Beyond the thermal model

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Topics

1. Introduction: Thermal equilibrium in nature: CMB; solar spectrum
2. Hadron production in particle and heavy-ion physics

   2.1 Particle production rates at LHC in the thermal model

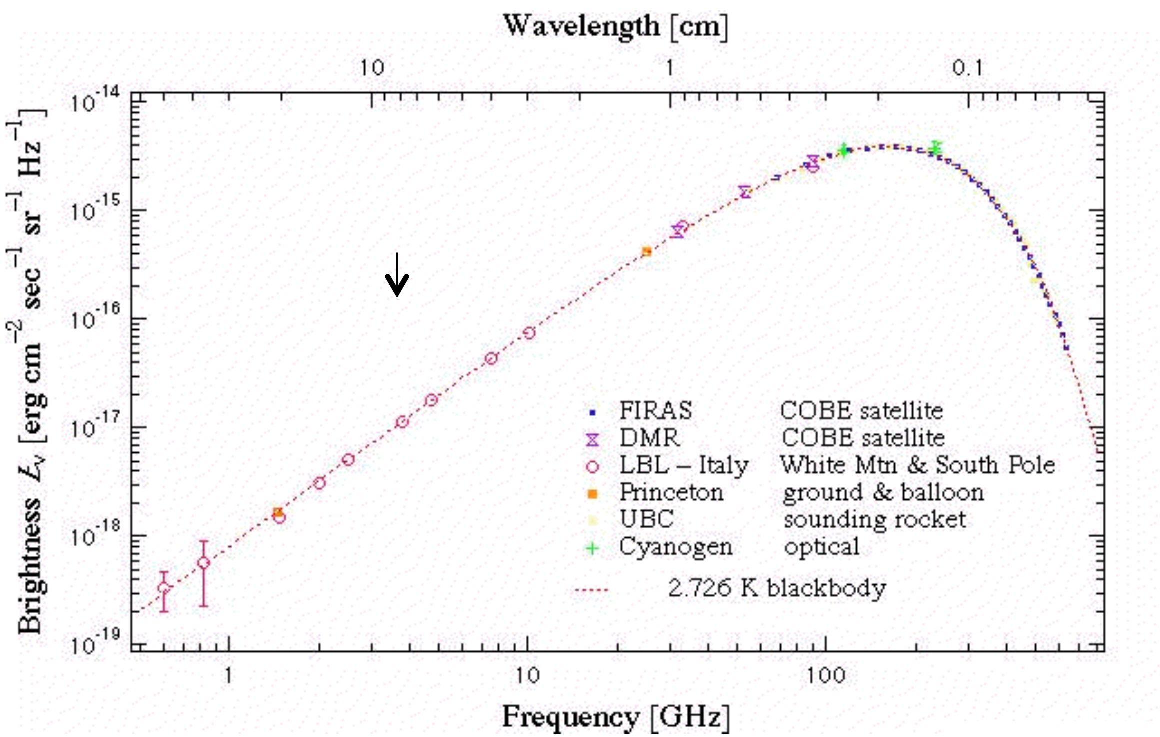
   2.2 Particle production rates vs. distribution functions

   2.3 Transverse energy distributions

   2.4 Stopping as an example of nonequilibrium distributions

   2.5 Pseudorapidity distributions of produced particles in a nonequilibrium-statistical model (RDM)
3. Conclusions

1. Introduction: Thermal equilibrium in nature
Spectrum of the cosmic microwave background (CMB)



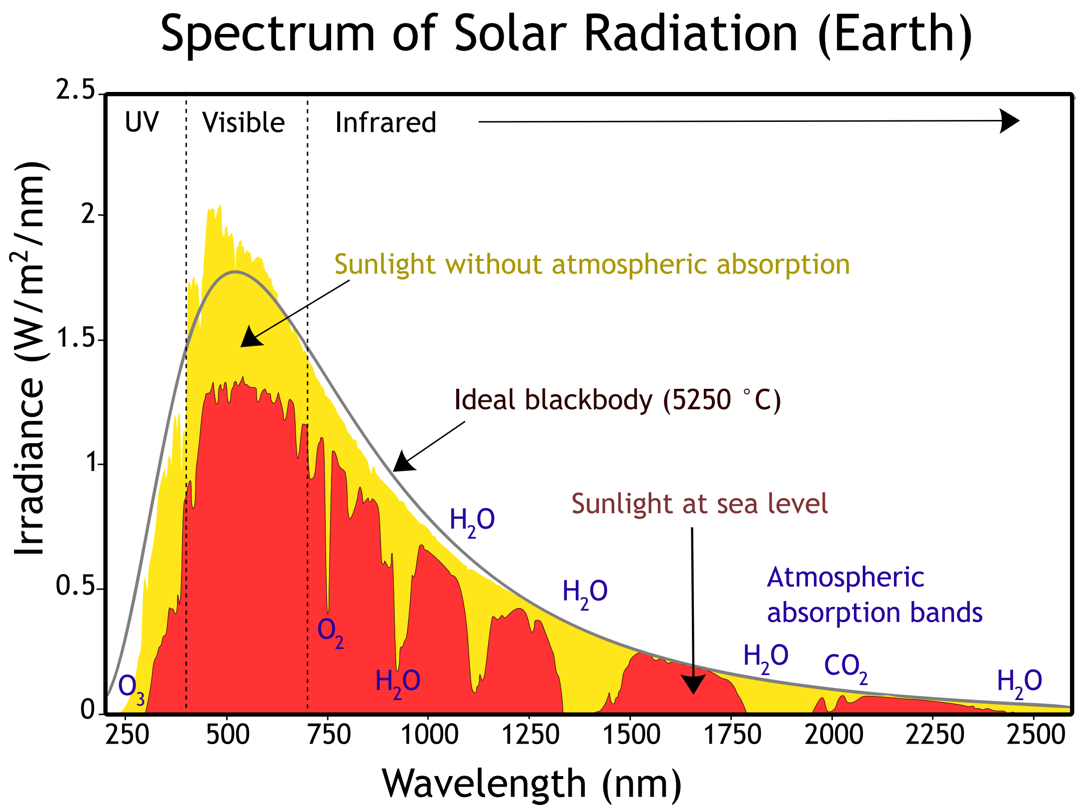
- Discovered by Arno Penzias und Robert Wilson 1964/5 @ 4.1 GHz (Physics Nobel prize 1978)
- The temperature dopped to 2,73 Kelvin due to expansion
- The energy spektrum is a Planck-distribution:
- **The CMB is in thermal equilibrium (apart from the fluctuations)**

$$U_\nu^o(\nu, T)\, d\nu = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{\left(\frac{h\nu}{kT}\right)} - 1} d\nu$$

The most precise blackbody spectrum realized in nature. Evidence for thermal equilibrium arises from the full distribution function, not from the value at a single frequency, or from the integral.
Single source: big bang/ recombination

source: COBE-Collaboration, 1992

The solar radiation …



Source: Wikipedia

...is also close to a blackbody spectrum, it is in thermal equilibrium.

