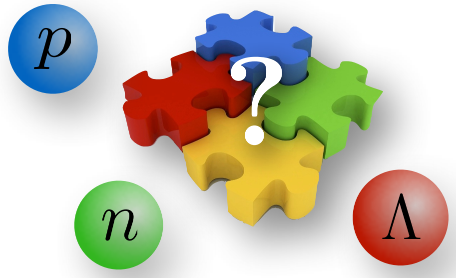
From hypernuclei to neutron stars: looking for the pieces of the puzzle



Diego Lonardoni

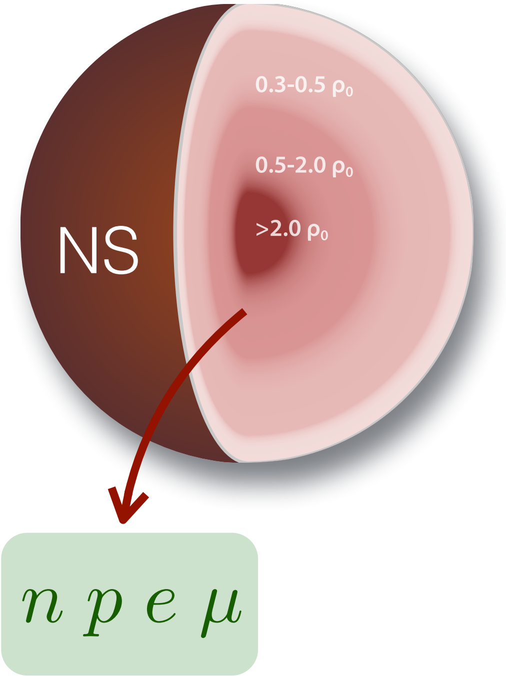
FRIB Theory Fellow

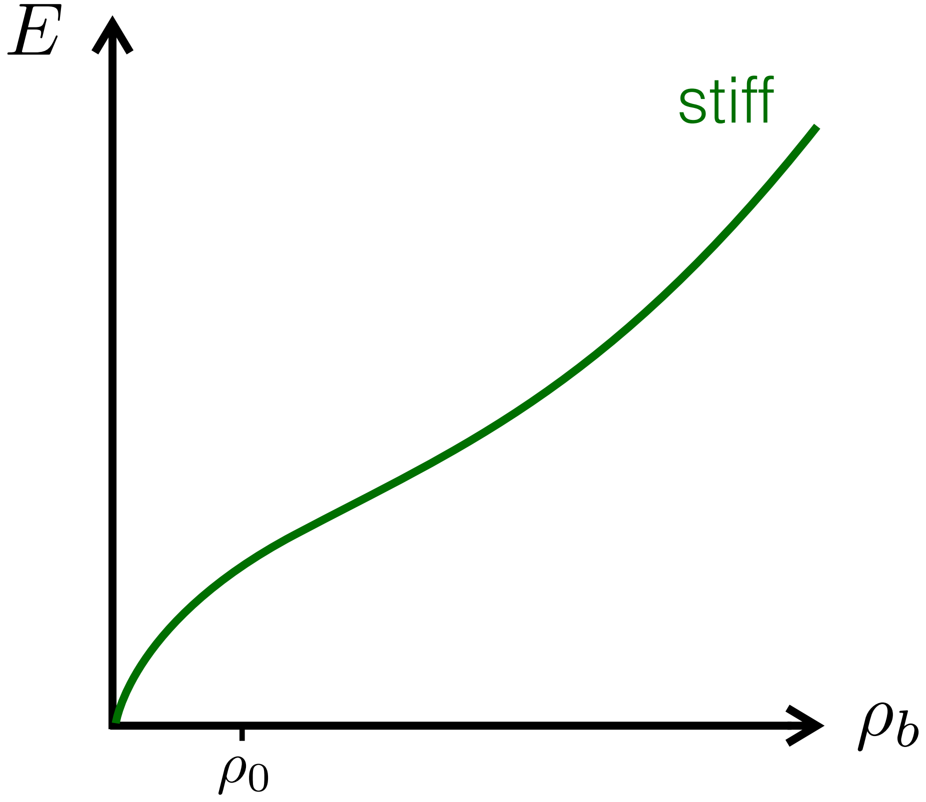
In collaboration with:

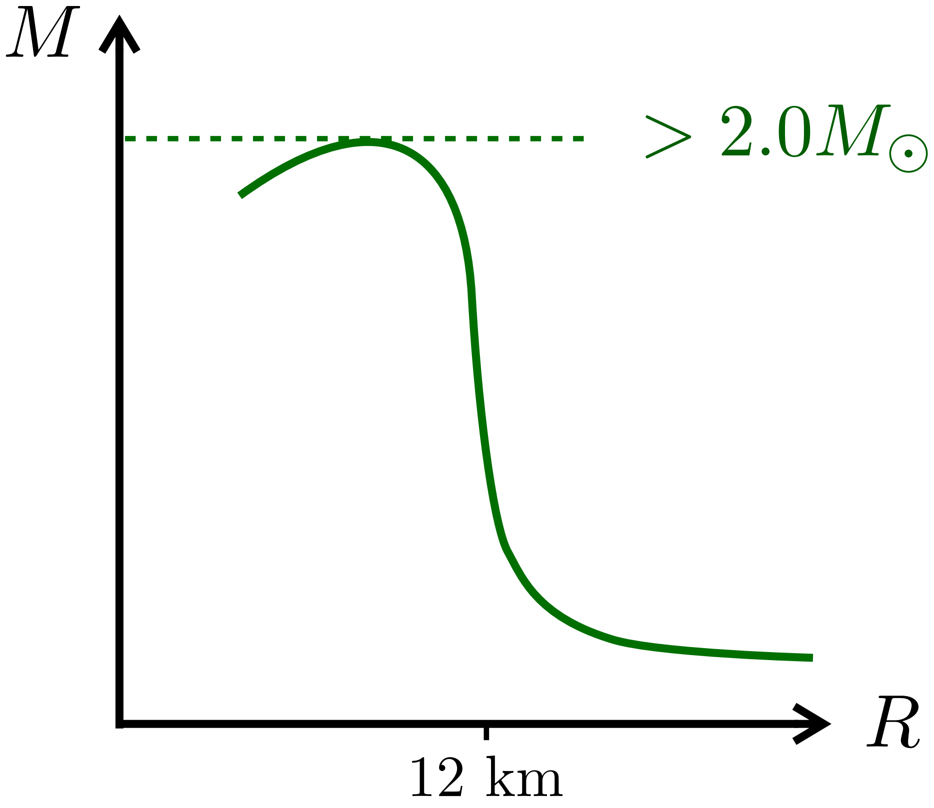
- ✓ Stefano Gandolfi, LANL
- ✓ Alessandro Lovato, ANL
- ✓ Francesco Pederiva, Trento
- ✓ Francesco Catalano, Uppsala

Bormio, January 28, 2016

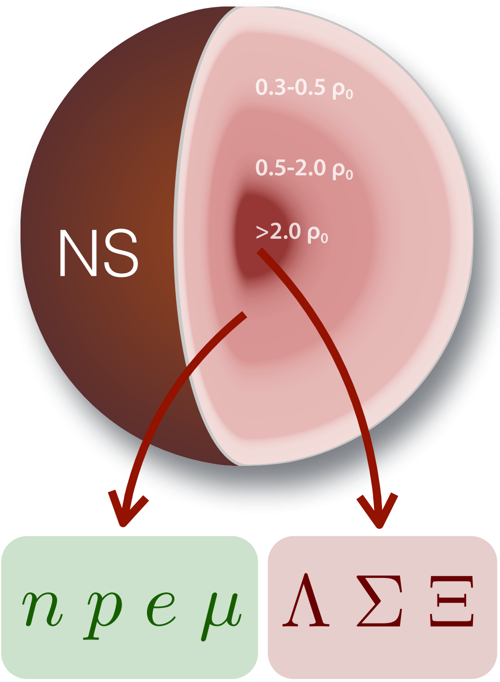
Strangeness in neutron stars: the hyperon puzzle

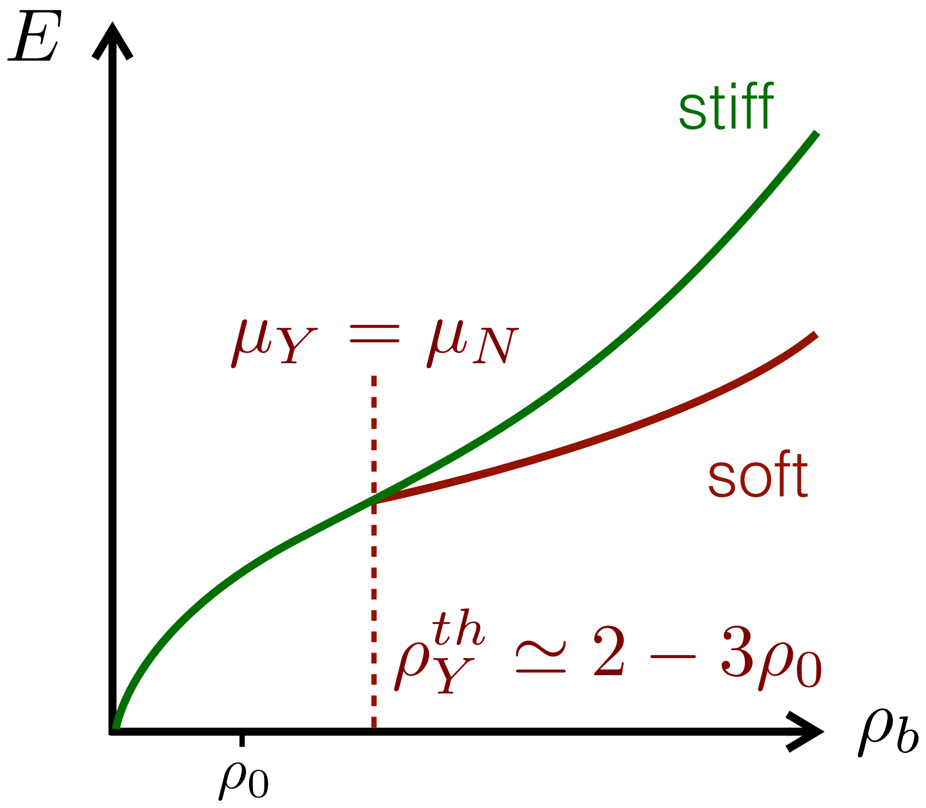


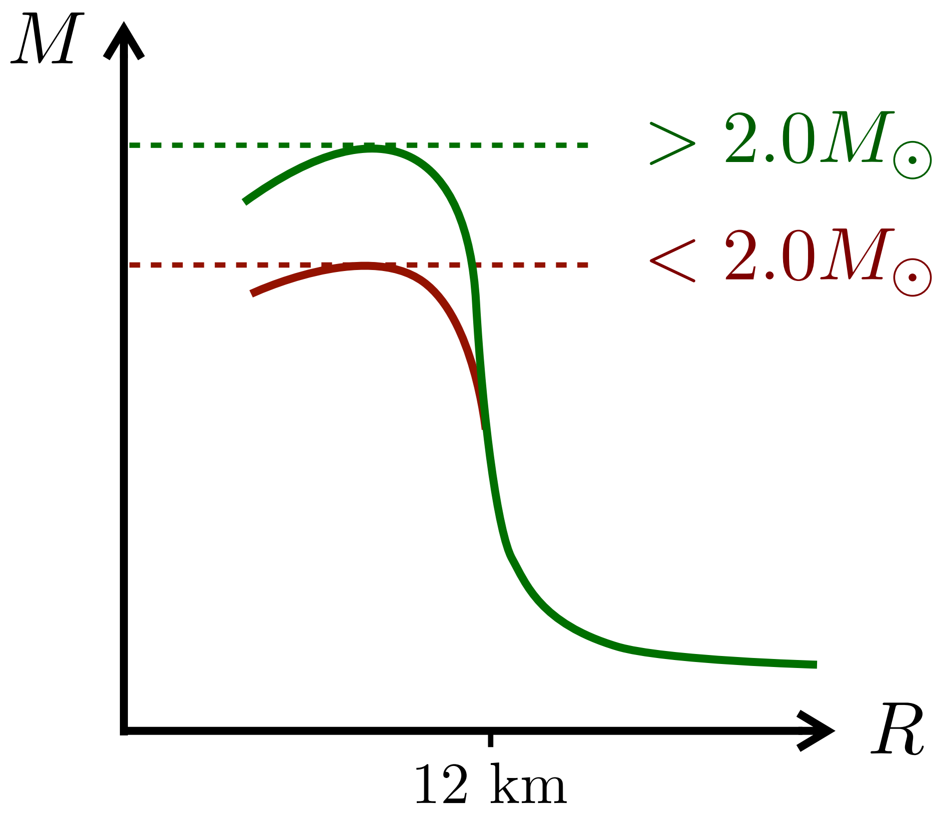




Strangeness in neutron stars: the hyperon puzzle







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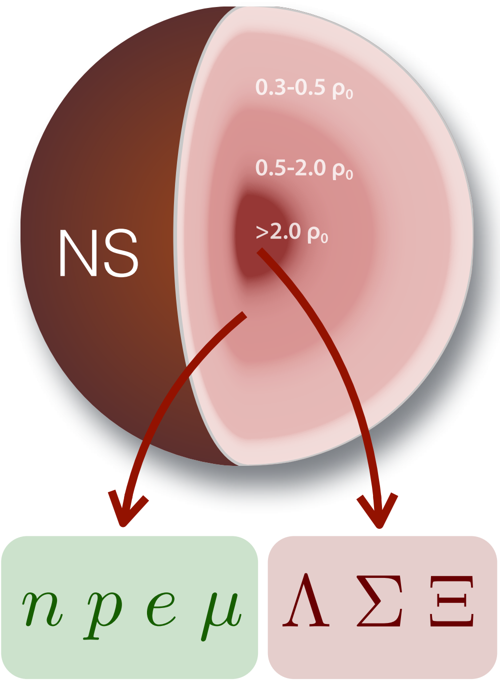

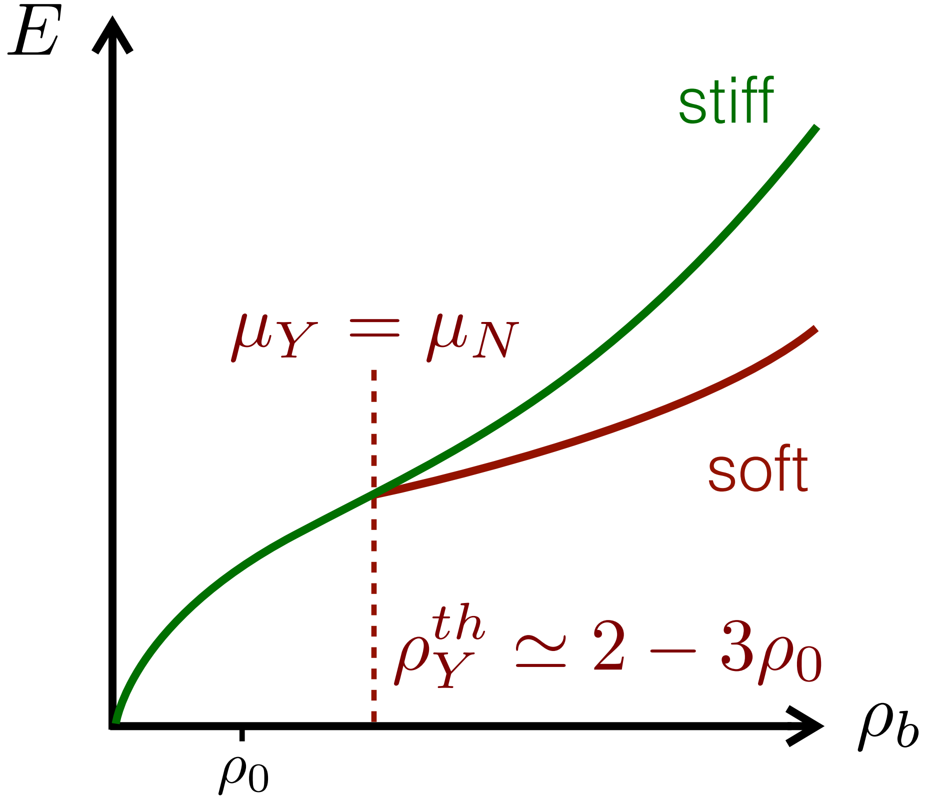

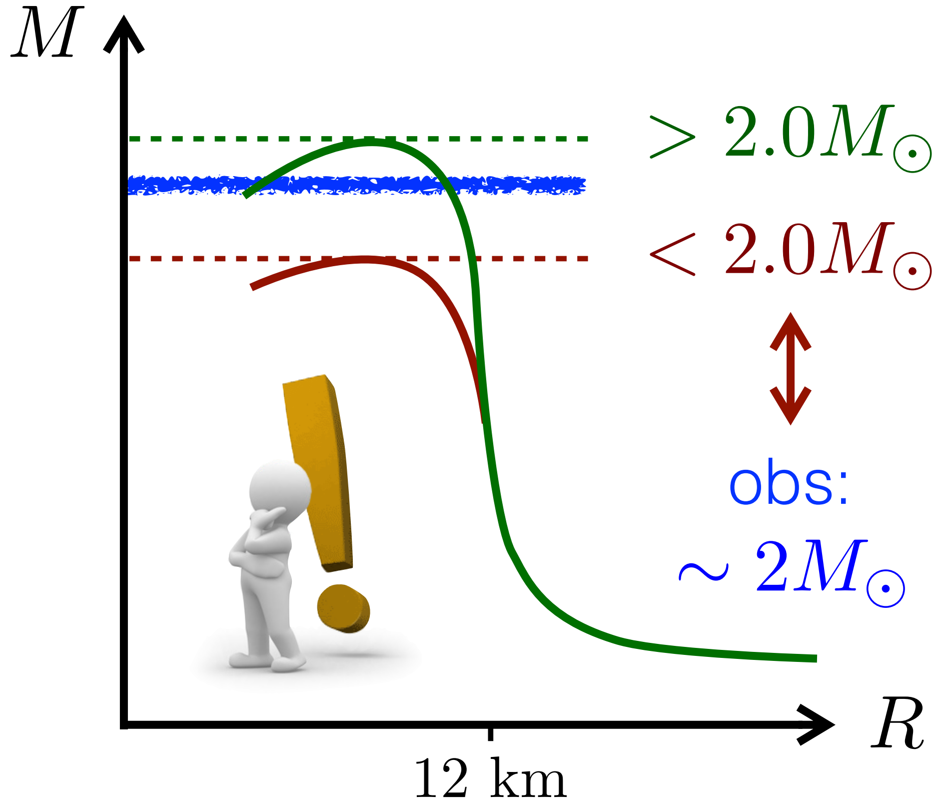

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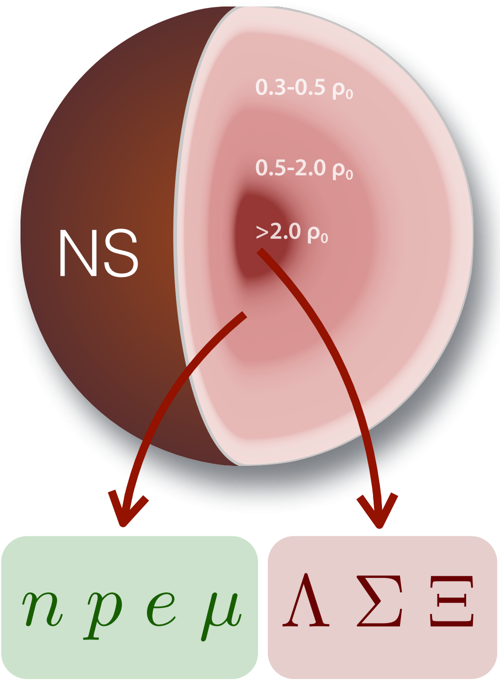
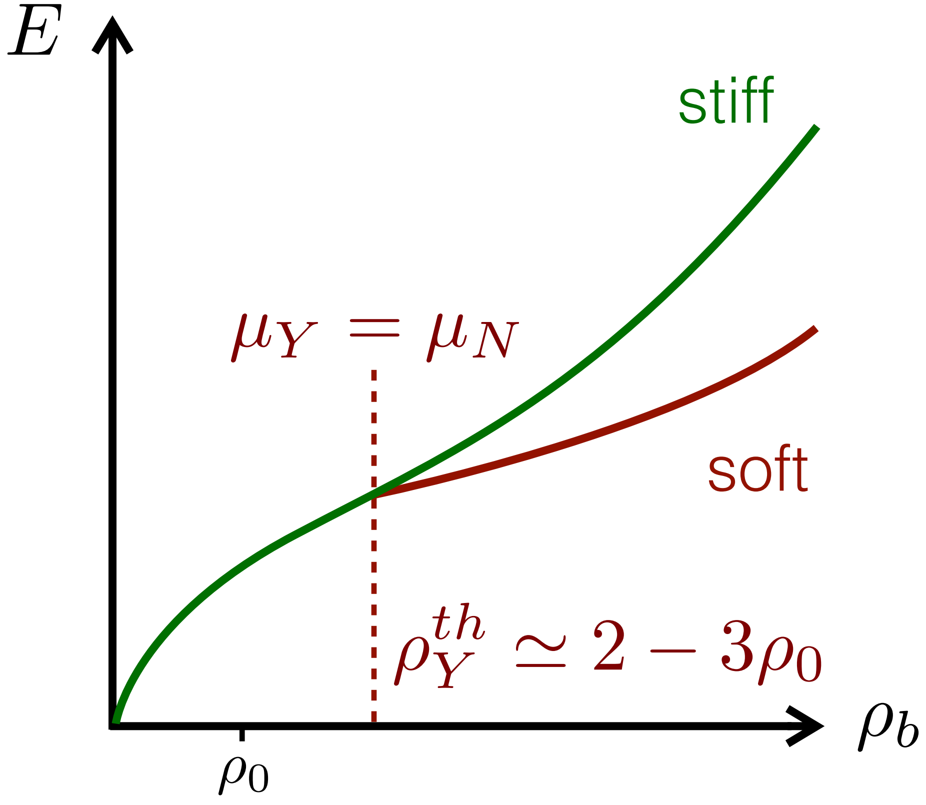
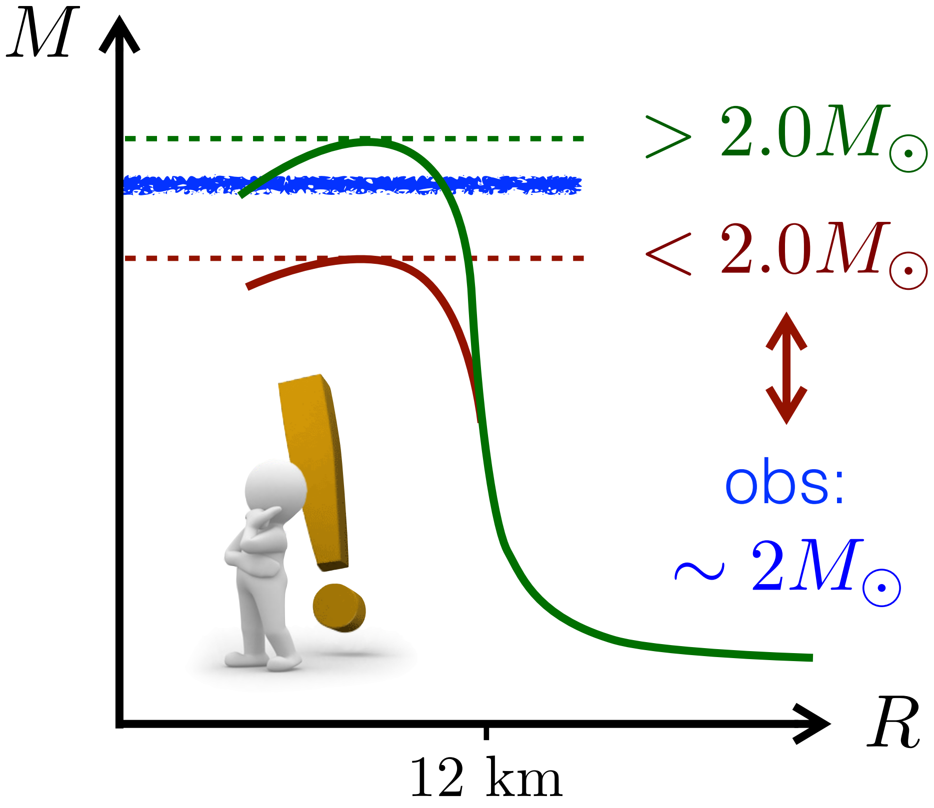


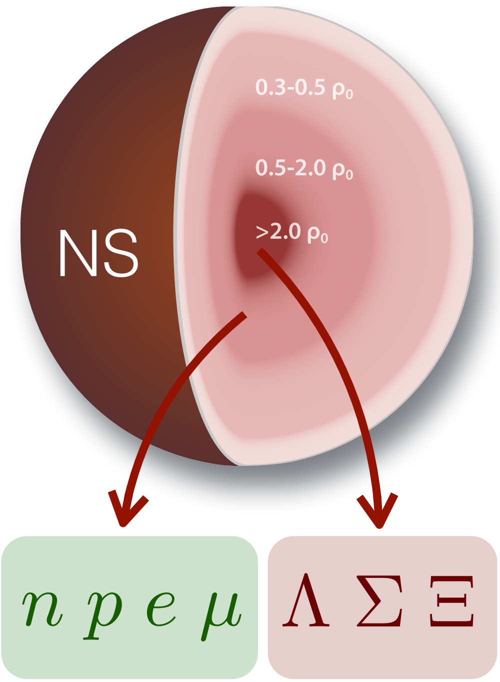


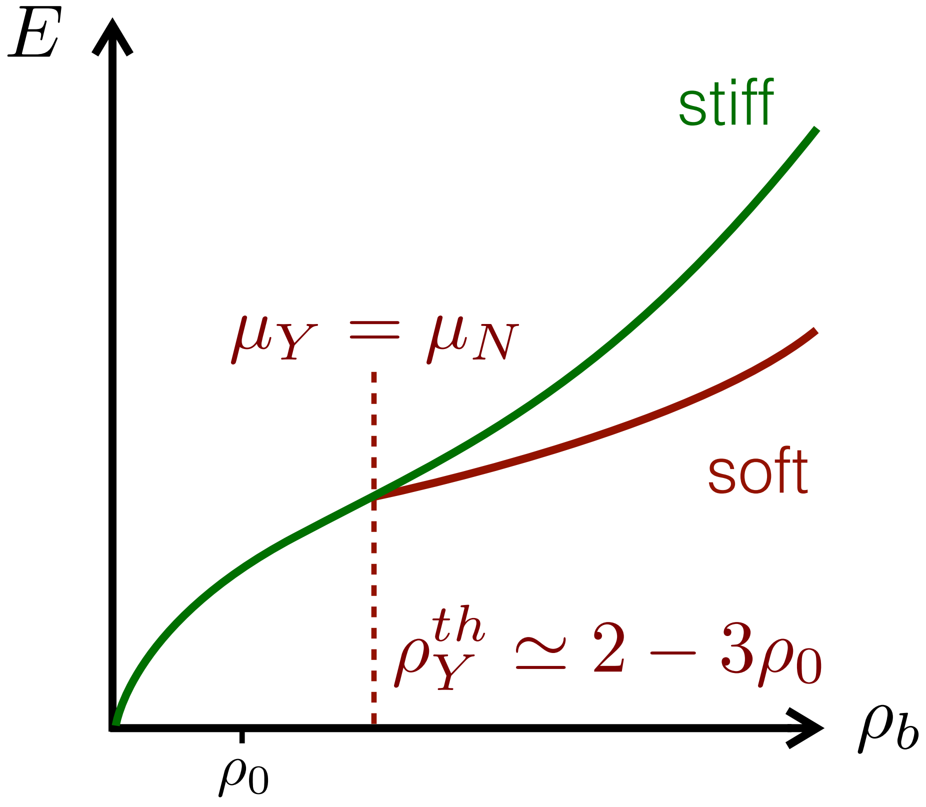


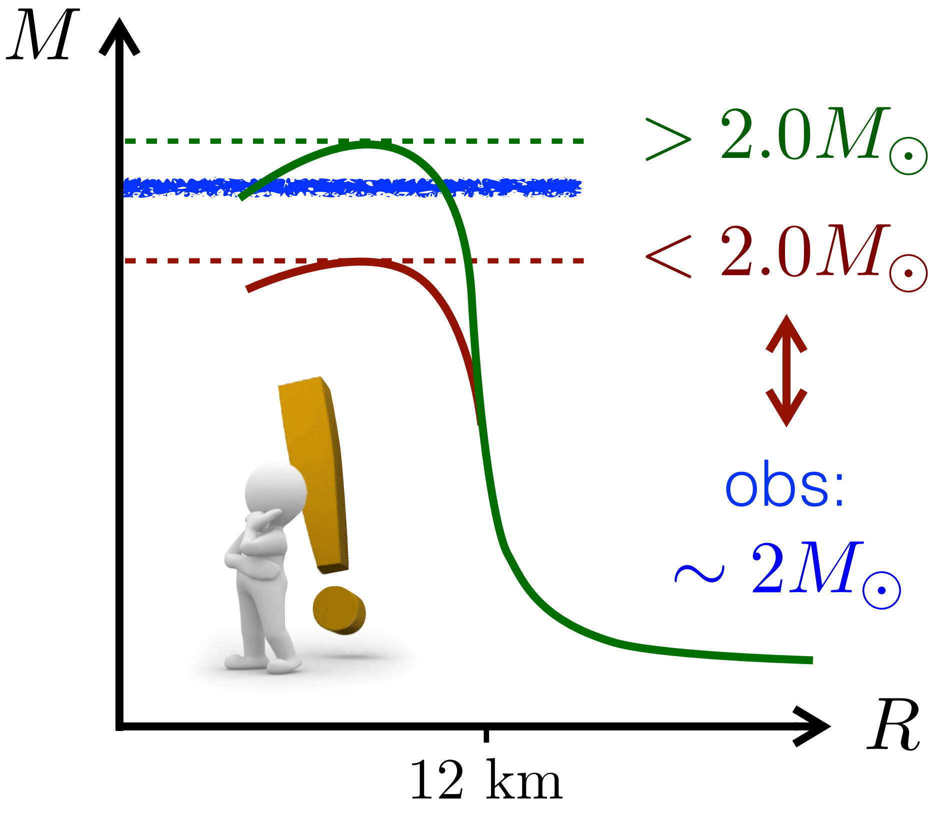
Hyperon puzzle

- ✓ Indication for the appearance of hyperons in NS core
- ✓ Apparent inconsistency between theoretical calculations and observations

Strangeness in neutron stars: the hyperon puzzle







Hyperon puzzle

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- ✓ Apparent inconsistency between theoretical calculations and observations

Quantum Monte Carlo → YN interaction

Strangeness in QMC calculations

Quantum Monte Carlo (Auxiliary Field Diffusion Monte Carlo)

$$-\frac{\partial}{\partial\tau}|\psi(\tau)\rangle = (H - E_0)|\psi(\tau)\rangle \qquad \tau = it/\hbar$$

imaginary time

$$\downarrow$$

$$|\psi(\tau)\rangle = \mathrm{e}^{-(H-E_0)\tau}|\psi(0)\rangle \xrightarrow{\tau \to \infty} c_0|\varphi_0\rangle$$

projection

Strangeness in QMC calculations

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projection

- ✓ nucleon-nucleon phenomenological interaction: Argonne & Urbana

$$H = \sum_i \frac{p_i^2}{2m_N} + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}$$

2B: NN scattering + deuteron

3B: nuclei + nuclear matter

Strangeness in QMC calculations

Quantum Monte Carlo (Auxiliary Field Diffusion Monte Carlo)

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projection

- ✓ nucleon-nucleon phenomenological interaction: Argonne & Urbana
- ✓ hyperon-nucleon phenomenological interaction: Argonne like

$$H = \sum_i \frac{p_i^2}{2m_N} + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}$$

$$+ \sum_\lambda \frac{p_\lambda^2}{2m_\Lambda} + \sum_{\lambda,i} v_{\lambda i} + \sum_{\lambda,i<j} v_{\lambda ij}$$

2B: Λp scattering + $A = 4$ CSB*

3B:

Strangeness in QMC calculations

Quantum Monte Carlo (Auxiliary Field Diffusion Monte Carlo)

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$$\downarrow$$

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3B: no unique fit

Strangeness in QMC calculations

Quantum Monte Carlo (Auxiliary Field Diffusion Monte Carlo)

