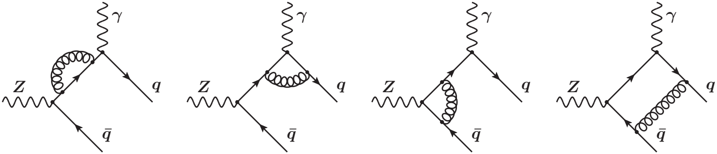
Exclusive Radiative Decays of Z Bosons in QCD Factorization

Stefan Alte, Johannes Gutenberg-Universität Mainz

Yuval Grossman, Matthias König and Matthias Neubert



MITP Summer School

1 August 2016

JG|U

PRISMA Cluster of Excellence
Precision Physics, Fundamental Interactions and Structure of Matter

Introduction

One of the **main challenges** to particle physics: **strongly coupled nature of QCD** at low energy scales

Introduction

One of the **main challenges** to particle physics: **strongly coupled nature of QCD** at low energy scales

Hard exclusive processes with individual hadrons in final state:
QCD factorization allows **factorization of the decay amplitude**

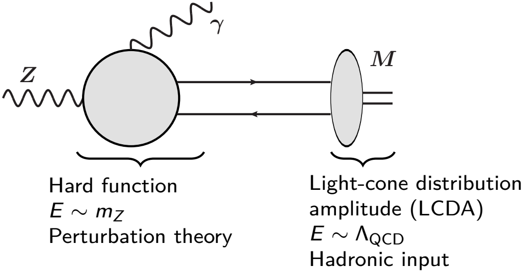
Brodsky, Lepage (1979); Efremov, Radyushkin (1980); Chernyak, Zhitnitsky (1984)

Introduction

One of the **main challenges** to particle physics: **strongly coupled nature of QCD** at low energy scales

Hard exclusive processes with individual hadrons in final state:
QCD factorization allows **factorization of the decay amplitude**

Brodsky, Lepage (1979); Efremov, Radyushkin (1980); Chernyak, Zhitnitsky (1984)



Hard function
$E \sim m_Z$
Perturbation theory

Light-cone distribution amplitude (LCDA)
$E \sim \Lambda_{\text{QCD}}$
Hadronic input

Introduction

Decay amplitude: organised as an **expansion in $\lambda = \Lambda_{\mathrm{QCD}}/m_Z$**

Introduction

Decay amplitude: organised as an **expansion in $\boldsymbol{\lambda = \Lambda_{\mathrm{QCD}}/m_Z}$**

- Power corrections **not negligible** in existing applications of QCD factorization

Introduction

Decay amplitude: organised as an **expansion in $\boldsymbol{\lambda = \Lambda_{\mathrm{QCD}}/m_Z}$**

- Power corrections **not negligible** in existing applications of QCD factorization
- **Benefit** for $Z \to M\gamma$: power corrections **negligible**

Introduction

Decay amplitude: organised as an **expansion in $\boldsymbol{\lambda = \Lambda_{\text{QCD}}/m_Z}$**

- Power corrections **not negligible** in existing applications of QCD factorization
- **Benefit** for $Z \to M\gamma$: power corrections **negligible**
- **Price to pay: small rates** → measurements will be challenging

Introduction

Decay amplitude: organised as an **expansion in $\lambda = \Lambda_{\text{QCD}}/m_Z$**

- Power corrections **not negligible** in existing applications of QCD factorization
- **Benefit** for $Z \to M\gamma$: power corrections **negligible**
- **Price to pay: small rates** → measurements will be challenging
- **But**: **enormous rates** of Z bosons expected in the future
    - HL-LHC: $\sim \mathbf{10^{11}}$ Z bosons
    - ILC or TLEP at the Z pole: $\mathbf{10^{12}}$ Z bosons per year

Introduction

Decay amplitude: organised as an **expansion in $\boldsymbol{\lambda = \Lambda_{\mathrm{QCD}}/m_Z}$**

- Power corrections **not negligible** in existing applications of QCD factorization
- **Benefit** for $Z \to M\gamma$: power corrections **negligible**
- **Price to pay: small rates** → measurements will be challenging
- **But**: **enormous rates** of Z bosons expected in the future
    - HL-LHC: $\sim \mathbf{10^{11}}$ Z bosons
    - ILC or TLEP at the Z pole: $\mathbf{10^{12}}$ Z bosons per year

"*Exclusive Radiative Decays of W and Z Bosons in QCD Factorization*", Grossman, König and Neubert (2015), JHEP **1504** (2015) 101, arXiv: **1501.06569**

"*Exclusive Radiative Z-Boson Decays to Mesons with Flavor-Singlet Components*", Alte, König and Neubert (2016), JHEP **1602** (2016) 162, arXiv: **1602.09135**

Factorization Formula

- Derivation using **soft-collinear effective theory (SCET)**

  Bauer, Pirjol, Stewart (2001); Bauer *et al.* (2002); Diehl, Feldmann (2002)

