^2$ and $M_{D^*(1+)} = 2.42 \text{ GeV}/c^2$ [14] for the vector and axial form factors, respectively. The ratios of these form factors, evaluated at $q^2 = 0$, $r_V = \frac{V(0)}{A_1(0)}$ and $r_2 = \frac{A_2(0)}{A_1(0)}$, are measured in this paper.