# Hidden flavor symmetries of SO(10) GUT

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*BB, A. Smirnov, 1605.07955*

# SO(10) Grand Unification

I will limit ourselves to supersymmetric SO(10) GUTs.

The Yukawa sector is generally given by

$$W_Y = 16_F^T \left( Y_{10} 10_H + Y_{126} 126_H + Y_{120} 120_H \right) 16_F$$

$${Y_{10,126}}^T = Y_{10,126} \quad , \quad {Y_{120}}^T = -Y_{120}$$

Why there are $N_g = 3$ gauge-equivalent generations? This calls for a flavor symmetry.

Can such a model have some flavor symmetry?

Flavor symmetry can be imposed from the beginning

*Aulakh, Khosa, '13*

*Björkeroth, de Anda, de Medeiros Varzielas, King, 15*

Here we will go the other way: take some existing fit and see if a flavor symmetry is hidden there. A real symmetry not expected, but maybe something less demanding → residual symmetry

*Lam,'07,'08,'09*

*Grimus, Lavoura, Ludl, '09*

*Grimus, Lavoura, '09*

# Residual symmetries

Look for some flavor structure in Yukawas

Start with just 2 Yukawa matrices: $Y_{10}, Y_{126}$, both symmetric:

$$16_F^T Y_{10} 16_F 10_H + 16_F^T Y_{126} 16_F 126_H$$

A real symmetry would mean $16_F \to U 16_F$:

$$\begin{aligned} U^T Y_{10} U &= Y_{10} \\ U^T Y_{126} U &= Y_{126} \end{aligned}$$

Nothing like that seems to exist (at least not for only 2 Yukawas)

Maybe less demanding: maybe $Y_{10}$ and $Y_{126}$ (i.e. their terms in the superpotential) have different symmetries.

For sure they have some:

$$Y^T_{10,126} = Y_{10,126}$$

From here

$$s_i^{d\,T} Y_{10}^d s_i^d = Y_{10}^d \qquad s_j^{d\,T} Y_{126}^d s_j^d = Y_{126}^d$$

$$s_i^d = s_1^d, s_2^d, s_3^d \quad , \quad (s_i^d)_{ab} = 2\delta_{ai}\delta_{bi} - \delta_{ab}$$

$$\begin{aligned} s_1^d &= \mathrm{diag}(+1,-1,-1) \\ s_2^d &= \mathrm{diag}(-1,+1,-1) \\ s_3^d &= \mathrm{diag}(-1,-1,+1) \end{aligned}$$

With unity this is the Klein group $Z_2 \times Z_2$

Of course $Y_{10}$ and $Y_{126}$ cannot be diagonal in the same basis.

$$\to Y_{10}^d \quad , \quad Y_{126} = U^* Y_{126}^d U^\dagger$$

So for sure

$$\begin{aligned} {s_i^d}^T Y_{10}^d s_i^d &= Y_{10}^d \\ \left(U s_j^d U^\dagger\right)^T \left(U^* Y_{126}^d U^\dagger\right) \left(U s_j^d U^\dagger\right) &= Y_{126} \end{aligned}$$

So far this trivial, always true

Each sector has its own symmetry:

$$\begin{aligned} \text{symmetry of } 16_F^T Y_{10} 16_F &: \quad (Z_2 \times Z_2)_{10} = \{\mathbb{I}, s_1^d, s_2^d, s_3^d\} \\ \text{symmetry of } 16_F^T Y_{126} 16_F &: \quad (Z_2 \times Z_2)_{126} = U\{\mathbb{I}, s_1^d, s_2^d, s_3^d\} U^\dagger \end{aligned}$$

*Lam, '14*

A bit less trivial: what if we check if 1 symmetry from $Y_{10}$ and 1 symmetry from $Y_{126}$ are remnants of an original bigger symmetry?

*Lam, '14*

Assumption: this symmetry will be discrete and finite.

$$\begin{aligned} {s_i^d}^T Y_{10}^T s_i^d &= Y_{10}^d \\ s_j^T Y_{126} s_j &= Y_{126} \quad , \quad s_j = U s_j^d U^\dagger \end{aligned}$$

$$\to s_i^d, s_j \in G \to \left(s_j s_i^d\right)^p = \mathbb{I}$$

*Hernandez, Smirnov, '12*

We will see that such an assumption gives a constraint on $U$.