It has a single source: the sun.

Note however, that many astrophysical processes are non-equilibrium – examples are jets in active galaxies or accretion discs of stellar objects

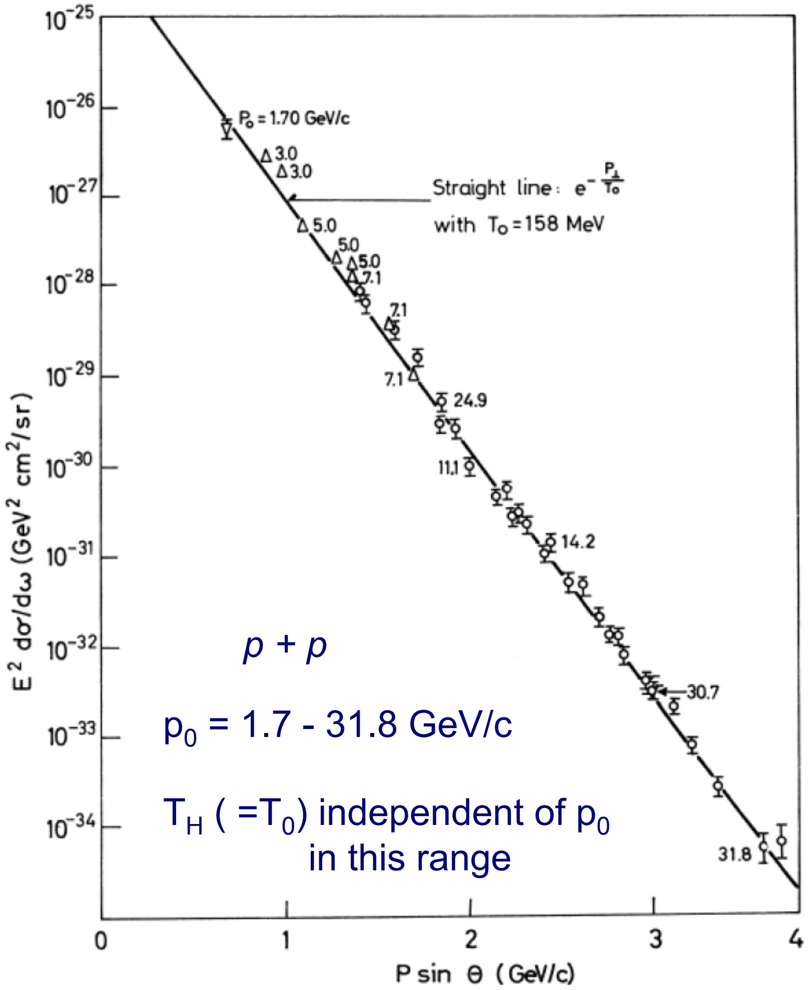
2. Thermalization in particle physics at the Large Hadron Collider

p+p @ 7,8,13,(14) TeV

p+Pb @ 5.02 TeV 2012/13

Pb+Pb @ 2.76 TeV 2011/12
@ 5.02 TeV Nov./ Dec. 2015
(design energy 5.52 TeV)

2.1 Thermal model: R. Hagedorn



SUPPLEMENTO AL NUOVO CIMENTO
VOLUME III N. 2, 1965

Statistical Thermodynamics of Strong Interactions at High Energies.

R. HAGEDORN
CERN - Geneva

(ricevuto il 12 Marzo 1965)

$$\mathbf{dN/dp_T \propto \exp\left(-\frac{\sqrt{p_T^2+m^2}}{T_0}\right)}$$

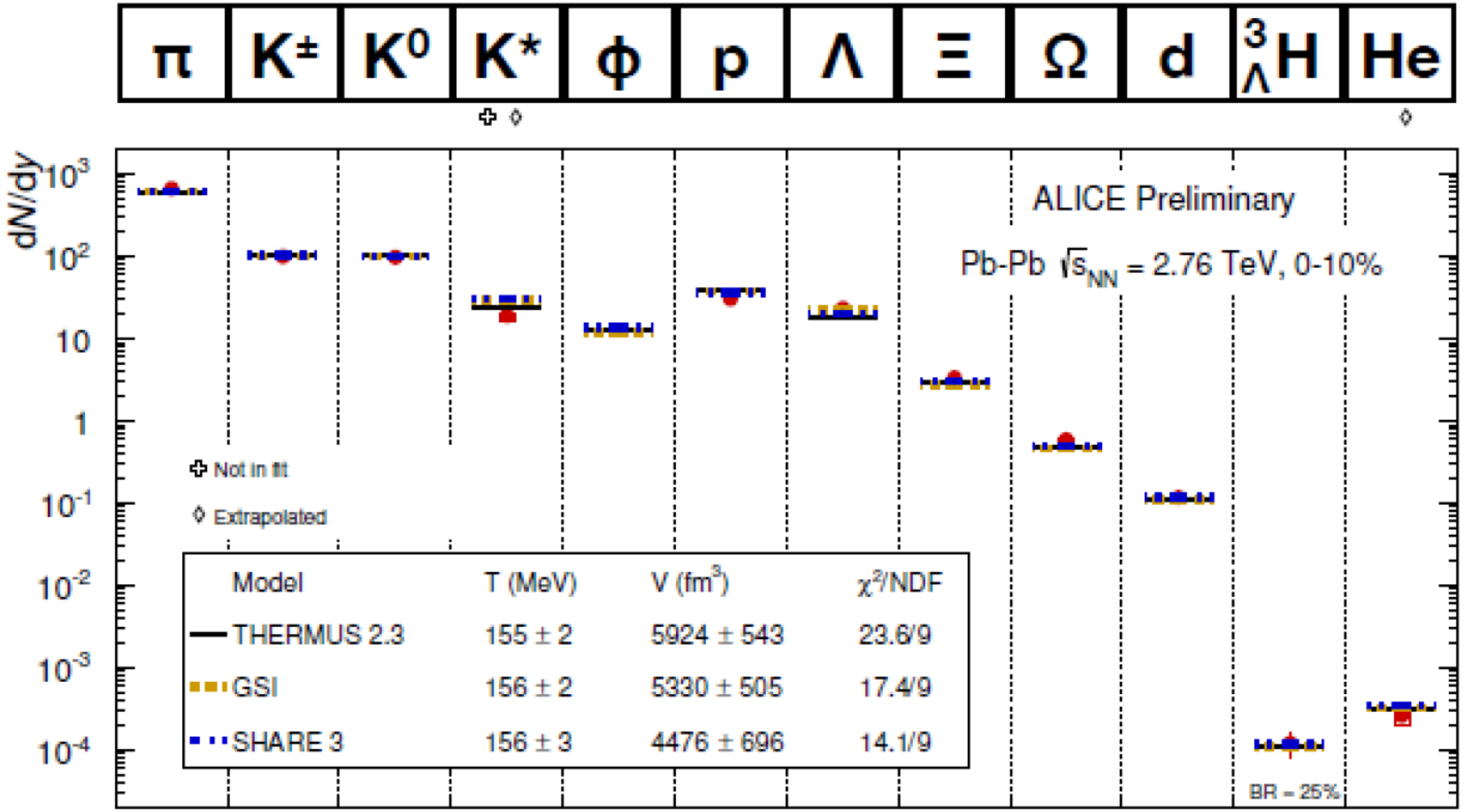
with T_0 = 158 MeV (later 160 ± 5 MeV)

