Lattice QCD: recent results and what to expect

Sinéad M. Ryan
Trinity College Dublin

54th International Winter Meeting on Nuclear Physics, Bormio, 29th January 2016

Plan

- Introduction and background
- A consumers guide to Lattice QCD
  - compromises and consequences
- Discussion and selected results
  - parallel tracks for progress
  - old challenges and new results
  - new and unexpected challenges and exploratory results
    - **precision spectroscopy of single hadron states including excited and exotic states**
    - **spectroscopy of scattering states - progress and challenges**
- Summary

Plan

- Introduction and background
- A consumers guide to Lattice QCD
    - compromises and consequences
- Discussion and selected results
    - parallel tracks for progress
    - old challenges and new results
    - new and unexpected challenges and exploratory results
        - **precision spectroscopy of single hadron states including excited and exotic states**
        - **spectroscopy of scattering states - progress and challenges**
- Summary

Many details and topics omitted for time constraints - APOLOGIES!

Why Lattice QCD ?

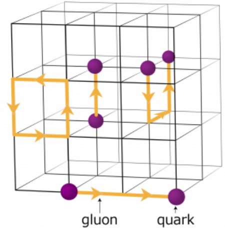
- A systematically-improvable non-perturbative formulation of QCD
  - Well-defined theory with the lattice a UV regulator
- Arbitrary precision is in principle possible
  - of course algorithmic and field-theoretic “wrinkles” can make this challenging!
- Starts from first principles - i.e. from the QCD Lagrangian
  - inputs are quark mass(es) and the coupling - can explore mass dependence and coupling dependence but getting to physical values can be hard!

A lattice QCD primer

Start from the QCD Lagrangian:

$$\mathcal{L} = \bar{\Psi}\left(i\gamma^{\mu} D_{\mu} - m\right)\Psi - \frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a}$$

Gluon fields on links of a hypercube;
Quark fields on sites: approaches to fermion discretisation - Wilson, Staggered, Overlap.;
Derivatives → finite differences.



Solve the QCD path integral on a finite lattice with spacing $a \neq 0$ estimated stochastically by Monte Carlo. Can only be done effectively in a Euclidean space-time metric (no useful importance sampling weight for the theory in Minkowski space).

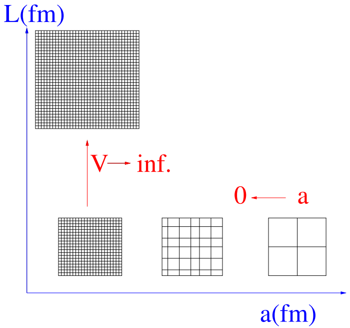
Observables determined from (Euclidean) path integrals of the QCD action

$$\langle \mathcal{O} \rangle = 1/Z \int \mathcal{D}U \mathcal{D}\bar{\Psi} \mathcal{D}\Psi \; \mathcal{O}[U, \bar{\Psi}, \Psi] e^{-S[U,\bar{\Psi},\Psi]}$$

Compromises and the Consequences

1. Working in a finite box at finite grid spacing

- Identify a "scaling window" where physics doesn't change with a or V. Recover continuum QCD by extrapolation.



A costly procedure but a regular feature in lattice calculations now

2. Simulating at physical quark masses

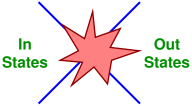
- Computational cost grows rapidly with decreasing quark mass $\rightarrow m_q = m_{u,d}$ costly. Care needed vis location of decay thresholds and identification of resonances.
- c-quark can be handled relativistically. b-quark with: NRQCD, FNAL etc.

Better algorithms for physical light quarks and/or chiral extrapolation. Relativistic m_b in reach

2. Breaking symmetry

- Lorentz symmetry broken at $a \neq 0$ so $SO(4)$ rotation group broken to discrete rotation group of a hypercube.
  Classify states by irreps of O_h and relate by subduction to J values of O_3. Lots of degeneracies in subduction for $J \geq 2$ and physical near-degeneracies. Complicates spin identification.

