Beyond the Isobar Model

M. Mikhasenko

HISKP, University of Bonn

January 27, 2016

Thanks to

- Bernhard Ketzer, HISKP, Uni Bonn;
- Adam Szczepaniak, JPAC;
- Andrey Sarantsev, Bonn-Gatchina group.

Spectrum of light mesons

Why have not we got bored?

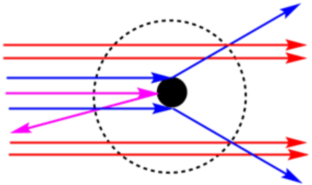
```
In[2]:= ParticleData["Meson"] // Length

Out[2]= 399
```

```
{π0, π-, π+, K-, K+, K0, K_L, K_S, K0-bar, η, ρ+(770), ρ0(770), ρ-(770), ω(782), f_0(600), K_0^*0(800), K_0^*+(800), K_0^*-(800), K_0^*0-bar(800), K*-(892), K*+(892), K*0(892), K*0-bar(892),
η^'(958), f_0(980), a_0^-(980), a_0^0(980), a_0^+(980), ϕ(1020), X(1070), X(1110), h_1(1170), b_1^-(1235), b_1^0(1235), b_1^+(1235), a_1^-(1260), a_1^0(1260), a_1^+(1260), K_1^0(1270),
K_1^+(1270), K_1^-(1270), K_1^0-bar(1270), f_2(1270), f_1(1285), η(1295), π-(1300), π0(1300), π+(1300), a_2^-(1320), a_2^0(1320), a_2^+(1320), f_0(1370), π_1^-(1400), π_1^0(1400), π_1^+(1400),
h_1(1380), K_1^0(1400), K_1^+(1400), K_1^-(1400), K_1^0-bar(1400), η(1405), K_0^*0(1430), K_0^*+(1430), K*0(1410), K*+(1410), K_0^*-(1430), K_0^*0-bar(1430), K*-(1410), K*0-bar(1410), X(1420),
ω(1420), K_2^*+(1430), K_2^*-(1430), f_1(1420), K_2^*0(1430), K_2^*0-bar(1430), f_2(1430), ρ-(1450), ρ0(1450), ρ+(1450), K0(1460), K+(1460), K-(1460), K0-bar(1460), a_0^-(1450), a_0^0(1450),
a_0^+(1450), η(1475), f_0(1600), f_0(1500), f_1(1510), f_2^'(1525), f_2(1565), K_2^0(1580), K_2^+(1580), K_2^-(1580), K_2^0-bar(1580), h_1(1595), X(1600), η_2(1645), K0(1630), K+(1630),
K-(1630), K0-bar(1630), f_2(1640), a_1^-(1640), a_1^0(1640), a_1^+(1640), K_1^0(1650), K_1^+(1650), K_1^-(1650), K_1^0-bar(1650), X(1650), π_1^-(1600), π_1^0(1600), π_1^+(1600), ω_3(1670),
ω(1650), π_2^-(1670), π_2^0(1670), π_2^+(1670), ϕ(1680), ρ_3^-(1690), ρ_3^0(1690), ρ_3^+(1690), K*0(1680), K*+(1680), K*-(1680), K*0-bar(1680), f_0(1710), ρ-(1700), ρ0(1700), ρ+(1700),
a_2^+(1700), a_2^-(1700), a_2^0(1700), X(1750), η(1760), X(1775), K_2^0(1770), K_2^+(1770), K_2^-(1770), K_2^0-bar(1770), K_3^*0(1780), K_3^*+(1780), K_3^*-(1780), K_3^*0-bar(1780), π-(1800),
π0(1800), π+(1800), f_2(1810), K_2^0(1820), K_2^+(1820), K_2^-(1820), K_2^0-bar(1820), K0(1830), K+(1830), K-(1830), K0-bar(1830), X(1835), η_2(1870), ϕ_3(1850), X(1855), ρ-(1900), ρ0(1900),
ρ+(1900), D0, D0-bar, D-, D+, a_3(1875), X(1870), π_2(1880), f_2(1910), a_1(1930), X(1935), ρ_2(1940), f_2(1950), K_0^*0(1950), K_0^*+(1950), ω_3(1945), K_0^*-(1950), K_0^*0-bar(1950),
b_1(1960), h_1(1965), ω(1960), ρ(1965), D_s, D_s-bar, X(1970), f_1(1970), K_2^*0(1980), K_2^*+(1980), X(1975), K_2^*-(1980), K_2^*0-bar(1980), ω_2(1975), ρ_3^-(1990), ρ_3^0(1990),
ρ_3^+(1990), f_0(2020), X0(2000), ρ(2000), a_4^-(2040), a_4^0(2040), a_4^+(2040), f_2(2000), a_2(1990), π_2(2005), D*(2007), D*-bar(2007), D*-bar(2010), D*(2010), η(2010), f_2(2010),
π_1(2015), X(2020), b_3(2025), f_4(2050), h_3(2025), a_0(2020), η_2(2030), K_4^*0(2045), K_4^*+(2045), K_4^*-(2045), K_4^*0-bar(2045), f_3(2050), a_2(2080), f_0(2060), a_3(2070), π(2070),
X(2075), X0(2080), π_2^-(2100), π_2^0(2100), π_2^+(2100), a_1(2095), η(2100), X(2100), f_0(2100), X(2110), D_s^*, D_s^*-bar, f_2(2140), ρ-(2150), ρ0(2150), ρ+(2150), ω(2145), X(2150),
f_2(2150), a_2(2175), f_0(2200), η(2190), ω_2(2195), ω(2205), X0(2210), η(2225), h_1(2215), f_J(2220), b_1(2240), π_2(2245), ρ_2(2240), ρ_4(2240), K_2^0(2250), K_2^+(2250), K_2^-(2250),
K_2^0-bar(2250), η_2(2250), ω_3(2255), ω_4(2250), π_4(2250), ρ_3^-(2250), ρ_3^0(2250), ρ_3^+(2250), ρ(2265), X(2260), a_2(2270), a_1(2270), h_3(2275), ω_3(2285), ρ(2280), η(2280), X(2290),
f_2(2300), a_4(2280), ρ_3(2300), f_3(2300), a_3(2310), f_1(2310), D_s0^*(2317), D_s0^*-bar(2317), K_3^0(2320), K_3^+(2320), K_3^-(2320), K_3^0-bar(2320), η_4(2320), ω(2330), ρ_5^-(2350),
ρ_5^0(2350), ρ_5^+(2350), f_4(2300), f_0(2330), f_2(2340), X(2340), a_1(2340), D_0^*0(2400), D_0^*0-bar(2400), π(2360), X(2360), K_5^*0(2380), K_5^*+(2380), K_5^*-(2380), K_5^*0-bar(2380),
D_0^*+(2400), D_0^*-(2400), D_1^0(2420), D_1^0-bar(2420), D_1^+(2420), D_1^-(2420), D_1(2430), D_1-bar(2430), X(2440), a_6^-(2450), a_6^0(2450), a_6^+(2450), D_s1(2460), D_s1-bar(2460),
D_2^*+(2460), D_2^*-(2460), f_6(2510), D_2^*0(2460), D_2^*0-bar(2460), K_4^0(2500), K_4^+(2500), K_4^-(2500), K_4^0-bar(2500), D_s1(2536), D_s1-bar(2536), D_s2(2573), D_s2-bar(2573), X(2632),
D*(2640), D*-bar(2640), X(2680), X(2710), X(2750), η_c(1S), K(3100), J/ψ(1S), f_6(3100), X0(3250), χ_c0(1P), χ_c1(1P), h_c(1P), χ_c2(1P), η_c(2S), ψ(2S), ψ(3770), X(3872), χ_c2(2P),
Y(3940), ψ(4040), ψ(4160), X(4260), ψ(4415), B-, B+, B0, B0-bar, B*-, B*+, B*0, B*0-bar, B_s, B_s-bar, B_s^*, B_s^*-bar, B_J^*0(5732), B_J^*+(5732), B_J^*-(5732), B_J^*0-bar(5732),
B_sJ^*(5850), B_sJ^*-bar(5850), B_c, B_c-bar, η_b(1S), Υ(1S), χ_b0(1P), χ_b1(1P), χ_b2(1P), Υ(2S), Υ(1D), χ_b0(2P), χ_b1(2P), χ_b2(2P), Υ(3S), Υ(4S), Υ(10860), Υ(11020)}
```

