Moduli Vacuum Misalignment and Precise Predictions in String Inflation

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Exploring the Energy Ladders of the Universe
Mainz Institute for Theoretical Physics
June, 2016

arXiv: 1604.08512[hep-th]

Michele Cicoli, Anshuman Maharana, Fernando Quevedo

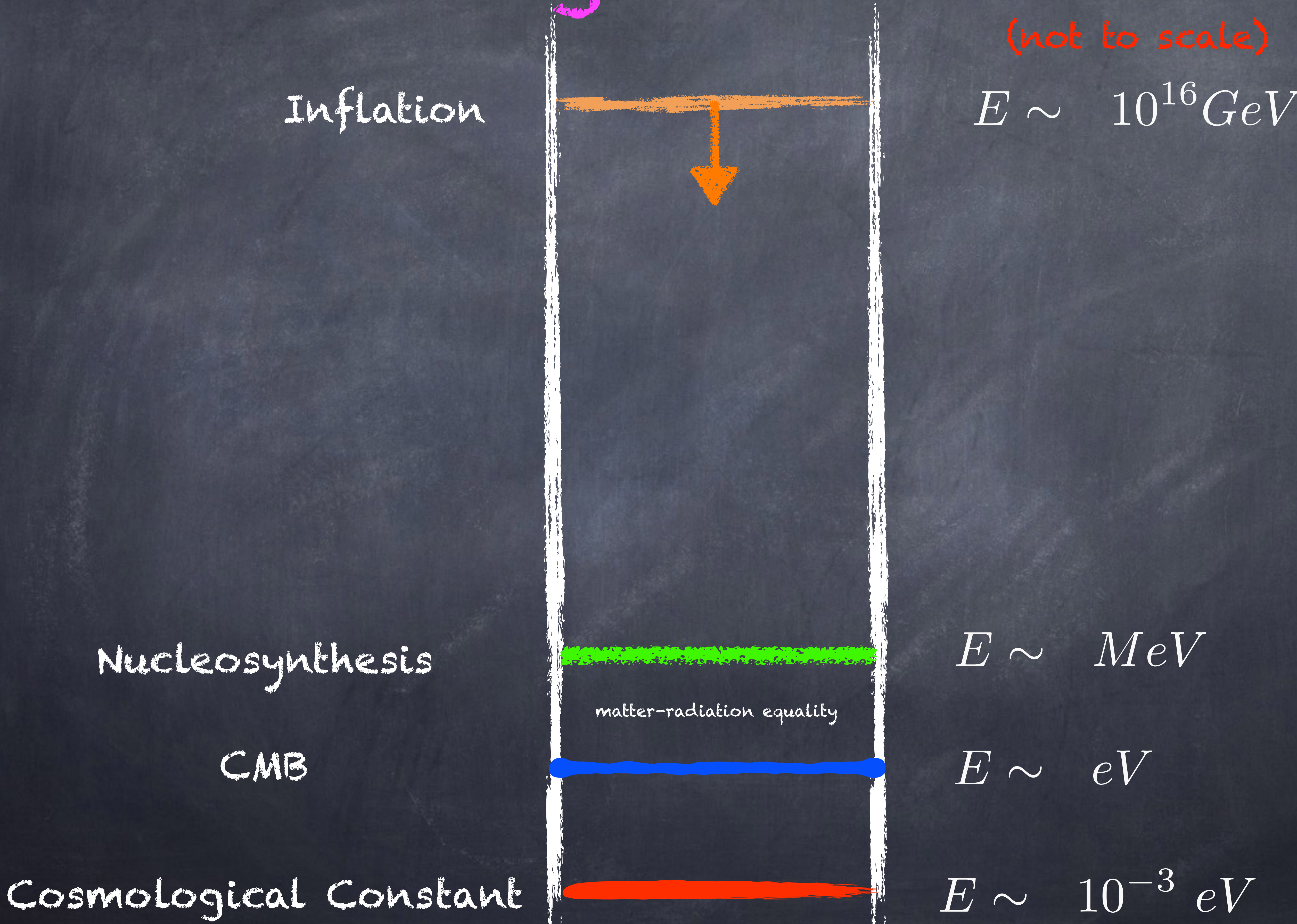
Kumar Das, Anshuman Maharana

arXiv:1506.05745[hep-ph]

Anshuman Maharana

arXiv:1409.7037[hep-ph]

Cosmological Ladders



Cosmological Ladders

(not to scale)

Inflation $E \sim \quad 10^{16} GeV$

We may also add 'particle physics ladder' on top if it:

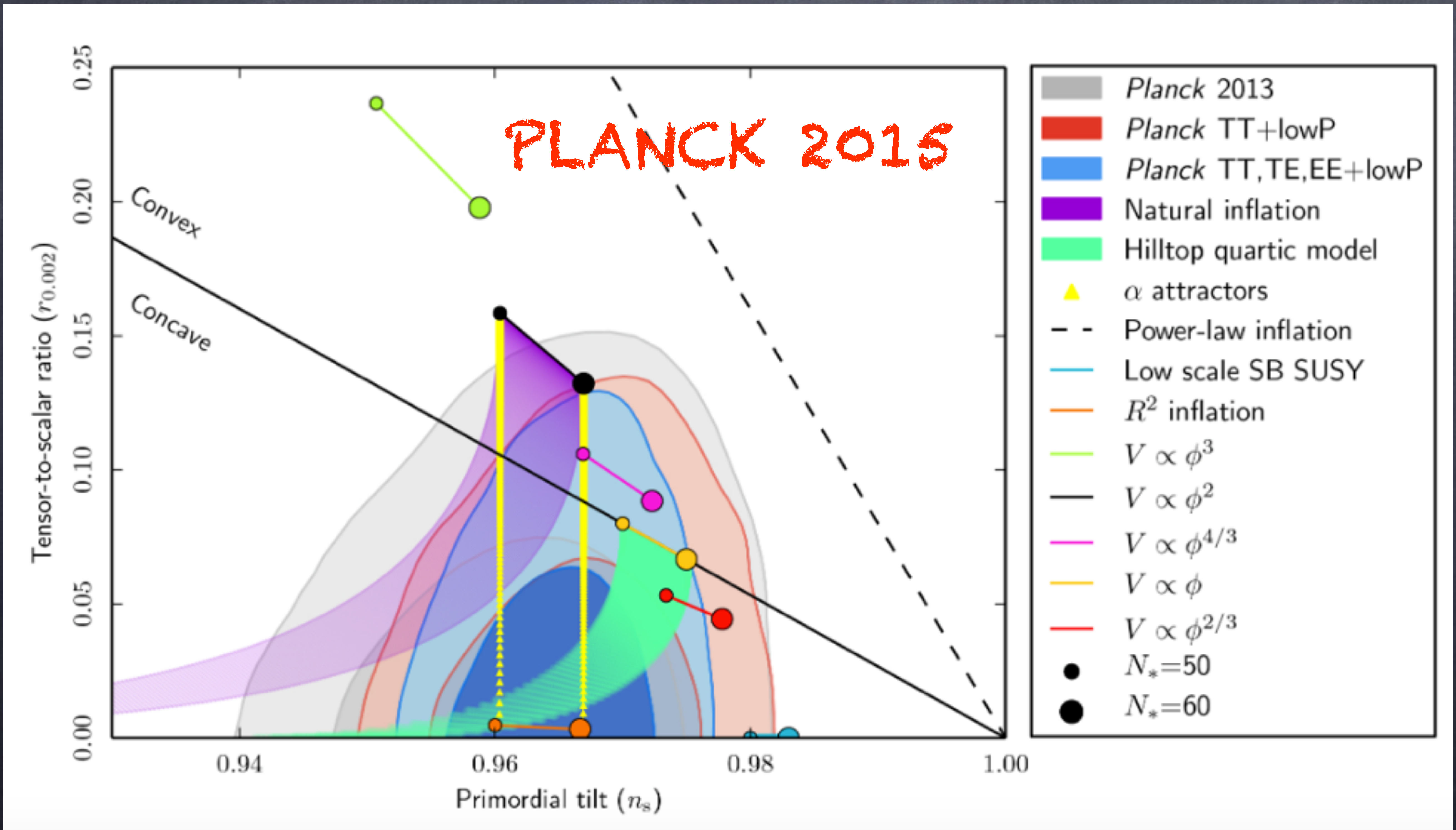
- GUT physics
- Lepto/Baryogenesis
- EW phase transition
- QCD phase transition
- SUSY
- String theory
- Dark Matter freeze out
- ....

Nucleosynthesis $E \sim \quad MeV$

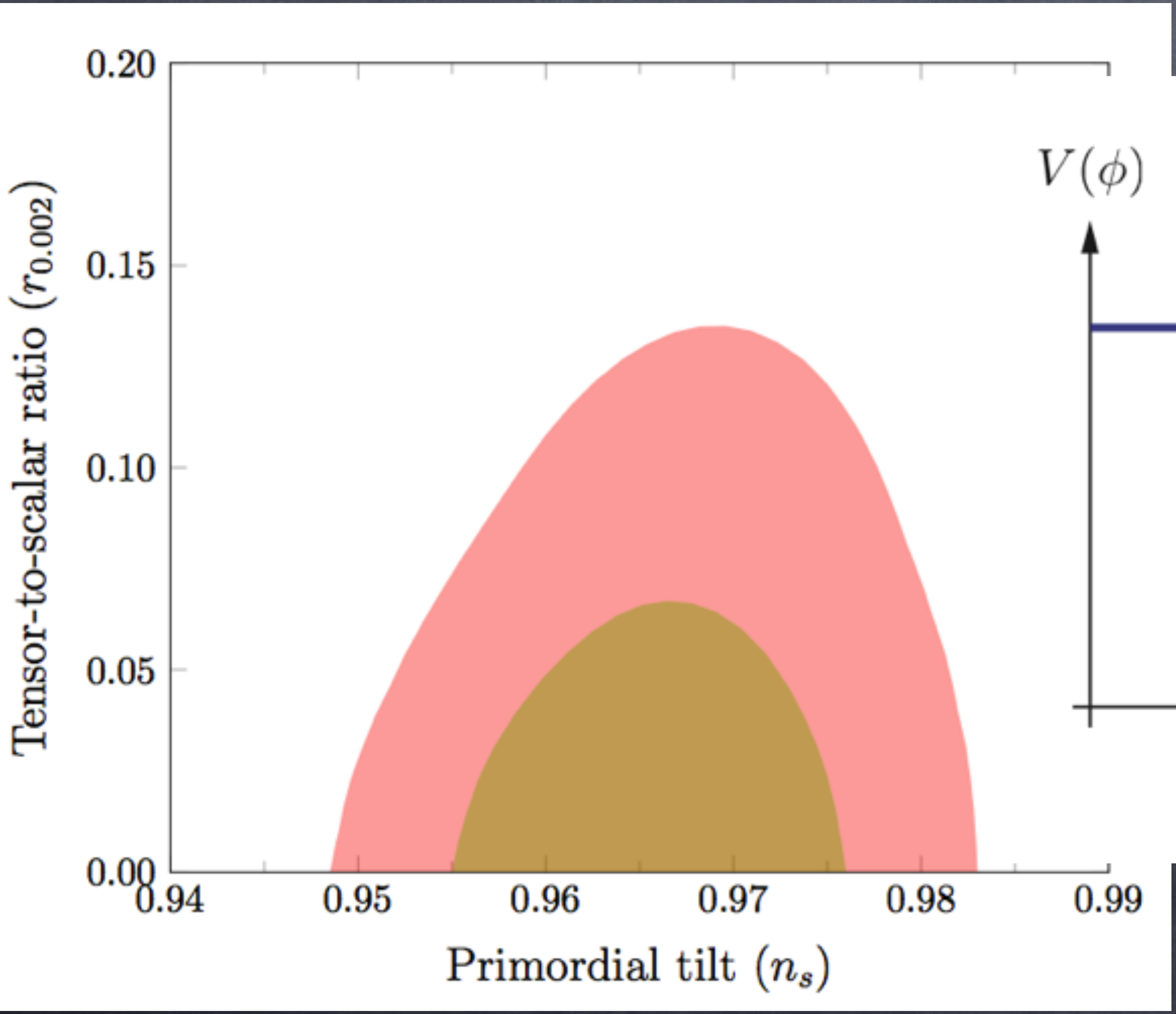
CMB $E \sim \quad eV$

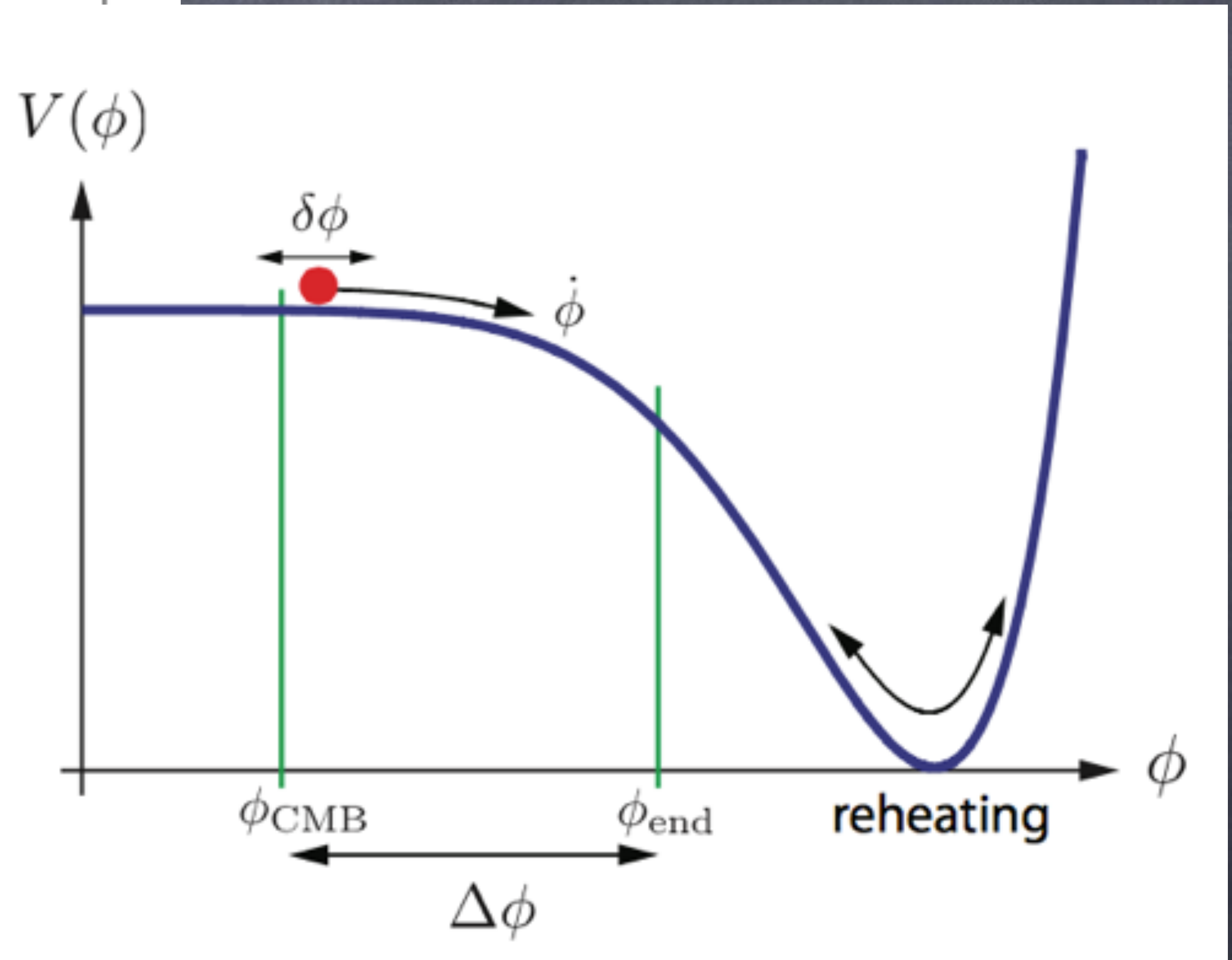
Cosmological Constant $E \sim \quad 10^{-3}\, eV$