Spin identification at finite lattice spacing: 0707.4162, 1204.5425



3. Working in Euclidean time.

- Scattering matrix elements not directly accessible from Euclidean QFT [*Maiani-Testa theorem*]. Scattering matrix elements: asymptotic $|\mathrm{in}\rangle$, $|\mathrm{out}\rangle$ states: $\langle \mathrm{out}|e^{i\hat{H}t}|\mathrm{in}\rangle \rightarrow \langle \mathrm{out}|e^{-\hat{H}t}|\mathrm{in}\rangle$. Euclidean metric: project onto ground state. Analytic continuation of numerical correlators an ill-posed problem.

Lüscher and generalisations of: method for indirect access.

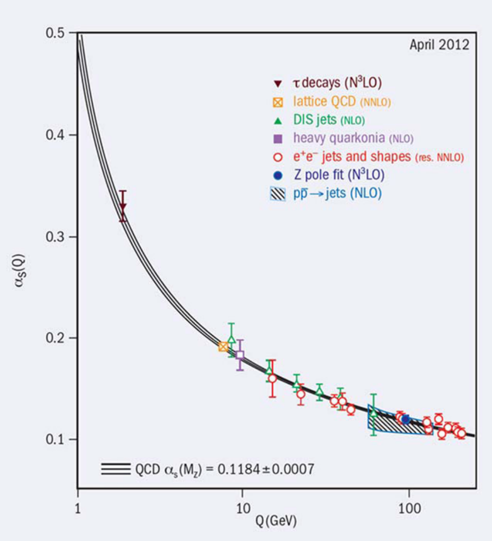
4. Quenching

No longer an issue: Simulations done with $N_f = 2, 2+1, 2+1+1$.

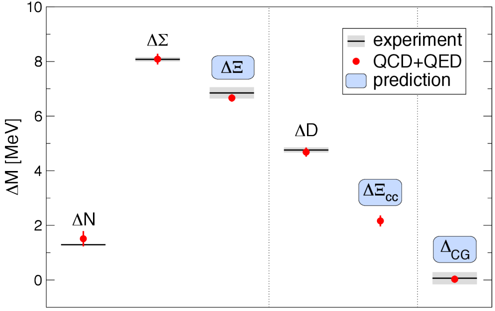
Validation:
can we reproduce known results and make verified predictions?

Validation

The running coupling, α_s



Baryon electromagnetic mass splittings

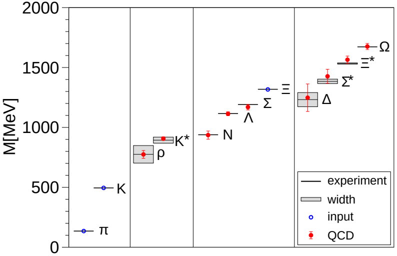


QED + QCD

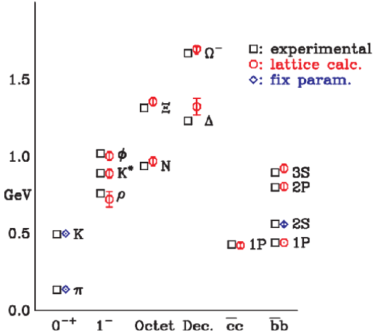
BMW Collab. Science 347 (2015) 1452

CONVERGENCE THROUGH UNIVERSALITY

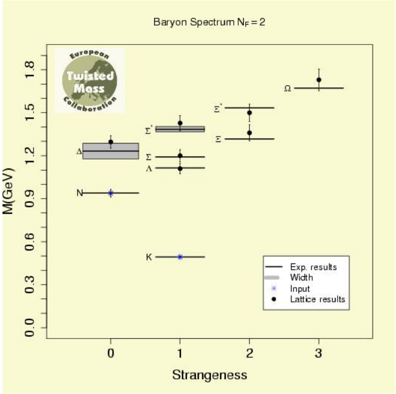
BMW Collaboration



MILC Collaboration



ETMC Collaboration



BMW: SW-Wilson [Science 322:1224-1227,2008.]