Or, the opposite: if $U$ satisfies a specific constraint $\to s_i^d$, $s_j$ compatible to originate from some finite discrete group.

Define

$$W_{ij} = s_i s_j^d = U s_i^d U^\dagger s_j^d \qquad\qquad i,j \text{ fixed}$$

Then

$$W_{ij}^\dagger = s_j^d U s_i^d U^\dagger = s_j^d W_{ij} s_j^d \qquad\qquad (s_j^d)^2 = \mathbb{I}$$

This means that

$$\begin{aligned} Tr\left(W_{ij}^\dagger\right) &= Tr\left(W_{ij}\right) \\ Tr\left(W_{ij}^\dagger\right)^2 &= Tr\left(W_{ij}\right)^2 \\ Tr\left(W_{ij}^\dagger\right)^3 &= Tr\left(W_{ij}\right)^3 \end{aligned}$$

$W_{ij}$ and $W_{ij}^\dagger$ have all eigenvalues equal

Define

$$Tr\,(W_{ij}) = a_p \qquad W_{ij}^p = \mathbb{I}$$

Then

$$a_p = 1 + \lambda + \lambda^* \qquad \text{with} \;\; \lambda = e^{i2\pi k/p} \;\; (\lambda^p = 1)$$

Finally

$$a_p = 4\cos^2(\pi k/p) - 1 \qquad 1 \leq k \leq p/2$$

On the other side one can calculate $a_p$ directly from the definition:

$$a_p = Tr\left(W_{ij}\right) = Tr\left(U s_i^d U^\dagger s_j^d\right)$$

From

$$\left(s_i^d\right)_{ab} = 2\delta_{ai}\delta_{bi} - \delta_{ab}$$

immediately follows

$$a_p = 4\left|U_{ji}\right|^2 - 1$$

i.e.

$$\left|U_{ji}\right| = \left|\cos\left(\pi k/p\right)\right|$$

*Hernandez, Smirnov, '12*

Two questions:

- any $k/p$ is ok?
- at which energy is this valid?

1) any $k/p$ is ok?

For any exp. value of $U_{ji}$ we could always find a ratio of integers $k/p$ which gives a close enough $\cos(\pi k/p)$.

We will not be interested in *any* such ratio, but only $p$ small enough.

2) at which energy is

$$|U_{ji}| = |\cos(\pi k/p)|$$

valid?

Here the status of supersymmetric SO(10) is particular.

Due to SO(10) the symmetricity of the Yukawas is built in, and all matter of a single generation live in the same multiplet.

Due to supersymmetry, there is only wave-function renormalization:

$$(Y_{10}^{ren})_{ij} = (Z_{16})_{ii'}\,(Z_{16})_{jj'}\,Z_{10}\,(Y_{10})_{i'j'}$$
$$(Y_{126}^{ren})_{ij} = (Z_{16})_{ii'}\,(Z_{16})_{jj'}\,Z_{126}\,(Y_{126})_{i'j'}$$

Up to differences in $Z_{10}$ and $Z_{126}$ renormalization of $Y_{10}$ and $Y_{126}$ the same. But $Z_{10}$ and $Z_{126}$ are over-all renormalizations and do not affect the flavor structure: they change the eigenvalues but not the eigenvectors of Yukawas. Thus they do not influence the relative mixing matrix $U$.

Conclusion: in supersymmetric SO(10) if there is a residual symmetry relation at one scale higher or equal to SO(10) breaking and supersymmetry breaking, it is true at any higher scale.

## $Det(s) = -1$

We assumed so far that $Det(s) = 1$ (having in mind that the original discrete flavor symmetry group $G$ is a subgroup of SU(3) . But this in principle not necessary.

