The study of nuclear structure far from stability

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Introduction

Stable nuclei are qualitatively described by "simple" models

- (semi-empirical) liquid-drop model
- (basic) shell model

New techniques enable ab-initio methods (A-body models)

What happens far from stability?

- Experimentally, Radioactive-Ion Beams (RIB) available since 80s
- \Rightarrow study of structure far from stability
- \Rightarrow discovery of exotic structures
 - super-heavy elements
 - halo nuclei
 - shell inversions

Nuclear Landscape



Basic features in nuclear structure

- Liquid-drop model
- Shell model
- 2 Ab-initio nuclear models
- Superheavy nuclei
- 4 Radioactive-Ion Beams
- Oddities far from stability : halo nuclei
- Experimental techniques
 - Knockout reactions
 - In-beam γ spectroscopy
 - Transfer reactions



Charge distributions in (stable) nuclei



- constant density ρ_0 out to the surface (saturation)
- same skin thickness t

(Stable) nuclei look like liquid drops of radius $R \propto A^{1/3}$

Bethe-Weizsäcker semi-empirical mass formula

 $B(Z, N) = a_v A$



Bethe-Weizsäcker semi-empirical mass formula

 $B(Z,N) = a_v A - a_s A^{2/3}$



Bethe-Weizsäcker semi-empirical mass formula

$$B(Z,N) = a_{\nu}A - a_{s}A^{2/3} - a_{C}\frac{Z(Z-1)}{A^{1/3}}$$



Bethe-Weizsäcker semi-empirical mass formula

$$B(Z,N) = a_v A - a_s A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_{\text{Sym}} \frac{(A-2Z)^2}{A}$$



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Excenergetic reactions :

- fission of heavy nuclei (nuclear power plants, atomic bomb)
- fusion of light nuclei (stars, thermonuclear weapons)



Variation from the semi-empirical mass formula



More bound systems at Z or N = 2, 8, 28, 50, 82, 126magic numbers

 \Rightarrow shell structure in nuclei as in atoms?

Two-nucleon separation energy

Same magic numbers in S_{2p} and S_{2n}

 \Rightarrow more bound at shell closure cf. ionisation energies of atoms



Shell model

Developed in 1949 by M. Goeppert Mayer, H. Jensen and E. Wigner

As electrons in atoms, nucleons in nuclei feel a mean field and arrange into shells

Spin-orbit coupling is crucial to get right ordering of shells



Example

Shell model explains the higher stability at some Z and N

It predicts the spin and parity of ground state of most nuclei and some of their excited levels, e.g. ¹⁷O and ¹⁷F



Confirmed within coupled-cluster calculation

[G. Hagen et al. PRL 104, 182501 (2010)]

Nowadays

Can we go beyond these models? Can we build <u>ab-initio</u> models? i.e. based on first principles

- nucleons as building blocks
- realistic N-N interaction

What happens away from stability?

- Is there an island of stability for heavy nuclei?
- Is nuclear density similar for radioactive nuclei?
- Are magic numbers conserved?

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A-body Hamiltonian

Nuclear-structure calculations : A nucleons (Z protons+N neutrons) Relative motion described by the A-body Hamiltonian

$$H = \sum_{i=1}^{A} T_i + \sum_{j>i=1}^{A} V_{i_j}$$

 \Rightarrow solve the A-body Schrödinger equation

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

 $\{E_n\}$ is the nucleus spectrum

Realistic *N*-*N* interactions

 V_{ij} not (yet) deduced from QCD \Rightarrow phenomenological potentials fitted on *N*-*N* observables : d binding energy, *N*-*N* phaseshifts

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Light nuclei calculations



[R. Wiringa, Argonne]

Three-body force

Need three-body forces to get it right...

$$H = \sum_{i=1}^{A} T_i + \sum_{j>i=1}^{A} V_{ij} + \sum_{k>j>i=1}^{A} V_{ijk} + \cdots$$

But there is no such thing as three-body force...

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But there is no such thing as three-body force...