ETMC: Twisted Mass [arXiv:0910.2419,0803.3190]

MILC: Staggered [arXiv:0903.3598]

Two strategies for progress

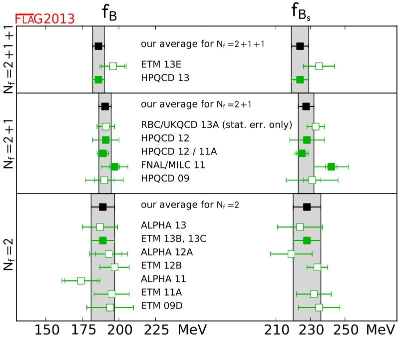
Gold-plated quantities

- e.g. single hadron states, or decays below thresholds
- phenomenologically relevant
- incremental progress
- robust/well-tested methods
- careful error budgeting

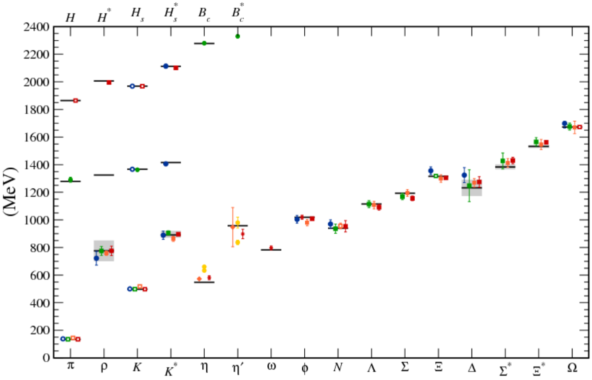
New directions

- new ideas - theoretical and algorithmic that open new avenues
- recent examples are scattering states, g-2, ...
- also improves gold-plated
- pioneering, error budgets not yet "robust"

Strategies for progress: gold plated quantities - a selection



FLAG 2013 itpwiki.unibe.ch/flag/



A. Kronfeld, Ann.Rev.Nucl.Part.Sci. 62 (2012)

- Stable single-hadron states, below thresholds
- Including continuum extrapolation, realistic quark masses, renormalisation etc

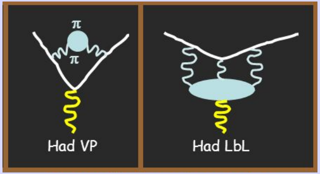
Strategies for progress: new directions - a selection

New ideas in hadron spectroscopy

- **Distillation** for quark propagation enabled isoscalars, precision spectroscopy, efficient calculation and motivated ...
- **Scattering and Coupled channels** new theoreticla ideas to tackle scattering states and study (X,Y,Z), resonance parameters in eg πK, $\pi\eta$ ...

New ideas for g-2

- Dominant uncertainty is in hadronic contributions - HVP and HLbL



- lots more!

Lattice Hadron Spectroscopy

precision & pioneering results

(i) Precision spectroscopy of single-hadron states

(ii) Exploratory studies of “exotic” and scattering states

A recipe for (meson) spectroscopy

- Construct a basis of local and non-local operators $\bar{\Psi}(x)\Gamma D_i D_j \ldots \Psi(x)$ from *distilled* fields [PRD80 (2009) 054506].
- Build a correlation matrix of two-point functions

$$C_{ij} = \langle 0|\mathcal{O}_i \mathcal{O}_j^\dagger|0\rangle = \sum_n \frac{Z_i^n Z_j^{n\dagger}}{2E_n} e^{-E_n t}$$

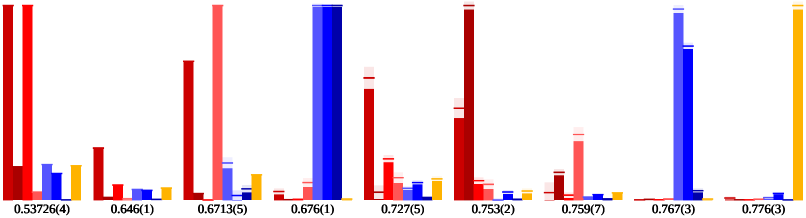
- Ground state mass from fits to $e^{-E_n t}$
- Beyond ground state: Solve generalised eigenvalue problem $C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$

- eigenvalues: $\lambda^{(n)}(t) \sim e^{-E_n t}\left[1 + O(e^{-\Delta E t})\right]$ - principal correlator
- eigenvectors: related to overlaps $Z_i^{(n)} = \sqrt{2E_n} e^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$

- operators of definite J^{PC} constructed in step 1 are subduced into the relevant irrep
- a subduced irrep carries a "memory" of continuum spin J from which it was subddduced - it **overlaps** predominantly with states of this J.

