Terrestrial Matter Effects
in reactor antineutrino oscillations

Zhi-zhong Xing

(IHEP, Beijing)

- ★ **The known and unknowns of ν's**
- ★ **Neutrino mass ordering matters**
- ★ **Terrestrial matter effects at JUNO / RENO-50**

In collaboration with Yu-Feng Li and Yifang Wang, 1605.00900, CPC

Exploring the Energy Ladder of the Universe, Mainz, 30/5 — 10/6, 16

Part A

Original idea of ν-mixing

Progress of Theoretical Physics, Vol. 28, No. 5, November 1962

The paper on μ-neutrino discovery was received by PRL on 15/6/1962

Remarks on the Unified Model of Elementary Particles

Ziro MAKI, Masami NAKAGAWA and Shoichi SAKATA

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta \sin^2\left(1.27\frac{\Delta m^2 L}{E}\right)$$

(Received June 25, 1962)

A particle mixture theory of neutrino is proposed assuming the existence of two kinds of neutrinos. Based on the neutrino-mixture theory, a possible unified model of elementary particles is constructed by generalizing the Sakata-Nagoya model.*) Our scheme gives a

$$\nu_e = \nu_1 \cos\delta - \nu_2 \sin\delta,$$

$$\nu_\mu = \nu_1 \sin\delta + \nu_2 \cos\delta.$$

Bruno Pontecorvo formulated neutrino oscillation in 1968.

Oscillations in 1998

Y. Suzuki
June 4

Solar ν's

Atmospheric ν's

T. Kajita
June 5

"Modest" Conclusions

(1) Flux: $\phi^{8B} = 2.44 \pm 0.05(\text{stat.})^{+0.09}_{-0.07}(\text{syst.}) \times 10^6/\text{cm}^2/\text{s}$
(0.368 for BP95, 0.47% for BP98)

(2) No seasonal variations.

(3) $(D-N)/(D+N) = -0.023 \pm 0.020(\text{stat}) \pm 0.014(\text{syst})$
no difference:
excluded regions
extended into "small angle sol
No core enhancement found.

(4) Day-Night + E-shape analysis.
(a) "No oscillation" is disfavoured @ 1~5% C.L.
(b) L.A. solution is disfavoured @ 1~5% C.L.
(c) V.O. regions are favoured (than MSW regions) @ 95% C.L.
(MSW is ok for 99% C.L.)

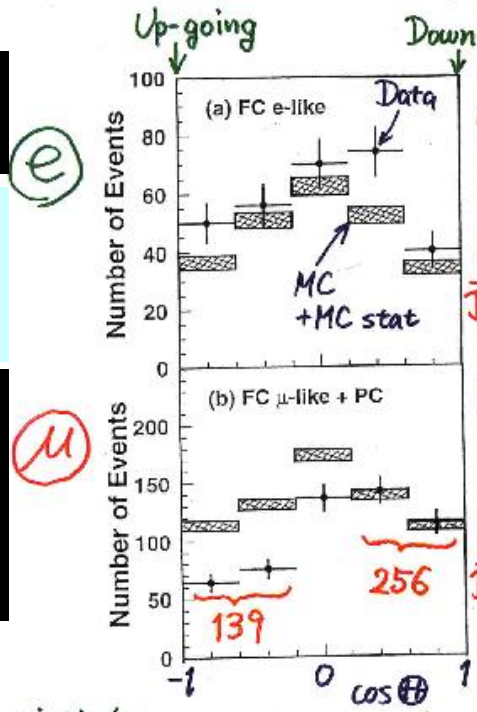
Super-K

Neutrino98
TAKAYAMA

Yes, two large mixing angles!
H. Fritzsch and Z.Z. Xing 1996:

$$\theta_{12} \simeq 42^\circ$$
$$\theta_{13} \simeq 4^\circ$$
$$\theta_{23} \simeq 52^\circ$$
$$\delta \simeq \pm 90^\circ$$

Zenith angle dependence
(Multi-GeV)



χ^2(shape) = 2.8/4 dof

$\frac{\text{Up}}{\text{Down}} = 0.93^{+0.13}_{-0.12}$

χ^2(shape) = 30/4 dof

$\frac{\text{Up}}{\text{Down}} = 0.54^{+0.06}_{-0.05}$

(6.2σ !!)

※ Up/Down syst. error for μ-like

Prediction (flux calculation ····· ≲ 1%, 1km rock above SK ···· 1.5%) 1.8%

Data (Energy calib. for ↑↓ ···· 0.7%, Non ν Background ····· < 2%) 2.1%

Bill Clinton's comments

REMARKS BY THE PRESIDENT
AT MIT 1998 COMMENCEMENT
June 5, 1998

Just yesterday in Japan, physicists announced a discovery that tiny neutrinos have mass. Now, that may not mean much to most Americans, but it may change our most fundamental theories -- from the nature of the smallest subatomic particles to how the universe itself works, and indeed how it expands.

This discovery was made, in Japan, yes, but it had the support of the investment of the U.S. Department of Energy. This discovery calls into question the decision made in Washington a couple of years ago to disband the super-conducting supercollider, and it reaffirms the importance of the work now being done at the Fermi National Acceleration Facility in Illinois.

The larger issue is that these kinds of findings have implications that are not limited to the laboratory. They affect the whole of society --- not only our economy, but our very view of life, our understanding of our relations with others, and our place in time....

NP2015 + BP2016

2016 Breakthrough Prize in Fundamental Physics (3MUS$)

Kam-Biu Luk and the Daya Bay Collaboration

Yifang Wang and the Daya Bay Collaboration

Koichiro Nishikawa and the K2K and T2K Collaboration

Atsuto Suzuki and the KamLAND Collaboration

Arthur B. McDonald and the SNO Collaboration

Takaaki Kajita and the Super K Collaboration

Yoichiro Suzuki and the Super K Collaboration

- **Kam-Biu Luk**: the Daya Bay Collaboration
- **Yifang Wang**: the Daya Bay Collaboration
- **Koichiro Nishikawa**: the K2K/T2K Collaboration
- **Atsuto Suzuki**: the KamLAND Collaboration
- **Arthur B. McDonald**: the SNO Collaboration
- **Takaaki Kajita**: the Super K Collaboration
- **Yoichiro Suzuki**: the Super K Collaboration

2015 Nobel Prize in Physics

1998 ——2015: Golden time

We've learnt a lot from ν oscillations:

$$\Delta m^2_{21}\ ,\ |\Delta m^2_{31}|\ ,\ \theta_{12}\ ,\ \theta_{13}\ ,\ \theta_{23}$$

It's more exciting that the SM is incomplete, although the Higgs has been discovered.

But a number of open questions:

- **the Majorana nature?**
- **the absolute ν mass scale?**
- **the ν mass hierarchy?**
- **the octant of θ_{23}?**
- **the Dirac phase δ?**
- **the Majorana phases? ……**

There are many other open questions about ν's in particle physics, cosmology, astrophysics …..

Global fit

F. Capozzi et al (2014) —— the standard parametrization:

Parameter	Best fit	1σ range	2σ range	3σ range
Normal neutrino mass ordering ($m_1 < m_2 < m_3$)				
$\Delta m^2_{21}/10^{-5}$ eV^2	7.54	7.32 — 7.80	7.15 — 8.00	6.99 — 8.18
$\Delta m^2_{31}/10^{-3}$ eV^2	2.47	2.41 — 2.53	2.34 — 2.59	2.26 — 2.65
$\sin^2\theta_{12}/10^{-1}$	3.08	2.91 — 3.25	2.75 — 3.42	2.59 — 3.59
$\sin^2\theta_{13}/10^{-2}$	2.34	2.15 — 2.54	1.95 — 2.74	1.76 — 2.95
$\sin^2\theta_{23}/10^{-1}$	4.37	4.14 — 4.70	3.93 — 5.52	3.74 — 6.26
$\delta/180°$	1.39	1.12 — 1.77	0.00 — 0.16 $\oplus$ 0.86 — 2.00	0.00 — 2.00
Inverted neutrino mass ordering ($m_3 < m_1 < m_2$)				
$\Delta m^2_{21}/10^{-5}$ eV^2	7.54	7.32 — 7.80	7.15 — 8.00	6.99 — 8.18
$\Delta m^2_{13}/10^{-3}$ eV^2	2.42	2.36 — 2.48	2.29 — 2.54	2.22 — 2.60
$\sin^2\theta_{12}/10^{-1}$	3.08	2.91 — 3.25	2.75 — 3.42	2.59 — 3.59
$\sin^2\theta_{13}/10^{-2}$	2.40	2.18 — 2.59	1.98 — 2.79	1.78 — 2.98
$\sin^2\theta_{23}/10^{-1}$	4.55	4.24 — 5.94	4.00 — 6.20	3.80 — 6.41
$\delta/180°$	1.31	0.98 — 1.60	0.00 — 0.02 $\oplus$ 0.70 — 2.00	0.00 — 2.00

The PMNS matrix looks so different from the CKM matrix!

Neutrino masses

The absolute ν-mass scale?

