

Flavor-Covariant Approach to Leptogenesis

BHUPAL DEV

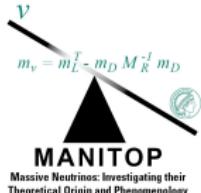
Max-Planck-Institut für Kernphysik, Heidelberg

BD, P. Millington, A. Pilaftsis and D. Teresi,
Nucl. Phys. B **886**, 569 (2014); Nucl. Phys. B **891**, 128 (2015);
Nucl. Phys. B **897**, 749 (2015); J. Phys. Conf. Ser. **631**, 012087 (2015).

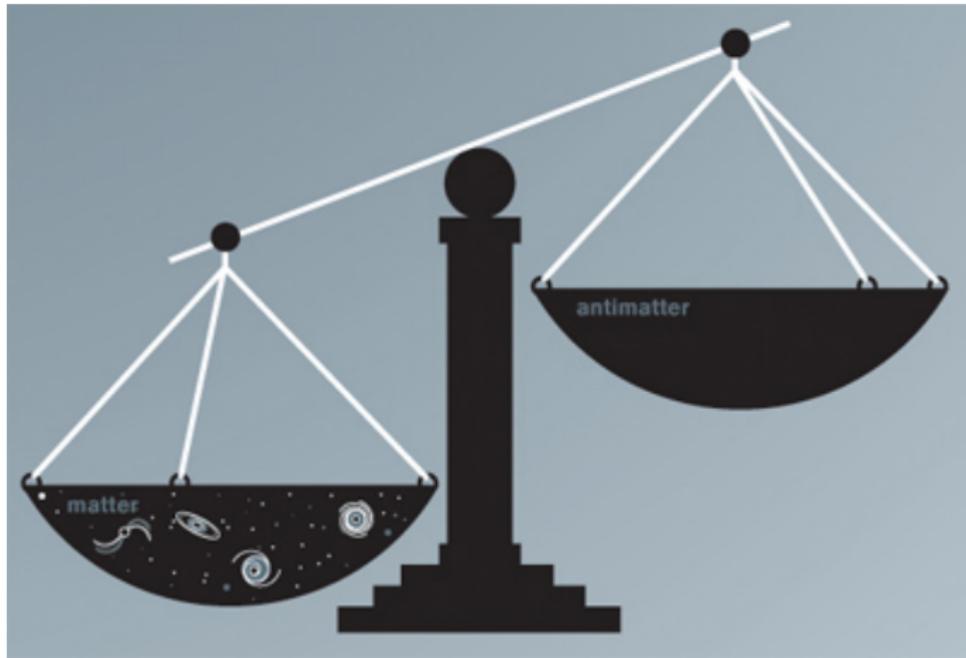
Exploring the Energy Ladder of the Universe

MITP, Johannes Gutenberg Universität, Mainz

June 3, 2016



Matter-Antimatter Asymmetry

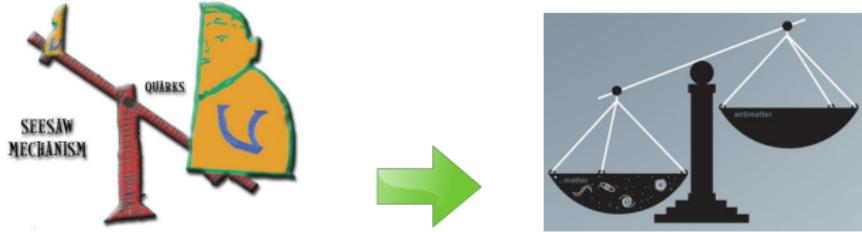


$$\eta^{\Delta B} \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.105^{+0.086}_{-0.081}) \times 10^{-10} \quad [\text{Planck (2015)}]$$

Baryogenesis

- Dynamical generation of baryon asymmetry.
- Basic **Sakharov conditions:** [Sakharov '67]
 - B violation: There must exist $X(B = 0) \rightarrow Y_1(B = 0) + Y_2(B \neq 0)$.
 - C and CP violation.
Otherwise, $\Gamma(X \rightarrow Y_1 + Y_2) = \Gamma(\bar{X} \rightarrow \bar{Y}_1 + \bar{Y}_2) \implies$ No net effect!
 - Out-of-equilibrium dynamics.
Otherwise, $\Gamma(X \rightarrow Y_1 + Y_2) = \Gamma(Y_1 + Y_2 \rightarrow X) \implies$ No net effect!
- Successful baryogenesis requires physics beyond the SM.
 - New sources of CP violation.
 - A departure from equilibrium (in addition to EWPT) or modify the EWPT itself.

Leptogenesis



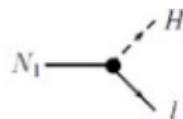
[Fukugita, Yanagida '86]

- **A cosmological consequence of the seesaw mechanism.**
- L violation due to the Majorana nature of heavy RH neutrinos.
- New source of CP violation through their complex Yukawa couplings (irrespective of the PMNS CP phases).
- Departure from equilibrium when $\Gamma_N \lesssim H$.
- $L \rightarrow \bar{B}$ through sphaleron interactions. [Kuzmin, Rubakov, Shaposhnikov '85]

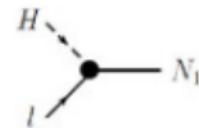
For Pedestrians

[Buchmüller, Di Bari, Plümacher '05]

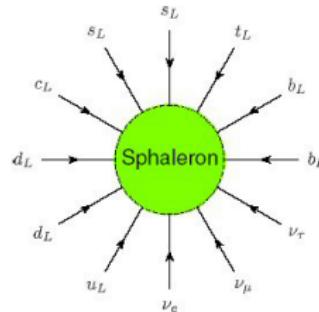
- ① Generation of L asymmetry by heavy Majorana neutrino decay:



- ② Partial washout of the asymmetry due to inverse decay (and scatterings):



- ③ Conversion of the left-over L asymmetry to B asymmetry at $T > T_{\text{sph}}$.



Boltzmann Equations

[Buchmüller, Di Bari, Plümacher '02]

$$\begin{aligned}\frac{dN_N}{dz} &= -(D + S)(N_N - N_N^{\text{eq}}), \\ \frac{dN_{B-L}}{dz} &= \varepsilon D(N_N - N_N^{\text{eq}}) - N_{B-L}W,\end{aligned}$$

(where $z = m_{N_1}/T$ and $D, S, W = \Gamma_{D,S,W}/Hz.$)

- Final baryon asymmetry:

$$\eta^{\Delta B} = d \cdot \varepsilon \cdot \kappa_f$$

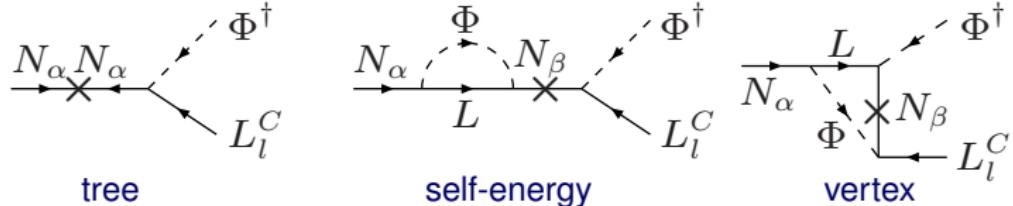
- $d \simeq 10^{-2}$ is the dilution factor.
- $\kappa_f \equiv \kappa(z_f)$ is the final efficiency factor, where

$$\kappa(z) = \int_{z_i}^z dz' \frac{D}{D + S} \frac{dn_{N_1}}{dz'} e^{-\int_{z'}^z dz'' W(z'')}$$

