

Little Warm Inflation

Cold inflation/Warm inflation

Dissipative coefficient: $\Upsilon(T) = C_T T$

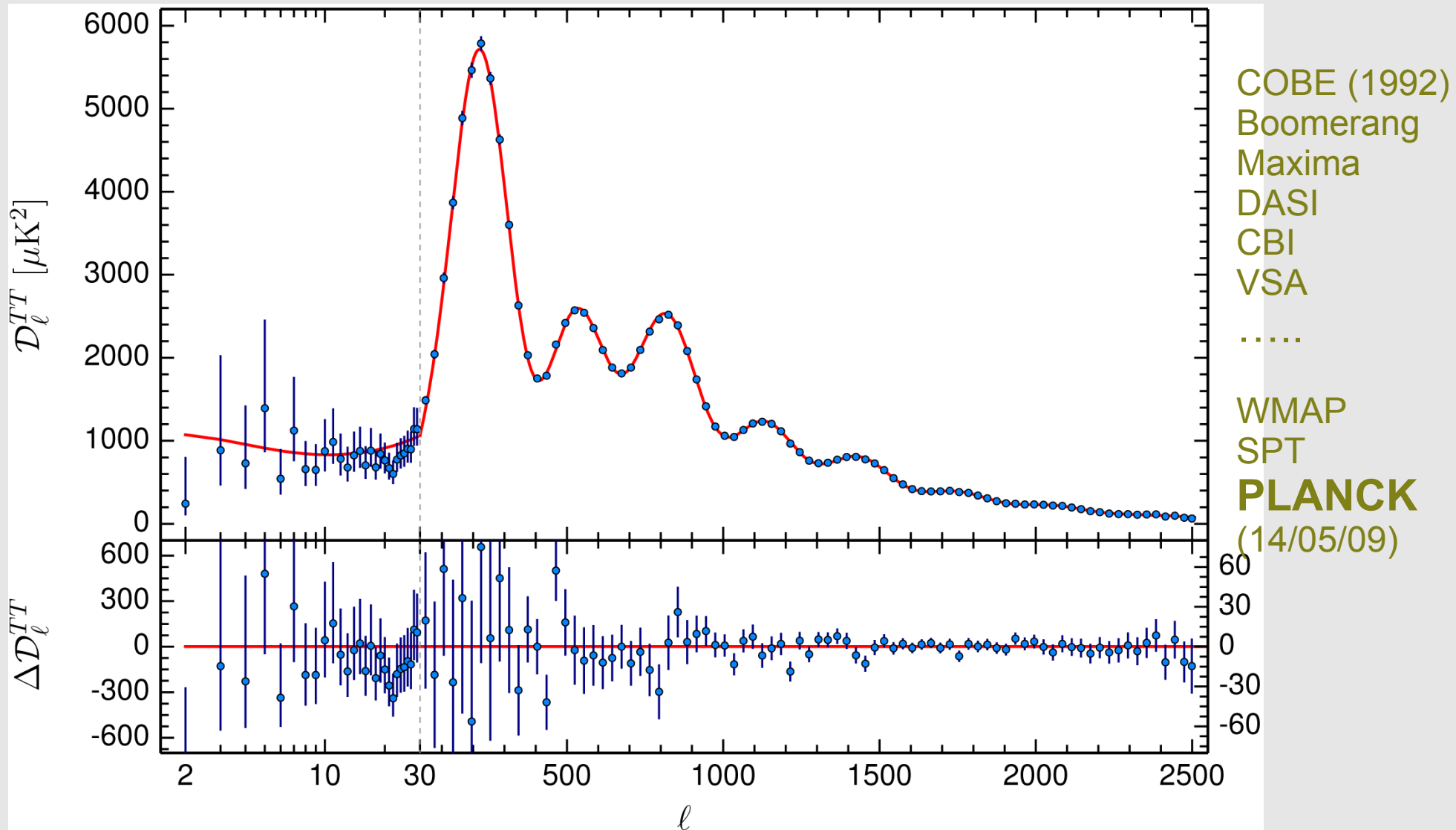
Primordial spectrum: Chaotic models $\lambda \phi^4$

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Work done in collab with: A. Berera, R. Ramos, J. Rosa [arXiv:1604.08838]

Cosmic microwave background radiation (CMB)