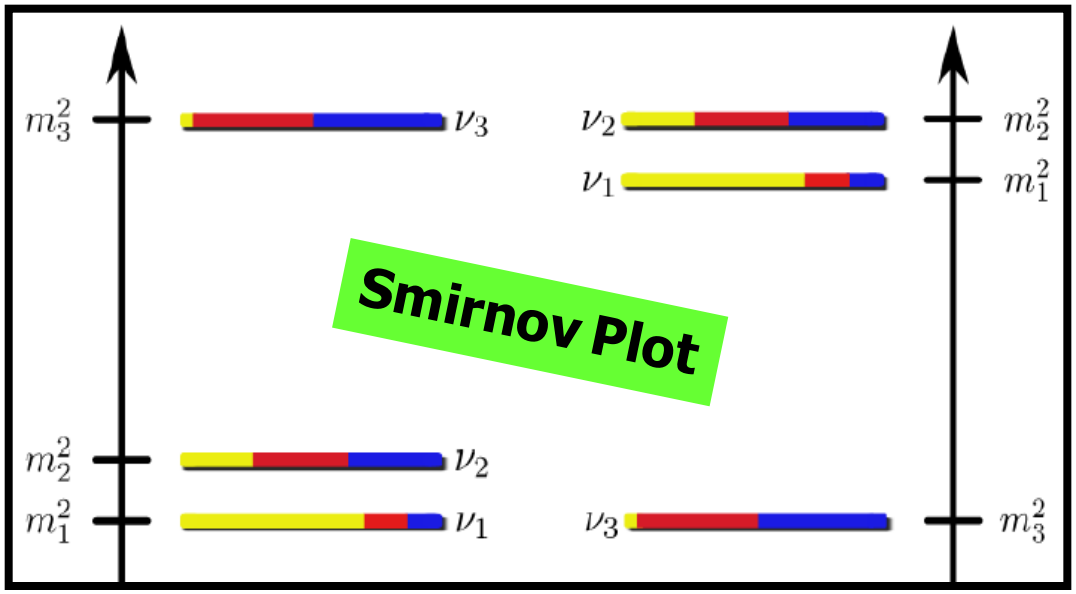
---- The β decay

---- The 0νββ decay

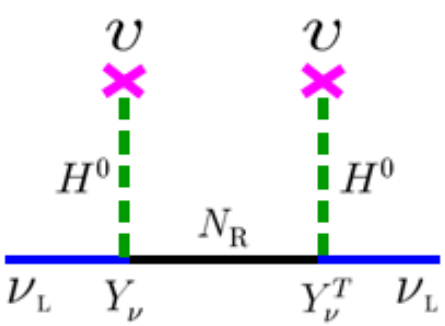
---- CMB + LSS constraints

Min (ν-mass) ~ 0?

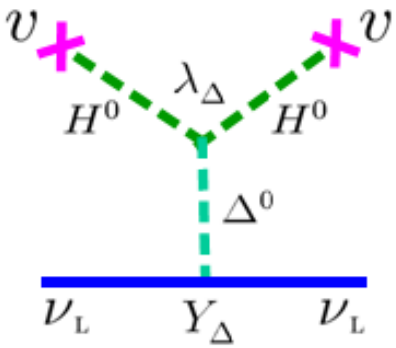
Max (ν-mass) ~ O(0.1) eV?



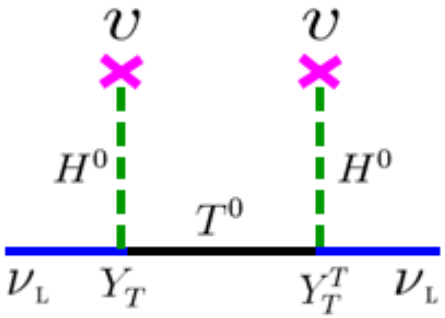
The origin of tiny ν masses: seesaw mechanisms?



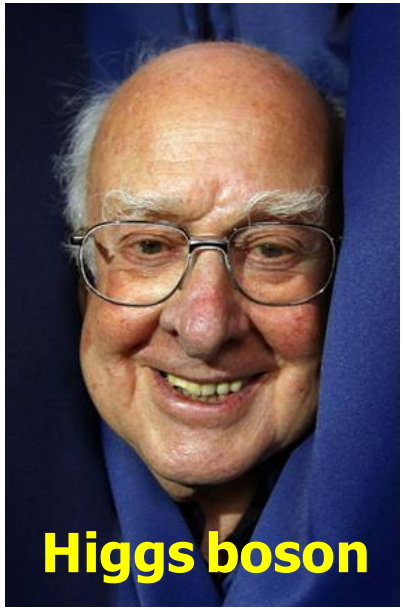
$$M_\nu \approx -v^2 Y_\nu \frac{1}{M_R} Y_\nu^T$$



$$M_\nu \approx \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta}$$



$$M_\nu \approx -v^2 Y_T \frac{1}{M_T} Y_T^T$$



Flavor mixing pattern

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}\left[\overline{(u\ \ c\ \ t)_L}\,\gamma^\mu\, V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_\mu^+ + \overline{(e\ \ \mu\ \ \tau)_L}\,\gamma^\mu\, U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W_\mu^-\right] + \text{h.c.}$$

CKM **PMNS**

Be careful when discussing something like the Q—L complementarity!

Quark mixing: hierarchy!

4/6 parameters

CKM

$$|V| = \begin{array}{c} \\ u \\ c \\ t \end{array}\!\!\begin{array}{c} \begin{array}{ccc} d & s & b \end{array} \\ \begin{bmatrix} \blacksquare & \blacksquare & \cdot \\ \blacksquare & \blacksquare & \blacksquare \\ \cdot & \blacksquare & \blacksquare \end{bmatrix} \end{array}$$

0.004

0.999

4 parameters

PMNS

$$|U| = \begin{array}{c} \\ e \\ \mu \\ \tau \end{array}\!\!\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{bmatrix} \end{array}$$

$\gtrsim 0.8$

Lepton mixing: anarchy?

Ordering or reordering?

All charged fermions: normal mass hierarchies. How about neutrinos?

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}}\left[\overline{(e\ \ \mu\ \ \tau)_{\rm L}}\,\gamma^\mu\, \underbrace{U}_{\textbf{PMNS}} \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}_{\rm L} W^-_\mu + \overline{(u\ \ c\ \ t)_{\rm L}}\,\gamma^\mu\, \underbrace{V}_{\textbf{CKM}} \begin{pmatrix}d\\ s\\ b\end{pmatrix}_{\rm L} W^+_\mu\right] + \text{h.c.}$$

Normal: $m_1 < m_2 < m_3$

Inverted: $m_3 < m_1 < m_2$

Given a weak basis, the mass ordering is fixed. A basis change is possible.

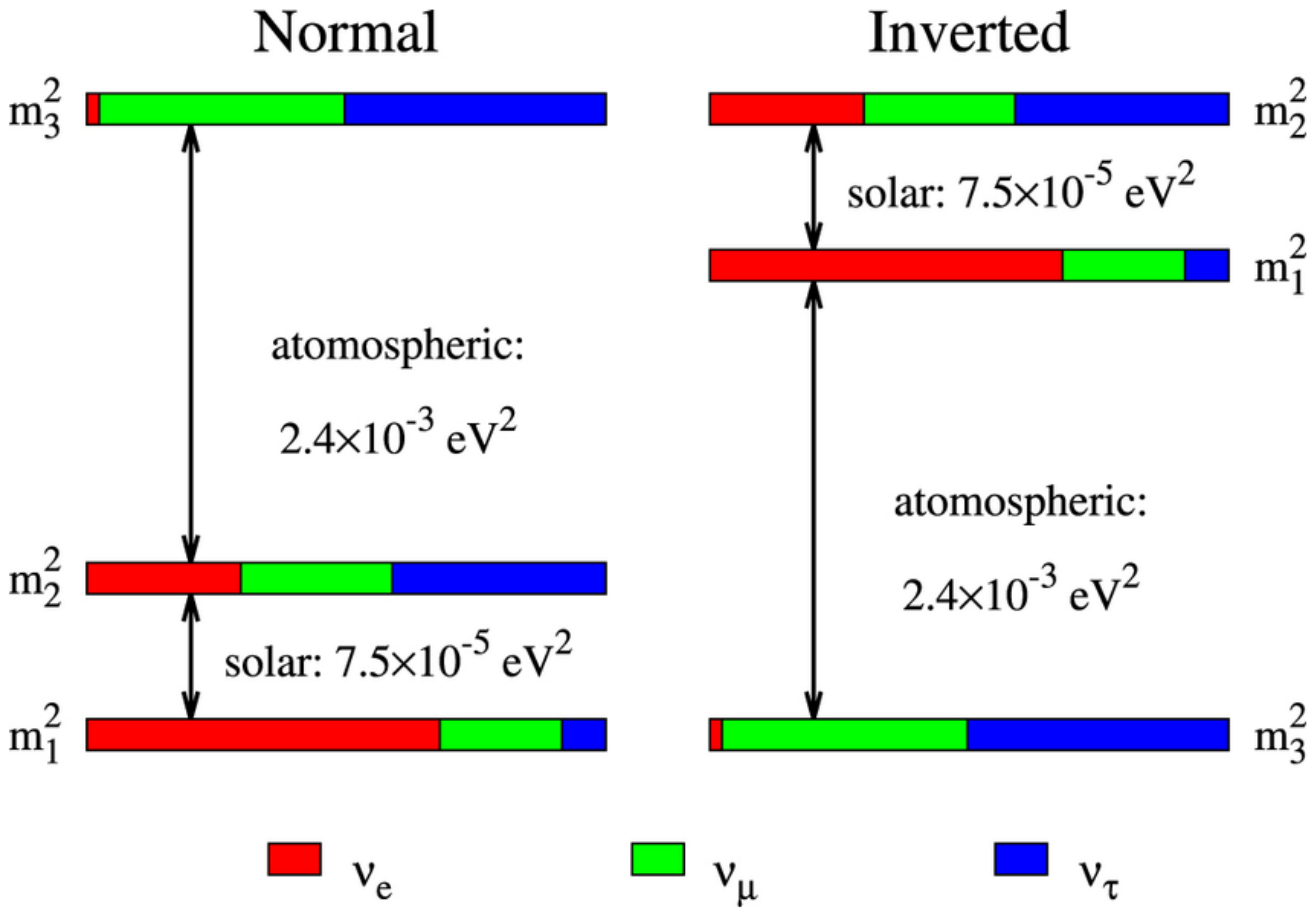
For instance: $(\nu_1, \nu_2, \nu_3) \to (\nu_2', \nu_3', \nu_1')$

Reorder: $m_1' < m_2' < m_3'$

$$|U| = \begin{pmatrix} 0.795 \to 0.846 & 0.513 \to 0.585 & 0.126 \to 0.178 \\ 0.205 \to 0.543 & 0.416 \to 0.730 & 0.579 \to 0.808 \\ 0.215 \to 0.548 & 0.409 \to 0.725 & 0.567 \to 0.800 \end{pmatrix}$$

$$|U'| = \begin{pmatrix} 0.126 \to 0.178 & 0.795 \to 0.846 & 0.513 \to 0.585 \\ 0.579 \to 0.808 & 0.205 \to 0.543 & 0.416 \to 0.730 \\ 0.567 \to 0.800 & 0.215 \to 0.548 & 0.409 \to 0.725 \end{pmatrix}$$

Normal or inverted?



