Part 3: The Soft / Collinear Limit of QCD

Last time: Jet Algorithms, workhorse of hadronic final states

Jet as proxy for parton → Fundamentally ambiguous but still essential

Anti-k_t is current standard → Circular jets, area $\sim \pi R^2$
IRC safe

Today: a jet is more than $\vec{p}_{jet}$!

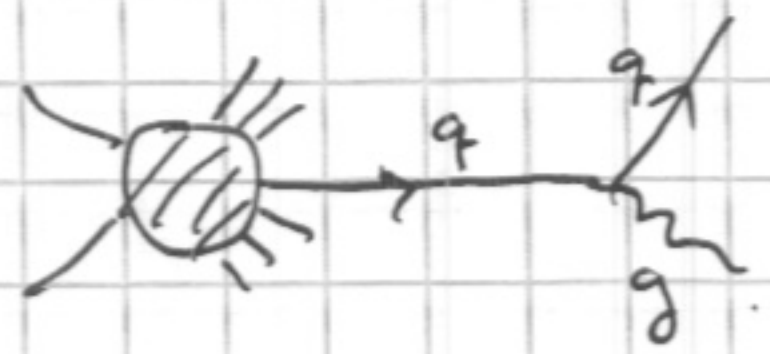
Jets have constituents and you can measure its substructure

Example: Using soft / collinear limit of QCD to understand quark / gluon discrimination

Pedagogical choice: I'm going to tell you a key fact, not derive it, and go from there.

Glosses over many cool aspects of QCD, but limited time, and I think this is best for lectures on jets.

Exercise: Show that in soft $\underline{\underline{\&}}$ collinear limit

$$\sum_{\text{polarizations}} \int d\Phi_{n+1} \left| \vphantom{\int} \right.$$  $$\left. \vphantom{\int} \right|^2$$

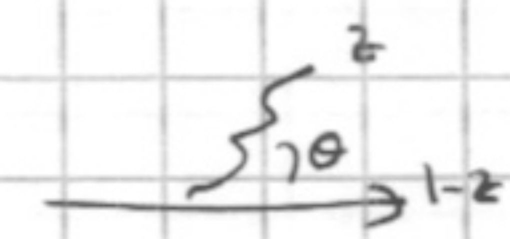
(with q and g in the same jet)

$$\simeq \sum_{\text{polarizations}} \int d\Phi_n \left| \vphantom{\int} \right.$$  $$\left. \vphantom{\int} \right|^2$$

$$\times \int_0^1 dz \int_0^R d\theta \; \frac{2\alpha_s}{\pi} \; C_F \; \frac{1}{z} \; \frac{1}{\theta}$$

color factor, swap to C_A for gluons — soft/collinear singularities

Energy fraction : $z = \dfrac{E_{gluon}}{E_{jet}}$ $\quad z \to 0$ soft limit



Splitting angle : $\theta = \theta_{qg}$ $\quad \theta \to 0$ collinear limit

Color factors: $\quad SU(N_c) \to SU(3)$

Quarks: $\sum_{a,j} t^a_{ij} t^a_{jk} = C_F \delta_{ik}$, $C_F = \dfrac{N_c^2 - 1}{2N_c} \to 4/3$