Spectrum of T fluctuations

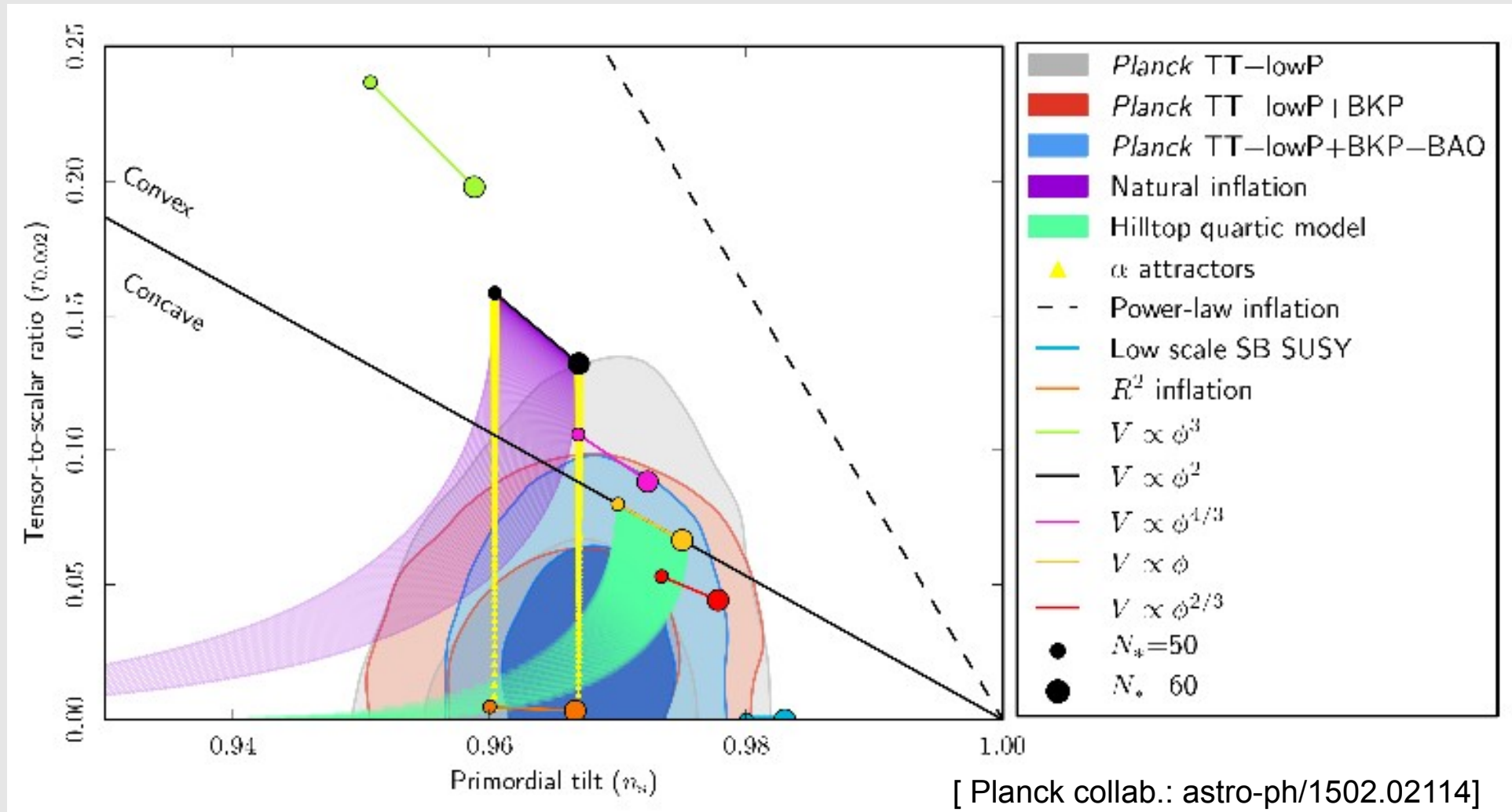


[Planck collab.: astro-ph/1502.01589]

Primordial spectrum: ~adiabatic, ~scale-invariant, gaussian?, tensors?

Primordial spectrum: $P_R = P_R(k_0) (k/k_0)^{n_s - 1}$ $k_0 = 0.002 \text{ Mpc}^{-1}$

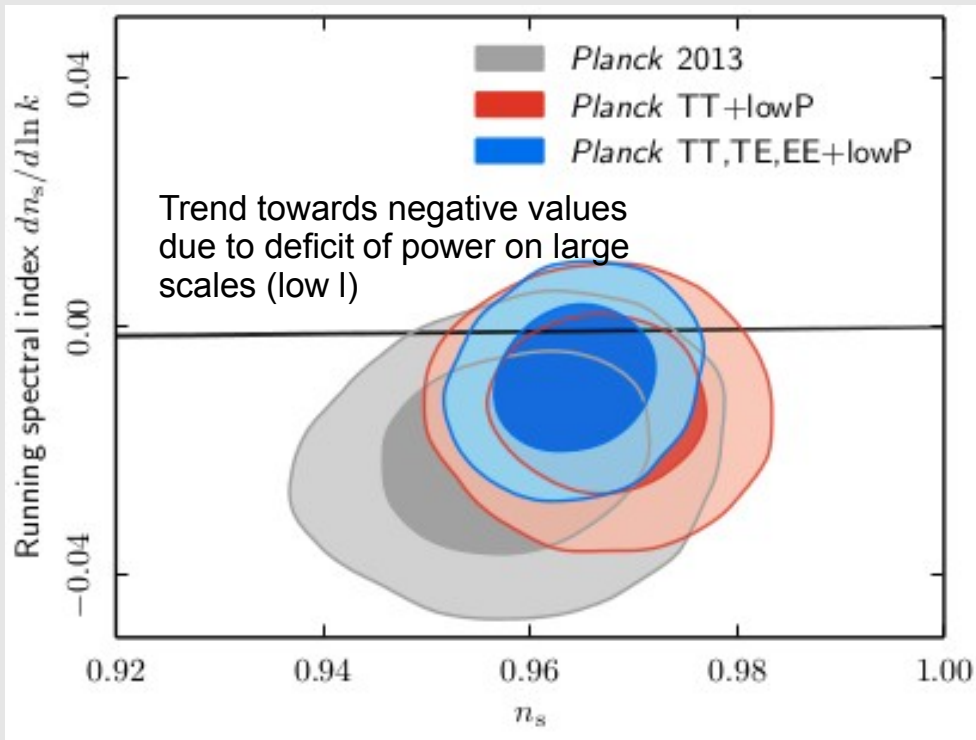
Tensor-to-scalar Ratio: $r = P_T / P_R$ $P_R = 2.2 \times 10^{-9}$



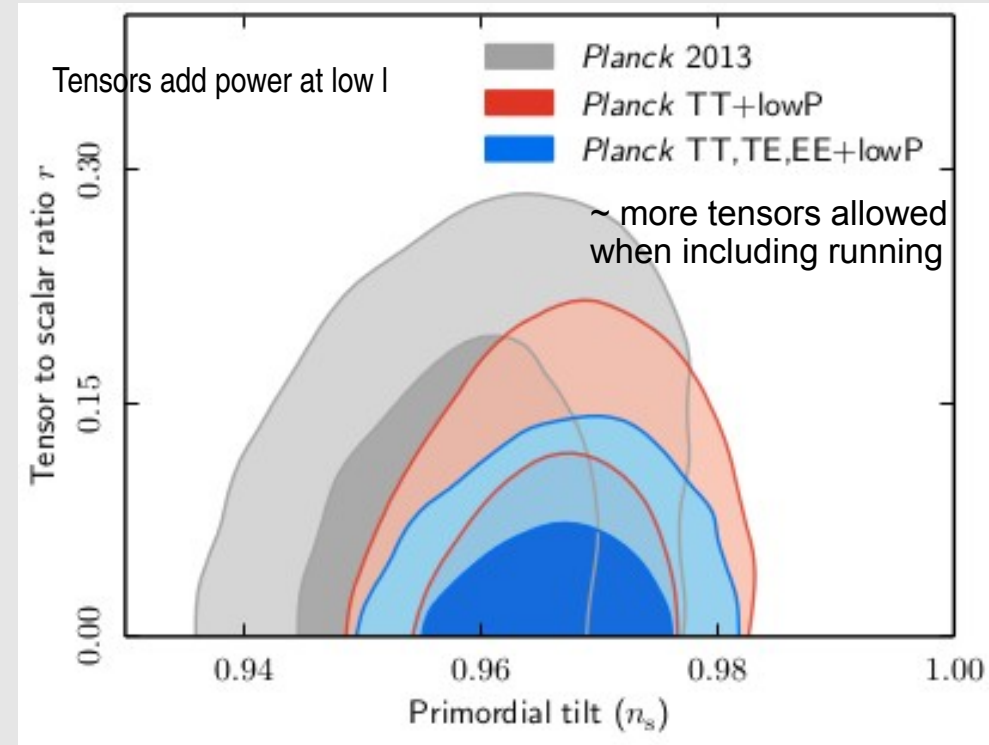
Primordial spectrum: $P_R = P_R(k_0) (k/k_0)^{n_s-1 + \frac{1}{2}\alpha_s \ln k/k_0 + \dots}$ $k_0 = 0.05 \text{ Mpc}^{-1}$