"…when T_0 is approached it becomes easier to create new particles than to increase the temperature."

$$w(p_\perp) \approx \mathrm{const}\cdot p_\perp \sqrt{T_0\sqrt{p_\perp^2+m^2}}\,\exp\left[-\frac{1}{T_0}\sqrt{p_\perp^2+m^2}\right] \to c\cdot p_\perp^{\frac{3}{2}}\exp\left[-\frac{p_\perp}{T_0}\right]$$

Indications for thermal equilibrium in PbPb @ LHC

.. are often inferred from (relative or absolute) particle production rates which are integrated over p_T



Source: R. Bellwied, for the ALICE Collab., NN2015 Catania

2.2 Particle production rates vs. distribution functions

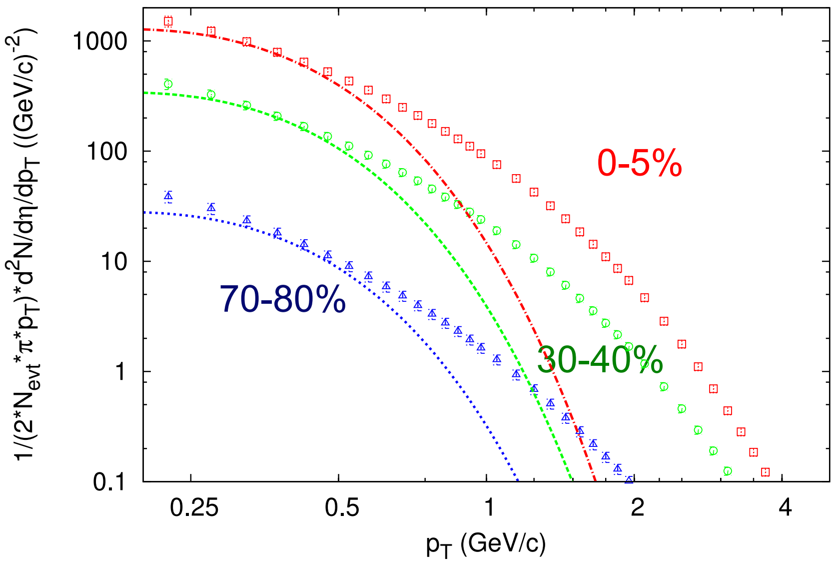
- The thermal model fits can be improved by e.g. excluding pions and protons (then the χ^2/ndf improves by a factor of 2, and the temperature for π / p becomes 148 MeV rather than 156 MeV): there is room for extra features, such as changing temperatures for different hadron species etc., and nonequilibrium effects.
- The agreement of measured production rates and thermal model results for suitable choices of the three parameters (T, μ, V) is a necessary, but not a sufficient condition for the system to be in thermal equilibrium.
- A necessary and sufficient condition for statistical equilibrium would be the agreement of the measured particle production distribution functions with thermal model predictions.

Deviations from thermal distribution functions in p_T and η (or y) indicate the necessity to *go beyond the thermal model*.

2.3 Transverse momentum distributions in 2.76 TeV PbPb

A purely thermal model deviates strongly from the data for produced charged hadrons *although the integrated production rates appear to be thermal*

(a much better agreement is achieved by including collective expansion e.g. in the blast wave model)



Jüttner distribution of a relativistic Maxwell gas

$$\gamma(p_T) = \sqrt{1 + (p_T/m)^2}$$

$$f(p_T) = \frac{1}{4\pi m^2 T K_2(m/T)} \exp\left(-\frac{\gamma(p_T)m}{T}\right)$$

Data: ALICE Collab., PLB 720, 52 (2013); Model: F. Jüttner, Annalen Phys. 34, 856 (1911)
Calc.: T. Kind & GW, unpublished