If we consider also the possibility $Det(s) = -1$:

$$|U_{ji}| = |\sin(\pi k/p)|$$

## More on minimal susy SO(10)

It has the particle content: $3 \times 16_F$ , $210_H$ , $10_H$ , $126_H$ , $\overline{126}_H$

*Aulakh, Mohapatra, '82 ; Clark, Kuo, Nakagawa, '82*

*Babu, Mohapatra,'92 ; Aulakh, BB, Melfo, Senjanović, Vissani, '03*

$$W_{Yukawa} = 16_F^T \left( Y_{10} 10_H + Y_{126} 126_H \right) 16_F$$

$$Y_{10,126}{}^T = Y_{10,126}$$

By SU(3) rotation of 3 generations of $16_F$'s always possible to real-diagonalize $Y_{10}$ (3 real parameters)

$Y_{126}$ has then $2 \times 6 = 12$ real parameters

MSSM Higgses live in both $10_H$ and $126_H$. Under $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{SU}(4)_C$

$$\begin{aligned} 10_H &= (2,2,1) + \ldots \\ 126_H &= (2,2,15) + \ldots \end{aligned}$$

$$\langle 10_H \rangle = \begin{pmatrix} v_{10}^d & 0 \\ 0 & v_{10}^u \end{pmatrix}_{SU(2)_L\times SU(2)_R} \bigotimes \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}_{SU(4)_C}$$

$$\langle 126_H \rangle = \begin{pmatrix} v_{126}^d & 0 \\ 0 & v_{126}^u \end{pmatrix}_{SU(2)_L\times SU(2)_R} \bigotimes \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}_{SU(4)_C}$$

On top of that extra vevs:

$$\langle 126_H \rangle = \ldots + \underbrace{\langle (1,3,10) \rangle}_{v_R} + \underbrace{\langle (3,1,\overline{10}) \rangle}_{v_L}$$

$$\begin{aligned} W_{Yukawa} &= u^T \left( v_{10}^u Y_{10} + v_{126}^u Y_{126} \right) u^c \\ &+ d^T \left( v_{10}^d Y_{10} + v_{126}^d Y_{126} \right) d^c \\ &+ e^T \left( v_{10}^d Y_{10} - 3 v_{126}^d Y_{126} \right) e^c \\ &+ \nu^T \left( v_{10}^u Y_{10} - 3 v_{126}^u Y_{126} \right) \nu^c \\ &+ \nu^{cT} \left( v_R Y_{126} \right) \nu^c + \nu^T \left( v_L Y_{126} \right) \nu \end{aligned}$$

All together

$$\begin{aligned} M_U &= v_{10}^u Y_{10} + v_{126}^u Y_{126} \\ M_D &= v_{10}^d Y_{10} + v_{126}^d Y_{126} \\ M_E &= v_{10}^d Y_{10} - 3 v_{126}^d Y_{126} \\ M_{\nu_D} &= v_{10}^u Y_{10} - 3 v_{126}^u Y_{126} \end{aligned}$$

neutrino mass from both type I and II contributions:

$$M_N = -M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu_D} + M_{\nu_L}$$

$$\begin{aligned} M_{\nu_L} &= v_L Y_{126} \\ M_{\nu_R} &= v_R Y_{126} \end{aligned}$$

The Higgs sector:

$$W_{Higgs} = m_{210} 210_H^3 + \lambda 210_H^3 + \overline{126}_H \left( m_{126} + \eta 210_H \right) 126_H + m_{10} 10_H^2 + \alpha 10_H 210_H 126_H + \bar{\alpha} 10_H 210_H \overline{126}_H$$

10 real parameters: 7 complex superpotential parameters but 4 phases unphysical (redefine 4 fields $10_H$, $210_H$, $126_H$, $\overline{126}_H$)

Also previous parameters ($v_{R,L}$, $v^{u,d}_{10,126}$) should be expressed through these ones.

$$\langle 210_H \rangle = \underbrace{\langle (1,1,1) \rangle}_{p} + \underbrace{\langle (1,1,15) \rangle}_{a} + \underbrace{\langle (1,3,15) \rangle}_{\omega}$$

$$\langle 126_H \rangle = \underbrace{\langle (1,3,\overline{10}) \rangle}_{\sigma} \quad , \quad \langle \overline{126}_H \rangle = \underbrace{\langle (1,3,10) \rangle}_{\bar{\sigma}}$$

Doublets live in $10_H$, $210_H$, $126_H$ and $\overline{126}_H$.

$$\begin{pmatrix}\bar{H}_{10} & \bar{H}_{\overline{126}} & \bar{H}_{126} & \bar{H}_{210}\end{pmatrix} \mathcal{M}_D \begin{pmatrix}H_{10} & H_{126} & H_{\overline{126}} & H_{210}\end{pmatrix}^T$$