Observables

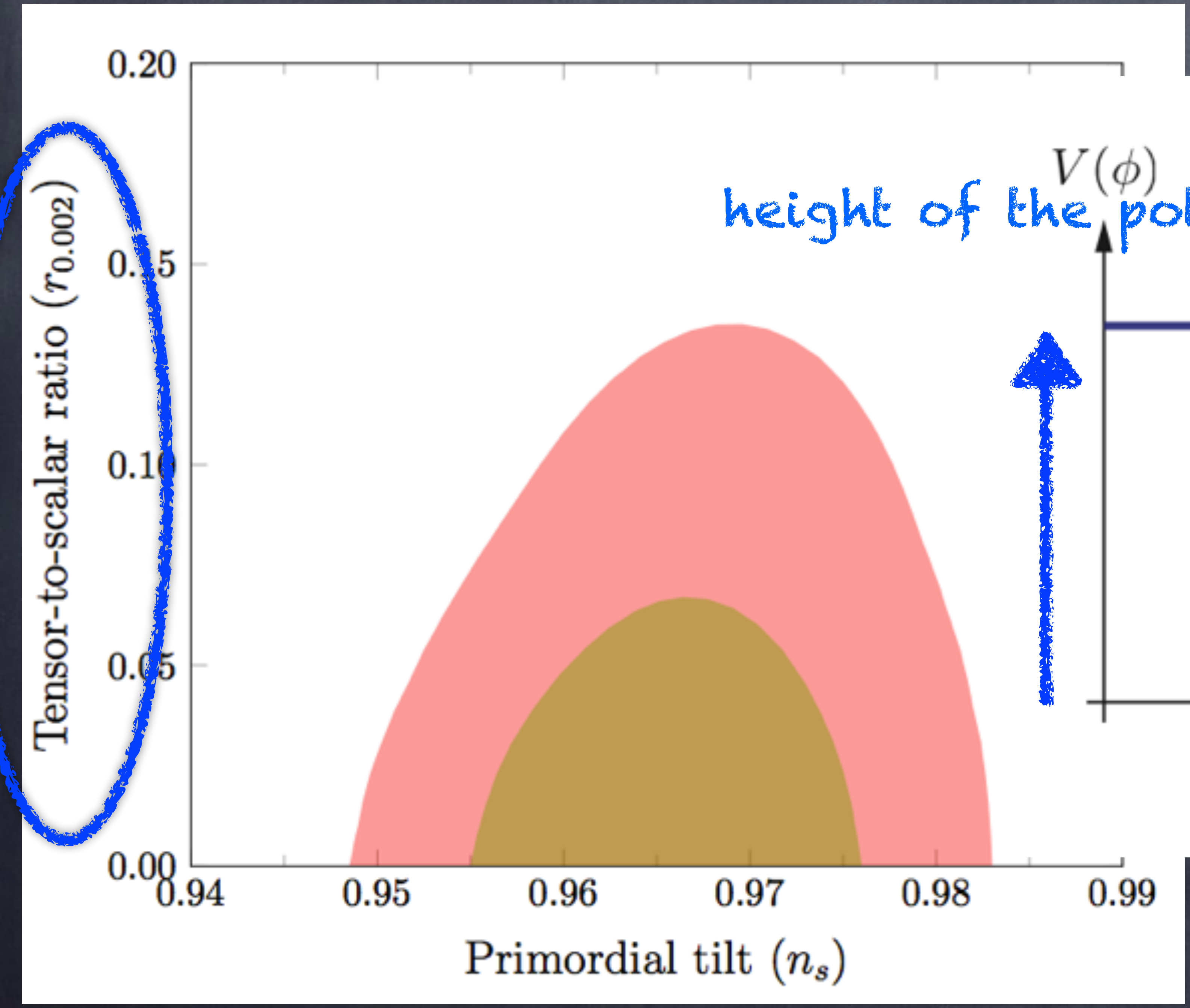


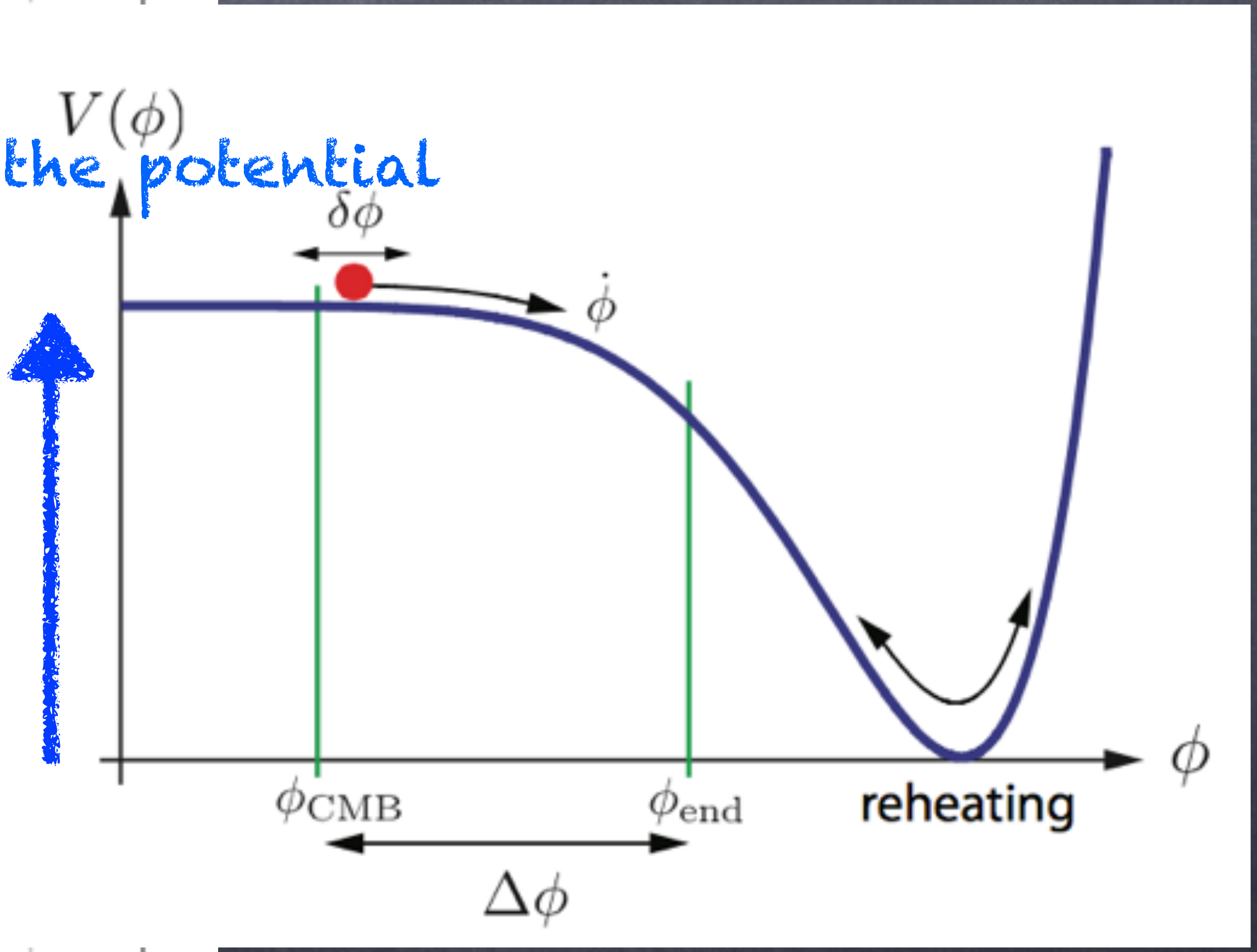
Observables



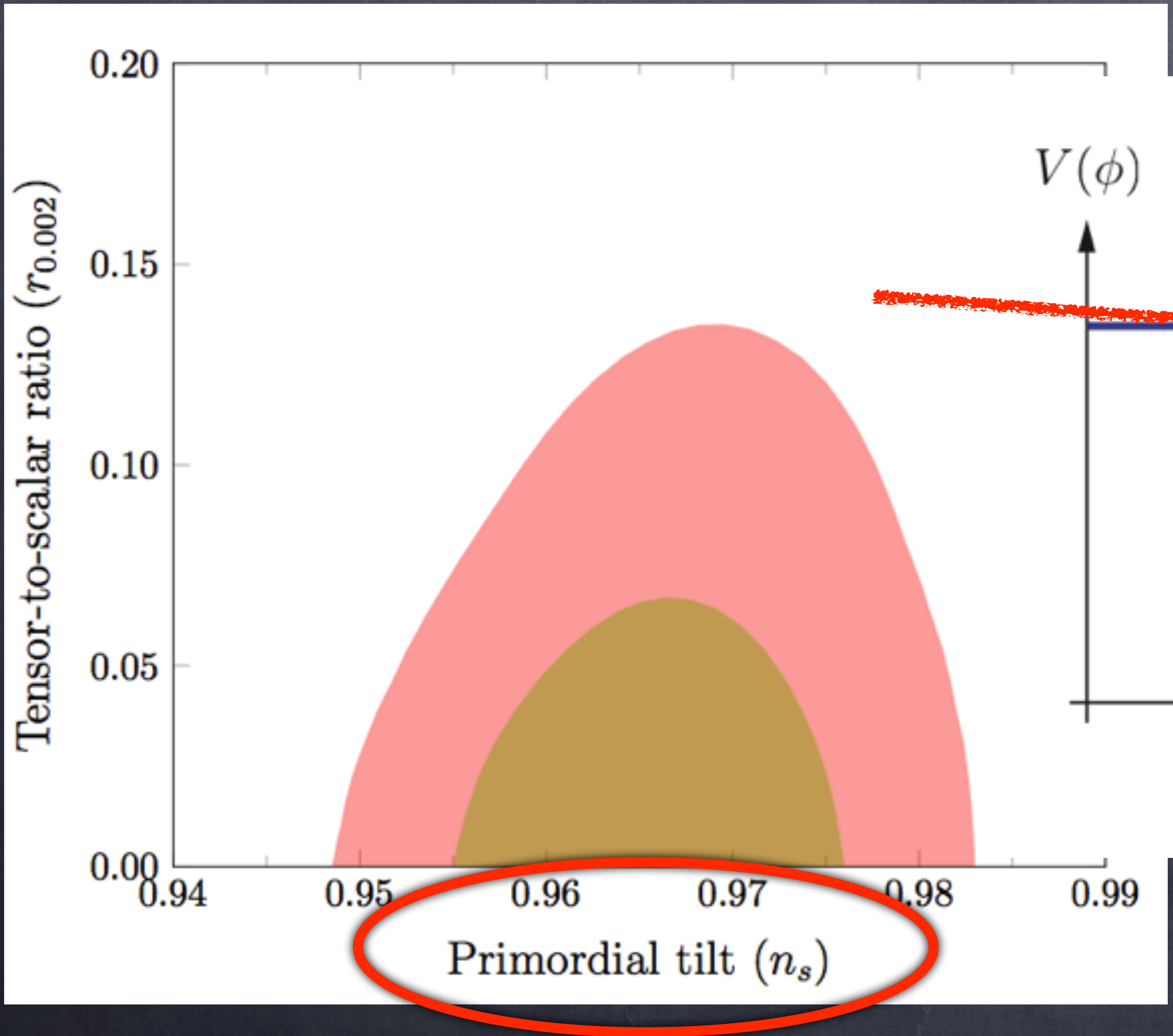


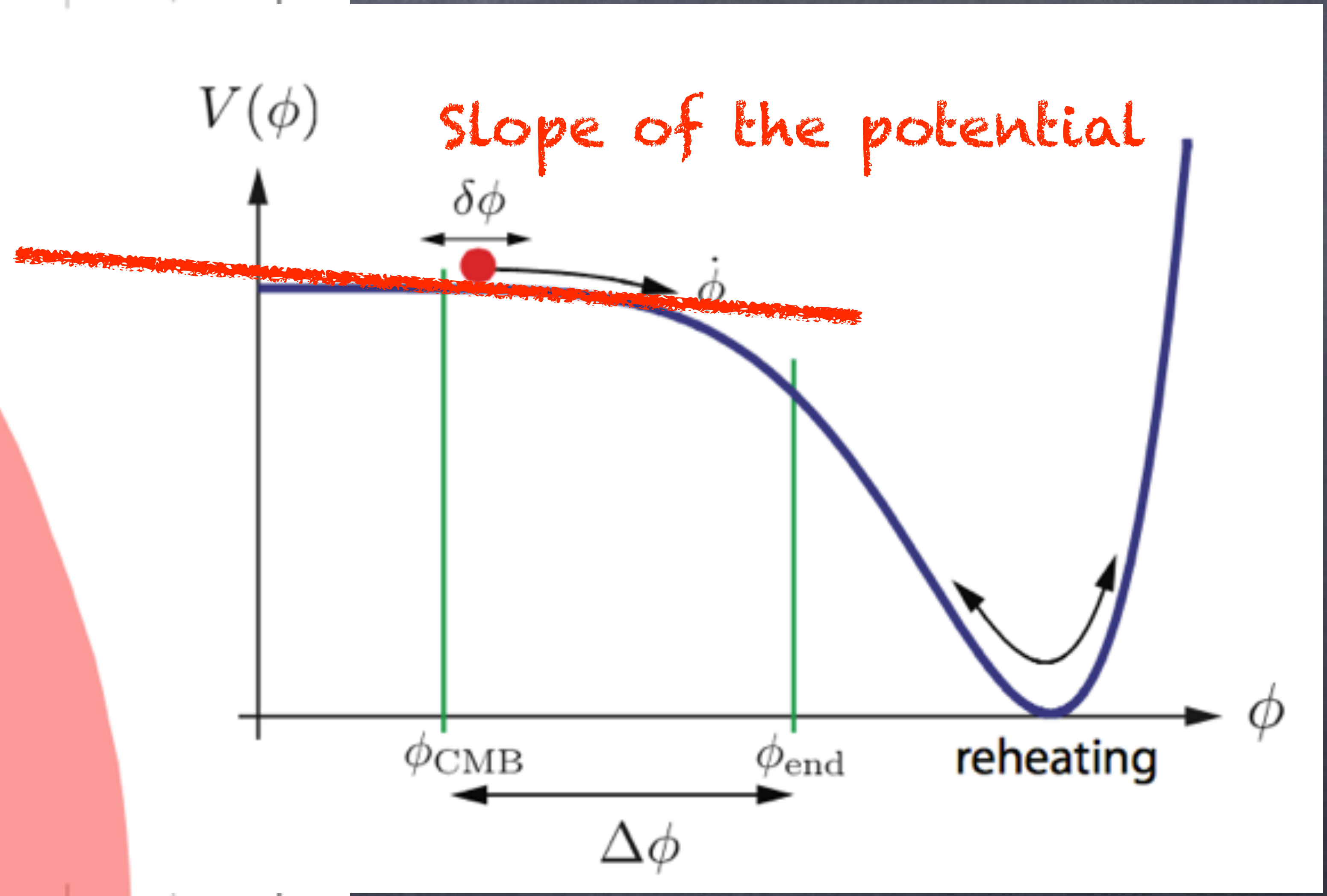
Observables



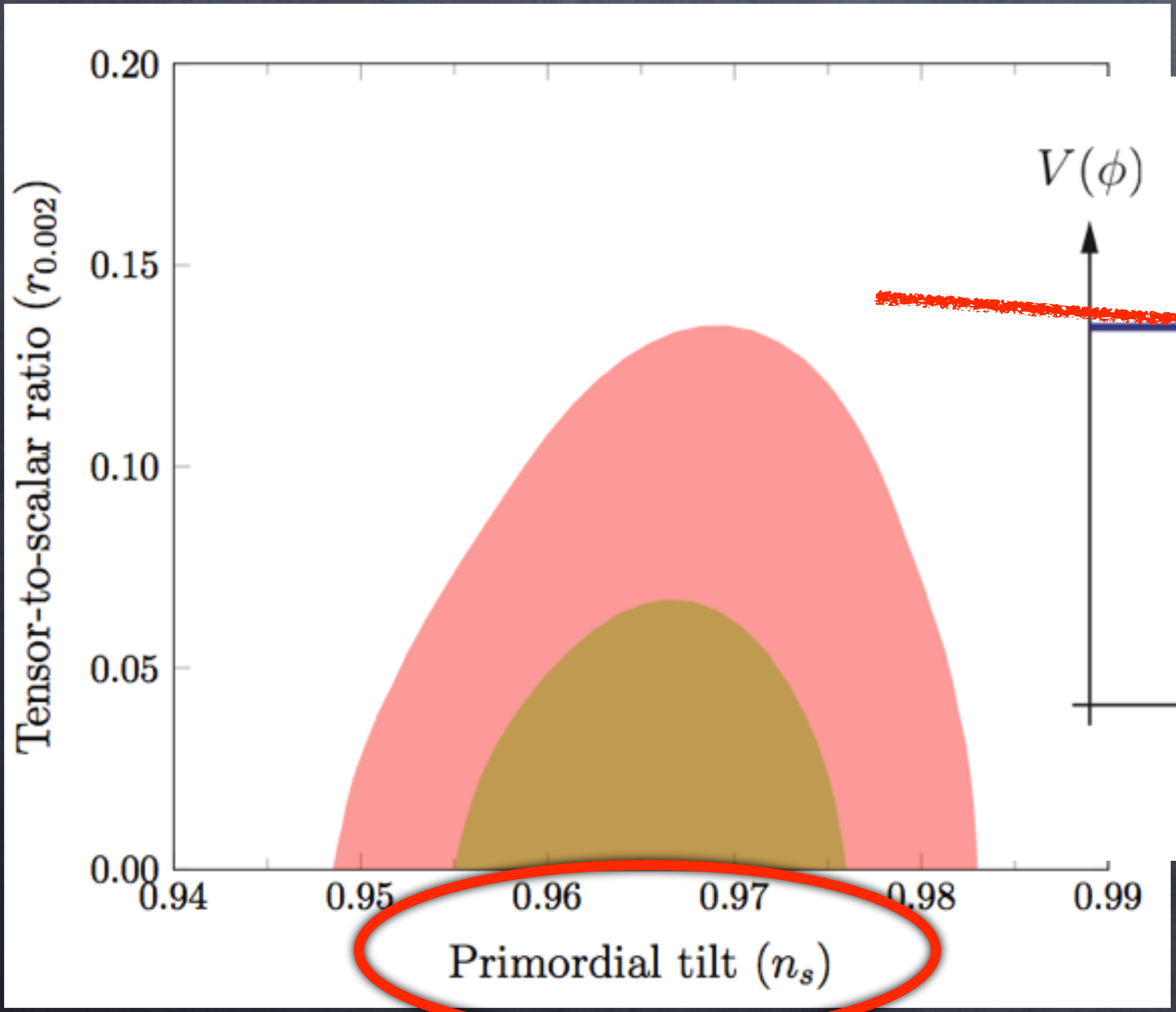


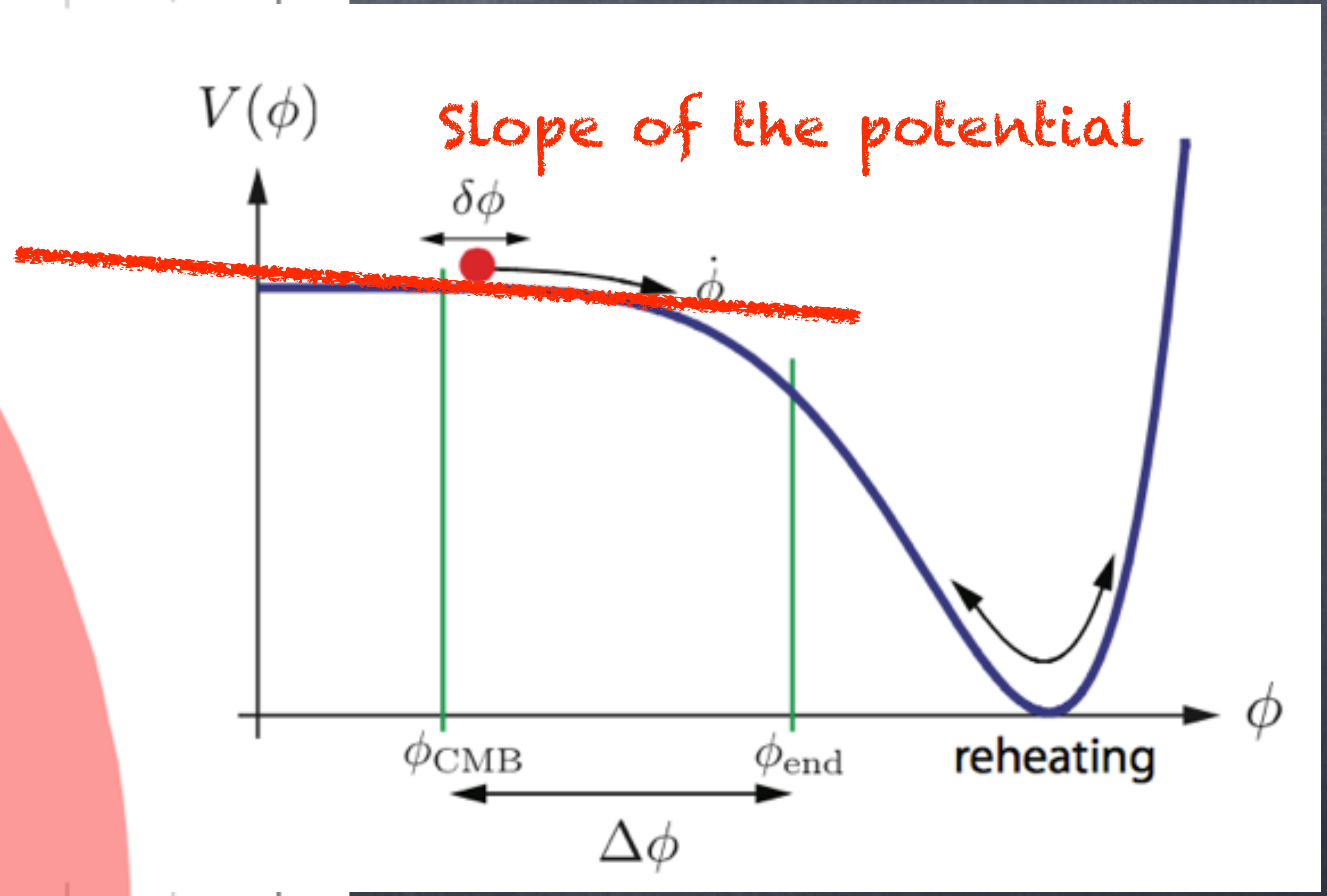
Observables





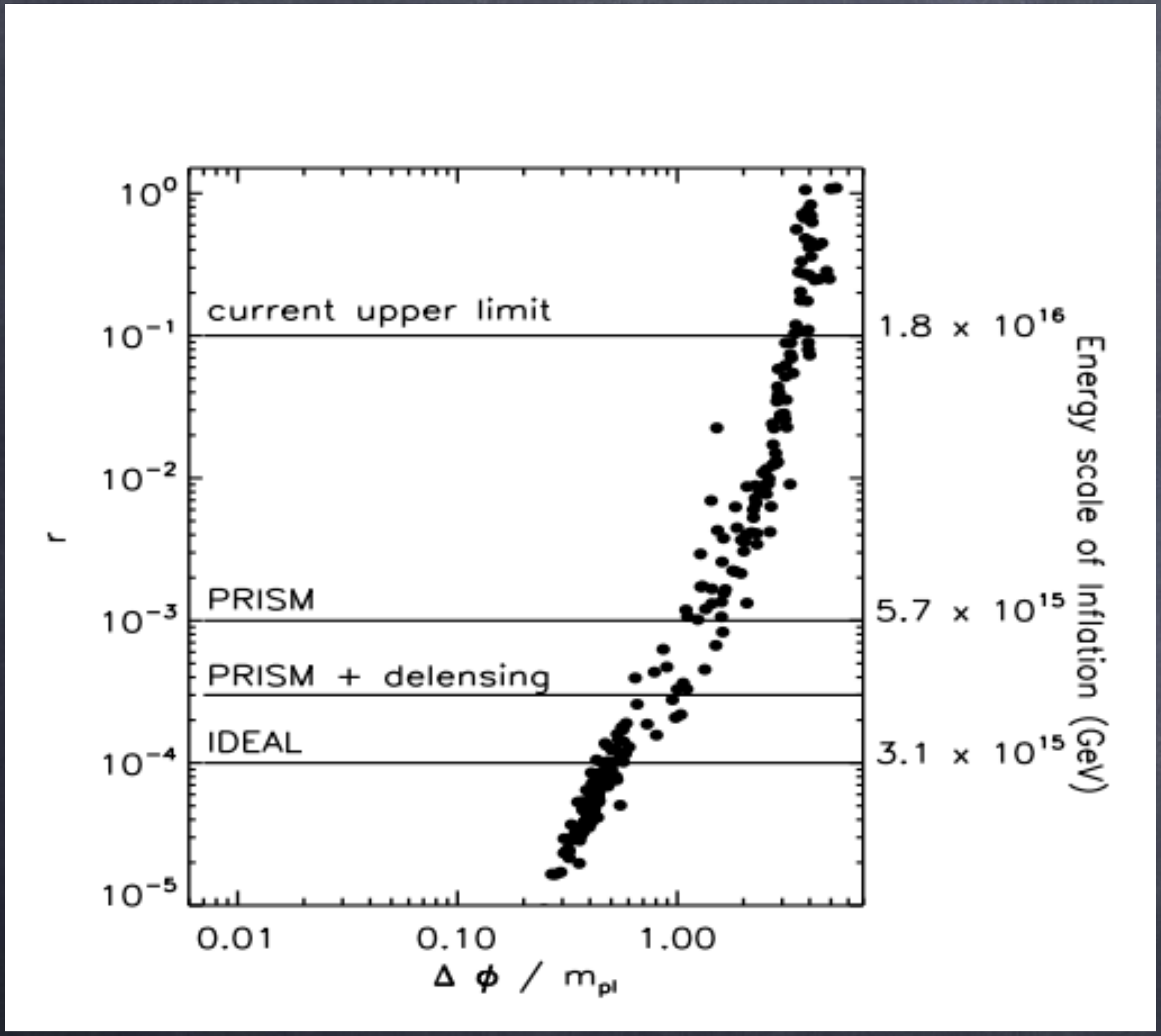
Observables





Few observables and other observables seem to be not required

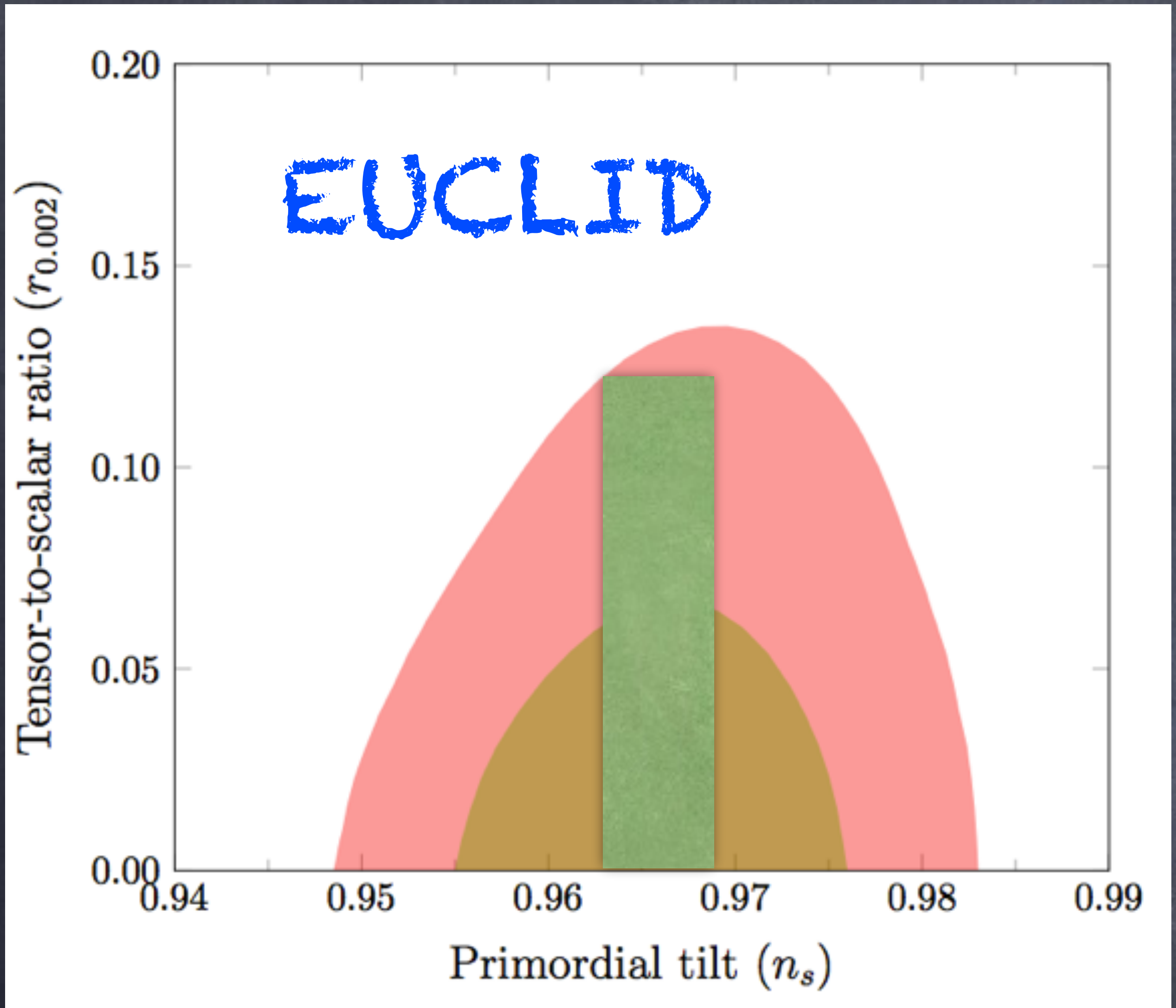
where we stand