adiabatic, gaussian, scale-invariant spectrum

**No evidence for:
non-gaussianity, isocurvature modes or running of the spectral index**



$$\alpha_s = \frac{dn_s}{d\ln k} = -0.0057 \pm 0.0071$$



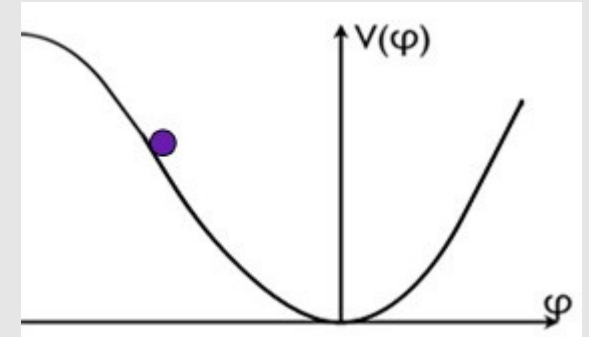
$$\alpha_s = -0.013 \pm 0.01, \quad r_{0.002} < 0.18$$

$$[n_t = -r/8 < 0]$$

Slow Roll Inflation

Scalar field rolling down its (flat) potential

$$P = \dot{\phi}^2/2 - V(\phi) \approx -V(\phi) \quad \text{negative pressure}$$



“Flat” potential

The curvature and the slope smaller than the (Hubble) expansion rate H

Kinetic energy \ll potential energy $H^2 \sim V/3m_p^2$ Hubble parameter ($H = \dot{a}/a$)
($a =$ scale factor)

Slow-roll parameters

$$|\eta_\phi| = m_p^2 \left| \frac{V''}{V} \right| < 1 \quad \epsilon_\phi = \frac{m_p^2}{2} \left(\frac{V'}{V} \right)^2 < 1 \quad \leftarrow$$

curvature

slope

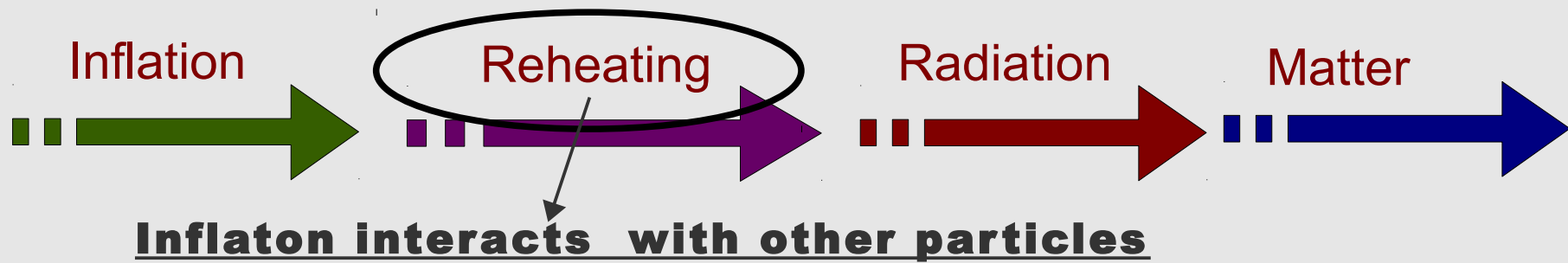
Slow-roll equation

$$\dot{\phi} \simeq -V'/3H$$

Primordial spectrum

$$P_R \simeq \left(\frac{H}{\dot{\phi}} \right)^2 \left(\frac{H}{2\pi} \right)^2 \quad n_s = 1 + 2\eta_\phi - 6\epsilon_\phi \quad r = 16\epsilon_\phi$$

$$\underline{V^{1/4} \sim 10^{16} \left(\frac{r}{0.1} \right)^{1/4} \text{ GeV}}$$

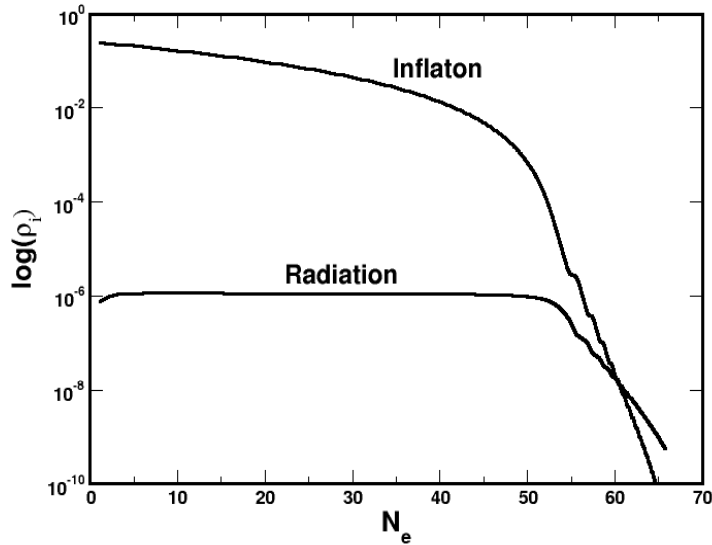


Interactions with cosmic plasma induce **dissipation**

$$\ddot{\phi} + (3H + Y)\dot{\phi} + V_{\phi} = 0$$

▲ “Decay” into light dof= extra friction

”Warm” inflation:



A (small) fraction of the vacuum energy is converted into radiation during inflation

$$\dot{\rho}_R + 4H\rho_R = Y\dot{\phi}^2 \quad \text{“Source term”}$$

Extra friction term: $Q = Y/(3H)$

- $Q \ll 1, T \ll H \quad \longrightarrow$ Standard **Cold Inflation**
- $Q < 1, T > H \quad \longrightarrow$ **Weak Dissipative Regime**

Standard slow-roll

- $Q > 1, T > H \quad \longrightarrow$ **Strong Dissipative Regime**

Slow-roll :

$$3H(1+Q)\dot{\phi} \simeq -V_{\phi}(\phi, T), \quad \rho_r \simeq \frac{3}{4}Q\dot{\phi}^2$$

(Thermal corrections)

$$|n_{\phi}| < (1+Q), \quad \epsilon_{\phi} < (1+Q), \quad \beta_Y < (1+Q), \quad \delta_T < 1$$

$$\beta_Y = m_P^2 (Y_{\phi} V_{\phi}) / (Y V)$$

$$\delta_T = T V_{T\phi} / V_{\phi}$$

- Q varies during inflation
- Extra friction prolongs inflation \longrightarrow Smaller ϕ values
- Dissipation induces thermal inflaton fluctuations