Gluon $\sum_{a,b} f^{abc} f^{abd} = C_A \delta^{cd}$, $C_A = N_c \to 3$

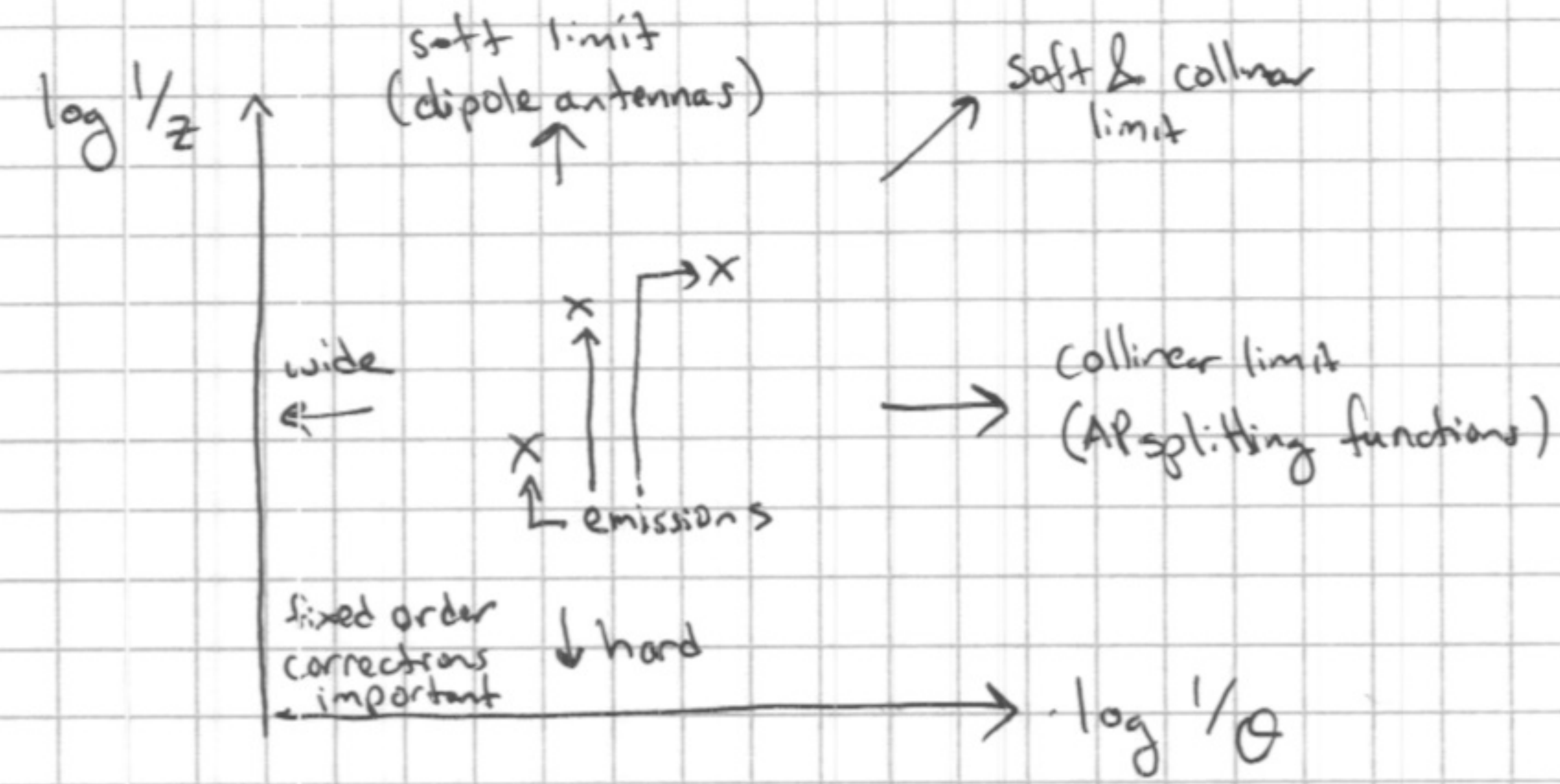
Note: Above only true for soft $\underline{\underline{\&}}$ collinear limit

In this extreme limit, splitting probability is:

$$dP_{i \to ig} = \frac{2\alpha_s}{\pi} \frac{dz}{z} \frac{d\theta}{\theta}$$

soft gluon

Uniform emissions in the $\left(\log \frac{1}{\theta}, \log 1/z\right)$ plane

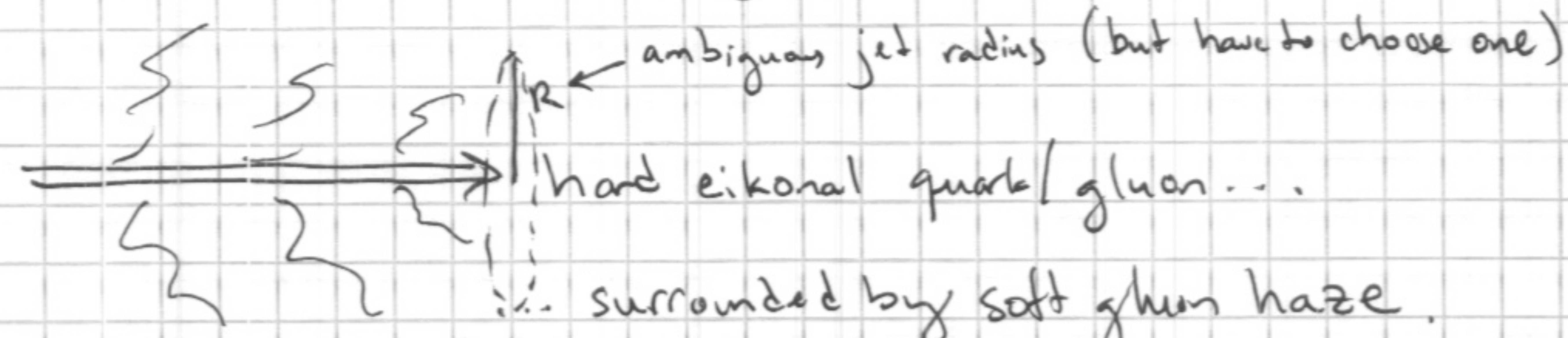


In "strongly-ordered" (or "leading log" or "double log") limit, we can do simple but insightful calculations.

Very low accuracy, but gives a flavor for questions worth asking.

Many approaches to systematically improving this picture, e.g. SCET, direct resummation, ...

The (simplest) picture of a jet



In fact, basis for parton shower, which recursively applies $p_i \to jk$ (for all parton flavors)

Benefit: Some information at <u>all</u> orders in α_s.

Often more realistic qualitatively than fixed-order calculations.

Caution: Have to think carefully about quantitative accuracy.

Hard work to convince yourself that it is systematically improvable.

Insight: Jet is clearly not a single parton

(Wilson-line-wrapped eikonal parton)

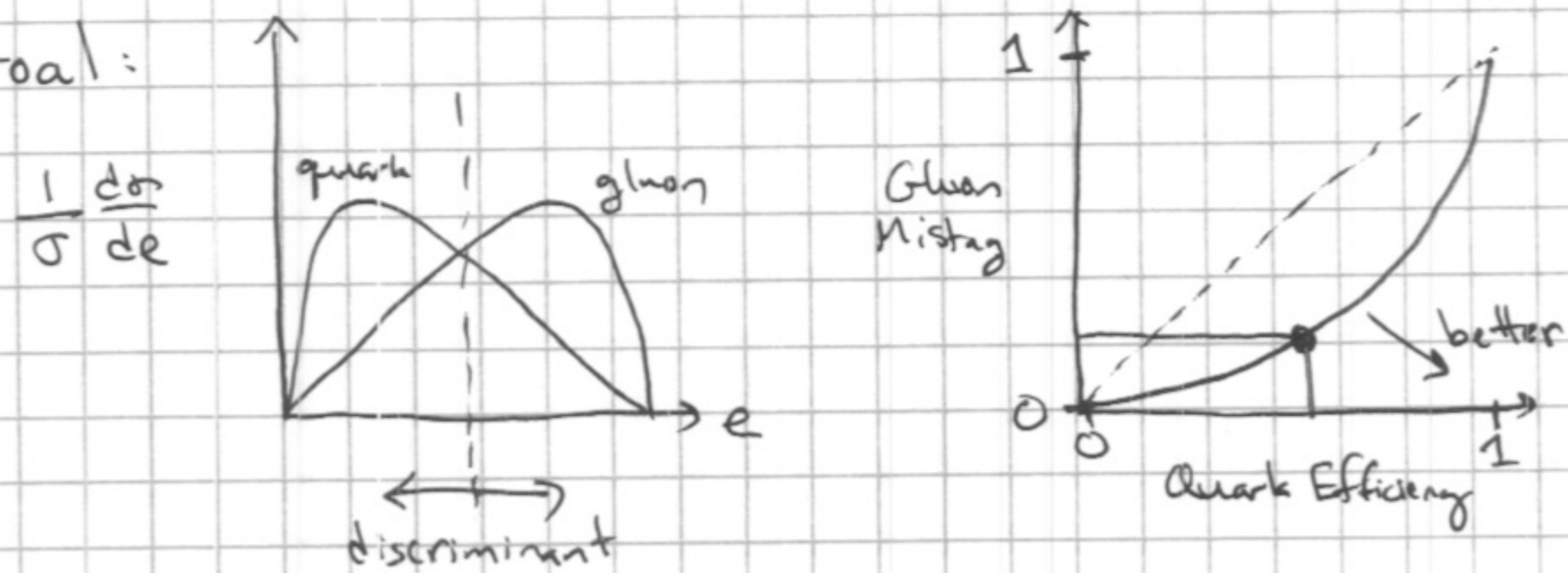
Very instructive to always visualize the haze.

Concrete question: Quark vs. Gluon Jets?