Factorization Formula

- Derivation using **soft-collinear effective theory (SCET)**

  Bauer, Pirjol, Stewart (2001); Bauer *et al.* (2002); Diehl, Feldmann (2002)

- Valid at **leading power** in the expansion in $\boldsymbol{\Lambda_{\mathrm{QCD}}/m_Z}$

Factorization Formula

- Derivation using **soft-collinear effective theory (SCET)**

  Bauer, Pirjol, Stewart (2001); Bauer *et al.* (2002); Diehl, Feldmann (2002)

- Valid at **leading power** in the expansion in $\boldsymbol{\Lambda_{\mathrm{QCD}}/m_Z}$
- Valid for mesons with **no flavour-singlet component**

Factorization Formula

- Derivation using **soft-collinear effective theory (SCET)**

  Bauer, Pirjol, Stewart (2001); Bauer *et al.* (2002); Diehl, Feldmann (2002)
- Valid at **leading power** in the expansion in $\boldsymbol{\Lambda_{\mathrm{QCD}}/m_Z}$
- Valid for mesons with **no flavour-singlet component**

$$\mathcal{A} = -i\, f_M\, E \int_0^1 \mathrm{d}x\, H_M(x,\mu)\, \phi_M(x,\mu)$$

Factorization Formula

- Derivation using **soft-collinear effective theory (SCET)**

  Bauer, Pirjol, Stewart (2001); Bauer *et al.* (2002); Diehl, Feldmann (2002)

- Valid at **leading power** in the expansion in $\boldsymbol{\Lambda_{\mathrm{QCD}}/m_Z}$
- Valid for mesons with **no flavour-singlet component**

$$\mathcal{A} = -i\, f_M\, E \int_0^1 \mathrm{d}x\, H_M(x,\mu)\, \phi_M(x,\mu)$$

Decay constant

Factorization Formula

- Derivation using **soft-collinear effective theory (SCET)**

  Bauer, Pirjol, Stewart (2001); Bauer *et al.* (2002); Diehl, Feldmann (2002)

- Valid at **leading power** in the expansion in $\boldsymbol{\Lambda_{\mathrm{QCD}}/m_Z}$
- Valid for mesons with **no flavour-singlet component**

$$\mathcal{A} = -i\, f_M\, E \int_0^1 \mathrm{d}x\, H_M(x,\mu)\, \phi_M(x,\mu)$$

Decay constant

Hard function

Factorization Formula

- Derivation using **soft-collinear effective theory (SCET)**

  Bauer, Pirjol, Stewart (2001); Bauer *et al.* (2002); Diehl, Feldmann (2002)
- Valid at **leading power** in the expansion in $\boldsymbol{\Lambda_{\mathrm{QCD}}/m_Z}$
- Valid for mesons with **no flavour-singlet component**

$$\mathcal{A} = -i\, f_M\, E \int_0^1 \mathrm{d}x\, H_M(x,\mu)\, \phi_M(x,\mu)$$

Decay constant

LCDA

Hard function

Light-Cone Distribution Amplitudes

$$\langle M(k)|\bar{q}(t\bar{n})\frac{\bar{\not{n}}}{2}(\gamma_5)[t\bar{n},0]q(0)|0\rangle = -i\, f_M\, E \int_0^1 \mathrm{d}x\, e^{ixt\bar{n}\cdot k}\phi_M(x,\mu)$$

The LCDA $\phi_M(x,\mu)$ can be interpreted as the **amplitude for finding a quark with longitudinal momentum fraction *x*** inside the meson

Light-Cone Distribution Amplitudes

$$\langle M(k)|\bar{q}(t\bar{n})\frac{\bar{\not{n}}}{2}(\gamma_5)[t\bar{n},0]q(0)|0\rangle = -i\, f_M\, E \int_0^1 \mathrm{d}x\, e^{ixt\bar{n}\cdot k}\phi_M(x,\mu)$$

The LCDA $\phi_M(x,\mu)$ can be interpreted as the **amplitude for finding a quark with longitudinal momentum fraction *x*** inside the meson

$$\phi_M(x,\mu) = 6x(1-x)\left[1 + \sum_{n=1}^{\infty} a_n^M(\mu) C_n^{(3/2)}(2x-1)\right]$$

- Expansion in Gegenbauer polynomials **diagonalises the scale evolution** at the leading order

Light-Cone Distribution Amplitudes

$$\langle M(k)|\bar{q}(t\bar{n})\frac{\bar{\not{n}}}{2}(\gamma_5)[t\bar{n},0]q(0)|0\rangle = -i\, f_M\, E \int_0^1 \mathrm{d}x\, e^{ixt\bar{n}\cdot k}\phi_M(x,\mu)$$

