QCD factorization for exclusive hadronic decays

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THEP, Johannes Gutenberg-University (Mainz)

MITP Summer School
New Physics on Trial
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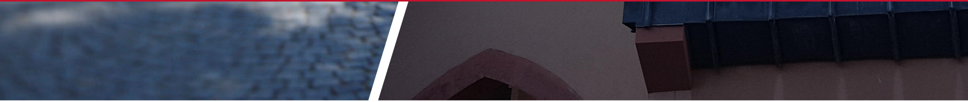

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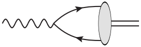

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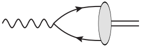

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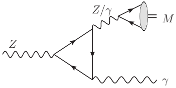

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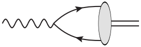 $\propto \langle M|\bar{q}\dots q|0\rangle \sim f_M$ *(decay constant)*

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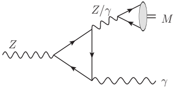

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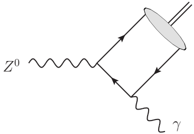

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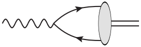 $\propto \langle M|\bar{q} \dots q|0\rangle \sim f_M$ *(decay constant)*

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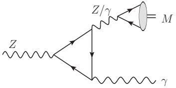

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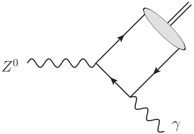
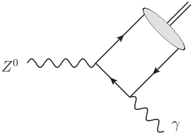


$\Rightarrow$ These diagrams **do exist** and are typically important.

What are the complications from this diagram?



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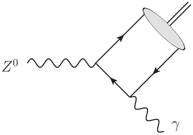

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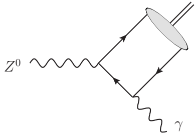

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2. The field operators are **not** evaluated **at the same space-time coordinate**:

$$J \sim \bar{q}(x)\ldots q(y)$$

Based on:

Exclusive Radiative Decays of W and Z Bosons in QCD Factorization
Yuval Grossman, MK, Matthias Neubert

JHEP 1504 (2015) 101, arXiv:1501.06569

Exclusive Radiative Z-Boson Decays to Mesons with Flavor-Singlet Components
Stefan Alte, MK, Matthias Neubert

JHEP 1602 (2016) 162, arXiv:1512.09135

Exclusive Radiative Higgs Decays as Probes of Light-Quark Yukawa Couplings
MK, Matthias Neubert

JHEP 1508 (2015) 012, arXiv:1505.03870

Exclusive Weak Radiative Higgs Decays
Stefan Alte, MK, Matthias Neubert

arXiv:160x.soonish

An effective theory approach

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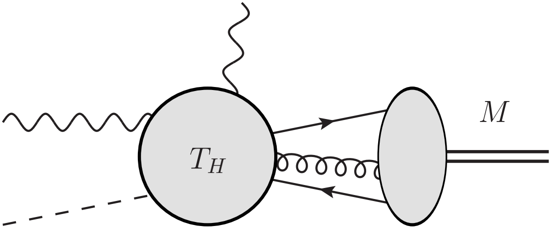

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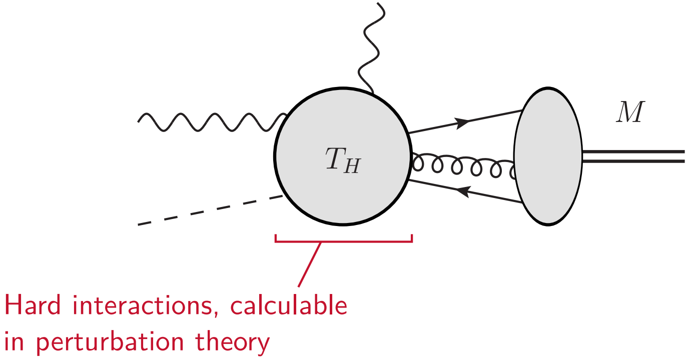

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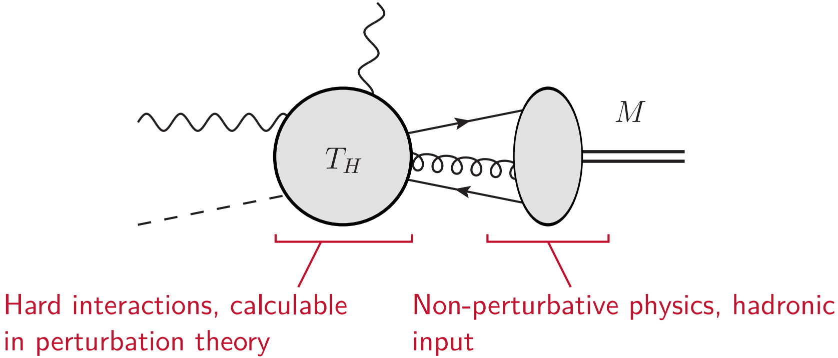