Four groups: **“standard” mesons**, **meson-molecules, exotics, something else(?)**.

Scattering experiment

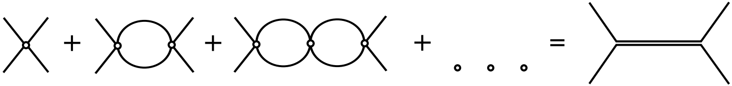


- We start with elastic scattering.
- We scan over energy of the system and find a preferable energy (peak of cross section)
- for a short time our particles prefer to form intermediate state

$$t(s) = \langle f|T|i\rangle = \frac{g^2}{m^2 - s - im\Gamma}, \quad \sigma(s) \sim |t(s)|^2 \rho(s).$$

ρ is two-body phase space. $\tilde{\rho}$ is scalar two-body loop expression.

$$K + K\left[ig^2\tilde{\rho}/2\right]K + K\left[ig^2\tilde{\rho}/2\right]K\left[ig^2\tilde{\rho}/2\right]K + \cdots = \frac{1}{K^{-1} - ig^2\tilde{\rho}/2}$$



Notice if $K = g^2/(m^2 - s)$, then we get Breit-Wigner formula.

Two-body unitarity

Conservation of probability gives the unitarity condition

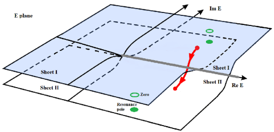
$$SS^\dagger = 1, \quad S = 1 + iT \;\; \Rightarrow \;\; T - T^\dagger = i\,T\,T^\dagger$$

$$t = \langle f|T|i\rangle, \quad \Delta t = i\,t^\star\,\rho\,t.$$

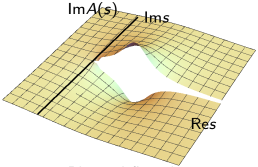
The general solution of the unitarity equation is

$$t(s) = \frac{1}{K^{-1}(s) - ig^2\tilde{\rho}(s)/2}, \text{ where } \tilde{\rho} = \frac{s}{i\pi}\int \frac{\rho(s')}{s'(s'-s)}\mathrm{d}s'.$$

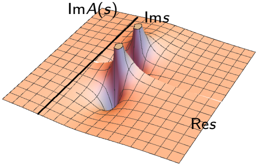
We find the reason the peak at second sheet, it is a pole of $t(s)$. Any resonance is associated with a pole.



[J. R. Pelaez, arXiv:1510.00653]

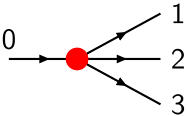


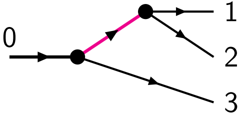
Physical first sheet



Unphysical second sheet

Multi-particle final states – Isobar model





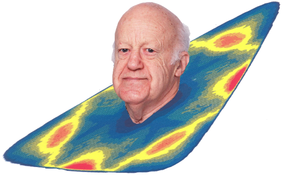


Image credit: Michael Pennington

- An amplitude summed over spins depends only on two invariants.
- Isobar model is suggested by data.

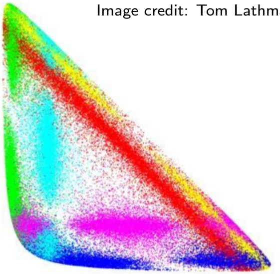
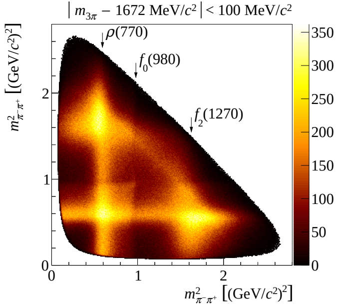


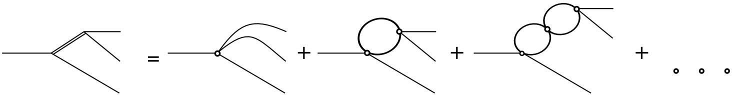
Image credit: Tom Lathman



Artificial example of $D \to K_s \pi^+ \pi^-$ and real measurements from COMPASS experiment $\pi_2(1670) \to \pi^- \pi^+ \pi^-$

Isobar model and rescattering

Isobars reflect the fact that the particles in every subchannel interact.



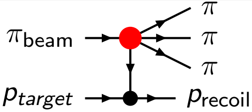
But isobar model does not satisfy subchannel unitarity.

Partial waves expansion

PW $= J^{PC}$ of the system $+$ Isobar $+$ L between the isobar and spectator.

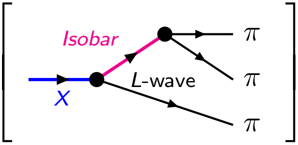
$$A = \sum_{\text{partial waves}} \overbrace{t^{\text{pw}}(m_{ij}^2)}^{\text{isobar dynamics}} \underbrace{D^{\text{pw}}(\Omega, \lambda, \dots)}_{\text{angular dependence}} \overbrace{g^{\text{pw}}}^{\text{coupling constants}}$$

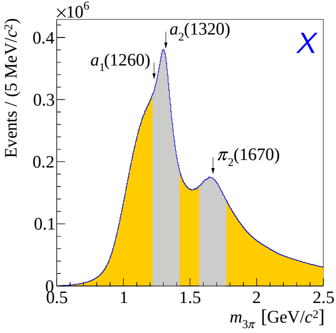
COMPASS/VES Partial Wave Analysis

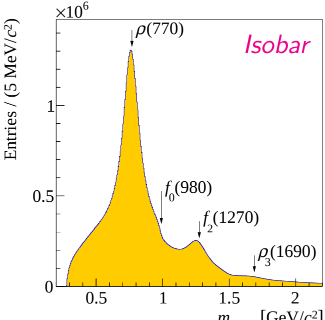


- COMPASS (VES) is a fixed-target experiment.
- 190 GeV (29 GeV) pion beam.