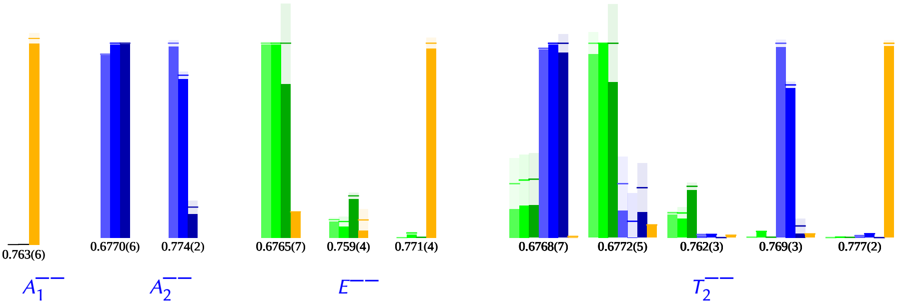
J	0	1	2	3	4
A_1	1	0	0	0	1
A_2	0	0	0	1	0
E	0	0	1	0	1
T_1	0	1	0	1	1
T_2	0	0	1	1	1

- Using $Z = \langle 0|\Phi|k\rangle$, helps to identify continuum spins
- For high spins, can look for agreement between irreps
- Data below for T_1^{--} irrep, colour-coding is **Spin 1**, **Spin 3** and **Spin 4**.



...the rest of the spin-4 state

- All polarisations of the spin-4 state are seen
- Spin labelling: **Spin 2**, **Spin 3** and **Spin 4**.

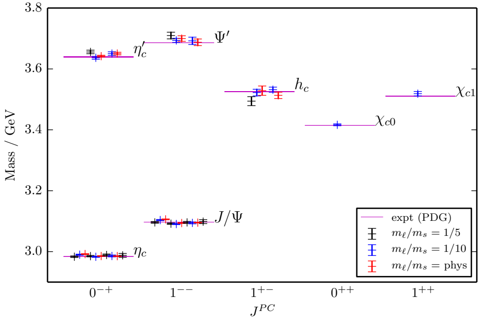


Precision Spectroscopy:
states below strong decay thresholds

Single hadron states: below threshold - "gold-plated"

- Methods: tested, validated.
- High statistics and improved actions for precise results.
- Different actions in agreement.
- Simulation at m_q^{phys} or extrapolation $m_q \to m_q^{\text{phys}}$.
- Discretisation errors $\mathcal{O}(am_c)$ and $\mathcal{O}(am_b)$ under control,

Charmonium, HPQCD 1411.1318



Continuum limit, physical quark masses

No disconnected diagrams in $c\bar{c}$ spectrum: OZI suppressed - assumed to be small
$\Rightarrow$ mixing with lighter states not included

Precision Spectroscopy:
single hadron states near/above thresholds

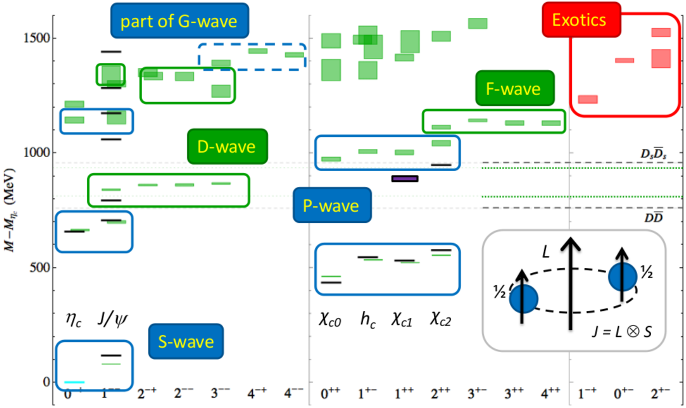
Single hadron states: above threshold

Precision calculation of high spin ($J \geq 2$) and exotic states is relatively new