$C_F < C_A$, so gluon jets should be "fatter"

Need to define observables sensitive to this difference

Goal:



Ideally: Predict from first principles QCD

Validate in Monte Carlo parton showers

Test in LHC data

Why do we care?

Experimentally: Different calibrations for different jet types

QCD Theory: Observables sensitive to color structure of QCD

BSM Theory: Often, BSM signals are quark-rich while backgrounds are gluon-rich.

Choice of discriminant observable?

→ IRC Safe (in order to perform perturbative calculations)

→ Not overly sensitive to hadronization or other non-perturbative effects.

Want $\delta e \sim \left(\frac{\Lambda_{QCD}}{E_{jet}}\right)^{\#}$

Simplest example I know: Energy-Energy Correlation Functions

$p_T \to E$ $\quad R \to \theta$ for simplicity

$$e_2^{(\beta)} = \frac{\sum_{i \neq j} E_i E_j \theta_{ij}^{\beta}}{\left(\sum_i E_i\right)^2}$$

e_2 ↑ 2-point correlator for 1-prong testing

$\beta > 0$ for IRC safety

Also known as $C_1^{(\beta)}$.

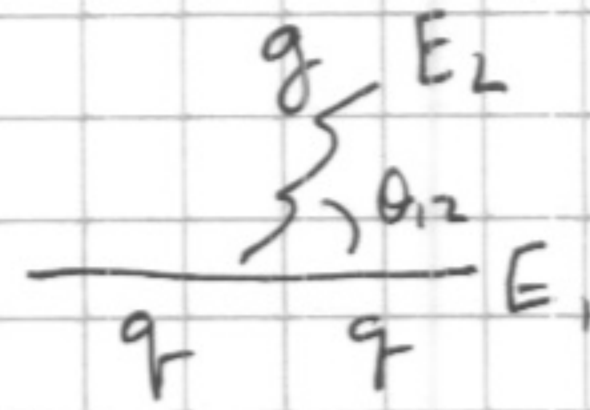
C_1 ↑ 1-prong testing with 2-point correlator

Soft safe? Yes, e_2 doesn't change if $E_i = 0$

Collinear safe? Yes, e_2 is additive, so $E \to E_1 + E_2$ has no effect (for $\beta > 0$)

What is quark/gluon discrimination power for e_2?

Go to extreme soft & collinear & strongly-ordered limit of QCD (borderline physical)