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projection

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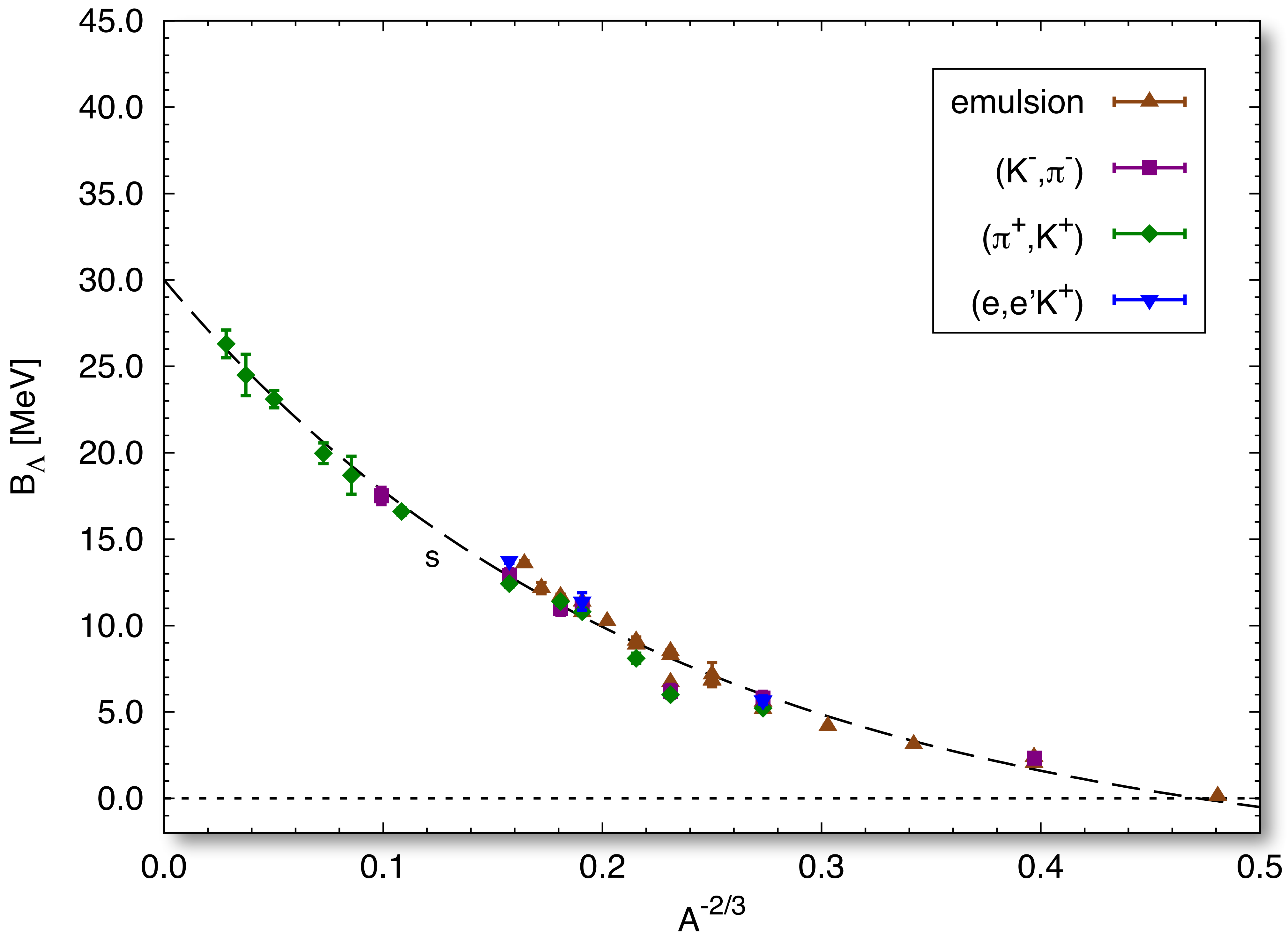
$$+ \sum_\lambda \frac{p_\lambda^2}{2m_\Lambda} + \sum_{\lambda,i} v_{\lambda i} + \sum_{\lambda,i<j} v_{\lambda ij}$$

use QMC to fit hyp. exp. data

$$B_\Lambda = E(^{A-1}Z) - E(^{A}_{\Lambda}Z)$$

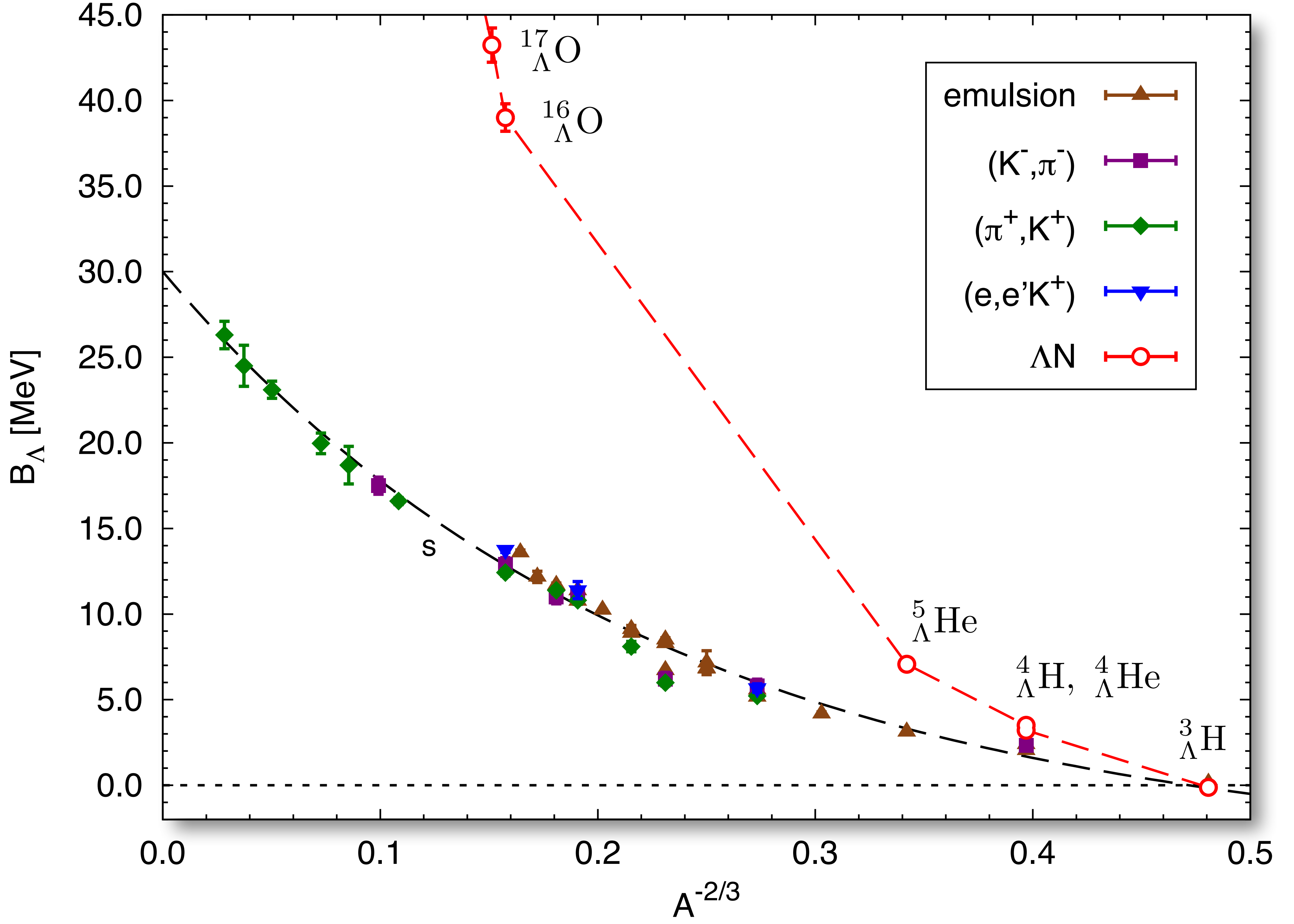
3B: no unique fit

Strangeness in nuclei



D. L., F. Pederiva, S. Gandolfi, Phys. Rev. C 89, 014314 (2014)

Strangeness in nuclei



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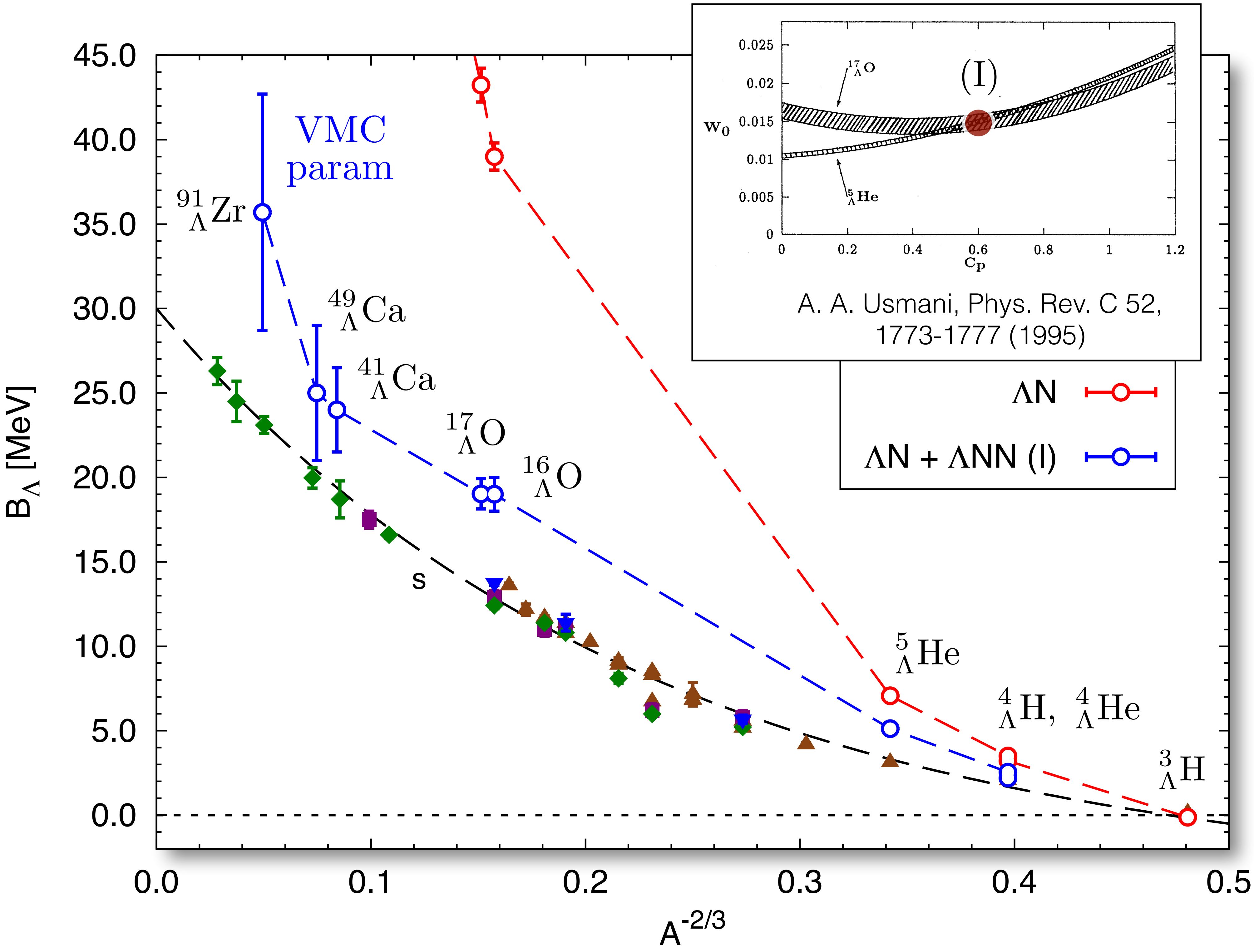

A. A. Usmani, Phys. Rev. C 52, 1773-1777 (1995)

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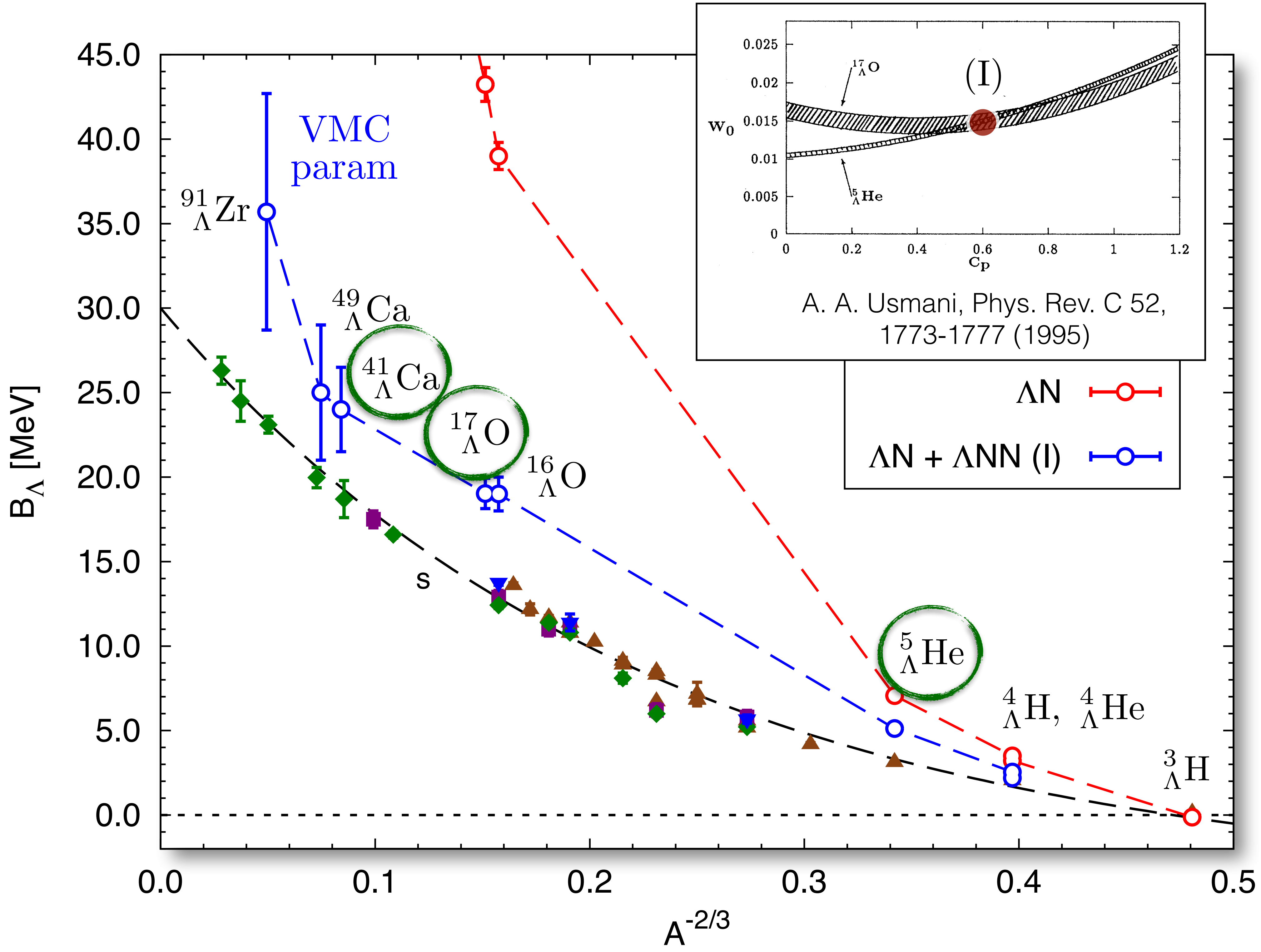

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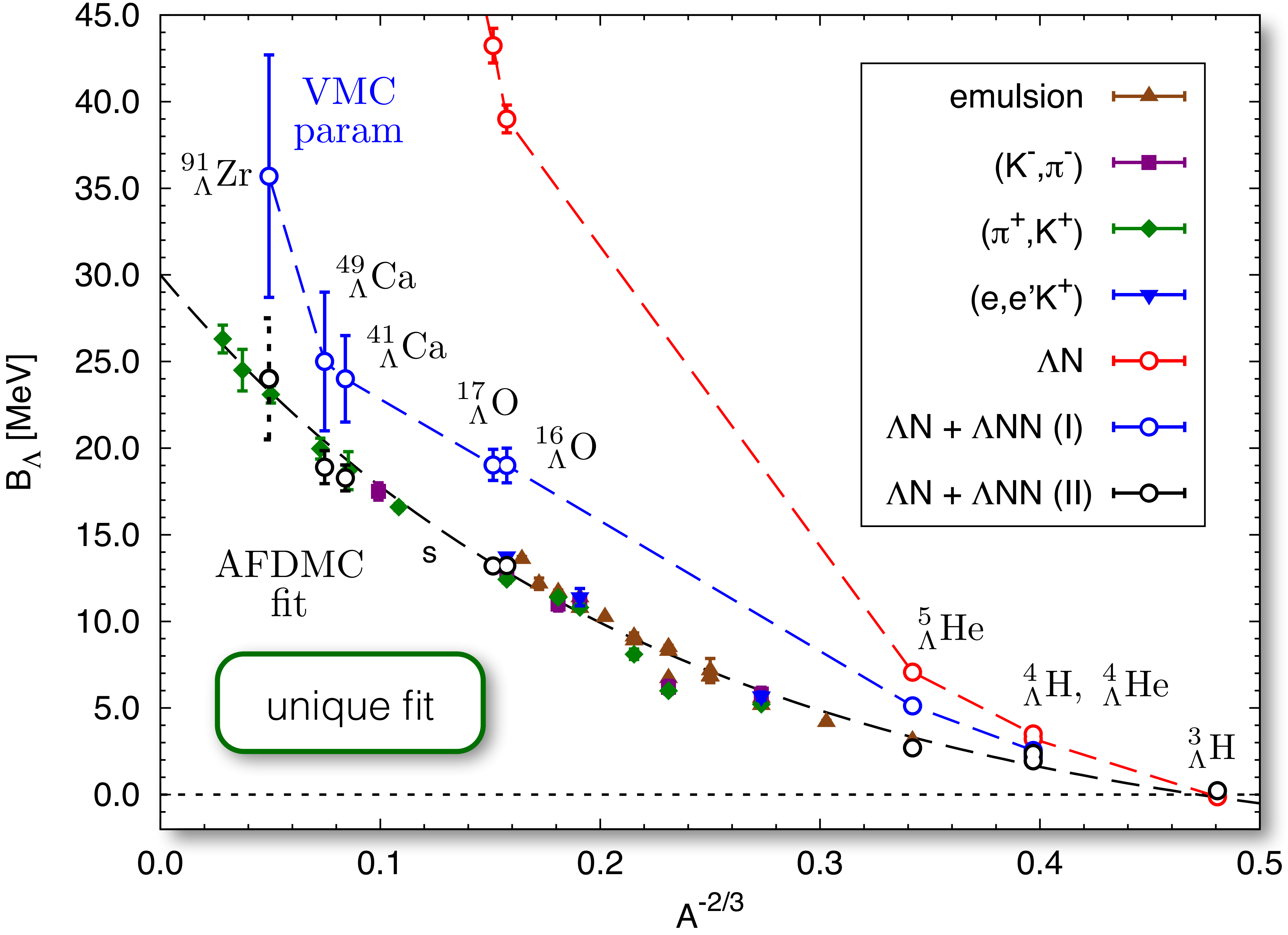

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F. Pederiva, F. Catalano, D. L., A. Lovato, S. Gandolfi, arXiv:1506.04042 (2015)

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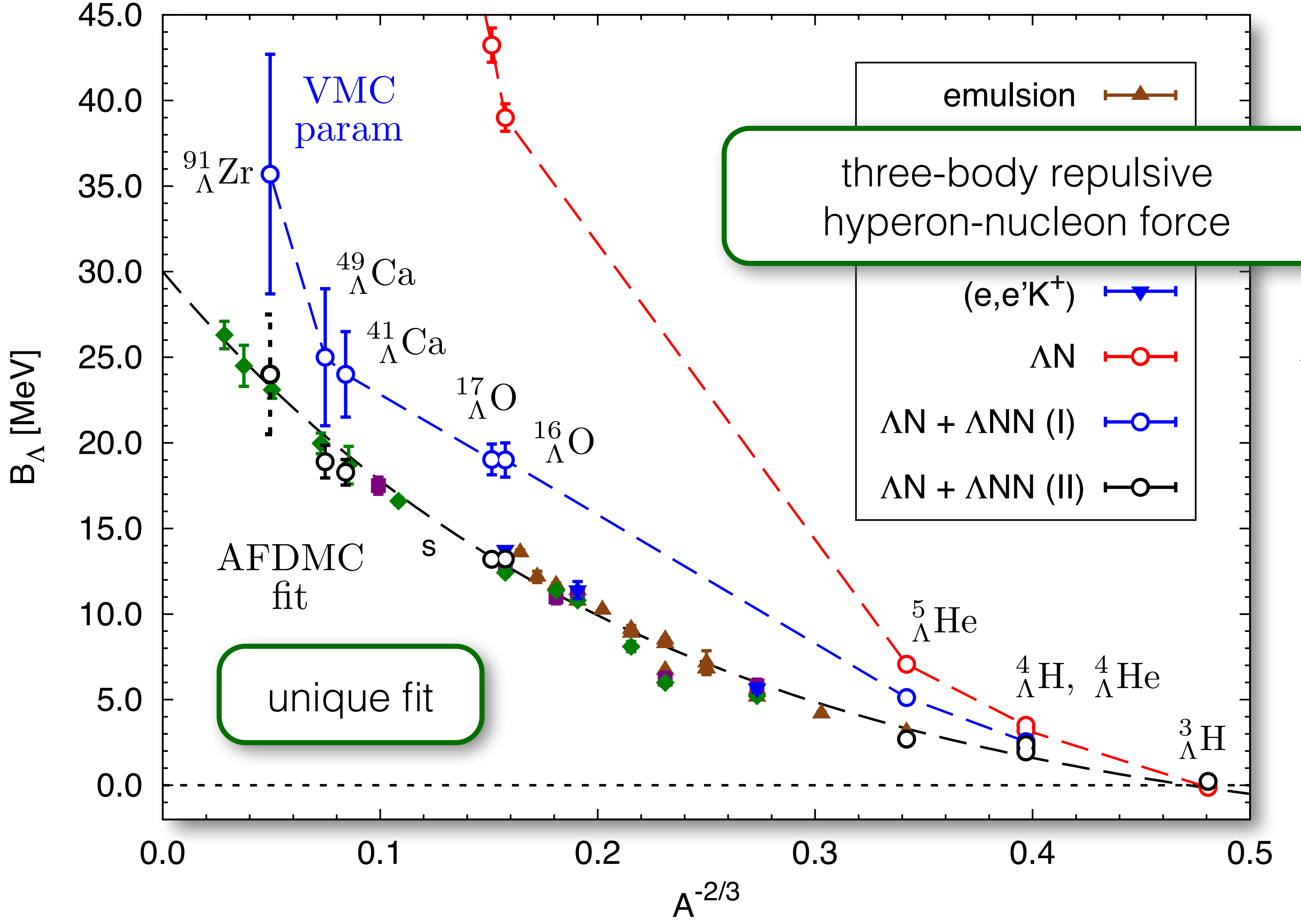

three-body repulsive
hyperon-nucleon force

unique fit

D. L., F. Pederiva, S. Gandolfi, Phys. Rev. C 89, 014314 (2014)

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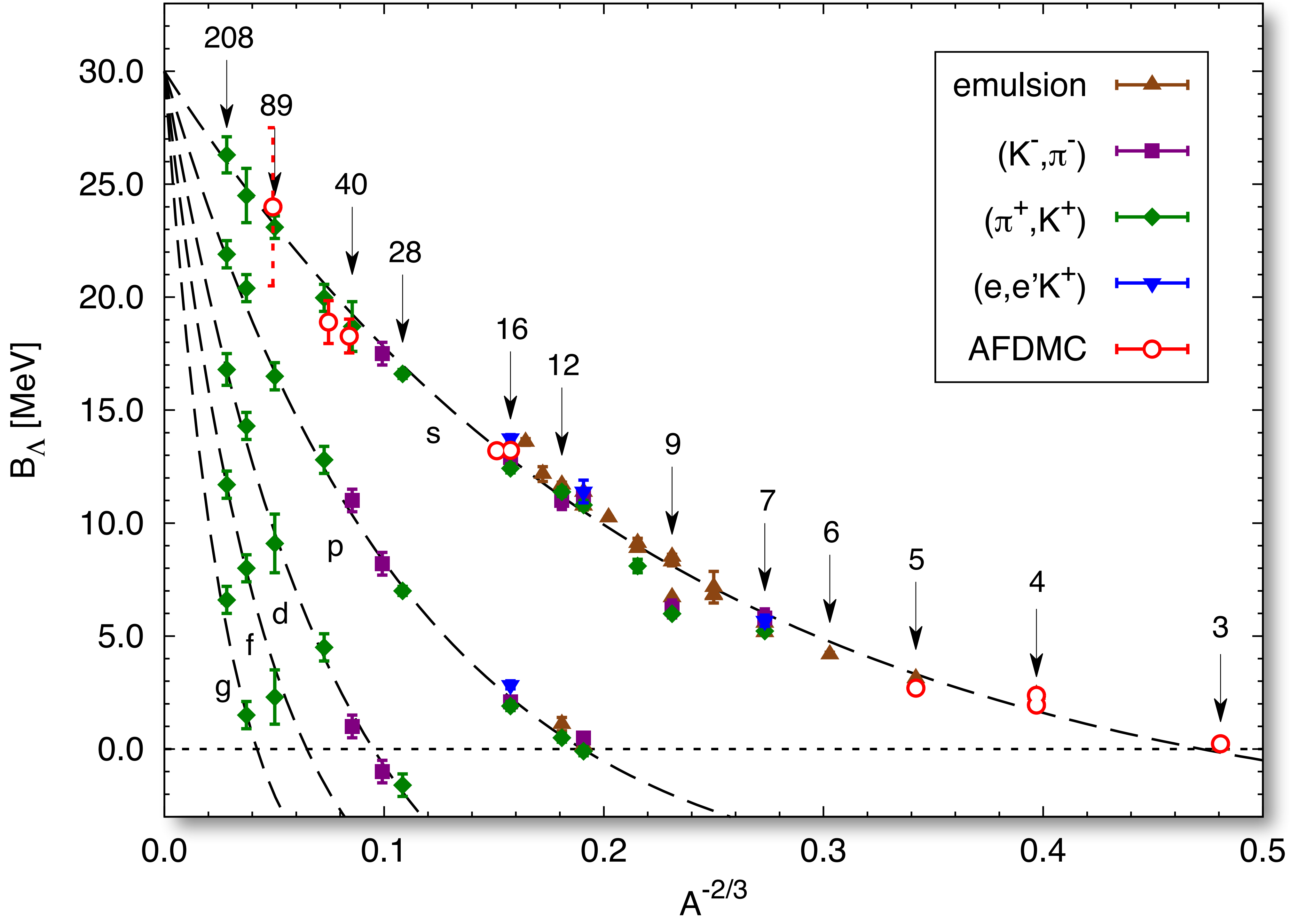

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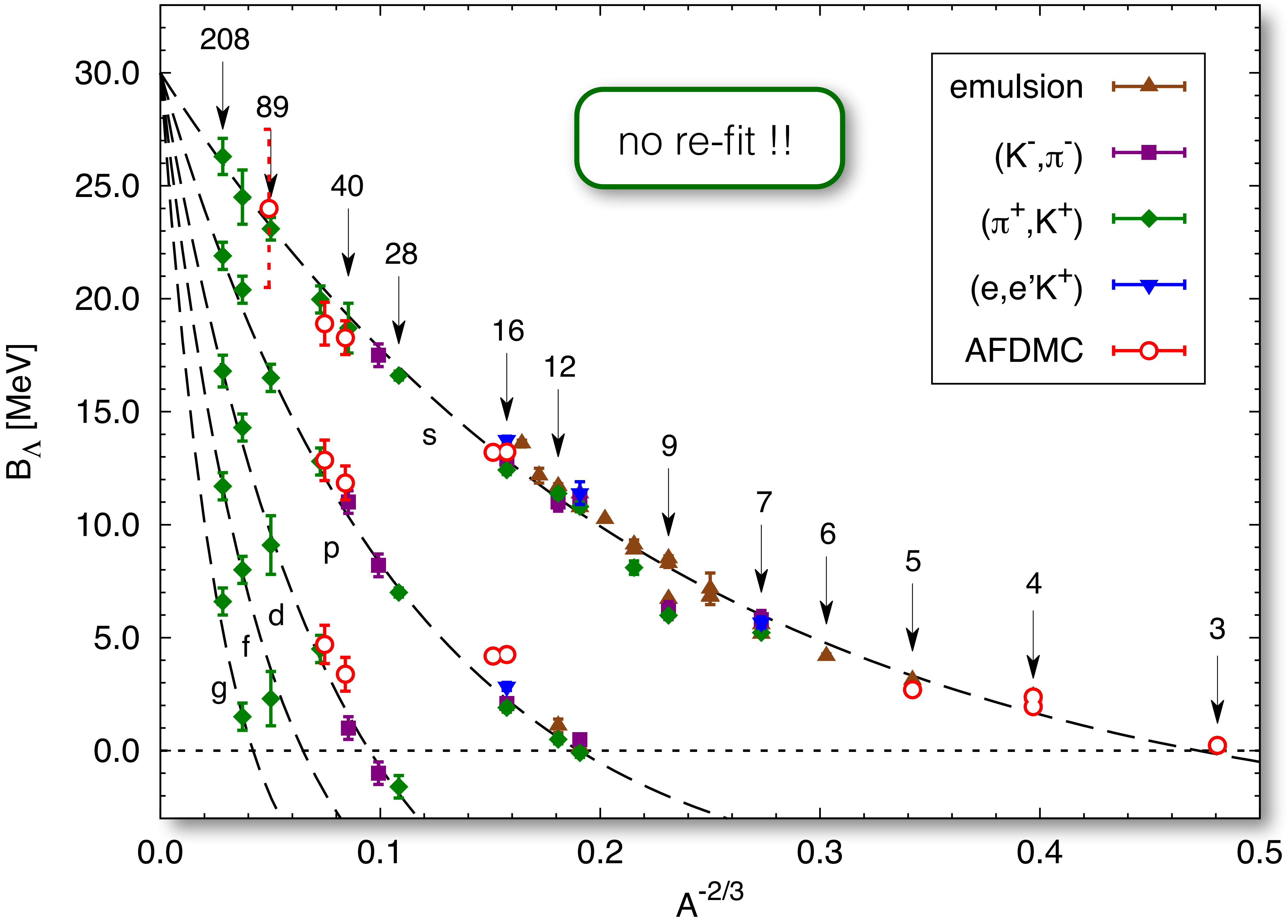

no re-fit !!