Caveat Emptor

- Only single-hadron operators
- Physics of multi-hadron states appears to need relevant operators
- No continuum extrapolation
- Relatively heavy pions ← already changing

Charmonium

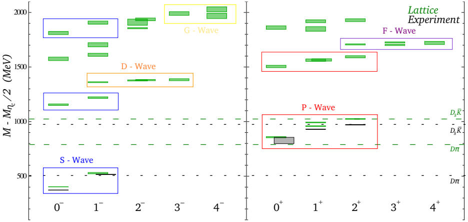


from HSC 2012

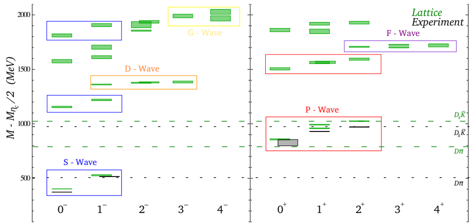
→ Expect improvements now methods established

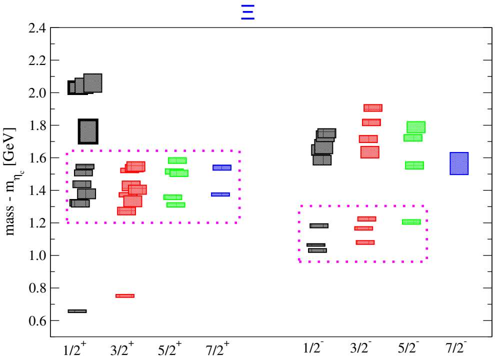
D Meson Spectrum - By J^P

[arXiv:1301.7670]

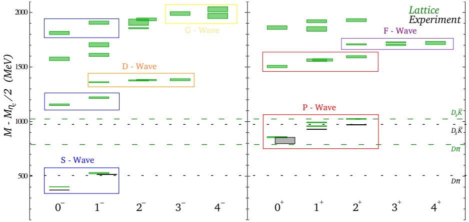


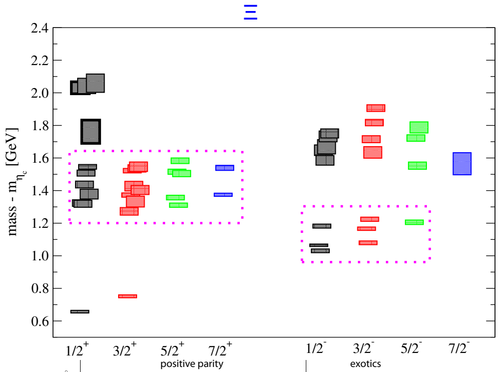
D Meson Spectrum - By J^P   *[arXiv:1301.7670]*





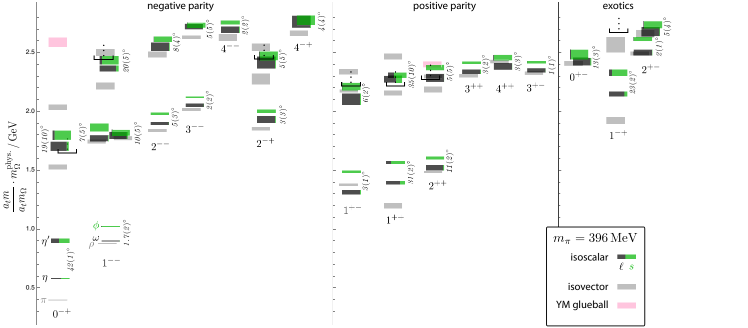
D Meson Spectrum - By J^P [arXiv:1301.7670]