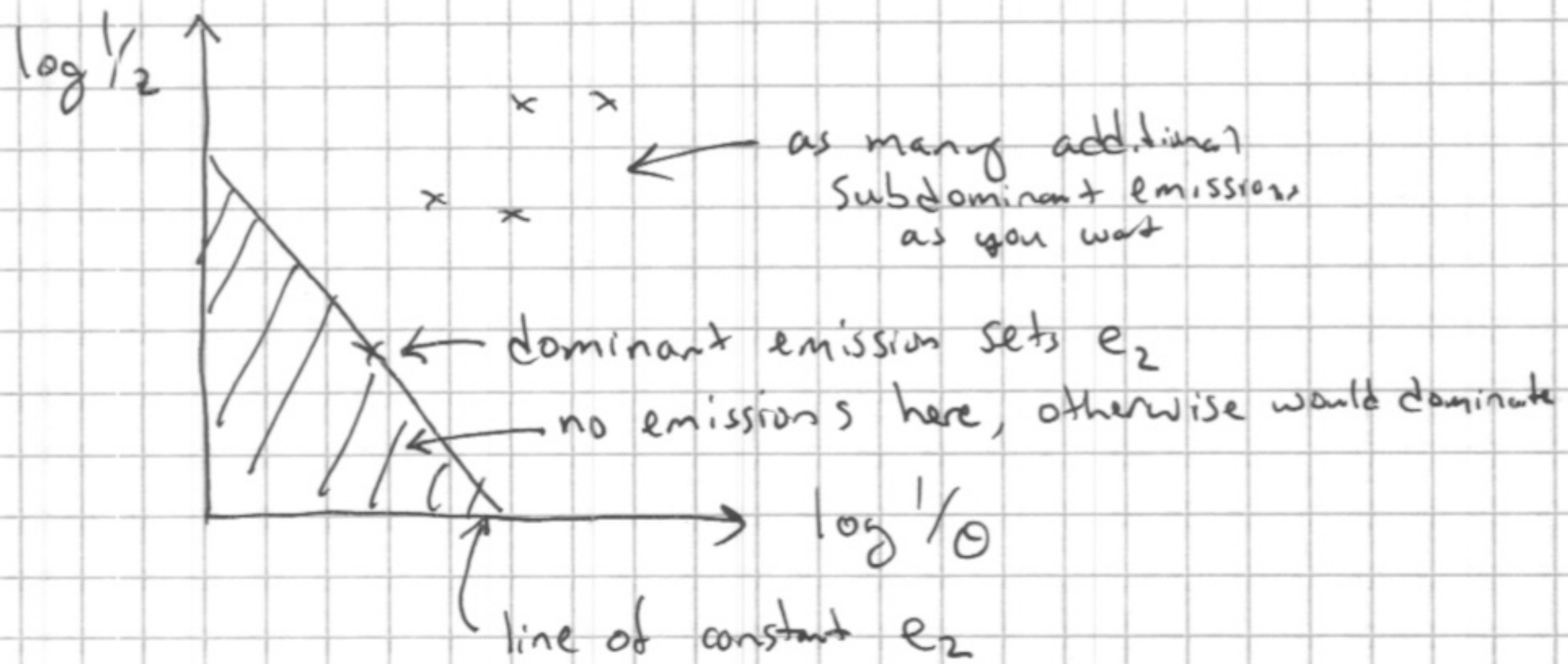


$$e_2 = \frac{E_1 E_2 \theta_{12}^{\beta}}{(E_1 + E_2)^2}$$

$$\approx z\,\theta^{\beta} \quad \text{for } E_2 \ll E_1$$

where $z = \dfrac{E_2}{E_1 + E_2}$

Recall: $dP_{i \to ig} = \dfrac{2\alpha_s}{\pi} C_i \dfrac{dz}{z} \dfrac{d\theta}{\theta}$



$$\log \frac{1}{e_2} = \log \frac{1}{z} + \beta \log \frac{1}{\theta}$$

How do you calculate cross section for e_2?

In particular, $dP_{i\to ig}$ is for real emissions.

Where are all the virtual diagrams?

Real:  Virtual: 

at finite z, θ effectively at $\left(\log\frac{1}{\theta}, \log\frac{1}{z}\right) \to (\infty,\infty)$

Easiest to think of in terms of probabilities.

$$P_{emit} + P_{no\text{-}emit} = 1$$

$\mathcal{O}(\alpha_s^0)$ $\quad P^{(0)}_{no\text{-}emit} = 1 \quad \xrightarrow{\text{just}}$

$\mathcal{O}(\alpha_s^1)$ $\quad P^{(1)}_{emit} + P^{(1)}_{no\text{-}emit} = 0$ 

weird!

has to be zero.

Fun exercise in probability: chance to get value of e_2 less than e_2^{max} is

$$\Sigma_q\left(e_2^{max}\right) = \int_0^{e_2^{max}} p(e_2)\, de_2$$

$$= \exp\left[-\frac{2\alpha_s}{\pi} C_F \left(\text{area of } \right)\right]$$ 

$\left(= 1 - \right.$  $+ \frac{1}{2}$  ^{2} $\left. \cdots\cdots \right)$

no emission @ α_s^0 | veto emission @ α_s^1 | subtleties @ α_s^2

from  $= -$ 

I assume you've never done a calculation like that before? Very standard in parton shower picture. Basis for MC generators.

Final result: $$\Sigma_q\left(e_2^{max}\right) = \exp\left[\frac{-\alpha_s}{\pi}\frac{C_F}{\beta}\log^2\frac{R^\beta}{e_2^{max}}\right]$$

For gluons swap $C_F \to C_A$.

Called Sudakov form factor.

double-logarithmic

Has some information (double-log enhanced) at all orders in α_s.

Systematically improvable in approaches to resummation like SCET.