$$A(m_X, m_{2\pi}, \Omega_X, \Omega_I) = \sum_{Isobars, L-waves}^{88} C_{I,L}$$



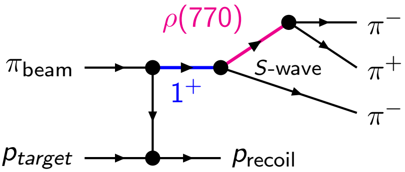


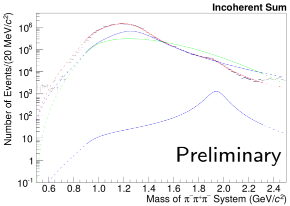


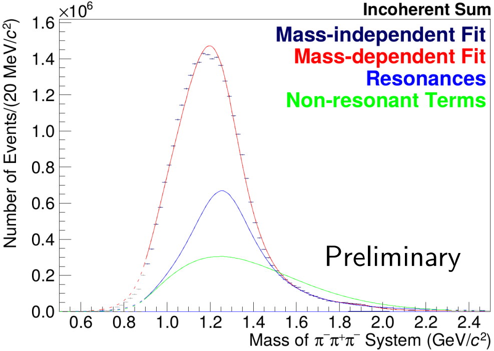
[ArXiv:1509.00992]

Example: $1^{++}\rho\pi\ S$

- Isobar: $\rho(770)$,
- ρ is in S-wave with bachelor pion,
- $J^P = 1^+$ states:
  - $a_1(1260)$
  - ? $a_1(1640)$
  - $a_1(1930)$
  - ? $a_1(2095)$

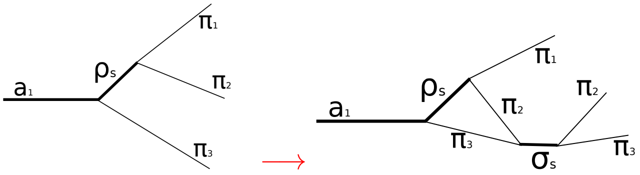






Corrections to the isobar model

Note, every isobar can rescatter to all others.



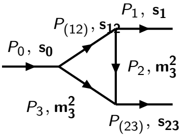
- First order is determined by the triangle loop diagram.
- Higher order diagram is hard to calculate.

The method to sum whole series of rescattering is known as Khuri-Treiman equations. It was applied to $\omega, \phi, \eta \to 3\pi$, $D \to K^-\pi^+\pi^+$.

- The amplitude of the "induced" isobars are given by the loop integral and primary coupling. No new parameters appear.

Triangle singularity on Dalitz plot

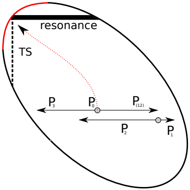
The properties of triangle loop diagram were studied extensively in past.



- Diverges $\sim \log(s - s_b)$ at one point. s_b dependents on all 5 invariants.
- Coleman-Norton theorem, i.e. "catch up" condition is satisfied at s_b.

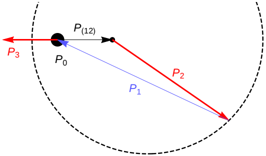
$$A(s_0, s_1, s_2) = g^3 \int \frac{d^4 k_1}{(2\pi)^4 i} \frac{1}{\Delta_1 \Delta_2 \Delta_3} = \frac{g^3}{16\pi^2} \int_0^1 \frac{\mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z}{D} \delta(1 - x - y - z),$$

$$\Delta_i = m_i^2 - k_i^2, \quad D = x\, m_1^2 + y\, m_2^2 + z\, m_3^2 - x\, y\, s_0 - z\, x\, s_1 - y\, z\, s_2.$$

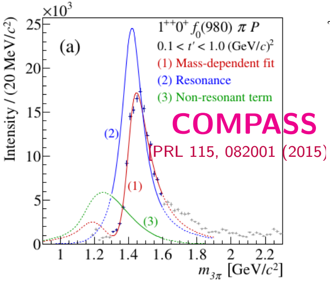


On the border of Dalitz plot the momenta are aligned, Particles are on mass shell.

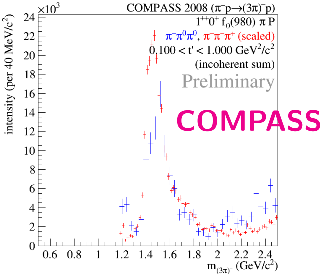
$\Rightarrow$ **singularity in s_{23} for fixed s_0.**



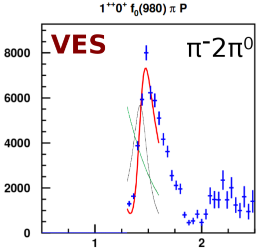
$a_1(1420)$ phenomenon - $1^{++}0^+ f_0(980)\pi\ P-$wave



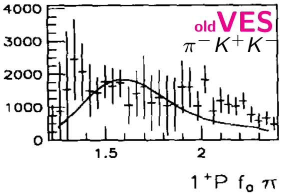
[PRL 115, 082001 (2015)]



[PoS **Hadron2013**, 087 (2013)]



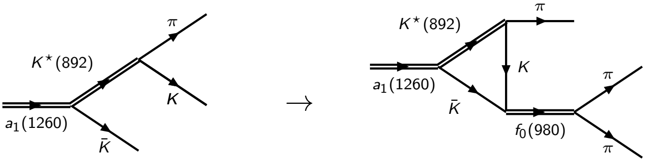
[PoS **Hadron2013**, 088 (2014)]



[Berdnikov *et al.*, Nuovo Cim. 107, 1941 (1994)]

The interpretations of $a_1(1420)$

- 4-quark state candidate [Hua-Xing Chen *et al.*, arXiv:1503.02597], [Zhi-Gang Wang, arXiv:1401.1134].
- $K^\star K$ molecule (similar to XYZ interpretation)
- Dynamic effect of interference with Deck [Basdevant *et al.*, arXiv:1501.04643] .
- Triangle singularity [Mikhasenko *et al.*, Phys. Rev. D91, 094015 (2015)] :

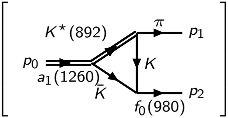


final state rescattering of almost real particles. Logarithmic singularity in the amplitude of the processes:

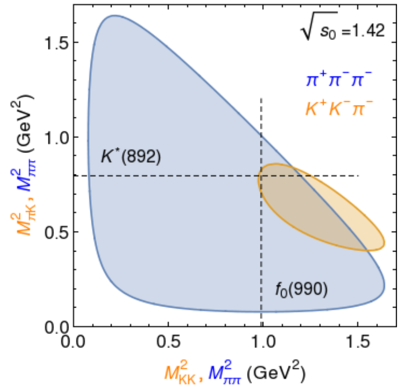
Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0\pi$

For the realistic decay, the amplitude is similar to the scalar case.

$\mathbb{M}^{(\mathrm{vpp})}_{a_1\to\pi f_0} =$

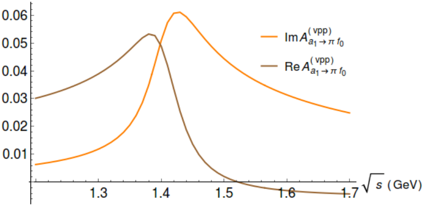


- Spin-Parity of particles.
- Width of $K^\star$



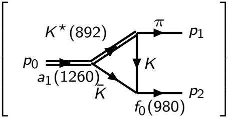
If one fixes mass of f_0, i.e. $p_{f_0}^2 = m_{f_0}^2$, then only $p_0^2 = s$ is variable.