F. Pederiva, F. Catalano, D. L., A. Lovato, S. Gandolfi, arXiv:1506.04042 (2015)

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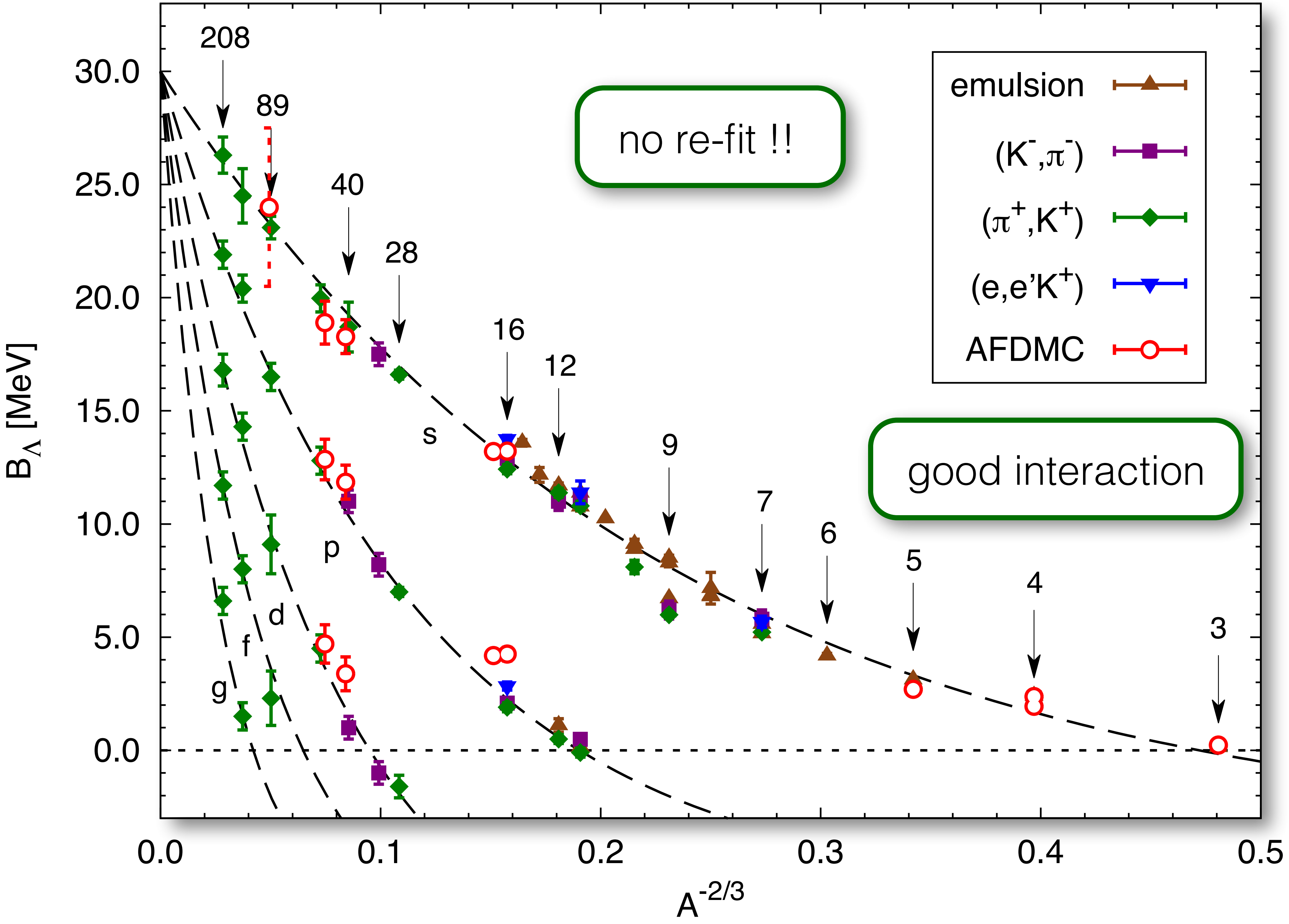
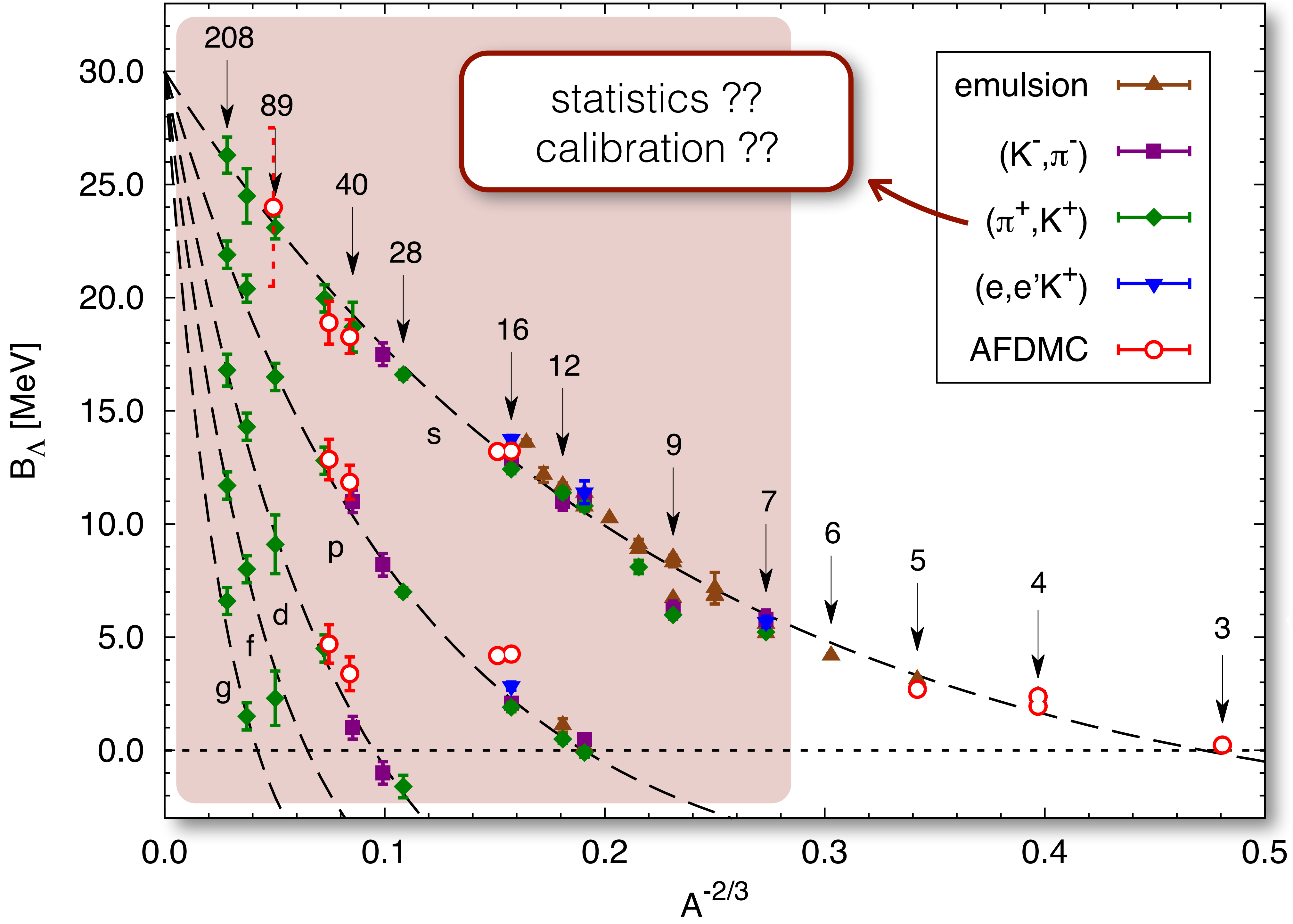


no re-fit !!

good interaction

F. Pederiva, F. Catalano, D. L., A. Lovato, S. Gandolfi, arXiv:1506.04042 (2015)

Strangeness in nuclei



statistics ??
calibration ??

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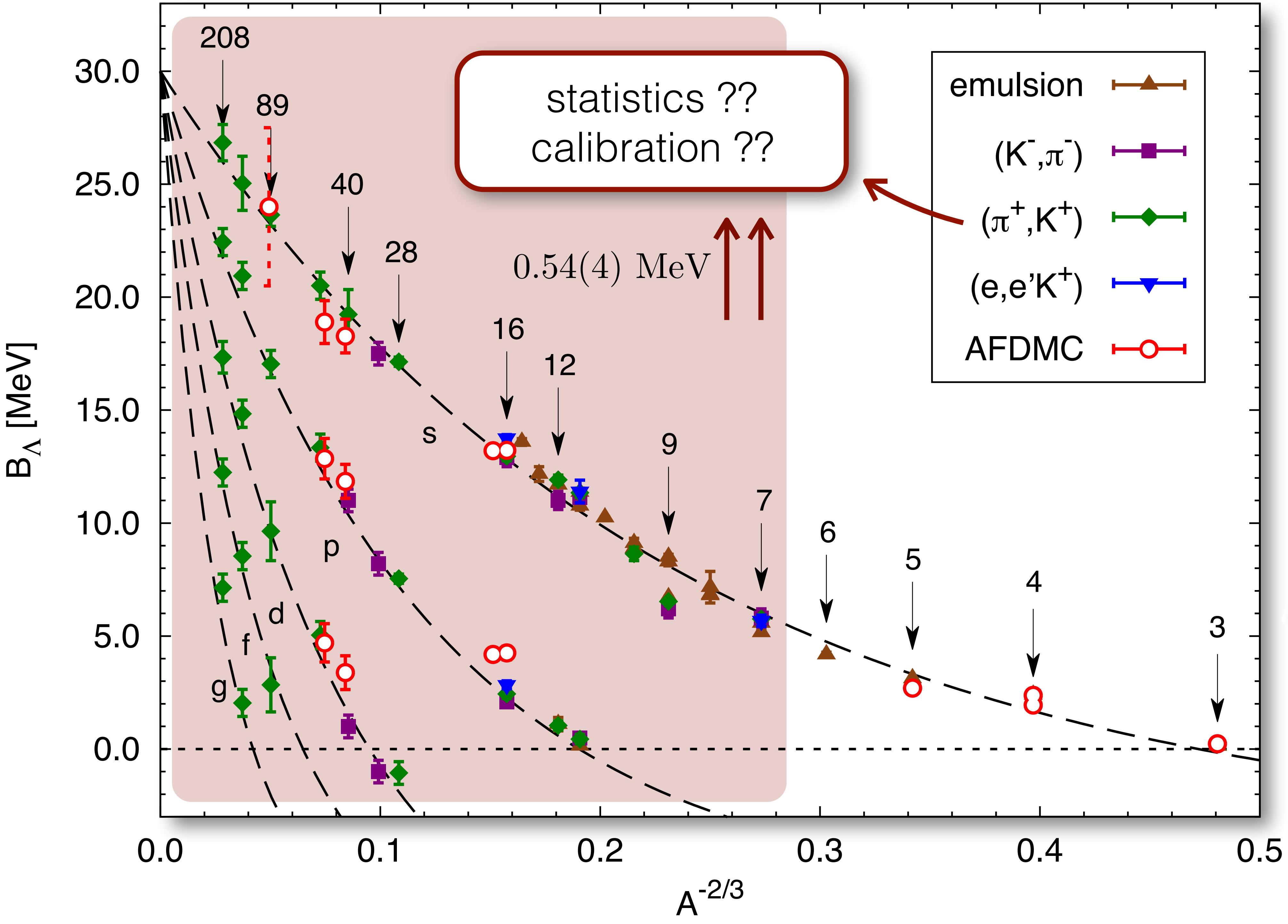

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T. Gogami, et al., arXiv:1511.04801 (2015), arXiv:1511.02472 (2015)

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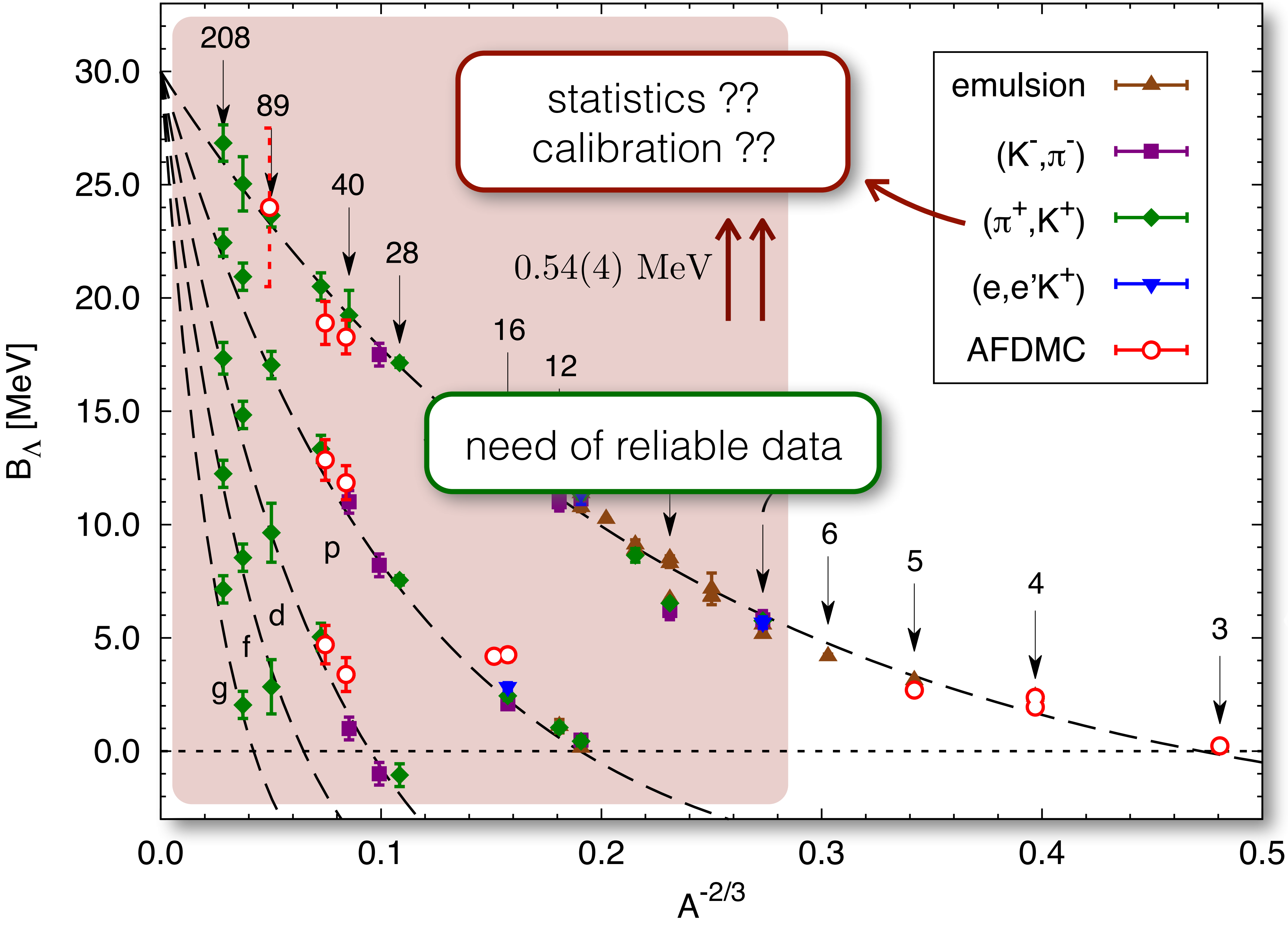


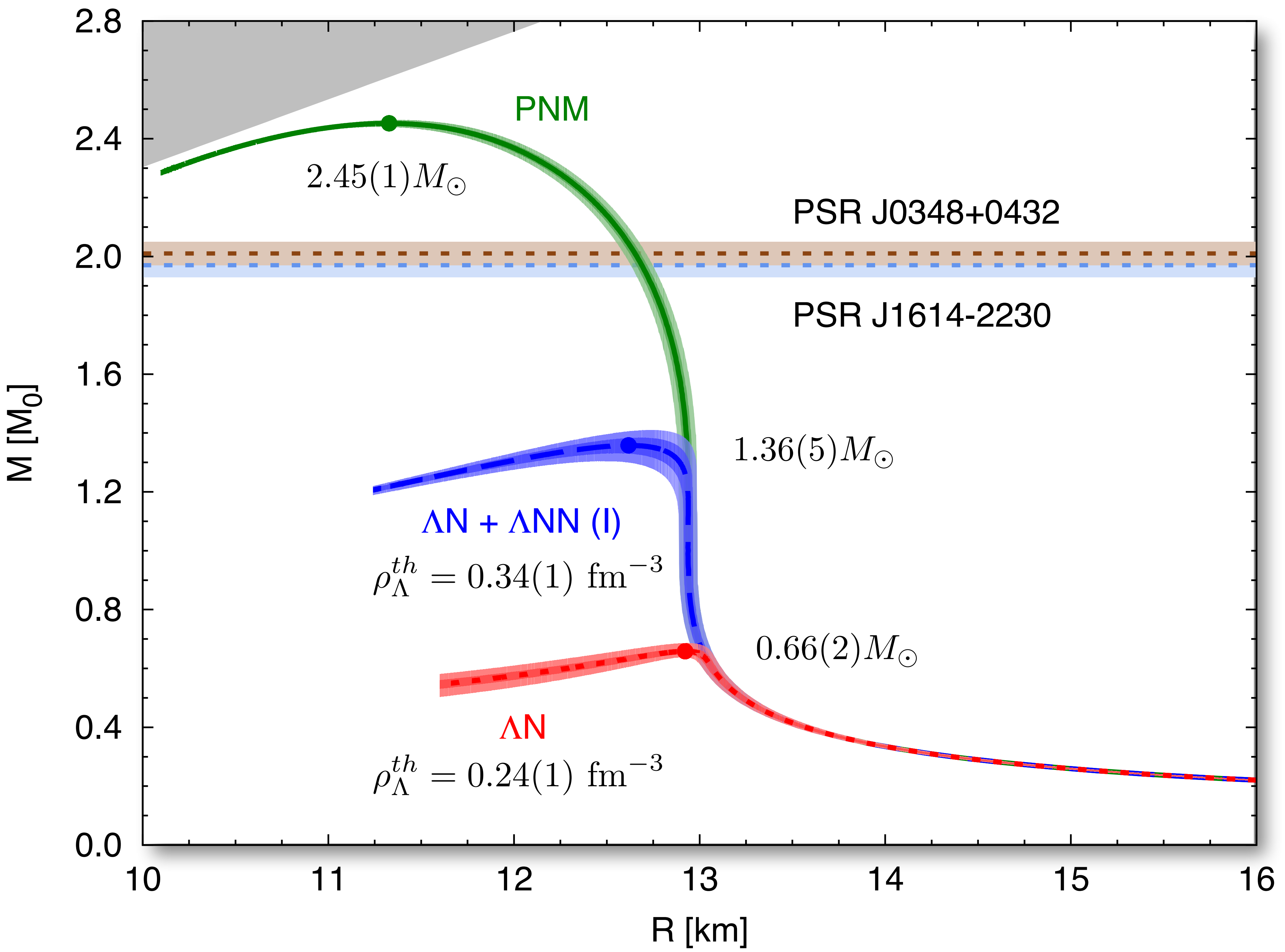
statistics ??
calibration ??

need of reliable data

F. Pederiva, F. Catalano, D. L., A. Lovato, S. Gandolfi, arXiv:1506.04042 (2015)

T. Gogami, et al., arXiv:1511.04801 (2015), arXiv:1511.02472 (2015)

Strangeness in neutron stars



D. L., A. Lovato, S. Gandolfi, F. Pederiva, Phys. Rev. Lett. 114, 092301 (2015)

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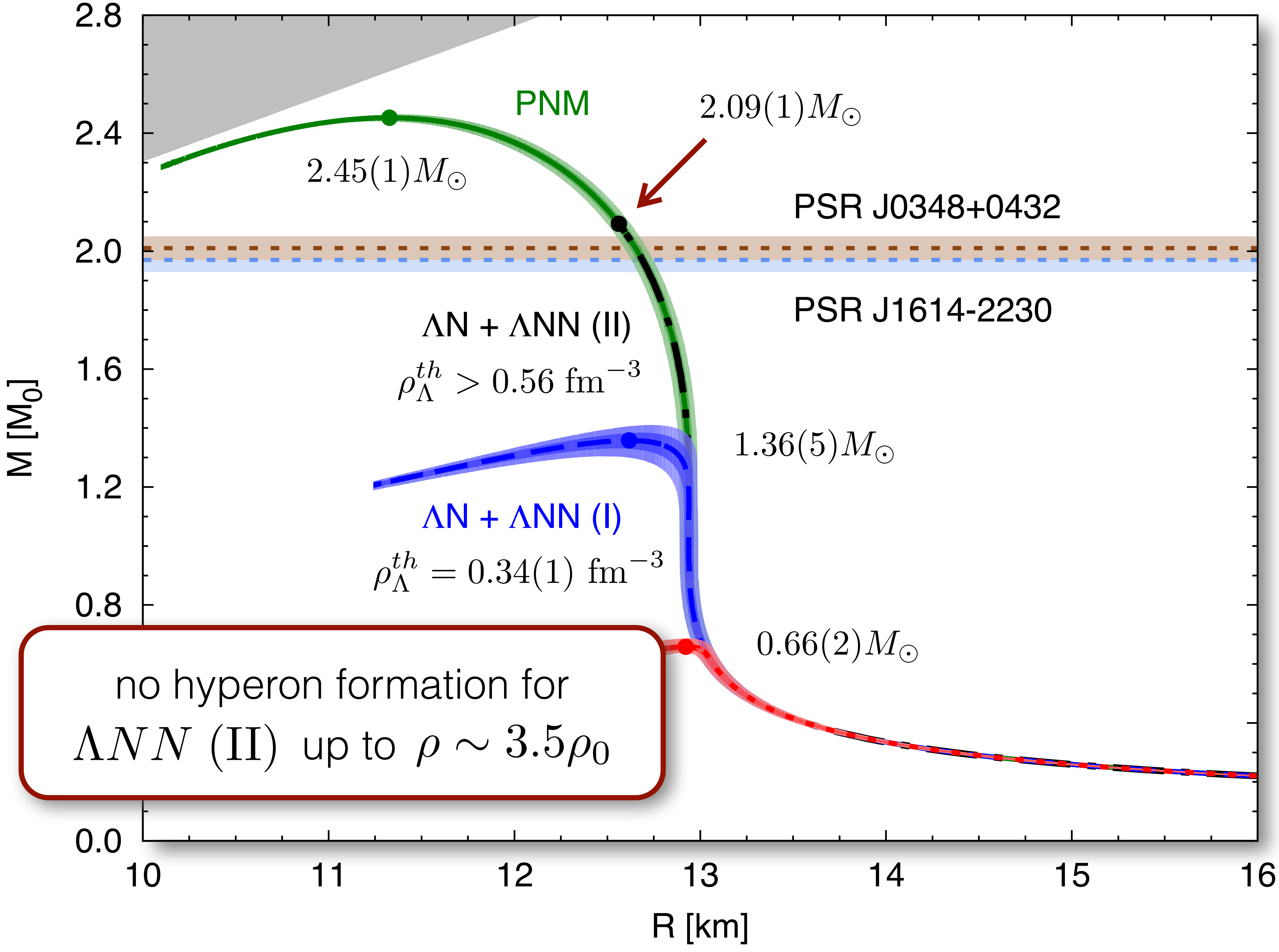
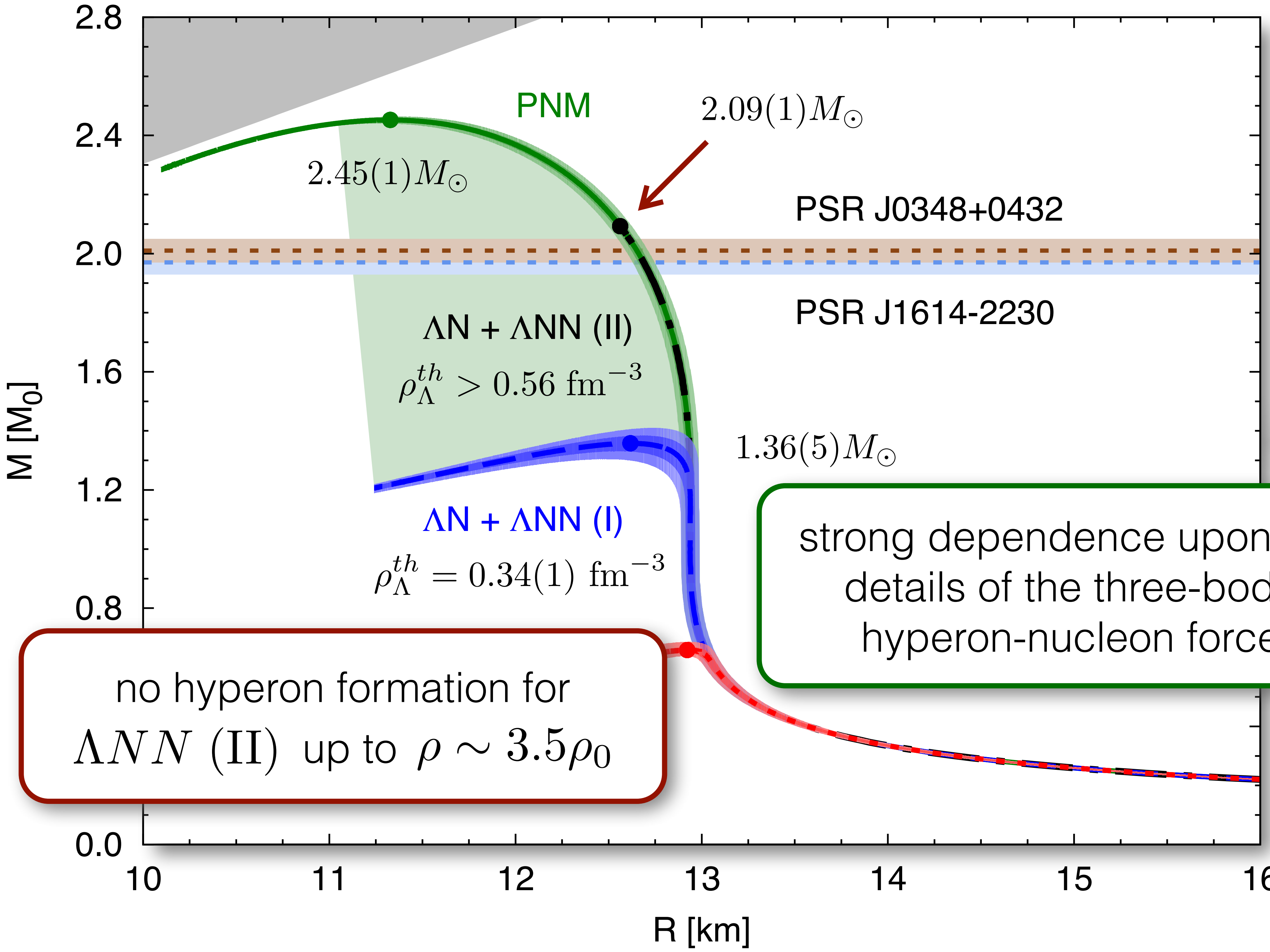


no hyperon formation for ΛNN (II) up to $\rho \sim 3.5\rho_0$

D. L., A. Lovato, S. Gandolfi, F. Pederiva, Phys. Rev. Lett. 114, 092301 (2015)

Strangeness in neutron stars



no hyperon formation for ΛNN (II) up to $\rho \sim 3.5 \rho_0$

strong dependence upon the details of the three-body hyperon-nucleon force

D. L., A. Lovato, S. Gandolfi, F. Pederiva, Phys. Rev. Lett. 114, 092301 (2015)

Strangeness in nuclei

3-body interaction → fit on symmetric hypernuclei

ΛNN force: no dependence on singlet or triplet nucleon isospin state

Strangeness in nuclei

3-body interaction → fit on symmetric hypernuclei

ΛNN force: no dependence on singlet or triplet nucleon isospin state

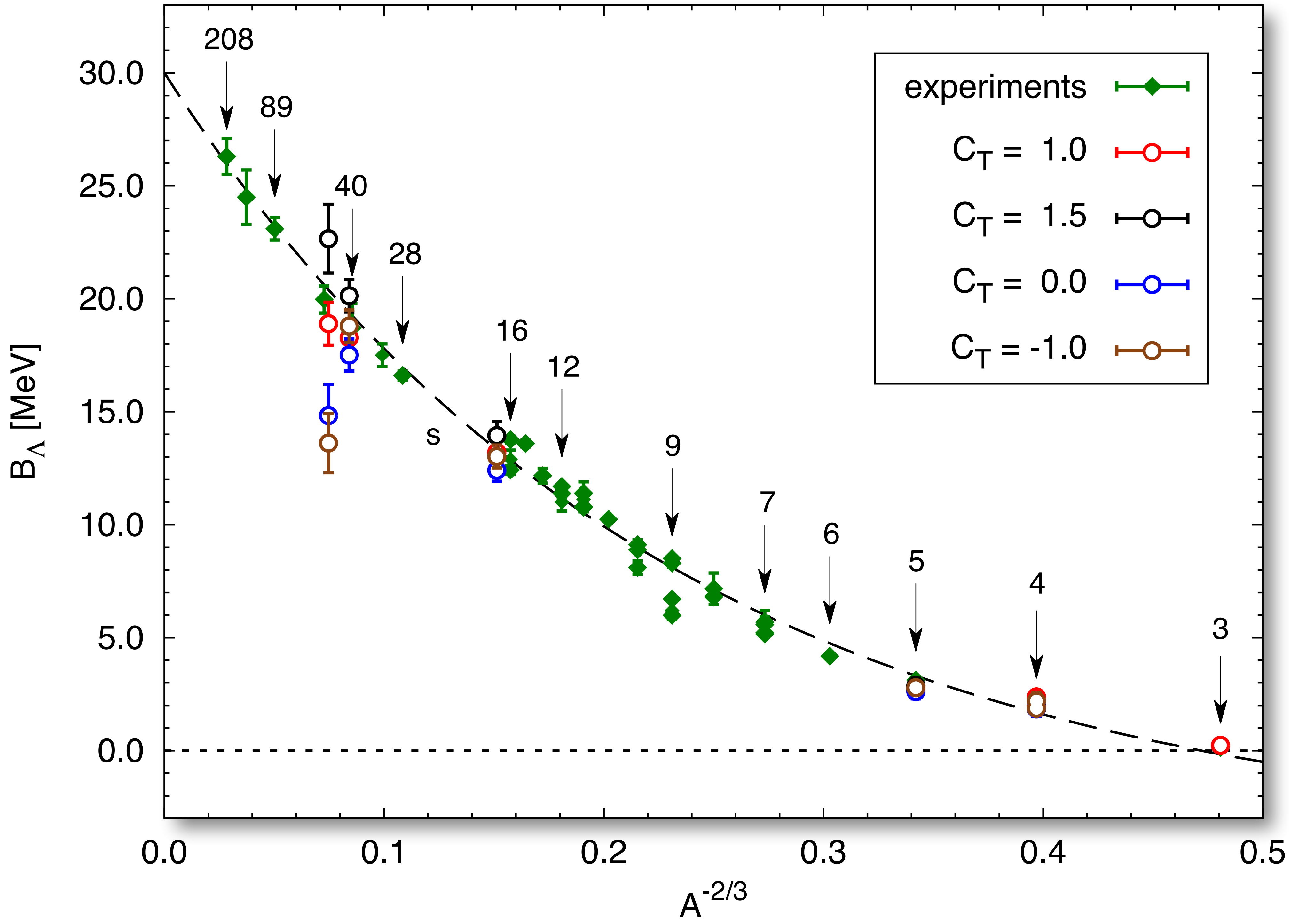
$$\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j = -3\,\mathcal{P}^{T=0} + \mathcal{P}^{T=1} \longrightarrow -3\,\mathcal{P}^{T=0} + C_T\,\mathcal{P}^{T=1}$$

isospin projectors

control parameter:
strength and sign of the nucleon isospin triplet channel

sensitivity study:
light- & medium-heavy hypernuclei

Strangeness in nuclei



F. Pederiva, F. Catalano, D. L., A. Lovato, S. Gandolfi, arXiv:1506.04042 (2015)

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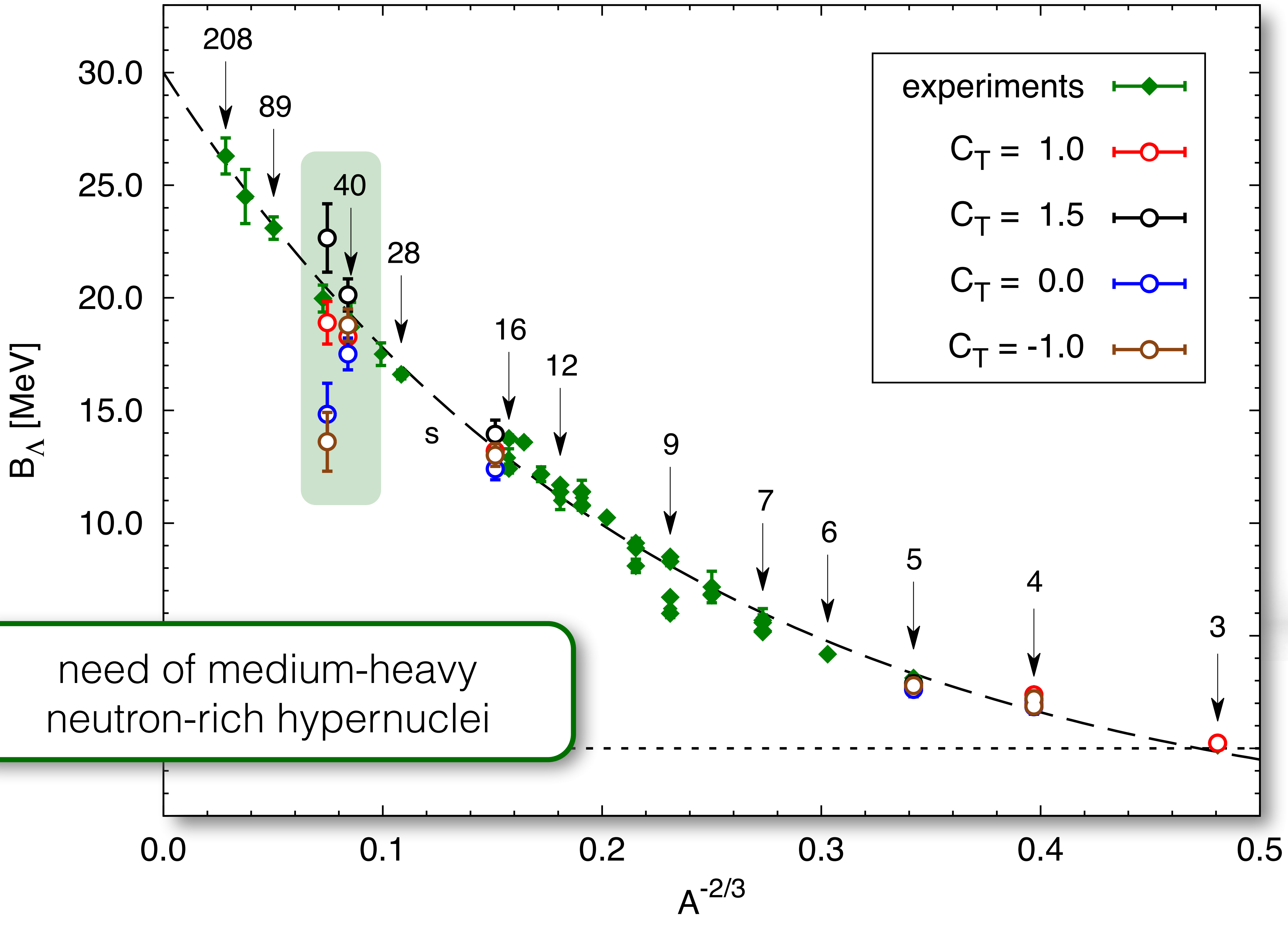


need of medium-heavy
neutron-rich hypernuclei

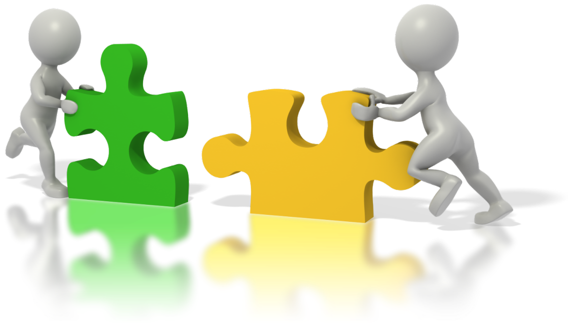
F. Pederiva, F. Catalano, D. L., A. Lovato, S. Gandolfi, arXiv:1506.04042 (2015)

Conclusions

- ✓ The observation of massive neutron stars reopened the debate about the presence of hyperons in the inner core
  - no general agreement among theoretical calculations
  - hyperon puzzle not yet solved: new hints?