Produced charged hadron distributions in 2.76 TeV PbPb

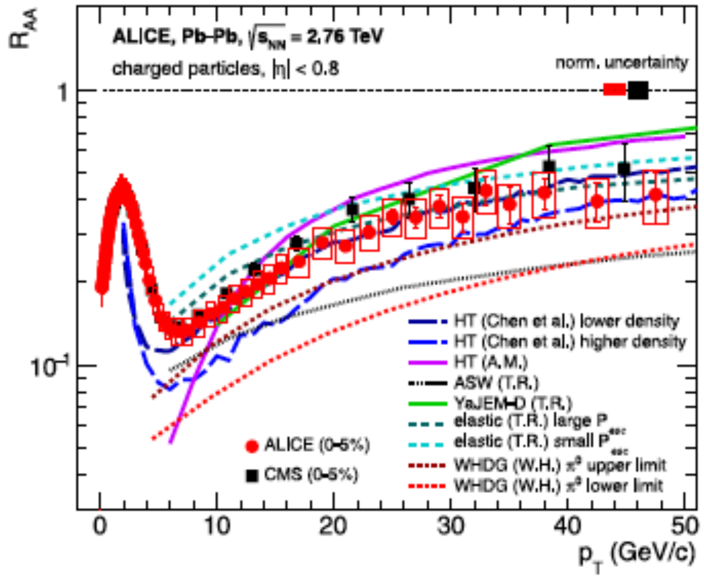
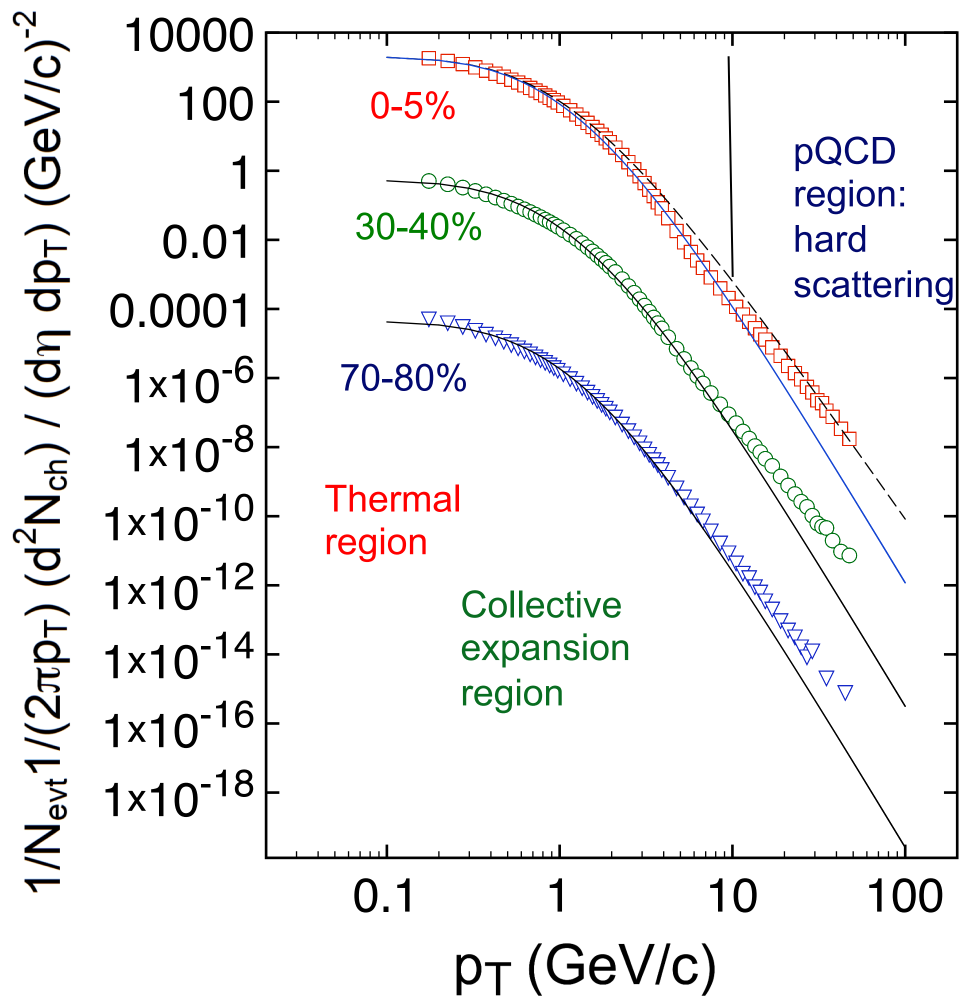


Fig. 4. Nuclear modification factor R_{AA} of charged particles measured by ALICE in the most central Pb-Pb collisions (0–5%) in comparison to results from CMS [25] and model calculations [26–31]. The boxes around the data denote p_T-dependent systematic uncertainties. For CMS statistical and systematic uncertainties on R_{AA} are added in quadrature. The systematic uncertainties on the normalization which are related to $\langle T_{AA}\rangle$ and the normalization of the pp data are added in quadrature and shown as boxes at $R_{AA} = 1$ (the right-most is for CMS).

Fits with 'non-extensivity parameter' q =1.12 that implicitly accounts for collective expansion

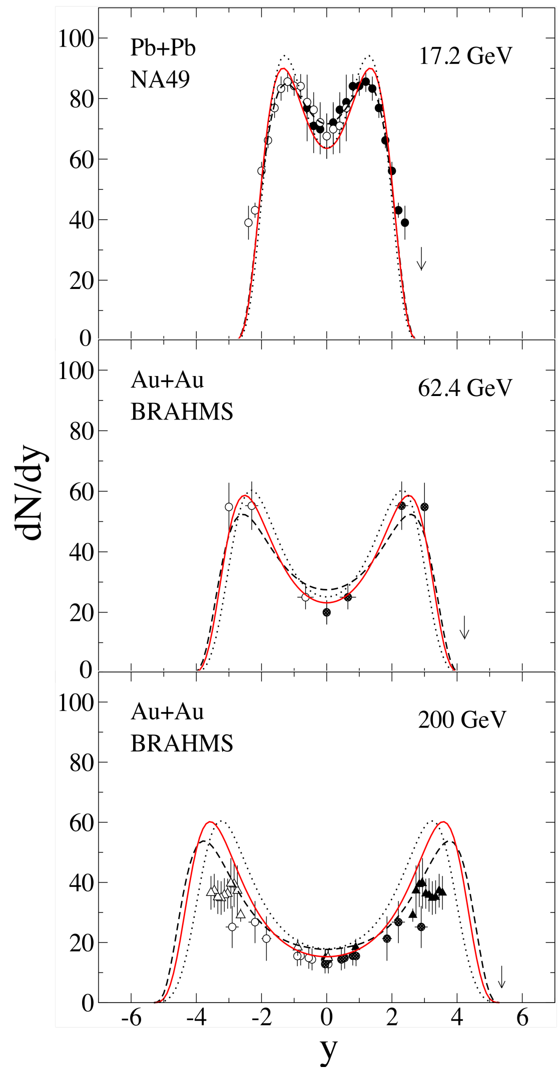


Thermal distribution with collective expansion valid up to $p_T \approx 8$ GeV/c; beyond that, pQCD

Data: ALICE Collab.PLB 720, 52 (2013); rhs. calc. GW

2.4 Stopping as an example of nonequilibrium rapidity distributions

Net-baryon dN/dy distributions at SPS, RHIC, and LHC



$$Q_s^2(x) \sim A^{1/3} x^{-\lambda}, \lambda \sim 0.3$$

- Central (0-5%) Pb+Pb (SPS) and Au+Au (RHIC) Collisions
- A larger gluon saturation scale produces more baryon stopping, as does a larger value of A.
- The saturation scale is $Q_s^2(x) = A^{1/3} Q_0^2 x^{-\lambda}$

Q_0^2 = 0.07 GeV^2

Calculations are in a QCD-inspired model
Y. Mehtar-Tani and GW, Phys. Rev. Lett. 102,182301 (2009).