- Strength of (inverse-decay) washout governed by

$$K = \frac{\Gamma_D(z \rightarrow \infty)}{H(z = 1)} \equiv \frac{\tilde{m}_1}{m_*}$$

CP Asymmetry



$$\varepsilon_{l\alpha} = \frac{\Gamma(N_\alpha \rightarrow L_l \Phi) - \Gamma(N_\alpha \rightarrow L_l^c \Phi^c)}{\sum_k [\Gamma(N_\alpha \rightarrow L_k \Phi) + \Gamma(N_\alpha \rightarrow L_k^c \Phi^c)]} = \frac{|\hat{\mathbf{h}}_{l\alpha}|^2 - |\hat{\mathbf{h}}_{l\alpha}^c|^2}{(\hat{\mathbf{h}}^\dagger \hat{\mathbf{h}})_{\alpha\alpha} + (\hat{\mathbf{h}}^{c\dagger} \hat{\mathbf{h}}^c)_{\alpha\alpha}}.$$

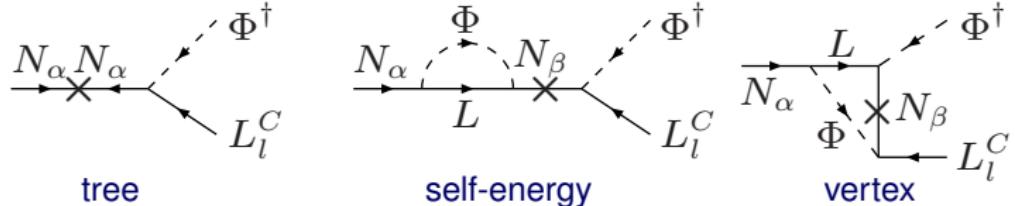
- In the two heavy-neutrino limit, self-energy term reduces to [Pilaftsis, Underwood '04]

$$\varepsilon_{l\alpha} \approx \frac{\text{Im} [(h_{l\alpha}^* h_{l\beta})(h^\dagger h)_{\alpha\beta}]}{(h^\dagger h)_{\alpha\alpha} (h^\dagger h)_{\beta\beta}} \underbrace{\frac{(m_{N_\alpha}^2 - m_{N_\beta}^2) m_{N_\alpha} \Gamma_{N_\beta}^{(0)}}{\left(m_{N_\alpha}^2 - m_{N_\beta}^2\right)^2 + \left(m_{N_\alpha} \Gamma_{N_\beta}^{(0)}\right)^2}}_{\text{regulator}}.$$

- There exist other expressions for the regulator [Buchmüller, Plümacher '98; Anisimov, Broncano, Plümacher '06; Garny, Kartavtsev, Hohenegger '11; Iso, Shimada, Yamanaka '13]

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A Simple Example

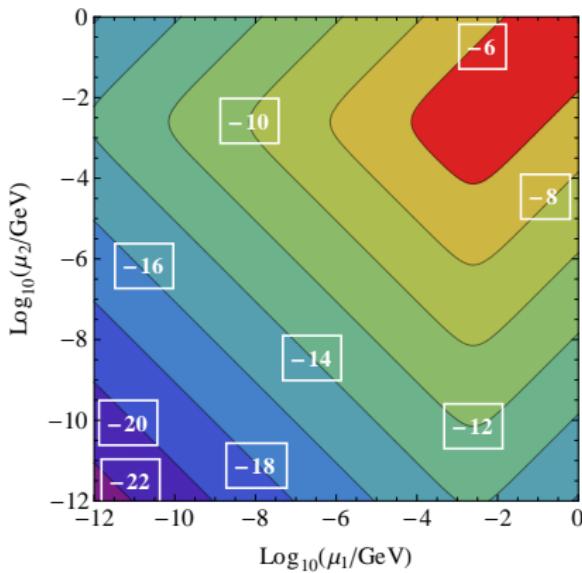
$$-\mathcal{L}_Y = Y_l \bar{L}_l \tilde{\Phi} N_1 + Y'_l \bar{L}_l \tilde{\Phi} N_2 + \frac{1}{2} \left(M \bar{N}_1 N_2^C + \mu_1 \bar{N}_1 N_1^C + \mu_2 e^{i\theta} \bar{N}_2 N_2^C \right) + \text{H.c.} .$$

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & \frac{v}{\sqrt{2}} Y & \frac{v}{\sqrt{2}} Y' \\ \frac{v}{\sqrt{2}} Y^\top & \mu_1 & M \\ \frac{v}{\sqrt{2}} Y'^\top & M & \mu_2 e^{i\theta} \end{pmatrix} .$$

$$\begin{aligned} \hat{h}_{l1} &\simeq \frac{i}{\sqrt{2}} e^{-i(\phi-\lambda)/2} \left[\left(1 + \frac{\mu_2^2 - \mu_1^2}{4M\mu} \right) e^{i\phi} Y_l - Y'_l \right], \\ \hat{h}_{l2} &\simeq \frac{1}{\sqrt{2}} e^{-i(\phi+\lambda)/2} \left[\left(1 - \frac{\mu_2^2 - \mu_1^2}{4M\mu} \right) e^{i\phi} Y_l + Y'_l \right], \end{aligned}$$

where $\mu e^{i\phi} = \mu_1 + \mu_2 e^{i\theta}$ and $\lambda = \sin \theta (\mu_1 \mu_2 / \mu M)$. [Blanchet, Hambye, Josse-Michaux '09]

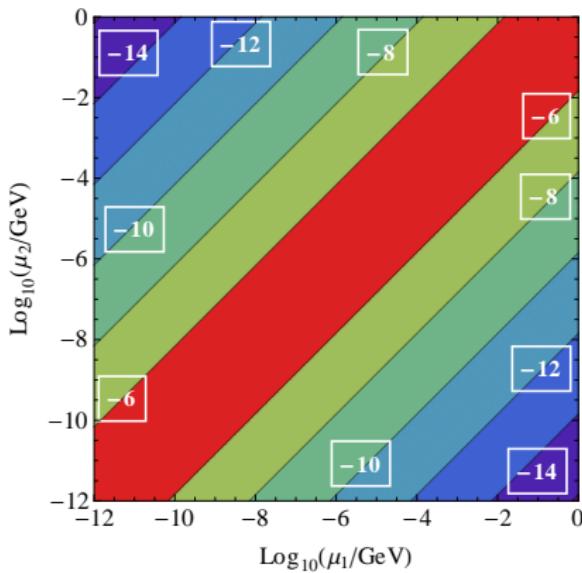
Regulator in the L -conserving limit



$$\frac{\left(m_{N_\alpha}^2 - m_{N_\beta}^2\right) m_{N_\alpha} \Gamma_{N_\beta}^{(0)}}{\left(m_{N_\alpha}^2 - m_{N_\beta}^2\right)^2 + \left(m_{N_\alpha} \Gamma_{N_\beta}^{(0)}\right)^2}$$

leads to the correct L -conserving limit. [BD, Millington, Pilaftsis, Teresi '14]

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does **not** lead to the correct L -conserving limit. [BD, Millington, Pilaftsis, Teresi '14]

Vanilla Leptogenesis

- Hierarchical heavy neutrino spectrum ($m_{N_1} \ll m_{N_2} < m_{N_3}$).
- Vertex correction diagram important.
- Maximal CP asymmetry is given by

$$\varepsilon_1^{\max} = \frac{3}{16\pi} \frac{m_{N_1}}{v^2} \sqrt{\Delta m_{\text{atm}}^2}$$