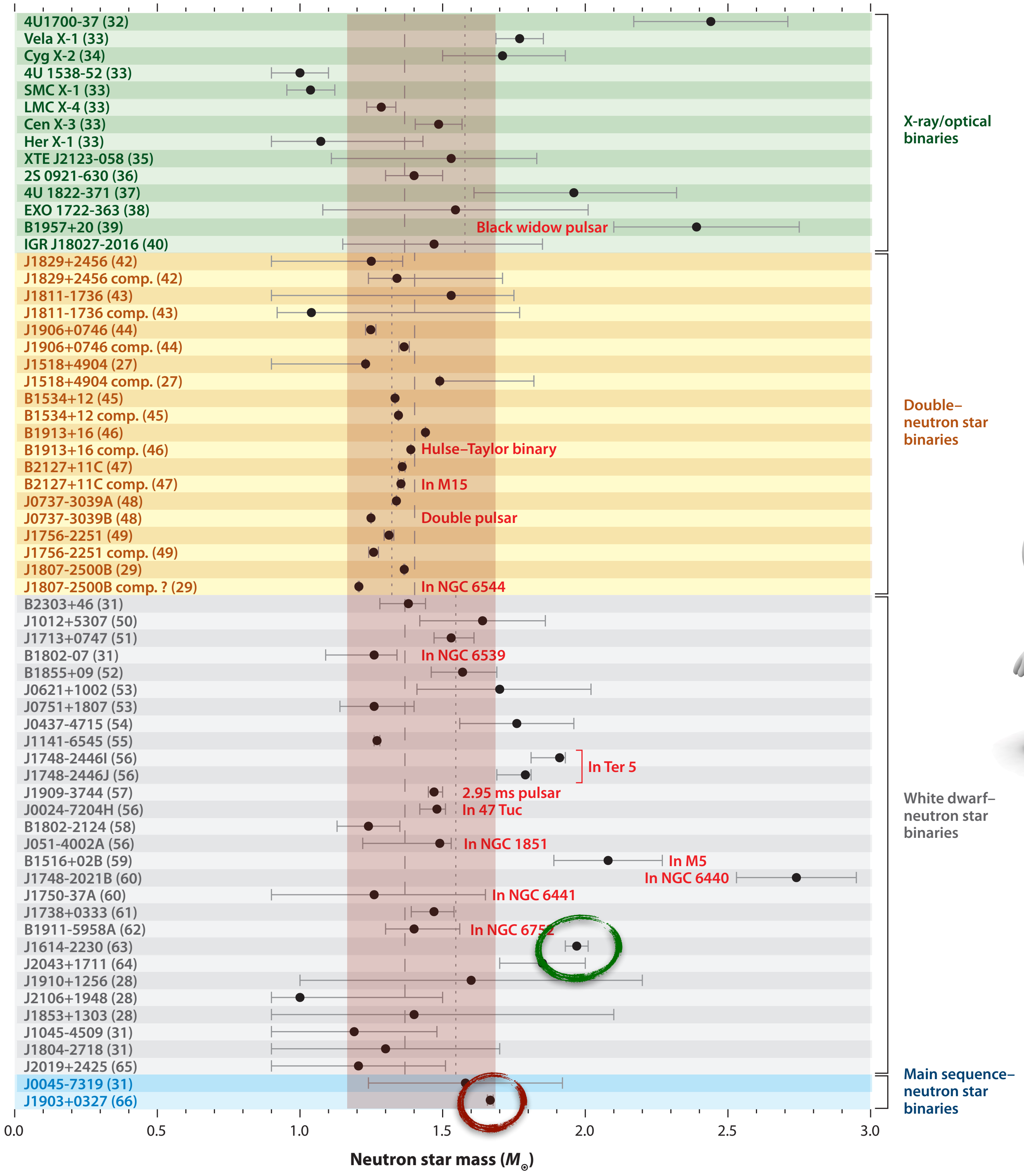
- ✓ We developed a quantum Monte Carlo algorithm to study finite and infinite hypernuclear systems:
  - a repulsive three-body ΛNN force is needed to reproduce the experimental Λ separation energies for light- and medium-heavy hypernuclei
  - the predicted neutron star equation of state and maximum mass strongly depend upon the details of the three-body ΛNN force

- ✓ Need of more constraints on hypernuclear interactions before drawing conclusions on the role played by hyperons in neutron stars
  - accurate experimental investigation: medium-heavy neutron-rich hypernuclei
  - accurate theoretical investigation



Thank you!!

Backup: the hyperon puzzle



J. M. Lattimer, Annu. Rev. Nucl. Part. Sci. 2012. 62:485–515

< 2010:

$$M_{\max} = 1.67(2) M_\odot$$

D. J. Champion et al.
Science 320, 1309 (2008)

2010:

$$M_{\max} = 1.97(4) M_\odot$$

P. B. Demorest et al.
Nature 467, 1081 (2010)

2013:

$$M_{\max} = 2.01(4) M_\odot$$

J. Antoniadis et al.
Science 340, 1233232 (2013)

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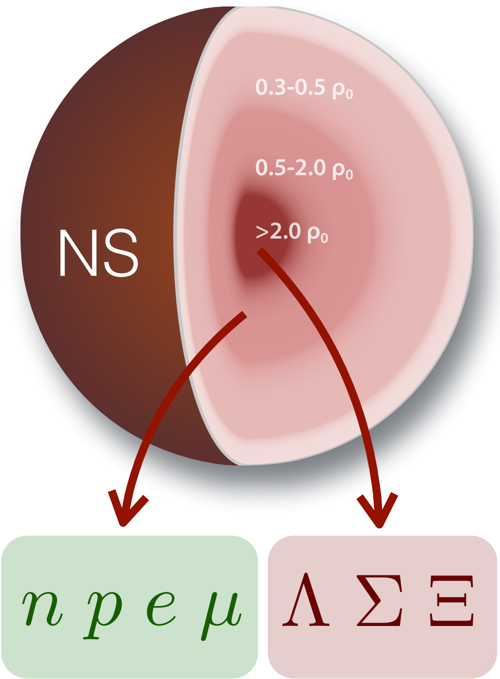
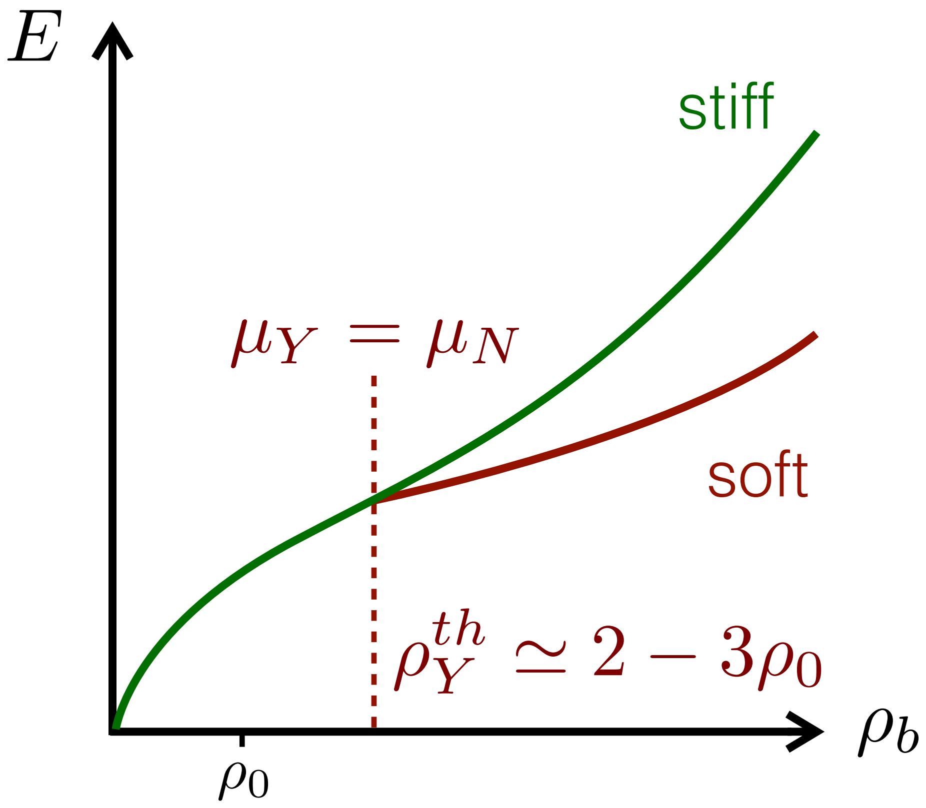
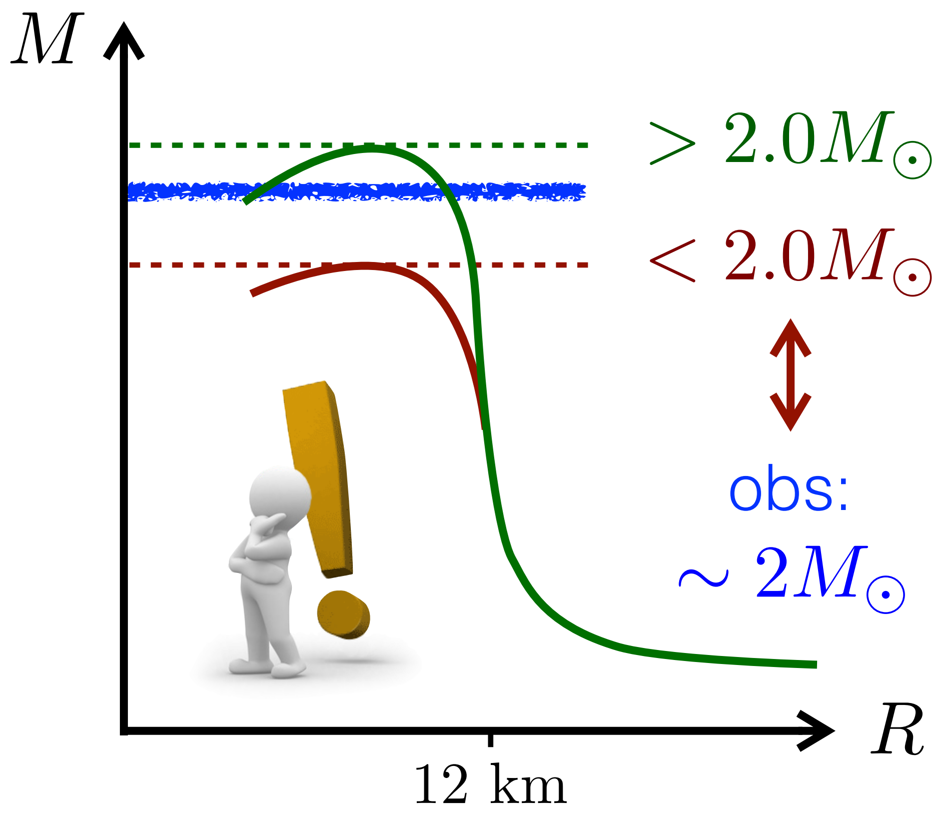






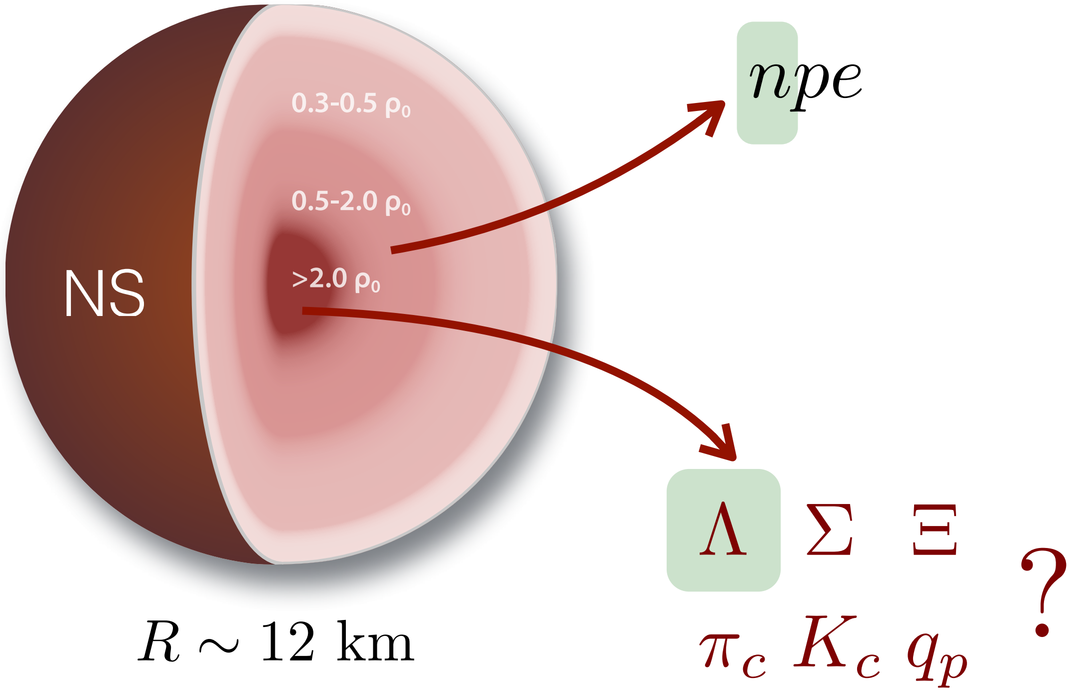
Hyperon puzzle

- ✓ Interactions poorly known
- ✓ Approximated theoretical many-body techniques

QMC → YN interaction

scattering data:	binding energies:
$NN : \sim 4300$	$\text{nuc} : \sim 3340$
$YN : \sim 52$	$\Lambda\ \text{hyp} : \sim 41$
	$\Lambda\Lambda\ \text{hyp} : \sim 5$
	$\Sigma\ \text{hyp} : \sim (1)$

Backup: the hyperon puzzle



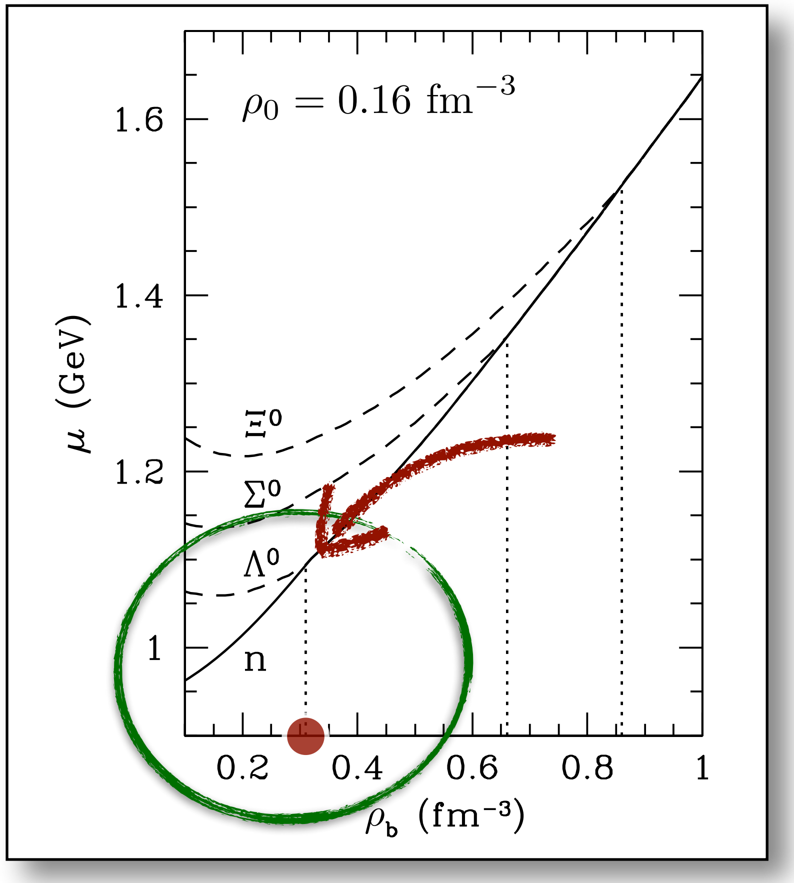
npe

$\Lambda \quad \Sigma \quad \Xi$

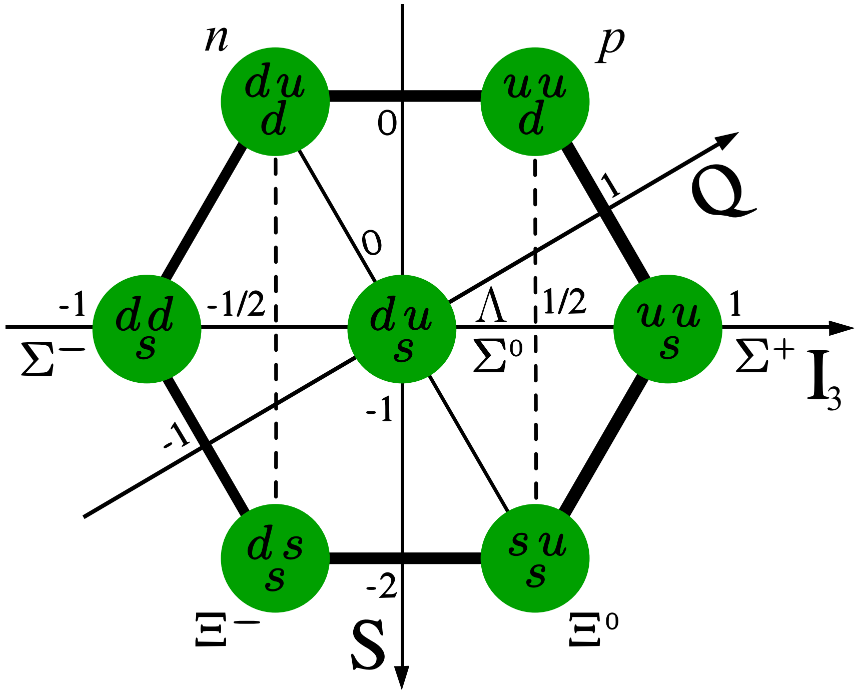
$\pi_c \; K_c \; q_p$?

$R \sim 12$ km

$M \sim 1.4\ M_\odot$



P. Haensel, A. Y. Potekhin, D. G. Yakovlev
Neutron Stars 1, Springer 2007



940 MeV

1116 MeV

1200 MeV

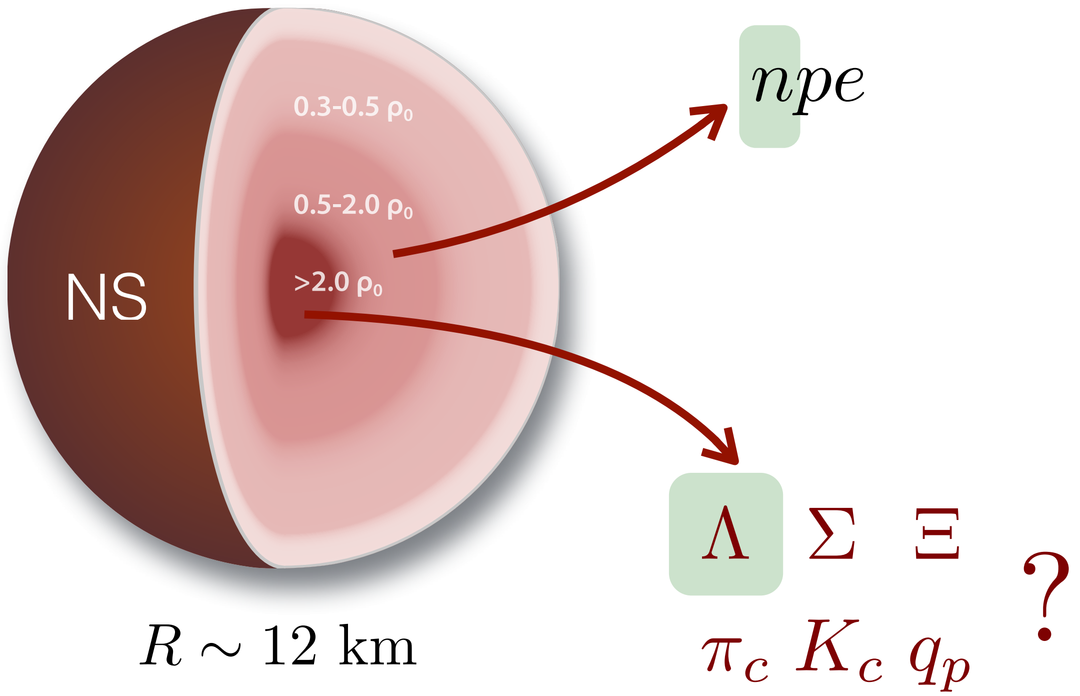
1300 MeV

$$Q = -1 \;:\; \mu_{b^-} = \mu_n + \mu_e$$

$$Q = \quad 0 \;:\; \mu_{b^0} = \mu_n$$

$$Q = +1 \;:\; \mu_{b^+} = \mu_n - \mu_e$$

Backup: the hyperon puzzle



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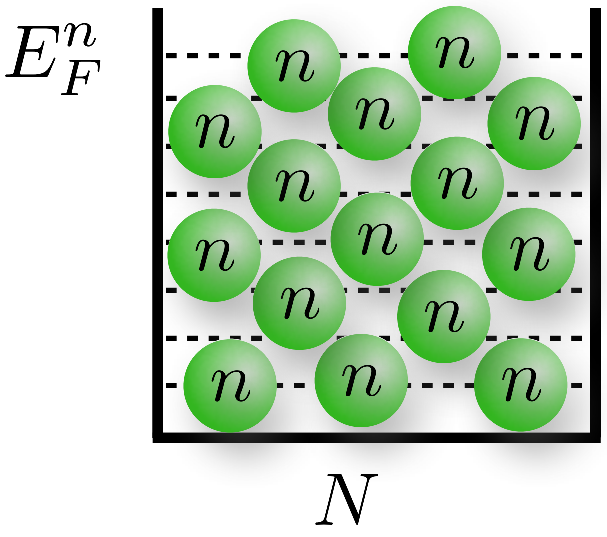
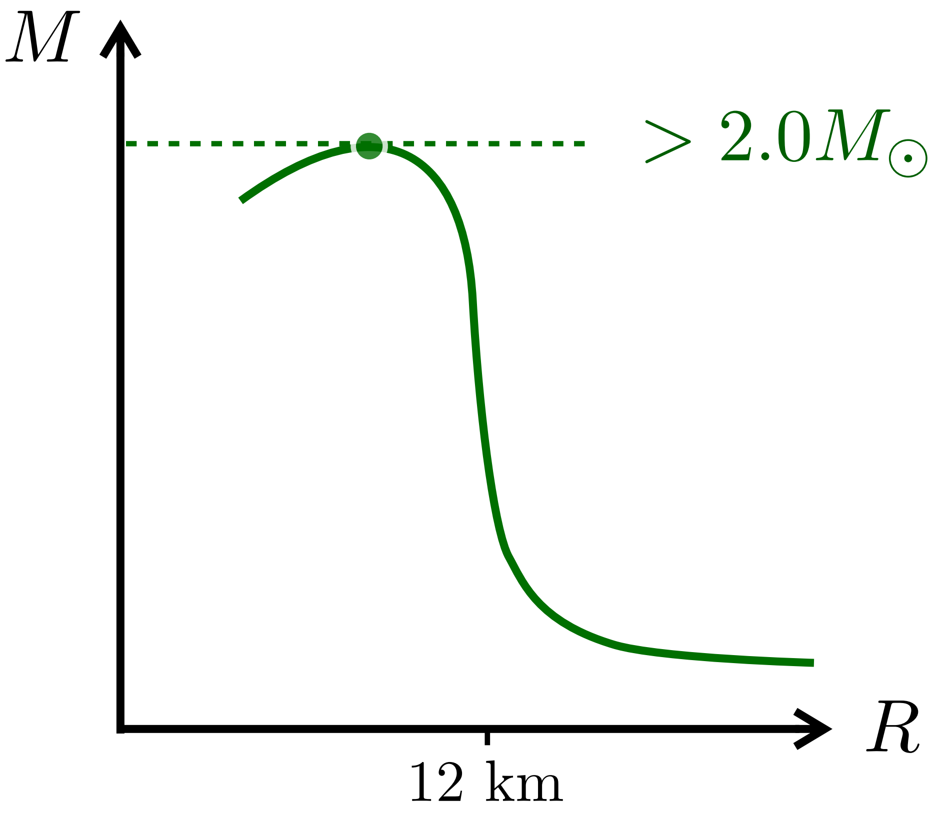
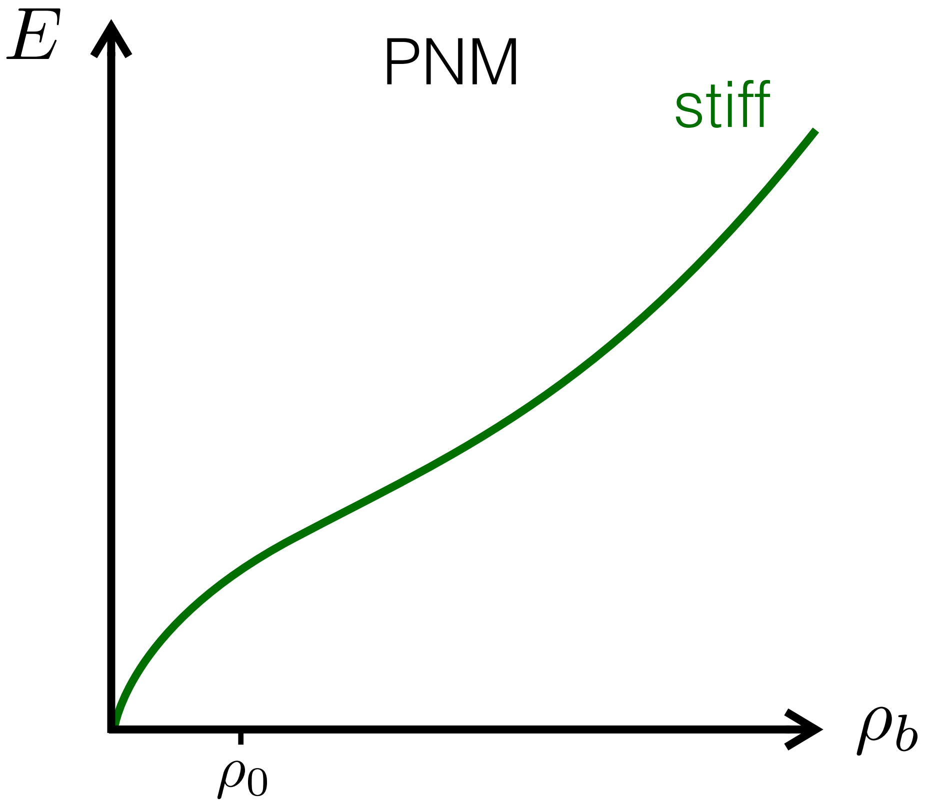


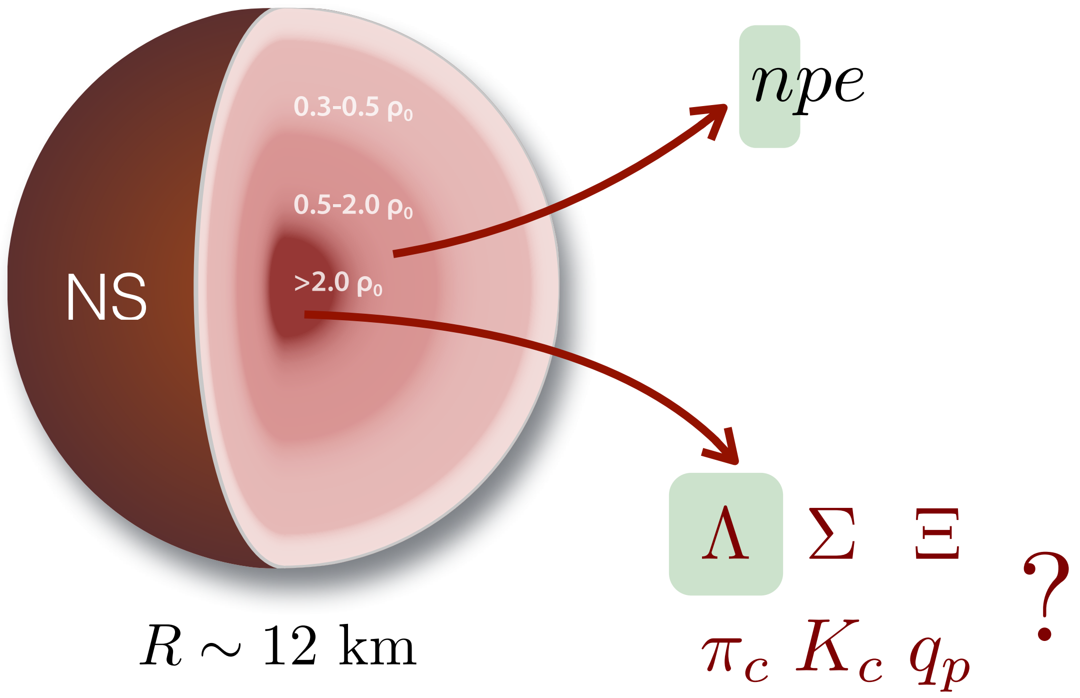


TOV

1:1

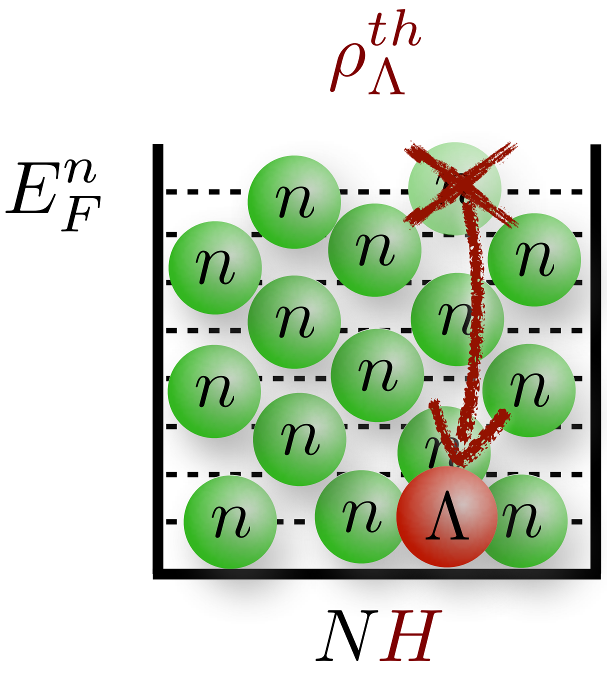


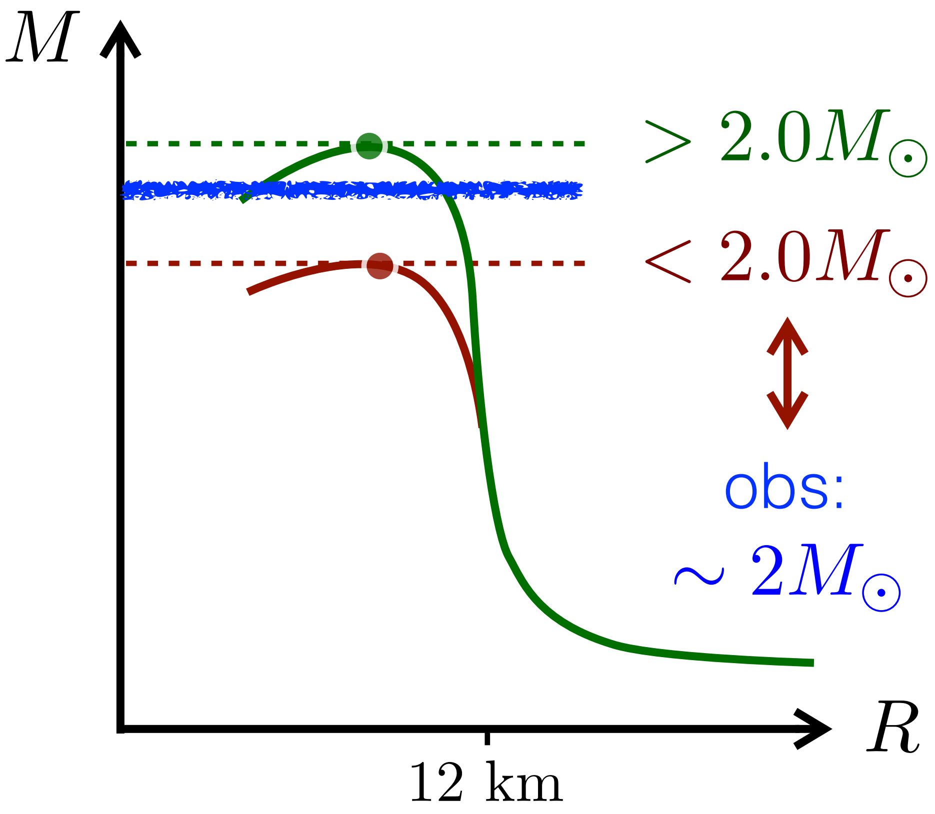
Backup: the hyperon puzzle



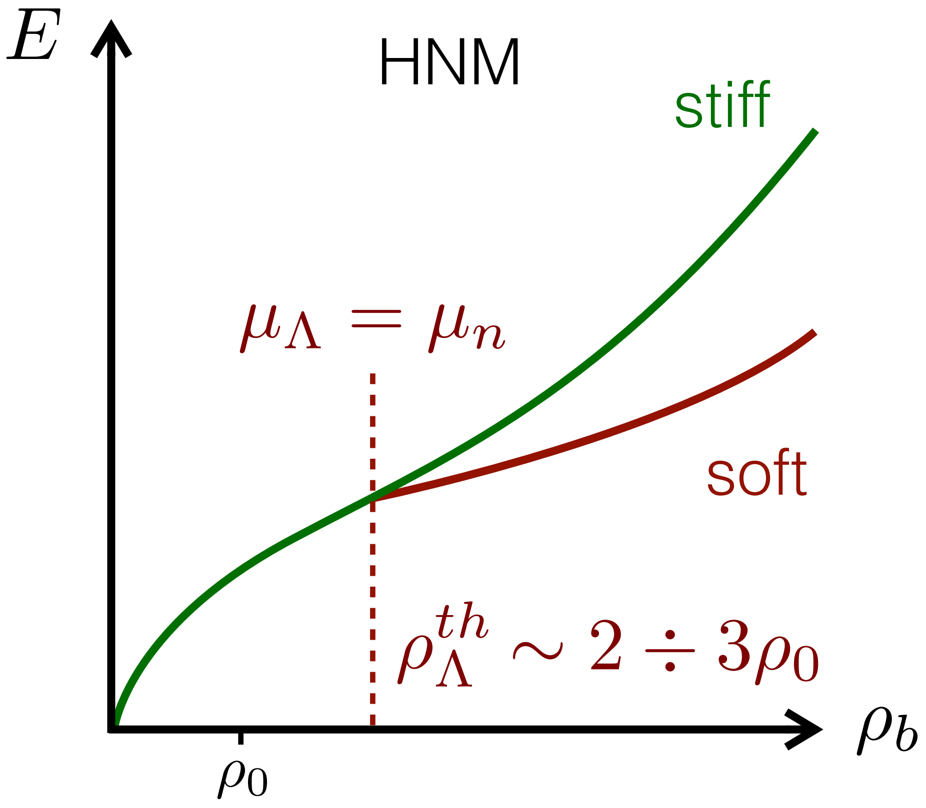
$R \sim 12$ km

$M \sim 1.4\ M_\odot$





$\xleftrightarrow[1:1]{\text{TOV}}$



Backup: the hyperon puzzle

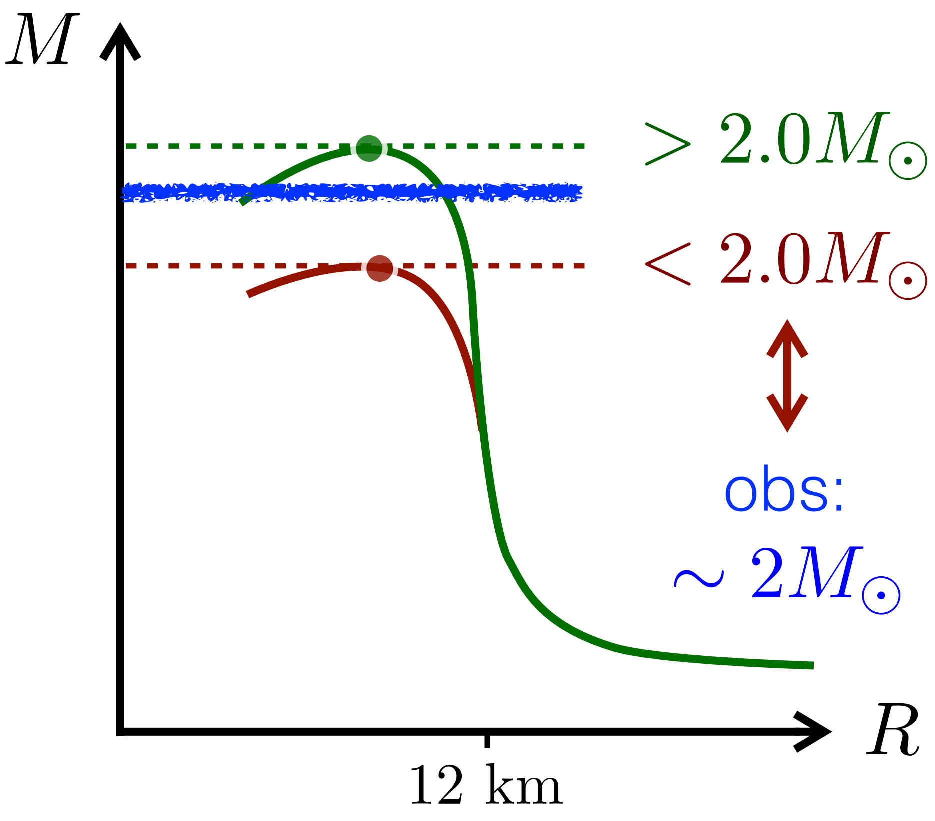
Hyperon puzzle

- ✓ Theoretical indication for hyperons in NS core: softening of the EOS
- ✓ Observation of massive NS: stiff EOS
- ✓ Magnitude of the softening: strongly model dependent