An equilibrium distribution would be ~Gaussian in y-space

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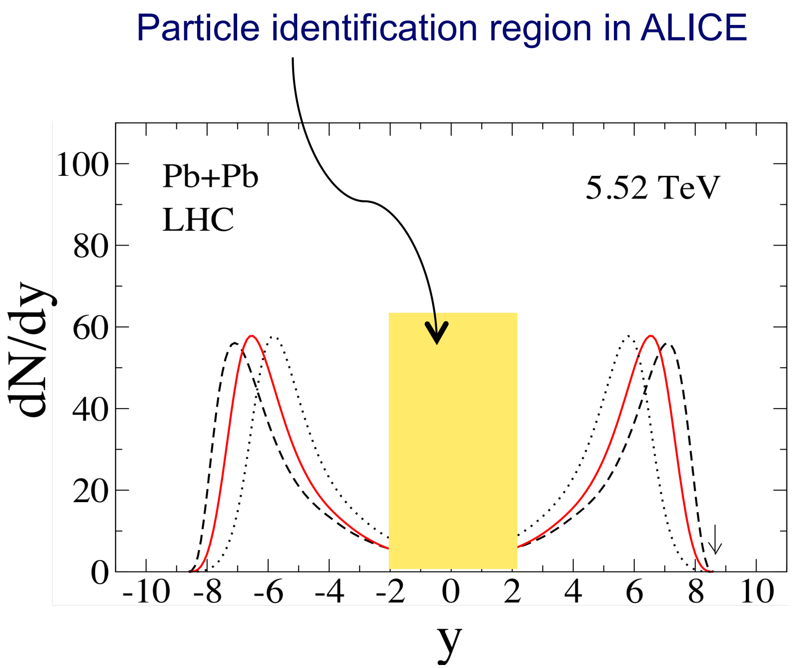
Net-baryon rapidity distributions at LHC: prediction



Y. Mehtar-Tani and GW
Phys. Rev. Lett. 102,182301 (2009)

- Central (0-5%) Pb+Pb collisions, $y_{beam} = 8.68$
- A larger gluon saturation scale produces more baryon stopping; the fragmentation peak position is sensitive to λ (dashed λ= 0, solid λ = 0.15, dotted λ= 0.3)
- The midrapidity value of the net-baryon distribution is small, but finite: dN/dy (y = 0) ≈ 4. The **total yield** is normalized to the number of baryon participants, $N_B \approx 357$.

Measurements with particle identification will be confined to the yellow region for the next years.

The fragmentation peaks seen in stopping are also relevant for particle production, where a central gluonic source arises that cancels out in the net baryon case: 3 sources for produced hadrons.

2.5 Produced particles: Relativistic Diffusion Model (RDM)

$$\frac{\partial}{\partial t}R(y,t) = -\frac{\partial}{\partial y}\Big[J(y)R(y,t)\Big] + D_y\frac{\partial^2}{\partial y^2}[R(y,t)]^{2-q}$$

R (y,t) Rapidity distribution function. The standard linear Fokker-Planck equation corresponds to q = 1, and a linear drift function. For the three components k = 1,2,3 of the rapidity distribution (fragmentation plus central sources),

$$\frac{\partial}{\partial t}R_k(y,t) = -\frac{1}{\tau_y}\frac{\partial}{\partial y}\Big[(y_{eq}-y)\cdot R_k(y,t)\Big] + D_y^k\frac{\partial^2}{\partial y^2}R_k(y,t)$$

Linear drift term with relaxation time τ_y Diffusion term, D_y=const.

Relaxation time and diffusion coefficient are related through a dissipation-fluctuation theorem. The broadening is enhanced due to collective expansion.

$$< y_{1,2}(t) >= y_{eq}[1-\exp(-t/\tau_y)] \mp y_{max}\exp(-t/\tau_y)$$ mean value

$$\sigma^2_{1,2,eq}(t) = D_y^{1,2,eq}\tau_y[1-\exp(-2t/\tau_y)]$$ variance

Linear Model: G. Wolschin, Eur. Phys. J. A5, 85 (1999); with 3 sources: Phys. Lett. B 569, 67 (2003); PLB 698, 411 (2011)

Diffusion of produced particles in η-space

Pseudorapidity distributions of produced particles are obtained through the Jacobian transformation

$$\frac{dN}{d\eta} = \frac{dN}{dy}\frac{dy}{d\eta} = \frac{p}{E}\frac{dN}{dy} \simeq J(\eta, \langle m\rangle/\langle p_T\rangle)\frac{dN}{dy}$$

GW, J.Phys. G40, 045104 (2013)
D. Roehrscheid & GW, Phys. Rev. C86, 024902 (2012)

$$J(\eta, \langle m\rangle/\langle p_T\rangle) = \cosh(\eta)\cdot \left[1 + (\langle m\rangle/\langle p_T\rangle)^2 + \sinh^2(\eta)\right]^{-1/2}.$$

with the rapidity distribution
in the three-sources model

$$\frac{dN_{ch}(y, t = \tau_{int})}{dy} = N_{ch}^1 R_1(y, \tau_{int}) + N_{ch}^2 R_2(y, \tau_{int}) + N_{ch}^{eq} R_{eq}(y, \tau_{int}).$$