Interactions & Dissipative coefficient

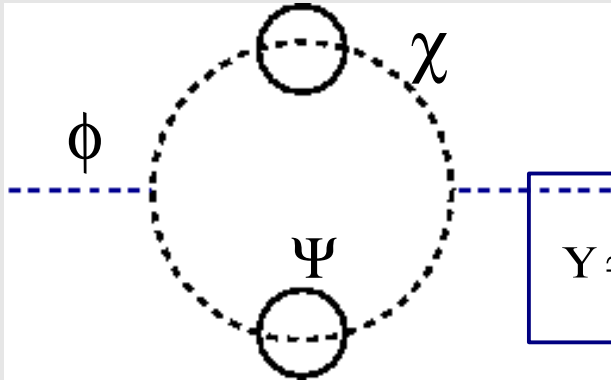
Low T regime:

$$L = \dots - \frac{1}{2} m_\phi^2 \phi^2 - \frac{g^2}{2} \phi^2 \chi^2 + h \chi \psi \bar{\psi} + \dots$$

light fermions

heavy $m_\chi = g\phi > H, T$

BG, Berera, Ramos & Rosa 2012



$$Y \simeq \frac{32}{\sqrt{2\pi}} \frac{g^2 N_\chi}{h^2 N_\psi} (m_\chi T)^{1/2} e^{-m_\chi/T} + 0.02 h^2 N_\psi N_\chi \left(\frac{T^3}{\phi^2}\right) \simeq C_\phi \frac{T^3}{\phi^2}$$

Adiabatic approximation:



$$T > H$$

$$\dot{\phi}/\phi, \quad H < \Gamma_\chi \simeq h^2 m_\chi / (8\pi)$$

Macroscopic

Microscopic

- Easy to fulfill for not too small values of h
- Thermal corrections under control (inflaton coupled to heavy fields) + susy to control T=0 corrections

Getting 50-60 e-fold of inflation typically requires $C_\phi \sim 10^6$

$$\frac{\Gamma_\chi}{\dot{\phi}/\phi} > \frac{\Gamma_\chi}{H} > \left(\frac{\Gamma_\chi}{m_\chi}\right) \left(\frac{m_\chi}{T}\right) \left(\frac{T}{H}\right) > 1$$

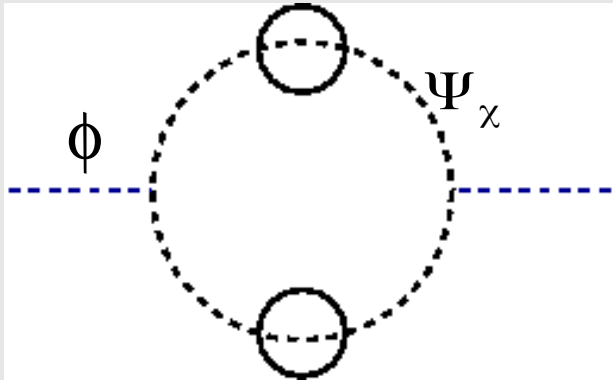
Interactions & Dissipative coefficient

High T regime:

$$L = \dots - \frac{1}{2} m_\phi^2 \phi^2 - g \phi \bar{\psi}_\chi \psi_\chi - h \sigma \bar{\psi}_\chi \psi_\chi + \dots$$

light scalar

light $m_\psi = g\phi < H, T, \quad g \ll 1$



$$Y \simeq \frac{3}{1 - 0.34 \log h} \frac{g^2}{h^2} T$$

Linear T coefficient

Adiabatic approximation:



$T > H$

$$\dot{\phi}/\phi, \quad H < \Gamma_\chi \simeq \frac{\pi}{512} h^4 \left(\frac{T}{H} \right)$$

Macroscopic

Microscopic

- Small g coupling to keep fermions light

- Not too small h because of adiabatic condition

- How to avoid thermal corrections to inflaton potential due to light fields?

Berera, Gleiser & Ramos PRD'98; Yokoyama & Linde PRD '98

Little warm inflation

- Consider a U(1) gauge theory spontaneously broken by two complex Higgs fields

$$\langle \phi_1 \rangle = \langle \phi_2 \rangle = M/\sqrt{2}$$

- One Nambu-Goldstone boson is “eaten” by the gauge field, and the other becomes the physical scalar inflaton field

$$\phi_1 = \frac{M}{\sqrt{2}} e^{\phi/M}, \quad \phi_2 = \frac{M}{\sqrt{2}} e^{-\phi/M}$$

- Couple the Higgses to charged and singlet Weyl fermions:

$$\begin{aligned} \mathcal{L} &= \frac{g}{\sqrt{2}} (\phi_1 + \phi_2) \bar{\Psi}_{1L} \Psi_{1R} - i \frac{g}{\sqrt{2}} (\phi_1 - \phi_2) \bar{\Psi}_{2L} \Psi_{2R} + \text{hc} \\ &= g M \cos(\phi/M) \bar{\psi}_1 \psi_1 - g M \sin(\phi/M) \bar{\psi}_2 \psi_2 \end{aligned}$$

With interchange symmetry: $\phi_1 \longleftrightarrow i\phi_2$ $\Psi_{1L,R} \longleftrightarrow \Psi_{2L,R}$

Fermion masses are bounded!!

- Light fermions: $gM < T < M$

Little warm inflation

High T regime:

Inflaton a PNCB of a broken U(1) symmetry + pair of fermions + exchange sym.

$$L = \dots - gM \cos(\phi/M) \bar{\psi}_1 \psi_1 - gM \sin(\phi/M) \bar{\psi}_2 \psi_2 - h \sigma \sum_{i=1,2} (\bar{\psi}_i \psi_\sigma + \bar{\psi}_\sigma \psi_i) + \dots$$

light Ψ : $gM < T < M$, $g \ll 1$

Thermal potential:

$$\Delta V_T = - \underbrace{\frac{\pi^2}{90} g_R}_{\text{Light dof}} T^4 + \underbrace{\frac{g^2 M^2}{12} T^2}_{\text{No thermal mass for the inflaton}} + \frac{g^4(\phi) M^4}{16 \pi^2} (\log \frac{\mu^2}{T^2} - c_f)$$

Total energy density:

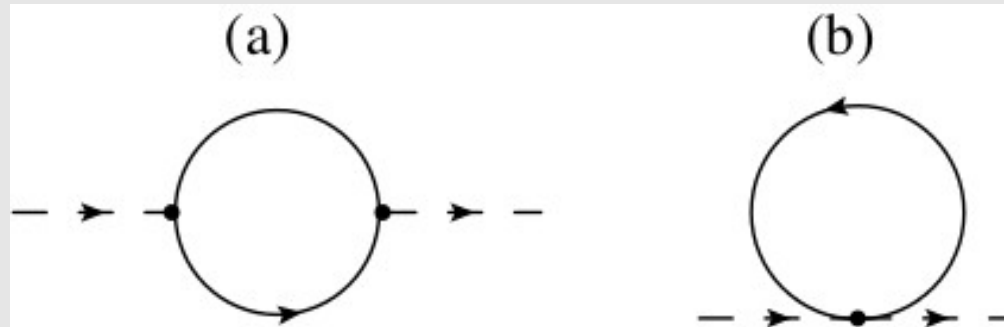
$$\rho_T = \frac{1}{2} \dot{\phi}^2 + V(\phi) + \underbrace{\Delta V_T - T \frac{d\Delta V_T}{dT}}_{\rho_R} = \frac{\pi^2}{30} g_R(\phi, T) T^4$$