- Lower bound on m_{N_1} : [Davidson, Ibarra '02; Buchmüller, Di Bari, Plümacher '02]

$$m_{N_1} > 6.4 \times 10^8 \text{ GeV} \left(\frac{\eta^{\Delta B}}{6 \times 10^{-10}} \right) \left(\frac{0.05 \text{ eV}}{\sqrt{\Delta m_{\text{atm}}^2}} \right) \kappa_f^{-1}$$

- Experimentally inaccessible mass range!
- Higgs naturalness implies $m_N \lesssim 10^7$ GeV. [Vissani '97; Clarke, Foot, Volkas '15]
- Also leads to a lower limit on the reheat temperature $T_{\text{rh}} \gtrsim 10^9$ GeV.
- In many supergravity scenarios, need $T_{\text{rh}} \lesssim 10^6 - 10^9$ GeV to avoid the Gravitino problem. [Khlopov, Linde '84; Ellis, Kim, Nanopoulos '84; Cyburt, Ellis, Fields, Olive '02; Kawasaki, Kohri, Moroi, Yotsuyanagi '08]

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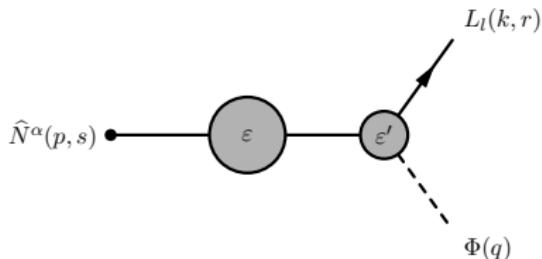
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Resonant Leptogenesis



- Heavy Majorana neutrino self-energy effects on the leptonic CP -asymmetry (ε -type) become dominant [Flanz, Paschos, Sarkar '95; Covi, Roulet, Vissani '96].
- Resonantly enhanced, even up to order 1, when $\Delta m_N \sim \Gamma_N \ll m_{N_{1,2}}$.
[Pilaftsis '97; Pilaftsis, Underwood '03]
- The quasi-degeneracy can be naturally motivated as due to approximate breaking of some symmetry in the leptonic sector.
- Heavy neutrino mass scale can be as low as the EW scale.
[Pilaftsis '04; Pilaftsis, Underwood '05]
- A potentially testable scenario of leptogenesis, with implications at both Energy and Intensity Frontiers. [Deppisch, Pilaftsis '10; BD, Millington, Pilaftsis, Teresi '14]

Flavor-diagonal Resonant Leptogenesis

$$\frac{n^\gamma H_N}{z} \frac{d\eta_\alpha^N}{dz} = \left(1 - \frac{\eta_\alpha^N}{\eta_{\text{eq}}^N}\right) \sum_l \gamma_{L_l \Phi}^{N_\alpha}$$

$$\frac{n^\gamma H_N}{z} \frac{d\delta\eta_l^L}{dz} = \sum_\alpha \left(\frac{\eta_\alpha^N}{\eta_{\text{eq}}^N} - 1 \right) \varepsilon_{l\alpha} \sum_k \gamma_{L_k \Phi}^{N_\alpha} - \frac{2}{3} \delta\eta_l^L \sum_k \left[\gamma_{L_k^c \Phi^c}^{L_l \Phi} + \gamma_{L_k \Phi}^{L_l \Phi} + \delta\eta_k^L (\gamma_{L_l^c \Phi^c}^{L_k \Phi} - \gamma_{L_l \Phi}^{L_k \Phi}) \right]$$

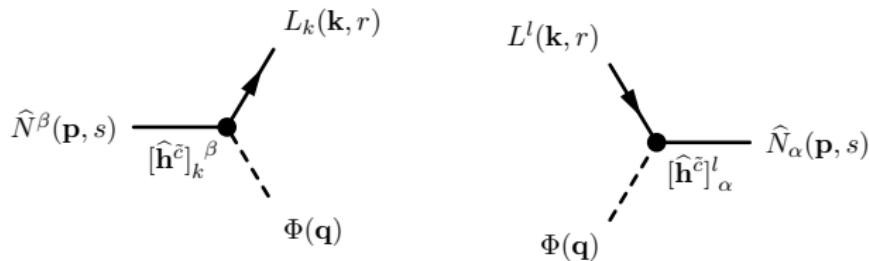


Figure : Decay and Inverse Decay.

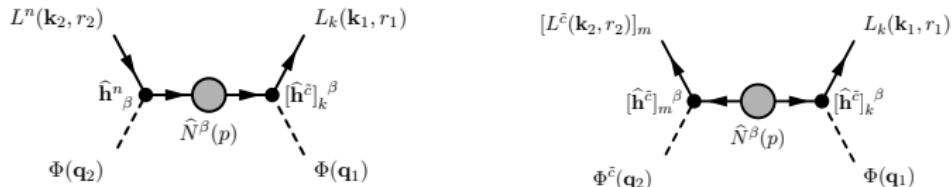
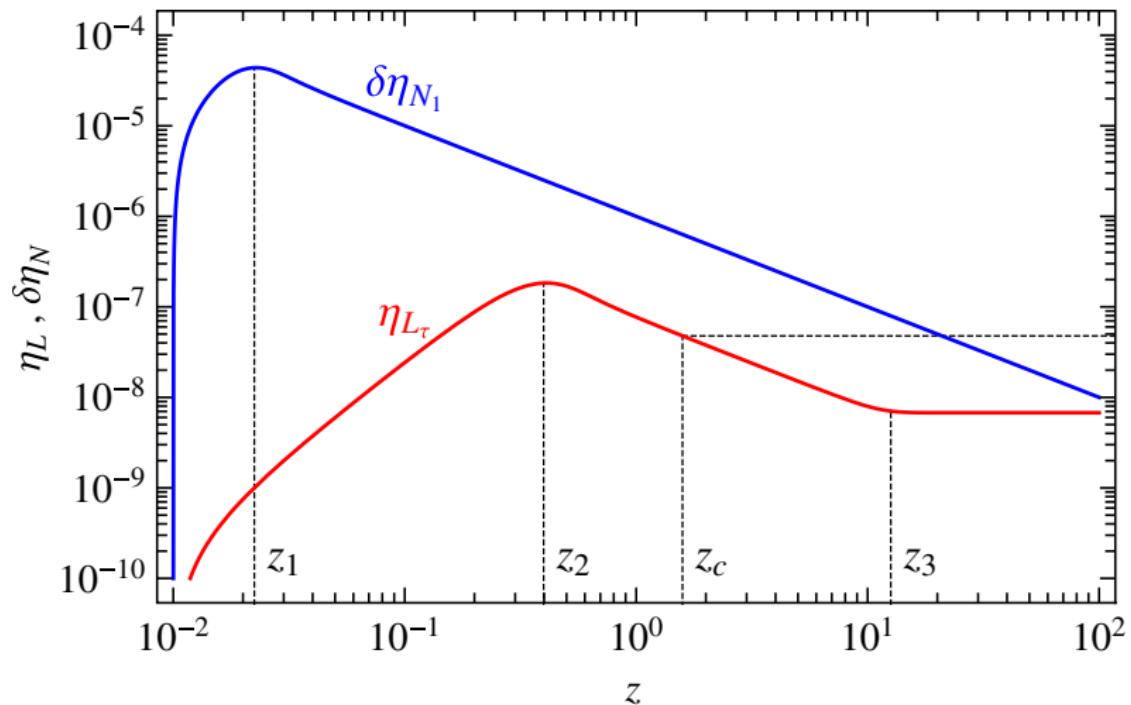


Figure : $\Delta L = 0$ and $\Delta L = 2$ scatterings.