They simulate the non-elementary character of nucleons \Rightarrow include virtual Δ resonances, \bar{N} ...

$$\begin{array}{c|c} \hline \pi,\rho,\omega \\ \hline \Delta,N^* \\ \hline \pi,\rho,\omega \end{array} \qquad \hline \hline \pi,\rho,\omega \\ \hline \hline \hline \hline \\ \pi,\rho,\omega \\ \hline \hline \end{array}$$

Phenomenological 3-body interaction fitted on A > 2 levels : IL2 Alternatively, derived from EFT

Effective Field Theory

EFT is an effective quantum field theory based on QCD symmetries with resolution scale Λ that selects appropriate degrees of freedom : nuclear physics is not built on quarks and gluons, but on nucleons and mesons

EFT provides the nuclear force with a systematic expansion in Q/Λ

• gives an estimate of theoretical uncertainty

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EFT provides the nuclear force with a systematic expansion in Q/Λ

- gives an estimate of theoretical uncertainty
- naturally includes many-body forces



[see G. Hagen's talk on Wednesday morning and J. Holt on Tuesday afternoon]

Expansion of the EFT force





Solving the Schrödinger equation

 $H |\Psi_n\rangle = E_n |\Psi_n\rangle$

 Ψ usually developed on a basis $\{|\Phi_{[\nu]}\rangle\}$:

$$|\Psi_n
angle = \sum_{[
u]} \langle \Phi_{[
u]} |\Psi_n
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$$\Psi_n \rangle = \sum_{[\nu]} \langle \Phi_{[\nu]} | \Psi_n \rangle | \Phi_{[\nu]} \rangle$$

Solving the Schrödinger equation reduces to matrix diagonalisation

$$\langle \Phi_{[\mu]} | H | \Psi_n \rangle = \sum_{[\nu]} \langle \Phi_{[\mu]} | H | \Phi_{[\nu]} \rangle \langle \Phi_{[\nu]} | \Psi_n \rangle$$

= $E_n \langle \Phi_{[\mu]} | \Psi_n \rangle$

No-core shell model

Slater determinants of 1-body mean-field wave functions ϕ_{ν_i}

 $\langle \xi_1 \xi_2 \dots \xi_A | \Phi_{[\nu]} \rangle = \mathcal{A} \phi_{\nu_1}(\xi_1) \phi_{\nu_2}(\xi_2) \dots \phi_{\nu_A}(\xi_A)$

But short-range correlations couple low and high momenta

No-core shell model

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 \Rightarrow requires large basis $|\Phi_{\nu}\rangle$

Similarity Renormalisation Group

Idea : apply a unitary transformation

$$\begin{split} |\widetilde{\Phi}_{[\nu]}\rangle &= U |\Phi_{[\nu]}\rangle \\ \Leftrightarrow H_{\rm eff} &= U^{\dagger} H U \end{split}$$

- keeps the same spectrum (unitary)
- keeps the same on-shell properties (phaseshifts)
- removes the short-range correlations

This has a costs : induces "unphysical" three-body forces



SRG : example on ⁴He SRG lowers correlations



[see G. Hagen's talk on Wednesday morning and J. Holt on Tuesday afternoon]



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What happens far from stability?

Liquid-drop and shell models are fair models of stable nuclei What happens away from stability ? Are there superheavy nuclei? Recent discovery of new elements (Z = 113, 115, 117, and 118)

In 80s Radioactive-Ion Beams were developed Enable study of nuclear structure [see A. Obertelli's talk on Friday]

- are radioactive nuclei compact?
- are shells conserved far from stability?

Reactions involving radioactive nuclei useful in astrophysics

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Superheavy nuclei

Does the stability end with U? Or is there an island of stability? Is $Z \sim 114 - 126$ a new magic number?



[Oganessian Rad. Act. 99, 429 (2011)]
Search elements heavier than U has started in the 40's Pu produced by U+d and U+n (identified by Seaborg in 1941) High flux of n on U or Pu \rightarrow 100 Fm

Z = 101-106 obtained by bombarding actinides by light elements Heavier elements (Z = 107-113) by cold fusion 208 Pb or 209 Bi + massive projectile ($A \ge 50$) \rightarrow high Z + n Nowadays, hot fusion : 48 Ca on actinide target \rightarrow SHE + 4–5 n

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Nowadays, hot fusion : 48 Ca on actinide target \rightarrow SHE + 4–5 n Z = 113, 115, 117, and 118 recently recognised by IUPAC



[Oganessian Rad. Act. 99, 429 (2011)]