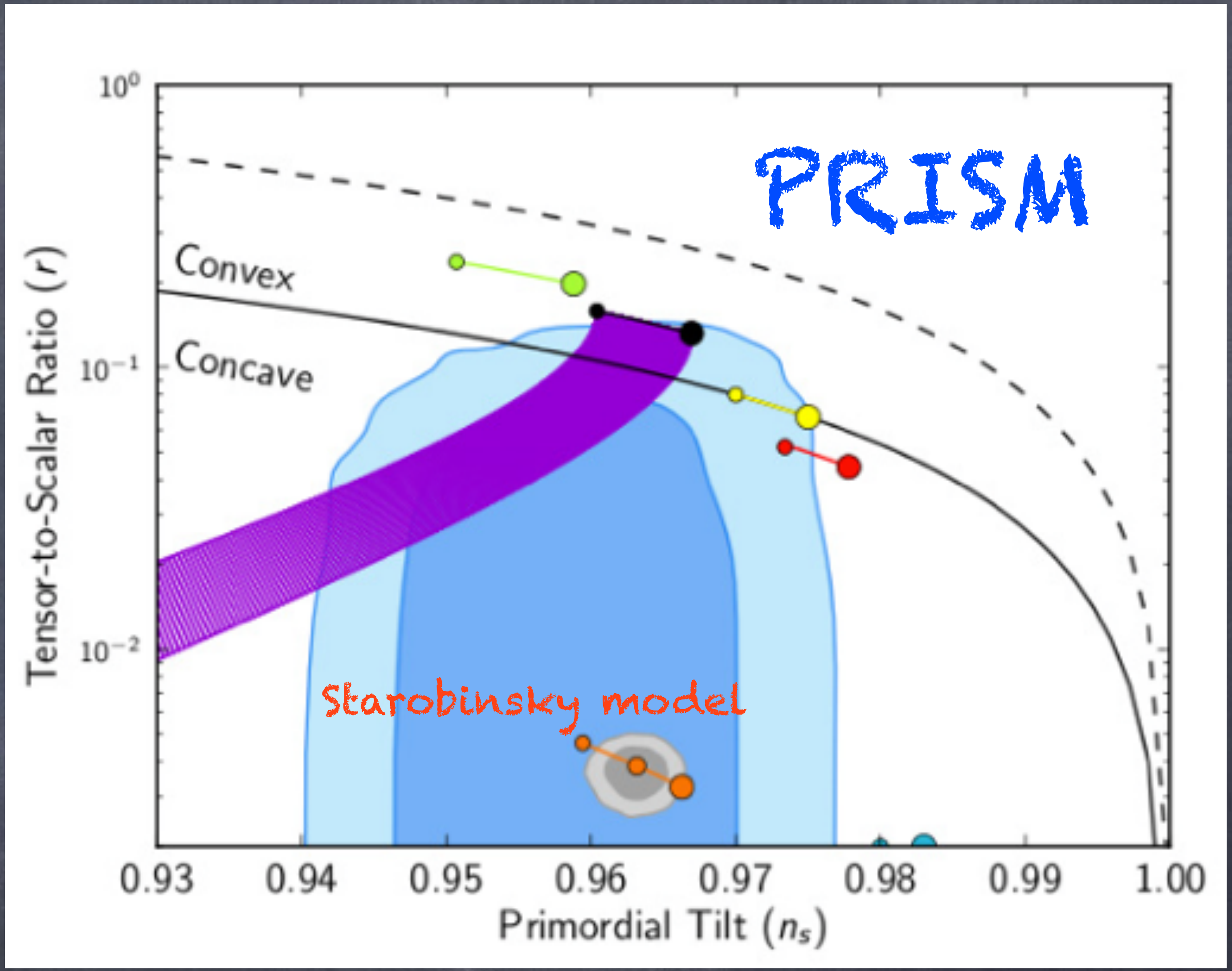


$$V^{1/4} \sim \left(\frac{r}{0.01}\right)^{1/4} \; 10^{16} \; GeV$$

We need some luck here!

where we hope to stand





precise measurements of spectral index is crucial!

Outline

- Cosmology with moduli

- Constraint on moduli mass/inflation

Dutta, Maharana (2014)

- Implications ..

Das, Dutta, Maharana (2015)

- An example: Kahler moduli inflation

Cicoli, Dutta, Maharana, Quevedo (2016)

Framework

'Modular Cosmology'