+ finite width of $K^\star$

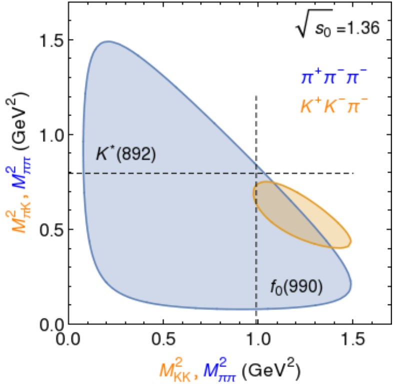


Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0 \pi$

For the realistic decay, the amplitude is similar to the scalar case.

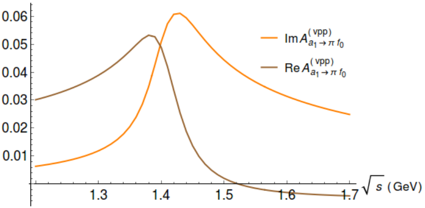
$\mathbb{M}^{(\text{vpp})}_{a_1 \to \pi f_0} =$ 

- Spin-Parity of particles.
- Width of $K^\star$



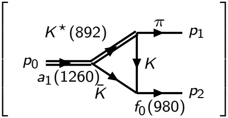
If one fixes mass of f_0, i.e. $p^2_{f_0} = m^2_{f_0}$, then only $p^2_0 = s$ is variable.

+ finite width of $K^\star$

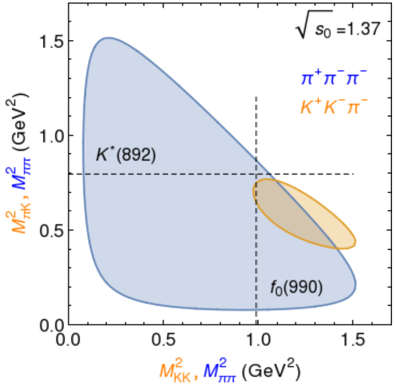


Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0\pi$

For the realistic decay, the amplitude is similar to the scalar case.

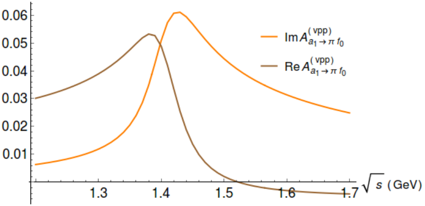
$\mathbb{M}^{(\mathrm{vpp})}_{a_1\to\pi f_0} =$ 

- Spin-Parity of particles.
- Width of $K^\star$



If one fixes mass of f_0, i.e. $p_{f_0}^2 = m_{f_0}^2$, then only $p_0^2 = s$ is variable.

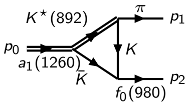
+ finite width of $K^\star$



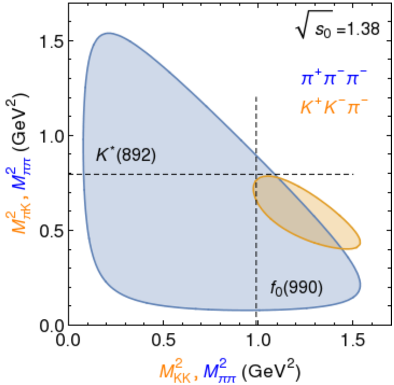
Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0\pi$

For the realistic decay, the amplitude is similar to the scalar case.

$\mathbb{M}^{(\mathrm{vpp})}_{a_1\to\pi f_0} =$

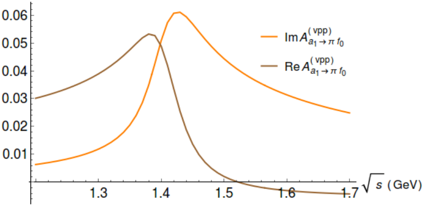


- Spin-Parity of particles.
- Width of $K^\star$



If one fixes mass of f_0, i.e. $p^2_{f_0} = m^2_{f_0}$, then only $p_0^2 = s$ is variable.

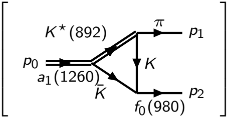
$+$ finite width of $K^\star$



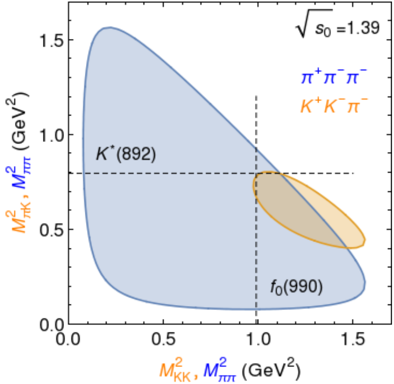
Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0\pi$

For the realistic decay, the amplitude is similar to the scalar case.

$\mathbb{M}^{(\mathrm{vpp})}_{a_1\to\pi f_0} =$

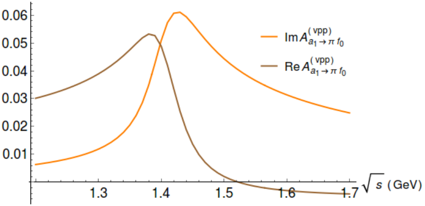


- Spin-Parity of particles.
- Width of $K^\star$



If one fixes mass of f_0, i.e. $p^2_{f_0} = m^2_{f_0}$, then only $p_0^2 = s$ is variable.

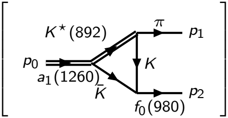
$+$ finite width of $K^\star$



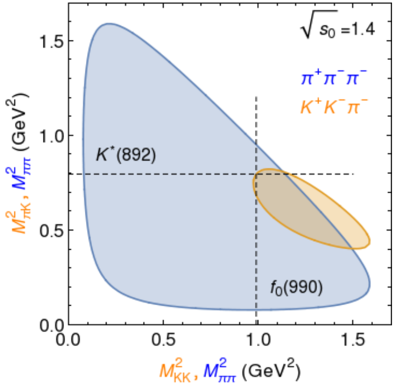
Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0 \pi$

For the realistic decay, the amplitude is similar to the scalar case.

$\mathbb{M}^{(\mathrm{vpp})}_{a_1 \to \pi f_0} =$

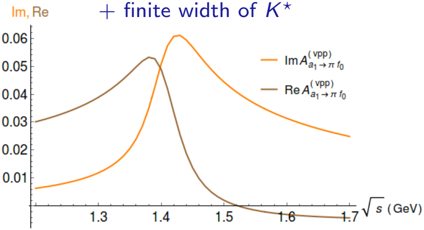


- Spin-Parity of particles.
- Width of $K^\star$



If one fixes mass of f_0, i.e. $p_{f_0}^2 = m_{f_0}^2$, then only $p_0^2 = s$ is variable.

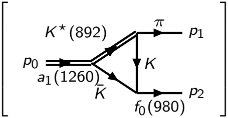
$+$ finite width of $K^\star$



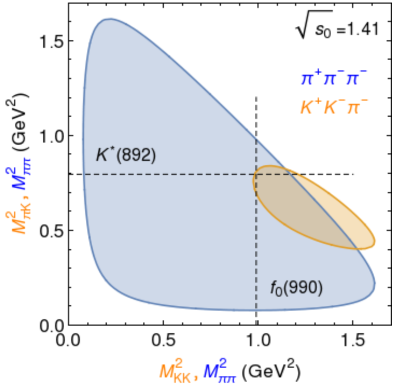
Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0\pi$

For the realistic decay, the amplitude is similar to the scalar case.