Identification by α decay chain



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• ISOL : Fire a proton at a heavy nucleus



• In-flight : Smash a heavy nucleus on a target



Where?





high-energy/intensity primary beam of light nuclei (e.g. protons) on thick target of heavy elements (Ta or UC_x) \Rightarrow spallation/fragmentation produces exotic fragments



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high-energy primary beam of heavy ions (e.g. ¹⁸O, ⁴⁸Ca, U...) on thin target of light elements (Be or C) \Rightarrow fragmentation/fission produces many exotic fragments at $\approx v_{\text{beam}}$



high-energy primary beam of heavy ions (e.g. ¹⁸O, ⁴⁸Ca, U...) on thin target of light elements (Be or C) \Rightarrow fragmentation/fission produces many exotic fragments at $\approx v_{beam}$ Sorted in fragment separator



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Used for high-energy reactions (KO, breakup...)



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Examples : NSCL (MSU), GSI, RIKEN, GANIL

Existing NSCL



Future : FRIB



Properties

ISOL

- Low beam energy may require post-acceleration
- Low beam intensity
- Not all elements produced
 - Slow
 - Chemically limited
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In-flight

High beam energy

 $v_{\rm fragments} \approx v_{\rm beam}$

- High beam intensity
- Efficient production
 - Fast
 - Chemically independent
- Many fragments in beam ⇒ need ion ID

Choose according what you want to measure



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Halo structure Seen as core + one or two neutrons at large distance

[P. G. Hansen and B. Jonson, Europhys. Lett. 4, 409 (1987)]

Peculiar structure of nuclei due to small S_n or S_{2n} \Rightarrow neutrons tunnel far from the core to form a halo

Halo only appears for low centrifugal barrier (low ℓ)



see F. Colomer's poster on Monday

Halo nuclei

- Light, neutron-rich nuclei
- small S_n or S_{2n}
- Iow-ℓ orbital

One-neutron halo ${}^{11}\text{Be} \equiv {}^{10}\text{Be} + n$ ${}^{15}\text{C} \equiv {}^{14}\text{C} + n$

Two-neutron halo ${}^{6}\text{He} \equiv {}^{4}\text{He} + n + n$ ${}^{11}\text{Li} \equiv {}^{9}\text{Li} + n + n$



Proton halces are possible but less probable : ⁸B, ¹⁷F

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Proton halces are possible but less probable : 8 B, 17 F Two-neutron halo nuclei are Borromean... c+n+n is bound but not two-body subsystems e.g. 6 He bound but not 5 He or 2 n

Borromean nuclei

Named after the Borromean rings...

[M. V. Zhukov et al. Phys. Rep. 231, 151 (1993)]



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Knockout reactions

Knockout reaction : direct removal of one nucleon For exotic nuclei, measured in inverse kinematics on light target (C or Be) at intermediate energy (70AMeV) ${}^{11}Be + {}^{9}Be \rightarrow {}^{10}Be + X$

Inclusive measurement (KO nucleon not measured) Used to probe the shell-model wave function of nuclei The valence nucleon occupies a shell outside a core :

 ${}^{A}Y(J^{\pi}) = {}^{A-1}X(J^{\pi}_{c}) \otimes \psi_{lm} + \dots$

Analysis of reaction within eikonal model (sudden approximation)

- Parallel-momentum distribution of the core gives nucleon shell
- Total σ_{-N} provides Spectroscopic Factor (SF)

${}^{9}\text{Be}({}^{11}\text{Be}, {}^{10}\text{Be} + \gamma)\text{X} @ 60A\text{MeV}$



[Aumann et al. PRL 84, 35 (2000)]

Shell model predicts

$${}^{11}\mathrm{Be} \equiv {}^{10}\mathrm{Be}(0^+) \otimes \nu_{0\mathrm{p}1/2}$$

• Experiment shows shell inversion

$${}^{11}\mathrm{Be} \equiv {}^{10}\mathrm{Be}(0^+) \otimes \nu_{1s1/2}$$

Narrow momentum distribution
 ⇔ extended spacial distribution
 ⇒ confirms the neutron halo

Study of nuclear spectroscopy

Reaction models rely on single-particle model :