Problems

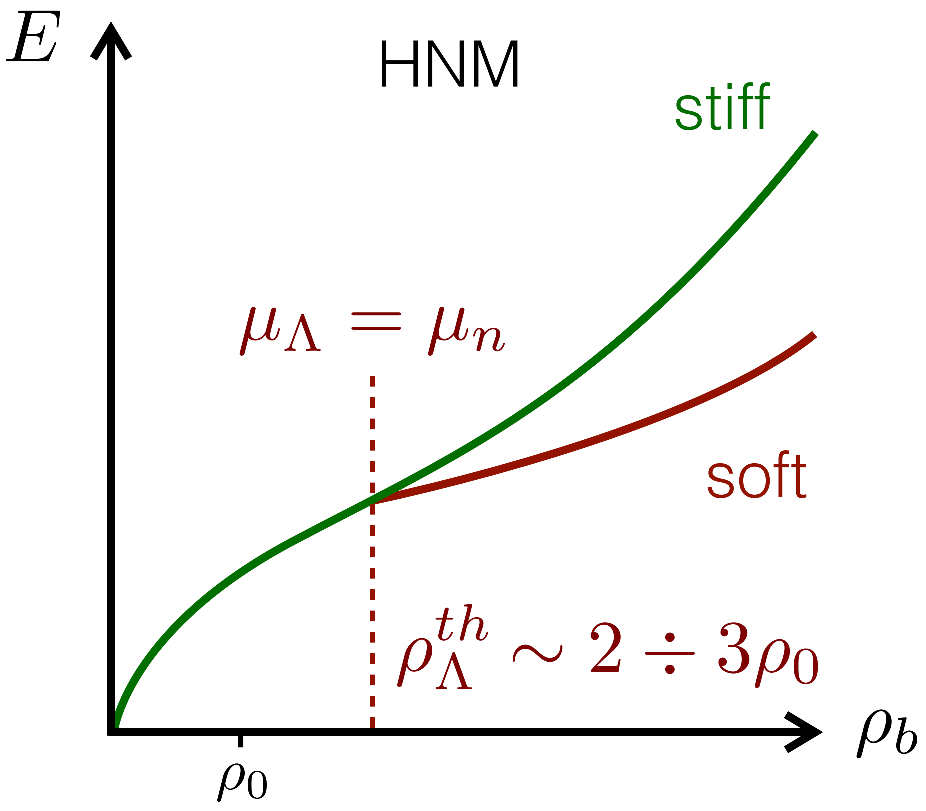
- ✓ Interactions poorly known
- ✓ Non trivial many-body problem: very dense system, strong interactions

QMC → HN interaction



TOV

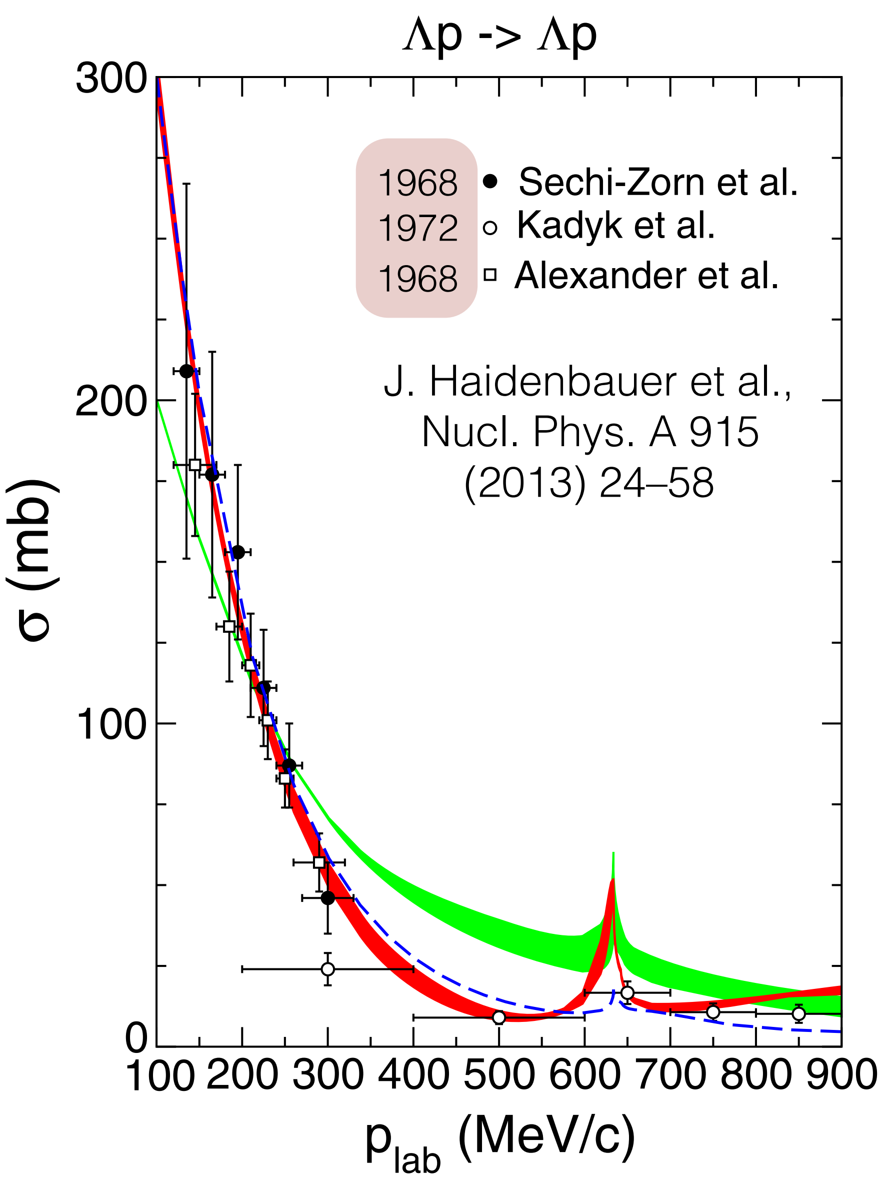
1:1



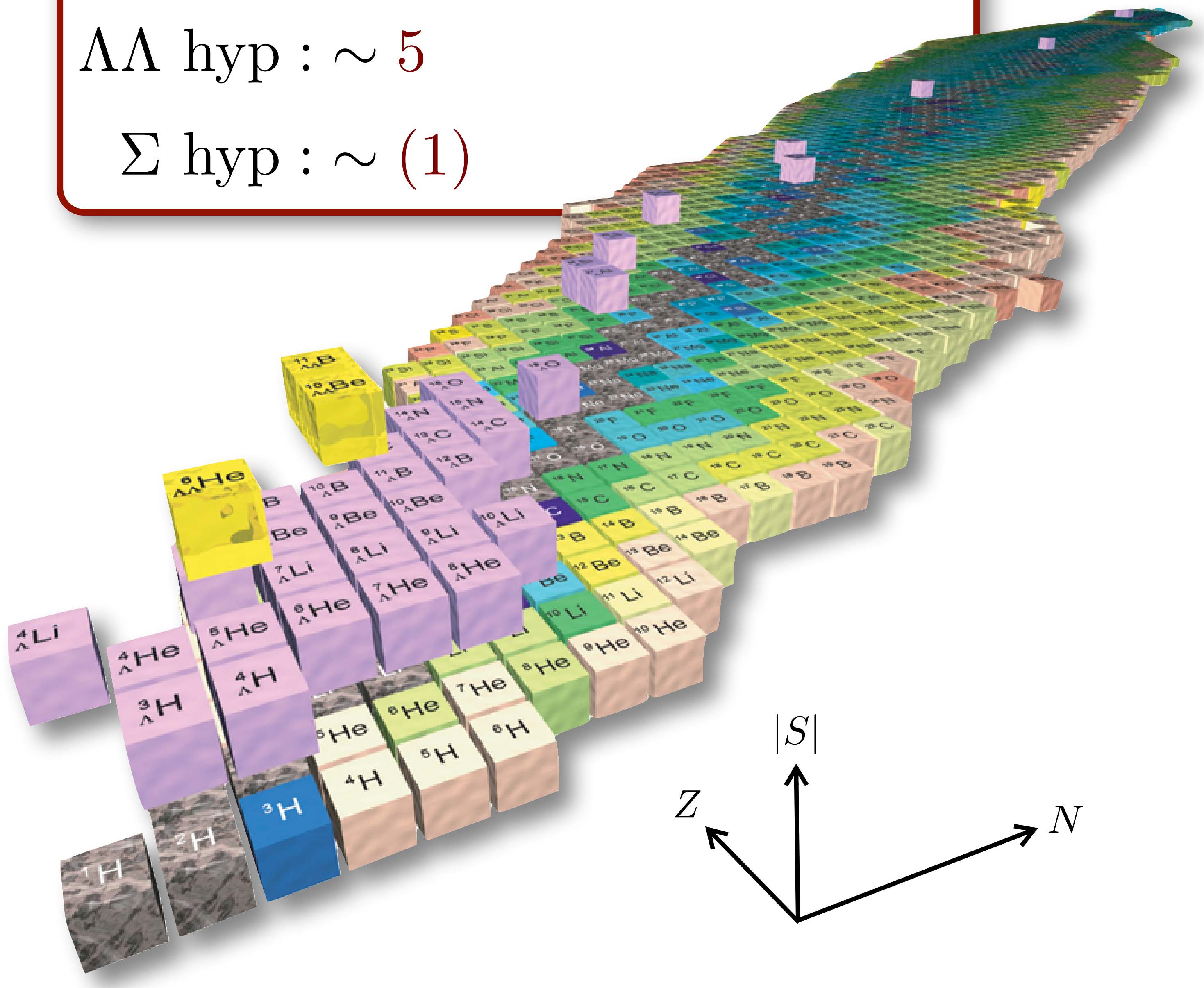
Backup: the hyperon puzzle

lack of experimental data !!

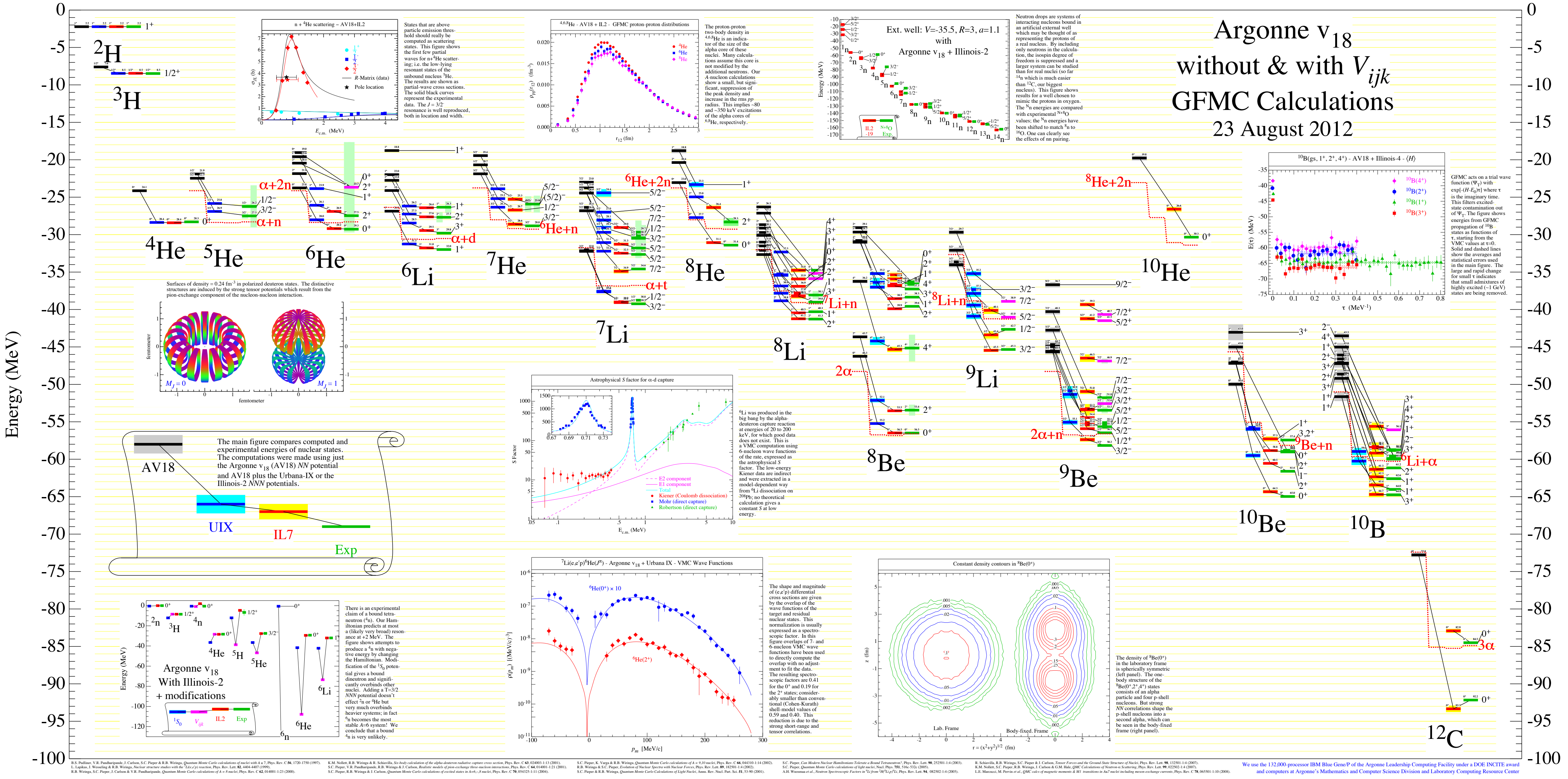
binding energies:	scattering data:
$\text{nuc} : \sim 3340$	$NN : \sim 4300$
$\Lambda\ \text{hyp} : \sim 41$	$HN : \sim 52$
$\Lambda\Lambda\ \text{hyp} : \sim 5$	
$\Sigma\ \text{hyp} : \sim (1)$	



J. Haidenbauer et al., Nucl. Phys. A 915 (2013) 24–58



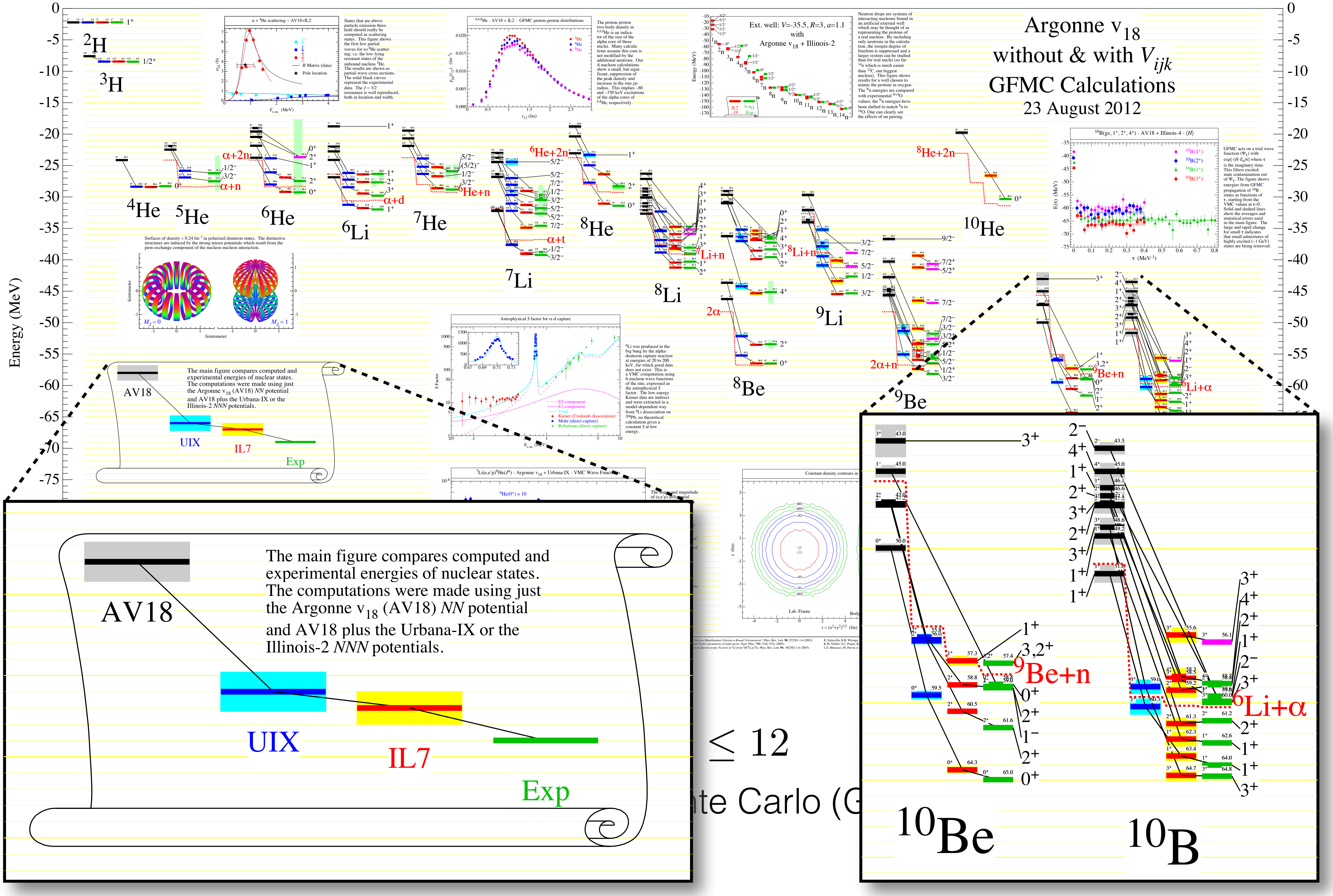
Backup: terrestrial experiments



nuclei $A \leq 12$

Green's function Monte Carlo (GFMC)

Backup: terrestrial experiments

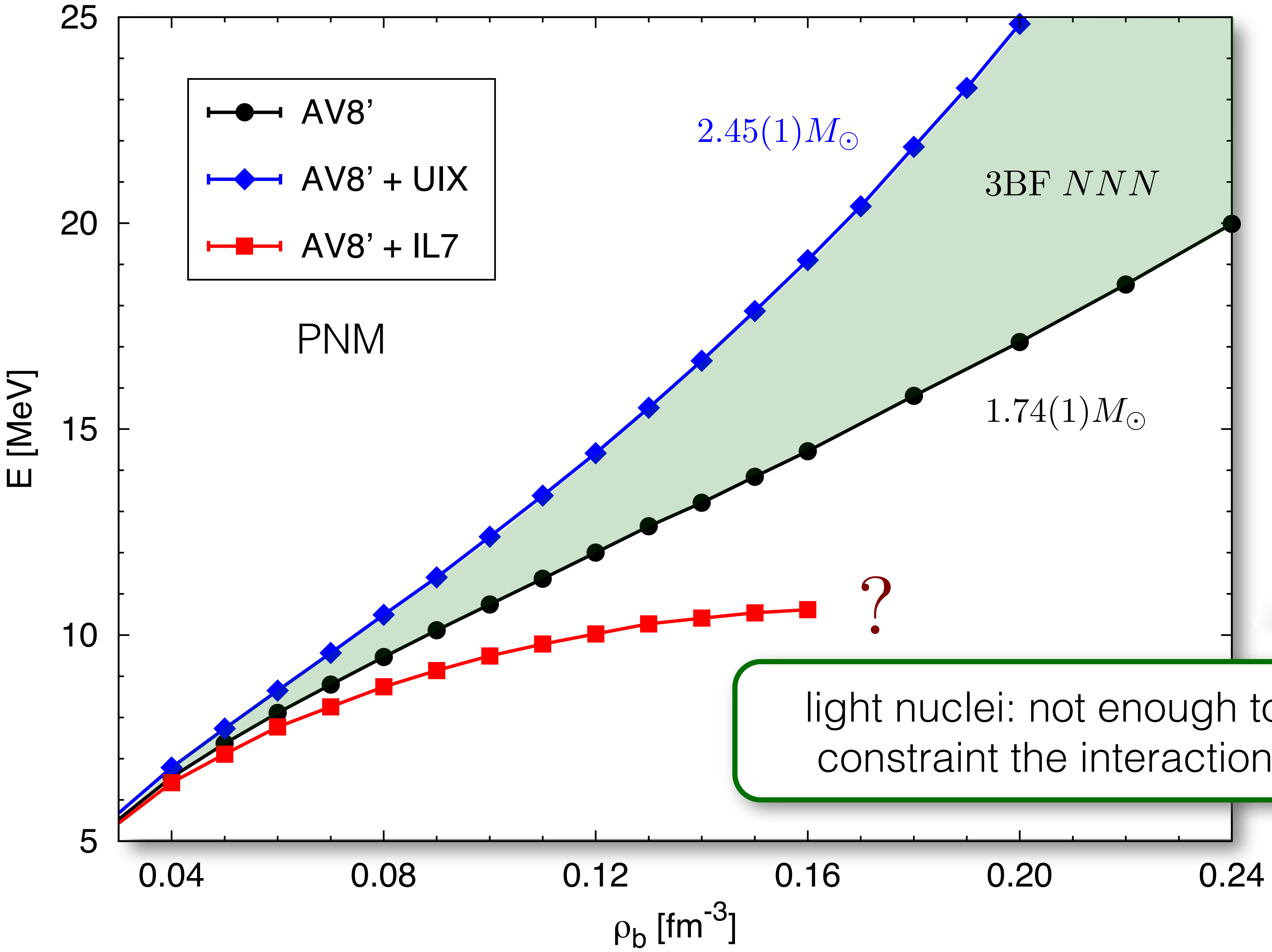


The main figure compares computed and experimental energies of nuclear states. The computations were made using just the Argonne v_{18} (AV18) *NN* potential and AV18 plus the Urbana-IX or the Illinois-2 *NNN* potentials.

≤ 12

te Carlo (G

Backup: terrestrial experiments



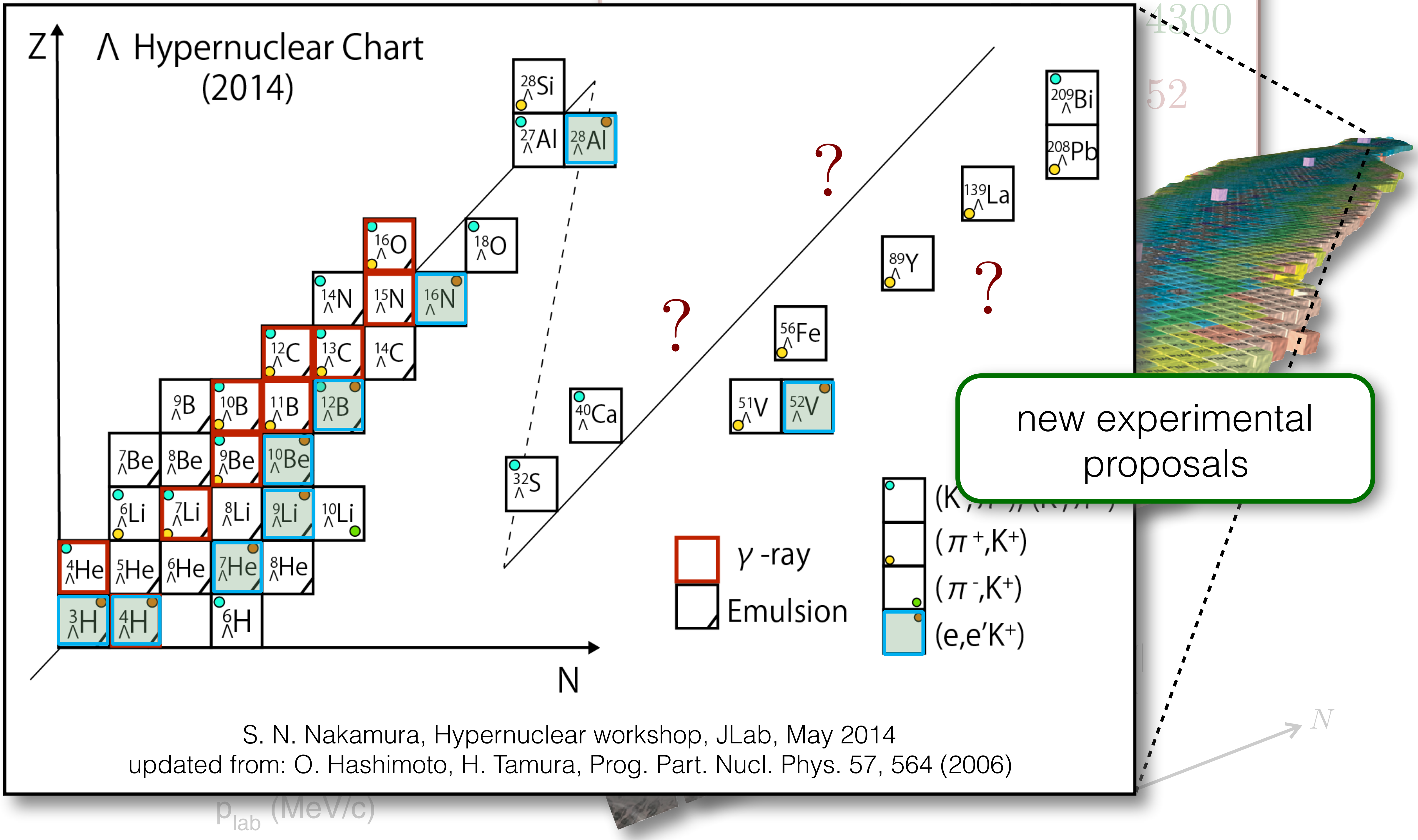
light nuclei: not enough to constraint the interaction

P. Maris, J. P. Vary, S. Gandolfi, J. Carlson, S. C. Pieper, Phys. Rev. C 87, 054318 (2013)

Backup: terrestrial experiments

lack of experimental data !!

binding energies: scattering data:



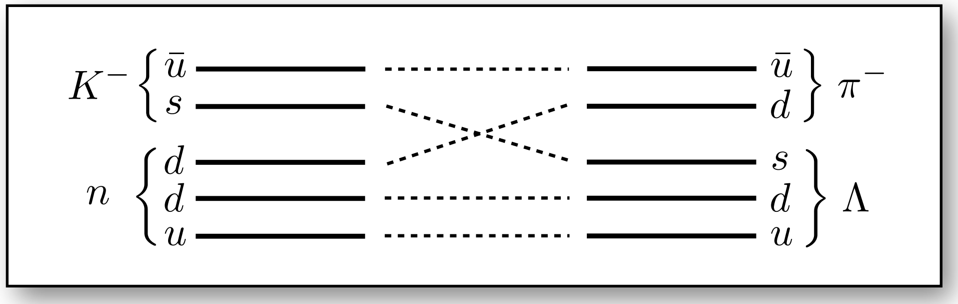
S. N. Nakamura, Hypernuclear workshop, JLab, May 2014
updated from: O. Hashimoto, H. Tamura, Prog. Part. Nucl. Phys. 57, 564 (2006)

new experimental proposals

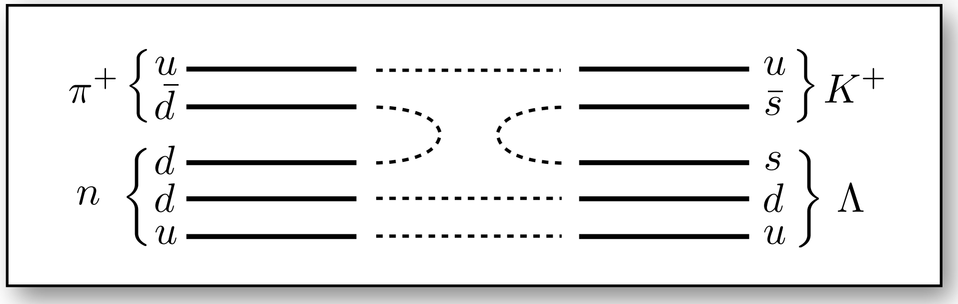
Backup: terrestrial experiments

✓ Charge conserving reactions

$${}^{A}Z\,(K^{-},\pi^{-})\,{}^{A}_{\Lambda}Z$$

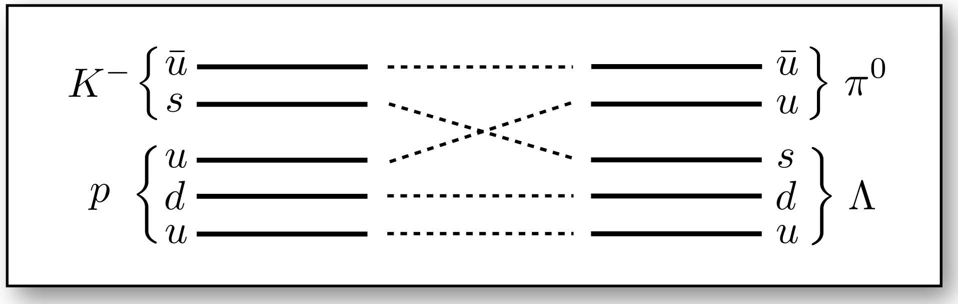


$${}^{A}Z\,(\pi^{+},K^{+})\,{}^{A}_{\Lambda}Z$$

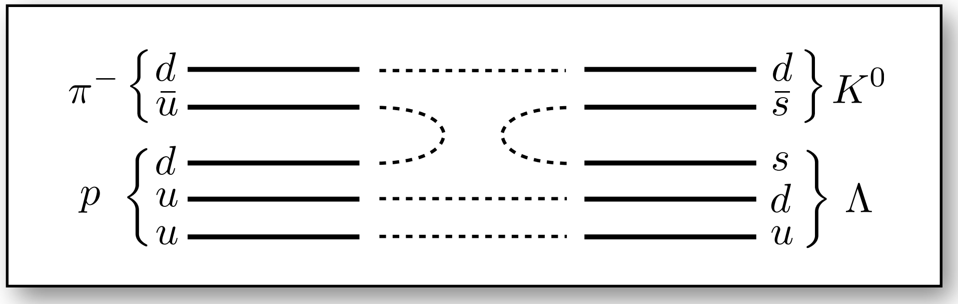


✓ Single charge exchange reactions (SCX)