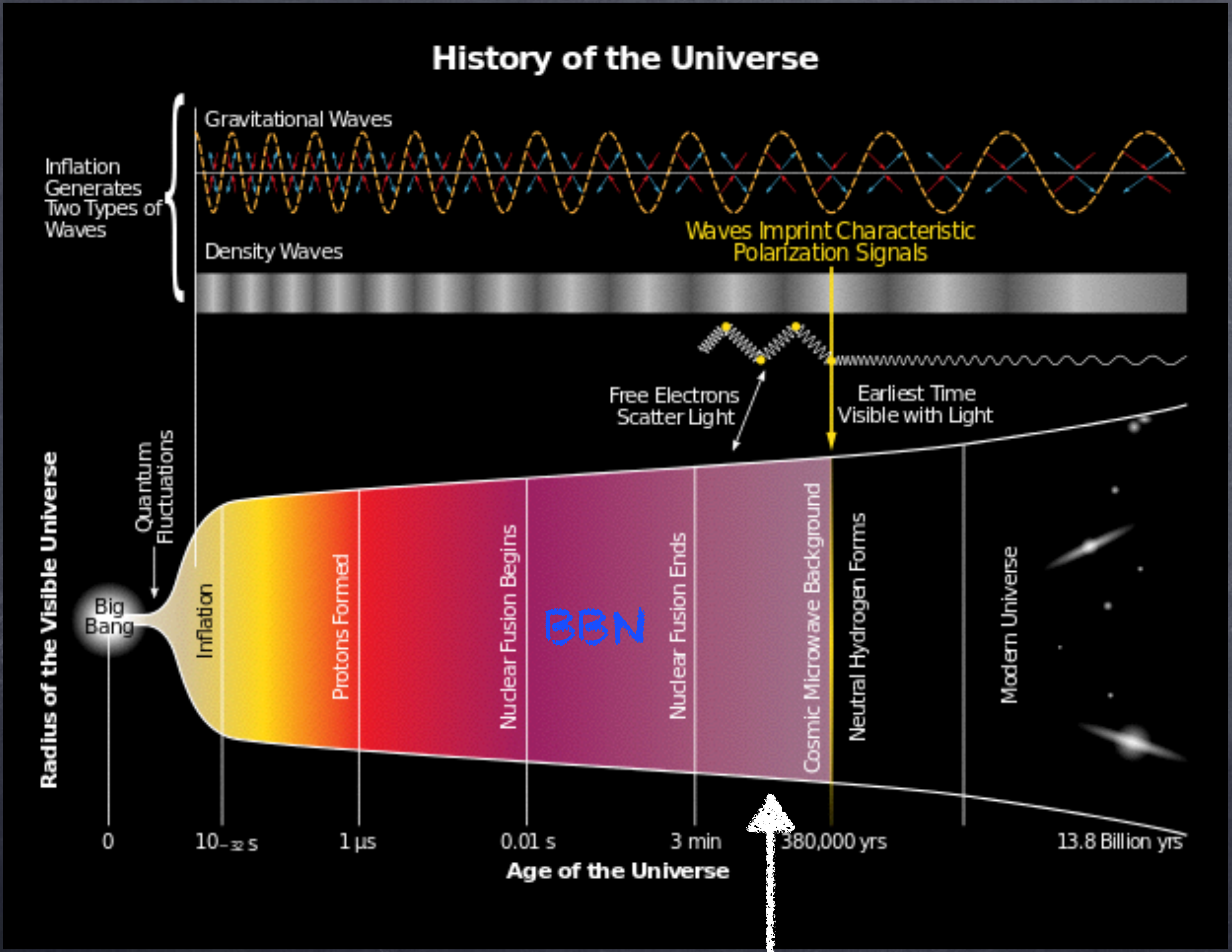
Moduli φ

$m < H_{inf}$

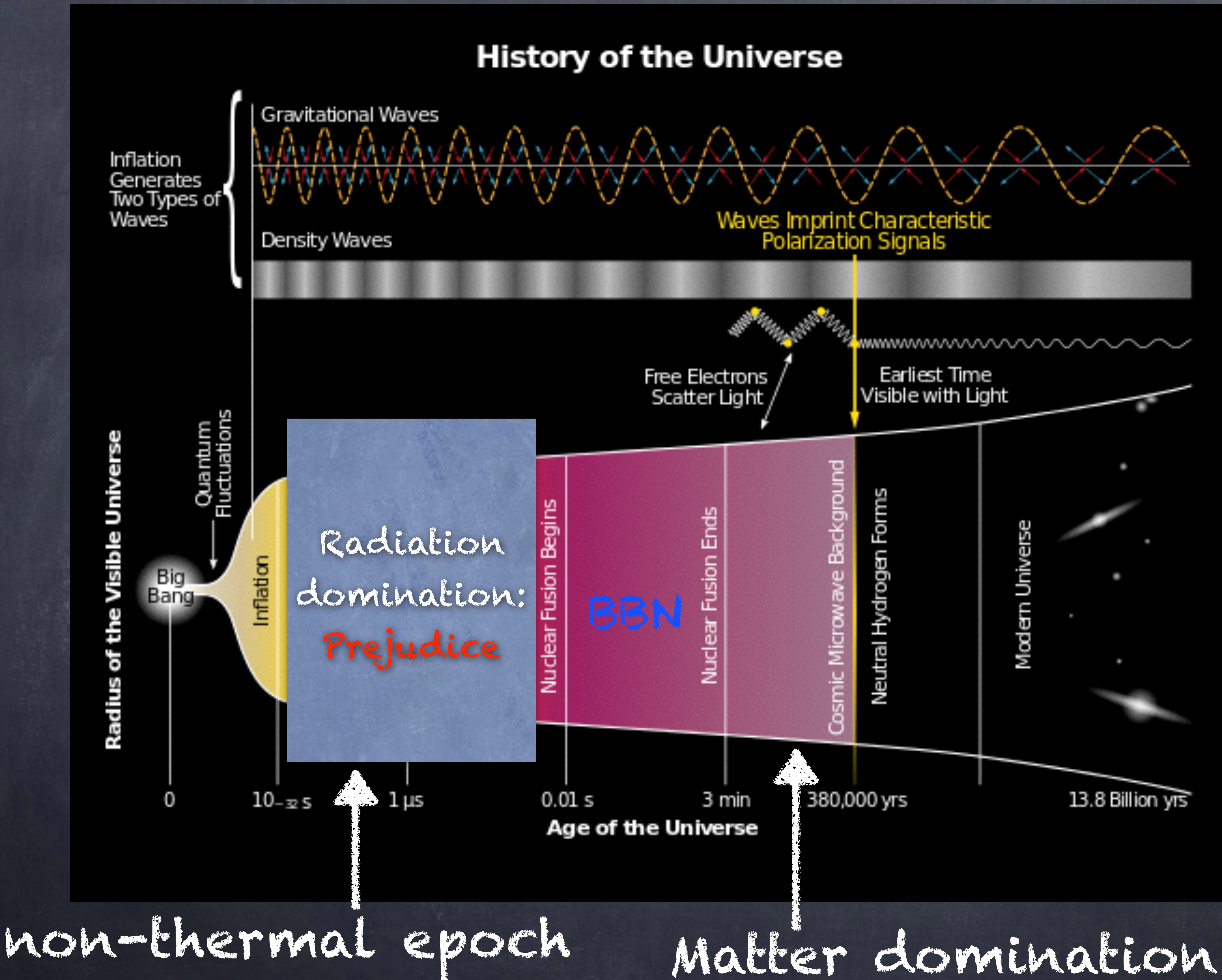
post-inflationary moduli mass

$$Y = \hat{\varphi}/M_{Pl} \sim 1$$

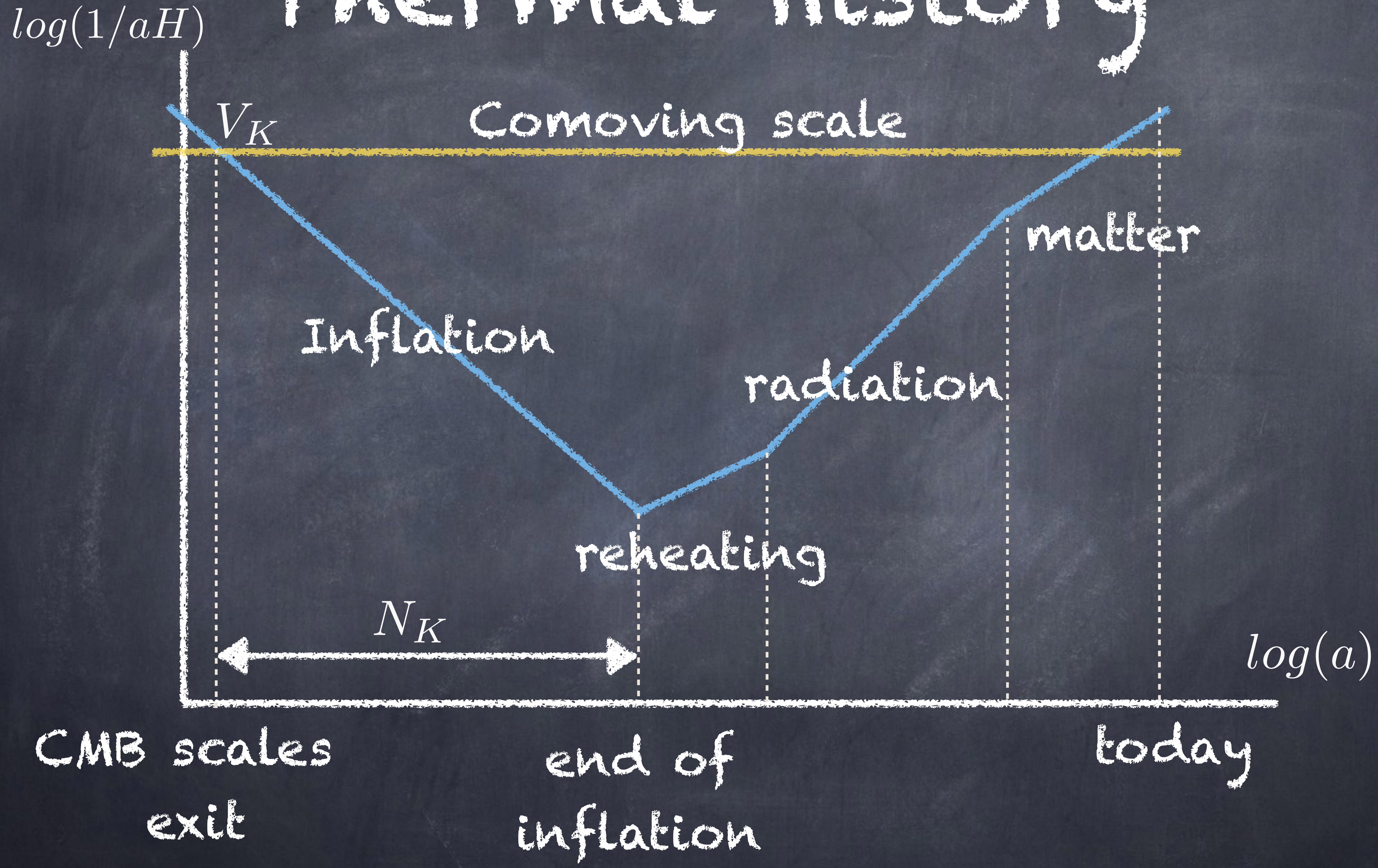
Watch out for talk by R. Allahverdi tomorrow



Matter domination

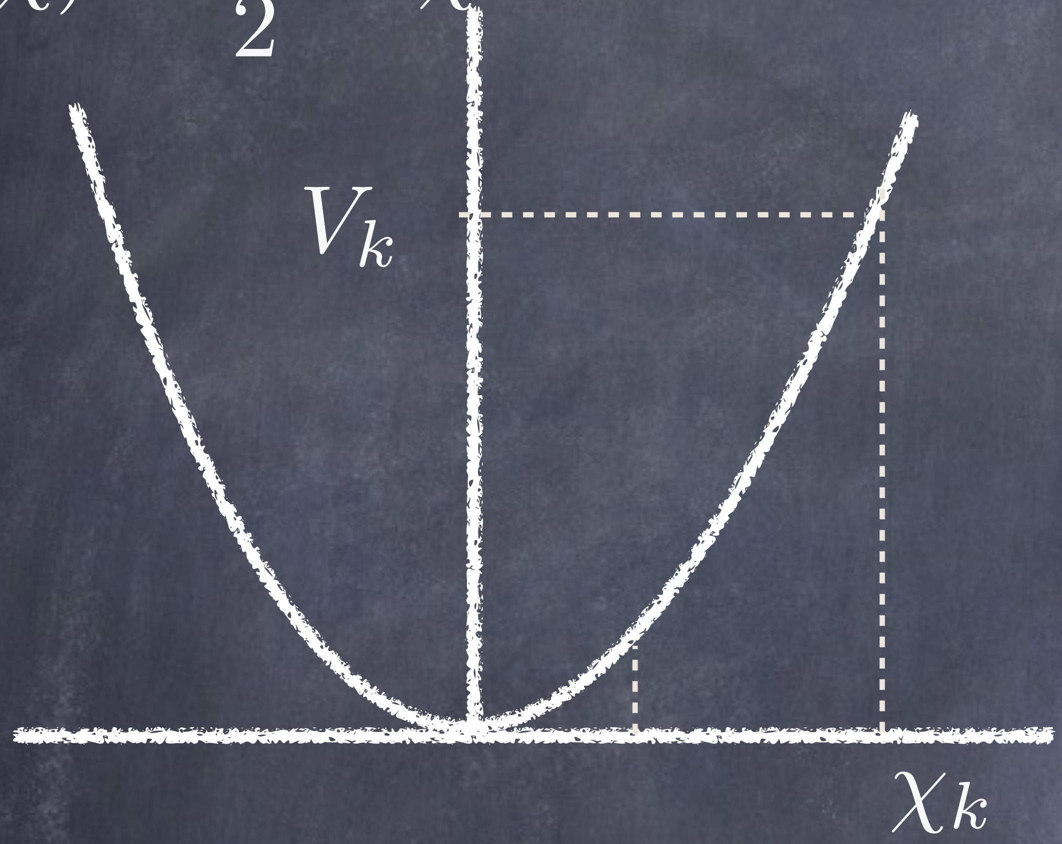

History of the Universe
Gravitational Waves
Inflation Generates Two Types of Waves
Density Waves
Waves Imprint Characteristic Polarization Signals
Free Electrons Scatter Light
Earliest Time Visible with Light
Radius of the Visible Universe
Quantum Fluctuations
Big Bang
Inflation
Radiation domination:
Prejudice
Nuclear Fusion Begins
BBN
Nuclear Fusion Ends
Cosmic Microwave Background
Neutral Hydrogen Forms
Modern Universe
0
10^{-32} s
1 µs
0.01 s
3 min
380,000 yrs
13.8 Billion yrs
Age of the Universe
non-thermal epoch
Matter domination

Thermal History



Inflation: Case Study

$$V(\chi) = \frac{1}{2}m^2\chi^2$$



$$N_k \simeq \frac{\chi_k^2}{4M_{Pl}^2}$$

$$n_s - 1 = -\frac{2}{N_k}$$

Inflation: Case Study

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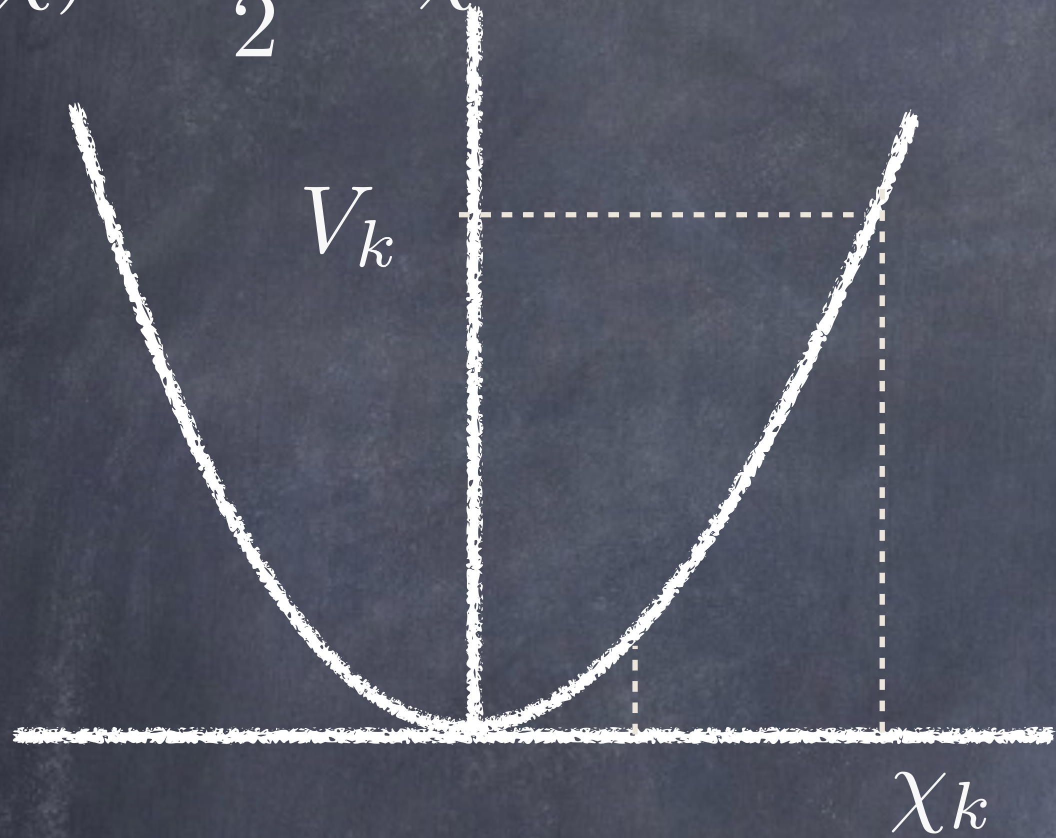


$$N_k \simeq \frac{\chi_k^2}{4M_{Pl}^2} \qquad n_s - 1 = -\frac{2}{N_k}$$

$$n_s = 0.968 \pm 0.006$$

PLANCK 2015

precision measurement of spectral index can pin down the e-folds during inflation

Inflation & Density Perturbations

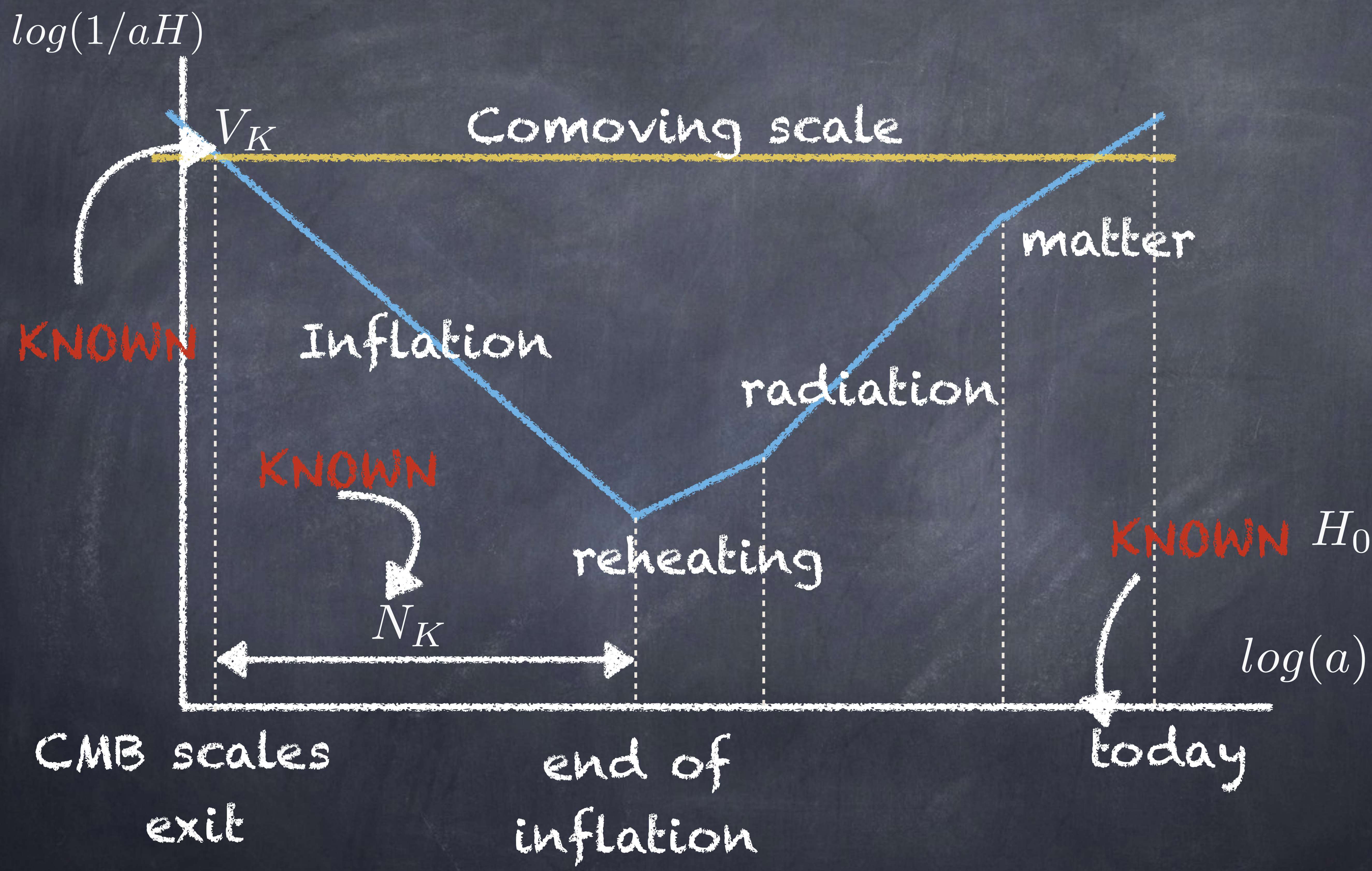
$$A_s = \frac{2}{3\pi^2 r}(\frac{\rho_k}{M_{Pl}^4})$$

Energy density at the time of horizon exit

strength of gravity wave

$$A_s = 2.2 \times 10^{-9} \quad @ \quad k = 0.05 Mpc^{-1}$$

knowing scalar amplitude and 'r' we know initial energy density



$log(1/aH)$
V_K
Comoving scale
KNOWN
Inflation
matter
radiation
KNOWN
reheating
KNOWN
H_0
N_K
$log(a)$
CMB scales exit
end of inflation
today

Consistency

V_k must be evolved to H_0

Any post inflationary phase must be evolved to the present energy density

Consistency Condition

$$N_{inf} + \frac{1}{4}(1 - 3w_{rh})N_{rh} = 55 + \frac{1}{4}lnr + \frac{1}{4}ln(\rho_k/\rho_{end})$$

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$$N_{inf} + \frac{1}{4}(1 - 3w_{rh})N_{rh} = 55 + \frac{1}{4}lnr + \frac{1}{4}ln(\rho_k/\rho_{end})$$