$\mathbb{M}^{(\mathrm{vpp})}_{a_1\to\pi f_0} =$

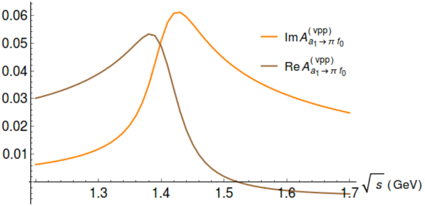


- Spin-Parity of particles.
- Width of $K^\star$



If one fixes mass of f_0, i.e. $p_{f_0}^2 = m_{f_0}^2$, then only $p_0^2 = s$ is variable.

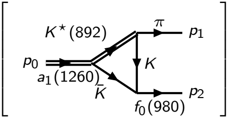
$+$ finite width of $K^\star$



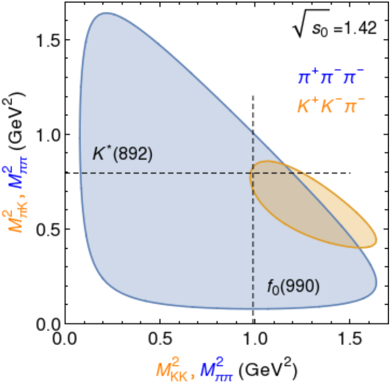
Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0\pi$

For the realistic decay, the amplitude is similar to the scalar case.

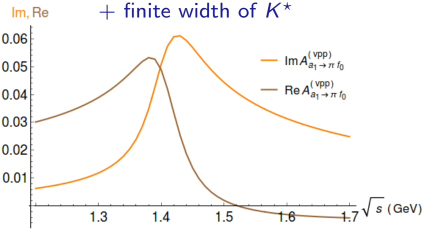
$\mathbb{M}^{(\mathrm{vpp})}_{a_1\to\pi f_0} =$



- Spin-Parity of particles.
- Width of $K^\star$



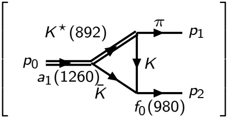
If one fixes mass of f_0, i.e. $p^2_{f_0} = m^2_{f_0}$, then only $p_0^2 = s$ is variable.



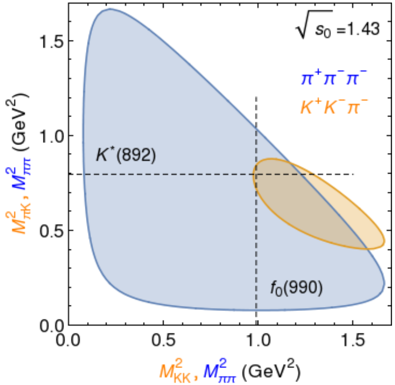
Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0 \pi$

For the realistic decay, the amplitude is similar to the scalar case.

$$\mathbb{M}^{(\mathrm{vpp})}_{a_1 \to \pi f_0} = \left[\quad \right]$$

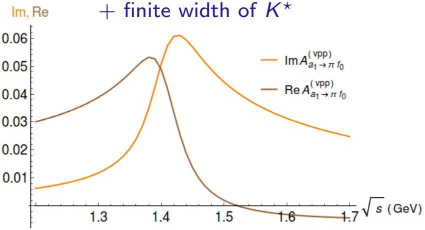


- Spin-Parity of particles.
- Width of $K^\star$



If one fixes mass of f_0, i.e. $p^2_{f_0} = m^2_{f_0}$, then only $p_0^2 = s$ is variable.

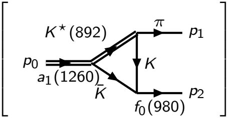
\+ finite width of $K^\star$



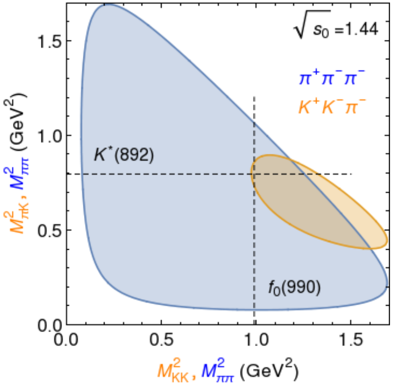
Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0\pi$

For the realistic decay, the amplitude is similar to the scalar case.

$\mathbb{M}^{(\mathrm{vpp})}_{a_1 \to \pi f_0} =$

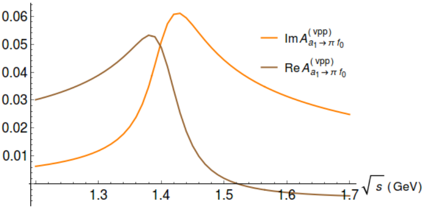


- Spin-Parity of particles.
- Width of $K^\star$



If one fixes mass of f_0, i.e. $p^2_{f_0} = m^2_{f_0}$, then only $p_0^2 = s$ is variable.

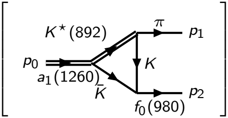
$+$ finite width of $K^\star$



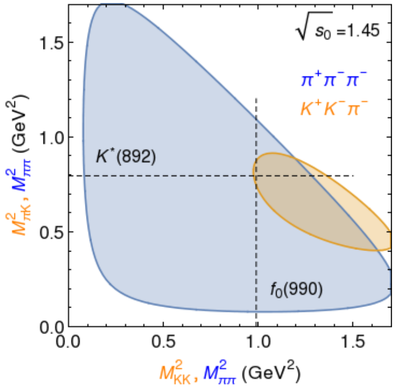
Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0\pi$

For the realistic decay, the amplitude is similar to the scalar case.

$\mathbb{M}^{(\mathrm{vpp})}_{a_1\to\pi f_0} =$

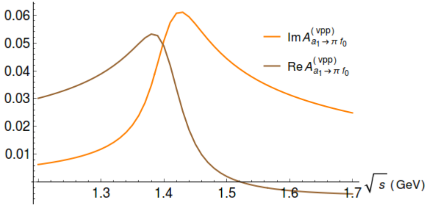


- Spin-Parity of particles.
- Width of $K^\star$



If one fixes mass of f_0, i.e. $p^2_{f_0} = m^2_{f_0}$, then only $p_0^2 = s$ is variable.

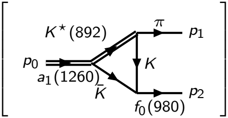
$+$ finite width of $K^\star$



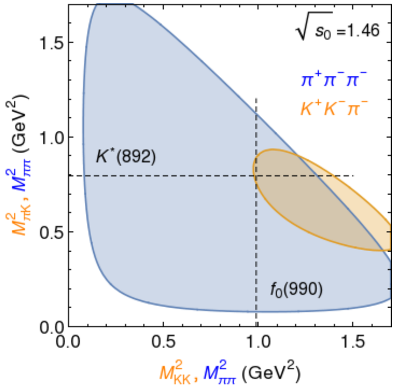
Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0 \pi$

For the realistic decay, the amplitude is similar to the scalar case.