For simplicity: set $R=1$ (or equivalently, rescale e_2)

To go from cumulative distribution Σ to probability distribution, just take a derivative:

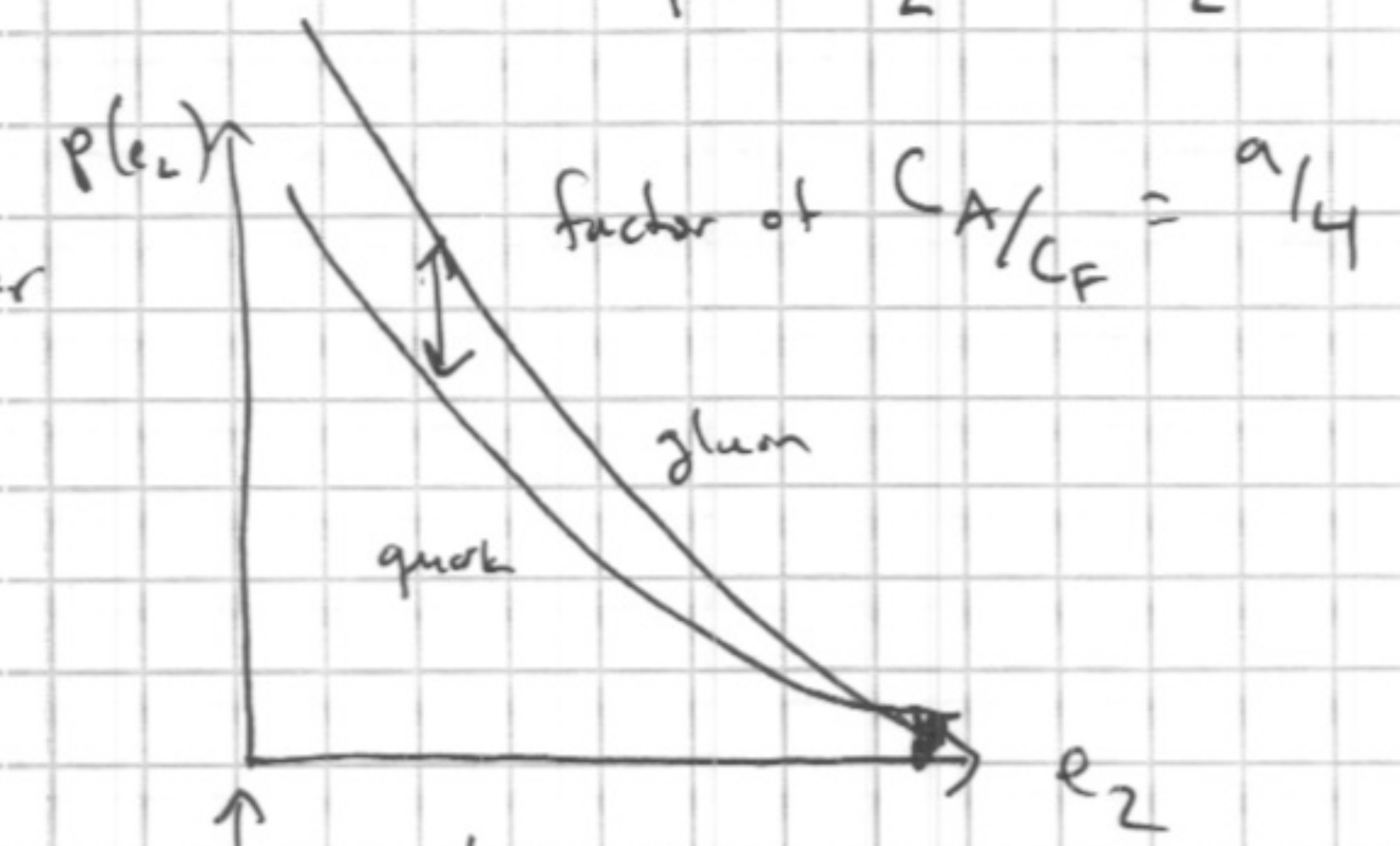
$$p(e_2) = \frac{1}{\sigma}\frac{d\sigma}{de_2} = \frac{\partial}{\partial e_2}\Sigma(e_2)$$

Time to interpret our result!

1) What is cross section?

$$p_q(e_2) = \frac{2\alpha_s}{\pi}\frac{C_F}{\beta}\frac{1}{e_2}\log\frac{1}{e_2}\exp\left[-\frac{\alpha_s}{\pi}\frac{C_F}{\beta}\log\frac{1}{e_2}\right]$$