Effective no. of dof:

$$g_R(\phi, T) \simeq g_R - \frac{5}{2\pi^2} \left(\frac{gM}{T}\right)^2 + \frac{15}{16\pi^4} \left(\frac{gM}{T}\right)^4 \left(3 + \cos\left(\frac{4\phi}{M}\right)\right)$$

Inflaton self-energy

$$L = -\sum_i (m_i + g_i \delta\phi + \frac{f_i}{2} \delta\phi^2 + \dots) \bar{\Psi}_i \Psi_i$$



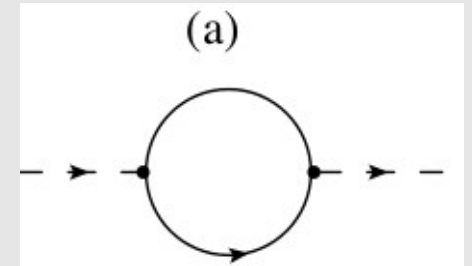
$$\Sigma_\phi(0) = \sum_i (g_i^2 + m_i f_i) I_T(0) = g^2 \left(-\cos\left(2\frac{\phi}{M}\right) + \cos\left(\frac{2\phi}{M}\right) \right) I_T(0) = 0$$

$$I_T(0) = -\frac{\Lambda^2}{2\pi^2} + \frac{T^2}{6}$$

Cancellations of quadratic divergences and thermal masses!!

Dissipation

Dissipation comes from non-local terms (diagram (a))



No cancellation of dissipative terms:

$$Y = \frac{4}{T} \sum_i \int \frac{d^3 p}{(2\pi)^3} \frac{m_i^2}{\Gamma_i \omega_p^2} n_F(1+n_F) \simeq \frac{3}{1-0.34 \log h} \frac{g^2}{h^2} T$$

Linear T coefficient

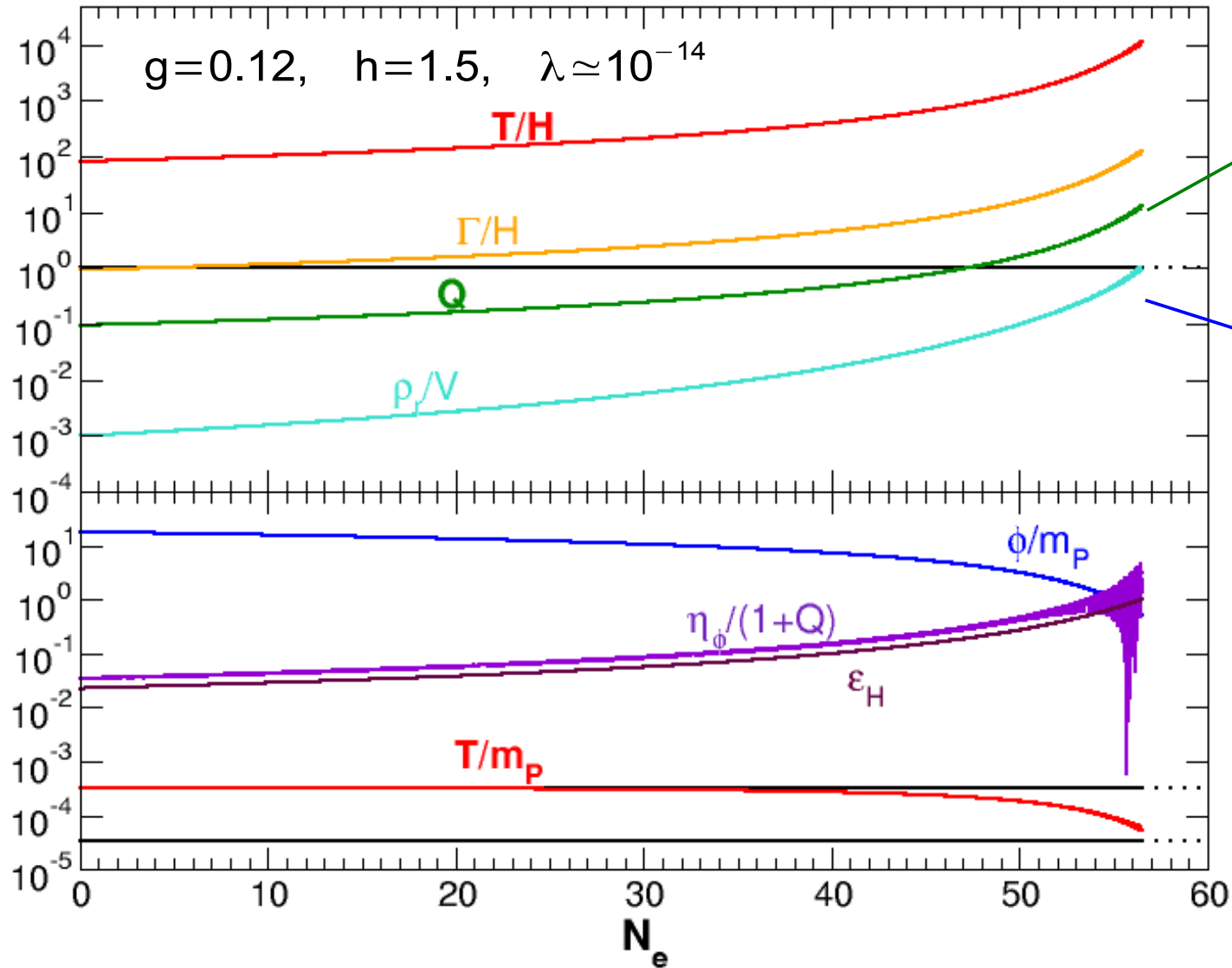
Decay rate $\Gamma_i = \frac{h^2}{16\pi} \frac{T^2 m_i^2}{\omega_p |p|} F_T(p/T, \omega_p/T)$ $[L = \dots - h\sigma \sum_i (\bar{\psi}_i \psi_\sigma + \bar{\psi}_\sigma \psi_i) + \dots]$

Additional Yukawa interactions with a massless field

Masses $m_i^2 \simeq h^2 T^2 / 8$

Background dynamics

Quartic potential: $V(\phi) = \frac{\lambda}{4} \phi^4$



Inflation ends in the strong dissipative regime

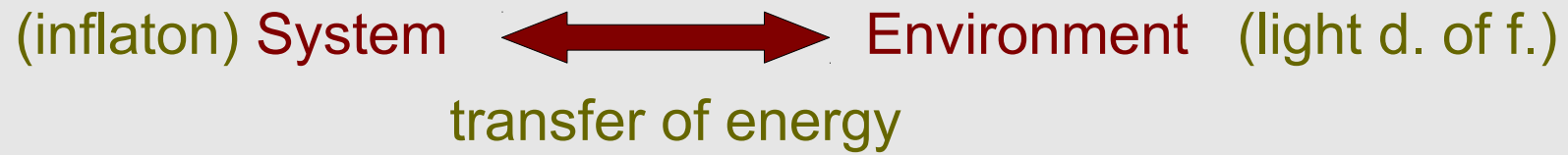
Radiation will dominate

$$\epsilon_H = -\frac{\dot{H}}{H^2}$$

$$M \simeq 10^{15} \text{ GeV}$$

Fluctuations & primordial spectrum

Thermal fluctuations become the source for the adiabatic perturbations



Dissipation Y

fluctuation force ξ

$$\delta \ddot{\phi}_k + (3H + Y) \delta \dot{\phi}_k + \dot{\phi} \delta Y + \left(\frac{k^2}{a^2} + V_{\phi\phi} \right) \delta \phi_k = \xi_k$$