Equivalent

$$N_* \approx 71.21 - \ln\left(\frac{k_*}{a_0 H_0}\right) + \frac{1}{4}\ln\left(\frac{V_{\rm hor}}{M_{\rm pl}^4}\right) + \frac{1}{4}\ln\left(\frac{V_{\rm hor}}{\rho_{\rm end}}\right) + \frac{1 - 3w_{\rm int}}{12(1 + w_{\rm int})}\ln\left(\frac{\rho_{\rm th}}{\rho_{\rm end}}\right),$$

PLANCK paper

Making predictions ..

$$N_{inf} + \frac{1}{4}(1 - 3w_{rh})N_{rh} = 55 + \frac{1}{4}lnr + \frac{1}{4}ln(\rho_k/\rho_{end})$$

$$N_{inf} = 55 \pm 5$$

Liddle, Leach (2003)

PLANCK (2016)

Making predictions ..

$$N_{inf} + \frac{1}{4}(1 - 3w_{rh})N_{rh} = 55 + \frac{1}{4}lnr + \frac{1}{4}ln(\rho_k/\rho_{end})$$

$$N_{inf} = 55 \pm 5$$

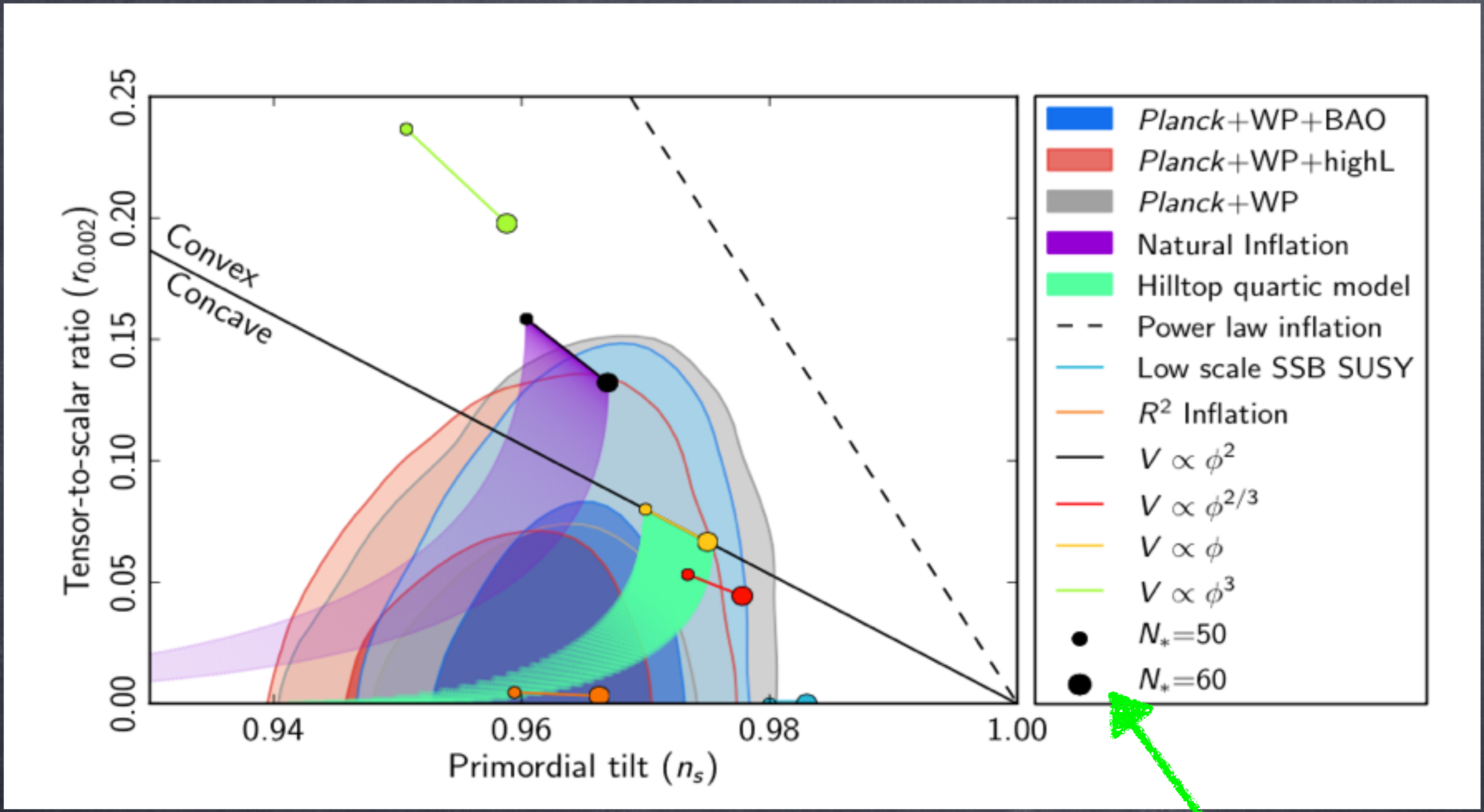
Liddle, Leach (2003)

PLANCK (2016)

'Theoretical prior'

compute observables in terms of N_{inf} and see whether it fits data for N = 50-60!

$$V(\chi) = \frac{1}{2}m^2\chi^2 \qquad n_s - 1 = -\frac{2}{N_k} \qquad r = 8/N_K$$



'Theoretical prior'

How does making predictions change for modular cosmology?

Moduli ..

- moduli: light scalar fields with Planck suppressed interactions

- at tree level effective Lagrangian of string theory/SUGRA, moduli are massless

- moduli must acquire masses (thus fixed vev) to become phenomenologically viable

- moduli stabilisation: KKLT etc...

Moduli ..

- Conservative approach: Make ALL modulus much heavier than the Hubble scale .. decouple from inflation

- Wishful ....

- In practice, few fields remain parametrically light in the post-inflationary vacua .. (e.g Many LVS constructions ..)   see later ..

A typical case

$$\mathcal{L} \supset -\frac{1}{2}m^2\varphi^2 - \frac{1}{2}H^2(\varphi - \hat{\varphi})^2 - V_{inf}(\chi)$$

$m < H_{inf}$

post-inflationary moduli mass

minima during inflation

A typical case

$$\mathcal{L} \supset -\frac{1}{2}m^2\varphi^2 - \frac{1}{2}H^2(\varphi - \hat{\varphi})^2 - V_{inf}(\chi)$$

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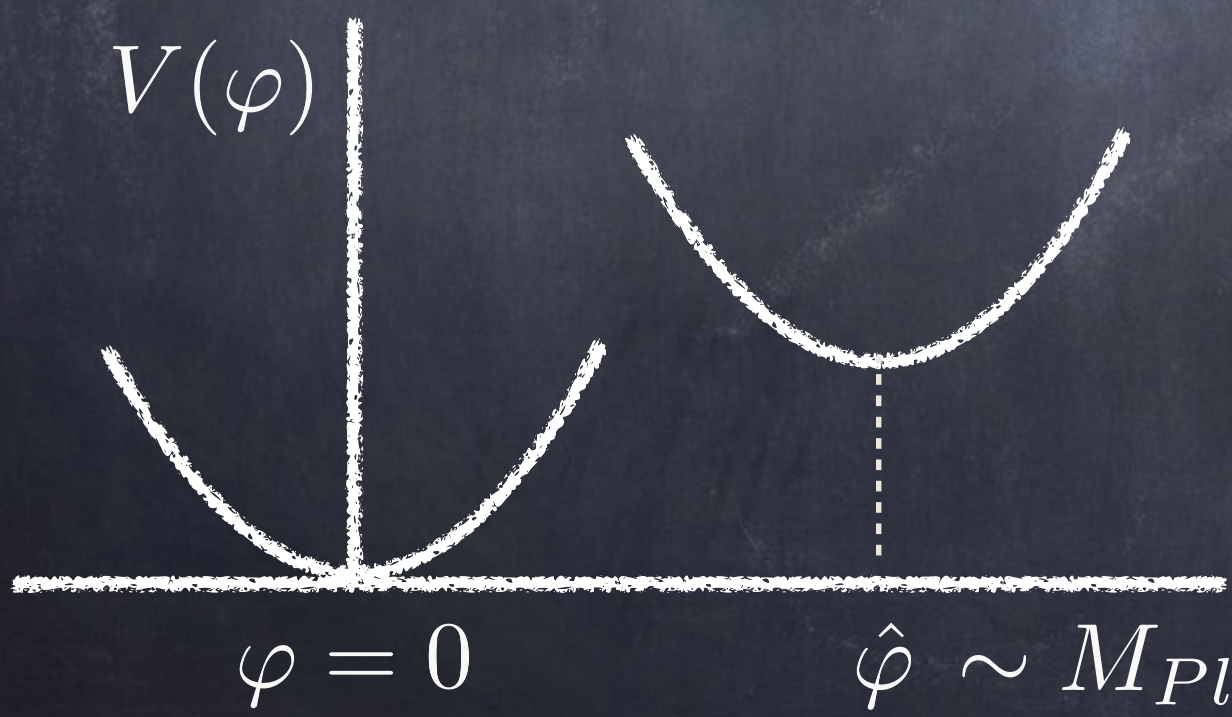
minima during inflation



$$Y = \hat{\varphi}/M_{Pl} \sim 1$$

Dine, Randall, Thomas

SUGRA ...

$$V = e^{K[\varphi,\bar{\varphi}]} V_0[\varphi,\chi] \sim H^2 M_{Pl}^2 f\left(\frac{\varphi}{M_{Pl}}\right)$$

$V'' \sim H^2$ $\eta - \text{problem}$

Scale of variations M_{Pl} $Y = \hat{\varphi}/M_{Pl} \sim 1$

Dine, Randall, Thomas

Dvali

SUGRA ...

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Scale of variations M_{Pl} $Y = \hat{\varphi}/M_{Pl} \sim 1$

Dine, Randall, Thomas

Dvali

Toy example ..

$$V = (m_{3/2}^2 - a^2 H^2)|\varphi|^2 + \frac{1}{2M_{Pl}^2}(m_{3/2}^2 + b^2 H^2)|\varphi|^4$$

$$\hat{\varphi} \sim (a/b) M_{Pl}$$

Sequence of events ..

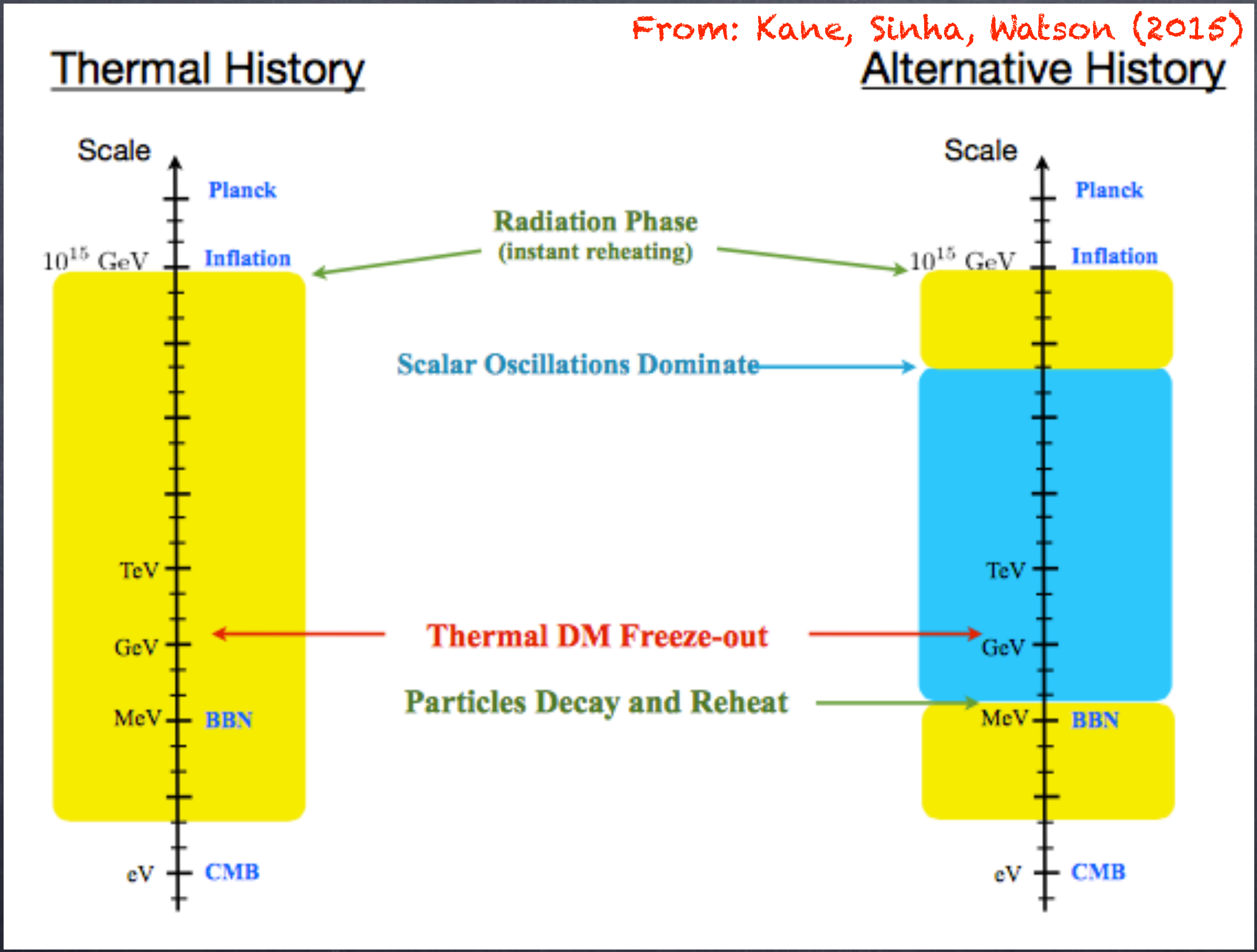
- when m ~ H_inf, moduli is stuck due to the Hubble friction

- inflation ends with $\varphi = \hat{\varphi}$

- When H < m, the field starts to move toward its post inflationary minima $\varphi = 0$

- Oscillations around the minima behaves as matter $\rho_\varphi \sim a^{-3}(t)$

Review: B. S. Acharya, G. Kane, and P. Kumar (2012)



See talk by R. Allahverdi

Consistency

V_k must be evolved to H_0

N_k is known

Any post inflationary evolution must be evolved to the present energy density

Decay of Modulus

moduli must decay so that it does not overclose the Universe

$$\Gamma_{mod} \sim \frac{m_\varphi^3}{16\pi M_{Pl}^2}$$

G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby and G. G. Ross 1984

Decay of Modulus

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G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby and G. G. Ross 1984

$$\rho_{mod}(t_{decay}) = g\frac{\pi^2}{30}T_{reheat}^4 \quad = 3H^2M_{Pl}^2$$

↑ decay happens

Γ_{mod}

$$T_{reheat} \sim \sqrt{\Gamma M_{Pl}}$$

$$T_{reheat} > MeV$$

successful BBN

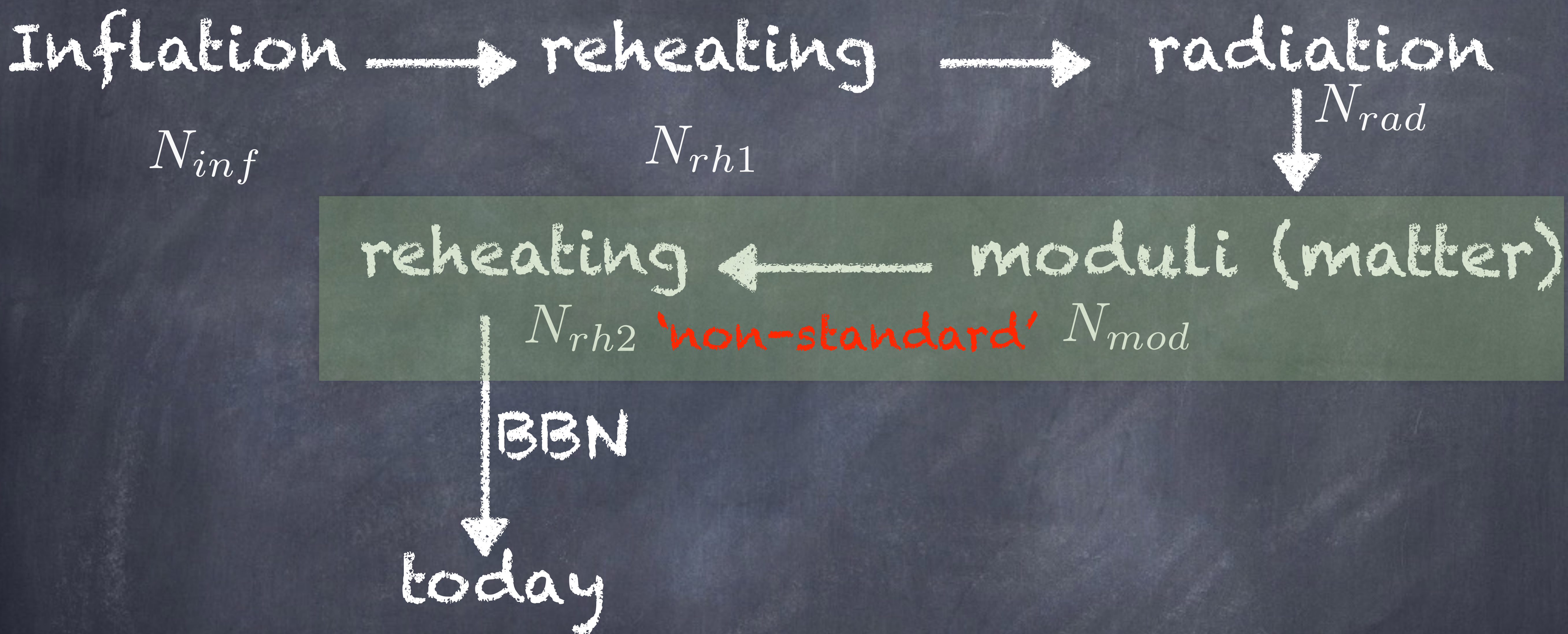
$$m_\varphi > 30\ TeV$$

BBN bound

phenomenological implications .. SUSY breaking ..