The standard parametrization of the PMNS matrix:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_\nu$$

SK + T2K hint

Summary

SK + T2K fit prefers normal hierarchy at $\chi^2_{IH} - \chi^2_{NH} = 3.2$

SK+T2K fit prefers δ_{cp}~240°

- both SK and T2K separately prefer δ_{cp}~240°, combination strengthens this preference
- sin δ_{cp}=0 is still allowed at 68% confidence

Improvements in mind

Improving Hierarchy Sensitivity

More Data

SK continues to run

T2K $\bar{\nu}$

NOνA

Reduce/Constrain Backgrounds

improve ν_e-ν_μ separation with better MutliGeV PID

constrain ν_τ background with neural net techniques from ν_τ appearance analysis

Improve ν-$\bar{\nu}$ Separation

n-H capture

n-Gd capture (with future addition of Gadolinium)

C. Kachulis

Boston U.

EPS 2015

NOνA hint

Summary

- NOvA has analyzed its first data set for both ν_μ disappearance and ν_e appearance
  - Made 6.5% measurement of Δm^2_{32}; θ_{23} is consistent with maximal mixing
  - Two methods for searching for ν_e events were used, the primary selector finds a signal at 3.3σ while the secondary sees one at 5.5σ
  - Both methods hint at a preference for the normal ordering with $\pi < \delta < 2\pi$
- Data set will be doubled by next summer - stay tuned!

Brian Rebel

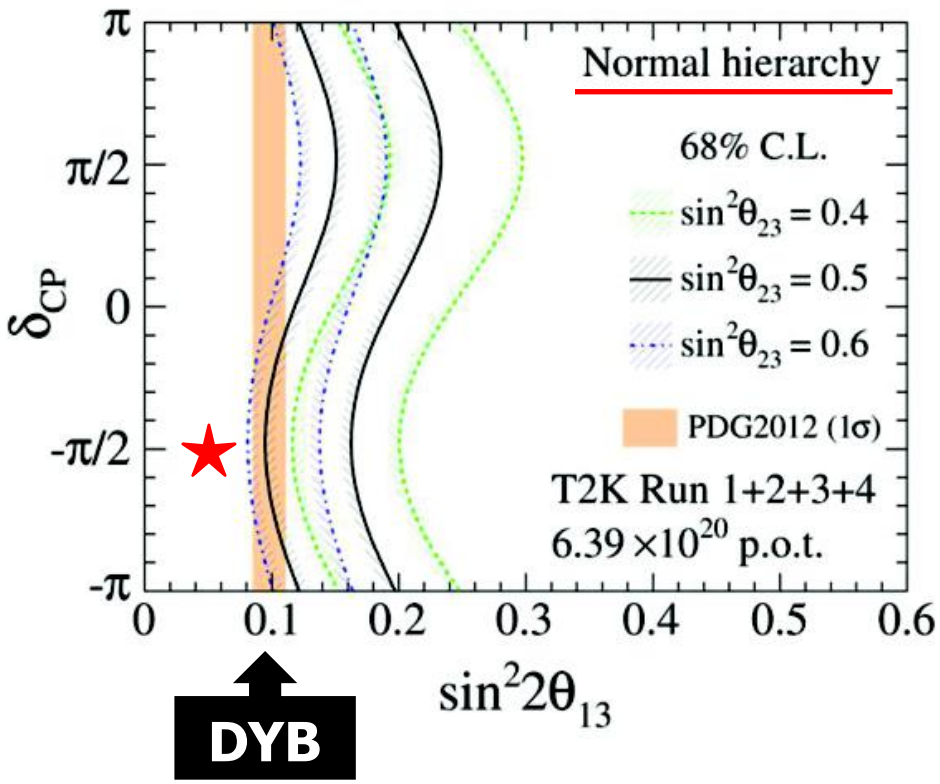
Fermilab

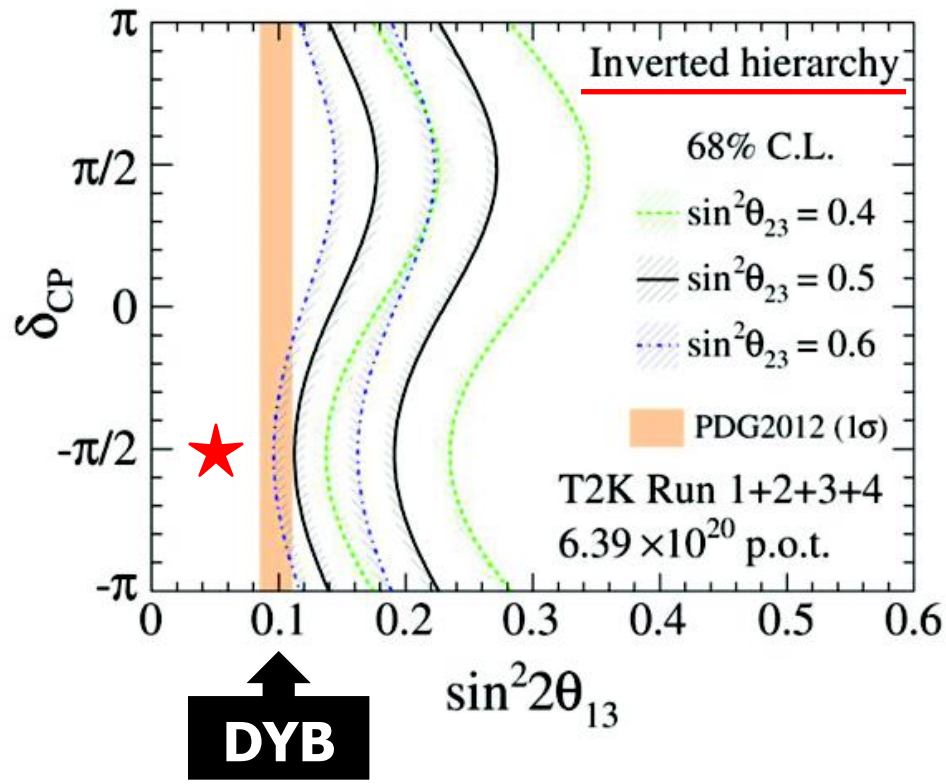
TAUP 2015

www-nova.fnal.gov

T2K + DYB hint

The T2K observation of a relatively strong $\nu_\mu \to \nu_e$ appearance plays a crucial role in the global fit to make θ_{13} consistent with the Daya Bay result and drive a slight but intriguing preference for $\delta \sim -\pi/2$.





DYB's good news: θ_{13} unsuppressed

T2K's good news: δ unsuppressed

Life is easier for probing CP violation, ν mass hierarchy ....

Cosmological hint (1)

Cosmological limits on neutrino unknowns versus low redshift priors

Eleonora Di Valentino,[1] Elena Giusarma,[2] Olga Mena,[3] Alessandro Melchiorri,[2] and Joseph Silk[1,4,5]

arXiv:1511.00975v1 [astro-ph.CO] 3 Nov 2015

Recent Cosmic Microwave Background (CMB) temperature and polarization anisotropy measurements from the Planck mission have significantly improved previous constraints on the neutrino masses as well as the bounds on extended models with massless or massive sterile neutrino states. However, due to parameter degeneracies, additional low redshift priors are mandatory in order to sharpen the CMB neutrino bounds. We explore here the role of different priors on low redshift quantities, such as the Hubble constant, the cluster mass bias, and the reionization optical depth τ. Concerning current priors on the Hubble constant and the cluster mass bias, the bounds on the neutrino parameters may differ appreciably depending on the choices adopted in the analyses. With regard to future improvements in the priors on the reionization optical depth, a value of $\tau = 0.05 \pm 0.01$, motivated by astrophysical estimates of the reionization redshift, would lead to $\sum m_\nu < 0.0993$ eV at 95% CL, thereby opening the window to unravel the neutrino mass hierarchy with existing cosmological probes.

	Planck pol +BAO+SZ+tau6	Planck pol +BAO+SZ+tau5	Planck pol H073p0+SZ+tau6	Planck pol H073p0+SZ+tau5	Planck pol+BAO +H073p0+SZ+tau6	Planck pol +BAO +H073p0+SZ+tau5
$\Omega_c h^2$	$0.1194^{+0.0021}_{-0.0021}$	$0.1195^{+0.0021}_{-0.0021}$	$0.1190^{+0.0026}_{-0.0025}$	$0.1192^{+0.0026}_{-0.0025}$	$0.1190^{+0.0020}_{-0.0020}$	$0.1192^{+0.0020}_{-0.0021}$
Σm_ν [eV]	< 0.122	< 0.116	< 0.112	< 0.107	< 0.104	< 0.0993
H_0 [km s^{-1} Mpc^{-1}]	$67.7^{+1.0}_{-1.0}$	$67.6^{+1.0}_{-1.0}$	$67.9^{+1.3}_{-1.4}$	$67.8^{+1.2}_{-1.4}$	$67.88^{+0.96}_{-0.98}$	$67.83^{+0.99}_{-0.98}$
σ_8	$0.823^{+0.022}_{-0.024}$	$0.818^{+0.022}_{-0.023}$	$0.824^{+0.022}_{-0.023}$	$0.819^{+0.021}_{-0.022}$	$0.824^{+0.021}_{-0.022}$	$0.819^{+0.021}_{-0.022}$
Ω_m	$0.311^{+0.013}_{-0.013}$	$0.311^{+0.014}_{-0.013}$	$0.307^{+0.018}_{-0.017}$	$0.309^{+0.018}_{-0.017}$	$0.308^{+0.013}_{-0.012}$	$0.308^{+0.013}_{-0.013}$
τ	$0.066^{+0.017}_{-0.017}$	$0.059^{+0.017}_{-0.017}$	$0.067^{+0.017}_{-0.017}$	$0.060^{+0.017}_{-0.017}$	$0.067^{+0.017}_{-0.017}$	$0.059^{+0.017}_{-0.017}$