Fluctuation-Dissipation rel.:


$$\langle \xi(\mathbf{k}) \xi(\mathbf{k}') \rangle = 2Y T a^{-3} (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}') \delta(t - t')$$

Fluctuations & primordial spectrum: coupled system

Field EOM:

$$\delta \ddot{\phi}_k^{\text{GI}} + (3H + Y) \delta \dot{\phi}_k^{\text{GI}} + \underbrace{\dot{\phi} \delta Y^{\text{GI}}}_{\text{coupled}} + \left(\frac{k^2}{a^2} + V_{\phi\phi} \right) \delta \phi_k^{\text{GI}} \simeq (2Y\mathcal{T})^{1/2} \hat{\xi}_k$$



$$\frac{\delta Y^{\text{GI}}}{Y} = \frac{1}{4} \frac{\delta \rho_r^{\text{GI}}}{\rho_r} \simeq \frac{\delta \mathcal{T}}{\mathcal{T}}$$


Coupled system
inflaton-radiation

Radiation (fluid stress energy-tensor): $T_{\text{rad}}^{\mu\nu} = (\rho_r + p_r) u^\mu u^\nu + p_r g^{\mu\nu}$

$$\delta \dot{\rho}_r^{\text{GI}} + 4H \delta \rho_r^{\text{GI}} \simeq \frac{k^2}{a^2} \Psi_r^{\text{GI}} + \underbrace{\dot{\phi}^2 \delta Y^{\text{GI}}}_{\text{coupled}} + 2\dot{\phi} Y \delta \dot{\phi}^{\text{GI}}$$

$$\dot{\Psi}_r^{\text{GI}} + 3H \Psi_r^{\text{GI}} \simeq -\delta \rho_r^{\text{GI}} / 3 - \dot{\phi} Y \delta \phi^{\text{GI}} \quad \text{Momentum density}$$

(Gauge invariant perturbations: $\delta \phi_k^{\text{GI}} = \delta \phi - \frac{H}{\dot{\phi}} \varphi$, φ : metric perturbation)

Fluctuations & primordial spectrum: coupled system

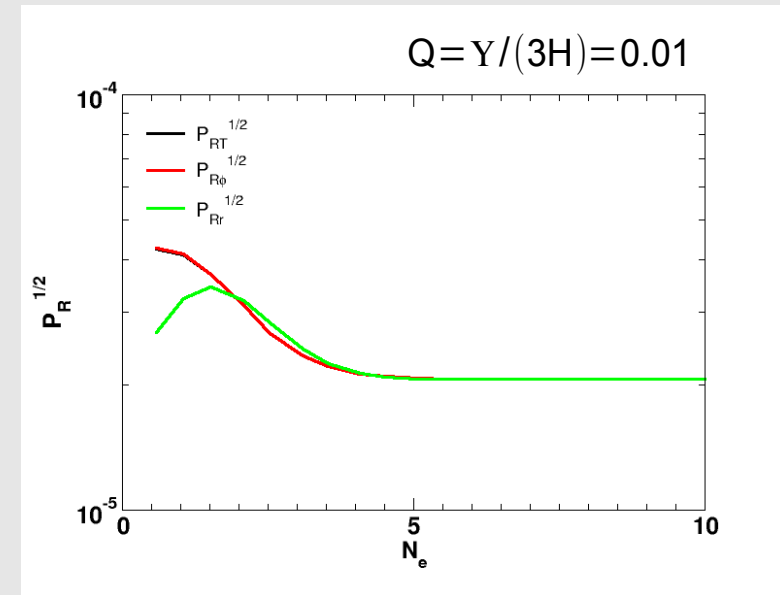
Weak dissipative regime ($Q=Y/H \ll 1$) : field decoupled from radiation

$$\delta \ddot{\phi}_k^{GI} + (3H + Y) \delta \dot{\phi}_k^{GI} + \left(\frac{k^2}{a^2} + V_{\phi\phi} \right) \delta \phi_k^{GI} \simeq (2YT)^{1/2} \hat{\xi}_k$$

$$P_{\delta\phi} \simeq \frac{HT}{2\pi} \frac{Q}{\sqrt{1+4\pi Q/3}}$$

Primordial spectrum: $P_R \simeq \left(\frac{H}{\dot{\phi}} \right)^2 P_{\delta\phi}$

R is constant after horizon crossing (freeze-out)



$$P_R \simeq (P_R)_{Q=0} \left(\underbrace{1 + 2N + \frac{T}{H} \frac{4\pi Q}{\sqrt{1+4\pi Q/3}}}_{\text{}} \right)$$

Dissipative processes may maintain a non-trivial distribution of inflaton particles:

$$N \simeq n_{BE} = (e^{k/aT} - 1)^{-1}$$

Fluctuations & primordial spectrum: coupled system

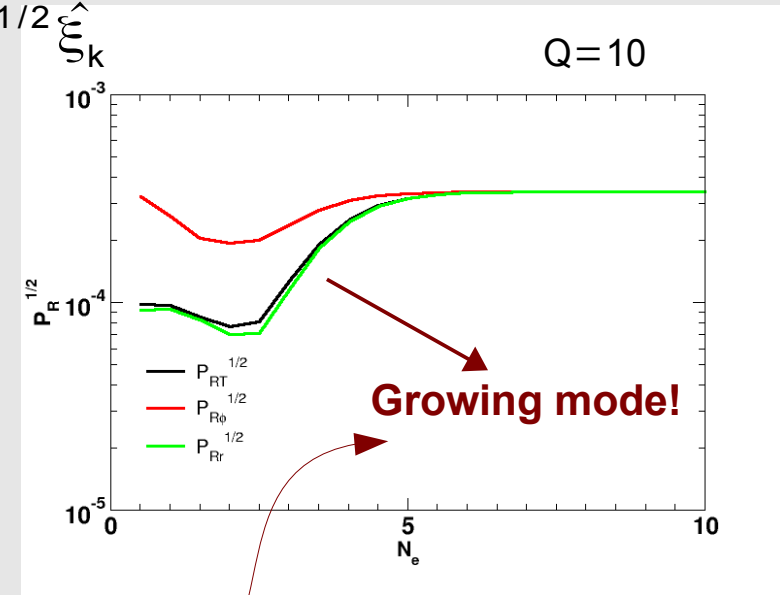
Strong dissipative regime ($Q=Y/H>1$) : coupled system

$$\delta \ddot{\phi}_k^{GI} + (3H + Y) \delta \dot{\phi}_k^{GI} + \dot{\phi} \delta Y^{GI} + \left(\frac{k^2}{a^2} + V_{\phi\phi} \right) \delta \phi_k^{GI} \simeq (2YT)^{1/2} \hat{\xi}_k$$