Doublet mass $\mathcal{M}_D \approx$

$$\begin{pmatrix} m_{10} & \alpha(a-\omega) & \bar{\alpha}(a+\omega) & \alpha\sigma \\ \bar{\alpha}(a-\omega) & m_{126}+\eta(a-\omega) & 0 & \eta\sigma \\ \alpha(a+\omega) & 0 & m_{126}+\eta(a+\omega) & 0 \\ \bar{\alpha}\bar{\sigma} & \eta\bar{\sigma} & 0 & m_{210}+\lambda(a-\omega) \end{pmatrix}$$

It depends on superpotential parameters $m_{10}$,$m_{126}$,$m_{210}$,$\alpha$, $\bar{\alpha}$,$\eta$,$\lambda$

plus vevs $p, a, \omega, \sigma, \bar{\sigma} = f(m_{10,126,210}, \alpha, \bar{\alpha}, \lambda, \eta)$

SU(2) breaking vevs are just components of left and right null eigenvectors:

$$\begin{aligned} \mathcal{M}_D \begin{pmatrix} v_{10}^u & v_{126}^u & v_{\overline{126}}^u & v_{210}^u \end{pmatrix}^T &= 0 \\ \mathcal{M}_D{}^\dagger \begin{pmatrix} v_{10}^d & v_{\overline{126}}^d & v_{126}^d & v_{210}^d \end{pmatrix}^T &= 0 \end{aligned}$$

This obviously possible only after fine-tuning (doublet-triplet splitting)

Normalization fixed by

$$\begin{aligned} |v_{10}^u|^2 + |v_{126}^u|^2 + |v_{\overline{126}}^u|^2 + |v_{210}^u|^2 &= v^2 \sin^2\beta \\ |v_{10}^d|^2 + |v_{\overline{126}}^d|^2 + |v_{126}^d|^2 + |v_{210}^d|^2 &= v^2 \cos^2\beta \end{aligned}$$

Similar constraints also on $v_{L,R}$.

## Before Higgs discovery

this model survived only with split supersymmetry:

$$m_\lambda \approx 10 - 100 \text{ TeV} \quad , \quad m_{\tilde{f}} \approx 10^{13} \text{ GeV}$$

*BB, Doršner, Nemevšek, '08*

## After Higgs discovery

split supersymmetry higher scale gets bounded by the Higgs mass:

$$m_{\tilde{f}} \lesssim 10^{8} \text{ GeV}$$

So this model does not have any known realistic solution at the moment.

A bit less ambitious is to check theories of Yukawas in SO(10) but without the full Higgs sector specified and taken into account.

This means that

- MSSM Higgs vevs ($v_{10,126}^{u,d}$) free parameters
- over-all neutrino mass scale ($v_{R,L}$) free parameters

Realaxing these constraints can help a lot: for example these constraints forced the previous model to have split supersymmetry (which is however ruled out by the Higgs mass) instead of low energy supersymmetry.

**Fit to Yukawa sector** (without taking into account the constraints on the Higgs sector) **successful**

*Bertolini, Malinsky, '04*

*Babu, Macesanu, '04*

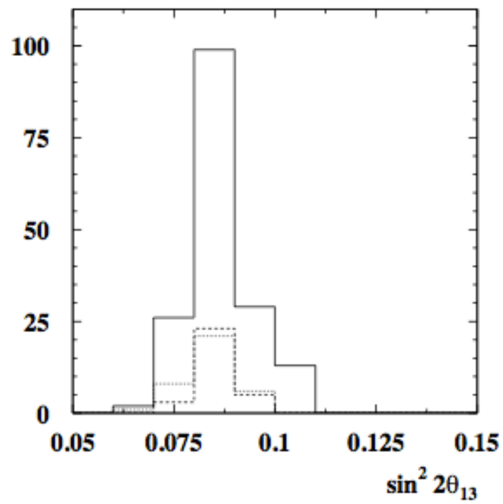
*Joshipura, Patel, 11*

*Dueck, Rodejohann, 13*

Prediction (in 2004):

$$\theta_{13}^l \approx 0.15 \pm 0.05$$

*Bertolini, Malinsky, '04*




*Babu, Macesanu, '04*

## Checks in realistic susy SO(10) examples

Are residual symmetries in the minimal susy SO(10) possible?