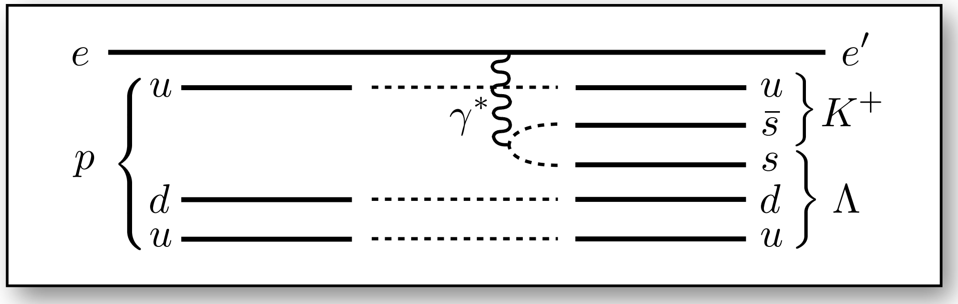
$${}^{A}Z\,(K^{-},\pi^{0})\,{}^{A}_{\Lambda}[Z-1]$$



$${}^{A}Z\,(\pi^{-},K^{0})\,{}^{A}_{\Lambda}[Z-1]$$



$${}^{A}Z\,(e,e'K^{+})\,{}^{A}_{\Lambda}[Z-1]$$



✓ Double charge exchange reactions (DCX)

$${}^{A}Z\,(\pi^{-},K^{+})\,{}^{A+1}_{\quad\Lambda}[Z-2]$$

$${}^{A}Z\,(K^{-},\pi^{+})\,{}^{A+1}_{\quad\Lambda}[Z-2]$$

Backup: terrestrial experiments

$^{89}\mathrm{Y}\left(\pi^+, K^+\right)^{89}_{\Lambda}\mathrm{Y}$

SKS spectrometer

KEK 12-GeV Proton Synchrotron

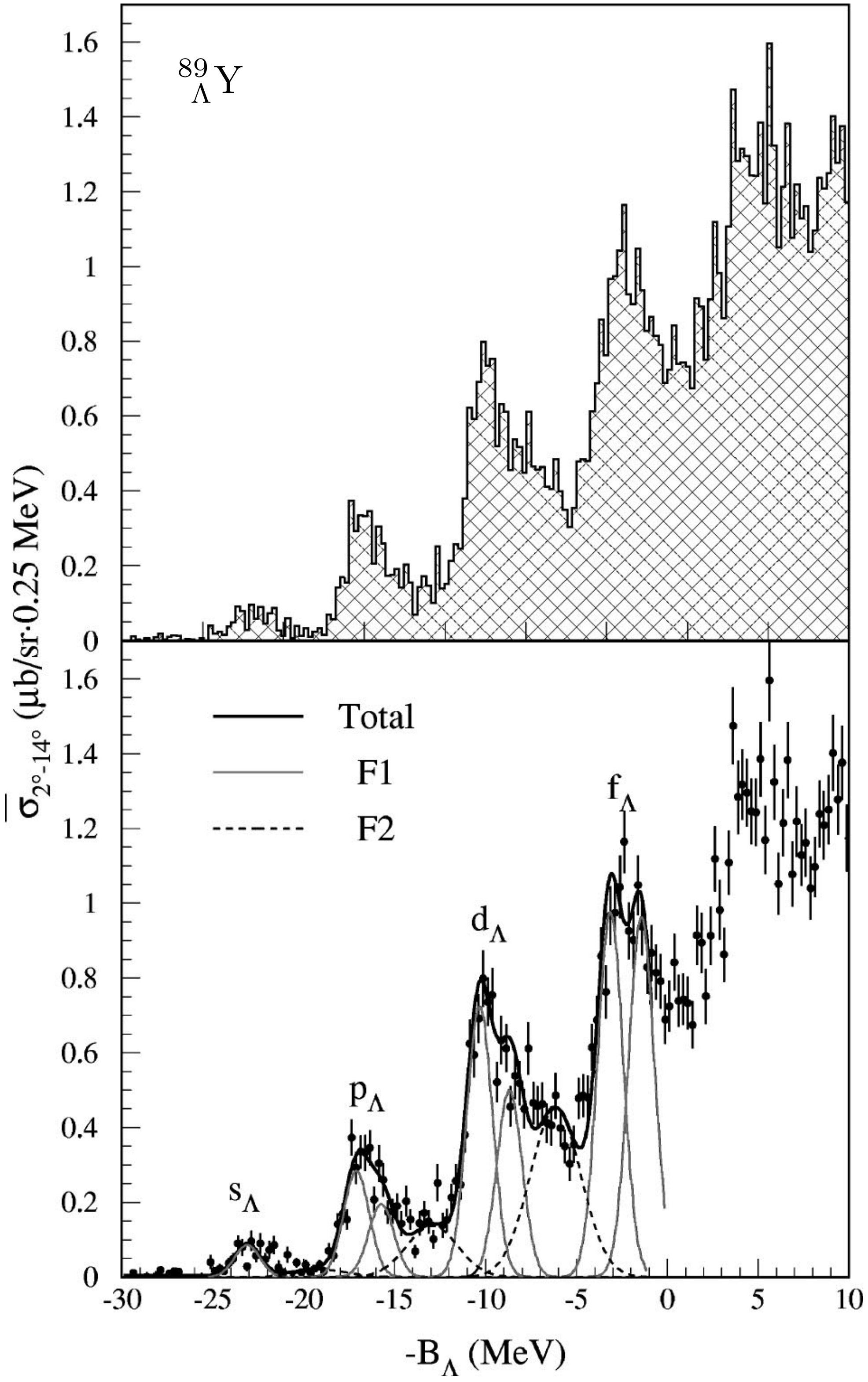
Japan

$$M_{HY} = \sqrt{(E_\pi + M_A - E_K)^2 - (p_\pi^2 + p_K^2 - 2p_\pi\, p_K \cos\theta)}$$

$$B_\Lambda = M_{A-1} + M_\Lambda - M_{HY}$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{A}{\rho_x \cdot N_\mathcal{A}} \cdot \frac{1}{N_{beam} \cdot f_{beam}} \cdot \frac{N_K}{\varepsilon_{\exp} \cdot d\Omega}$$

$$\bar{\sigma}_{2^\circ - 14^\circ} = \int_{\theta=2^\circ}^{\theta=14^\circ} \left(\frac{d\sigma}{d\Omega}\right) d\Omega \bigg/ \int_{\theta=2^\circ}^{\theta=14^\circ} d\Omega$$



H. Hotchi et al., Phys. Rev. C 64, 044302 (2001)

Backup: hyperon-nucleon interaction

hyperon-nucleon interaction ?

- ✓ one boson exchange model Nijmegen & Jülich

  Th. A. Rijken, M. M. Nagels, Y. Yamamoto,
  Few-Body Syst. (2013) 54, 801

  J. Haidenbauer, Ulf-G. Meißner,
  Phys. Rev. C 72, 044005 (2005)

- ✓ χ-EFT (NLO)

  J. Haidenbauer, S. Petschauer, N. Kaiser,
  U. -G. Meißner, A. Nogga, W. Weise,
  Nucl. Phys. A 915 (2013) 24–58

- ✓ effective - mean field models
  - cluster approach

    E. Hiyama,Y. Yamamoto,
    Prog. Theor. Phys. (2012) 128 (1) 105

  - Skyrme-Hartree-Fock

    H.-J. Schulze, E. Hiyama
    Phys. Rev. C 90, 047301 (2014)

- ✓ phenom. pion exchange model Argonne-Urbana like

  A. A. Usmani, F. C. Khanna, J. Phys. G: Nucl. Part. Phys. 35 (2008) 025105

  → good for QMC

Backup: hyperon-nucleon interaction

- ✓ 2-body interaction: AV18 & Usmani

$$NN \left\{ \begin{aligned} v_{ij} &= \sum_{p=1,18} v_p(r_{ij})\,\mathcal{O}_{ij}^p \\ \mathcal{O}_{ij}^{p=1,8} &= \Big\{1, \sigma_{ij}, S_{ij}, \boldsymbol{L}_{ij}\cdot\boldsymbol{S}_{ij}\Big\} \otimes \Big\{1, \tau_{ij}\Big\} \end{aligned} \right.$$

NN scattering

deuteron

$$\Lambda N \left\{ \begin{aligned} v_{\lambda i} &= \sum_{p=1,4} v_p(r_{\lambda i})\,\mathcal{O}_{\lambda i}^p \\ \mathcal{O}_{\lambda i}^{p=1,4} &= \Big\{1, \sigma_{\lambda i}\Big\} \otimes \Big\{1, \tau_i^z\Big\} \end{aligned} \right.$$

Λp scattering

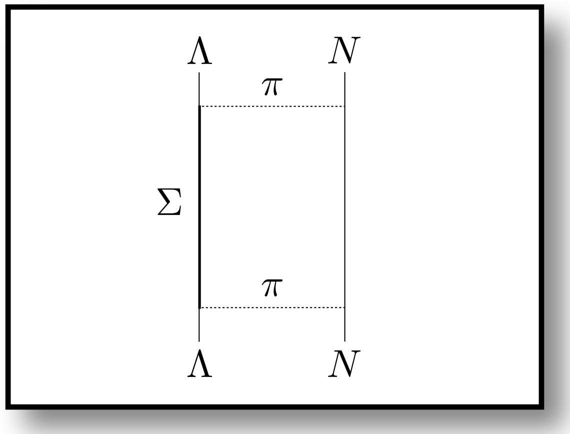
$A = 4$ CSB

Note: ~~$\Lambda\pi\Lambda$ vertex~~

forbidden

$\Lambda\pi\Sigma$ vertex

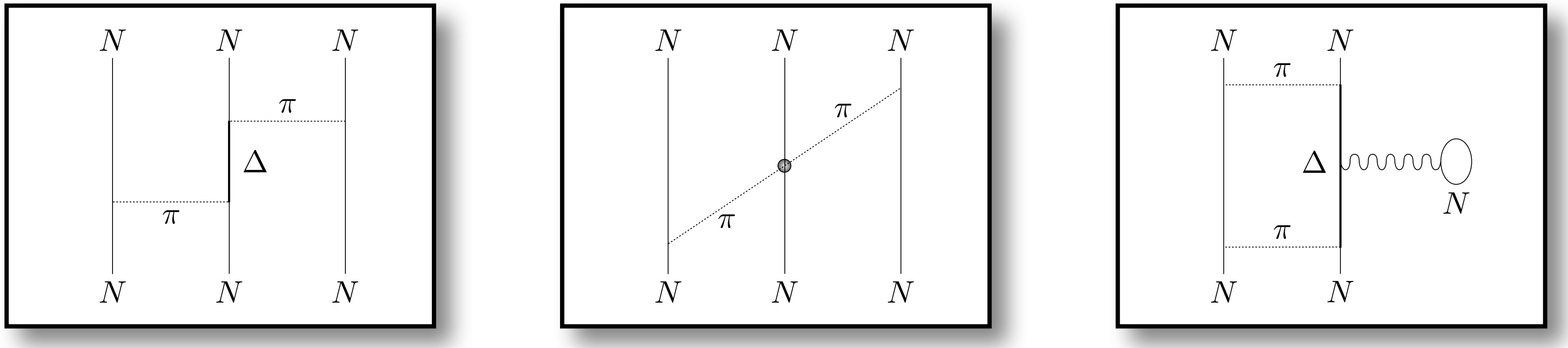
2π exchange



Backup: hyperon-nucleon interaction

- ✓ 3-body interaction: Urbana IX & Usmani

NNN

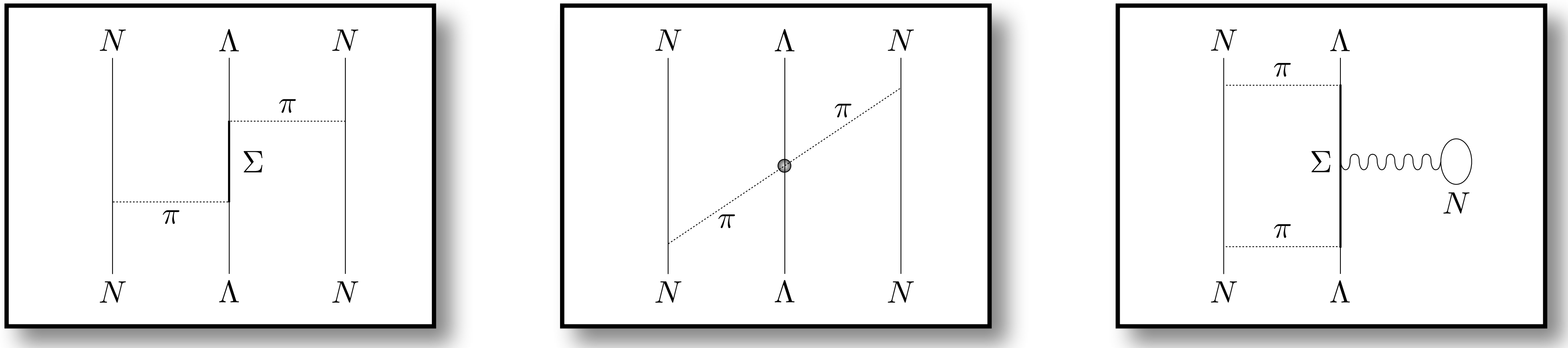


nuclei

nuclear matter

$$v_{ijk} = A_{2\pi}^{P}\,\mathcal{O}_{ijk}^{2\pi,P} + A_{2\pi}^{S}\,\mathcal{O}_{ijk}^{2\pi,S} + A_R\,\mathcal{O}_{ijk}^{R}$$

ΛNN



VMC calc.

no unique fit

$$v_{\lambda ij} = C_P\,\mathcal{O}_{\lambda ij}^{2\pi,P} + C_S\,\mathcal{O}_{\lambda ij}^{2\pi,S} + W_D\,\mathcal{O}_{\lambda ij}^{R}$$

Backup: hyperon-nucleon interaction

✓ 2-body interaction

$$v_{\lambda i} = v_0(r_{\lambda i}) + \frac{1}{4} v_\sigma T_\pi^2(r_{\lambda i})\, \boldsymbol{\sigma}_\lambda \cdot \boldsymbol{\sigma}_i$$

charge symmetric

$$v_{\lambda i}^{CSB} = C_\tau\, T_\pi^2\,(r_{\lambda i})\, \tau_i^z$$

charge symmetry breaking
(spin independent)

A. R. Bodmer, Q. N. Usmani, Phys.Rev.C 31, 1400 (1985)

✓ 3-body interaction

$$v_{\lambda ij} = v_{\lambda ij}^{2\pi,P} + v_{\lambda ij}^{2\pi,S} + v_{\lambda ij}^{D}$$

$$\begin{cases} v_{\lambda ij}^{2\pi,P} = -\dfrac{C_P}{6}\Big\{X_{i\lambda}\,, X_{\lambda j}\Big\}\, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \\ v_{\lambda ij}^{2\pi,S} = C_S\, Z\,(r_{\lambda i})\, Z\,(r_{\lambda j})\;\, \boldsymbol{\sigma}_i \cdot \hat{\boldsymbol{r}}_{i\lambda}\, \boldsymbol{\sigma}_j \cdot \hat{\boldsymbol{r}}_{j\lambda}\, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \\ v_{\lambda ij}^{D} = W_D\, T_\pi^2\,(r_{\lambda i})\, T_\pi^2\,(r_{\lambda j}) \Big[1 + \dfrac{1}{6}\boldsymbol{\sigma}_\lambda \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\Big] \end{cases}$$

use QMC to fit on
hyp. exp. data

Backup: hyperon-nucleon interaction

$$v_0(r) = v_c(r) - \bar{v}\, T_\pi^2(r)$$

$$v_c(r) = W_c \left(1 + \mathrm{e}^{\frac{r-\bar{r}}{a}}\right)^{-1}$$

$$\bar{v} = (v_s + 3v_t)/4 \quad v_\sigma = v_s - v_t$$

$$Y_\pi(r) = \frac{\mathrm{e}^{-\mu_\pi r}}{\mu_\pi r} \xi_Y(r)$$

$$T_\pi(r) = \left[1 + \frac{3}{\mu_\pi r} + \frac{3}{(\mu_\pi r)^2}\right] \frac{\mathrm{e}^{-\mu_\pi r}}{\mu_\pi r} \xi_T(r)$$

$$\mu_\pi = \frac{m_\pi}{\hbar} = \frac{1}{\hbar} \frac{m_{\pi^0} + 2\, m_{\pi^\pm}}{3}$$

$$\xi_Y(r) = \xi_T^{1/2}(r) = 1 - \mathrm{e}^{-cr^2}$$

$$Z_\pi(r) = \frac{\mu_\pi r}{3} \Big[Y_\pi(r) - T_\pi(r)\Big]$$

$$X_{\lambda i} = Y_\pi(r_{\lambda i})\, \boldsymbol{\sigma}_\lambda \cdot \boldsymbol{\sigma}_i + T_\pi(r_{\lambda i})\, S_{\lambda i}$$

$$S_{\lambda i} = 3\, (\boldsymbol{\sigma}_\lambda \cdot \hat{\boldsymbol{r}}_{\lambda i})\, (\boldsymbol{\sigma}_i \cdot \hat{\boldsymbol{r}}_{\lambda i}) - \boldsymbol{\sigma}_\lambda \cdot \boldsymbol{\sigma}_i$$

Constant	Value	Unit
W_c	2137	MeV
$\bar{r}$	0.5	fm
a	0.2	fm
v_s	6.33, 6.28	MeV
v_t	6.09, 6.04	MeV
$\bar{v}$	6.15(5)	MeV
v_σ	0.24	MeV
c	2.0	fm^{-2}
C_τ	-0.050(5)	MeV
C_P	$0.5 \div 2.5$	MeV
C_S	$\simeq 1.5$	MeV
W_D	$0.002 \div 0.058$	MeV

Backup: strangeness in QMC calculations

- ✓ diffusion Monte Carlo

$$-\frac{\partial}{\partial\tau}|\psi(\tau)\rangle = (H - E_0)|\psi(\tau)\rangle$$

$\tau = it/\hbar$ imaginary time

$$\Big\downarrow$$

$$|\psi(\tau)\rangle = \mathrm{e}^{-(H-E_0)\tau}|\psi(0)\rangle$$

$$|\psi(0)\rangle = |\psi_T\rangle = \sum_{n=0}^{\infty} c_n|\varphi_n\rangle$$

$$= \sum_{n=0}^{\infty} \mathrm{e}^{-(E_n-E_0)\tau} c_n|\varphi_n\rangle \xrightarrow{\tau\to\infty} \boxed{c_0|\varphi_0\rangle}$$

projection

$$\Big\downarrow$$

⇨ $$E = \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} \xrightarrow{\tau\to\infty} \boxed{E_0}$$

ground state

Backup: strangeness in QMC calculations

- ✓ diffusion Monte Carlo

imaginary time evolution: $\tau = \mathcal{M} d\tau \qquad d\tau \ll 1$

$$\langle SR|\psi(\tau + d\tau)\rangle = \int dR' dS' \langle SR|\mathrm{e}^{-(H-E_0)d\tau}|R'S'\rangle\langle S'R'|\psi(\tau)\rangle$$

final walkers

propagator

initial walkers

$$\{\boldsymbol{r}^*, s^*\}_w \longleftarrow \{\boldsymbol{r}, s\}_w$$

Backup: strangeness in QMC calculations

✓ diffusion Monte Carlo

imaginary time evolution: $\tau = \mathcal{M} d\tau \qquad d\tau \ll 1$

$$\langle SR|\psi(\tau + d\tau)\rangle = \int dR' dS' \langle SR|\mathrm{e}^{-(H-E_0)d\tau}|R'S'\rangle\langle S'R'|\psi(\tau)\rangle$$

final walkers

propagator

initial walkers

$$\{\boldsymbol{r}_0, s_0\}_w \xleftarrow{\infty \leftarrow \tau \quad \infty \leftarrow \mathcal{M}} \{\boldsymbol{r}, s\}_w$$

propagator: $H = T \longrightarrow$ diffusion in coordinate space

$+ V(\boldsymbol{r}) \longrightarrow$ branching of configurations

$+ V(s) \longrightarrow$ problem !!

Backup: strangeness in QMC calculations

- ✓ auxiliary field

many body $|S\rangle : \quad 2^A \frac{A!}{(A-Z)!Z!}$ components GFMC: $A \leq 12$

single particle $|S\rangle = \bigotimes_i |S\rangle_i : \quad 4A$ components AFDMC: $A \sim 90$

$$\mathcal{P} \sim \mathrm{e}^{-\frac{1}{2}\gamma d\tau \mathcal{O}^2} \quad \longrightarrow \quad \mathrm{e}^{-\frac{1}{2}\gamma d\tau \mathcal{O}^2} \bigotimes_i |S\rangle_i \neq \bigotimes_i |\tilde{S}\rangle_i$$

Idea: Hubbard-Stratonovich transformation

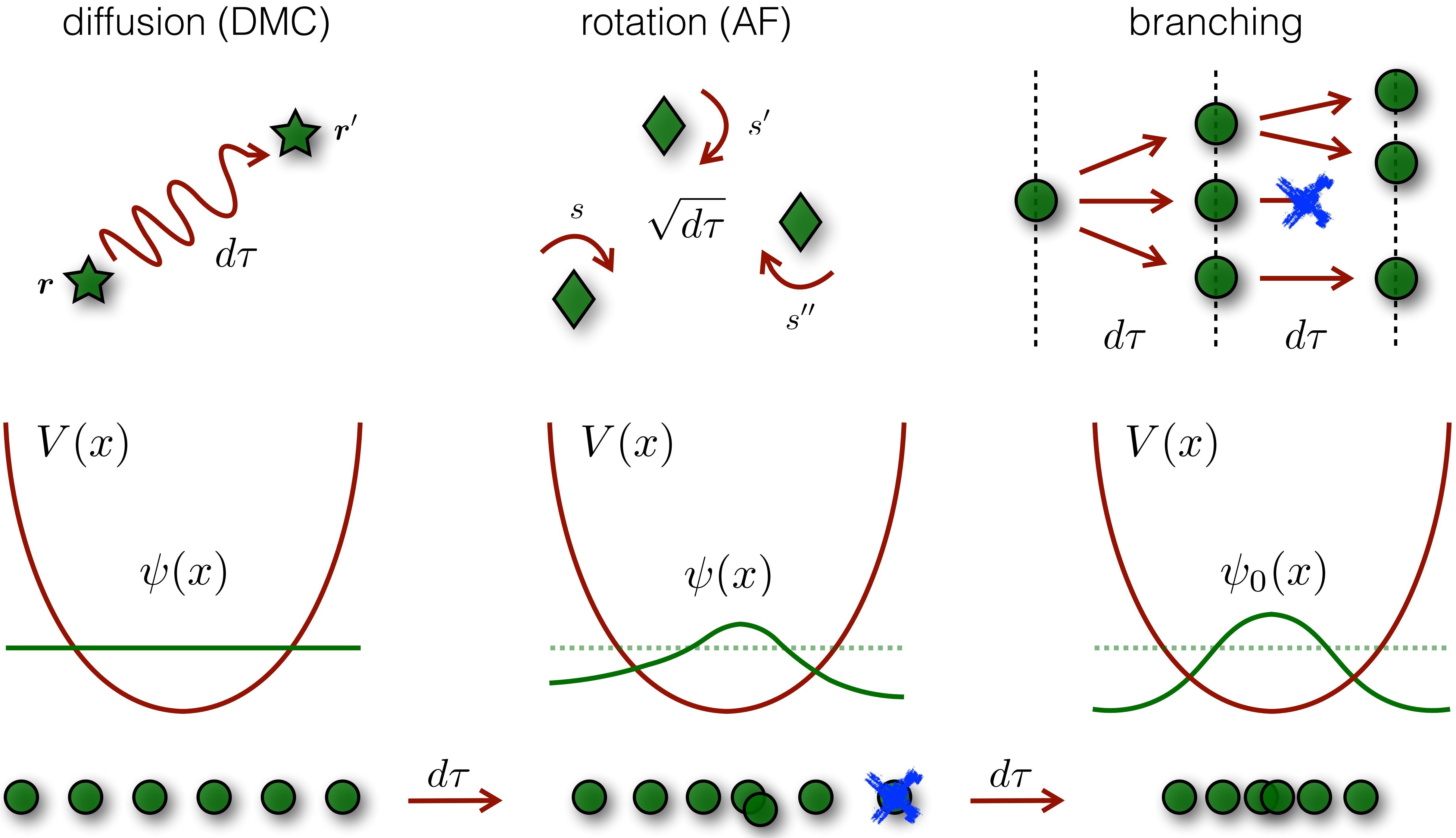
$$\mathrm{e}^{-\frac{1}{2}\gamma d\tau \mathcal{O}^2} = \frac{1}{\sqrt{2\pi}} \int dx \; \mathrm{e}^{-\frac{x^2}{2} + \sqrt{-\gamma d\tau} x \mathcal{O}}$$

auxiliary field

rotation over spin-isospin configurations

Backup: strangeness in QMC calculations

- ✓ auxiliary field diffusion Monte Carlo



Backup: strangeness in QMC calculations

- ✓ auxiliary field diffusion Monte Carlo
  - imaginary time projection → exact ground state
  - stochastic method → error estimate: $\sigma \sim 1/\sqrt{\mathcal{N}}$

$$H = \sum_i \frac{p_i^2}{2m_N} + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk} + \sum_\lambda \frac{p_\lambda^2}{2m_\Lambda} + \sum_{\lambda,i} v_{\lambda i} + \sum_{\lambda,i<j} v_{\lambda ij}$$

non-strange

strange

- ✓ extended wavefunction: nucleons + hyperons
- ✓ new propagation:
  - hyperon diffusion
  - nucleon & hyperon spinor rotations

Backup: strangeness in QMC calculations

✓ auxiliary field diffusion Monte Carlo

- imaginary time projection → exact ground state
- stochastic method → error estimate: $\sigma \sim 1/\sqrt{\mathcal{N}}$

$$H = \sum_i \frac{p_i^2}{2m_N} + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk} + \sum_\lambda \frac{p_\lambda^2}{2m_\Lambda} + \sum_{\lambda,i} v_{\lambda i} + \sum_{\lambda,i<j} v_{\lambda ij}$$

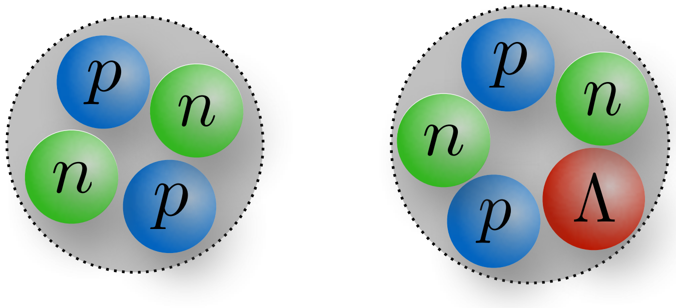
non-strange

strange

$$B_\Lambda = E\left({}^{A-1}Z\right) - E\left({}^{A}_{\Lambda}Z\right) > 0$$

↔ fit on exp. values

ex: $B_\Lambda\left({}^{5}_{\Lambda}\mathrm{He}\right) = E\left({}^{4}\mathrm{He}\right) - E\left({}^{5}_{\Lambda}\mathrm{He}\right)$



Backup: strangeness in QMC calculations

$$\psi_T(R,S) = \prod_{\lambda i} f_c^{\Lambda N}(r_{\lambda i})\,\psi_T^N(R_N,S_N)\,\psi_T^\Lambda(R_\Lambda,S_\Lambda)$$

$$\begin{cases} \psi_T^\kappa(R_\kappa,S_\kappa) = \displaystyle\prod_{i<j} f_c^{\kappa\kappa}(r_{ij})\,\Phi_\kappa(R_\kappa,S_\kappa) \\[2ex] \Phi_\kappa(R_\kappa,S_\kappa) = \mathcal{A}\left[\displaystyle\prod_{i=1}^{\mathcal{N}_\kappa}\varphi_\epsilon^\kappa(\boldsymbol{r}_i,s_i)\right] = \det\Big\{\varphi_\epsilon^\kappa(\boldsymbol{r}_i,s_i)\Big\} \end{cases} \qquad \kappa = N,\Lambda$$

s.p. orbitals   plane waves

$$s_i = \begin{pmatrix} a_i \\ b_i \\ c_i \\ d_i \end{pmatrix}_i = a_i|p\uparrow\rangle_i + b_i|p\downarrow\rangle_i + c_i|n\uparrow\rangle_i + d_i|n\downarrow\rangle_i$$

$$s_\lambda = \begin{pmatrix} u_\lambda \\ v_\lambda \end{pmatrix}_\lambda = u_\lambda|\Lambda\uparrow\rangle_\lambda + v_\lambda|\Lambda\downarrow\rangle_\lambda$$

Backup: strangeness in QMC calculations

$$
\begin{aligned}
V_{NN}^{SD} + V_{\Lambda N}^{SD} = & \frac{1}{2}\sum_{n=1}^{3\mathcal{N}_N} \lambda_n^{[\sigma]} \left(\mathcal{O}_n^{[\sigma]}\right)^2 \\
& + \frac{1}{2}\sum_{n=1}^{3\mathcal{N}_N}\sum_{\alpha=1}^{3} \lambda_n^{[\sigma\tau]} \left(\mathcal{O}_{n\alpha}^{[\sigma\tau]}\right)^2 \\
& + \frac{1}{2}\sum_{n=1}^{\mathcal{N}_N}\sum_{\alpha=1}^{3} \lambda_n^{[\tau]} \left(\mathcal{O}_{n\alpha}^{[\tau]}\right)^2 \\
& + \frac{1}{2}\sum_{n=1}^{\mathcal{N}_\Lambda}\sum_{\alpha=1}^{3} \lambda_n^{[\sigma\Lambda]} \left(\mathcal{O}_{n\alpha}^{[\sigma\Lambda]}\right)^2 \\
& + \frac{1}{2}\sum_{n=1}^{\mathcal{N}_N\mathcal{N}_\Lambda}\sum_{\alpha=1}^{3} B_n^{[\sigma]} \left(\mathcal{O}_{n\alpha}^{[\sigma\Lambda N]}\right)^2 \\
& + \sum_{i=1}^{\mathcal{N}_N} B_i^{[\tau]}\, \tau_i^z
\end{aligned}
$$

$A_{i\alpha,j\beta}^{[\sigma]}$

$A_{i\alpha,j\beta}^{[\sigma\tau]}$

$A_{ij}^{[\tau]}$

$C_{\lambda\mu}^{[\sigma]}$

diagonalization:

λ_n eigenvalues

ψ_n eigenvectors

$\mathcal{O}_n = \sigma_n\, \psi_n$

direct calculation

Backup: strangeness in QMC calculations

computing time

- 5000 configurations, 3 time steps: nucleus & hypernucleus
- 10 nodes @ Edison (NERSC)
- 2 socket 12-core Intel "Ivy Bridge" processor @ 2.4 GHz