Cosmological hint (2)

Neutrino mass limits: robust information from the power spectrum of galaxy surveys

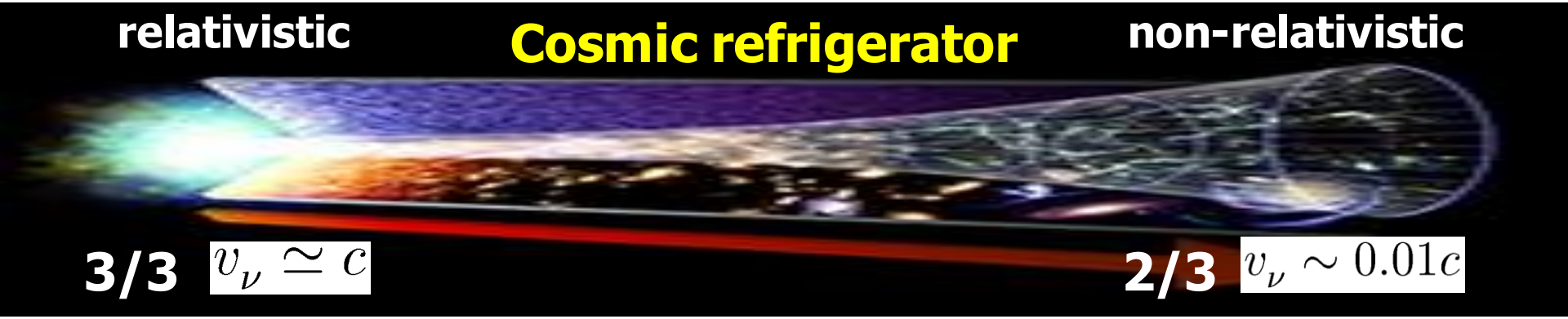
Antonio J. Cuesta[a*], Viviana Niro[b†], Licia Verde[acde‡]

We present cosmological upper limits on the sum of active neutrino masses using large-scale power spectrum data from the WiggleZ Dark Energy Survey and from the Sloan Digital Sky Survey -Data Release 7 (SDSS-DR7) sample of Luminous Red Galaxies (LRG). Combining measurements on the Cosmic Microwave Background temperature and polarisation anisotropies by the Planck satellite together with WiggleZ power spectrum results in a neutrino mass bound of 0.43 eV at 95% C.L., while replacing WiggleZ by the SDSS-DR7 LRG power spectrum, the 95% C.L. bound on the sum of neutrino masses improves to 0.17 eV. Adding Baryon Acoustic Oscillation (BAO) distance scale measurements, the neutrino mass upper limits greatly improve, since BAO data break degeneracies in parameter space. Within a ΛCDM model, we find an upper limit of 0.11eV (0.15 eV) at 95% C.L., when using SDSS-DR7 LRG (WiggleZ) together with BAO and Planck. The addition of BAO data makes the neutrino mass upper limit robust, showing only a weak dependence on the power spectrum used. We also quantify the dependence of neutrino mass limit reported here on the CMB lensing information. The tighter upper limit (0.11 eV) obtained with SDSS-DR7 LRG is very close to that recently obtained using Lyman-alpha clustering data, yet uses a completely different probe and redshift range, further supporting the robustness of the constraint. This constraint puts under some pressure the inverted mass hierarchy and favours the normal hierarchy.

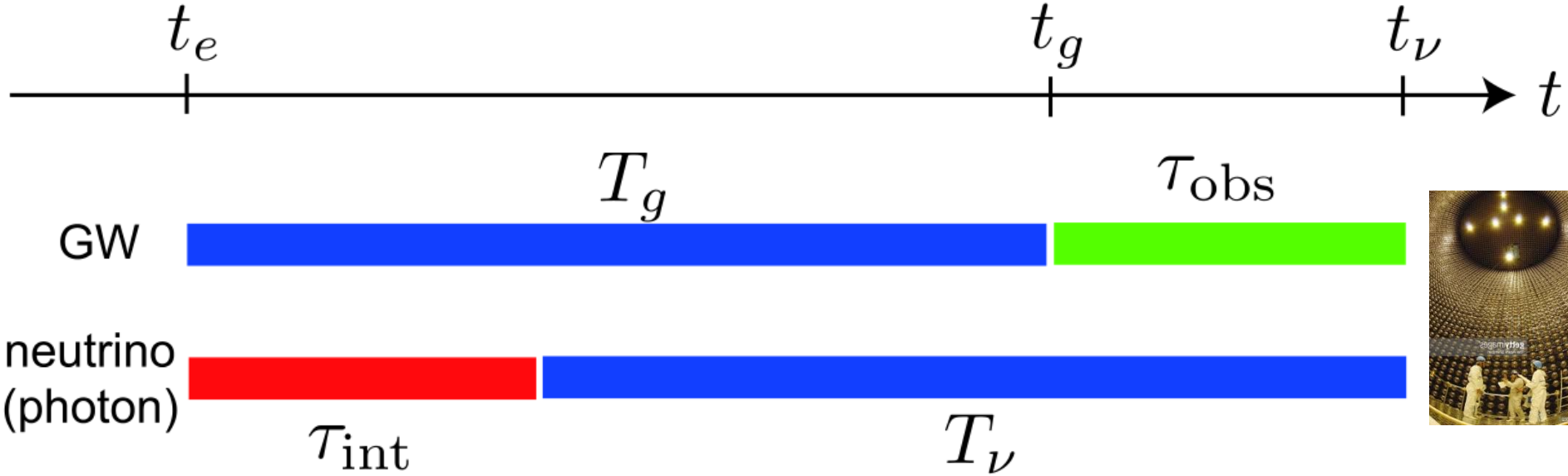
arXiv:1511.05983v1 [astro-ph.CO] 18 Nov 2015

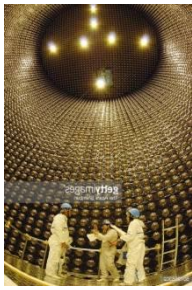
CνB or astrophysical ν's

In today's Universe, at least 2/3 of relic neutrinos (T ~ 1.9 K ~ 0.0005 eV) are not hot anymore, depending on the mass hierarchy.



Gravitational waves, neutrinos, photons from compact binary mergers and supernovae (Nishizawa, Nakamura 14; Langaeble, Meroni, Sannino 16)



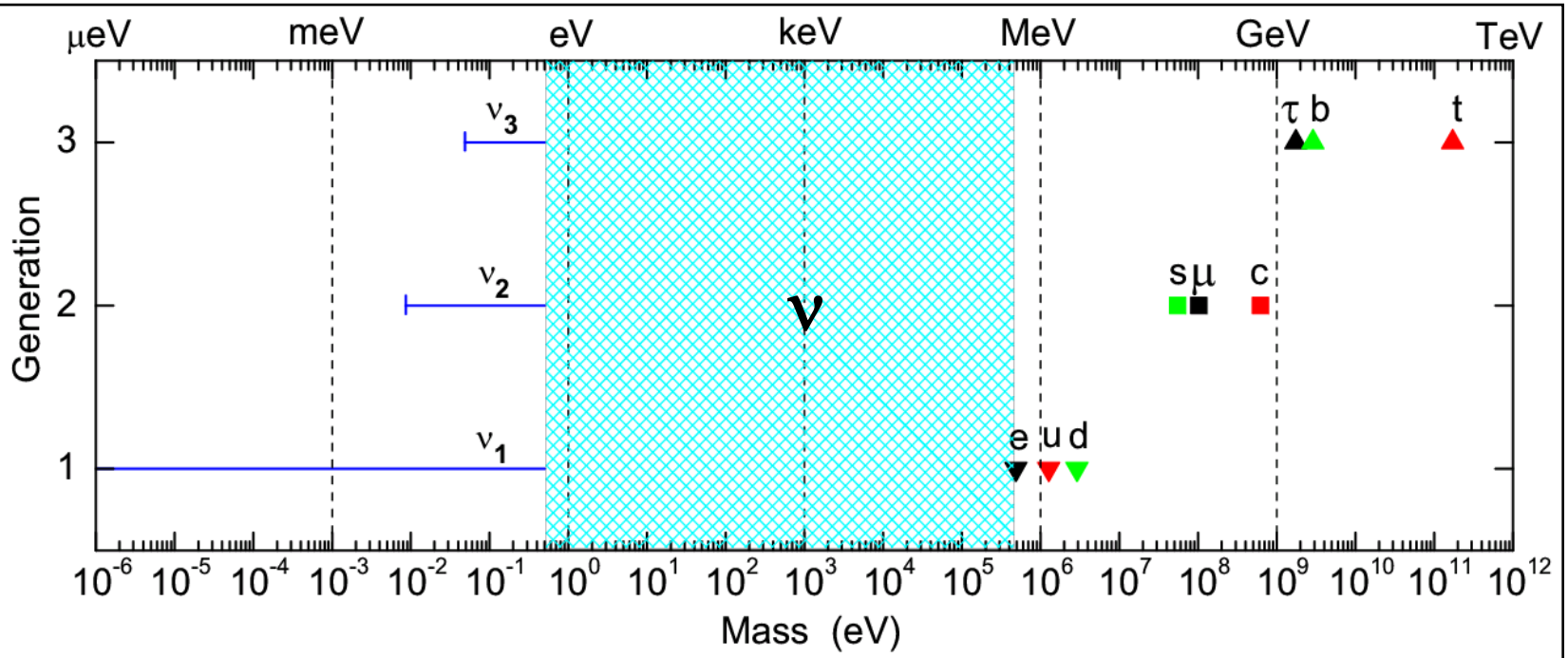


The seesaw portal

The split seesaw: 2 heavy right-handed neutrinos + 1 keV right-handed neutrino (Kusenko, Yanagida 2010, 2016).

Existence of life **requires: a) primordial antimatter disappeared; b) dark matter mattered ---- anthropic arguments.**

Split seesaw + thermal leptogenesis + warm DM ---- one stone kills 3 birds.