Tom Banks, David B. Kaplan, and Ann E. Nelson

B. de Carlos, J. Casas, F.Quevedo, E.Roulet



Inflation
N_{inf}
reheating
N_{rh1}
radiation
N_{rad}
moduli (matter)
N_{mod}
reheating
N_{rh2}
'non-standard'
BBN
today

Inflation → reheating → radiation

N_{inf} N_{rh1} ↓ N_{rad}

reheating ← moduli (matter)

N_{rh2} 'non-standard' N_{mod}

↓ BBN

today

K.D, Maharana

arXiv:1409.7037[hep-ph]

$$N_{inf} + \frac{1}{4}(1 - 3w_{rh1})N_{rh1} + \frac{1}{4}N_{mod} + \frac{1}{4}(1 - 3w_{rh2})N_{rh2}$$

$$= 55.43 + \frac{1}{4} ln\ r + \frac{1}{4} ln(\frac{\rho_k}{\rho_{end}})$$

non-thermal history

constraint ..

$$\Gamma_{mod} \sim \frac{m_\varphi^3}{16\pi M_{Pl}^2}$$

$$N_{mod} \sim \frac{2}{3} ln(\frac{16\pi M_{Pl}^2 Y^4}{m_\phi^2})$$

initial displacement $Y = \hat{\varphi}/M_{Pl}$

constraint ..

$$\Gamma_{mod} \sim \frac{m_\varphi^3}{16\pi M_{Pl}^2} \qquad N_{mod} \sim \frac{2}{3} ln(\frac{16\pi M_{Pl}^2 Y^4}{m_\phi^2})$$

initial displacement $Y = \hat{\varphi}/M_{Pl}$

post-inflationary details

$$\frac{1}{6} ln(\frac{16\pi M_{Pl}^2 Y^4}{m_\varphi^2}) + \frac{1}{4}(1 - 3w_{rh1})N_{re1} + \frac{1}{4}(1 - 3w_{rh2})N_{re2}$$

$$= 55.43 - N_{inf} + \frac{1}{4} ln\ r + \frac{1}{4} ln(\frac{\rho_k}{\rho_{end}})$$

inflationary details

inflationary potentials

Implications: I

Central value of e-folding shifts

$$N_{inf} = 55 \pm 5$$

$$N_{inf} = \left(55 - \frac{N_{mod}}{4}\right) \pm 5$$

$$\Gamma_{mod} \sim \frac{m_\varphi^3}{16\pi M_{Pl}^2}$$

$$N_{inf} = \left(55 - \frac{1}{3} ln\left(\frac{\sqrt{16\pi} M_{pl} Y^2}{m_\varphi}\right)\right) \pm 5$$

$$N_{inf} = \left(55 - \frac{1}{3} ln\left(\frac{\sqrt{16\pi} M_{pl} Y^2}{m_\varphi}\right)\right) \pm 5$$

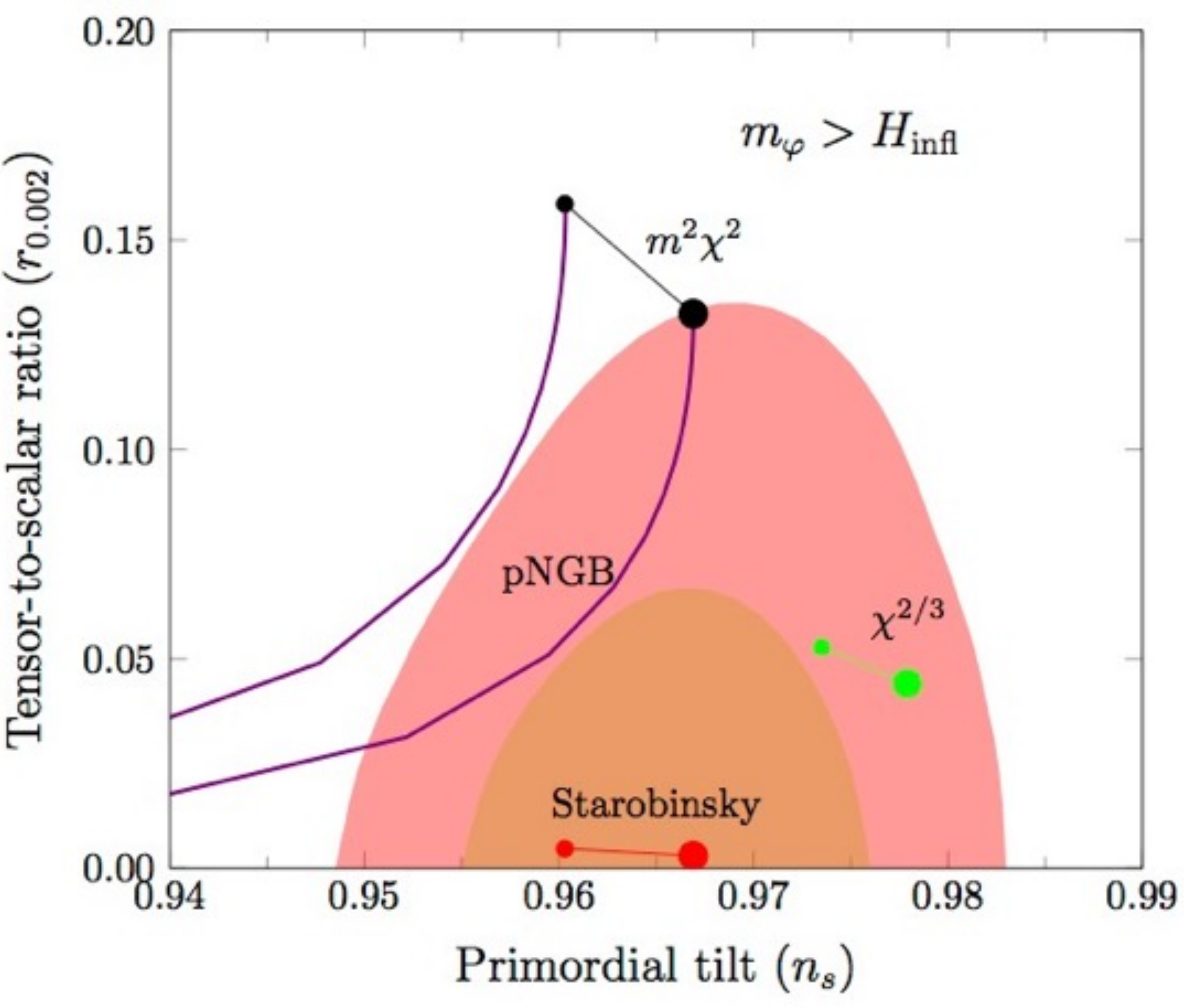
Central value of e folding shifts

For $m_\varphi \sim 10^3$ TeV : $N_{inf} = 41 \quad - \quad 51$

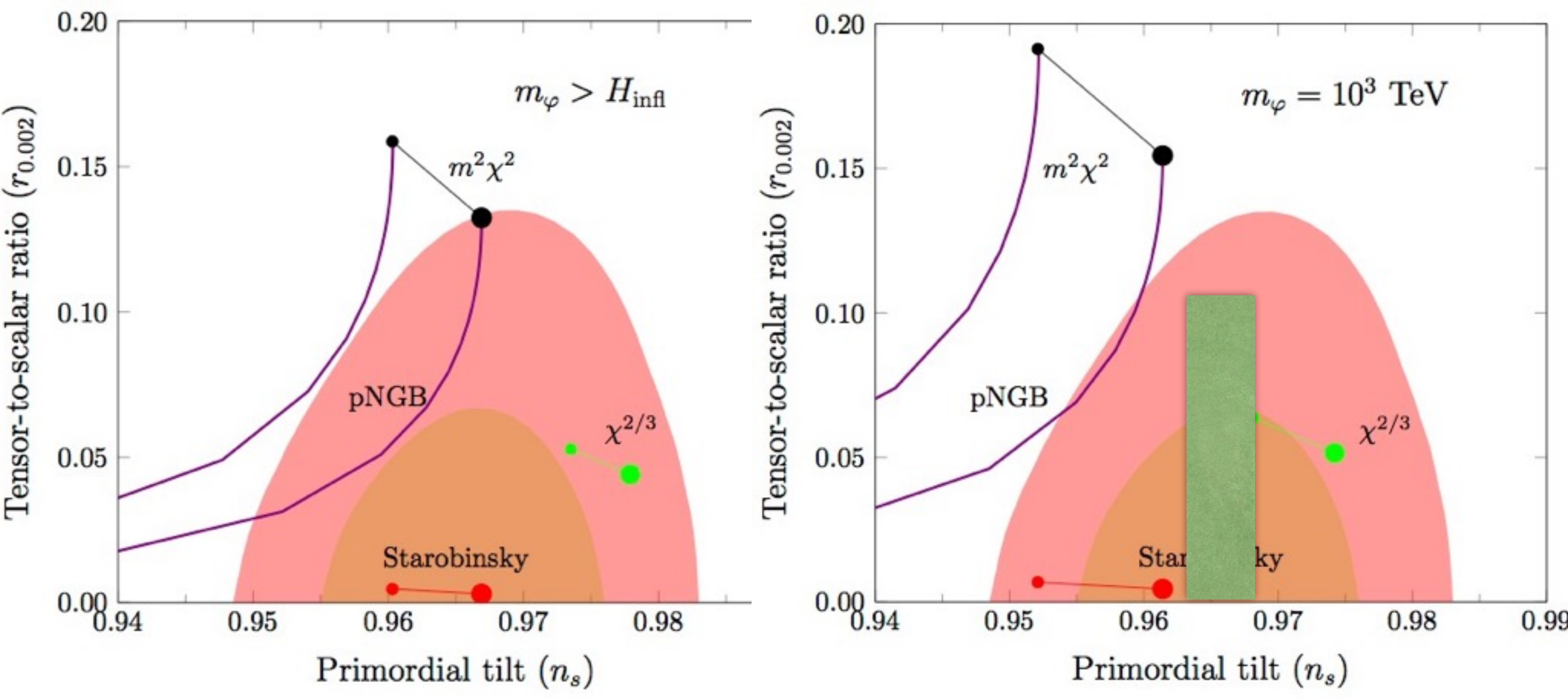
For $m_\varphi \sim 10^6$ TeV : $N_{inf} = 43 \quad - \quad 53$

(used to be 50 - 60)

(Y ~ 0.1 assumed)



$m_\varphi > H_{\text{infl}}$
$m^2\chi^2$
pNGB
$\chi^{2/3}$
Starobinsky
Tensor-to-scalar ratio ($r_{0.002}$)
Primordial tilt (n_s)
0.20
0.15
0.10
0.05
0.00
0.94
0.95
0.96
0.97
0.98
0.99



Das, K.D, Maharana

sensitivity $n_s \sim 10^{-3}$

EUCLID/PRISM

Gravity mediated models

- moduli masses tied to soft masses in SUSY

- typical moduli mass 100/1000 TeV

Preferred value of inflation e-folds

$$\hat{N}_{inf} = 55 - \frac{1}{3}\left(\frac{\sqrt{16\pi} M_{Pl} Y^2}{m_\varphi}\right) \simeq 45$$

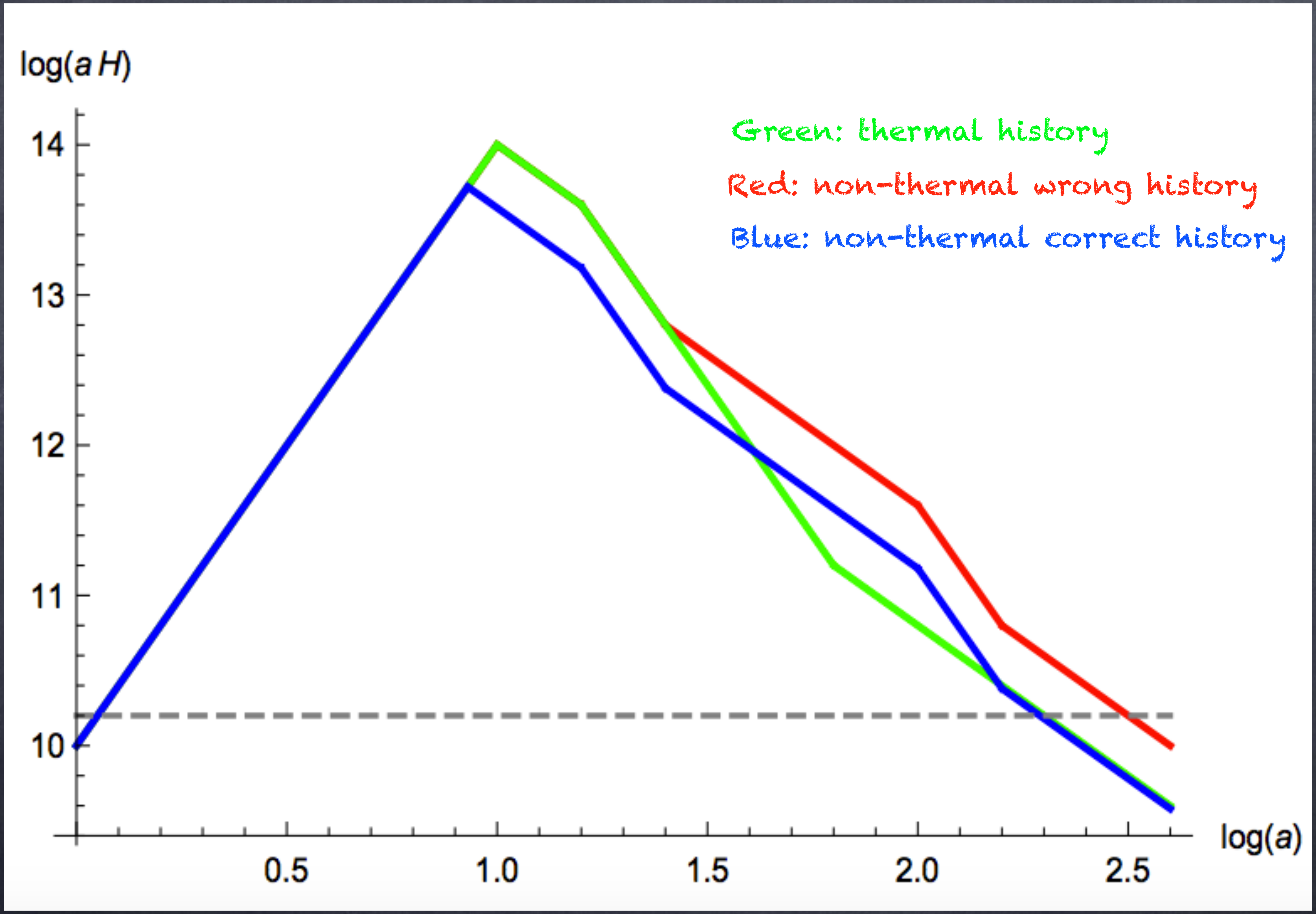
very different from usual

inflation ⟷ SUSY breaking

The central value reaches N = 50 for

$$m_\varphi \sim 10^{10} GeV$$

The effects of modulus mass must be taken for inflation models for $m_\varphi \lesssim 10^{10} GeV$


log(a H)
14
13
12
11
10
Green: thermal history
Red: non-thermal wrong history
Blue: non-thermal correct history
0.5
1.0
1.5
2.0
2.5
log(a)

constraint on modulus mass

$$\frac{1}{6}ln(\frac{16\pi M_{Pl}^2 Y^4}{m_\varphi^2}) + \frac{1}{4}(1-3w_{rh1})N_{re1} + \frac{1}{4}(1-3w_{rh2})N_{re2}$$

usually positive definite

$$= 55.43 - N_{inf} + \frac{1}{4}ln\ r + \frac{1}{4}ln(\frac{\rho_k}{\rho_{end}})$$

analytical/numerical understanding of reheating: $w_{re} < 1/3$

Ellis, Garcia, Nanopoulos, Olive (2015)

constraint ..

$$m_\varphi \gtrsim \sqrt{16\pi} M_{\rm pl} Y^2 \; e^{-3\left(55.43 - N_k + \frac{1}{4}\ln(\frac{\rho_{\rm k}}{\rho_{\rm end}}) + \frac{1}{4}\ln r\right)}$$

- Dependence correlated

Das, K.D, Maharana

- larger the value of N_{inf}, stronger the bound

- smaller the value of 'r' stronger the bound

- bound depends on the nature of inflationary potentials via the ratio of energy densities

small field models

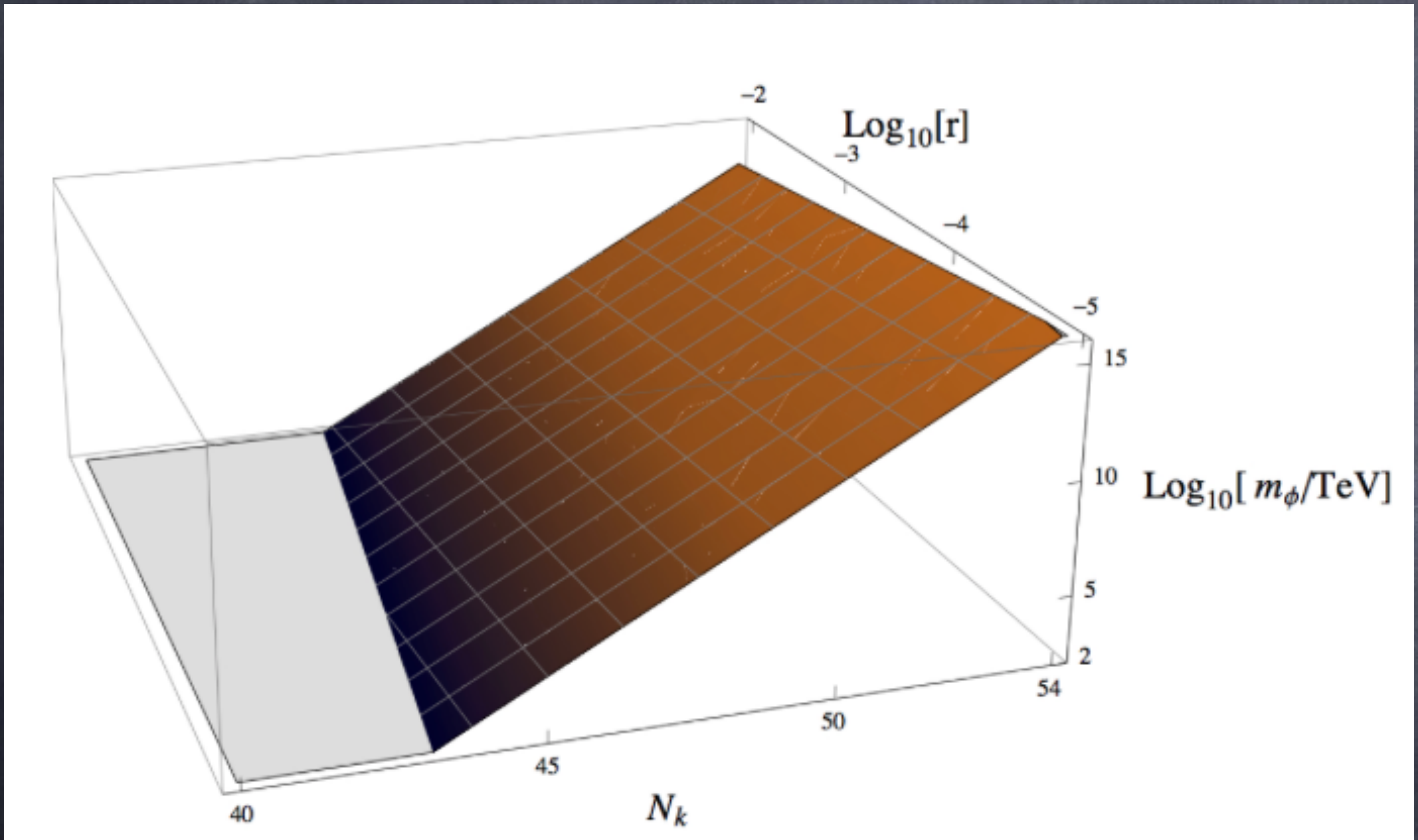
$$m_\varphi \gtrsim \sqrt{16\pi} M_{\rm pl} Y^2 \; e^{-3\left(55.43 - N_k + \frac{1}{4}\ln(\frac{\rho_{\rm k}}{\rho_{\rm end}}) + \frac{1}{4}\ln r\right)}$$