and the rapidity

$$y = 0.5 \cdot \ln((E + p)/(E - p))$$

$$\eta = -\ln\left[\tan(\theta/2)\right]$$

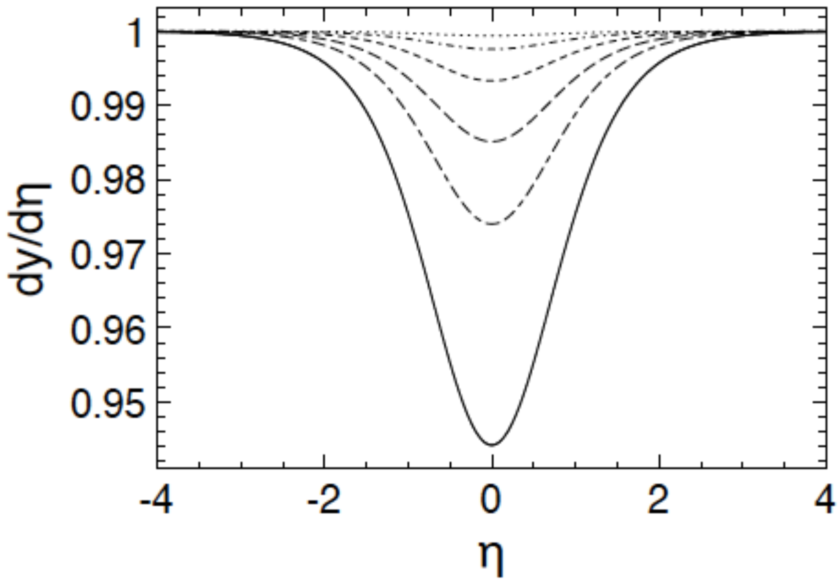
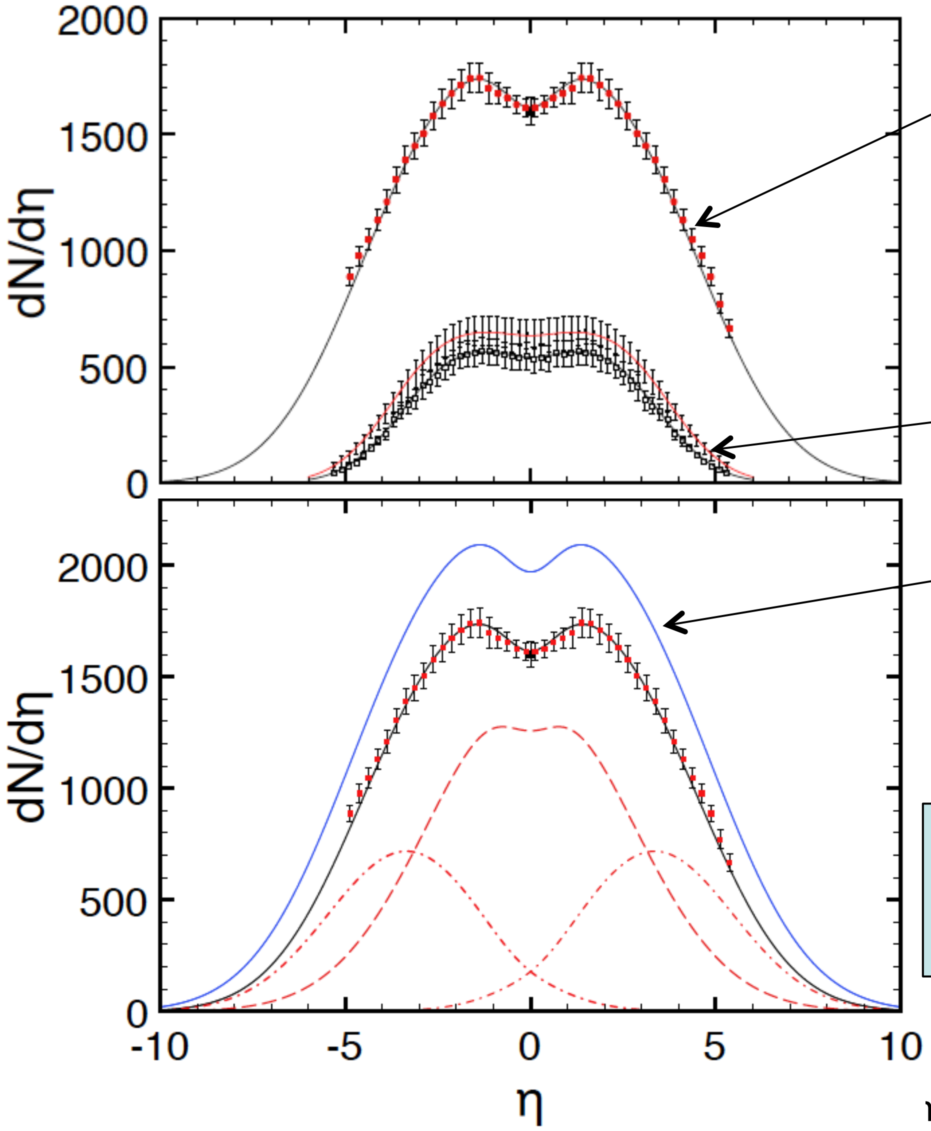


Figure 1: The Jacobian $dy/d\eta$ for $< m >= m_\pi$ and average transverse momenta (bottom to top) $< p_T >= 0.4, 0.6, 0.8, 1.2, 2$ and 4 GeV/c.

Comparing data with the RDM calculation for produced charged hadrons



Central PbPb @ 2.76 TeV
(ALICE data; RDM calc.):
3 sources

RHIC data
(PHOBOS)
130 and 200 GeV

Prediction at 5.52 TeV

3 sources: 2 symmetric fragmentation sources
1 midrapidity gluon-gluon source, modified by the Jacobian
RDM parameters determined in χ^2 minimizations

For t -> ∞ the 3 sources would merge into a single thermal equilibrium distribution

GW, J. Phys. G40, 045104 (2013)

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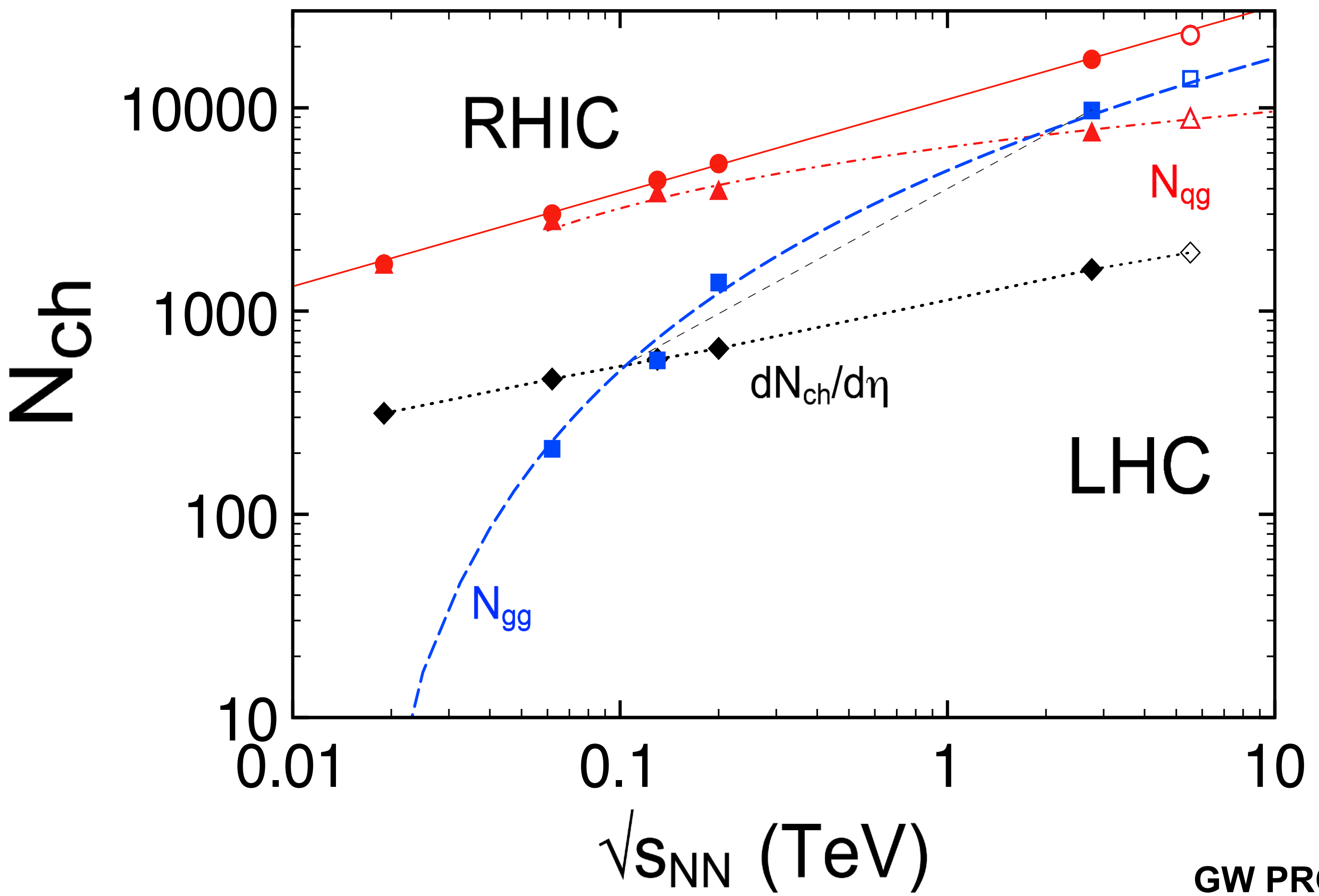
Data:
[1] ALICE collab., PRL 105, 252301 (2010)
[2] ALICE collab., PRL 106, 032301 (2011)
[3] B.B. Back et al.,PHOBOS coll., PRL 87, 102303 (2001); PRL 91, 052303 (2003); PRC 83, 024913 (2011)