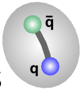


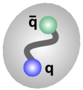
HadSpec results

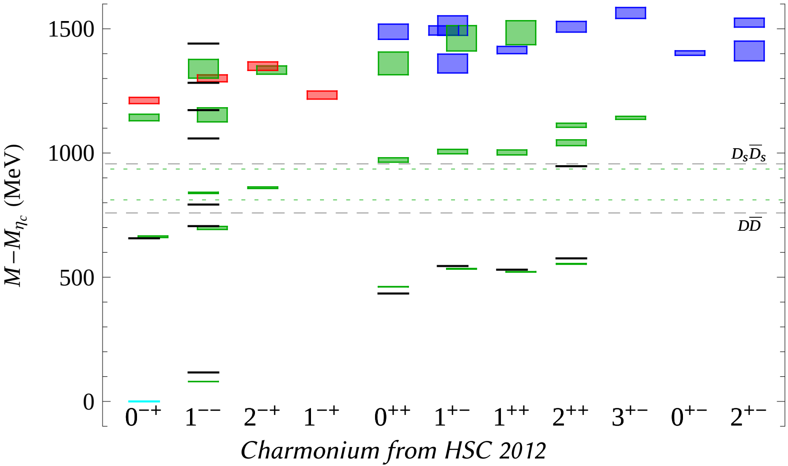
light mesons



Hybrids







Lightest hybrid supermultiplet and excited hybrid supermultiplet same pattern and scale as in open charm and light[*HadSpec*:1106.5515] sectors.

Exploratory studies of scattering states

Characterised by

- New methods (developed/applied in last 5 years)
  - algorithmic: distillation allows access to all elements of propagators *and* construction of sophisticated basis of operators.
  - theoretical: spin-identification; construction of multi-hadron operators and mesons in flight; scattering below inelastic thresholds; coupled-channels (new in '14).
- Generally high statistics, improved actions etc - results can be very precise.
- Systematic errors not all controlled in exploratory studies: e.g. no continuum extrapolation, relatively heavy pions ...

Rapid progress in the last 5 years!

Scattering in a Euclidean theory

Lose direct access to scattering in (Euclidean) lattice calculations

Lüscher found a way to extract scattering information in the elastic region from **LQCD** .
[NPB354, 531-578 (1991)]

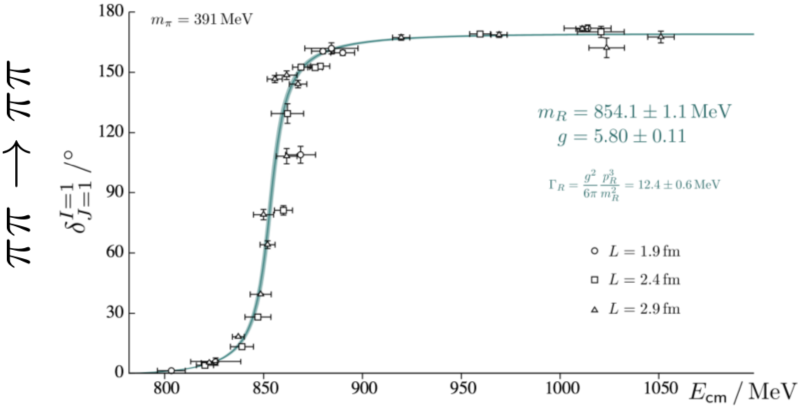
- related lattice energy levels in a finite volume to a decomposition of the scattering amplitude in partial waves in infinite volume

$$\det\left[\cot\delta(E_n^*) + \cot\phi(E_n, \vec{P}, L)\right] = 0$$

  and $\cot\phi$ a known function (containing a generalised zeta function).
- The idea dates from the quenched era. To use it in a dynamical simulation need energy levels at extraordinary precision. This is why it has taken a while ...

Using Lüscher's idea

Now in use to determine resonance parameters



$m_\pi = 391$ MeV

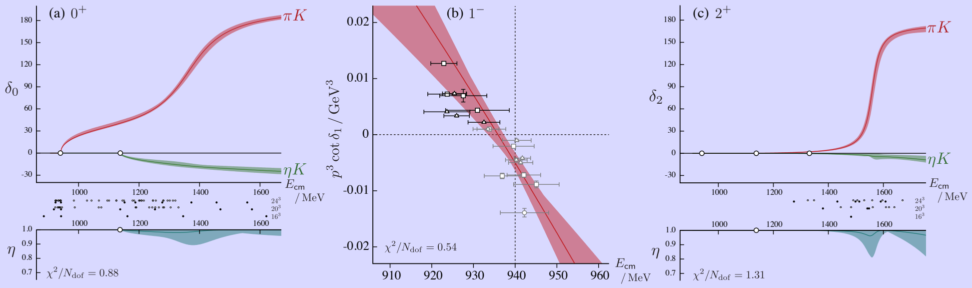
from Dudek, Edwards, Thomas in *Phys.Rev.* D87 (2013) 034505