... EeV...

ν

ν

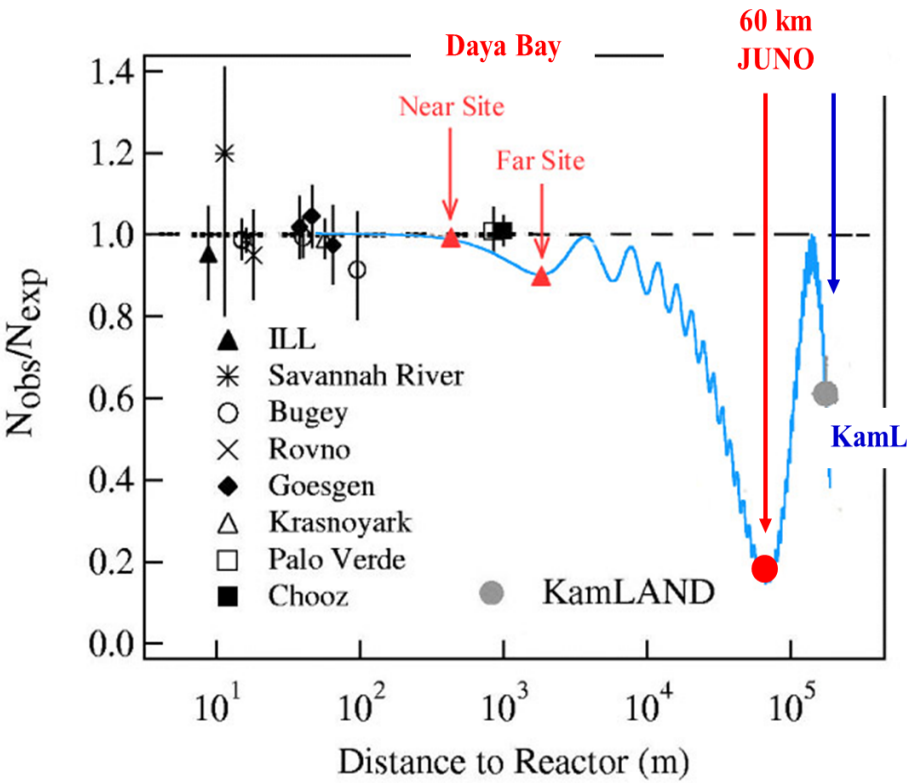
Oscillation experiments

Accelerator/atmospheric: terrestrial matter effects play crucial roles.

$$\Delta m^2_{31} \mp 2\sqrt{2} G_F N_e E$$

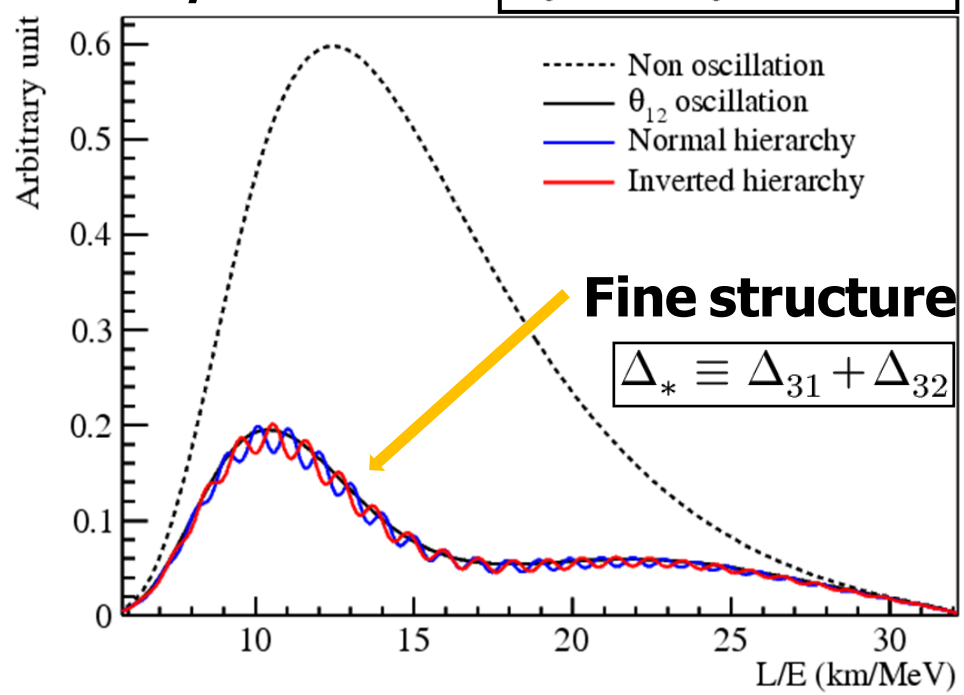
T2K, NOνA, SK, PINGU, INO, …

Reactor (JUNO): Optimum baseline at the valley of Δm^2_{21} oscillations, corrected by the fine structure of Δm^2_{31} oscillations.



JUNO / RENO-50

$$F_{ji} \equiv \Delta_{ji} L/(4E)$$



$$P(\overline{\nu}_e \to \overline{\nu}_e) = 1 - \sin^2 2\theta_{12} \cos^4\theta_{13} \sin^2 F_{21} - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \cos F_* \cos F_{21} + \cos 2\theta_{12} \sin F_* \sin F_{21}\right)$$

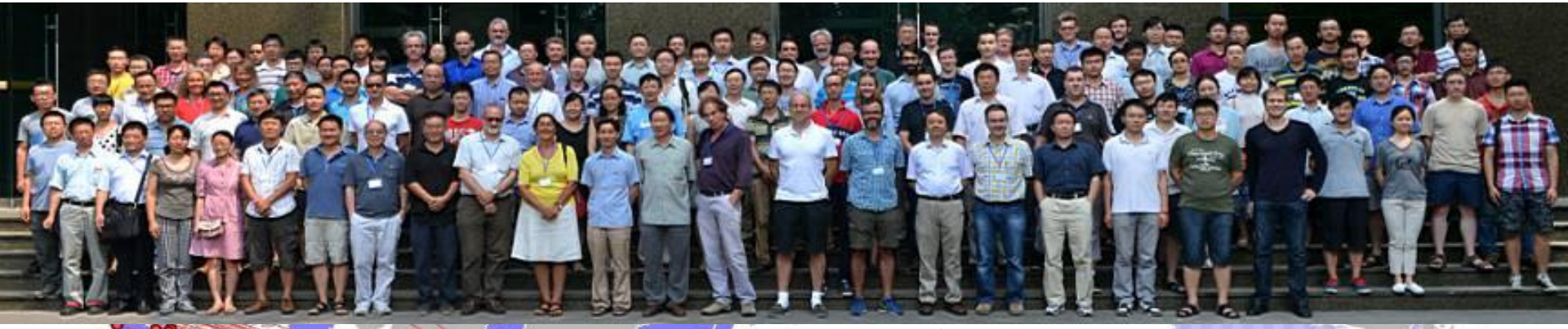
JUNO in progress

	Daya Bay	Yangjiang	Taishan
Status	running	construction	construction
Power/GW	17.4	17.4	18.4

Roman Mythology

- Idea in 2008
- 20 kton LS detector
  3% E-resolution
- Approved in 2/2013
  ~ 2 billion CNY

JUNO collaboration



Europe (24)

France(5)
- APC Paris
- CPPM Marseille
- IPHC Strasbourg
- LLR Paris
- Subatech Nantes

Czech(1)
- Charles U

Finland(1)
- U.Oulu

Russia(2)
- INR Moscow
- JINR

Italy(7)
- INFN-Frascati
- INFN-Ferrara
- INFN-Milano
- INFN-Mi-Bicocca
- INFN-Padova
- INFN-Perugia
- INFN-Roma 3

Armenia(1)
- Yerevan Phys. Inst.

Belgium(1)
- ULB

Germany(6)
- FZ Julich
- RWTH Aachen
- TUM
- U.Hamburg
- U.Mainz
- U.Tuebingen

America(3)