Parameters of the 3-sources RDM at RHIC and LHC energies

Table 1. Three-sources RDM-parameters τ_{int}/τ_y, $\Gamma_{1,2}$, Γ_{gg}, and N_{gg}. N_{ch}^{1+2} is the total charged-particle number in the fragmentation sources, N_{gg} the number of charged particles produced in the central source. Results for $\langle y_{1,2}\rangle$ are calculated from y_{beam} and τ_{int}/τ_y. Values are shown for 0–5% PbPb at LHC energies of 2.76 and 5.52 TeV in the lower two lines, with results at 2.76 TeV from a χ^2-minimization with respect to the preliminary ALICE data [2], and using limited fragmentation as constraint. Corresponding parameters for 0–6% AuAu at RHIC energies are given for comparison in the upper four lines based on PHOBOS results [1]. Parameters at 5.52 TeV denoted by * are extrapolated. Experimental midrapidity values (last column) are from PHOBOS [1] for $|\eta| < 1$, 0–6% at RHIC energies and from ALICE [13] for $|\eta| < 0.5$, 0–5% at 2.76 TeV.

$\sqrt{s_{NN}}$ (TeV)	y_{beam}	τ_{int}/τ_y	$\langle y_{1,2}\rangle$	$\Gamma_{1,2}$	Γ_{gg}	N_{ch}^{1+2}	N_{gg}	$\frac{dN}{d\eta}\|_{\eta\simeq 0}$
0.019	$\mp$3.04	0.97	$\mp$1.16	2.83	0	1704	–	314 ± 23 [1]
0.062	$\mp$4.20	0.89	$\mp$1.72	3.24	2.05	2793	210	463 ± 34 [1]
0.13	$\mp$4.93	0.89	$\mp$2.02	3.43	2.46	3826	572	579 ± 23 [1]
0.20	$\mp$5.36	0.82	$\mp$2.40	3.48	3.28	3933	1382	655 ± 49 [1]
2.76	$\mp$7.99	0.87	$\mp$3.34	4.99	6.24	7624	9703	1601 ± 60 [13]
5.52	$\mp$8.68	0.85*	$\mp$3.70	5.16*	7.21*	8889*	13903*	1940*

Content of the sources as function of energy



Centrality 0-5%

(RHIC 0-6%)

Fragmentation sources

$$\propto \ln(s_{NN}/s_0)$$

Central source

$$\propto \ln^3(s_{NN}/s_0)$$

GW PRC 91, 014905 (2015)

Content of the central source as function of energy

Rise of the cross section with energy in the central distribution is driven by the growth of the gluon density at small *x*, but is "..suppressed by the quantum-classical interaction from the dense medium"

$$\frac{dN}{d\eta}|^{gg}_{\eta\simeq 0} \propto \ln^2(s_{NN}/s_0)$$

cf. M.F. Cheung and C.B. Chiu, arXiv:1111.6957; analogous to Froissart bound of the total cross section (approximate solution to leading order)

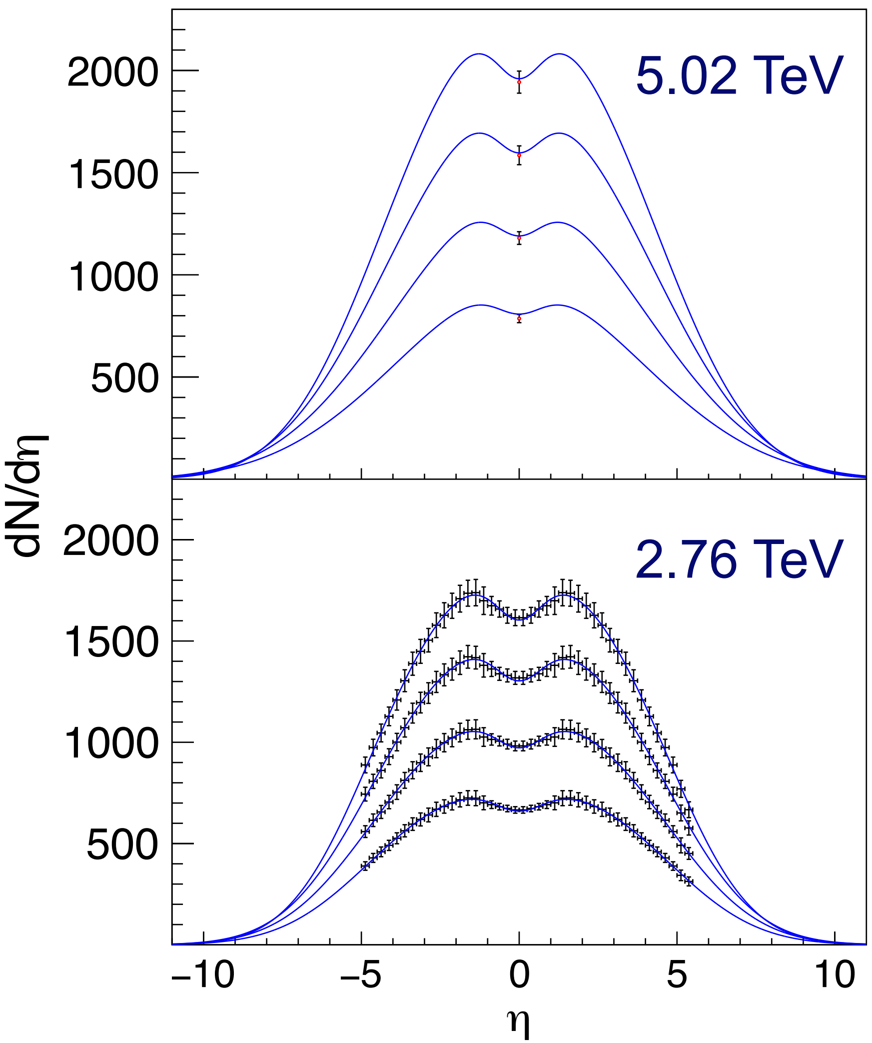
Total particle content in the *gg*-source by integrating over η:

$$\int_{-y_{\rm beam}}^{y_{\rm beam}} \frac{dN}{d\eta}|^{gg}_{\eta\simeq 0} d\eta \propto \ln^3(s_{NN}/s_0)$$

since $y_{\rm beam} = \ln(\sqrt{s_{NN}}/m_p)$

$$\rightarrow N^{gg}_{tot} \propto \ln^3(s_{NN}/s_0)$$

as inferred from the phenomenological analysis: Room for further theoretical development.