$\mathbb{M}^{(\mathrm{vpp})}_{a_1 \to \pi f_0} =$

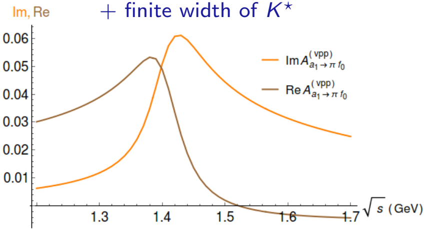


- Spin-Parity of particles.
- Width of $K^\star$



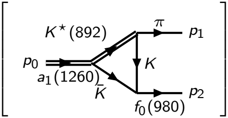
If one fixes mass of f_0, i.e. $p^2_{f_0} = m^2_{f_0}$, then only $p_0^2 = s$ is variable.

$+$ finite width of $K^\star$

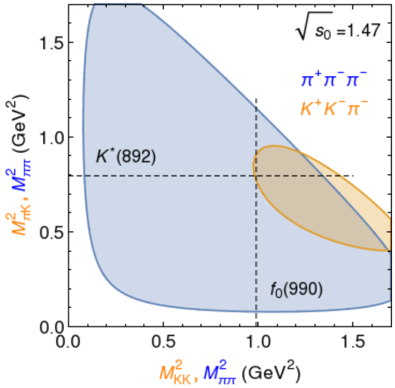


Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0\pi$

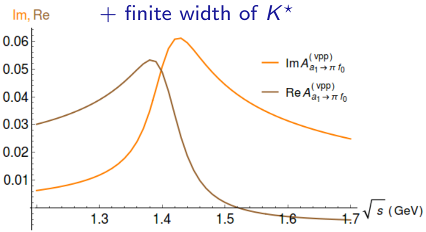
For the realistic decay, the amplitude is similar to the scalar case.

$\mathbb{M}^{(\mathrm{vpp})}_{a_1 \to \pi f_0} =$ 

- Spin-Parity of particles.
- Width of $K^\star$

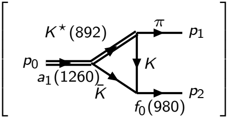


If one fixes mass of f_0, i.e. $p^2_{f_0} = m^2_{f_0}$, then only $p_0^2 = s$ is variable.

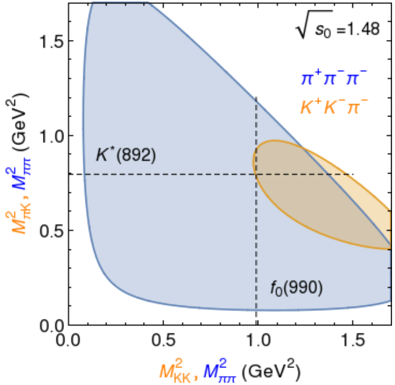


Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0\pi$

For the realistic decay, the amplitude is similar to the scalar case.

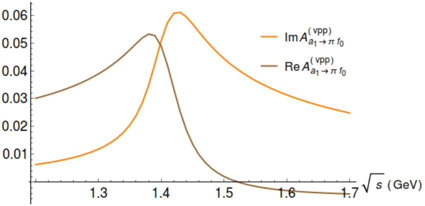
$\mathbb{M}^{(\text{vpp})}_{a_1\to\pi f_0} =$ 

- Spin-Parity of particles.
- Width of $K^\star$



If one fixes mass of f_0, i.e. $p^2_{f_0} = m^2_{f_0}$, then only $p_0^2 = s$ is variable.

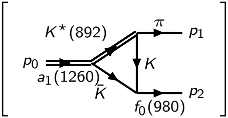
\+ finite width of $K^\star$



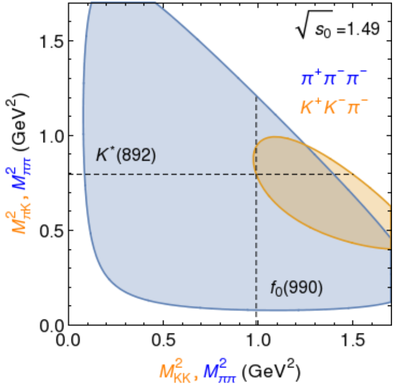
Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0 \pi$

For the realistic decay, the amplitude is similar to the scalar case.

$$\mathbb{M}^{(\mathrm{vpp})}_{a_1 \to \pi f_0} = \left[\quad \right]$$

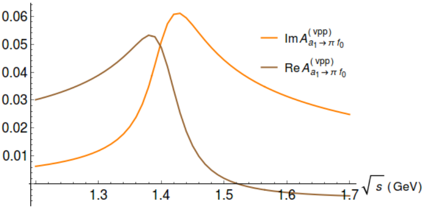


- Spin-Parity of particles.
- Width of $K^\star$



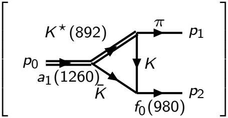
If one fixes mass of f_0, i.e. $p_{f_0}^2 = m_{f_0}^2$, then only $p_0^2 = s$ is variable.

+ finite width of $K^\star$

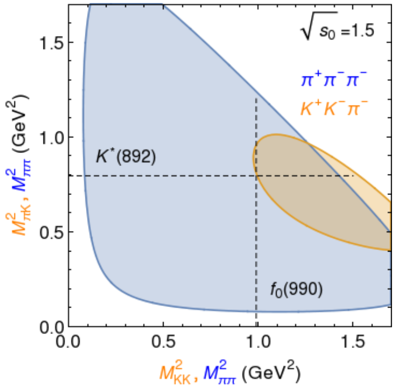


Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0 \pi$

For the realistic decay, the amplitude is similar to the scalar case.

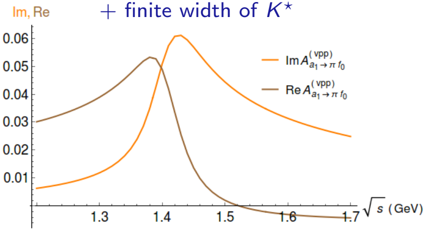
$\mathbb{M}^{(\mathrm{vpp})}_{a_1 \to \pi f_0} =$ 

- Spin-Parity of particles.
- Width of $K^\star$



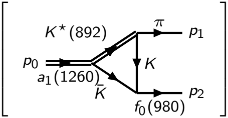
If one fixes mass of f_0, i.e. $p^2_{f_0} = m^2_{f_0}$, then only $p_0^2 = s$ is variable.

$+$ finite width of $K^\star$

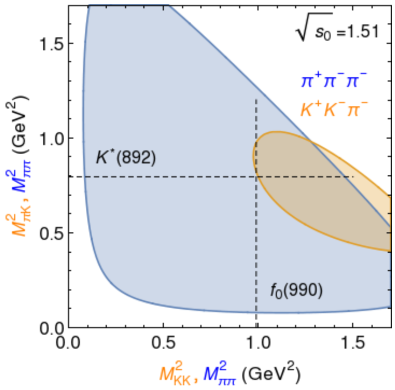


Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0 \pi$

For the realistic decay, the amplitude is similar to the scalar case.

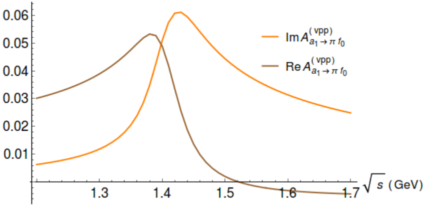
$\mathbb{M}^{(\text{vpp})}_{a_1 \to \pi f_0} =$ 

- Spin-Parity of particles.
- Width of $K^\star$



If one fixes mass of f_0, i.e. $p^2_{f_0} = m^2_{f_0}$, then only $p_0^2 = s$ is variable.