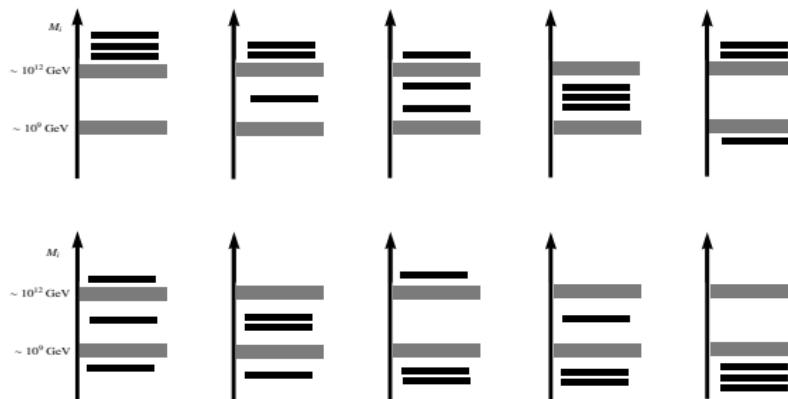
Analytic Solution



[Deppisch, Pilaftsis '11]

Flavordynamics of RL

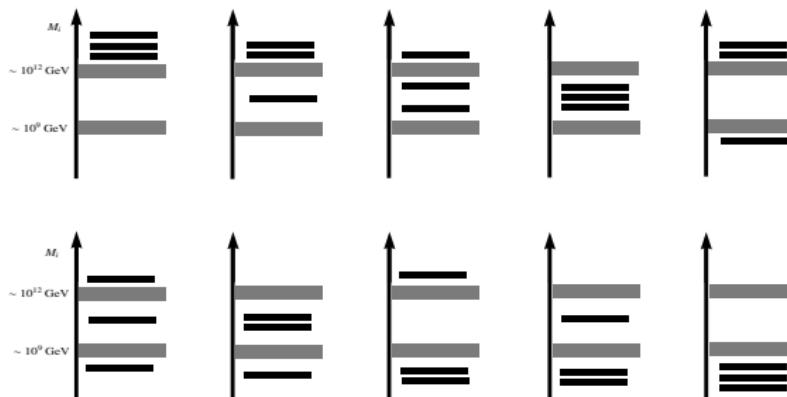
- Flavor effects important for the time-evolution of lepton asymmetry in RL models.
[Abada, Davidson, Ibarra, Josse-Michaux, Losada, Riotto '06; Nardi, Nir, Roulet, Racker '06;
Blanchet, Di Bari '06; De Simone, Riotto '06; Blanchet, Di Bari, Jones, Marzola '12]



- Two sources of flavor effects:
 - Heavy neutrino Yukawa couplings h_l^α [Pilaftsis '04; Endoh, Morozumi, Xiong '04]
 - Charged lepton Yukawa couplings y_l^k [Barbieri, Creminelli, Strumia, Tetradi '00]
- Lead to *three* distinct physical phenomena: **mixing**, **oscillation** and **decoherence**.
- In Boltzmann approach, captured by the 'density matrix' formalism. [Sigl, Raffelt '93]
- Fully* flavor-covariant formalism to consistently include all flavor effects in a single framework. [BD, Millington, Pilaftsis, Teresi '14]

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Flavor Transformations

$$-\mathcal{L}_N = h_l^\alpha \bar{L}^l \tilde{\Phi} N_{R,\alpha} + \frac{1}{2} \bar{N}_{R,\alpha}^C [M_N]^{\alpha\beta} N_{R,\beta} + \text{H.c.} .$$

- Under $U(\mathcal{N}_L) \otimes U(\mathcal{N}_N)$,

$$L_l \rightarrow L'_l = V_l^m L_m, \quad L^l \equiv (L_l)^\dagger \rightarrow L'^l = V_m^l L^m,$$

$$N_{R,\alpha} \rightarrow N'_{R,\alpha} = U_\alpha^\beta N_{R,\beta}, \quad N_R^\alpha \equiv (N_{R,\alpha})^\dagger \rightarrow N_R'^\alpha = U_\beta^\alpha N_R^\beta.$$

$$h_l^\alpha \rightarrow h'_l{}^\alpha = V_l^m U_\beta^\alpha h_m^\beta, \quad [M_N]^{\alpha\beta} \rightarrow [M'_N]^{\alpha\beta} = U_\gamma^\alpha U_\delta^\beta [M_N]^{\gamma\delta}.$$

- Number densities:

$$[n_{s_1 s_2}^L(\mathbf{p}, t)]_l^m \equiv \frac{1}{\mathcal{V}_3} \langle b^m(\mathbf{p}, s_2, \tilde{t}) b_l(\mathbf{p}, s_1, \tilde{t}) \rangle_t,$$

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- Total number density:

$$n^N(t) \equiv \sum_{r=-,+} \int_{\mathbf{k}} n_{rr}^N(\mathbf{k}, t), \quad n^L(t) \equiv \text{Tr}_{\text{iso}} \sum_{s=-,+} \int_{\mathbf{p}} n_{ss}^L(\mathbf{p}, t).$$

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Master Equation for Transport Phenomena

- In quantum statistical mechanics,

$$\mathbf{n}^X(t) \equiv \langle \check{\mathbf{n}}^X(\tilde{t}; \tilde{t}_i) \rangle_t = \text{Tr} \left\{ \rho(\tilde{t}; \tilde{t}_i) \check{\mathbf{n}}^X(\tilde{t}; \tilde{t}_i) \right\} .$$

- Differentiate w.r.t. the macroscopic time $t = \tilde{t} - \tilde{t}_i$:

$$\frac{d\mathbf{n}^X(t)}{dt} = \text{Tr} \left\{ \rho(\tilde{t}; \tilde{t}_i) \frac{d\check{\mathbf{n}}^X(\tilde{t}; \tilde{t}_i)}{d\tilde{t}} \right\} + \text{Tr} \left\{ \frac{d\rho(\tilde{t}; \tilde{t}_i)}{d\tilde{t}} \check{\mathbf{n}}^X(\tilde{t}; \tilde{t}_i) \right\} \equiv \mathcal{I}_1 + \mathcal{I}_2 .$$

- Use the Heisenberg EoM for \mathcal{I}_1 and Liouville-von Neumann equation for \mathcal{I}_2 .
- **Markovian master equation** for the number density matrix:

$$\frac{d}{dt} \mathbf{n}^X(\mathbf{k}, t) \simeq i \langle [H_0^X, \check{\mathbf{n}}^X(\mathbf{k}, t)] \rangle_t - \frac{1}{2} \int_{-\infty}^{+\infty} dt' \langle [H_{\text{int}}(t'), [H_{\text{int}}(t), \check{\mathbf{n}}^X(\mathbf{k}, t)]] \rangle_t .$$

Flavor Covariant Transport Equations for RL

Explicitly, for charged-lepton and heavy-neutrino matrix number densities,

$$\frac{d}{dt} [n_{s_1 s_2}^L(\mathbf{p}, t)]_l^m = -i [E_L(\mathbf{p}), n_{s_1 s_2}^L(\mathbf{p}, t)]_l^m + [C_{s_1 s_2}^L(\mathbf{p}, t)]_l^m$$

$$\frac{d}{dt} [n_{r_1 r_2}^N(\mathbf{k}, t)]_\alpha^\beta = -i [E_N(\mathbf{k}), n_{r_1 r_2}^N(\mathbf{k}, t)]_\alpha^\beta + [C_{r_1 r_2}^N(\mathbf{k}, t)]_\alpha^\beta + G_{\alpha \lambda} [\bar{C}_{r_2 r_1}^N(\mathbf{k}, t)]_\mu^\lambda G^{\mu \beta}$$