In principle we should redo a fit assuming some constraints on $|U_{ji}|$ for one or more pairs $(j, i)$.

This not easy, here will take a simpler route: check if known fits get $|U_{ji}|$ among those predicted from low $p$.

For small enough $p \leq 5$ we need matrix elements

| $p$ | $k$ | $\lvert\cos(\pi k/p)\rvert$ | $\lvert\sin(\pi k/p)\rvert$ |
|---|---|---|---|
| 2 | 1 | 0 | 1 |
| 3 | 1 | 0.5 | 0.866 |
| 4 | 1 | 0.707 | 0.707 |
| 5 | 1 | 0.809 | 0.588 |
| 5 | 2 | 0.309 | 0.951 |

Is any of these numbers realized in some fit?

Minimal susy SO(10) Yukawa sector (2 Yukawas, no Higgs sector constraints)

*Dueck, Rodejohan, '13*

Example 1: $\tan\beta = 50$:

$$|U_{ji}| = \begin{pmatrix} 0.919 & 0.392 & 0.037 \\ 0.362 & 0.812 & 0.458 \\ 0.156 & 0.432 & 0.888 \end{pmatrix}$$

Look to element $|U_{22}| = 0.812 \approx 0.809 = |\cos(\pi/5)|$

$$(W_{22})^5 = \begin{pmatrix} 0.999 & 0.035 & 0.001 \\ 0.035 & 0.999 & 0.039 \\ 0.001 & 0.039 & 0.999 \end{pmatrix} \approx \mathbb{I}$$

Example 2: $\tan\beta = 10$

$$|U_{ji}| = \begin{pmatrix} 0.958 & 0.285 & 0.033 \\ 0.262 & 0.917 & 0.301 \\ 0.116 & 0.280 & 0.953 \end{pmatrix}$$

Look to element $|U_{23}| = 0.301 \approx 0.309 = |\cos(2\pi/5)|$:

$$(W_{32})^5 = \begin{pmatrix} 1.000 & 0.003 & 0.000 \\ 0.003 & 0.997 & 0.080 \\ 0.000 & 0.080 & 0.997 \end{pmatrix} \approx \mathbb{I}$$

# More matrix elements?

What happens if more matrix elements are known?

Imagine that more than one element $|U_{ji}|$ can be found, i.e. for two pairs of $(j,i)$. Is the matrix found automatically unitary?

The answer is no, and a constraints will follow.

Remember

$$s_1^d = \begin{pmatrix} +1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad , \quad s_2^d = \begin{pmatrix} -1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad , \quad \ldots$$

Then

$$\sum_{i=1}^{3} s_i^d = -\mathbb{I} \quad , \quad Tr\left(s_i^d\right) = -1$$

On the other side

$$
\begin{aligned}
Tr\left(W_{1j}\right) &= a_{p_1} \qquad & (W_{1j}^{p_1} = \mathbb{I}) \\
Tr\left(W_{2j}\right) &= a_{p_2} \qquad & (W_{2j}^{p_2} = \mathbb{I}) \\
Tr\left(W_{3j}\right) &= a_{p_3} \qquad & (W_{3j}^{p_3} = \mathbb{I})
\end{aligned}
$$

So

$$a_{p_1} + a_{p_2} + a_{p_3} = 1$$

Unitarity constraint

$$\sum_{i=1}^{3} \left|U_{ji}\right|^2 = 1$$

satisfied providing consistent choice of $(k_1/p_1, k_2/p_2, k_3/p_3)$:

$$\cos^2\left(\pi k_1/p_1\right) + \cos^2\left(\pi k_2/p_2\right) + \cos^2\left(\pi k_3/p_3\right) = 1$$