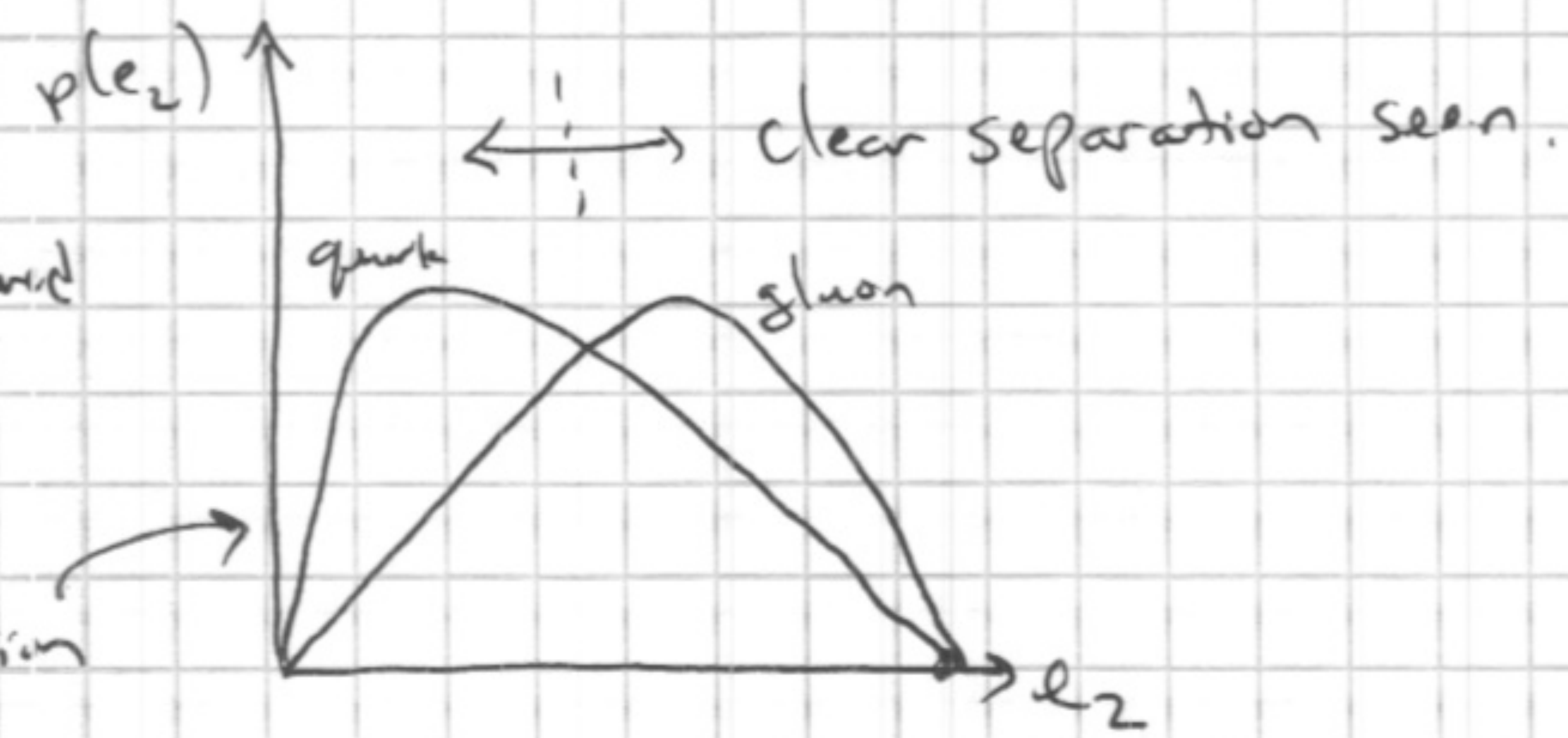
At fixed-order $\mathcal{O}(\alpha_s^1)$



plus/delta function from virtual diagrams at origin to ensure normalized distributions.

In strongly-ordered limit to all orders in α_s

Sudakov suppression



This resummed calculation is much closer to what is seen experimentally.

2) Can we quantify discrimination power?