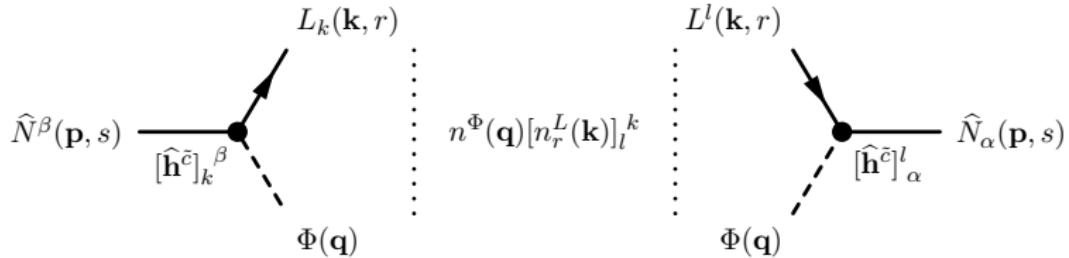
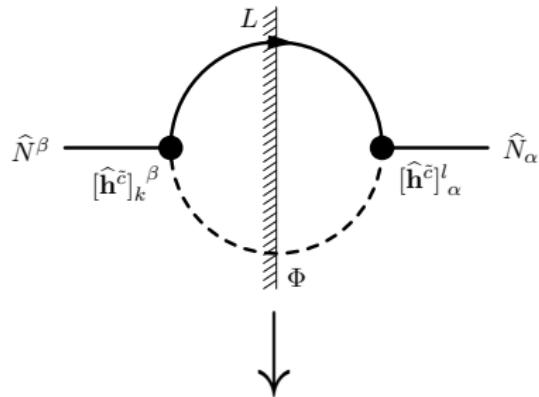
Collision terms are of the form

$$[C_{s_1 s_2}^L(\mathbf{p}, t)]_l^m \supset -\frac{1}{2} [\mathcal{F}_{s_1 s_2 r_1 r_2}(\mathbf{p}, \mathbf{q}, \mathbf{k}, t)]_l^n{}_\alpha^\beta [\Gamma_{s_1 s_2 r_1 r_2}(\mathbf{p}, \mathbf{q}, \mathbf{k})]_n{}^\mu{}_\beta^\alpha,$$

where \mathcal{F} are statistical tensors, and Γ are the rank-4 absorptive rate tensors describing heavy neutrino decays and inverse decays.

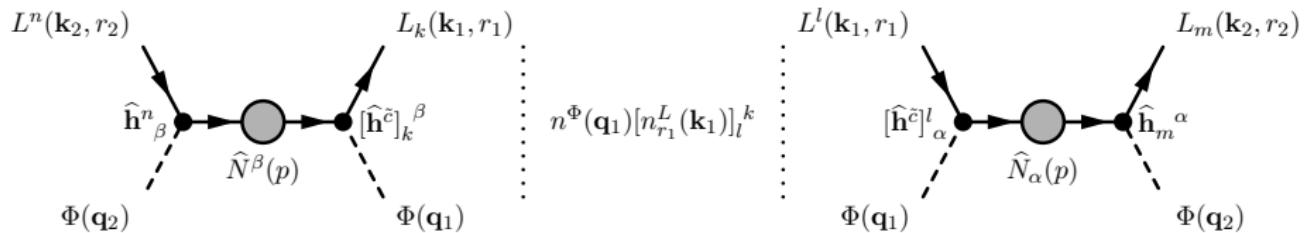
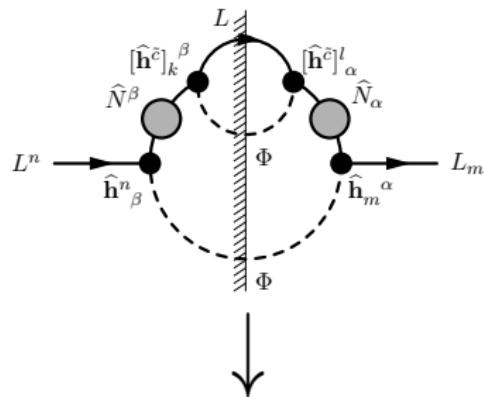
Collision Rates for Decay and Inverse Decay

$$n^\Phi [n^L]_l^k [\gamma(L\Phi \rightarrow N)]_k^l {}_\alpha^\beta$$



Collision Rates for $2 \leftrightarrow 2$ Scattering

$$n^\Phi [n^L]_l^k [\gamma(L\Phi \rightarrow L\Phi)]_{k m}^{l \ n}$$



Final Rate Equations

$$\frac{H_N n^\gamma}{z} \frac{d[\underline{\eta}^N]_\alpha^\beta}{dz} = -i \frac{n^\gamma}{2} \left[\mathcal{E}_N, \delta \eta^N \right]_\alpha^\beta + \left[\widetilde{\text{Re}}(\gamma_{L\Phi}^N) \right]_\alpha^\beta - \frac{1}{2 \eta_{\text{eq}}^N} \left\{ \underline{\eta}^N, \widetilde{\text{Re}}(\gamma_{L\Phi}^N) \right\}_\alpha^\beta$$

$$\begin{aligned} \frac{H_N n^\gamma}{z} \frac{d[\delta \eta^N]_\alpha^\beta}{dz} &= -2i n^\gamma \left[\mathcal{E}_N, \underline{\eta}^N \right]_\alpha^\beta + 2i \left[\widetilde{\text{Im}}(\delta \gamma_{L\Phi}^N) \right]_\alpha^\beta - \frac{i}{\eta_{\text{eq}}^N} \left\{ \underline{\eta}^N, \widetilde{\text{Im}}(\delta \gamma_{L\Phi}^N) \right\}_\alpha^\beta \\ &\quad - \frac{1}{2 \eta_{\text{eq}}^N} \left\{ \delta \eta^N, \widetilde{\text{Re}}(\gamma_{L\Phi}^N) \right\}_\alpha^\beta \end{aligned}$$

$$\begin{aligned} \frac{H_N n^\gamma}{z} \frac{d[\delta \eta^L]_l^m}{dz} &= -[\delta \gamma_{L\Phi}^N]_l^m + \frac{[\underline{\eta}^N]_\beta^\alpha}{\eta_{\text{eq}}^N} [\delta \gamma_{L\Phi}^N]_l^m{}_\alpha^\beta + \frac{[\delta \eta^N]_\beta^\alpha}{2 \eta_{\text{eq}}^N} [\gamma_{L\Phi}^N]_l^m{}_\alpha^\beta \\ &\quad - \frac{1}{3} \left\{ \delta \eta^L, \gamma_{L\bar{c}\Phi\bar{c}}^{L\Phi} + \gamma_{L\Phi}^{L\Phi} \right\}_l^m - \frac{2}{3} [\delta \eta^L]_k^n \left([\gamma_{L\bar{c}\Phi\bar{c}}^{L\Phi}]_n^k {}_l^m - [\gamma_{L\Phi}^{L\Phi}]_n^k {}_l^m \right) \\ &\quad - \frac{2}{3} \left\{ \delta \eta^L, \gamma_{\text{dec}} \right\}_l^m + [\delta \gamma_{\text{dec}}^{\text{back}}]_l^m \end{aligned}$$