- conservative estimate r ~ 0.01 stronger the bound
- potential plateau like .. ratio of energy densities negligible
- take Y = 0.1, then for N = 50

$$m_\varphi \gtrsim 4.5 \times 10^6 TeV$$

much stronger than BBN bound

small field models



for N > 48, the bound much stronger than BBN bound

Das, Maharana, K.D

large field models

$$m_\varphi \gtrsim \sqrt{16\pi} M_{\rm pl} Y^2 \; e^{-3\left(55.43 - N_k + \frac{1}{4}\ln(\frac{\rho_{\rm k}}{\rho_{\rm end}}) + \frac{1}{4}\ln r\right)}$$

chaotic inflation
axion monodormy

$$V_\chi = m^{4-\alpha}\chi^\alpha$$

$$m_\phi \gtrsim$$

$$\sqrt{16\pi} M_{\rm pl} Y^2 e^{-3\left(55.85 - \frac{(2+\alpha)}{2(1-n_s)} + \frac{\alpha}{8}\ln 2 + \frac{1}{8}(\alpha-2)\ln\left(\frac{2+\alpha}{\alpha(1-n_s)}\right)\right)}$$

$\alpha = 2 \quad m_\varphi >> 10^{10}$ TeV   PLANCK: Central value

$\alpha = 2/3$ Bound insignificant

Implications

- guiding principle for modular cosmology (Independent from CMP bound)

- modulus mass related to soft masses in SUSY (gravity mediated SUSY breaking)

- Large SUSY breaking scale ...

- for many models N_k > 50, and the bound is much stronger than BBN bound for PLANCK central value ..

What to calculate now?

Cicoli, K.D, Maharana, Quevedo

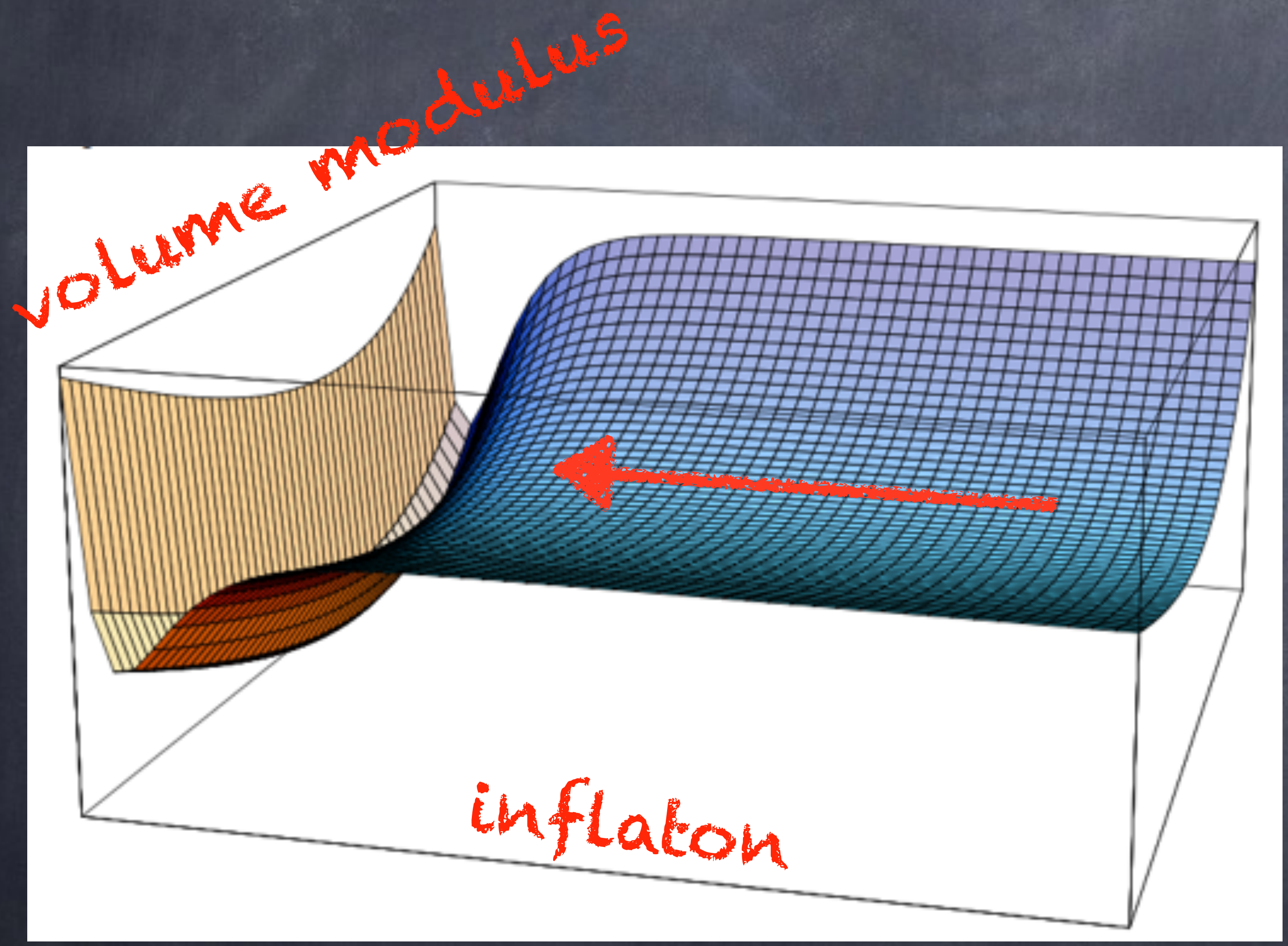
$$N_{mod} \sim \frac{2}{3} ln(\frac{16\pi M_{Pl}^2 Y^4}{m_\phi^2})$$

$$\frac{1}{6} ln(\frac{16\pi M_{Pl}^2 Y^4}{m_\varphi^2}) + \frac{1}{4}(1 - 3w_{rh1})N_{re1} + \frac{1}{4}(1 - 3w_{rh2})N_{re2}$$

$$= 55.43 - N_{inf} + \frac{1}{4} ln\ r + \frac{1}{4} ln(\frac{\rho_k}{\rho_{end}})$$

Kahler moduli Inflation

Conlon, Quevedo (2005)



- In LVS scenario of string theory.
- A concrete set-up where inflationary potential is known
- Post-inflationary modulus domination happens

$$\Gamma_{\tau_n} \simeq 0.1 \frac{m_{\tau_n}^3}{M_s^2} \gg \Gamma_\nu \simeq \frac{m_\nu^3}{16\pi M_{pl}^2}$$

Cicoli, Mazumdar (2010)

Details ..

Volume $\mathcal{V} = \alpha\left(\tau_1^{3/2} - \sum_{i=2}^{n} \lambda_i \tau_i^{3/2}\right)$

$$V = \sum_{i=2}^{n} \frac{8(a_i A_i)^2 \sqrt{\tau_i}}{3\mathcal{V}\lambda_i} e^{-2a_i\tau_i} - \sum_{i=2}^{n} \frac{4a_i A_i W_0}{\mathcal{V}^2} \tau_i e^{-a_i\tau_i} + \frac{3\hat{\xi} W_0^2}{4\mathcal{V}^3} + \frac{D}{\mathcal{V}^\gamma}$$

$$m_{\tau_i}^2 \simeq \frac{W_0^2 (\ln \mathcal{V})^2 M_{\rm pl}^2}{\mathcal{V}^2} \gg m_{\mathcal{V}}^2 \simeq \frac{W_0^2 M_{\rm pl}^2}{\mathcal{V}^3 \ln \mathcal{V}}$$

Details ..

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taking all small moduli at minima: two field potential

$$V_{\rm inf} = -\frac{3W_0^2}{2\mathcal{V}^3} \left(\sum_{i=2}^{n-1} \left[\frac{\lambda_i \alpha}{a_i^{3/2}} \right] (\ln \mathcal{V})^{3/2} - \frac{\xi}{2} \right) + \frac{D}{\mathcal{V}^\gamma} - \frac{4 a_n A_n W_0}{\mathcal{V}^2} \tau_n e^{-a_n \tau_n}$$

modulus inflaton

Inflationary Phenomenology

$$V = V_0 - \frac{4W_0 a_n A_n}{\mathcal{V}_{\rm in}^2}\left(\frac{3\mathcal{V}_{\rm in}}{4\lambda}\right)^{2/3}\sigma^{4/3}\exp\left[-a_n\left(\frac{3\mathcal{V}_{\rm in}}{4\lambda}\right)^{2/3}\sigma^{4/3}\right]$$

Canonical inflaton field

$$V(\sigma) = C_0(1 - e^{-b\sigma})$$

Effective single field dynamics

Volume modulus is stabilised during inflation and heavy

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$$\epsilon = \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 = \frac{32\mathcal{V}_{\rm in}^3}{3\beta^2 W_0^2\lambda_n} a_n^2 A_n^2\sqrt{\tau_n}\,(1 - a_n\tau_n)^2\, e^{-2a_n\tau_n},$$

$$\eta = M_{\rm pl}^2\frac{V''}{V} = -\frac{4\mathcal{V}_{\rm in}^2}{3\beta W_0\lambda_n\sqrt{\tau_n}} a_n A_n\left[\left(1 - 9a_n\tau_n + 4a_n^2\tau_n^2\right)e^{-a_n\tau_n}\right].$$

Inflationary Phenomenology

$$N_e(\sigma) = \int_{\sigma_{\rm end}}^{\sigma} \frac{1}{\sqrt{2\epsilon(\sigma)}}\, d\sigma \;\simeq\; \frac{3\beta W_0 \lambda_n}{16\mathcal{V}_{\rm in}^2 a_n^{3/2} A_n} \frac{e^{a_n\tau_n}}{(a_n\tau_n)^{3/2}}$$

$$\epsilon \simeq \left(\frac{3\lambda_n}{8a_n^{3/2}\mathcal{V}_{\rm in}}\right) \frac{1}{N_e^2\sqrt{a_n\tau_n}} \quad \ll \quad \eta \simeq -\frac{1}{N_e}$$

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COBE normalisation: $\quad \dfrac{V^{3/2}}{M_{\rm pl}^3\, V'} = 5.2 \times 10^{-4}$

$$\tau_n \simeq 7.31 \cdot 10^{-14} \left(\frac{6\pi\lambda_n}{g_s\, \beta\, e^{K_{\rm cs}}}\right)^2 \left(\frac{\mathcal{V}_{\rm in}^4}{W_0^4\, a_n^4}\right) \frac{1}{N_e^4}$$

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$$r = 16\epsilon \simeq 16\times 3.7\cdot 10^6\left(\frac{g_s\,\beta\, e^{K_{\rm cs}}}{16\pi}\right)\left(\frac{W_0^2}{\mathcal{V}_{\rm in}^3}\right) \ll 10^{-4} N_e^{-3}$$

$$n_s = 1 + 2\eta - 6\epsilon \simeq 1 - \frac{2}{N_e}$$

spectral index depends only on N_e

Predictions ..

$$N_e + \approx 57 + \frac{1}{4}\ln r + \frac{1}{4}\ln\left(\frac{\rho_*}{\rho_{\text{end}}}\right) - \frac{1}{4}N_{\text{mod}}$$

$$\mathcal{V}_{\text{in}} \sim 10^5 - 10^6$$

$$r = 16\epsilon \sim 10^{-10} - 10^{-11}$$

$$N_e \simeq 50 - \frac{1}{4}N_{\text{mod}}$$

Qualitative estimate:

$$N_{mod} \sim \frac{2}{3}ln(\frac{16\pi M_{Pl}^2 Y^4}{m_\phi^2})$$

$$Y \simeq \mathcal{O}(0.1 - 1)$$

Shift in Volume Modulus

Potential experienced by the volume modulus depends on the inflaton: Vacuum misalignment

Other modulus are not shifted and having masses much larger than the Hubble scale.

$$V = -\frac{3W_0^2}{2\mathcal{V}^3}\left(\sum_{i=2}^{n}\left[\frac{\lambda_i \alpha}{a_i^{3/2}}\right](\ln \mathcal{V})^{3/2} - \frac{\hat{\xi}}{2}\right) + \frac{D}{\mathcal{V}^\gamma}$$

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$$V(\mathcal{V}_*) = \frac{\partial V(\mathcal{V}_*)}{\partial \mathcal{V}} = 0 \qquad -\frac{3W_0^2}{2}\, e^{-3\phi_*}\left(P\phi_*^{3/2} - \frac{\hat{\xi}}{2}\right) + D\, e^{-2\phi_*} = 0$$

$$\frac{3W_0^2}{2}\, e^{-3\phi_*}\left(3P\phi_*^{3/2} - \frac{3}{2}P\phi_*^{1/2} - \frac{3\hat{\xi}}{2}\right) - 2D\, e^{-2\phi_*} = 0$$

$$\phi \equiv \ln \mathcal{V} \quad \text{and} \quad P \equiv \alpha \sum_{i=2}^{n} \lambda_i\, a_i^{-3/2} = \frac{\alpha}{R}\, \lambda_n\, a_n^{-3/2} \qquad R \equiv \frac{\lambda_n\, a_n^{-3/2}}{\sum_{i=2}^{n} \lambda_i\, a_i^{-3/2}} \ll 1$$

Shift in Volume Modulus

$$\phi_*^{3/2} - \frac{3}{2}\phi_*^{1/2} - \frac{\hat{\xi}}{2P} = 0$$

$$D = \frac{9W_0^2}{4} P e^{-\phi_*} \phi_*^{1/2}$$

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$$V_{\rm in}(\phi) = -\frac{3W_0^2}{4} e^{-3\phi}\left[2P\left(1-R\right)\phi^{3/2} - \hat{\xi} - 3P\phi_*^{1/2} e^{(\phi-\phi_*)}\right]$$

$$\left(1-R\right)\phi_{\rm in}^{3/2} - \frac{1}{2}\left(1-R\right)\phi_{\rm in}^{1/2} - e^{(\phi_{\rm in}-\phi_*)}\phi_*^{1/2} - \frac{\hat{\xi}}{2P} = 0$$

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$$V_{\rm in}(\phi) = V(\phi) + \delta V(\phi)$$

$$\delta V(\phi) = \frac{3W_0^2}{2} e^{-3\phi} P R\, \phi^{3/2}$$

$$\delta\phi = -\frac{\delta V'(\phi_*)}{V''(\phi_*)} = 4R\,\frac{\phi_* + \frac{\hat{\xi}}{2P}\,\phi_*^{1/2}}{2\phi_* - 1} \simeq 2R\phi_*$$

$$\delta\phi = \phi_{\rm in} - \phi_*$$

$$Y = \frac{\delta\varphi}{M_{pl}} = \sqrt{\frac{2}{3}}\delta\phi \simeq 2\sqrt{\frac{2}{3}}R\phi_* \simeq 0.1 - 1$$

We validate the generic arguments with explicit calculations

e-foldings

t1: end of inflation

$$\rho_{\tau_n}(t_1) \approx \frac{M_{\rm pl}^4 W_0^2 \beta}{\mathcal{V}^3}$$

Barnaby, Bond, Huang, Kofman (2009)

$$H(t_1) \approx \frac{M_{\rm pl} W_0 \beta^{1/2}}{\mathcal{V}^{3/2}} \simeq m_{\mathcal{V}}$$

Volume modulus starts to oscillate immediately at the end of inflation

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$$\rho_{\mathcal{V}}(t_1) \approx m_{\mathcal{V}}^2 \varphi_{\rm in}^2 \approx \frac{M_{\rm pl}^4 W_0^2 Y^2}{\mathcal{V}^3 \ln \mathcal{V}} \qquad \frac{\rho_{\mathcal{V}}(t_1)}{\rho_{\tau_n}(t_1)} \approx \frac{Y^2}{\beta \ln \mathcal{V}} \equiv \theta^2 \ll 1$$

short matter dominated epoch until inflaton decays

$$N_{\rm mod1} = \ln\left(\frac{a(t_2)}{a(t_1)}\right) = \frac{1}{3}\ln\left(\frac{\rho_{\tau_n}(t_1)}{\rho_{\tau_n}(t_2)}\right) \simeq \frac{2}{3}\ln\left(\frac{H(t_1)}{\Gamma_{\tau_n}}\right) \simeq \frac{2}{3}\ln\left(\frac{10\beta^{1/2}\mathcal{V}^{1/2}}{W_0^2(\ln \mathcal{V})^3}\right)$$

t2: inflation decay time

e-foldings

Inflaton may decay to hidden sectors Cicoli, Mazumdar (2010)

When inflaton decays, radiation domination

$$H(t_2) = H(t_1)\left(\frac{a(t_1)}{a(t_2)}\right)^{3/2} = H(t_1)\, e^{-\frac{3}{2}N_{\rm mod1}} \simeq \frac{H(t_1) W_0^2 (\ln \mathcal{V})^3}{10\beta^{1/2}\mathcal{V}^{1/2}}$$