Not easy to satisfy, only few cases work:

$$\left(\cos\left(\pi\frac{k_1}{p_1}\right), \cos\left(\pi\frac{k_2}{p_2}\right), \cos\left(\pi\frac{k_3}{p_3}\right)\right)$$

$$= \quad \left(\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{\phi}{2}, \frac{1}{2\phi}\right), (\cos\alpha, \sin\alpha, 0)$$

with

$$\phi \quad = \quad \frac{\sqrt{5}+1}{2}$$

$$\alpha \quad = \quad \pi k_0/p_0 \quad , \quad 1 \leq k_0 \leq p_0/2 \quad , \quad k_0 \in \mathbb{Z}$$

which correspond to

$$\left(\frac{k_1}{p_1}, \frac{k_2}{p_2}, \frac{k_3}{p_3}\right) = \left(\frac{1}{4}, \frac{1}{3}, \frac{1}{3}\right), \left(\frac{1}{3}, \frac{1}{5}, \frac{2}{5}\right), \left(\frac{k_0}{p_0}, \frac{p_0 - 2k_0}{2p_0}, \frac{1}{2}\right)$$

*Fonseca, Grimus, '14, '15*

*Byakti, Pal, '16*

You can check it explicitly. For example for $p_1, p_2 \leq 10$ the only possibilities are given by:

| $p_1/k_1$ | $p_2/k_2$ | $p_3/k_3$ |
|---|---|---|
| 3 | 3 | 4 |
| 3 | 4 | 3 |
| 3 | 5 | 5/2 |
| 3 | 5/2 | 5 |
| 5 | 5/2 | 3 |

All these cases fit with the previous rule.

Is any of these cases found in the known fits?

# Hint for a symmetry or numerical coincidence?

Can such a special number be just a numerical coincidence?

Back of the envelope estimate:

there are $3 \times 3 = 9$ matrix elements

for "small" $p = 2, 3, 4, 5$ there are $1 + 1 + 1 + 2 = 5$ different special numbers

$$|\cos(k\pi/p)| \quad , \quad k = 1, \ldots, p/2$$

Taking a possible discrepancy of 0.008 (twice the difference between our best solution and the real one) one gets the estimate

probability that it is a coincidence $\approx 9 \times 5 \times 0.008 \approx 0.36$

Of course the matrix $U$ is unitary and so the matrix elements are not independent. A correct computation gives (assuming CP)

probability that it is a coincidence $\approx 0.3$

So a single number is not to take too seriously.

But we have more than that: the second row in Example 1:

$$|U_{ji}| = \begin{pmatrix} 0.919 & 0.392 & 0.037 \\ 0.362 & 0.812 & 0.458 \\ 0.156 & 0.432 & 0.888 \end{pmatrix}$$

$$\begin{aligned}(0.362, 0.812, 0.458) &\approx (0.309, 0.809, 0.5) \\ &= (\cos(2\pi/5), \cos(\pi/5), \cos(\pi/3))\end{aligned}$$

This is harder to reproduce by coincidence

## 120

Imagine now to have

$$W = 16_F^T \left(10_H Y_{10} + 120_H Y_{120} + \ldots\right) 16_F$$

One anti-symmetric Yukawa ($Y_{120}$) and (at least) one symmetric Yukawa ($Y_{10}$). Since

$$Y_{10}^T = Y_{10}$$

we can always diagonalize it by

$$16_F \to U_{10} 16_F \to U_{10}^T Y_{10} U_{10} = Y_{10}^d$$

We can never diagonalize the anti-symmetric $Y_{120}$ but can put it into canonical form:

$$16_F \to U_{120} 16_F \to U_{120}^T Y_{120} U_{120} = Y_{120}^c$$

with

$$Y_{120}^c = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & x \\ 0 & -x & 0 \end{pmatrix}$$

Then the term

$$16_F^T Y_{120}^c 16_F$$

is symmetric under SU(2)

$$16_F \to g_\phi 16_F$$

$$g_\phi = \begin{pmatrix} 1 & 0 \\ 0 & \exp\left(i\vec{\phi}\vec{\tau}\right) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \cos\phi + i\frac{\vec{\phi}\vec{\tau}}{\phi}\sin\phi \end{pmatrix}$$

($\phi \equiv \sqrt{\vec{\phi}.\vec{\phi}}$ , $\vec{\tau}$ Pauli matrices)

Assume that $g_\phi$ is a residual symmetry, then

$$g_\phi^p = \mathbb{I} \to \phi = \frac{2\pi}{p}$$

If $g_\phi$ is a residual symmetry, so are $g_\phi^2, \ldots, g_\phi^{p-1}$.