Final Rate Equations: Mixing

$$\frac{H_N n^\gamma}{z} \frac{d[\underline{\eta}^N]_\alpha^\beta}{dz} = -i \frac{n^\gamma}{2} \left[\mathcal{E}_N, \delta \eta^N \right]_\alpha^\beta + \left[\widetilde{\text{Re}}(\gamma_{L\Phi}^N) \right]_\alpha^\beta - \frac{1}{2 \eta_{\text{eq}}^N} \left\{ \underline{\eta}^N, \widetilde{\text{Re}}(\gamma_{L\Phi}^N) \right\}_\alpha^\beta$$

$$\begin{aligned} \frac{H_N n^\gamma}{z} \frac{d[\delta \eta^N]_\alpha^\beta}{dz} &= -2i n^\gamma \left[\mathcal{E}_N, \underline{\eta}^N \right]_\alpha^\beta + 2i \left[\widetilde{\text{Im}}(\delta \gamma_{L\Phi}^N) \right]_\alpha^\beta - \frac{i}{\eta_{\text{eq}}^N} \left\{ \underline{\eta}^N, \widetilde{\text{Im}}(\delta \gamma_{L\Phi}^N) \right\}_\alpha^\beta \\ &\quad - \frac{1}{2 \eta_{\text{eq}}^N} \left\{ \delta \eta^N, \widetilde{\text{Re}}(\gamma_{L\Phi}^N) \right\}_\alpha^\beta \end{aligned}$$

$$\begin{aligned} \frac{H_N n^\gamma}{z} \frac{d[\delta \eta^L]_l^m}{dz} &= -[\delta \gamma_{L\Phi}^N]_l^m + \frac{[\underline{\eta}^N]_\beta^\alpha}{\eta_{\text{eq}}^N} [\delta \gamma_{L\Phi}^N]_l^m \delta_\alpha^\beta + \frac{[\delta \eta^N]_\beta^\alpha}{2 \eta_{\text{eq}}^N} [\gamma_{L\Phi}^N]_l^m \delta_\alpha^\beta \\ &\quad - \frac{1}{3} \left\{ \delta \eta^L, \gamma_{L\bar{c}\Phi\bar{c}}^{L\Phi} + \gamma_{L\Phi}^{L\Phi} \right\}_l^m - \frac{2}{3} [\delta \eta^L]_k^n \left([\gamma_{L\bar{c}\Phi\bar{c}}^{L\Phi}]_n^k \delta_l^m - [\gamma_{L\Phi}^{L\Phi}]_n^k \delta_l^m \right) \\ &\quad - \frac{2}{3} \left\{ \delta \eta^L, \gamma_{\text{dec}} \right\}_l^m + [\delta \gamma_{\text{dec}}^{\text{back}}]_l^m \end{aligned}$$

Final Rate Equations: Oscillation

$$\frac{H_N n^\gamma}{z} \frac{d[\underline{\eta}^N]_\alpha^\beta}{dz} = -i \frac{n^\gamma}{2} [\mathcal{E}_N, \delta\eta^N]_\alpha^\beta + [\widetilde{\text{Re}}(\gamma_{L\Phi}^N)]_\alpha^\beta - \frac{1}{2\eta_{\text{eq}}^N} \left\{ \underline{\eta}^N, \widetilde{\text{Re}}(\gamma_{L\Phi}^N) \right\}_\alpha^\beta$$

$$\begin{aligned} \frac{H_N n^\gamma}{z} \frac{d[\delta\eta^N]_\alpha^\beta}{dz} &= -2i n^\gamma [\mathcal{E}_N, \underline{\eta}^N]_\alpha^\beta + 2i [\widetilde{\text{Im}}(\delta\gamma_{L\Phi}^N)]_\alpha^\beta - \frac{i}{\eta_{\text{eq}}^N} \left\{ \underline{\eta}^N, \widetilde{\text{Im}}(\delta\gamma_{L\Phi}^N) \right\}_\alpha^\beta \\ &\quad - \frac{1}{2\eta_{\text{eq}}^N} \left\{ \delta\eta^N, \widetilde{\text{Re}}(\gamma_{L\Phi}^N) \right\}_\alpha^\beta \end{aligned}$$

$$\begin{aligned} \frac{H_N n^\gamma}{z} \frac{d[\delta\eta^L]_l^m}{dz} &= -[\delta\gamma_{L\Phi}^N]_l^m + \frac{[\underline{\eta}^N]_\beta^\alpha}{\eta_{\text{eq}}^N} [\delta\gamma_{L\Phi}^N]_l^m {}_\alpha^\beta + \frac{[\delta\eta^N]_\beta^\alpha}{2\eta_{\text{eq}}^N} [\gamma_{L\Phi}^N]_l^m {}_\alpha^\beta \\ &\quad - \frac{1}{3} \left\{ \delta\eta^L, \gamma_{L\bar{c}\Phi\bar{c}}^{L\Phi} + \gamma_{L\Phi}^{L\Phi} \right\}_l^m - \frac{2}{3} [\delta\eta^L]_k^n \left([\gamma_{L\bar{c}\Phi\bar{c}}^{L\Phi}]_n^k {}_l^m - [\gamma_{L\Phi}^{L\Phi}]_n^k {}_l^m \right) \\ &\quad - \frac{2}{3} \left\{ \delta\eta^L, \gamma_{\text{dec}} \right\}_l^m + [\delta\gamma_{\text{dec}}^{\text{back}}]_l^m \end{aligned}$$

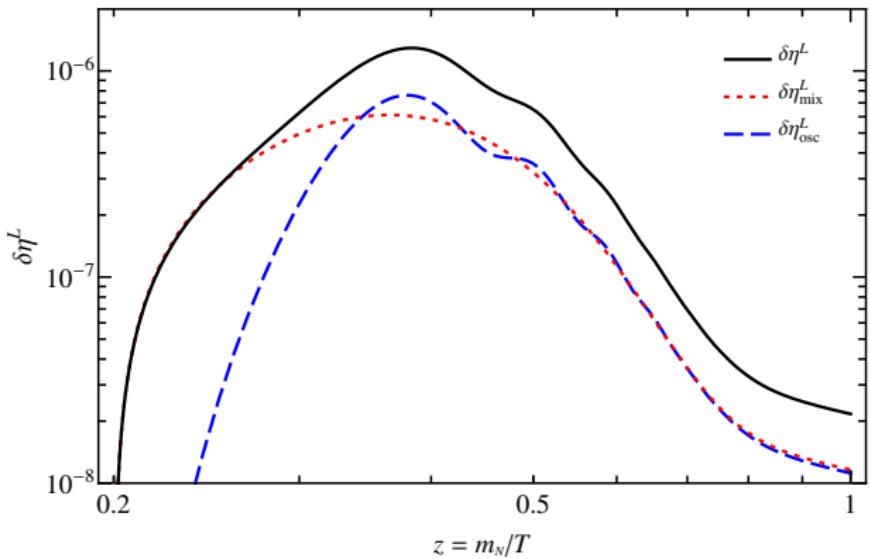
Final Rate Equations: Charged Lepton Decoherence

$$\frac{H_N n^\gamma}{z} \frac{d[\underline{\eta}^N]_\alpha^\beta}{dz} = -i \frac{n^\gamma}{2} [\mathcal{E}_N, \delta\eta^N]_\alpha^\beta + [\widetilde{\text{Re}}(\gamma_{L\Phi}^N)]_\alpha^\beta - \frac{1}{2\eta_{\text{eq}}^N} \left\{ \underline{\eta}^N, \widetilde{\text{Re}}(\gamma_{L\Phi}^N) \right\}_\alpha^\beta$$

$$\begin{aligned} \frac{H_N n^\gamma}{z} \frac{d[\delta\eta^N]_\alpha^\beta}{dz} &= -2i n^\gamma [\mathcal{E}_N, \underline{\eta}^N]_\alpha^\beta + 2i [\widetilde{\text{Im}}(\delta\gamma_{L\Phi}^N)]_\alpha^\beta - \frac{i}{\eta_{\text{eq}}^N} \left\{ \underline{\eta}^N, \widetilde{\text{Im}}(\delta\gamma_{L\Phi}^N) \right\}_\alpha^\beta \\ &\quad - \frac{1}{2\eta_{\text{eq}}^N} \left\{ \delta\eta^N, \widetilde{\text{Re}}(\gamma_{L\Phi}^N) \right\}_\alpha^\beta \end{aligned}$$