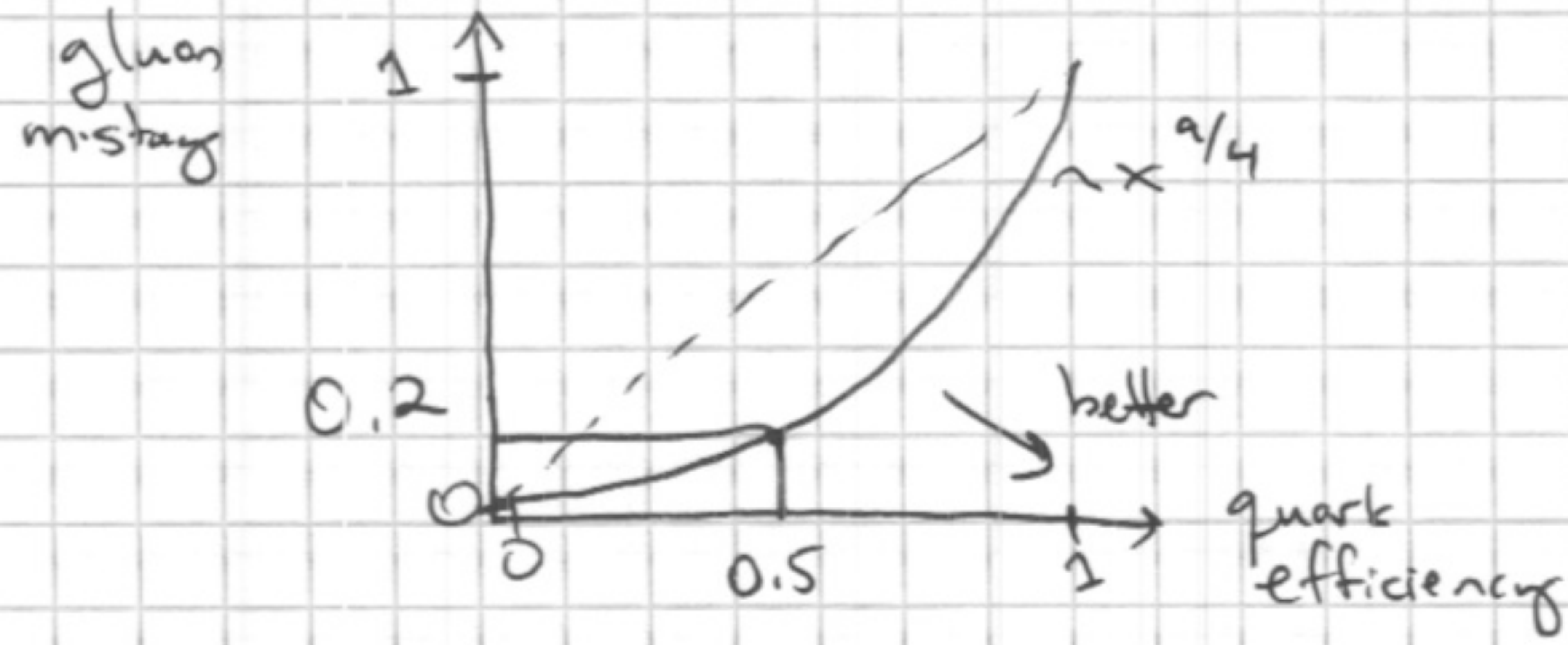
Place a cut of e_2^{cut} $\quad$ $e_2 < e_2^{cut} \Rightarrow$ "quark"

$e_2 > e_2^{cut} \Rightarrow$ "gluon"

Want to know mistag rate $\left(\Sigma_g(e_2^{cut})\right)$ as a function of efficiency $\left(\Sigma_q(e_2^{cut})\right)$.

Key: $\quad \Sigma_g = \left(\Sigma_q\right)^{C_A/C_F \to 9/4}$

Called "Casimir Scaling" (violated at higher orders)



In strongly-ordered limit, independent of β!

All* quark/gluon discriminants alike, because only difference at this order is C_F vs C_A.

Open question: How best to separate quarks from gluons using higher-order information?

Many important higher-order effects

- → Multiple emissions
- → Color coherence
- → Subleading terms in splitting function
- → Fixed-order corrections
- → Running α_s
- → Non-global logarithms
- → Hadronization Effects
- → ...

Hope you've gained some appreciation for value of working to all orders in α_s.

Next time: The Substructure (R)evolution.

Multi-prong Jets & Jet Cleaning

(more complicated observables for W/Z/H/top)

(dealing with challenges of jet contamination.

General Strategy for Jet Substructure

→ Figure out your goal

"Discriminate Quarks vs. Gluons"

→ Identify Underlying Physics

"C_F vs. C_A"

→ Construct "clean" observable to probe that physics

$$\text{"} e_2^{(\beta)} = \sum_{i \neq j} z_i z_j \theta_{ij}^{\beta} \text{"}$$

→ Use analytics | Monte Carlo generators to estimate distributions and discrimination power.

→ Convince experimentalists to apply your method to data (or if desperate, do yourself)

Ask me about CMS Open Data