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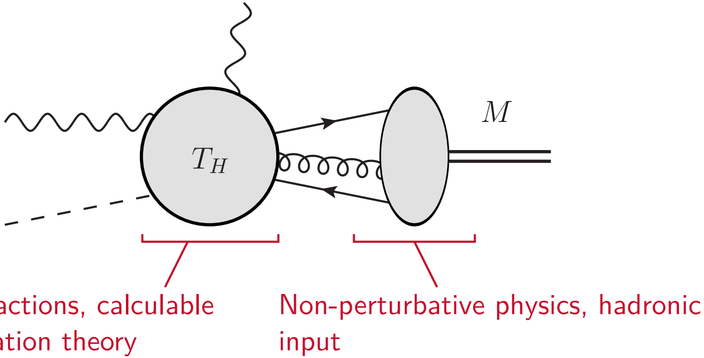
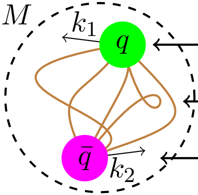


The **scale separation** in the case at hand **calls for an effective theory** description!

An effective theory approach

The simplest way to imagine the meson in its rest frame:



valence quark, momentum $k_1^\mu \sim \mathcal{O}(\Lambda_{\mathrm{QCD}})$

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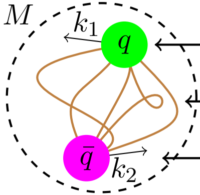

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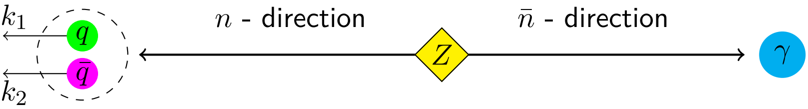

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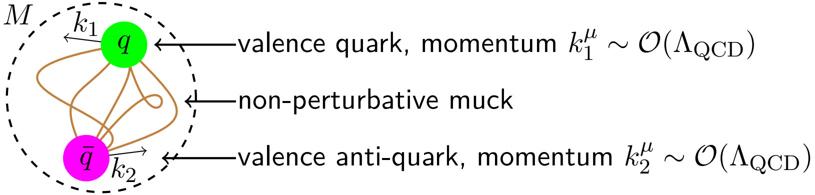

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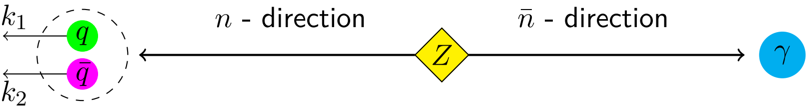

In the Z boson's rest frame, the two quarks move collinear with momenta

$$k_i^\mu = \frac{m_Z}{2}\left(x_i n^\mu + \lambda\, n_{i\perp}^\mu\right) \qquad \lambda = \frac{\Lambda_{\mathrm{QCD}}}{m_Z}$$

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In an **expansion** in λ our list of operators **starts with two collinear quarks** at leading power and contributions with **three or more particles** are power-suppressed - and in our case completely **negligible**.

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$$J_q \quad \sim \quad \bar{q}_c \ldots q_c \; + \; \bar{q}_c \ldots (\bar{n} \cdot \partial) q_c \; + \; \bar{q}_c \ldots (\bar{n} \cdot \partial)^2 q_c + \quad \ldots$$

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The result is that the current **operator** can be **non-local along the light-cone** - a typical feature of SCET.

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With our effective operator $J_q(t) = \bar{q}_c(t\bar{n})\Gamma[t\bar{n}, 0]q_c(0)$ the amplitude for $X \to M + V$ is then given by:

$$i\mathcal{A} = \int \mathcal{C}(t, \dots)\langle M(k)|\, J_q(t, \dots)\, |0\rangle dt$$

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$$\langle M(k)|\, J_q(t, \dots)\, |0\rangle \sim f_M \int e^{i(t\bar{n})\cdot(xk)} \phi_M^q(x) dx$$

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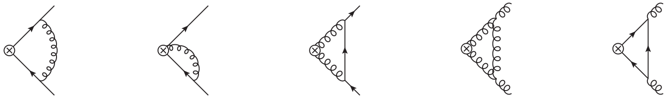
The **Wilson coefficients** $\mathcal{C}$ contain the hard scattering processes that are integrated out at the factorization scale.