The LCDA $\phi_M(x,\mu)$ can be interpreted as the **amplitude for finding a quark with longitudinal momentum fraction x** inside the meson

$$\phi_M(x,\mu) = 6x(1-x)\left[1+\sum_{n=1}^{\infty} a_n^M(\mu)\, C_n^{(3/2)}(2x-1)\right]$$

- Expansion in Gegenbauer polynomials **diagonalises the scale evolution** at the leading order
- Gegenbauer coefficients are extracted from **data, QCD sum rules or lattice QCD** at $\mu_0 = 1$ **GeV**

Renormalisation Group Evolution

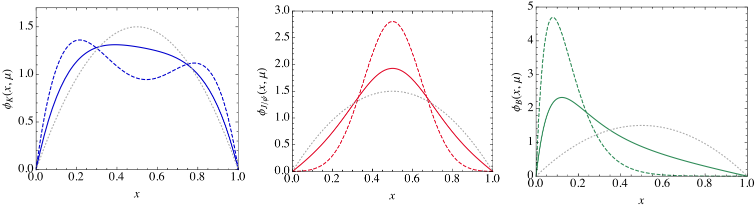
$$\phi_M(x, \mu \to \infty) = 6x(1 - x)$$

The LCDAs approach **asymptotic form** when evolving from the low hadronic scale μ_0 up to the factorization scale $\mu = m_Z$

Renormalisation Group Evolution

$$\phi_M(x, \mu \to \infty) = 6x(1-x)$$

The LCDAs approach **asymptotic form** when evolving from the low hadronic scale μ_0 up to the factorization scale $\mu = m_Z$

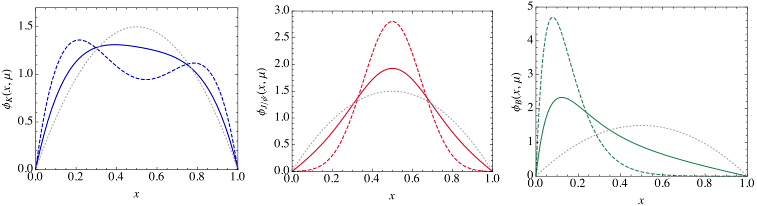


Dashed: $\mu = 1$ GeV; solid: $\mu = m_Z$; dotted: asymptotic

Renormalisation Group Evolution

$$\phi_M(x, \mu \to \infty) = 6x(1-x)$$

The LCDAs approach **asymptotic form** when evolving from the low hadronic scale μ_0 up to the factorization scale $\mu = m_Z$

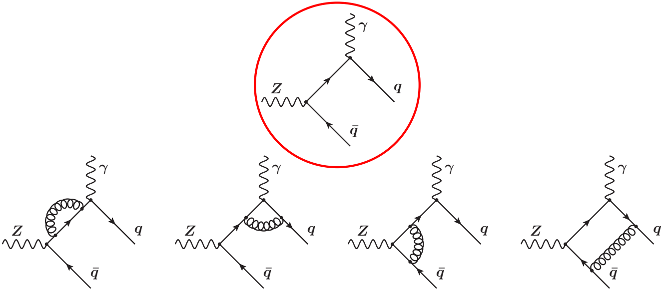


Dashed: $\mu = 1$ GeV; solid: $\mu = m_Z$; dotted: asymptotic

Profit from high factorization scale: less sensitive to the poorly known hadronic parameters

Result for the Decay Amplitude

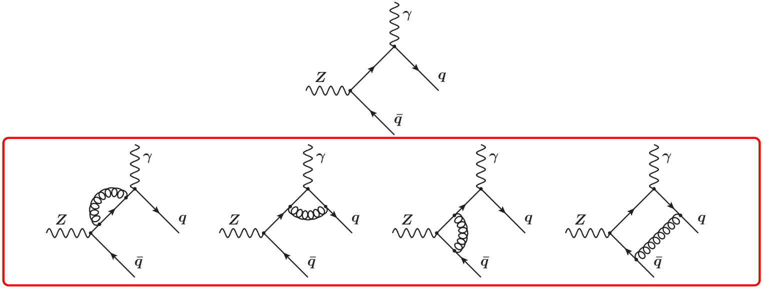
$$i\mathcal{A}(Z \to M\gamma) = \pm \frac{e\,g\,f_M}{2\cos\theta_W}\left[i\epsilon_{\mu\nu\alpha\beta}\frac{k^\mu q^\nu \varepsilon_Z^\alpha \varepsilon_\gamma^{*\beta}}{k\cdot q}F_1^M - \left(\varepsilon_Z\cdot\varepsilon_\gamma^* - \frac{q\cdot\varepsilon_Z\, k\cdot\varepsilon_\gamma^*}{k\cdot q}\right)F_2^M\right]$$