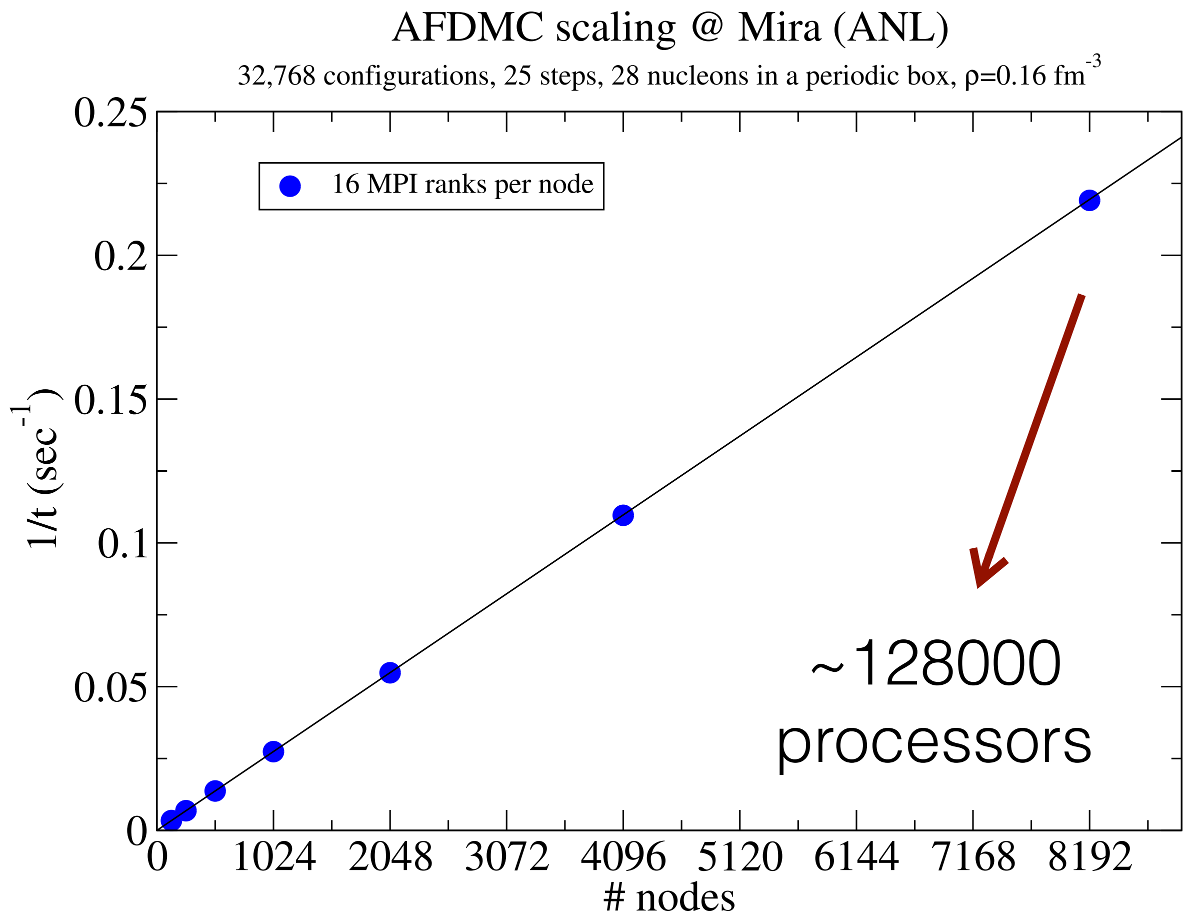
→ 240 processors

system	CPU time	B_Λ error
${}^{41}_{\Lambda}$Ca - ^{40}Ca	$\sim 30\,$k hrs	~ 0.75 MeV
${}^{49}_{\Lambda}$Ca - ^{48}Ca	$\sim 55\,$k hrs	~ 0.75 MeV
${}^{91}_{\Lambda}$Zr - ^{90}Zr	$\sim 350\,$k hrs	~ 0.75 MeV
${}^{209}_{\Lambda}$Pb - ^{208}Pb	$\sim 4.2\,$M hrs	~ 0.75 MeV
AFDMC	$\sim A^3$	$\sigma \sim 1/\sqrt{\mathcal{N}}$

↓

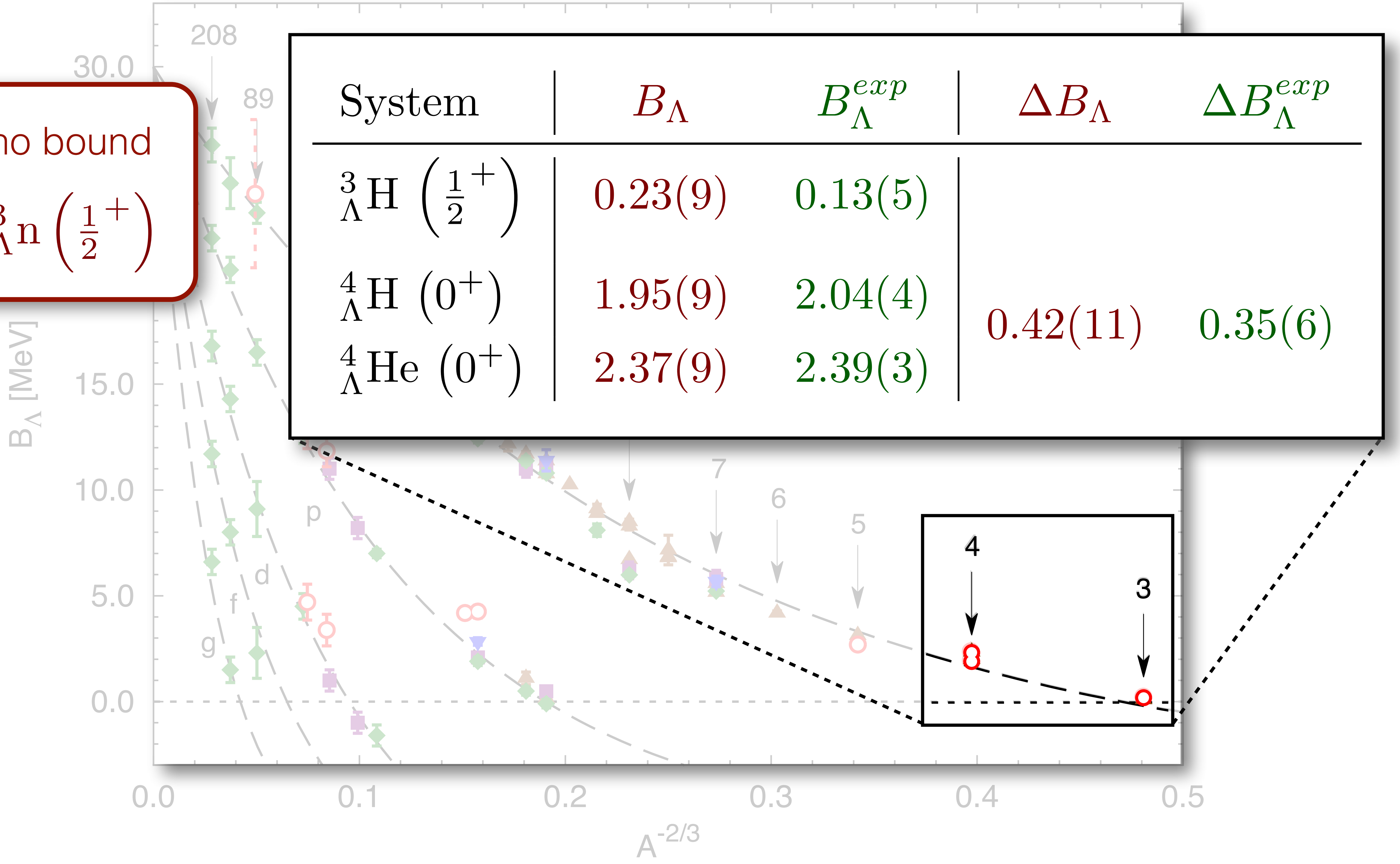
calculation accessible

B_Λ in all waves, $A \pm 1$



S. Gandolfi, unpublished

Backup: strangeness in nuclei



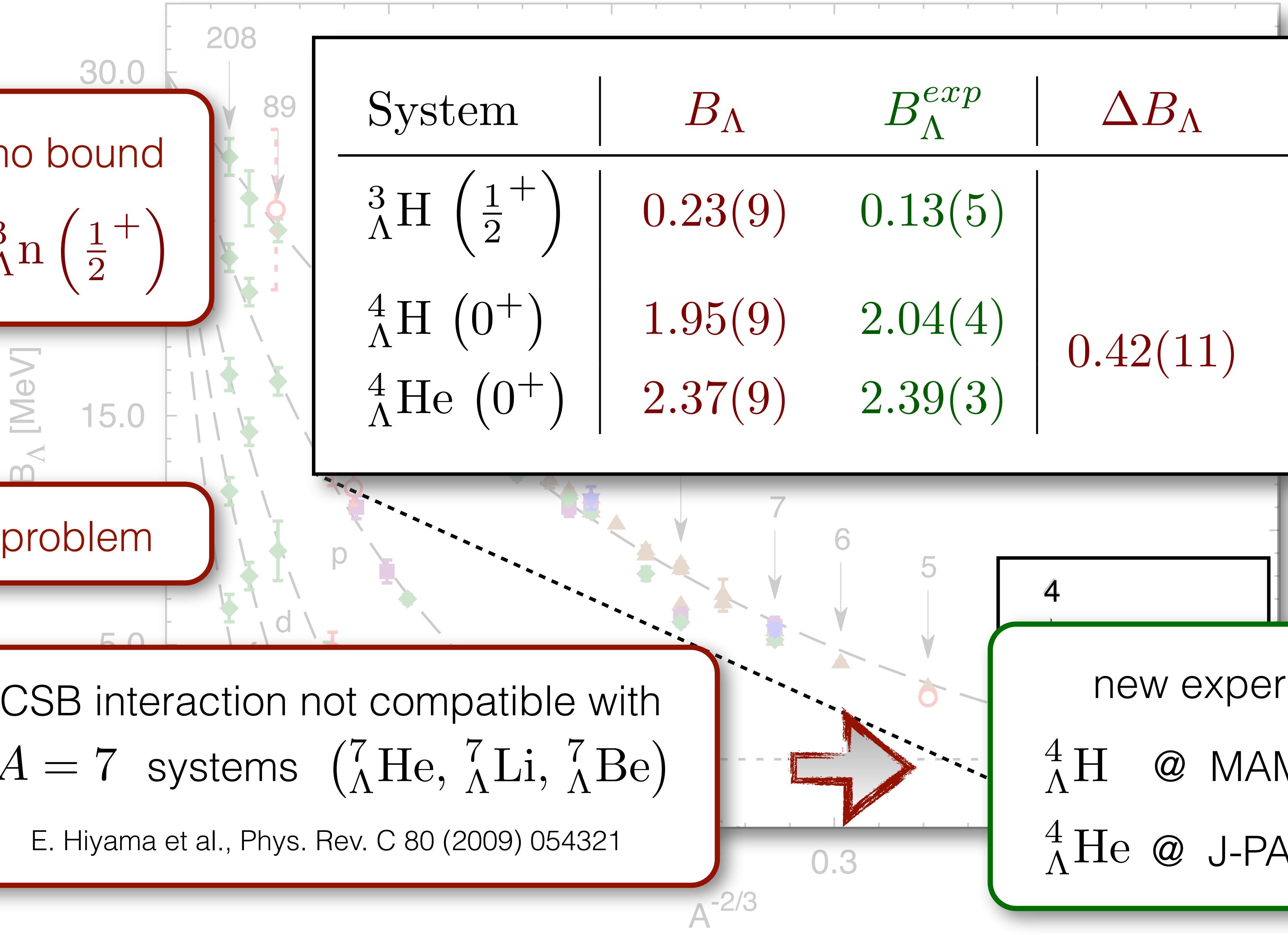
no bound

${}^{3}_{\Lambda}\mathrm{n}\left(\frac{1}{2}^{+}\right)$

System	B_Λ	B_Λ^{exp}	ΔB_Λ	ΔB_Λ^{exp}
${}^{3}_{\Lambda}\mathrm{H}\left(\frac{1}{2}^{+}\right)$	0.23(9)	0.13(5)		
${}^{4}_{\Lambda}\mathrm{H}\ (0^{+})$	1.95(9)	2.04(4)	0.42(11)	0.35(6)
${}^{4}_{\Lambda}\mathrm{He}\ (0^{+})$	2.37(9)	2.39(3)		

D. L., A. Lovato, S. Gandolfi, F. Pederiva, arXiv:1508.04722 (2015)

Backup: strangeness in nuclei



no bound

${}^{3}_{\Lambda}\mathrm{n}\ \left(\frac{1}{2}^{+}\right)$

problem

System	B_Λ	B_Λ^{exp}	ΔB_Λ	ΔB_Λ^{exp}
${}^{3}_{\Lambda}\mathrm{H}\ \left(\frac{1}{2}^{+}\right)$	0.23(9)	0.13(5)		
${}^{4}_{\Lambda}\mathrm{H}\ (0^{+})$	1.95(9)	2.04(4)	0.42(11)	0.35(6)
${}^{4}_{\Lambda}\mathrm{He}\ (0^{+})$	2.37(9)	2.39(3)		

CSB interaction not compatible with

$A = 7$ systems $({}^{7}_{\Lambda}\mathrm{He},\ {}^{7}_{\Lambda}\mathrm{Li},\ {}^{7}_{\Lambda}\mathrm{Be})$

E. Hiyama et al., Phys. Rev. C 80 (2009) 054321

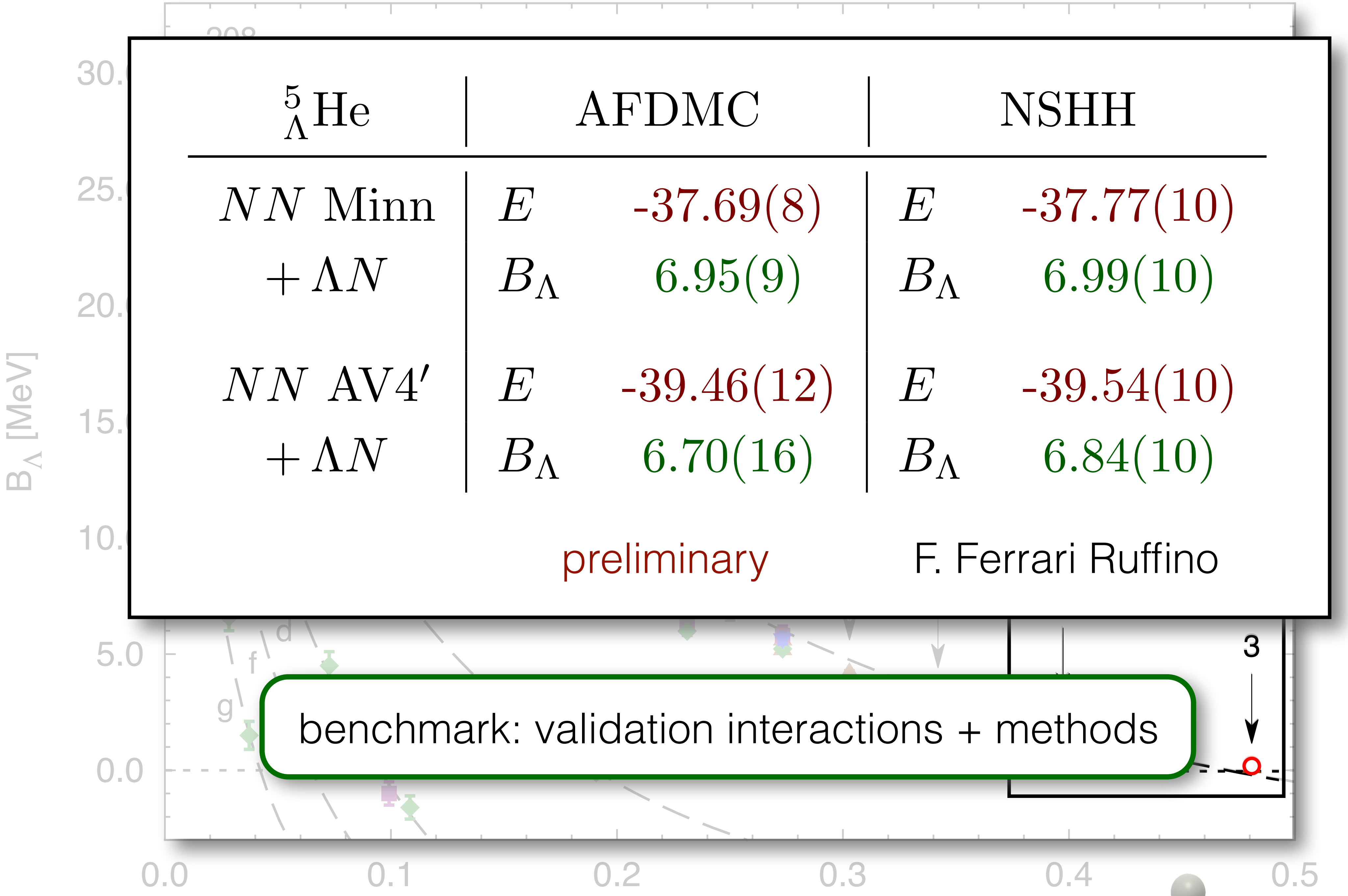
new experiments:

${}^{4}_{\Lambda}\mathrm{H}$ @ MAMI, JLab

${}^{4}_{\Lambda}\mathrm{He}$ @ J-PARC

D. L., A. Lovato, S. Gandolfi, F. Pederiva, arXiv:1508.04722 (2015)

Backup: strangeness in nuclei



${}^5_\Lambda$He	AFDMC		NSHH	
NN Minn $+\Lambda N$	E	-37.69(8)	E	-37.77(10)
	B_Λ	6.95(9)	B_Λ	6.99(10)
NN AV4′ $+\Lambda N$	E	-39.46(12)	E	-39.54(10)
	B_Λ	6.70(16)	B_Λ	6.84(10)

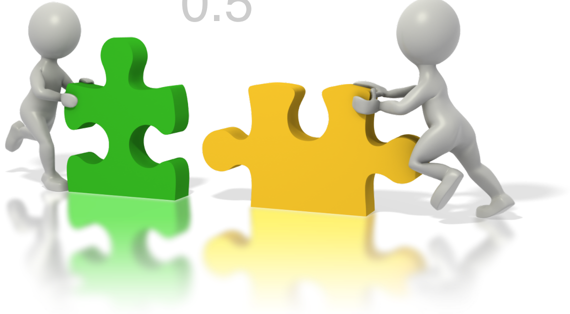
preliminary

F. Ferrari Ruffino

$A^{-2/3}$

benchmark: validation interactions + methods

D. L., A. Lovato, S. Gandolfi, F. Pederiva, arXiv:1508.04722 (2015)



Backup: strangeness in nuclei

$$B_\Lambda = E(^{A-1}Z) - E(^{A}_{\Lambda}Z)$$

Idea: nuclear effects cancel at most ✓

NN potential	$^{5}_{\Lambda}\mathrm{He}$		$^{17}_{\Lambda}\mathrm{O}$	
	$V_{\Lambda N}$	$V_{\Lambda N} + V_{\Lambda NN}$	$V_{\Lambda N}$	$V_{\Lambda N} + V_{\Lambda NN}$
Argonne V4’	7.1(1)	5.1(1)	43(1)	19(1)
Argonne V6’	6.3(1)	5.2(1)	34(1)	21(1)
Minnesota	7.4(1)	5.2(1)	50(1)	17(2)
Expt.	3.12(2)		13.0(4)	

D. L., S. Gandolfi, F. Pederiva, Phys. Rev. C 87, 041303(R) (2013)

Backup: strangeness in nuclei

double Λ hypernuclei

$$v_{\lambda\mu} = \sum_{k=1}^{3} \left(v_0^{(k)} + v_\sigma^{(k)} \, \boldsymbol{\sigma}_\lambda \cdot \boldsymbol{\sigma}_\mu \right) \mathrm{e}^{-\mu^{(k)} r_{\lambda\mu}^2}$$

E. Hiyama, et al., Phys. Rev. C 66, 024007 (2002)

System	E	$B_{\Lambda(\Lambda)}$	$\Delta B_{\Lambda\Lambda}$
^{4}He	-32.67(8)	—	—
$^5_\Lambda$He	-35.89(12)	3.22(14)	—
$^{6}_{\Lambda\Lambda}$He	-40.6(3)	7.9(3)	1.5(4)
$^{6}_{\Lambda\Lambda}$He	Expt.	$7.25 \pm 0.19^{+0.18}_{-0.11}$	$1.01 \pm 0.20^{+0.18}_{-0.11}$

D. L., F. Pederiva, S. Gandolfi, Phys. Rev. C 89, 014314 (2014)

Backup: strangeness in nuclei

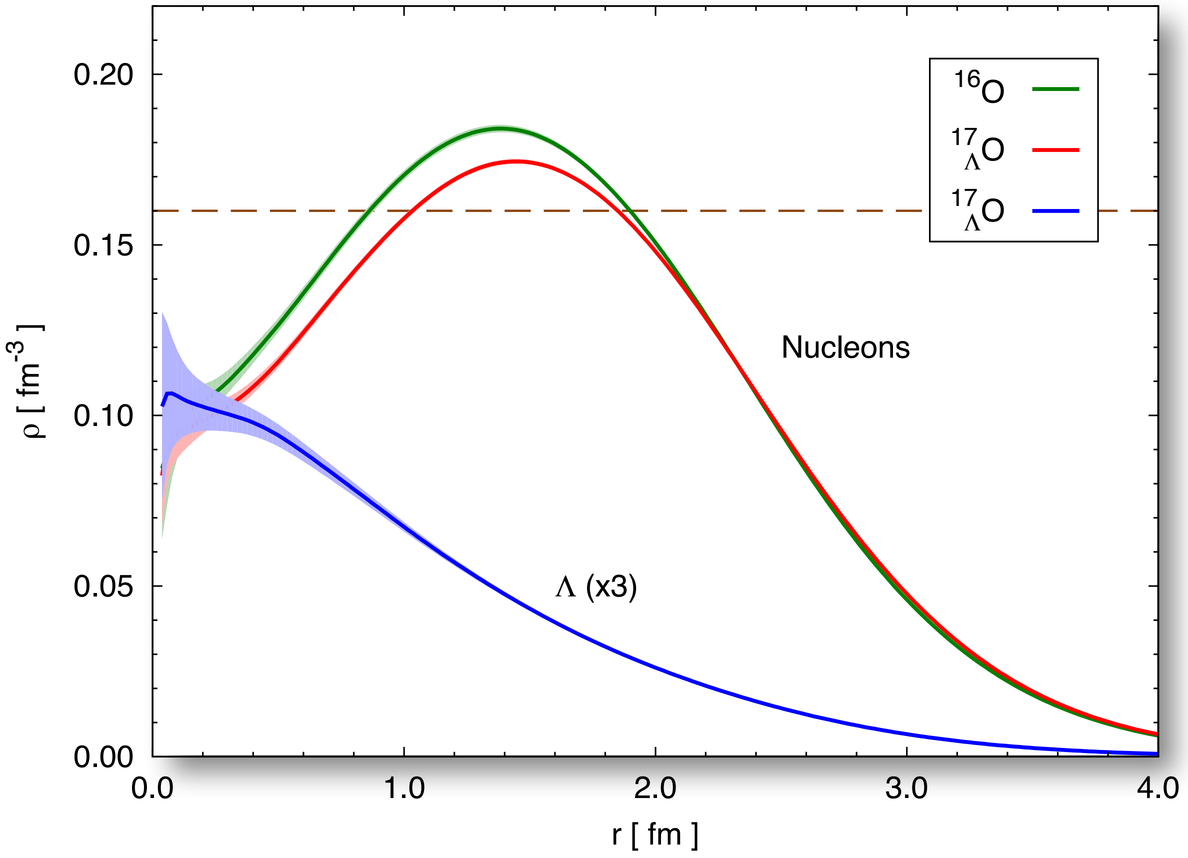
	AV4′	AV4′+UIX$_c$	AV6′	AV7′	exp
^{4}He	-32.83(5)	-26.63(3)	-27.09(3)	-25.7(2)	-28.295
^{16}O	-180.1(4)	-119.9(2)	-115.6(3)	-90.6(4)	-127.619
^{40}Ca	-597(3)	-382.9(6)	-322(2)	-209(1)	-342.051
^{48}Ca	-645(3)	-414.2(6)	–	–	-416.001

	Hamiltonian	AFDMC	GFMC
^{4}He	AV4′	-32.83(5)	-32.88(6)
	AV4′+UIX$_c$	-26.63(3)	-26.82(8)
	AV6′	-27.09(3)	-26.85(2)
	AV7′	-25.7(2)	-26.2(1)
	N^{2}LO ($R_0 = 1.0$ fm)	-24.41(3)	-24.56(1)
	N^{2}LO ($R_0 = 1.2$ fm)	-25.77(2)	-25.75(1)

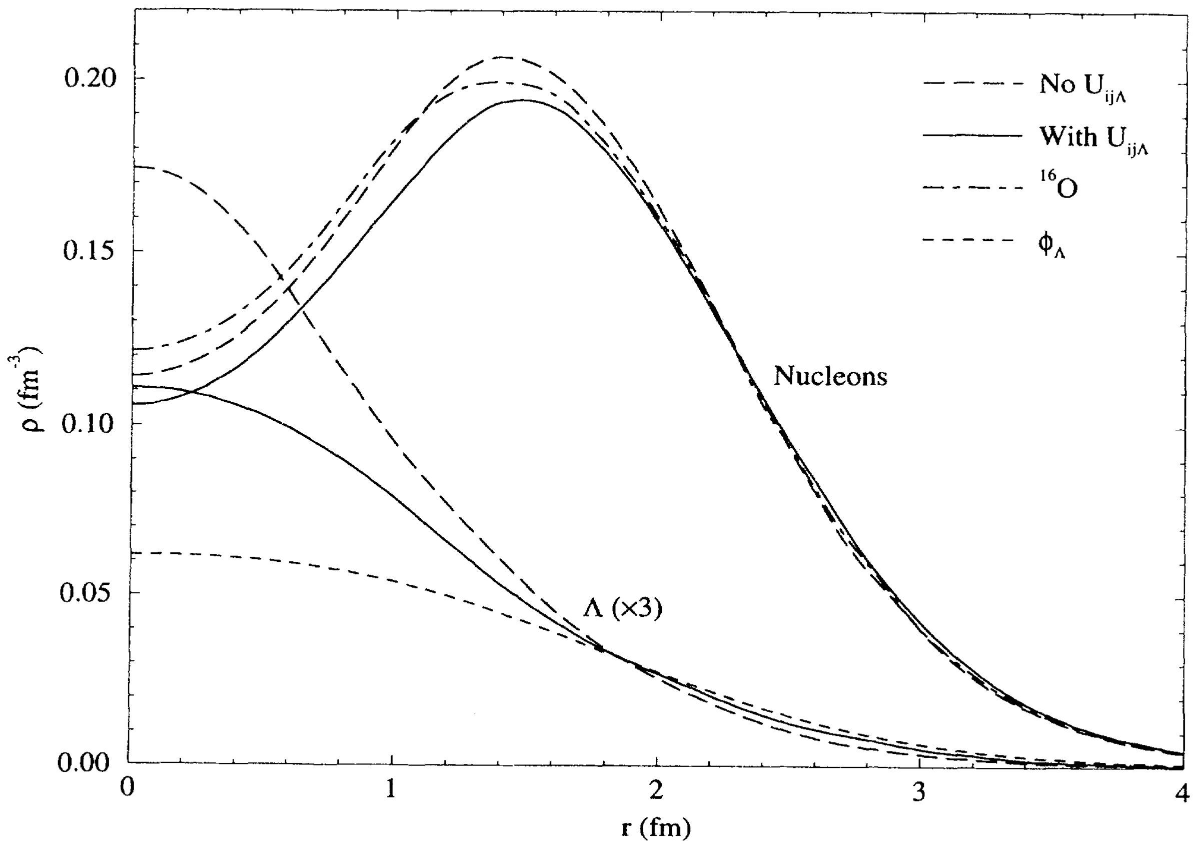
S. Gandolfi, A. Lovato, J. Carlson, K. E. Schmidt, Phys. Rev. C 90, 061306(R) (2014)
F. Pederiva, F. Catalano, D. L., A. Lovato, S. Gandolfi, arXiv:1506.04042 (2015)

Backup: strangeness in nuclei

single particle densities and radii



unpublished



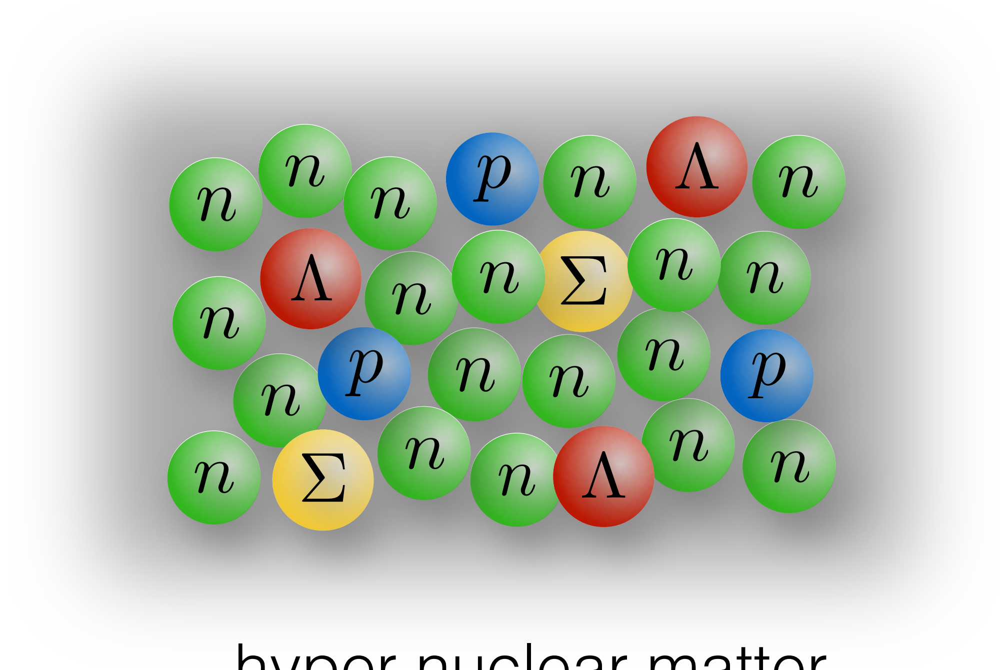
A. A. Usmani, S. C. Pieper, Q. N. Usmani,
Phys. Rev. C 51, 2347-2355 (1995)

$$p\left(^{16}\text{O}\right) = 2.50(2)\ \text{fm} \longrightarrow exp: 2.79\ \text{fm}$$

$$rms: \quad p\left(^{17}_{\Lambda}\text{O}\right) = 2.52(3)\ \text{fm}$$

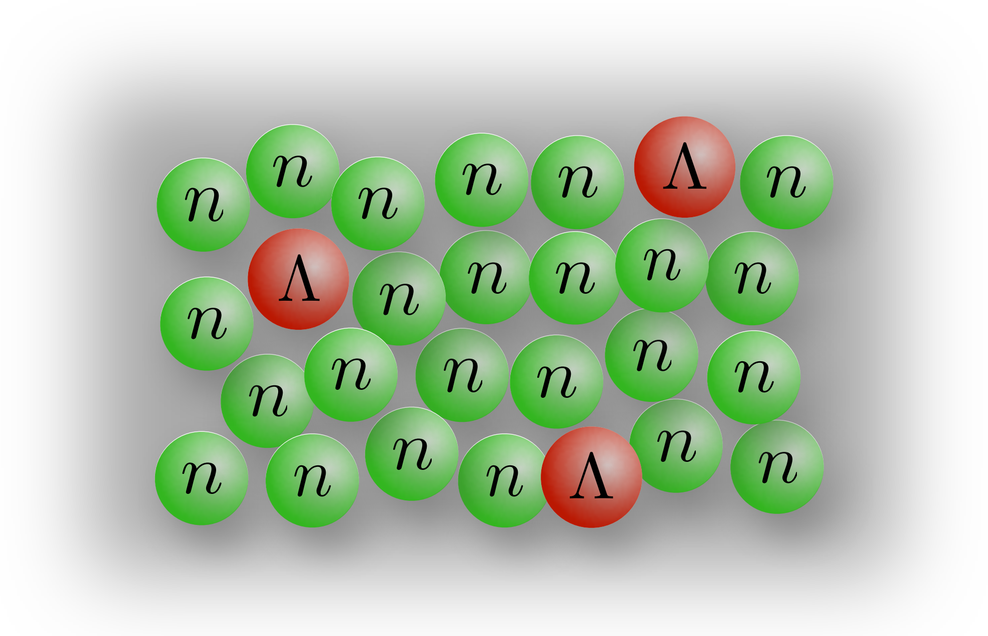
$$\Lambda\left(^{17}_{\Lambda}\text{O}\right) = 2.2(1)\ \text{fm}$$

Backup: strangeness in neutron stars



hyper-nuclear matter

Backup: strangeness in neutron stars



lambda-neutron matter

PNM → hyperon fraction → energy per particle

equilibrium condition: chemical potentials

$$\mu_\Lambda(\rho_b, x_\Lambda) = \mu_n(\rho_b, x_\Lambda)$$

$$\text{EOS} \left\{ \begin{array}{lcl} E_{\text{HNM}} & \equiv & E_{\text{HNM}}(\rho_b) \\ \mathcal{E}_{\text{HNM}} & \equiv & \mathcal{E}_{\text{HNM}}(\rho_b) \\ P_{\text{HNM}} & \equiv & P_{\text{HNM}}(\rho_b) \end{array} \right. \quad \Rightarrow \quad \text{TOV} \left\{ \begin{array}{l} M(R) \\ M_{\max} \end{array} \right.$$

$E_{\text{HNM}} \equiv E_{\text{HNM}}(\rho_b, x_\Lambda)$ ↔ AFDMC calculations neutrons + lambdas

Backup: strangeness in neutron stars

neutrons
+
lambdas

$$\begin{cases} \rho_b = \rho_n + \rho_\Lambda \\ x_\Lambda = \dfrac{\rho_\Lambda}{\rho_b} \end{cases} \qquad \begin{cases} \rho_n = (1 - x_\Lambda)\rho_b \\ \rho_\Lambda = x_\Lambda \rho_b \end{cases}$$

$$E_{\mathrm{HNM}}(\rho_b, x_\Lambda) = \Big[E_{\mathrm{PNM}}((1-x_\Lambda)\rho_b) + m_n\Big](1 - x_\Lambda) + \Big[E_\Lambda^F(x_\Lambda \rho_b) + m_\Lambda\Big]x_\Lambda + f(\rho_b, x_\Lambda)$$