Centrality dependence PbPb @ 5.02 TeV

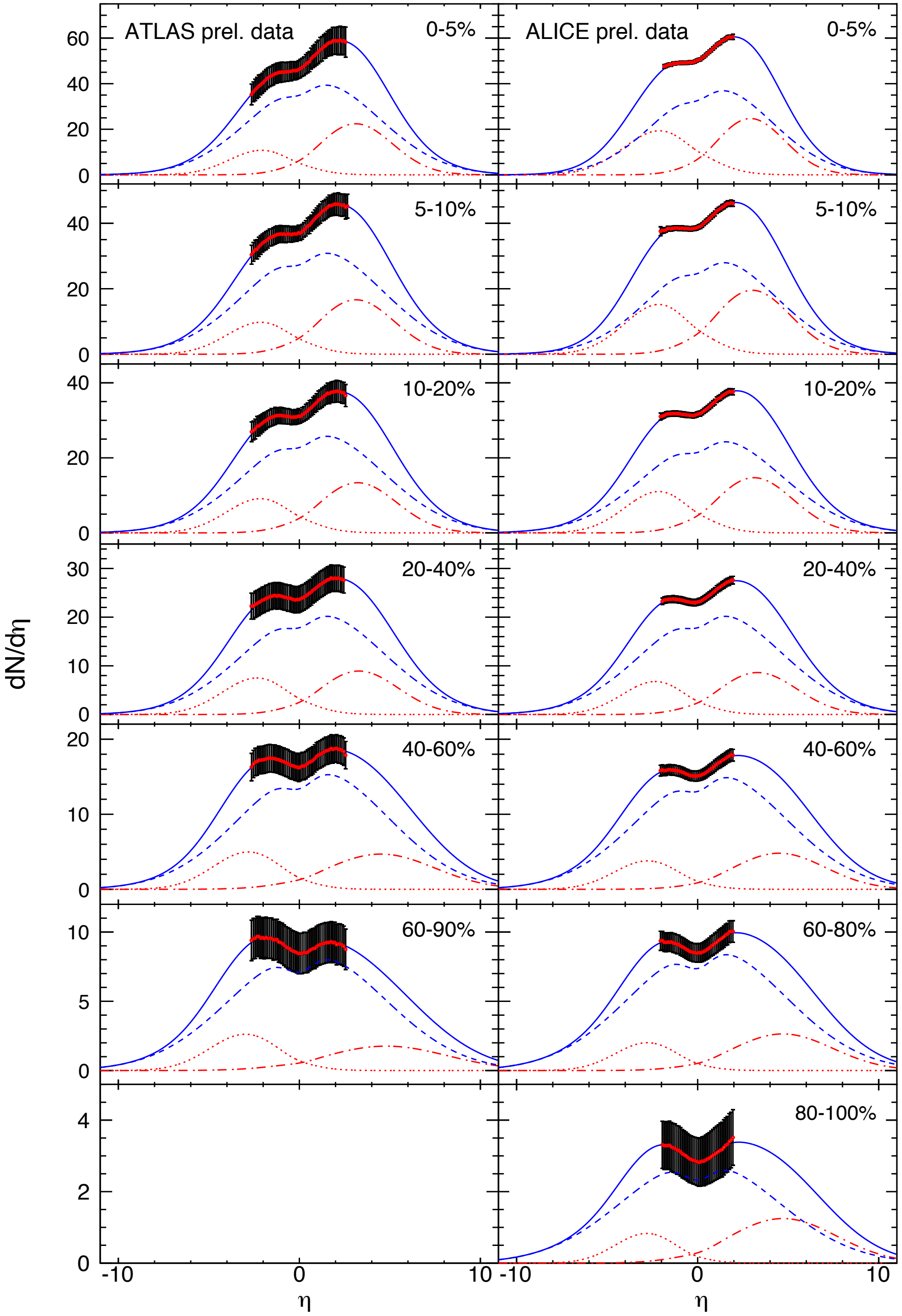
- Good agreement of RDM prediction and recent ALICE results at midrapidity
- The midrapidity gluon-gluon source is the largest hadron production source at all centralities
- $(dN/d\eta)|_{\eta \approx 0}$ decreases from ≈ 1959 at 0-5 % to ≈ 800 at 20-30 %

GW J. Phys. G 40, 045104 (2013)
χ^2- minimizations with respect to the ALICE 2.76 TeV PbPb data in the RDM.

ALICE Data: E. Abbas et al., ALICE Collab., Phys. Lett. B726, 610 (2013); arXiv:1512.06104

RDM-Parameters for prediction @ 5.02 TeV PbPb:
GW Phys. Rev. C 91, 014905 (2015)

This plot from F. Vogel, BSc thesis (HD 2015)



Centrality dependence *p* Pb @ 5.02 TeV

- Good agreement of preliminary ATLAS and ALICE results for most centralities
- The midrapidity gluon-gluon source remains the largest hadron production source at all centralities, the *p*-like source the smallest
- (dN/dη)_max decreases from ≈ 60 at 0-5 % to ≈ 3 at 80-100 %

Calculations: P. Schulz & GW EPJA 51, 18 (2015) χ^2- minimizations with respect to the (prel.) data in the RDM.

Final ALICE Data: J. Adam et al., ALICE Collab., Phys. Rev. C91, 064905 (2015)

Prel. ATLAS data: B.Cole et al.

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Conclusion

- Hadron production rates in relativistic collisions agree with the thermal model: A necessary condition for thermal equilibrium
- Deviations from thermal distribution functions in p_T and η (or y) suggest to go beyond the thermal model
- In transverse momentum distributions the high-p_T region ($p_T > 8$ GeV/c) requires a pQCD approach
- The nonequilibrium contributions from the fragmentation regions in rapidity and pseudorapidity distributions are accounted for in a three-sources relativistic diffusion model (RDM)

❖ Thank you for your attention,

and for organizing Bormio 2016 !