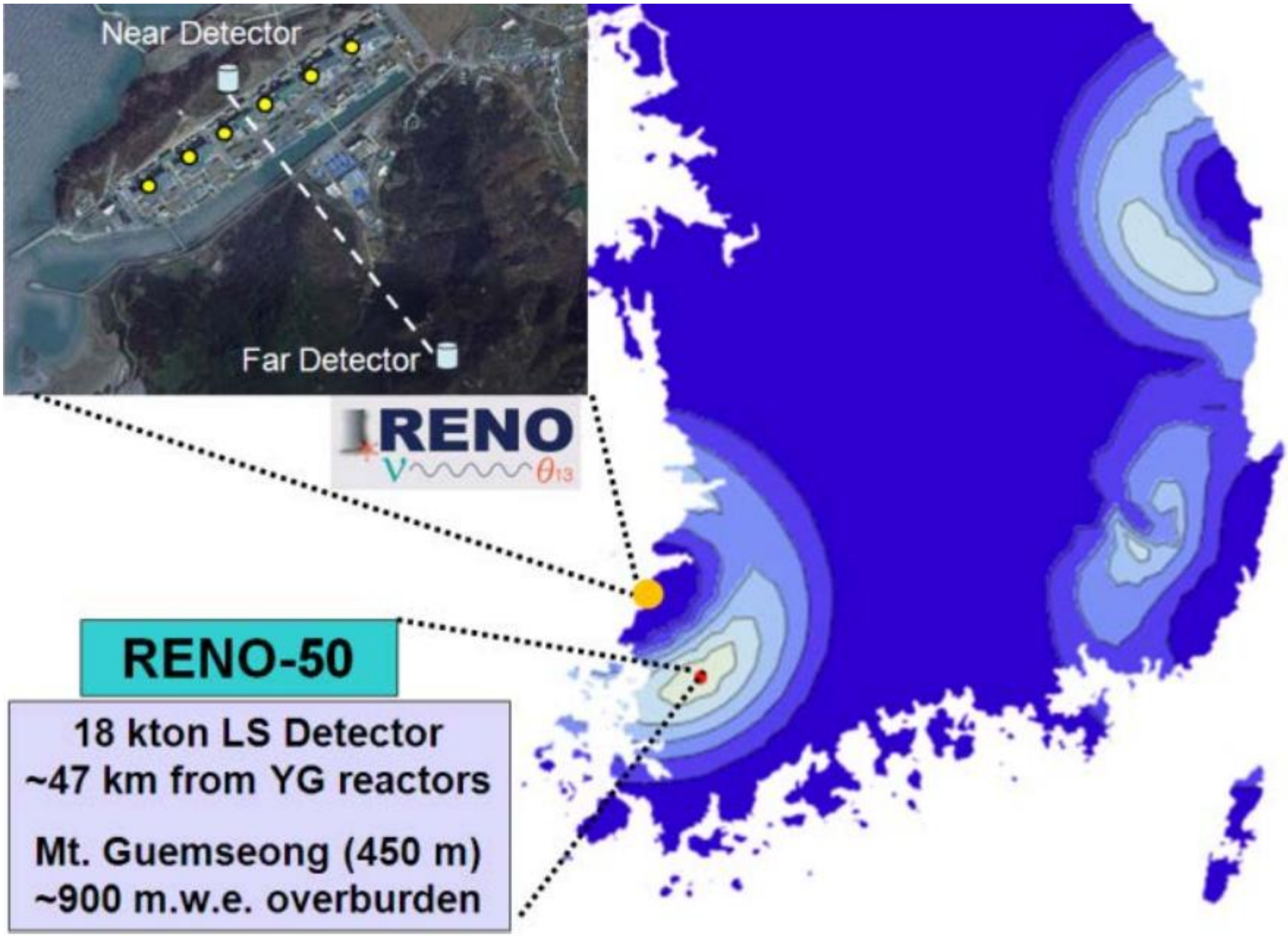
US(2)
- UMD
- UMD-Geo

Chile(1)
- Catholic Univ. of Chile

Asia (28)

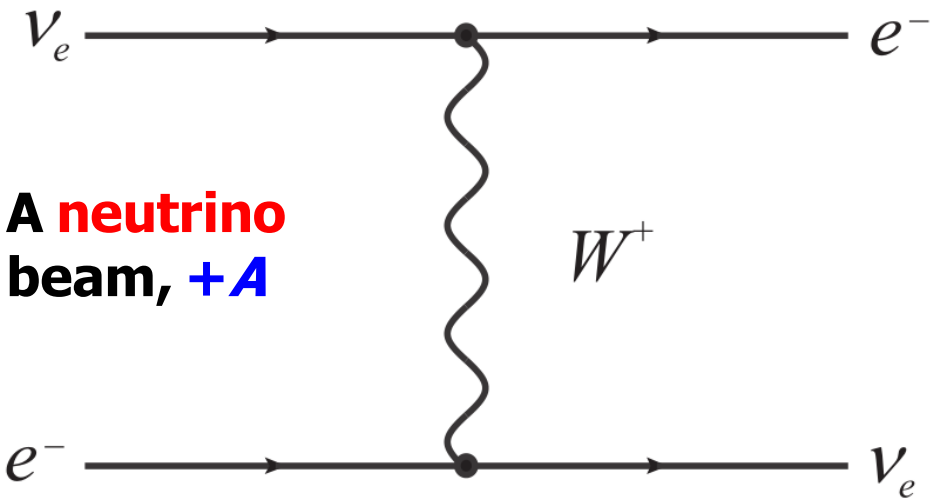
- BJ Nor. U.
- CAGS
- Chongqing U.
- CIAE
- DGUT
- ECUST
- Guangxi U.
- HIT
- IHEP
- Jilin U.
- Nanjing U.
- Nankai U.
- Chiao-Tung U.
- Taiwan U.
- United U.
- NCEPU
- Pekin U.
- Shandong U.
- Shanghai JT U.
- Sichuan U.
- SYSU
- Tsinghua U.
- UCAS
- USTC
- Wuhan U.
- Wuyi U.
- Xi'an JT U.
- Xiamen U.

RENO-50 under proposal

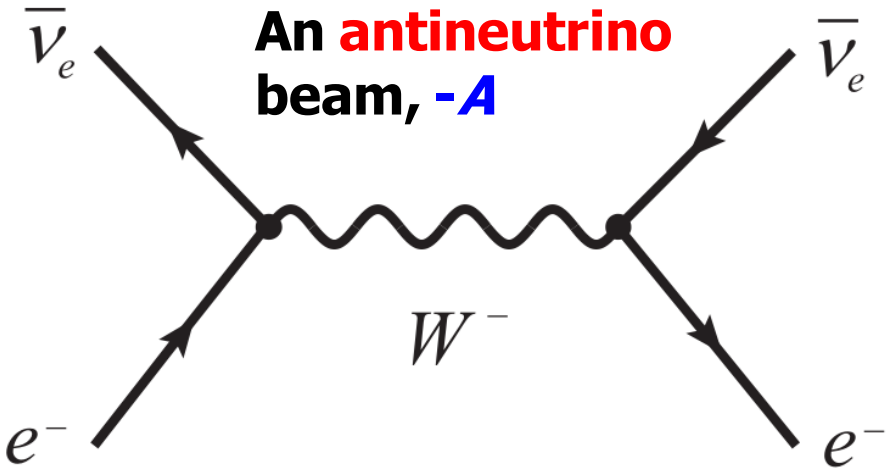


Oscillations in matter

In accelerator / reactor-based (anti)neutrino oscillation experiments, it makes sense to check terrestrial matter effects.



A neutrino beam, +*A*



An antineutrino beam, -*A*

Coherent forward scattering from weak charged-current interactions.
The effective Hamiltonian for antineutrino propagation in matter:

$$\widetilde{\mathcal{H}}_{\rm eff} = \frac{1}{2E}\left[\widetilde{U}\begin{pmatrix}\widetilde{m}_1^2 & 0 & 0\\ 0 & \widetilde{m}_2^2 & 0\\ 0 & 0 & \widetilde{m}_3^2\end{pmatrix}\widetilde{U}^\dagger\right] = \frac{1}{2E}\left[U\begin{pmatrix}m_1^2 & 0 & 0\\ 0 & m_2^2 & 0\\ 0 & 0 & m_3^2\end{pmatrix}U^\dagger - \begin{pmatrix}A & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right]$$

in matter **in vacuum** **correction**

One may establish the relations between the effective & fundamental parameters.

$$A = 2\sqrt{2}\, G_{\rm F} N_e E$$

Matter effects

The effective antineutrino mass-squared differences in matter:

$\Delta_{31} > 0$

$$\begin{aligned}
\tilde{\Delta}_{21} &= \frac{2}{3}\sqrt{x^2 - 3y}\sqrt{3\left(1 - z^2\right)} \\
\tilde{\Delta}_{31} &= \frac{1}{3}\sqrt{x^2 - 3y}\left[3z + \sqrt{3\left(1 - z^2\right)}\right] \\
\tilde{\Delta}_{32} &= \frac{1}{3}\sqrt{x^2 - 3y}\left[3z - \sqrt{3\left(1 - z^2\right)}\right]
\end{aligned}$$

$\Delta_{31} < 0$

$$\begin{aligned}
\tilde{\Delta}_{21} &= \frac{1}{3}\sqrt{x^2 - 3y}\left[3z - \sqrt{3\left(1 - z^2\right)}\right] \\
\tilde{\Delta}_{31} &= -\frac{2}{3}\sqrt{x^2 - 3y}\sqrt{3\left(1 - z^2\right)} \\
\tilde{\Delta}_{32} &= -\frac{1}{3}\sqrt{x^2 - 3y}\left[3z + \sqrt{3\left(1 - z^2\right)}\right]
\end{aligned}$$

$$\begin{aligned}
x &= \Delta_{31}\left(1 + \alpha - \beta\right) \\
y &= \Delta_{31}^2\left[\alpha - \beta\left(|U_{e1}|^2 + |U_{e2}|^2\right) - \alpha\beta\left(1 - |U_{e2}|^2\right)\right]
\end{aligned}$$

$$\alpha \equiv \Delta_{21}/\Delta_{31}$$

$$\beta \equiv A/\Delta_{31}$$

$$z = \cos\left[\frac{1}{3}\arccos\frac{2x^3 - 9xy - 27\Delta_{31}^3\alpha\beta|U_{e1}|^2}{2\sqrt{\left(x^2 - 3y\right)^3}}\right]$$

Barger et al, 80
Zaglauer, Schwarzer, 88
ZZX, 00

Matter parameter:

$$A \sim 1.52 \times 10^{-4}\ \mathrm{eV}^2\ Y_e\left(\rho/\mathrm{g/cm}^3\right)\left(E/\mathrm{GeV}\right) \simeq 1.98 \times 10^{-4}\ \mathrm{eV}^2\left(E/\mathrm{GeV}\right)$$

$Y_e \simeq 0.5$ (electron fraction), $\rho \simeq 2.6\ \mathrm{g/cm}^3$ (matter density of Earth's crust)

Sum rules

The effective Hamiltonian with a constant matter profile:

$$\widetilde{\mathcal{H}}_{\rm eff} = \frac{1}{2E}\left[\widetilde{U}\begin{pmatrix}\widetilde{m}_1^2 & 0 & 0\\ 0 & \widetilde{m}_2^2 & 0\\ 0 & 0 & \widetilde{m}_3^2\end{pmatrix}\widetilde{U}^\dagger\right] = \frac{1}{2E}\left[U\begin{pmatrix}m_1^2 & 0 & 0\\ 0 & m_2^2 & 0\\ 0 & 0 & m_3^2\end{pmatrix}U^\dagger - \begin{pmatrix}A & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right]$$

in matter **in vacuum** **correction**

It allows us to establish sum rules between matter and vacuum, from which one may calculate the moduli of the PMNS matrix elements.