Renormalization of the LCDAs

Remember, we are dealing with a **huge scale hierarchy**: m_Z vs. Λ_{QCD}

⇒ Large logarithms $\alpha_s \log(m_Z/\Lambda_{\text{QCD}})$ need to be resummed.

Examples of corrections to the LCDAs at $\mathcal{O}(\alpha_s)$:



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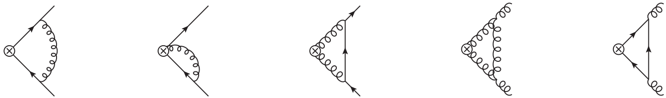

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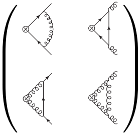

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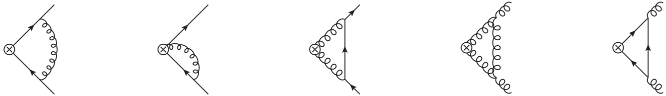

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$$\begin{pmatrix} \phi_q^{\text{ren}}(x,\mu) \\ \phi_g^{\text{ren}}(x,\mu) \end{pmatrix} = \int_0^1 \left[\mathbf{1} \cdot \delta(x-y) \; + \; \frac{\alpha_s(\mu)}{4\pi\epsilon} \begin{pmatrix} V_{qq}(x,y) & V_{qg}(x,y) \\ V_{gq}(x,y) & V_{gg}(x,y) \end{pmatrix} \right] \begin{pmatrix} \phi_q^{\text{bare}}(y) \\ \phi_g^{\text{bare}}(y) \end{pmatrix} dy$$

[Brodsky, Lepage (*1980*), Phys. Rev. D 22, 2157]

[Terentev (*1981*), Sov. J. Nucl. Phys. 33, 911]

[Ohrndorf (*1981*), Nucl. Phys. B 186, 153]

[Shifman, Vysotsky (*1981*), Nucl. Phys. B 186, 475]

[Baier, Grozin (*1981*), Nucl.Phys. B192 476-488]

Renormalization

The LCDAs are expanded in the eigenfunctions of the evolution Kernels:

$$\phi_M^q(x,\mu) = 6x\,\bar{x}\left[1 + \sum_{n=1}^{\infty} a_n^M(\mu) C_n^{(3/2)}(2x-1)\right]$$

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At higher orders, moments of order n mix with moments of order $k < n$.

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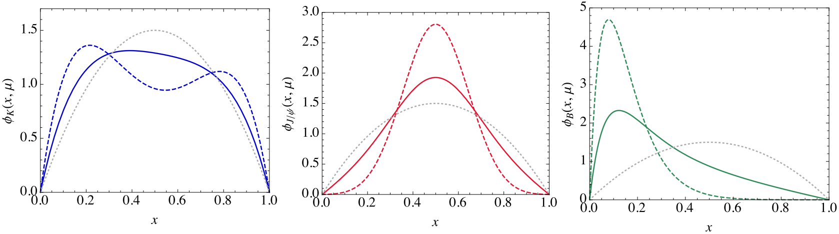

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LCDAs for mesons at different scales, dashed lines: $\phi_M(x, \mu = \mu_0)$, solid lines: $\phi_M(x, \mu = m_Z)$, grey dotted lines: $\phi_M(x, \mu \to \infty)$

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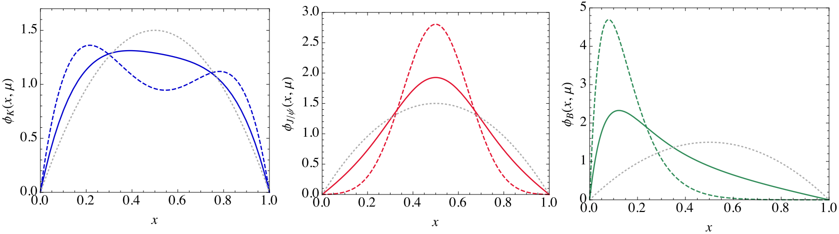

a) K LCDA b) J/ψ LCDA c) B LCDA

LCDAs for mesons at different scales, dashed lines: $\phi_M(x, \mu = \mu_0)$, solid lines: $\phi_M(x, \mu = m_Z)$, grey dotted lines: $\phi_M(x, \mu \rightarrow \infty)$

At high scales compared to Λ_{QCD} (e.g. $\mu \sim m_Z$) the sensitivity to poorly-known a_n^M, b_n^M is greatly reduced!

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