LO and NLO diagrams

Result for the Decay Amplitude

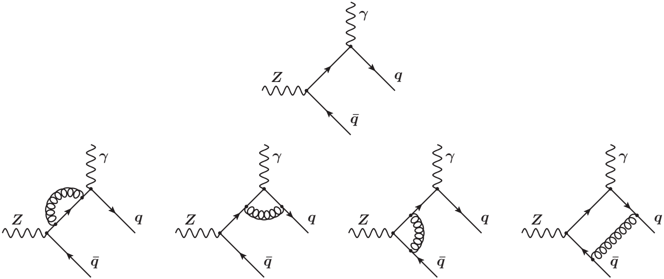
$$i\mathcal{A}(Z\to M\gamma) = \pm\frac{e\,g\,f_M}{2\cos\theta_W}\left[i\epsilon_{\mu\nu\alpha\beta}\frac{k^\mu q^\nu \varepsilon_Z^\alpha \varepsilon_\gamma^{*\beta}}{k\cdot q}F_1^M - \left(\varepsilon_Z\cdot\varepsilon_\gamma^* - \frac{q\cdot\varepsilon_Z\, k\cdot\varepsilon_\gamma^*}{k\cdot q}\right)F_2^M\right]$$



LO and NLO diagrams

Result for the Decay Amplitude

$$i\mathcal{A}(Z\to M\gamma) = \pm\frac{e\,g\,f_M}{2\cos\theta_W}\left[i\epsilon_{\mu\nu\alpha\beta}\frac{k^\mu q^\nu \varepsilon_Z^\alpha \varepsilon_\gamma^{*\beta}}{k\cdot q}\, F_1^M - \left(\varepsilon_Z\cdot\varepsilon_\gamma^* - \frac{q\cdot\varepsilon_Z\, k\cdot\varepsilon_\gamma^*}{k\cdot q}\right) F_2^M\right]$$



LO and NLO diagrams

Result for the Decay Amplitude

Form factors in terms of Gegenbauer moments

$$F_1^M = \mathcal{Q}_M \boxed{\sum_{n=0}^{\infty} C_{2n}^{(+)}(m_Z,\mu)\, a_{2n}^M(\mu)} \qquad \text{even moments}$$

$$F_2^M = -\mathcal{Q}_M \boxed{\sum_{n=0}^{\infty} C_{2n+1}^{(-)}(m_Z,\mu)\, a_{2n+1}^M(\mu)} \qquad \text{odd moments}$$

Result for the Decay Amplitude

Form factors in terms of Gegenbauer moments

$$F_1^M = \mathcal{Q}_M \boxed{\sum_{n=0}^{\infty} C_{2n}^{(+)}(m_Z,\mu)\, a_{2n}^M(\mu)} \qquad \text{even moments}$$

$$F_2^M = -\mathcal{Q}_M \boxed{\sum_{n=0}^{\infty} C_{2n+1}^{(-)}(m_Z,\mu)\, a_{2n+1}^M(\mu)} \quad \text{odd moments}$$

Hard functions in moment space

Tree level

$$C_n^{\pm}(m_Z,\mu) = \boxed{1} + \frac{C_F\,\alpha_S(\mu)}{4\pi} c_n^{(\pm)}\left(\frac{m_Z}{\mu}\right) + \mathcal{O}(\alpha_S^2)$$

$$c_n^{(\pm)}\left(\frac{m_Z}{\mu}\right) = \left[\frac{2}{(n+1)(n+2)} - 4H_{n+1} + 3\right]\left(\log\frac{m_Z^2}{\mu^2} - i\pi\right) + 4H_{n+1}^2 - \frac{4(H_{n+1}-1)\pm 1}{(n+1)(n+2)} + \frac{2}{(n+1)^2(n+2)^2} - 9$$

Result for the Decay Amplitude

Form factors in terms of Gegenbauer moments

$$F_1^M = \mathcal{Q}_M \boxed{\sum_{n=0}^{\infty} C_{2n}^{(+)}(m_Z,\mu)\, a_{2n}^M(\mu)} \qquad \text{even moments}$$

$$F_2^M = -\mathcal{Q}_M \boxed{\sum_{n=0}^{\infty} C_{2n+1}^{(-)}(m_Z,\mu)\, a_{2n+1}^M(\mu)} \text{odd moments}$$

Hard functions in moment space

One loop QCD

$$C_n^{\pm}(m_Z,\mu) = 1 + \frac{C_F \boxed{\alpha_S(\mu)}}{4\pi} c_n^{(\pm)}\left(\frac{m_Z}{\mu}\right) + \mathcal{O}(\alpha_S^2)$$

$$c_n^{(\pm)}\left(\frac{m_Z}{\mu}\right) = \left[\frac{2}{(n+1)(n+2)} - 4H_{n+1} + 3\right]\left(\log \frac{m_Z^2}{\mu^2} - i\pi\right) + 4H_{n+1}^2 - \frac{4(H_{n+1}-1)\pm 1}{(n+1)(n+2)} + \frac{2}{(n+1)^2(n+2)^2} - 9$$