Many talks at Lattice 2015 & 2016

Even more recent progress

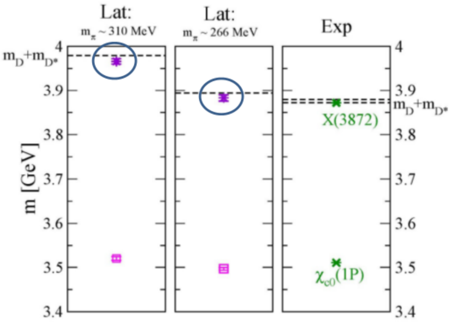
- Generalised for: moving frames; non-identical particles; multiple two-particle channels, particles with spin, by many authors.
- The precision and robustness of some numerical implementations is now very impressive. *[See talks at Lattice 2015 & 2016]*
- First coupled-channel resonance in a lattice calculation

$\pi K \to \eta K$ by D. Wilson et al 1406.4158 and 1507.02599



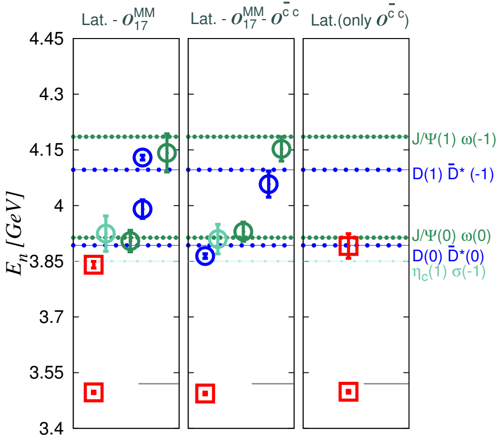
X(3872) - a first look

Prelovsek & Leskovec 1307.5172



ground state: $\chi_{c1}(1P)$
$D\bar{D}^*$ scattering mx: pole just below thr.
Threshold $\sim m_{u,d}$ *and* m_c *discretisation?*

Padmanath, Lang, Prelovsek 1503.03257

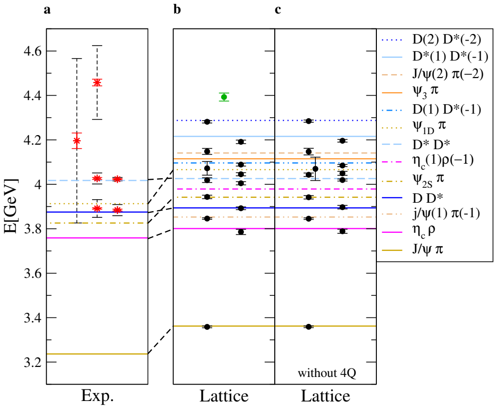


X(3872) not found if $c\bar{c}$ not in basis.

Also results from Lee et al 1411.1389
State is within 1MeV of $D^0\bar{D}^{0*}$ and 8MeV of D^+D^* thresholds: isospin breaking effects important?

Z_c^+ - First look on the lattice

Prelovsek, Lang, Leskovec, Mohler: 1405.7615



- 13 expected 2-meson e'states found (black)
- no additional state below 4.2GeV
- no Z_c^+ candidate below 4.2GeV

Similar conclusion from Lee et al [1411.1389] and Chen et al [1403.1318]

Why no eigenstate for Zc? Is Z_c^+ a coupled channel effect? What can other groups say? Work needed!

Z_c^+

An "exotic" hadron i.e. does not fit in the quark model picture.

There are a number of exploratory calculations on the lattice.

Challenges:

- The Z_c^+ (and most of the XYZ states) lies above several thresholds and so decay to several two-meson final states
- requires a coupled-channel analysis for a rigorous treatment
- on a lattice the number of relevant coupled-channels is large for high energies.