$$\sum_{i=1}^{3}\widetilde{m}_i^2\widetilde{U}_{\alpha i}\widetilde{U}^*_{\beta i} = \sum_{i=1}^{3} m_i^2 U_{\alpha i}U^*_{\beta i} - A\delta_{\alpha e}\delta_{e\beta}$$

$$\sum_{i=1}^{3}\widetilde{m}_i^4\widetilde{U}_{\alpha i}\widetilde{U}^*_{\beta i} = \sum_{i=1}^{3} m_i^2\left[m_i^2 - A\left(\delta_{\alpha e}+\delta_{e\beta}\right)\right]U_{\alpha i}U^*_{\beta i} + A^2\delta_{\alpha e}\delta_{e\beta}$$

$$\sum_{i=1}^{3}\widetilde{U}_{\alpha i}\widetilde{U}^*_{\beta i} = \sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i} = \delta_{\alpha\beta}$$

$$\sum_{i=1}^{3}\widetilde{m}_i^2 = \sum_{i=1}^{3} m_i^2 - A$$

The first row of $U(1)$

Relations between the vacuum and matter elements (Zhang, Xing 05)

$$|\widetilde{U}_{e1}|^2 = +\frac{\Delta'_{21}\Delta'_{31}}{\widetilde{\Delta}_{21}\widetilde{\Delta}_{31}}|U_{e1}|^2 + \frac{\Delta'_{11}\Delta'_{31}}{\widetilde{\Delta}_{21}\widetilde{\Delta}_{31}}|U_{e2}|^2 + \frac{\Delta'_{11}\Delta'_{21}}{\widetilde{\Delta}_{21}\widetilde{\Delta}_{31}}|U_{e3}|^2$$

$$|\widetilde{U}_{e2}|^2 = -\frac{\Delta'_{22}\Delta'_{32}}{\widetilde{\Delta}_{21}\widetilde{\Delta}_{32}}|U_{e1}|^2 - \frac{\Delta'_{12}\Delta'_{32}}{\widetilde{\Delta}_{21}\widetilde{\Delta}_{32}}|U_{e2}|^2 - \frac{\Delta'_{12}\Delta'_{22}}{\widetilde{\Delta}_{21}\widetilde{\Delta}_{32}}|U_{e3}|^2$$

$$|\widetilde{U}_{e3}|^2 = +\frac{\Delta'_{23}\Delta'_{33}}{\widetilde{\Delta}_{31}\widetilde{\Delta}_{32}}|U_{e1}|^2 + \frac{\Delta'_{13}\Delta'_{33}}{\widetilde{\Delta}_{31}\widetilde{\Delta}_{32}}|U_{e2}|^2 + \frac{\Delta'_{13}\Delta'_{23}}{\widetilde{\Delta}_{31}\widetilde{\Delta}_{32}}|U_{e3}|^2$$

$$\Delta_{ij} \equiv m_i^2 - m_j^2$$

$$\widetilde{\Delta}_{ij} \equiv \widetilde{m}_i^2 - \widetilde{m}_j^2$$

$$\Delta'_{ij} \equiv m_i^2 - \widetilde{m}_j^2$$

Analytical expansions in terms of two small parameters:

$$\alpha \equiv \frac{\Delta_{21}}{\Delta_{31}} \simeq 3.1 \cdot 10^{-2} \cdot \frac{\Delta_{21}}{7.5 \cdot 10^{-5}\ \mathrm{eV}^2} \cdot \frac{\pm 2.4 \cdot 10^{-3}\ \mathrm{eV}^2}{\Delta_{31}}$$

$$\beta \equiv \frac{A}{\Delta_{31}} \simeq 3.3 \cdot 10^{-4} \cdot \frac{E}{4\ \mathrm{MeV}} \cdot \frac{\pm 2.4 \cdot 10^{-3}\ \mathrm{eV}^2}{\Delta_{31}}$$

$$\widetilde{\Delta}_{21} \simeq \Delta_{31}\left(1 - \frac{3}{2}|U_{e3}|^2\beta\right)\epsilon$$

$$\widetilde{\Delta}_{31} \simeq \Delta_{31}\left[1 - \frac{1}{2}\alpha + \frac{1}{2}\left(1 - 3|U_{e3}|^2\right)\beta + \frac{1}{2}\epsilon + \frac{3}{4}|U_{e3}|^2\beta\left(2\beta - \epsilon\right)\right]$$

$$\widetilde{\Delta}_{32} \simeq \Delta_{31}\left[1 - \frac{1}{2}\alpha + \frac{1}{2}\left(1 - 3|U_{e3}|^2\right)\beta - \frac{1}{2}\epsilon + \frac{3}{4}|U_{e3}|^2\beta\left(2\beta + \epsilon\right)\right]$$

$$\rho \simeq 2.6\ \mathrm{g/cm}^3$$

matter density

The first row of $U(2)$

Xing, Zhu, arXiv: 1603.02002

$$\Delta'_{11} \simeq -\Delta_{31}\left[\frac{1}{2}\alpha - \frac{1}{2}\left(1-|U_{e3}|^2\right)\beta - \frac{1}{2}\epsilon - \frac{1}{4}|U_{e3}|^2\beta\left(2\beta-3\epsilon\right)\right]$$

$$\Delta'_{12} \simeq -\Delta_{31}\left[\frac{1}{2}\alpha - \frac{1}{2}\left(1-|U_{e3}|^2\right)\beta + \frac{1}{2}\epsilon - \frac{1}{4}|U_{e3}|^2\beta\left(2\beta+3\epsilon\right)\right]$$

$$\Delta'_{13} \simeq -\Delta_{31}\left(1-|U_{e3}|^2\beta+|U_{e3}|^2\beta^2\right) \ ,$$

$$\Delta'_{21} \simeq +\Delta_{31}\left[\frac{1}{2}\alpha + \frac{1}{2}\left(1-|U_{e3}|^2\right)\beta + \frac{1}{2}\epsilon + \frac{1}{4}|U_{e3}|^2\beta\left(2\beta-3\epsilon\right)\right]$$

$$\Delta'_{22} \simeq +\Delta_{31}\left[\frac{1}{2}\alpha + \frac{1}{2}\left(1-|U_{e3}|^2\right)\beta - \frac{1}{2}\epsilon + \frac{1}{4}|U_{e3}|^2\beta\left(2\beta+3\epsilon\right)\right]$$

$$\Delta'_{23} \simeq -\Delta_{31}\left(1-\alpha-|U_{e3}|^2\beta+|U_{e3}|^2\beta^2\right) \ ,$$

$$\Delta'_{31} \simeq +\Delta_{31}\left[1-\frac{1}{2}\alpha + \frac{1}{2}\left(1-|U_{e3}|^2\right)\beta + \frac{1}{2}\epsilon + \frac{1}{4}|U_{e3}|^2\beta\left(2\beta-3\epsilon\right)\right]$$

$$\Delta'_{32} \simeq +\Delta_{31}\left[1-\frac{1}{2}\alpha + \frac{1}{2}\left(1-|U_{e3}|^2\right)\beta - \frac{1}{2}\epsilon + \frac{1}{4}|U_{e3}|^2\beta\left(2\beta+3\epsilon\right)\right]$$

$$\Delta'_{33} \simeq +\Delta_{31}\left(|U_{e3}|^2\beta+|U_{e3}|^2\beta^2\right) \ ,$$

where we have defined another small quantity:

$$\boxed{\epsilon \equiv \sqrt{\alpha^2 + 2\left(|U_{e1}|^2-|U_{e2}|^2\right)\alpha\beta + \left(1-2|U_{e3}|^2\right)\beta^2}}$$

The first row of $U(3)$

The smallness of β, as compared with α, leads to

$$\epsilon \simeq \alpha + \beta \cos 2\theta_{12}$$

$$|\widetilde{U}_{e1}|^2 \simeq \frac{\alpha + \beta \cos^2\theta_{12}}{\alpha + \beta \cos 2\theta_{12}} |U_{e1}|^2 + \frac{\beta \cos^2\theta_{12}}{\alpha + \beta \cos 2\theta_{12}} |U_{e2}|^2$$

$$|\widetilde{U}_{e2}|^2 \simeq \frac{\alpha - \beta \sin^2\theta_{12}}{\alpha + \beta \cos 2\theta_{12}} |U_{e2}|^2 - \frac{\beta \sin^2\theta_{12}}{\alpha + \beta \cos 2\theta_{12}} |U_{e1}|^2$$

$$|\widetilde{U}_{e3}|^2 \simeq (1 - 2\beta)\, |U_{e3}|^2 \longrightarrow \widetilde{\theta}_{13} \simeq \theta_{13}$$

$$|U_{e1}| = \cos\theta_{12} \cos\theta_{13}$$
$$|U_{e2}| = \sin\theta_{12} \cos\theta_{13}$$
$$|U_{e3}| = \sin\theta_{13}$$

$$\sin^2 2\widetilde{\theta}_{12} \simeq \sin^2 2\theta_{12} \left(1 - 2\frac{A}{\Delta_{21}} \cos 2\theta_{12}\right)$$

$$\cos 2\widetilde{\theta}_{12} \simeq \cos 2\theta_{12} + \frac{A}{\Delta_{21}} \sin^2\theta_{12}$$

$$\frac{A}{\Delta_{21}} \simeq 1.05 \times 10^{-2} \times \frac{E}{4\ \mathrm{MeV}} \times \frac{7.5 \times 10^{-5}\ \mathrm{eV}^2}{\Delta_{21}}$$

About 1% correction!