Result for the Decay Amplitude

Form factors in terms of Gegenbauer moments

$$F_1^M = \mathcal{Q}_M \boxed{\sum_{n=0}^{\infty} C_{2n}^{(+)}(m_Z,\mu)\, a_{2n}^M(\mu)} \qquad \text{even moments}$$

$$F_2^M = -\mathcal{Q}_M \boxed{\sum_{n=0}^{\infty} C_{2n+1}^{(-)}(m_Z,\mu)\, a_{2n+1}^M(\mu)} \text{ odd moments}$$

Hard functions in moment space

$$C_n^{\pm}(m_Z,\mu) = 1 + \frac{C_F\,\alpha_S(\mu)}{4\pi} c_n^{(\pm)}\left(\frac{m_Z}{\mu}\right) + \mathcal{O}(\alpha_S^2)$$

$$c_n^{(\pm)}\left(\frac{m_Z}{\mu}\right) = \left[\frac{2}{(n+1)(n+2)} - 4H_{n+1} + 3\right]\left(\log \boxed{\frac{m_Z^2}{\mu^2}} - i\pi\right) + 4H_{n+1}^2 - \frac{4(H_{n+1}-1)\pm 1}{(n+1)(n+2)} + \frac{2}{(n+1)^2(n+2)^2} - 9$$

Resummation of large logarithms by choosing $\mu = m_Z$

Branching Ratios

Decay mode	Branching ratio	asymptotic
$Z^0 \to \pi^0\gamma$	$(9.80\,^{+0.09}_{-0.14}\,{}_{\mu} \pm 0.03_f \pm 0.61_{a_2} \pm 0.82_{a_4}) \cdot 10^{-12}$	7.71
$Z^0 \to \rho^0\gamma$	$(4.19\,^{+0.04}_{-0.06}\,{}_{\mu} \pm 0.16_f \pm 0.24_{a_2} \pm 0.37_{a_4}) \cdot 10^{-9}$	3.63
$Z^0 \to \omega\gamma$	$(2.82\,^{+0.03}_{-0.04}\,{}_{\mu} \pm 0.15_f \pm 0.28_{a_2} \pm 0.25_{a_4}) \cdot 10^{-8}$	2.48
$Z^0 \to \phi\gamma$	$(1.04\,^{+0.01}_{-0.02}\,{}_{\mu} \pm 0.05_f \pm 0.07_{a_2} \pm 0.09_{a_4}) \cdot 10^{-8}$	0.86
$Z^0 \to J/\psi\,\gamma$	$(8.02\,^{+0.14}_{-0.15}\,{}_{\mu} \pm 0.20_f\,^{+0.39}_{-0.36}\,{}_{\sigma}) \cdot 10^{-8}$	10.48
$Z^0 \to \Upsilon(1S)\,\gamma$	$(5.39\,^{+0.10}_{-0.10}\,{}_{\mu} \pm 0.08_f\,^{+0.11}_{-0.08}\,{}_{\sigma}) \cdot 10^{-8}$	7.55
$Z^0 \to \Upsilon(4S)\,\gamma$	$(1.22\,^{+0.02}_{-0.02}\,{}_{\mu} \pm 0.13_f\,^{+0.02}_{-0.02}\,{}_{\sigma}) \cdot 10^{-8}$	1.71
$Z^0 \to \Upsilon(nS)\,\gamma$	$(9.96\,^{+0.18}_{-0.19}\,{}_{\mu} \pm 0.09_f\,^{+0.20}_{-0.15}\,{}_{\sigma}) \cdot 10^{-8}$	13.96

- Ratios from 10^{-11} for $Z^0 \to \pi^0\gamma$ to 10^{-7} for $Z^0 \to J/\psi\gamma$

Branching Ratios

Decay mode	Branching ratio	asymptotic
$Z^0 \to \pi^0\gamma$	$(9.80\,^{+0.09}_{-0.14}\,{}_{\mu} \pm 0.03_f \pm 0.61_{a_2} \pm 0.82_{a_4}) \cdot 10^{-12}$	7.71
$Z^0 \to \rho^0\gamma$	$(4.19\,^{+0.04}_{-0.06}\,{}_{\mu} \pm 0.16_f \pm 0.24_{a_2} \pm 0.37_{a_4}) \cdot 10^{-9}$	3.63
$Z^0 \to \omega\gamma$	$(2.82\,^{+0.03}_{-0.04}\,{}_{\mu} \pm 0.15_f \pm 0.28_{a_2} \pm 0.25_{a_4}) \cdot 10^{-8}$	2.48
$Z^0 \to \phi\gamma$	$(1.04\,^{+0.01}_{-0.02}\,{}_{\mu} \pm 0.05_f \pm 0.07_{a_2} \pm 0.09_{a_4}) \cdot 10^{-8}$	0.86
$Z^0 \to J/\psi\,\gamma$	$(8.02\,^{+0.14}_{-0.15}\,{}_{\mu} \pm 0.20_f\,^{+0.39}_{-0.36}\,{}_{\sigma}) \cdot 10^{-8}$	10.48
$Z^0 \to \Upsilon(1S)\,\gamma$	$(5.39\,^{+0.10}_{-0.10}\,{}_{\mu} \pm 0.08_f\,^{+0.11}_{-0.08}\,{}_{\sigma}) \cdot 10^{-8}$	7.55
$Z^0 \to \Upsilon(4S)\,\gamma$	$(1.22\,^{+0.02}_{-0.02}\,{}_{\mu} \pm 0.13_f\,^{+0.02}_{-0.02}\,{}_{\sigma}) \cdot 10^{-8}$	1.71
$Z^0 \to \Upsilon(nS)\,\gamma$	$(9.96\,^{+0.18}_{-0.19}\,{}_{\mu} \pm 0.09_f\,^{+0.20}_{-0.15}\,{}_{\sigma}) \cdot 10^{-8}$	13.96