Problem1: limitation in x_Λ due to simulation box

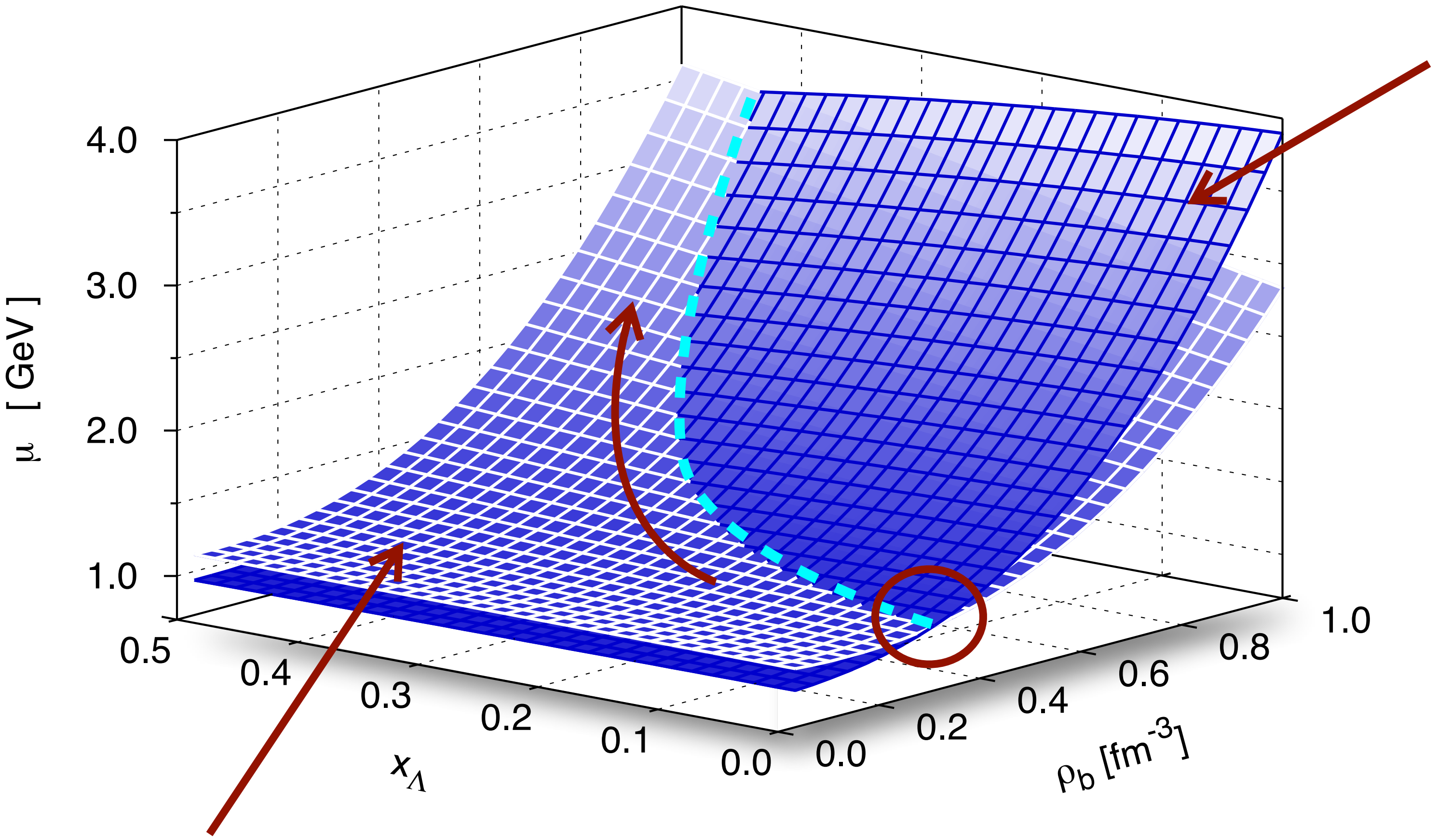
Problem2: finite size effects

Problem3: fitting procedure

$f(\rho_b, x_\Lambda)$ cluster expansion $\dfrac{\rho_\Lambda \rho_n}{\rho_b}$, $\dfrac{\rho_\Lambda \rho_n \rho_n}{\rho_b}$, $\cancel{\dfrac{\rho_\Lambda \rho_\Lambda \rho_n}{\rho_b}}$, $\cancel{\dfrac{\rho_\Lambda \rho_n \rho_n \rho_n}{\rho_b}}$

Backup: strangeness in neutron stars

$$\begin{cases} \mu_n(\rho_b, x_\Lambda) = E_{\rm PNM}(\rho_n) + \rho_n \dfrac{\partial E_{\rm PNM}}{\partial \rho_n} + m_n + f(\rho_b, x_\Lambda) + \rho_b \dfrac{\partial f}{\partial \rho_n} \\ \mu_\Lambda(\rho_b, x_\Lambda) = \quad E^F_\Lambda(\rho_\Lambda) \quad + \quad \rho_\Lambda \dfrac{\partial E^F_\Lambda}{\partial \rho_\Lambda} \quad + m_\Lambda + f(\rho_b, x_\Lambda) + \rho_b \dfrac{\partial f}{\partial \rho_\Lambda} \end{cases}$$



$\mu_n(\rho_b, x_\Lambda)$

$\mu_\Lambda(\rho_b, x_\Lambda)$

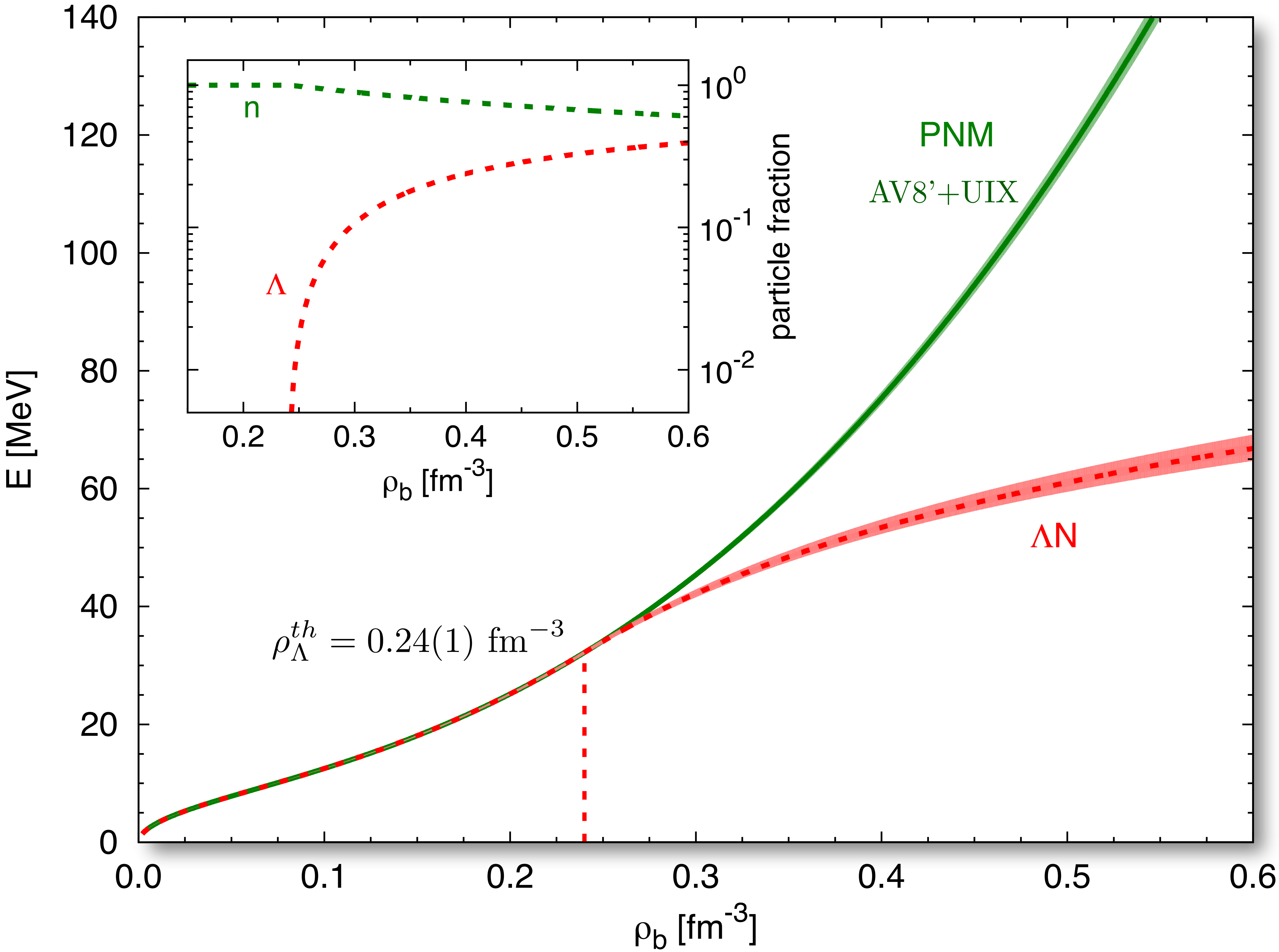
equilibrium condition:

$$\mu_\Lambda(\rho_b, x_\Lambda) = \mu_n(\rho_b, x_\Lambda)$$

$$\rho_\Lambda^{th} \quad @ \quad x_\Lambda \to 0$$

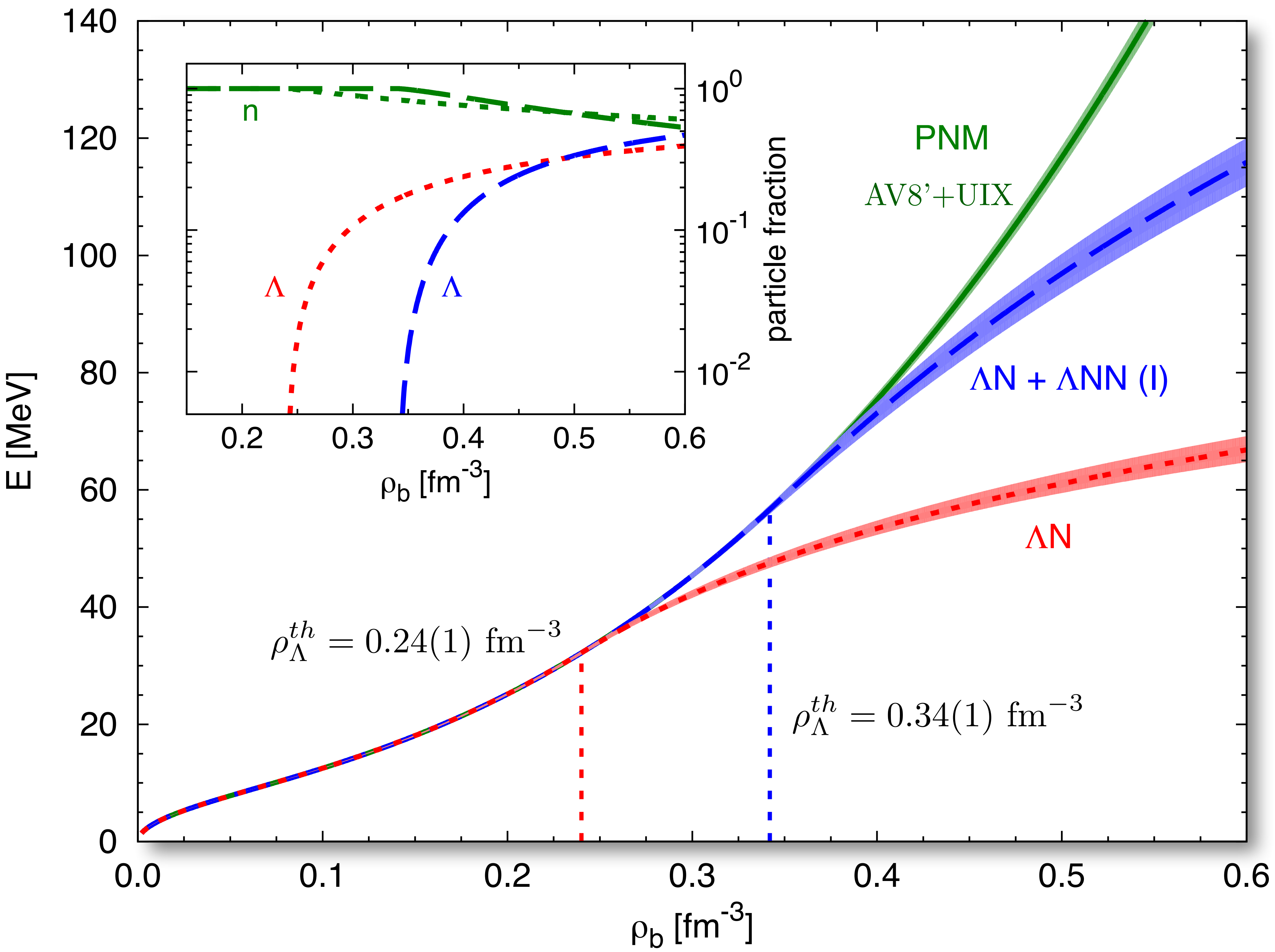
$$x_\Lambda \equiv x_\Lambda(\rho_b)$$

Backup: strangeness in neutron stars



D. L., A. Lovato, S. Gandolfi, F. Pederiva, Phys. Rev. Lett. 114, 092301 (2015)

Backup: strangeness in neutron stars



D. L., A. Lovato, S. Gandolfi, F. Pederiva, Phys. Rev. Lett. 114, 092301 (2015)

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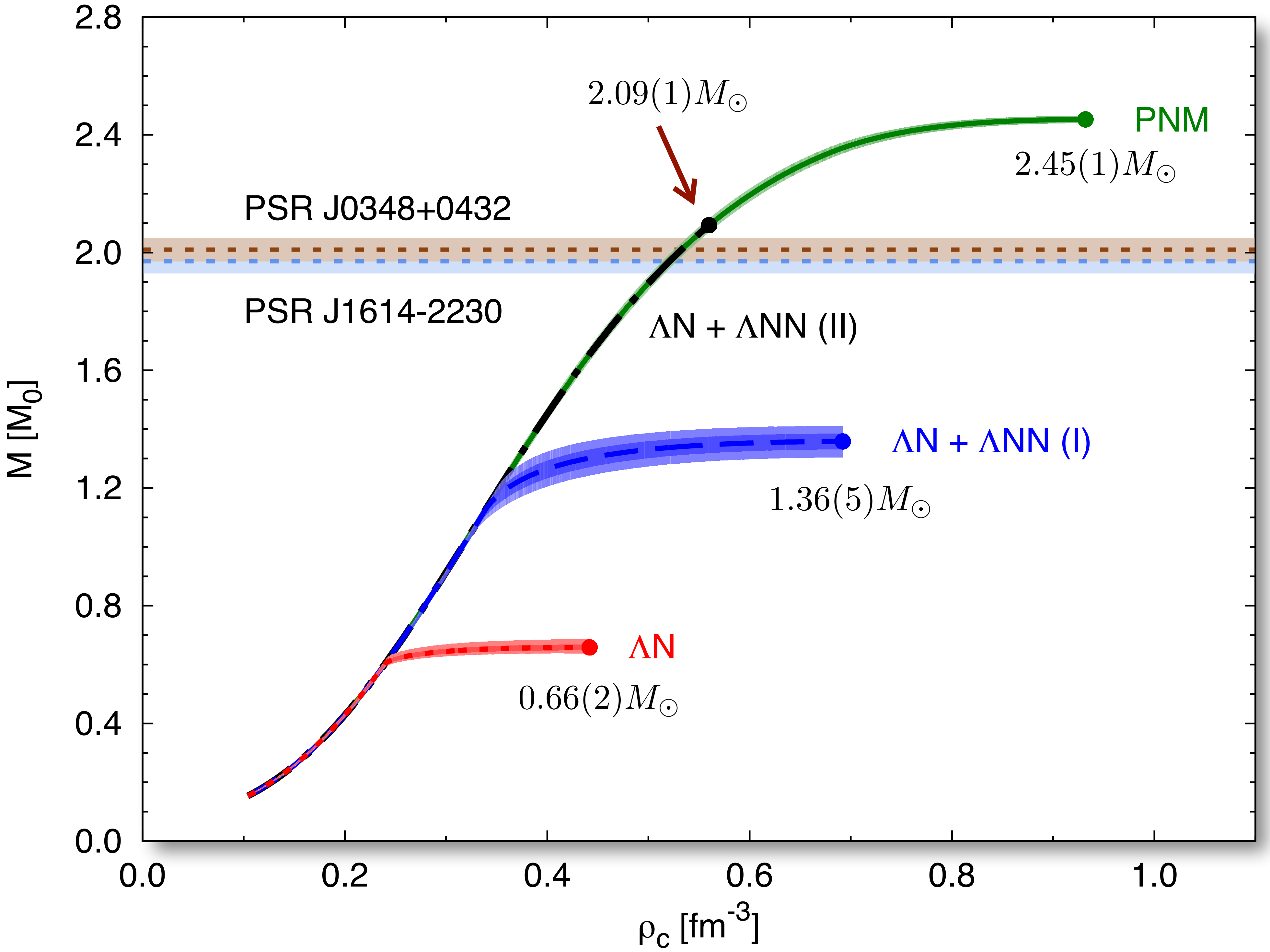

D. L., A. Lovato, S. Gandolfi, F. Pederiva, Phys. Rev. Lett. 114, 092301 (2015)

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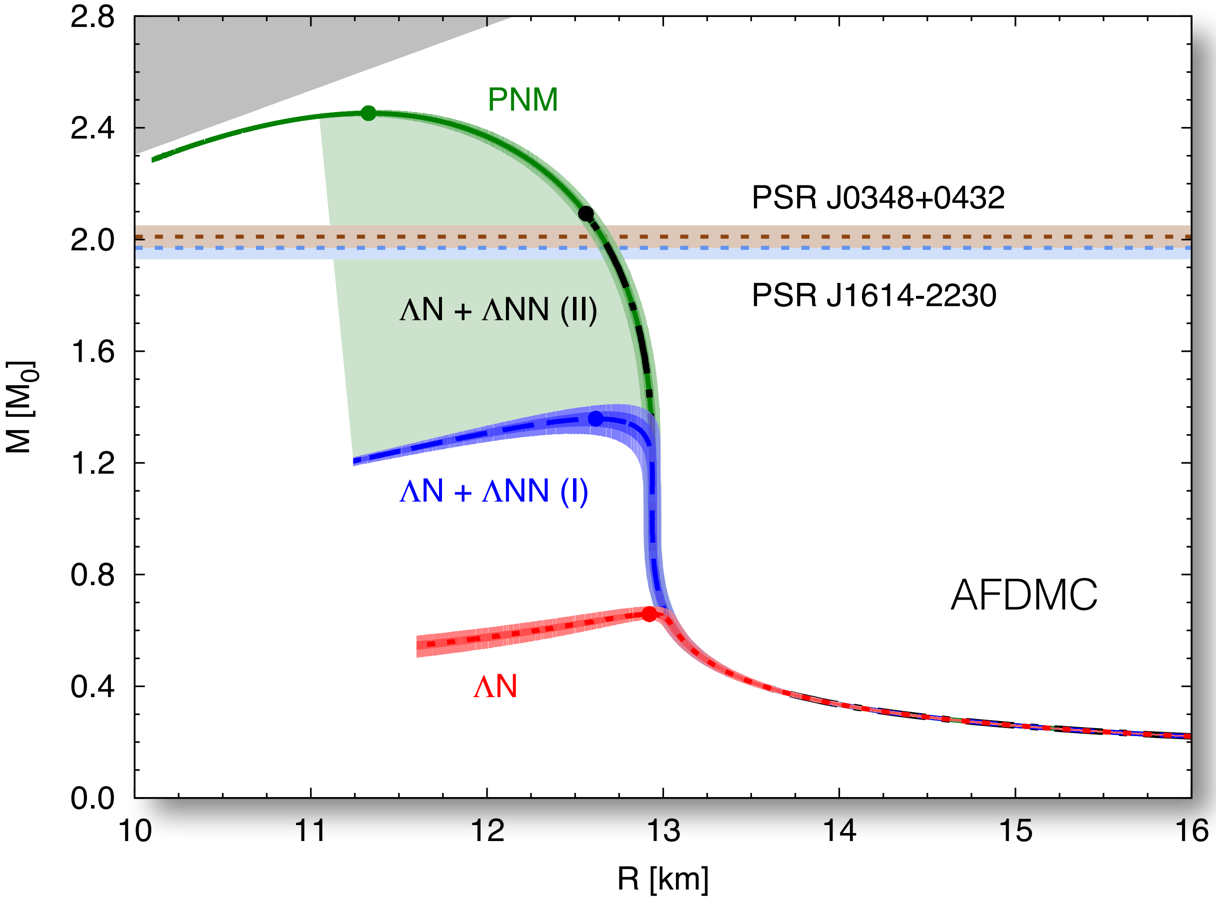
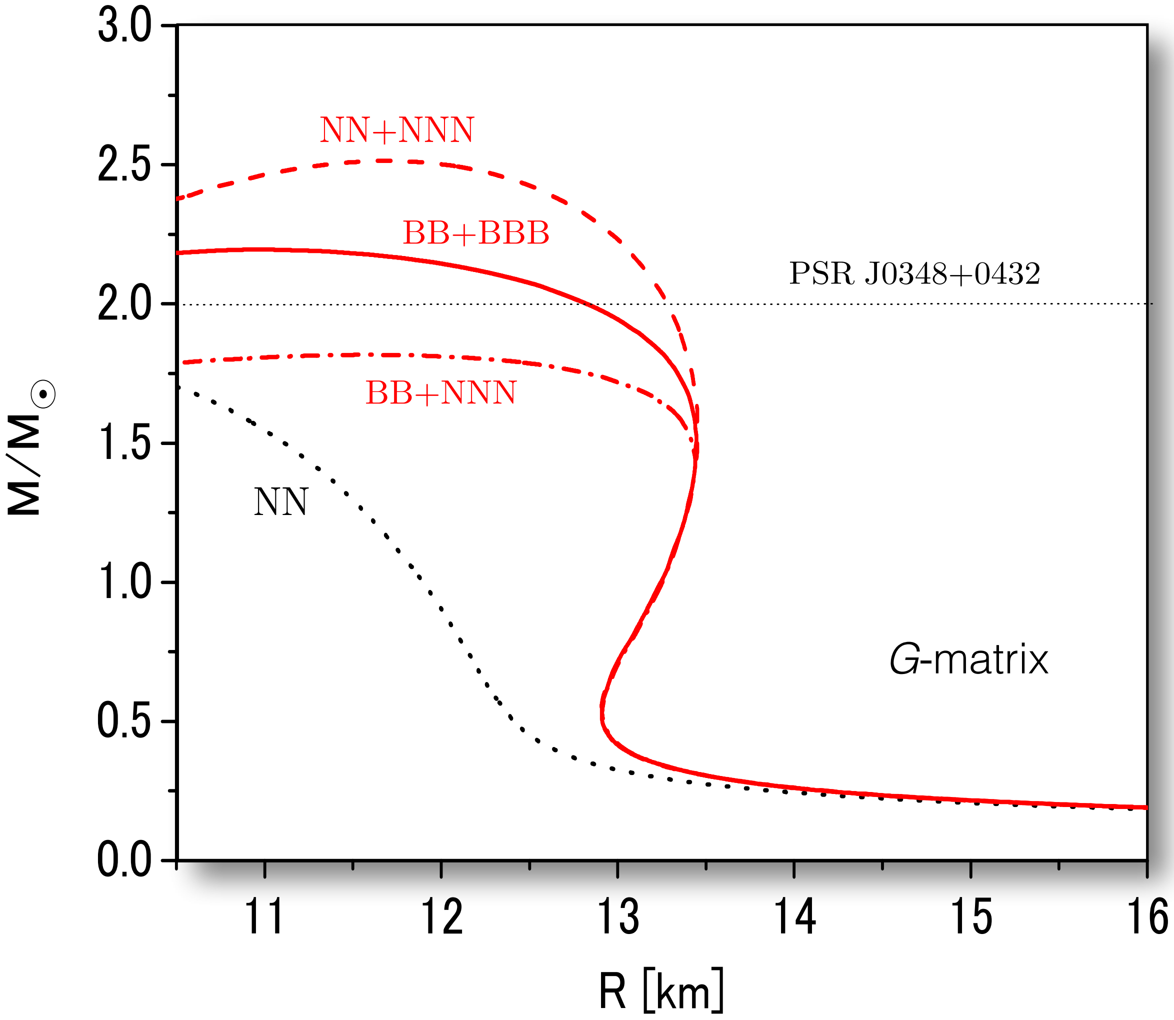
Phys. Rev. Lett. 114, 092301 (2015)



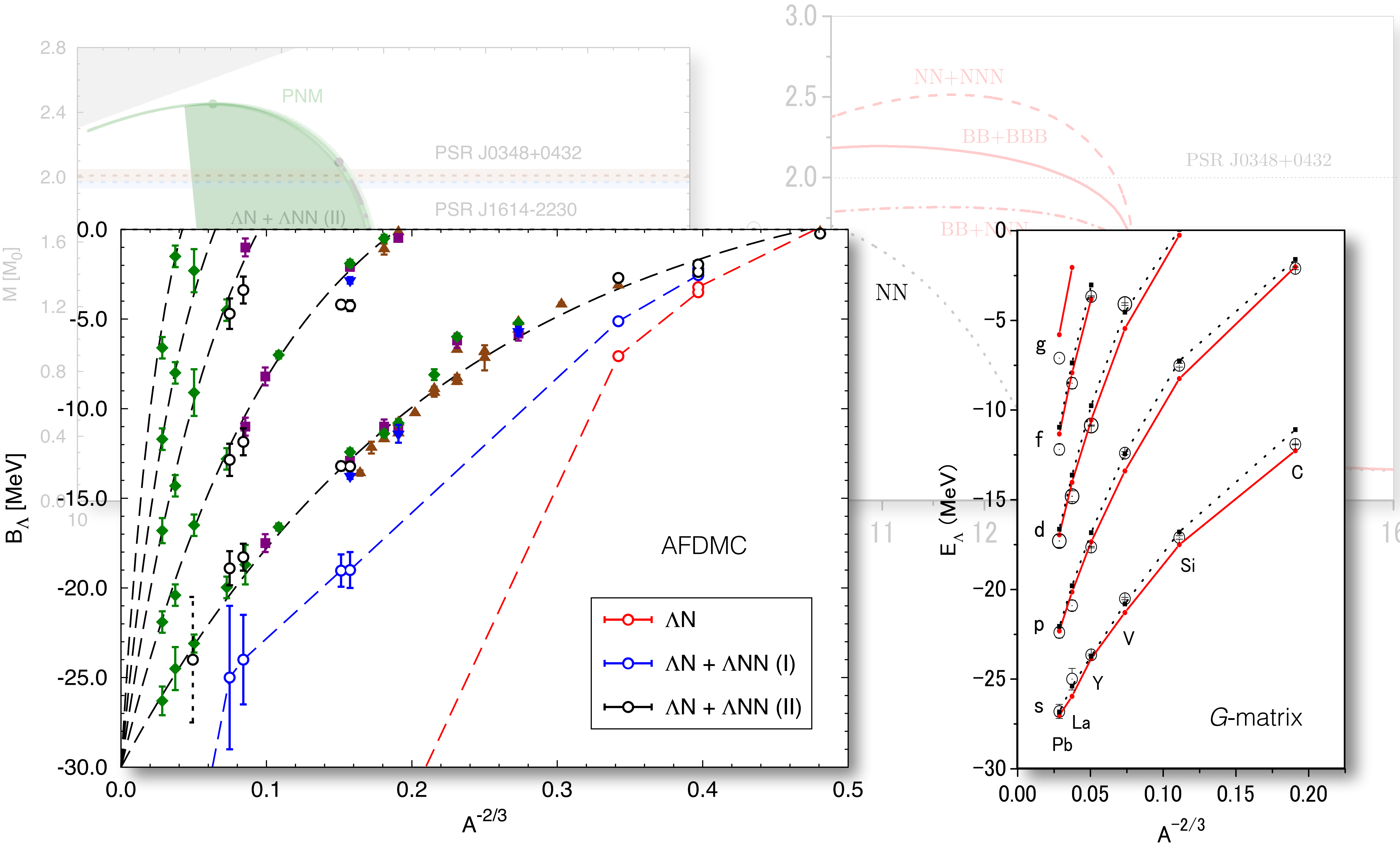
Phys. Rev. C 90, 045805 (2014)



Backup: strangeness in neutron stars

Phys. Rev. Lett. 114, 092301 (2015)

Phys. Rev. C 90, 045805 (2014)



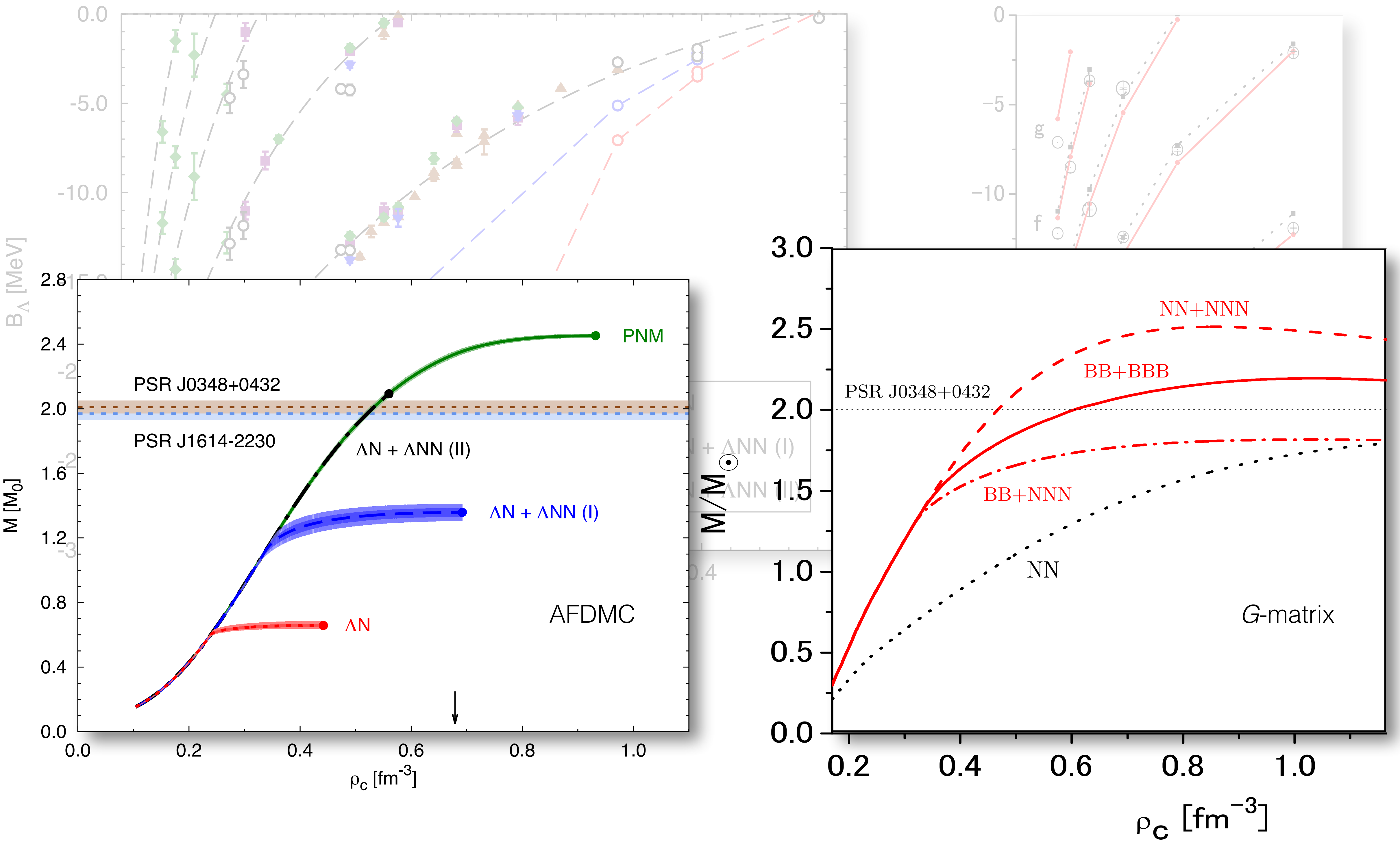
Phys. Rev. C 89, 014314 (2014), arXiv:1506.04042 (2015)

Phys. Rev. C 90, 045805 (2014)

Backup: strangeness in neutron stars

Phys. Rev. C 89, 014314 (2014), arXiv:1506.04042 (2015)

Phys. Rev. C 90, 045805 (2014)



Phys. Rev. Lett. 114, 092301 (2015)

Phys. Rev. C 90, 045805 (2014)

Backup: strangeness in nuclei

- ✓ 3-body interaction ⟶ fit on symmetric hypernuclei

$$v_{\lambda ij} = v^{2\pi,P}_{\lambda ij} + v^{2\pi,S}_{\lambda ij} + v^{D}_{\lambda ij}$$

$$\begin{cases} v^{2\pi,P}_{\lambda ij} = -\frac{C_P}{6}\Big\{X_{i\lambda}\,,X_{\lambda j}\Big\}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j \\ v^{2\pi,S}_{\lambda ij} = C_S\,Z\left(r_{\lambda i}\right)Z\left(r_{\lambda j}\right)\,\boldsymbol{\sigma}_i\cdot\hat{\boldsymbol{r}}_{i\lambda}\,\boldsymbol{\sigma}_j\cdot\hat{\boldsymbol{r}}_{j\lambda}\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j \\ v^{D}_{\lambda ij} = W_D\,T^2_\pi\left(r_{\lambda i}\right)T^2_\pi\left(r_{\lambda j}\right)\Big[1+\frac{1}{6}\boldsymbol{\sigma}_\lambda\cdot\left(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j\right)\Big] \end{cases}$$

isospin projectors

$$\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j = -3\,\mathcal{P}^{T=0}+\mathcal{P}^{T=1} \longrightarrow -3\,\mathcal{P}^{T=0}+C_T\,\mathcal{P}^{T=1}$$

control parameter:
strength and sign of the nucleon
isospin triplet channel

⟵

sensitivity study:
light- & medium-heavy hypernuclei