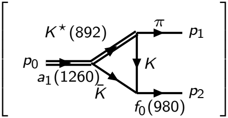
$+$ finite width of $K^\star$



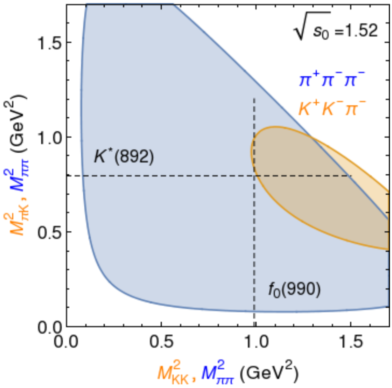
Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0 \pi$

For the realistic decay, the amplitude is similar to the scalar case.

$\mathbb{M}^{(\mathrm{vpp})}_{a_1 \to \pi f_0} =$

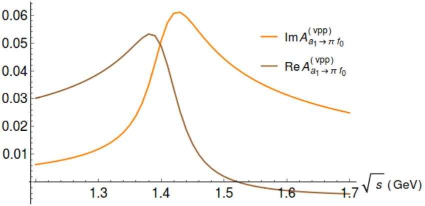


- Spin-Parity of particles.
- Width of $K^\star$

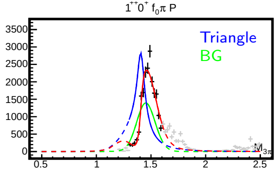


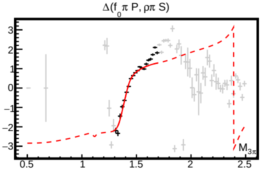
If one fixes mass of f_0, i.e. $p_{f_0}^2 = m_{f_0}^2$, then only $p_0^2 = s$ is variable.

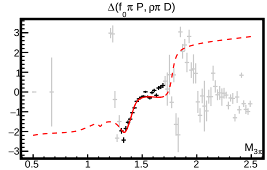
$+$ finite width of $K^\star$



Fit with one triangle amplitude

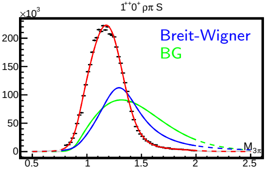


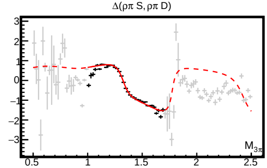


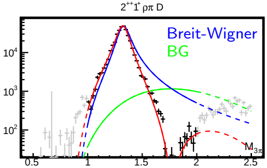


Very preliminary.

One t' bin



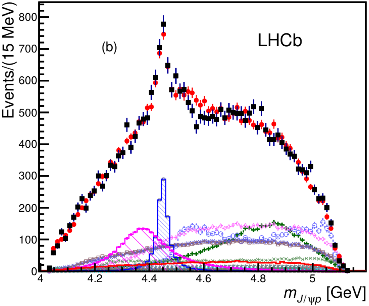


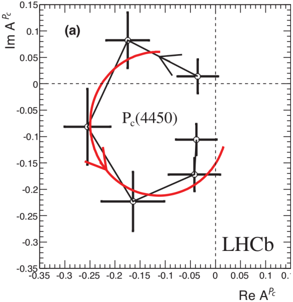


LHCb analysis and observation of $J/\Psi\, p$ peak

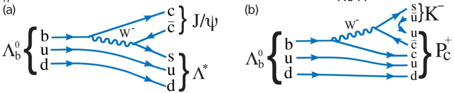
An extensive analysis has been done. Several isobar $\Lambda(\Sigma)$ with different LS couplings (up to 6) were used in $K\,p$ channel and the model is not able to reproduce the peak in $J/\Psi\, p$ projection.

[LHCb collaboration. Phys. Lett. (2015)]



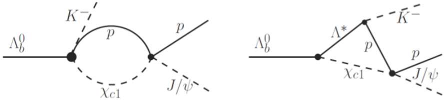


- Prominent peak and phase.
- Quark content $uudc\bar{c}$.

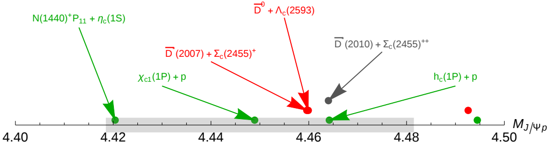


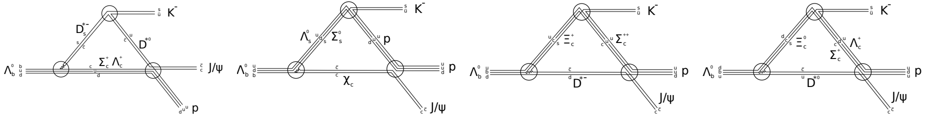
Relevant thresholds

- [F.K. Guo, U.-G. Meissner *et al*, arXiv: 1507.04950]



- [X.-H. Liu, Q. Wang *et al*, arXiv: 1507.05359]
- [M. Mikhasenko, arXiv: 1507.06552]

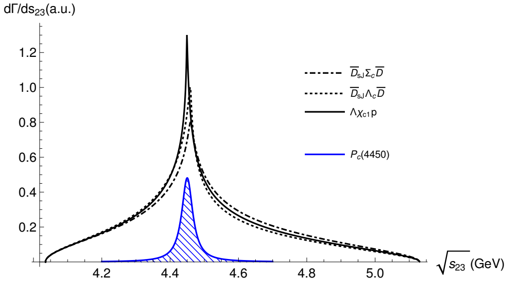


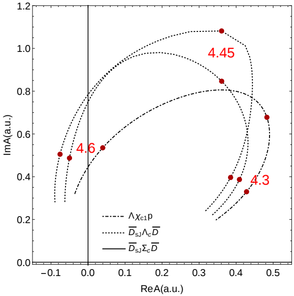


Mass distribution

Calculate amplitudes for the processes:

- $\Lambda_b \to D_{sJ}(2860)\,\Lambda_c(2593) \to K^-\,\bar{D}^0\,\Lambda_c \to K^-\,J/\Psi\,p,$
- $\Lambda_b \to D_{sJ}(3040)\,\Sigma_c^+(2455) \to K^-\,\bar{D}^{\star 0}(2007)\,\Sigma_c^+ \to K^-\,J/\Psi\,p,$
- $\Lambda_b \to \Lambda(1890)\,\chi_{c1} \to K^-\,p\,\chi_{c1} \to K^-\,J/\Psi\,p.$





$$\frac{\mathrm{d}\Gamma_{\Lambda_b\to K^- J/\Psi p}}{\mathrm{d}s_{23}} = \frac{1}{2\sqrt{s_0}}\left|A_{\Lambda_b\to K^- J/\Psi p}\right|^2\frac{\mathrm{d}\Phi_3}{\mathrm{d}s_{23}}$$

Conclusions

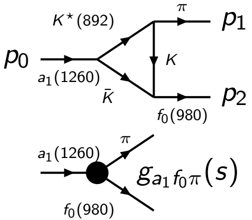
- The **isobar model** is often a good approximation, while in special cases it may lead to the **incorrect interpretation**.
- The final state interaction in the system of three particles can produce an enhancement in the Dalitz plot. The effect of the singularity in the triangle diagram appears as a **peak** with a noticeable **phase motion**. (Possibly, it can explain several XYZ states, pentaquarks candidates).
- The rescattering processes in coupled channel system cause a migration between the systems **changing the three-body dynamics**. (likely $a_1(1420)$ is an example)
- The rescattering series can be taking into account while the two-body interaction is assumed to be known.