- Ratios from 10^{-11} for $Z^0 \to \pi^0\gamma$ to 10^{-7} for $Z^0 \to J/\psi\gamma$
- Dominant uncertainties from LCDA shape parameters and decay constants

Branching Ratios

Decay mode	Branching ratio	asymptotic
$Z^0 \to \pi^0\gamma$	$(9.80\,{}^{+0.09}_{-0.14}\,{}_{\mu} \pm 0.03_f \pm 0.61_{a_2} \pm 0.82_{a_4}) \cdot 10^{-12}$	7.71
$Z^0 \to \rho^0\gamma$	$(4.19\,{}^{+0.04}_{-0.06}\,{}_{\mu} \pm 0.16_f \pm 0.24_{a_2} \pm 0.37_{a_4}) \cdot 10^{-9}$	3.63
$Z^0 \to \omega\gamma$	$(2.82\,{}^{+0.03}_{-0.04}\,{}_{\mu} \pm 0.15_f \pm 0.28_{a_2} \pm 0.25_{a_4}) \cdot 10^{-8}$	2.48
$Z^0 \to \phi\gamma$	$(1.04\,{}^{+0.01}_{-0.02}\,{}_{\mu} \pm 0.05_f \pm 0.07_{a_2} \pm 0.09_{a_4}) \cdot 10^{-8}$	0.86
$Z^0 \to J/\psi\,\gamma$	$(8.02\,{}^{+0.14}_{-0.15}\,{}_{\mu} \pm 0.20_f\,{}^{+0.39}_{-0.36}\,{}_{\sigma}) \cdot 10^{-8}$	10.48
$Z^0 \to \Upsilon(1S)\,\gamma$	$(5.39\,{}^{+0.10}_{-0.10}\,{}_{\mu} \pm 0.08_f\,{}^{+0.11}_{-0.08}\,{}_{\sigma}) \cdot 10^{-8}$	7.55
$Z^0 \to \Upsilon(4S)\,\gamma$	$(1.22\,{}^{+0.02}_{-0.02}\,{}_{\mu} \pm 0.13_f\,{}^{+0.02}_{-0.02}\,{}_{\sigma}) \cdot 10^{-8}$	1.71
$Z^0 \to \Upsilon(nS)\,\gamma$	$(9.96\,{}^{+0.18}_{-0.19}\,{}_{\mu} \pm 0.09_f\,{}^{+0.20}_{-0.15}\,{}_{\sigma}) \cdot 10^{-8}$	13.96

- Ratios from 10^{-11} for $Z^0 \to \pi^0\gamma$ to 10^{-7} for $Z^0 \to J/\psi\gamma$
- Dominant uncertainties from LCDA shape parameters and decay constants
- Asymptotic LCDAs $6x(1-x)$ provide good approximation