Primordial spectrum:

$$P_R = \frac{h_\phi}{h_T} P_{R_\phi} + \frac{h_r}{h_T} P_{R_r} \simeq P_{R_r} \simeq P_{R_\phi}, \quad (h_i = \rho_i + p_i)$$

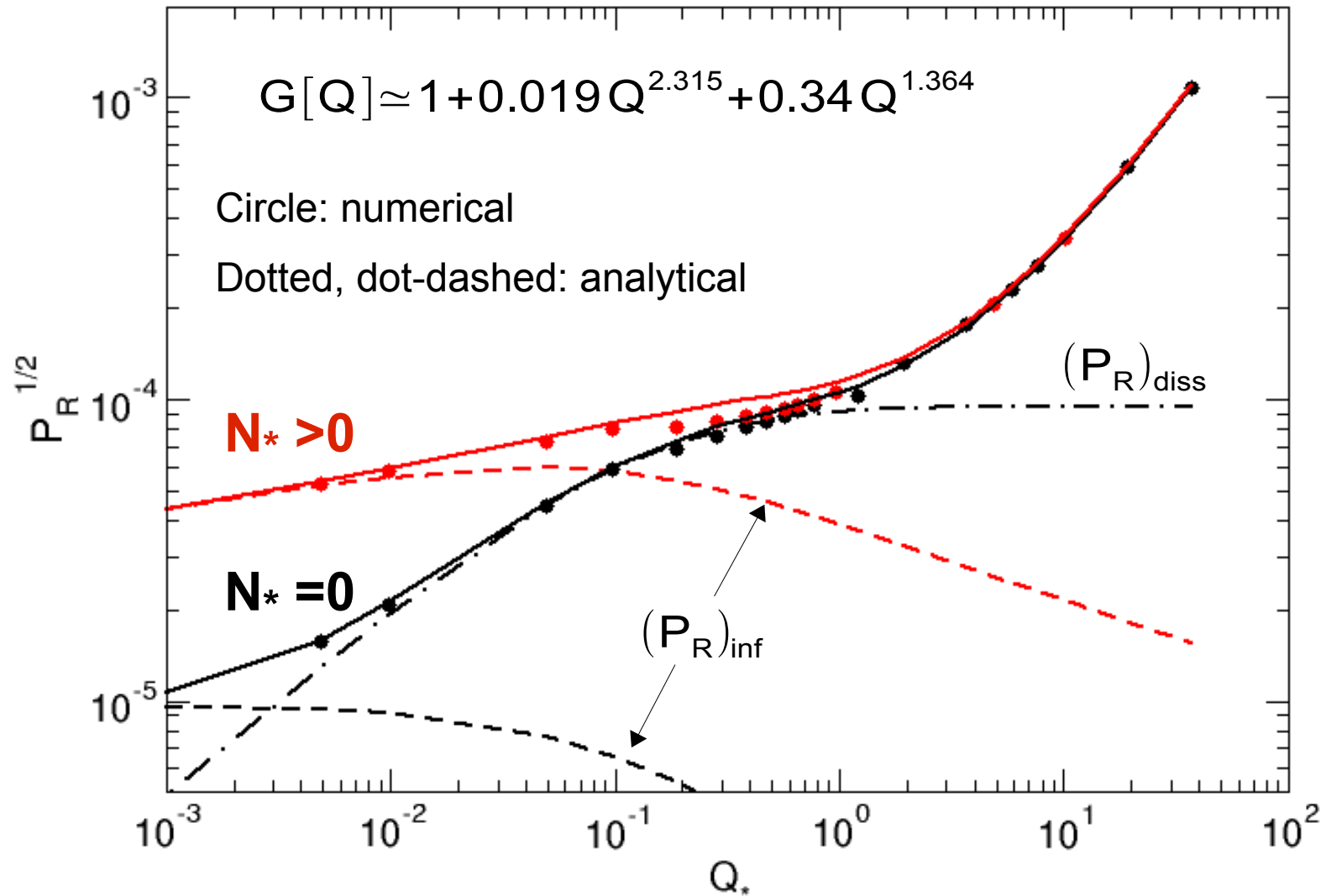
R is constant after horizon crossing (freeze-out)



$$P_R \simeq \left(\frac{H}{\dot{\phi}} \right)^2 \left(\frac{H}{2\pi} \right)^2 \left(1 + 2N + \frac{T}{H} \frac{4\pi Q}{\sqrt{1 + 4\pi Q/3}} \right) \times G[Q], \quad Q = Y/(3H)$$

Primordial spectrum

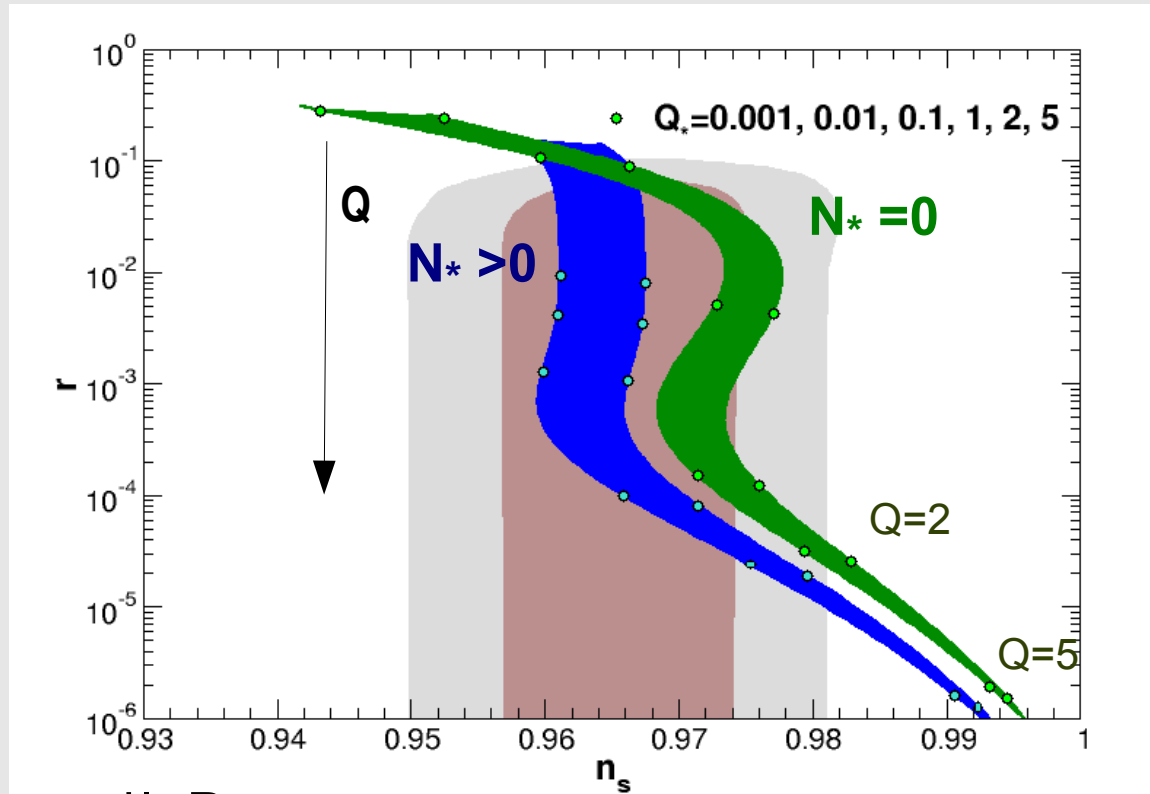
$$P_R \simeq ((P_R)_{\text{vac}} + (P_R)_{\text{diss}}) G[Q]$$