 $[T_r + V_{cf}(r) - \epsilon]\phi_{nlm}(\mathbf{r}) = 0 \qquad \text{with } \|\phi_{nlm}\| = 1$

In reality, there is admixture of configurations :

 ${}^{A}Y(J^{\pi}) = {}^{A-1}X(J^{\pi}_{c}) \otimes \psi_{lm} + \dots$

where ψ_{lm} is the overlap wave function Spectroscopic Factor : $S_l = ||\psi_{lm}||^2$

Single-particle approximation $\equiv \psi_{lm} = \sqrt{S_l} \phi_{nlm}$ \Rightarrow usual idea : $S_l = \sigma_{bu}^{exp} / \sigma_{bu}^{th}$

Reduction of the SF Predicted shell-model SF too high for deeply-bound nucleons



[Gade et al. PRC 77, 044306 (2008)]

- Something missing in usual shell-model calculations?
- Problem in the reaction modelling?

Knockout reactions

Quenching of SF



[Jensen *et al.* PRL 107, 032501 (2011)]; see also G. Hagen's talk on Wednesday Inclusion of continuum in shell-model basis reduces SF \Rightarrow solves partly the problem

KO reaction dynamics

- For loosely-bound nucleons $S_n({}^{16}C) = 4.25 \text{ MeV}$ $S_p({}^{14}O) = 4.63 \text{ MeV}$ eikonal model works fine
- For deeply-bound nucleons $S_p(^{16}C) = 22.6 \text{ MeV}$ $S_n(^{14}O) = 23.2 \text{ MeV}$ energy conservation must be taken into account



[Flavigny *et al.* PRL 108, 252501 (2012)] see A. Obertelli's talk on Friday

 \Rightarrow solves another part of the problem ?

New magic numbers ?
New magic
$$Z = 14$$
 in 42 Si ?
 44 S + 9 Be $\rightarrow {}^{42}$ Si + X @ 99AMeV
 ${}^{N-20}$
 46 Ca 56 Ca 51 Ca 52 Ca 53 Ca 54 Ca 55 Ca 56 Ca 46 Ca 56 Ca 51 Ca 52 Ca 53 Ca 54 Ca 55 Ca 56 Ca ${$

Analysis within shell-model calculation suggests Z = 14 to be magic far from stability...

However... Measurement of γ in coincidence with proton stripping see A. Obertelli's and L. Atar's talks on Friday $^{44}\text{S} + {}^{12}\text{C} \rightarrow {}^{42}\text{Si} + \gamma + X @ 60AMeV$ 770(19) 6 Counts / 30 keV 2 [B. Bastin et al. PRL 99, 022503 (2007)] Indicates a low-lying state in ${}^{42}Si \Rightarrow Z = 14$ not magic

and collapse of N = 28 far from stability

Confirmed magic numbers

Transfer reaction can be used to study shell structure



[K. Jones et al. Nature 465, 454 (2010)]

Confirms the single-particle structure of ¹³³Sn and hence that ¹³²Sn is magic (i.e. N = 82 at Z = 50)

Summary

Liquid-drop and shell model describe qualitatively stable nuclei Nowadays <u>ab-initio</u> nuclear-structure models from first principles

RIB enable study nuclear structure far from stability Low intensities require new experimental techniques : KO reactions, in-beam γ spectroscopy,...

- discovery of halo nuclei diffuse halo around a compact core
- shell inversions or shell collapse

RIB can be used to study reactions of astrophysical interest

Combined with a gas stopper



- can use thin target in ISOL
- can study low-energy reaction with in-flight fragmentation

Summary

$C(^{31}Ne,^{30}Ne + \gamma)X @ 230AMeV$

Shell model predicts ³¹Ne ground state to be $7/2^{-}$ (³⁰Ne $\otimes v_{0f7/2}$) One-neutron knock-out measured at RIKEN

$$^{31}\text{Ne} + \text{C} \rightarrow ^{30}\text{Ne} + \text{X} @ 230A\text{MeV}$$

[T. Nakamura et al. PRL 103, 262501 (2009)]

Theoretical analysis suggests $3/2^-$ (³⁰Ne $\otimes v_{p3/2}$)



[W. Horiuchi et al. PRC 81, 024606 (2010)]

Systematic study



[Sauvan et al. PLB 491, 1 (2000)]