$$\begin{aligned} \frac{H_N n^\gamma}{z} \frac{d[\delta\eta^L]_l^m}{dz} &= -[\delta\gamma_{L\Phi}^N]_l^m + \frac{[\underline{\eta}^N]_\beta^\alpha}{\eta_{\text{eq}}^N} [\delta\gamma_{L\Phi}^N]_l^m {}_\alpha^\beta + \frac{[\delta\eta^N]_\beta^\alpha}{2\eta_{\text{eq}}^N} [\gamma_{L\Phi}^N]_l^m {}_\alpha^\beta \\ &\quad - \frac{1}{3} \left\{ \delta\eta^L, \gamma_{L\bar{c}\Phi\bar{c}}^{L\Phi} + \gamma_{L\Phi}^{L\Phi} \right\}_l^m - \frac{2}{3} [\delta\eta^L]_k^n \left([\gamma_{L\bar{c}\Phi\bar{c}}^{L\Phi}]_n^k {}_l^m - [\gamma_{L\Phi}^{L\Phi}]_n^k {}_l^m \right) \\ &\quad - \frac{2}{3} \left\{ \delta\eta^L, \gamma_{\text{dec}} \right\}_l^m + [\delta\gamma_{\text{dec}}^{\text{back}}]_l^m \end{aligned}$$

Key Result



$$\delta\eta_{\text{mix}}^L \simeq \frac{g_N}{2} \frac{3}{2Kz} \sum_{\alpha \neq \beta} \frac{\Im(\hat{h}^\dagger \hat{h})_{\alpha\beta}^2}{(\hat{h}^\dagger \hat{h})_{\alpha\alpha} (\hat{h}^\dagger \hat{h})_{\beta\beta}} \frac{(M_{N,\alpha}^2 - M_{N,\beta}^2) M_N \widehat{\Gamma}_{\beta\beta}^{(0)}}{(M_{N,\alpha}^2 - M_{N,\beta}^2)^2 + (M_N \widehat{\Gamma}_{\beta\beta}^{(0)})^2},$$

$$\delta\eta_{\text{osc}}^L \simeq \frac{g_N}{2} \frac{3}{2Kz} \sum_{\alpha \neq \beta} \frac{\Im(\hat{h}^\dagger \hat{h})_{\alpha\beta}^2}{(\hat{h}^\dagger \hat{h})_{\alpha\alpha} (\hat{h}^\dagger \hat{h})_{\beta\beta}} \frac{(M_{N,\alpha}^2 - M_{N,\beta}^2) M_N (\widehat{\Gamma}_{\alpha\alpha}^{(0)} + \widehat{\Gamma}_{\beta\beta}^{(0)})}{(M_{N,\alpha}^2 - M_{N,\beta}^2)^2 + M_N^2 (\widehat{\Gamma}_{\alpha\alpha}^{(0)} + \widehat{\Gamma}_{\beta\beta}^{(0)})^2} \frac{\Im[(\hat{h}^\dagger \hat{h})_{\alpha\beta}]^2}{(\hat{h}^\dagger \hat{h})_{\alpha\alpha} (\hat{h}^\dagger \hat{h})_{\beta\beta}}$$

A Minimal Model of RL

- Resonant ℓ -genesis (RL_ℓ). [Pilaftsis (PRL '04); Deppisch, Pilaftsis '10]
- Minimal model: $O(N)$ -symmetric heavy neutrino sector at a high scale μ_X .
- Small mass splitting at low scale from RG effects.

$$\boldsymbol{M}_N = m_N \mathbf{1} + \Delta \boldsymbol{M}_N^{\text{RG}}, \quad \text{with} \quad \Delta \boldsymbol{M}_N^{\text{RG}} = -\frac{m_N}{8\pi^2} \ln\left(\frac{\mu_X}{m_N}\right) \text{Re} \left[\boldsymbol{h}^\dagger(\mu_X) \boldsymbol{h}(\mu_X) \right].$$

- An example of RL_τ with $U(1)_{L_e+L_\mu} \times U(1)_{L_\tau}$ flavor symmetry:

$$\boldsymbol{h} = \begin{pmatrix} 0 & ae^{-i\pi/4} & ae^{i\pi/4} \\ 0 & be^{-i\pi/4} & be^{i\pi/4} \\ 0 & 0 & 0 \end{pmatrix} + \delta \boldsymbol{h},$$

$$\delta \boldsymbol{h} = \begin{pmatrix} \epsilon_e & 0 & 0 \\ \epsilon_\mu & 0 & 0 \\ \epsilon_\tau & \kappa_1 e^{-i(\pi/4-\gamma_1)} & \kappa_2 e^{i(\pi/4-\gamma_2)} \end{pmatrix},$$

A Next-to-minimal RL_ℓ Model

[BD, Millington, Pilaftsis, Teresi '15]

- Asymmetry vanishes at $\mathcal{O}(h^4)$ in minimal RL_ℓ .
- Add an additional flavor-breaking ΔM_N :

$$M_N = m_N \mathbf{1} + \Delta M_N + \Delta M_N^{\text{RG}}, \quad \text{with } \Delta M_N = \begin{pmatrix} \Delta M_1 & 0 & 0 \\ 0 & \Delta M_2/2 & 0 \\ 0 & 0 & -\Delta M_2/2 \end{pmatrix},$$

$$\mathbf{h} = \begin{pmatrix} 0 & a e^{-i\pi/4} & a e^{i\pi/4} \\ 0 & b e^{-i\pi/4} & b e^{i\pi/4} \\ 0 & c e^{-i\pi/4} & c e^{i\pi/4} \end{pmatrix} + \begin{pmatrix} \epsilon_e & 0 & 0 \\ \epsilon_\mu & 0 & 0 \\ \epsilon_\tau & 0 & 0 \end{pmatrix}.$$

- Light neutrino mass constraint:

$$M_\nu \simeq -\frac{v^2}{2} \mathbf{h} M_N^{-1} \mathbf{h}^\top \simeq \frac{v^2}{2m_N} \begin{pmatrix} \frac{\Delta m_N}{m_N} a^2 - \epsilon_e^2 & \frac{\Delta m_N}{m_N} ab - \epsilon_e \epsilon_\mu & -\epsilon_e \epsilon_\tau \\ \frac{\Delta m_N}{m_N} ab - \epsilon_e \epsilon_\mu & \frac{\Delta m_N}{m_N} b^2 - \epsilon_\mu^2 & -\epsilon_\mu \epsilon_\tau \\ -\epsilon_e \epsilon_\tau & -\epsilon_\mu \epsilon_\tau & -\epsilon_\tau^2 \end{pmatrix},$$