Chaotic model: $V(\phi) = \lambda \phi^4 / 4$, $\lambda = 10^{-14}$, $N_e = 50$

Primordial spectrum: quartic chaotic model

$$V(\phi) = \frac{\lambda}{4} \phi^4, \quad N_e = 50 - 60$$



$$n_s - 1 = \frac{d \ln P_R}{d N_e} = (n_s - 1)_N + (n_s - 1)_{\text{diss}} + (n_s - 1)_G, \quad (n_s - 1)_G > 0$$

$$r \simeq \frac{16 \epsilon_\phi}{(1 + 2N + \Delta_Q) G[Q]} \leq 16 \epsilon_\phi$$

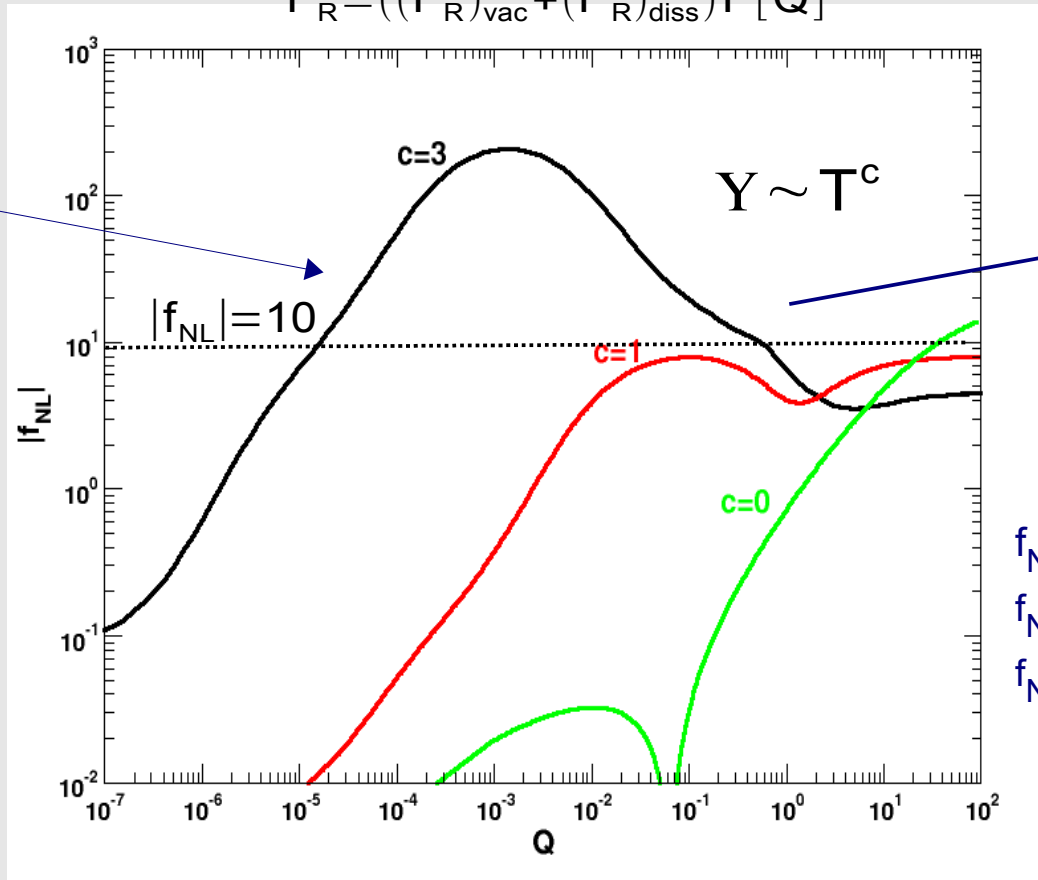
Quartic:

$$N \neq 0, Q < 1: \quad n_s \simeq 1 - 2/N_e, \quad r \simeq 16 \epsilon_\phi \left(\frac{H}{T} \right) \ll 0.1$$

Warm inflation & Non-gaussianity : T dependent diss. coefficient

- Bispectrum:** $B_R(k_1, k_2, k_3) = \sum_{\text{cyc}} \langle R_1(k_1) R_1(k_2) R_2(k_3) \rangle = A_B(k) \bar{B}(k_1, k_2, k_3)$
- $f_{\text{NL}} = \frac{18}{5} \frac{A_B(k)}{P_R(k)^2}$
- Non-linear parameter**
- ↘ **shape**

$$P_R \simeq ((P_R)_{\text{vac}} + (P_R)_{\text{diss}}) F[Q]$$



(Planck: $f_{\text{NL}} < \sim O(10)$)

$f_{\text{NL}}^{\text{local}} = 2.7 \pm 5.8$	$(k_3 \ll k_1 \sim k_2)$
$f_{\text{NL}}^{\text{equil}} = -42 \pm 75$	$(k_3 \sim k_1 \sim k_2)$
$f_{\text{NL}}^{\text{warm}} = 4 \pm 33$	

Summary

- Dissipative effects due to decaying fields can be relevant during inflation, and modify the inflationary predictions

- “Low T” regime for dissipation (massive scalar χ decaying into light dof): thermal corrections under control, but required large number of fields $N_\chi \sim 10^6$

- “High T” regime for dissipation (light fermion ψ decaying into light dof): $Y = C_T T$

Inflaton a PNCB of a broken U(1) symmetry + pair of fermions + exchange sym.

Light fermions: $gM < T$ + thermal corrections under control + minimal matter content

$\lambda\phi^4$ compatible with data, $Q^* \sim 0.01-1$, $r \sim 0.1-10^{-4}$

- For a T dependent dissipative coefficient, the field and radiation perturbation EOM form a coupled system: Field fluctuations are amplified before freeze-out ($Q < 1$)

Blue-tilted spectrum for $Q \gg 1$

- Non-gaussianity compatible with observations for both weak and strong dissipative regime, with a characteristic shape