(product of two elements of a group is again element of the group)

Together

$$s_j^{dT} Y_{10}^d s_j^d = Y_{10}^d$$
$$g_\phi^T Y_{120}^c g_\phi = Y_{120}^c$$

Define

$$U = U_{10}^\dagger U_{120}$$

Then both elements $s_j^d$ and $g_\phi^n$ are in the same flavor group iff

$$\left[W_{j\phi}^n\right]^p = \mathbb{I} \quad , \quad n = 1, \ldots, p-1$$

with

$$W_{j\phi}^n = U g_\phi^n U^\dagger s_j^d$$

Possible to satisfy all $(p-1)$ constraints only because $p, k, l$ different for different $n$:

$$a_{p_n}(k_n, l_n) = Tr\left(W_{j\phi}^n\right)$$

where (assuming again $Det(W) = 1$)

$$a_p(k,l) = e^{2\pi i(k/p)} + e^{2\pi i(l/p)} + e^{-2\pi i(k/p + l/p)}$$

As before we expand

$$
\begin{aligned}
Tr\left(W_{j\phi}^{n}\right) &= 2|U_{j1}|^2 - 1 - 2\cos\left(2\pi n/p\right)|U_{j1}|^2 \\
&+ i\, 2\sin\left(2\pi n/p\right)\left(1 - |U_{j1}|^2\right)\hat{\phi}\hat{e}
\end{aligned}
$$

with

$$
\begin{aligned}
\hat{e} &= \frac{1}{1-|U_{j1}|^2}\left(2Re\left(U_{j2}U_{j3}^*\right), 2Im\left(U_{j2}U_{j3}^*\right), |U_{j2}|^2 - |U_{j3}|^2\right) \\
\hat{\phi} &= \frac{\vec{\phi}}{\phi}
\end{aligned}
$$

So there are $2(p-1)$ real equations to be satisfied:

$$
\begin{aligned}
Re(a_{p_n}(k_n,l_n)) &= -1+2|U_{j1}|^2(1-\cos(2\pi n/p)) \\
Im(a_{p_n}(k_n,l_n)) &= 2\sin(2\pi n/p)(1-|U_{j1}|^2)\hat{e}\hat{\phi}
\end{aligned}
$$

for the $2p$ unknowns $|U_{j1}|$, $\hat{e}\hat{\phi}$, triples $T_n$

$$
T_n \equiv (k_n, l_n, -(k_n+l_n) \bmod p_n)/p_n \quad , \quad n=1,\ldots p-1
$$

Some degeneracy of the equations needed.

Lower $p$ ($\leq 5$) solutions:

| $\lvert (U_{10-120})_{j1} \rvert$ | $\hat{e}\hat{\phi}$ |
|---|---|
| $0$ | $0$ |
| $\sqrt{\frac{1}{3}} = 0.577$ | $0$ |
| $\sqrt{\frac{2}{3}} = 0.816$ | $0$ |
| $\sqrt{\frac{3+\sqrt{5}}{6}} = 0.934$ | $0$ |
| $\sqrt{\frac{3-\sqrt{5}}{6}} = 0.357$ | $0$ |
| $\sqrt{\frac{1}{2}} = 0.707$ | $0$ |
| $0$ | $1$ |
| $0$ | $-1$ |
| $\sqrt{\frac{5+\sqrt{5}}{10}} = 0.851$ | $0$ |
| $\sqrt{\frac{5-\sqrt{5}}{10}} = 0.526$ | $0$ |

Taking data for $|U|$ (for demonstration) from

*Aulakh, Garg, Khosa, 13*

$$U = \begin{pmatrix} 0.951 & 0.310 & 0 \\ 0.306 & 0.939 & 0.158 \\ 0.049 & 0.150 & 0.987 \end{pmatrix}$$

we see that

$$|U_{11}| = 0.951 \approx 0.934 = \sqrt{\frac{3+\sqrt{5}}{6}}$$

This is obtained for $p = 3$ and triples $T_1 = T_2 = (0,1,4)/5$

Other solutions can give other special values.

## Conclusions

- Minimal susy SO(10) models have no room for real flavor symmetries
- all we can hope is to have some residual symmetry (remnants)
- situation unique in susy SO(10): existence of residual symmetry RGE invariant
- residual symmetries predict particular values of $|U_{ji}|$; such values can be found in existent fits

Homework:

- new fits (with residual symmetries assumed from the beginning)
- UV theory?