where

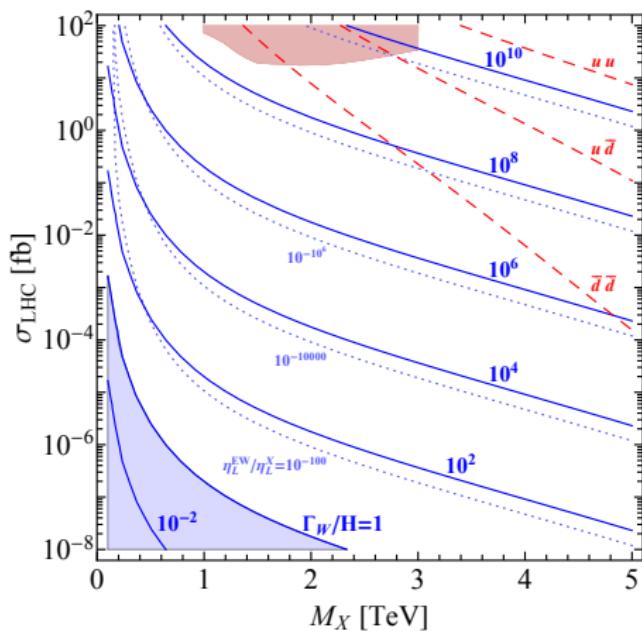
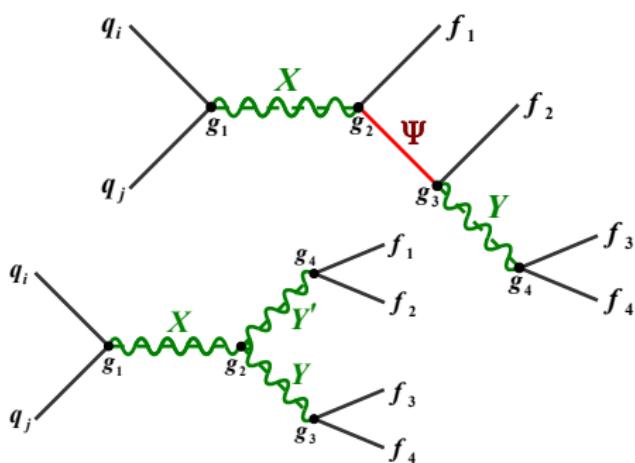
$$\Delta m_N \equiv 2 [\Delta M_N]_{23} + i ([\Delta M_N]_{33} - [\Delta M_N]_{22}) = -i \Delta M_2.$$

Benchmark Points

Parameters	BP1	BP2	BP3
m_N	120 GeV	400 GeV	5 TeV
c	2×10^{-6}	2×10^{-7}	2×10^{-6}
$\Delta M_1/m_N$	-5×10^{-6}	-3×10^{-5}	-4×10^{-5}
$\Delta M_2/m_N$	$(-1.59 - 0.47i) \times 10^{-8}$	$(-1.21 + 0.10i) \times 10^{-9}$	$(-1.46 + 0.11i) \times 10^{-8}$
a	$(5.54 - 7.41i) \times 10^{-4}$	$(4.93 - 2.32i) \times 10^{-3}$	$(4.67 - 4.33i) \times 10^{-3}$
b	$(0.89 - 1.19i) \times 10^{-3}$	$(8.04 - 3.79i) \times 10^{-3}$	$(7.53 - 6.97i) \times 10^{-3}$
ϵ_e	$3.31i \times 10^{-8}$	$5.73i \times 10^{-8}$	$2.14i \times 10^{-7}$
ϵ_μ	$2.33i \times 10^{-7}$	$4.30i \times 10^{-7}$	$1.50i \times 10^{-6}$
ϵ_τ	$3.50i \times 10^{-7}$	$6.39i \times 10^{-7}$	$2.26i \times 10^{-6}$

Observables	BP1	BP2	BP3	Current Limit
$\text{BR}(\mu \rightarrow e\gamma)$	4.5×10^{-15}	1.9×10^{-13}	2.3×10^{-17}	$< 4.2 \times 10^{-13}$
$\text{BR}(\tau \rightarrow \mu\gamma)$	1.2×10^{-17}	1.6×10^{-18}	8.1×10^{-22}	$< 4.4 \times 10^{-8}$
$\text{BR}(\tau \rightarrow e\gamma)$	4.6×10^{-18}	5.9×10^{-19}	3.1×10^{-22}	$< 3.3 \times 10^{-8}$
$\text{BR}(\mu \rightarrow 3e)$	1.5×10^{-16}	9.3×10^{-15}	4.9×10^{-18}	$< 1.0 \times 10^{-12}$
$R_{\mu \rightarrow e}^{\text{Ti}}$	2.4×10^{-14}	2.9×10^{-13}	2.3×10^{-20}	$< 6.1 \times 10^{-13}$
$R_{\mu \rightarrow e}^{\text{Au}}$	3.1×10^{-14}	3.2×10^{-13}	5.0×10^{-18}	$< 7.0 \times 10^{-13}$
$R_{\mu \rightarrow e}^{\text{Pb}}$	2.3×10^{-14}	2.2×10^{-13}	4.3×10^{-18}	$< 4.6 \times 10^{-11}$
$ \Omega _{e\mu}$	5.8×10^{-6}	1.8×10^{-5}	1.6×10^{-7}	$< 7.0 \times 10^{-5}$

Falsifying (High-scale) Leptogenesis at the LHC

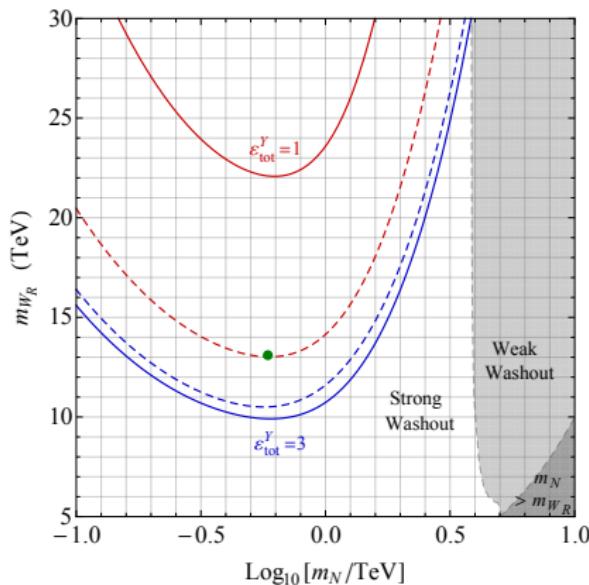


[Deppisch, Harz, Hirsch (PRL '14)]

Falsifying (Low-scale) Leptogenesis?

- One example: Left-Right Symmetric Model. [Pati, Salam '74; Mohapatra, Pati '75; Senjanović, Mohapatra 75]
- Common lore: $M_{W_R} > 18$ TeV for leptogenesis. [Frere, Hambye, Vertongen '09]
- Mainly due to additional $\Delta L = 1$ washout effects induced by W_R .

- True only with generic $Y_N \lesssim 10^{-11/2}$.
- Somewhat weaker in a class of low-scale LRSM with larger Y_N . [BD, Lee, Mohapatra '13]
- A lower limit of $M_{W_R} \gtrsim 10$ TeV.
- **A Discovery of M_{W_R} at the LHC rules out leptogenesis in LRSM.** [BD, Lee, Mohapatra '14, '15; Dhuria, Hati, Rangarajan, Sarkar '15]



Conclusion and Outlook

- Leptogenesis provides an attractive link between neutrino mass and baryon asymmetry.
- Resonant Leptogenesis provides a way to test this idea in laboratory experiments.
- Flavor effects play a crucial role in the calculation of lepton asymmetry.
- We discussed a flavor-covariant formalism to consistently capture all flavor effects.
- Approximate analytic solutions are available for a quick phenomenological analysis.
- Possible future directions/unsettled issues:
 - A better understanding of the interplay between mixing and oscillation effects.
 - Model building: Motivating the quasi-degeneracy from flavor symmetry.
 - Dependence of lepton asymmetry on Dirac and Majorana CP phases.
 - Possible correlations of the lepton asymmetry with low-energy observables, such as LFV, EDM and $0\nu\beta\beta$ to enhance its testability.

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