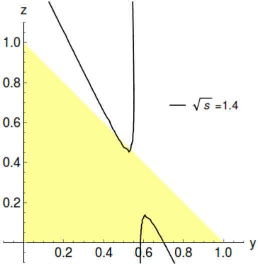
Ongoing studies:

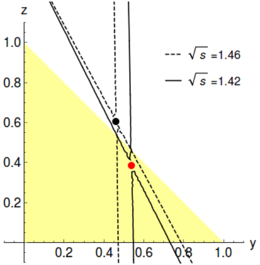
- The general case of the Khuri-Treiman equations (arbitrary quantum numbers), KTA-PWA.
- The rescattering equations for coupled channel problem in application to $\pi\pi\pi/\pi KK$ systems.

Backup slides

The pinch singularity







$$g_{a_1 f_0 \pi}(s|_{s=p_0^2}) = g^3 \int \frac{d^4 k_1}{(2\pi)^4 i} \frac{1}{\Delta_1 \Delta_2 \Delta_3} = \frac{g^3}{16\pi^2} \int_0^1 \mathrm{d}y \int_0^{1-y} \mathrm{d}z \, \frac{1}{D},$$

$$\Delta_i = m_i^2 - k_i^2 - i\epsilon, \quad D = (1-y-z)m_1^2 + y m_2^2 + z m_3^2 - y(1-y-z)p_0^2 - z(1-z-y)p_1^2 - yz p_2^2.$$

If Landau conditions [Nucl. Phys. **13**, 181 (1959)] are satisfied, $g_{a_1 f_0 \pi} \sim \log(s - s_0)$.

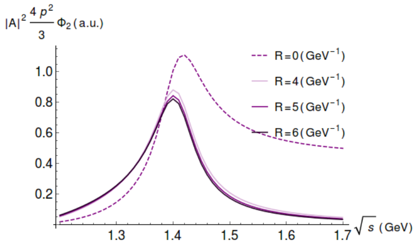
For a **box loop**, $A \sim (s - s0)^{-1/2}$, for **5-leg loop** $A \sim (s - s_0)^{-1}$ (pole).

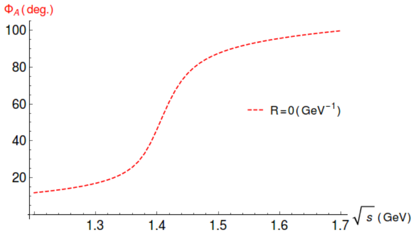
Further corrections to the amplitude

$K^\star(892) \to K\pi$, P-wave decay gives tail to the amplitude.
A left-hand singularity is introduced to correct the amplitude
$g_{K^\star K\pi} \to g_{K^\star K\pi} \times F(k_1)$. k_1 is $K^\star$ four-momentum.

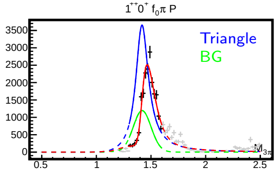
$$F(k_1) = \frac{M^2 - m_{K^\star}^2}{M^2 - k_1^2}, \quad M^2 = (m_\pi + m_K)^2 - \frac{4}{R^2} \ .$$

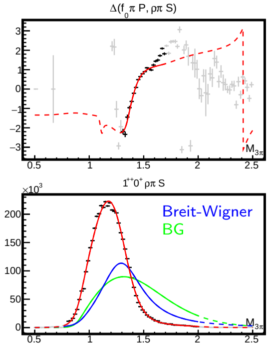
M is position of the left singularity, it corresponds to the size of $K^\star$:
$F \approx (1 + R^2|\vec{p_0}|^2)/(1 + R^2|\vec{p}|^2)$. $|\vec{p}|$ is $K^\star \to K\pi$ break up momentum.

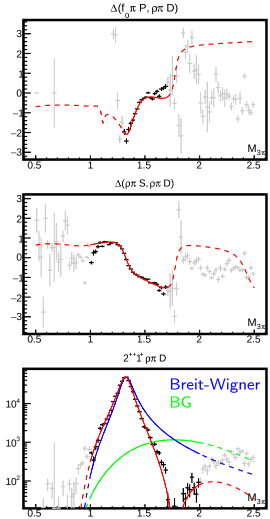




Fit with a Breit-Wigner signal







Very preliminary.

One t' bin

KT: Basic idea

- Amplitude for every sub-channel is the sum of "isobars".

$$A(s,t,u) = A^{(s)} + A^{(t)} + A^{(u)}, \quad A^{(s)}(s,t,u) = \sum a_l^{(s)} P_l(z_s)$$

- Two contributions to the projection on the partial sub-channel amplitude.

$$f_l^{(s)} = \underbrace{a_l^{(s)}}_{\text{isobar}} + \underbrace{b_l^{(s)}}_{\text{projections}} \qquad A(s,t,u) = \sum f_l^{(s)} P_l(z_s)$$

- The projections to the sub-channel from cross sub-channels.

$$b_l^{(s)} = \int \mathrm{d}z_s \left[A^{(t)} + A^{(u)}\right] P_l(z_s)$$

- Unitarity consistency equation.

$$a_l^{(s)} = \underbrace{t_l^{(s)}}_{\text{2b. interaction}} \left(c^{(s)} + \frac{1}{\pi} \int \mathrm{d}s' \, \frac{b_l^{(s)}(s')\rho^{(s)}(s')}{s'-s} \right)$$

KT: solution of equation

We have integral equations for $b_l^{(i)}$, $i \in \{s, t, u\}$. All sub-channels are coupled.

$$\begin{pmatrix} b_l^{(s)} \\ \vdots \\ b_l^{(t)} \\ \vdots \\ b_l^{(u)} \end{pmatrix} = L \times \begin{pmatrix} c_l^{(s)} \\ \vdots \\ c_l^{(t)} \\ \vdots \\ c_l^{(u)} \end{pmatrix} + \underbrace{\hat{K}}_{\text{int.op.}} \begin{pmatrix} b_l^{(s)} \\ \vdots \\ b_l^{(t)} \\ \vdots \\ b_l^{(u)} \end{pmatrix}.$$

If we solve it then our **corrected isobar amplitude** is

$$a_l^{(j)} = \overbrace{t_l^{(j)} (c_l^{(j)}}^{\text{isobar}} + \overbrace{\underbrace{\hat{M}}_{\text{int.op.}} b_l^{(j)}}^{\text{correction to IM}})$$

KT: Input and output

Input

- fixed number of channel(s,t,u) and partial waves $l = 0, 1, 2$;
- elastic two body interaction $t_l^{(j)}$ parameterizated by
  - a phase shift (Omnés approach),
  - a pole position (Breit-Wigner approach);
- couplings as parameters $c_l^{(j)}$

Output: corrected isobar amplitudes $a_l^{(j)}$.

$$A(s,t,u) = A^{(s)} + A^{(t)} + A^{(u)}, \quad A^{(s)}(s,t,u) = \sum a_l^{(s)} P_l(z_s)$$

$\Rightarrow$ for every set of parameters need to solve integral equations. (Every step of the fit) Not good. Too slow.

Pentaquark Dalitz plot