State of the art in coupled-channel analysis:

- Lüscher: $K\pi$, $K\eta$ [HSC 2014,2015]
- HALQCD: Z_c [preliminary results]

Many other states being investigated

Tetraquarks:

- Double charm tetraquarks ($J^P = 1^+$, $I = 0$) by HALQCD [PLB712 (2012)]
  - attractive potential, no bound tetraquark state
- Charm tetraquarks: variational method with DD^*, $D^* D^*$ and tetraquark operators finds no candidate.

Y(4140)

- Ozaki and Sasaki [1211.5512] - no resonant Y(4140) structure found
- Padmanath, Lang, Prelovsek [1503.03257] considered operators: $c\bar{c}$, $(\bar{c}s)(\bar{s}c)$, $(\bar{c}c)(\bar{s}s)$, $[\bar{c}\bar{s}][cs]$ in $J^P = 1^+$. Expected 2-particle states found and χ_{c1}, $X(3872)$ **not** Y(4140).

⋮

See Prelovsek @ Charm2015 for more

Challenges

There have been many successes the last 5 years including

- "Gold-plated" quantities increasingly well-determined
- New ideas have led to rapid progress in spectroscopy - precision excited and exotic states and scattering analyses
- New results in g-2, finite temperature continue to flow - not discussed here.

Many challenges remain

- ♦ Improving existing calculations - understanding the effect of lighter light quarks on thresholds etc, simulations at multiple and larger volumes
- ♦ Handling the large number of coupled-channels that emerged on larger volumes
- ♦♦ A general framework for coupled channels for scattering involving more than 2 hadrons. Some progress [M. Hansen @ Lattice 2015]
- Haven't discussed the many other open problems including finite density, BSM, ... !

CHALLENGES

There have been many successes the last 5 years including

- "Gold-plated" quantities increasingly well-determined
- New ideas have led to rapid progress in spectroscopy - precision excited and exotic states and scattering analyses
- New results in g-2, finite temperature continue to flow - not discussed here.

Many challenges remain

- ♦ Improving existing calculations - understanding the effect of lighter light quarks on thresholds etc, simulations at multiple and larger volumes
- ♦ Handling the large number of coupled-channels that emerged on larger volumes
- ♦♦ A general framework for coupled channels for scattering involving more than 2 hadrons. Some progress [M. Hansen @ Lattice 2015]
- Haven't discussed the many other open problems including finite density, BSM, ... !

Thanks for listening!

Backup Slides

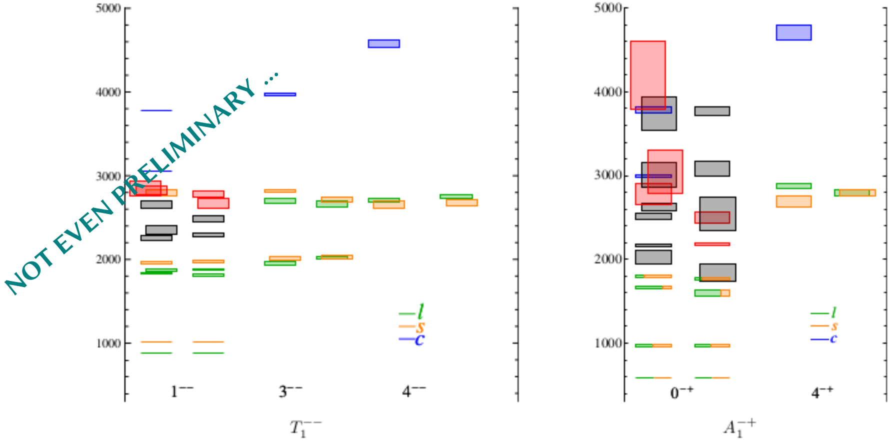
Last comment on single-hadron spectrum

Disconnected diagrams a remaining uncertainty in most $c\bar{c}$ calculations.

Distillation allows precision determination. BUT it's a can of worms!

Last comment on single-hadron spectrum

Disconnected diagrams a remaining uncertainty in most $c\bar{c}$ calculations.
Distillation allows precision determination. BUT it's a can of worms!



$\Delta(1^{--}) = -17(16)\text{MeV}$

from HadSpec