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At t_eq, radiation and volume modulus oscillations density become equal

$$\rho_{\rm rad}(t_2)\left(\frac{a(t_2)}{a(t_{\rm eq})}\right)^4 = \rho_{\mathcal{V}}(t_2)\left(\frac{a(t_2)}{a(t_{\rm eq})}\right)^3 \qquad H(t_{\rm eq}) = \frac{H(t_1)W_0^2(\ln\mathcal{V})^3\theta^4}{10\beta^{1/2}\ \mathcal{V}^{1/2}}$$

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$$N_{\rm mod2} \simeq \frac{2}{3}\ln\left(\frac{H(t_{\rm eq})}{\Gamma_{\mathcal{V}}}\right)$$

Similar to the previous expressions

$$N_{\rm mod2} \approx \frac{2}{3}\ln\left(\frac{16\pi\mathcal{V}^{5/2}(\ln\mathcal{V})^{5/2}Y^4}{10\beta^2}\right) \approx \frac{2}{3}\ln\left(\frac{16\pi\mathcal{V}^{5/2}Y^4}{10P^2R^2(\ln\mathcal{V})^{1/2}}\right)$$

A benchmark example

$$W_0 = \alpha = \lambda_i = 1, a_i = 2\pi, g_s = 0.06$$

$$\mathcal{V}_{\rm in} \simeq 1.38 \cdot 10^5, \beta \simeq 3.88$$

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plays important role

$$m_{\mathcal{V}} \sim 10^8 - 10^9 \quad GeV$$

$$N_{\rm mod1} \simeq 0.99 \qquad \text{and} \qquad N_{\rm mod2} \simeq 25.4$$

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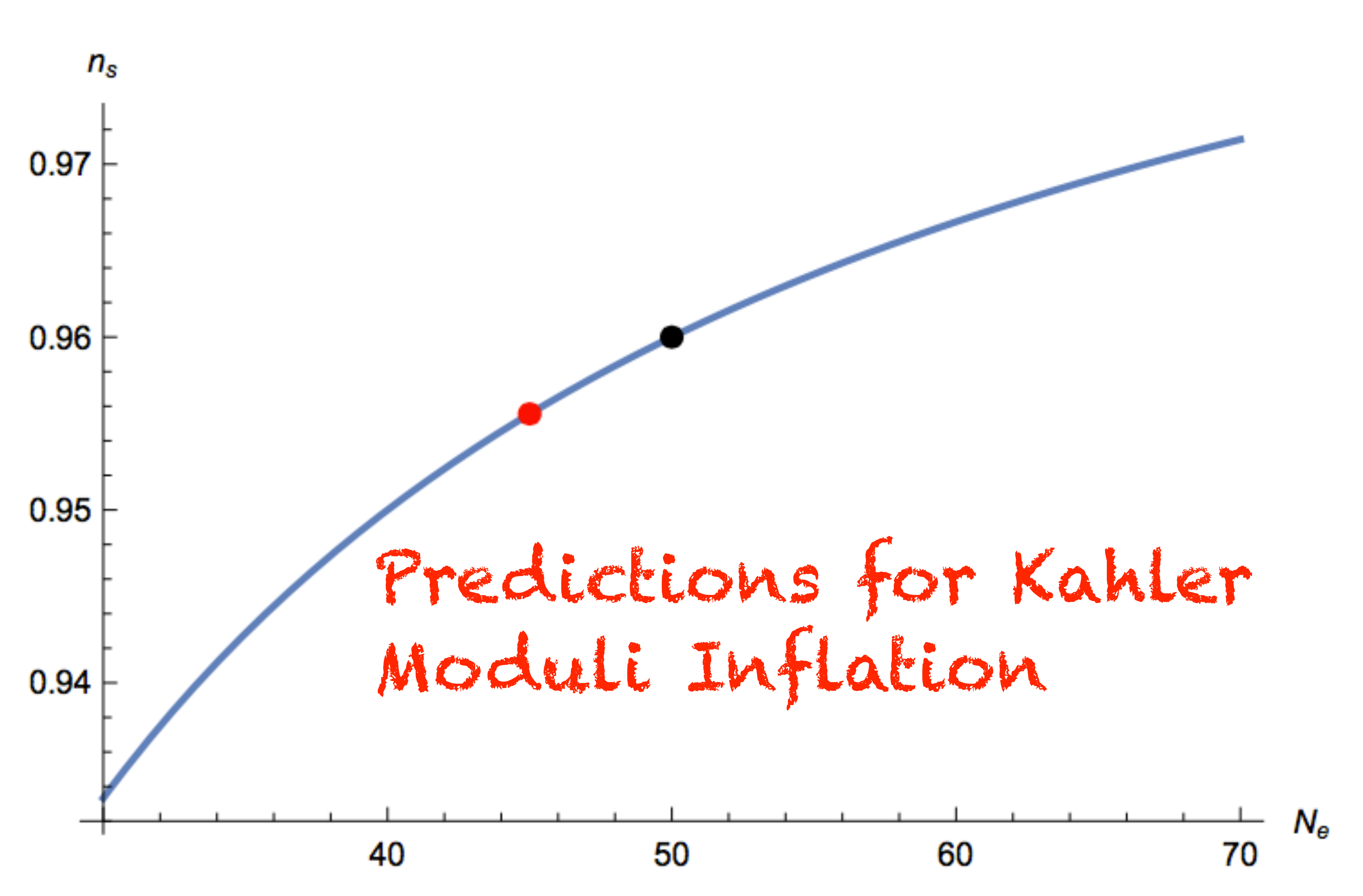
$$N_e \simeq 44.65 + \frac{1}{4} \ln\left(\frac{\rho_*}{\rho_{\rm end}}\right) \simeq 45 \qquad \Rightarrow \qquad \tau_n \simeq 27.3 \qquad \text{and} \qquad n_s \simeq 0.955$$

$$T_{\rm rh} \gtrsim 10^3 \quad GeV$$

OK with BBN bound


n_s
0.97
0.96
0.95
0.94
40
50
60
70
N_e
Predictions for Kahler
Moduli Inflation

Conclusions

- modulus dominated cosmology is a generic feature of string/sugra motivated scenario

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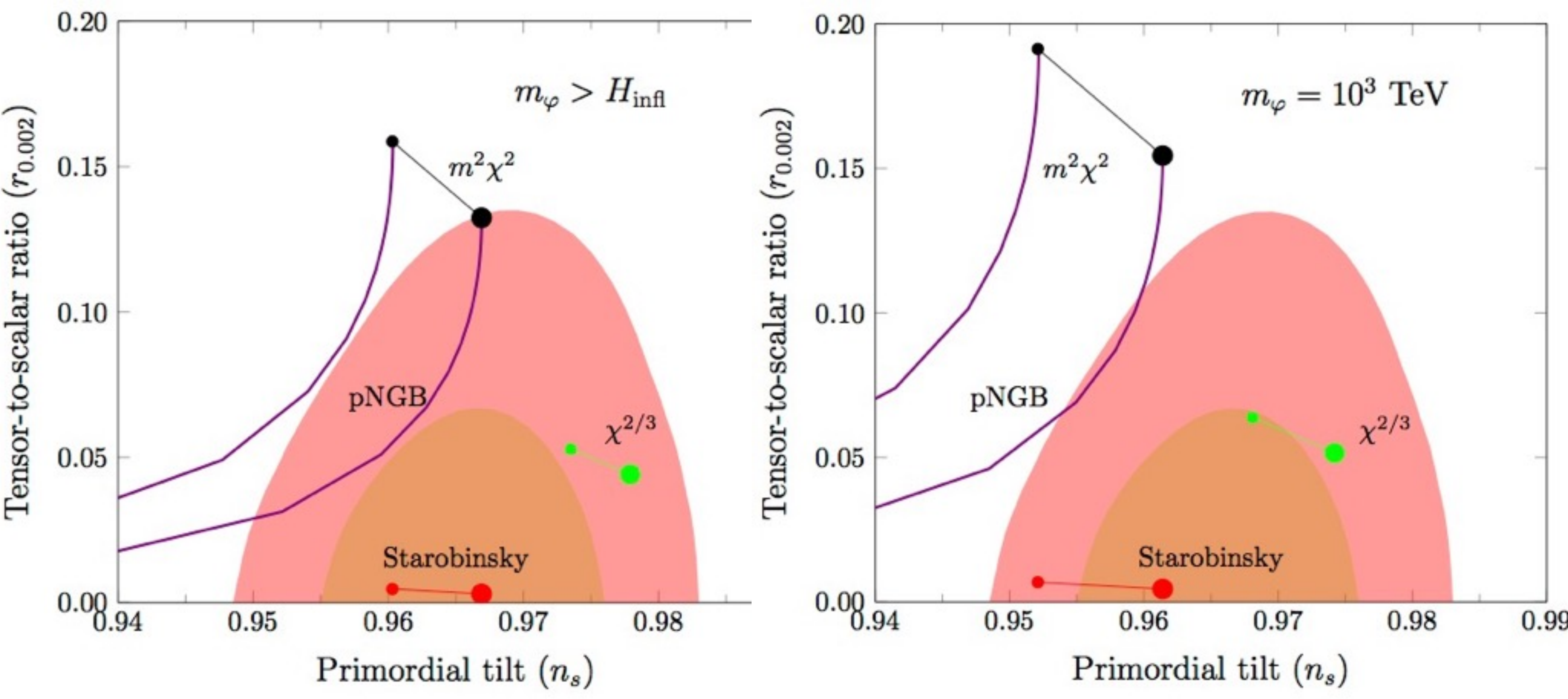
$$\frac{1}{6}ln(\frac{16\pi M_{Pl}^2 Y^4}{m_\varphi^2}) + \frac{1}{4}(1-3w_{rh1})N_{re1} + \frac{1}{4}(1-3w_{rh2})N_{re2}$$

$$= 55.43 - N_{inf} + \frac{1}{4}ln\ r + \frac{1}{4}ln(\frac{\rho_k}{\rho_{end}})$$

Conclusions

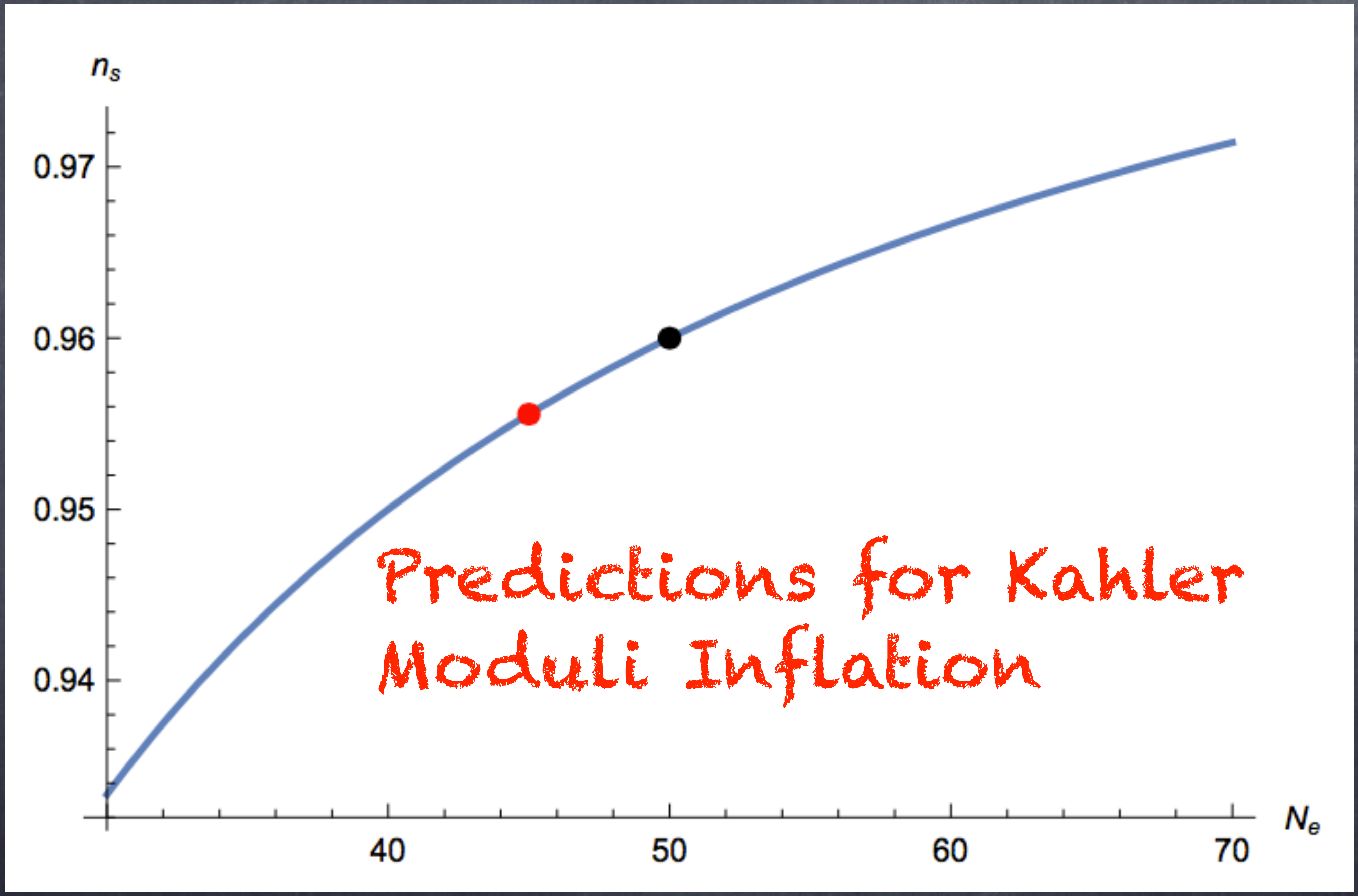
- modulus dominated cosmology is a generic feature of string/sugra motivated scenario

$$\hat{N}_{inf} = 55 - \frac{1}{3}\left(\frac{\sqrt{16\pi}M_{Pl}Y^2}{m_\varphi}\right)$$



Das, K.D, Maharana

sensitivity $n_s \sim 10^{-3}$ DECIGO/PRISM/21 cm


n_s
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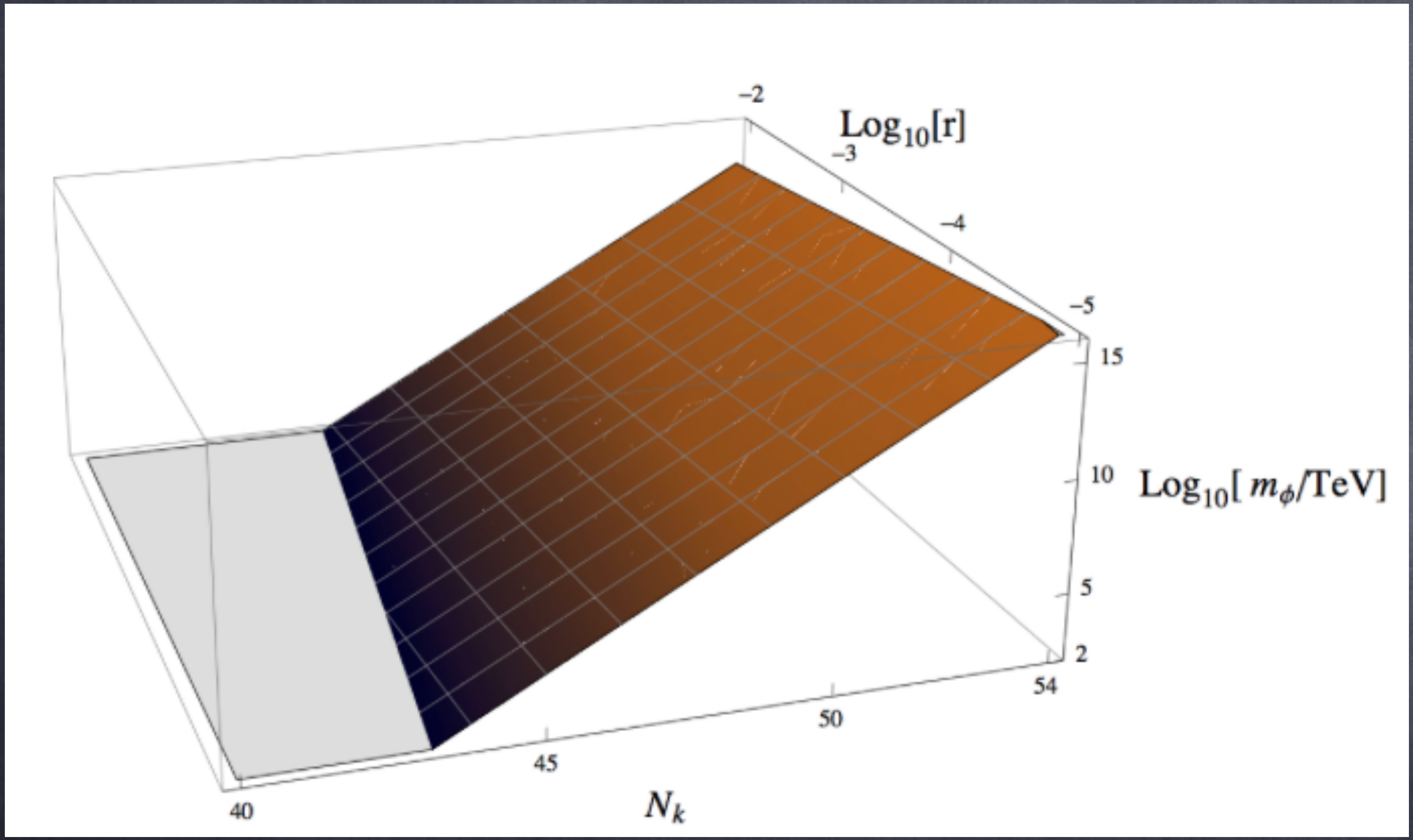
Bound..



for N > 48, the bound much stronger than BBN bound

Conclusions

- modulus dominated cosmology is a generic feature of string/sugra motivated scenario

$$\hat{N}_{inf} = 55 - \frac{1}{3}\left(\frac{\sqrt{16\pi}M_{Pl}Y^2}{m_\varphi}\right) \simeq 45$$

- Independent constraint on modulus mass derived using precision CMB data

- Explicit calculations for Kahler Moduli inflation

Thank You