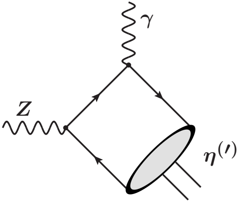
Branching Ratios

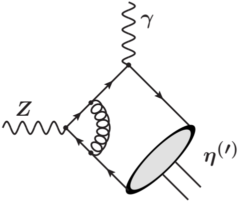
Decay mode	Branching ratio	asymptotic
$Z^0 \to \pi^0\gamma$	$(9.80\,^{+\,0.09}_{-\,0.14}\,{}_{\mu} \pm 0.03_f \pm 0.61_{a_2} \pm 0.82_{a_4}) \cdot 10^{-12}$	7.71
$Z^0 \to \rho^0\gamma$	$(4.19\,^{+\,0.04}_{-\,0.06}\,{}_{\mu} \pm 0.16_f \pm 0.24_{a_2} \pm 0.37_{a_4}) \cdot 10^{-9}$	3.63
$Z^0 \to \omega\gamma$	$(2.82\,^{+\,0.03}_{-\,0.04}\,{}_{\mu} \pm 0.15_f \pm 0.28_{a_2} \pm 0.25_{a_4}) \cdot 10^{-8}$	2.48
$Z^0 \to \phi\gamma$	$(1.04\,^{+\,0.01}_{-\,0.02}\,{}_{\mu} \pm 0.05_f \pm 0.07_{a_2} \pm 0.09_{a_4}) \cdot 10^{-8}$	0.86
$Z^0 \to J/\psi\,\gamma$	$(8.02\,^{+\,0.14}_{-\,0.15}\,{}_{\mu} \pm 0.20_f\,^{+\,0.39}_{-\,0.36}\,{}_{\sigma}) \cdot 10^{-8}$	10.48
$Z^0 \to \Upsilon(1S)\,\gamma$	$(5.39\,^{+\,0.10}_{-\,0.10}\,{}_{\mu} \pm 0.08_f\,^{+\,0.11}_{-\,0.08}\,{}_{\sigma}) \cdot 10^{-8}$	7.55
$Z^0 \to \Upsilon(4S)\,\gamma$	$(1.22\,^{+\,0.02}_{-\,0.02}\,{}_{\mu} \pm 0.13_f\,^{+\,0.02}_{-\,0.02}\,{}_{\sigma}) \cdot 10^{-8}$	1.71
$Z^0 \to \Upsilon(nS)\,\gamma$	$(9.96\,^{+\,0.18}_{-\,0.19}\,{}_{\mu} \pm 0.09_f\,^{+\,0.20}_{-\,0.15}\,{}_{\sigma}) \cdot 10^{-8}$	13.96

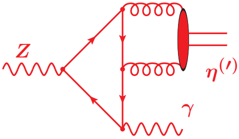
- Ratios from 10^{-11} for $Z^0 \to \pi^0\gamma$ to 10^{-7} for $Z^0 \to J/\psi\gamma$
- Dominant uncertainties from LCDA shape parameters and decay constants
- Asymptotic LCDAs $6x(1-x)$ provide good approximation

What about $Z^0 \to \eta^{(\prime)}\gamma$?

Decays $Z^0 \to \eta^{(\prime)}\gamma$

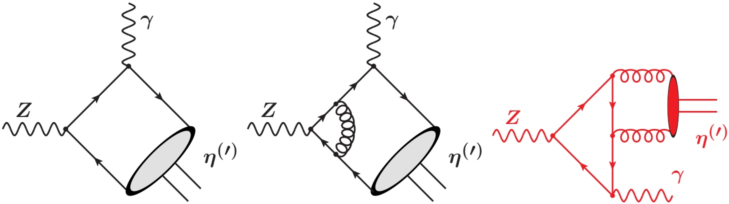






The decays $Z^0 \to \eta^{(\prime)}\gamma$ are special since the **decay amplitude receives an additional contribution** where the meson is formed from a **two-gluon state**

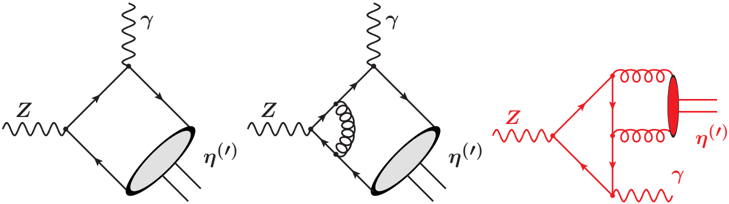
Decays $Z^0 \to \eta^{(\prime)}\gamma$



The decays $Z^0 \to \eta^{(\prime)}\gamma$ are special since the **decay amplitude receives an additional contribution** where the meson is formed from a **two-gluon state**

- Additional term in the factorization formula

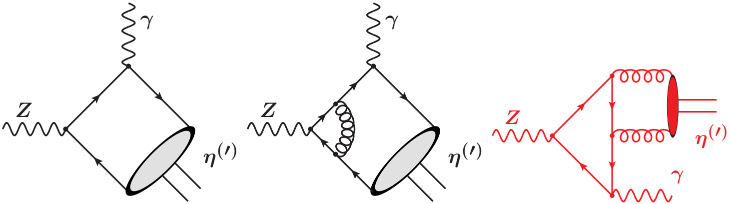
Decays $Z^0 \to \eta^{(\prime)}\gamma$



The decays $Z^0 \to \eta^{(\prime)}\gamma$ are special since the **decay amplitude receives an additional contribution** where the meson is formed from a **two-gluon state**

- Additional term in the factorization formula
- Complicated RG running due to mixing of quark and gluon contribution

Decays $Z^0 \to \eta^{(\prime)}\gamma$



The decays $Z^0 \to \eta^{(\prime)}\gamma$ are special since the **decay amplitude receives an additional contribution** where the meson is formed from a **two-gluon state**

- Additional term in the factorization formula
- Complicated RG running due to mixing of quark and gluon contribution
- Flavour structure of η and η': FKS scheme Feldmann et al. (1998)

Decays $Z^0 \to \eta^{(\prime)}\gamma$

Two different sets for the **mixing parameters**

$f_q = (1.07 \pm 0.02)\, f_\pi\,, \quad f_s = (1.34 \pm 0.06)\, f_\pi\,, \quad \varphi = 39.3^\circ \pm 1.0^\circ$ Feldmann et al. (1998)

$f_q = (1.09 \pm 0.03)\, f_\pi\,, \quad f_s = (1.66 \pm 0.06)\, f_\pi\,, \quad \varphi = 40.7^\circ \pm 1.4^\circ$ Escribano, Frere (2005)

Decays $Z^0 \to \eta^{(\prime)}\gamma$

Two different sets for the **mixing parameters**

$f_q = (1.07 \pm 0.02)\, f_\pi\,, \quad f_s = (1.34 \pm 0.06)\, f_\pi\,, \quad \varphi = 39.3^\circ \pm 1.0^\circ$ Feldmann et al. (1998)

$f_q = (1.09 \pm 0.03)\, f_\pi\,, \quad f_s = (1.66 \pm 0.06)\, f_\pi\,, \quad \varphi = 40.7^\circ \pm 1.4^\circ$ Escribano, Frere (2005)

Different sets of **Gegenbauer moments** for the LCDAs ϕ_q, ϕ_s and ϕ_g are extracted from data for the $\gamma^*\gamma \to \eta^{(\prime)}$ **transition form factor** taken by CLEO (1998) and BaBar (2011)
Agaev et al. (2014); Kroll and Passek (2013)

Decays $Z^0 \to \eta^{(\prime)}\gamma$

Model	(i)	(ii)	(iii)
$10^9 \mathrm{Br}(Z \to \eta\gamma)$	0.16 ± 0.05	0.17 ± 0.05	0.16 ± 0.05
$10^9 \mathrm{Br}(Z \to \eta'\gamma)$	4.70 ± 0.23	4.77 ± 0.24	4.73 ± 0.24

Model	(iv)	(v)	(vi)
$10^9 \mathrm{Br}(Z \to \eta\gamma)$	0.11 ± 0.03	0.10 ± 0.03	$0.010\,^{+0.014}_{-0.010}$
$10^9 \mathrm{Br}(Z \to \eta'\gamma)$	3.43 ± 0.17	3.08 ± 0.15	4.84 ± 0.23

Models (i)-(iii): Agaev et al. (2014); Models (iv)-(vi): Kroll and Passek (2013)

Decays $Z^0 \to \eta^{(\prime)}\gamma$

Model	(i)	(ii)	(iii)
$10^9\mathrm{Br}(Z \to \eta\gamma)$	0.16 ± 0.05	0.17 ± 0.05	0.16 ± 0.05
$10^9\mathrm{Br}(Z \to \eta'\gamma)$	4.70 ± 0.23	4.77 ± 0.24	4.73 ± 0.24

Model	(iv)	(v)	(vi)
$10^9\mathrm{Br}(Z \to \eta\gamma)$	0.11 ± 0.03	0.10 ± 0.03	$0.010\,^{+0.014}_{-0.010}$
$10^9\mathrm{Br}(Z \to \eta'\gamma)$	3.43 ± 0.17	3.08 ± 0.15	4.84 ± 0.23

Models (i)-(iii): Agaev et al. (2014); Models (iv)-(vi): Kroll and Passek (2013)

Measurements of the decays $\boldsymbol{Z^0 \to \eta^{(\prime)}\gamma}$ at a **future *Z*-factory** could provide **valuable information** about the **hadronic input parameters** and the **gluon LCDA** in particular

Summary and Outlook

The **decays $Z^0 \to M\gamma$** provide a perfect playground to **test the QCD factorization approach**. Measurements could yield valuable **information about the LCDAs**.

Summary and Outlook

The **decays $Z^0 \to M\gamma$** provide a perfect playground to **test the QCD factorization approach**. Measurements could yield valuable **information about the LCDAs**.

The **gluon LCDA** could be accessed in the **decays $Z^0 \to \eta^{(\prime)}\gamma$**.

Summary and Outlook

The **decays $Z^0 \to M\gamma$** provide a perfect playground to **test the QCD factorization approach**. Measurements could yield valuable **information about the LCDAs**.

The **gluon LCDA** could be accessed in the **decays $Z^0 \to \eta^{(\prime)}\gamma$**.

"*Exclusive Radiative Higgs Decays as Probes of Light-Quark Yukawa Couplings*", König, Neubert (2015), arXiv: **1503.03870**

"*Exclusive Weak Radiative Higgs Decays and Flavor-Changing Higgs-Top Couplings*", Alte, König and Neubert, in preparation