Matter corrections to neutrino mass-squared differences:

$$\widetilde{\Delta}_{21} \simeq \Delta_{21} + A \cos 2\theta_{12}$$
$$\widetilde{\Delta}_{*} \simeq \Delta_{*} + A$$

~1%

~ 0.02%

$$\Delta_{21} \simeq 7.5 \times 10^{-5}\ \mathrm{eV}^2$$
$$\Delta_{*} \simeq \pm 4.8 \times 10^{-3}\ \mathrm{eV}^2$$
$$A \simeq 7.9 \times 10^{-7}\ \mathrm{eV}^2\, (E/4\ \mathrm{MeV})$$

How small is small

The effective probability of reactor antineutrino oscillations in matter:

$$\widetilde{P}(\overline{\nu}_e \to \overline{\nu}_e) = 1 - \sin^2 2\widetilde{\theta}_{12} \cos^4 \widetilde{\theta}_{13} \sin^2 \widetilde{F}_{21} - \frac{1}{2} \sin^2 2\widetilde{\theta}_{13} \left[1 - \cos \widetilde{F}_* \cos \widetilde{F}_{21} + \cos 2\widetilde{\theta}_{12} \sin \widetilde{F}_* \sin \widetilde{F}_{21}\right]$$

about 1% correction!

$$\sin^2 2\widetilde{\theta}_{12} \simeq \sin^2 2\theta_{12} \left(1 - 2\frac{A}{\Delta_{21}} \cos 2\theta_{12}\right)$$

$$\cos 2\widetilde{\theta}_{12} \simeq \cos 2\theta_{12} + \frac{A}{\Delta_{21}} \sin^2 \theta_{12}$$

$$\widetilde{\Delta}_{21} \simeq \Delta_{21} + A \cos 2\theta_{12} \,, \qquad \widetilde{\Delta}_* \simeq \Delta_* + A$$

Terrestrial matter effects at JUNO or RENO-50:

$$\widetilde{P}(\overline{\nu}_e \to \overline{\nu}_e) - P(\overline{\nu}_e \to \overline{\nu}_e) \simeq -A \sin^2 2\theta_{12} \cos 2\theta_{12} \cos^4 \theta_{13} \left(\underline{1267 \frac{L}{E}} \sin 2F_{21} - \frac{2}{\Delta_{21}} \sin^2 F_{21}\right)$$

$$-\frac{1}{2} A \sin^2 2\theta_{13} \left\{\underline{1267 \frac{L}{E}} \left[\left(1 + \cos^2 2\theta_{12}\right) \sin F_* \cos F_{21} + 2 \cos 2\theta_{12} \cos F_* \sin F_{21}\right] + \frac{1}{\Delta_{21}} \sin^2 \theta_{12} \sin F_* \sin F_{21}\right\}$$

L **— in unit of km**

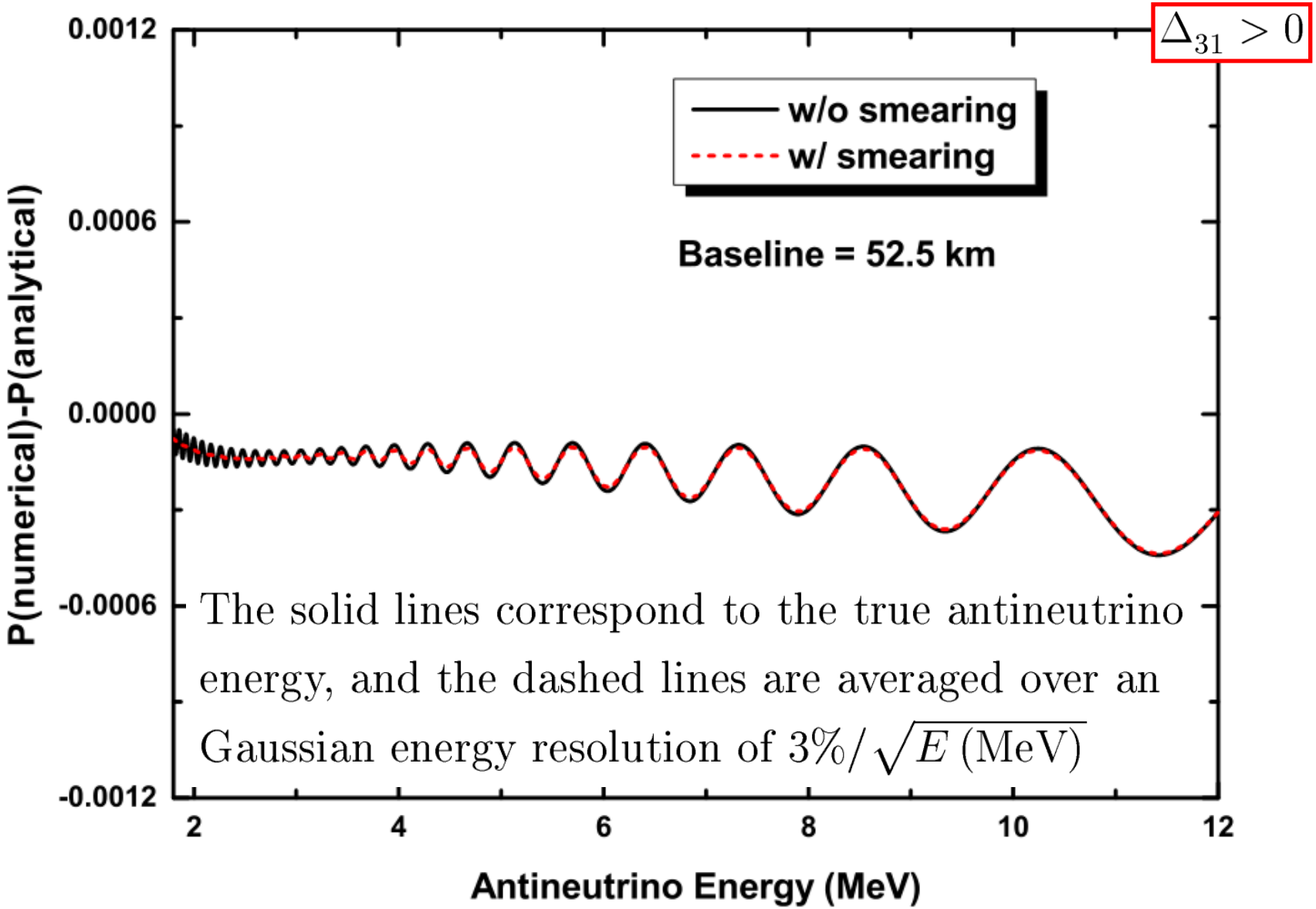
E **— in unit of MeV**

where $F_{21} = 1267 \Delta_{21} L/E \sim \pi/2$ (or equivalently, $L \sim 50$ km)

Matter effects do not invalidate the determination of ν mass ordering!

How good approximation (1)

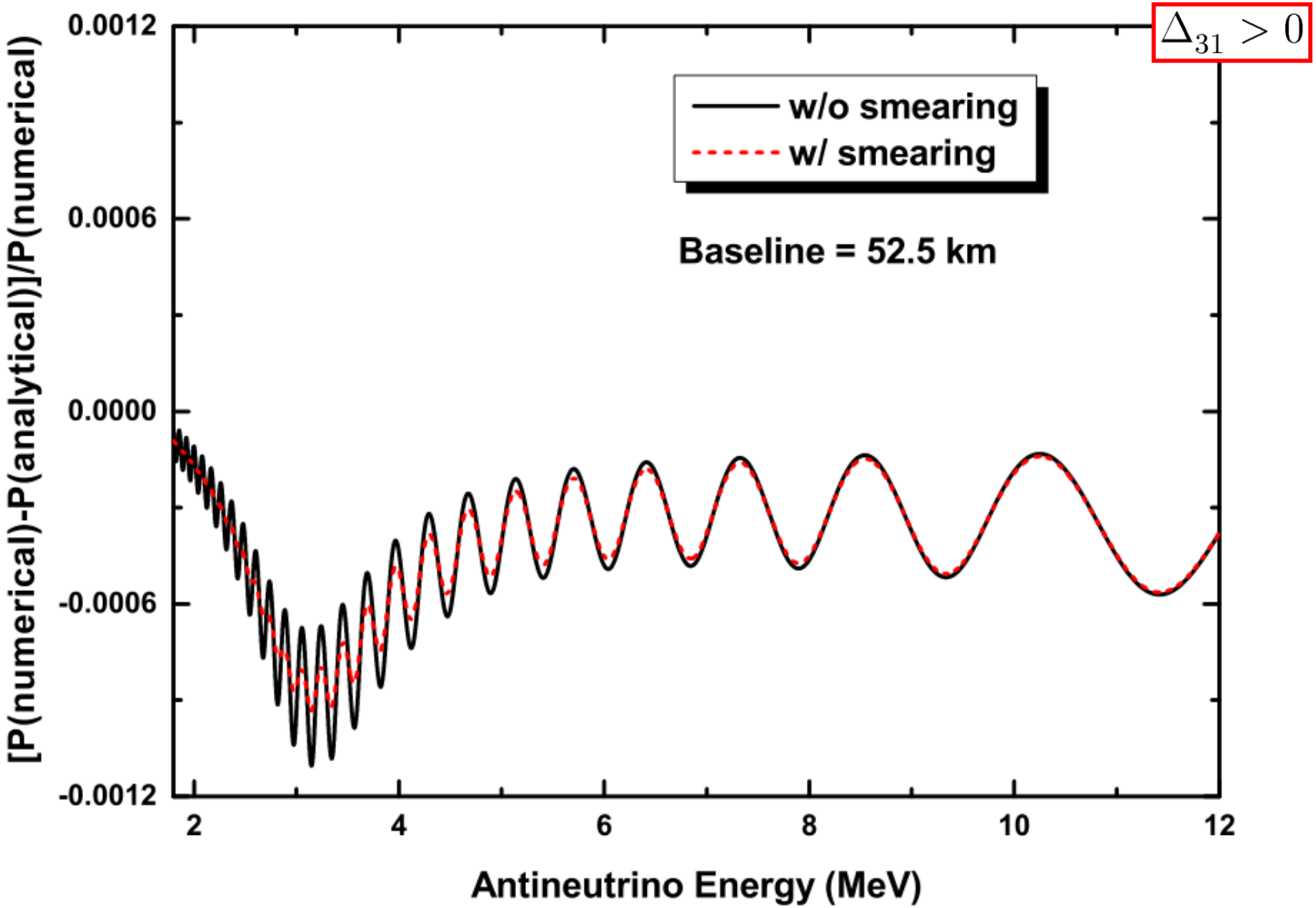
The exact numerical result vs the one from analytical approximations.



The solid lines correspond to the true antineutrino energy, and the dashed lines are averaged over an Gaussian energy resolution of $3\%/\sqrt{E\,(\mathrm{MeV})}$

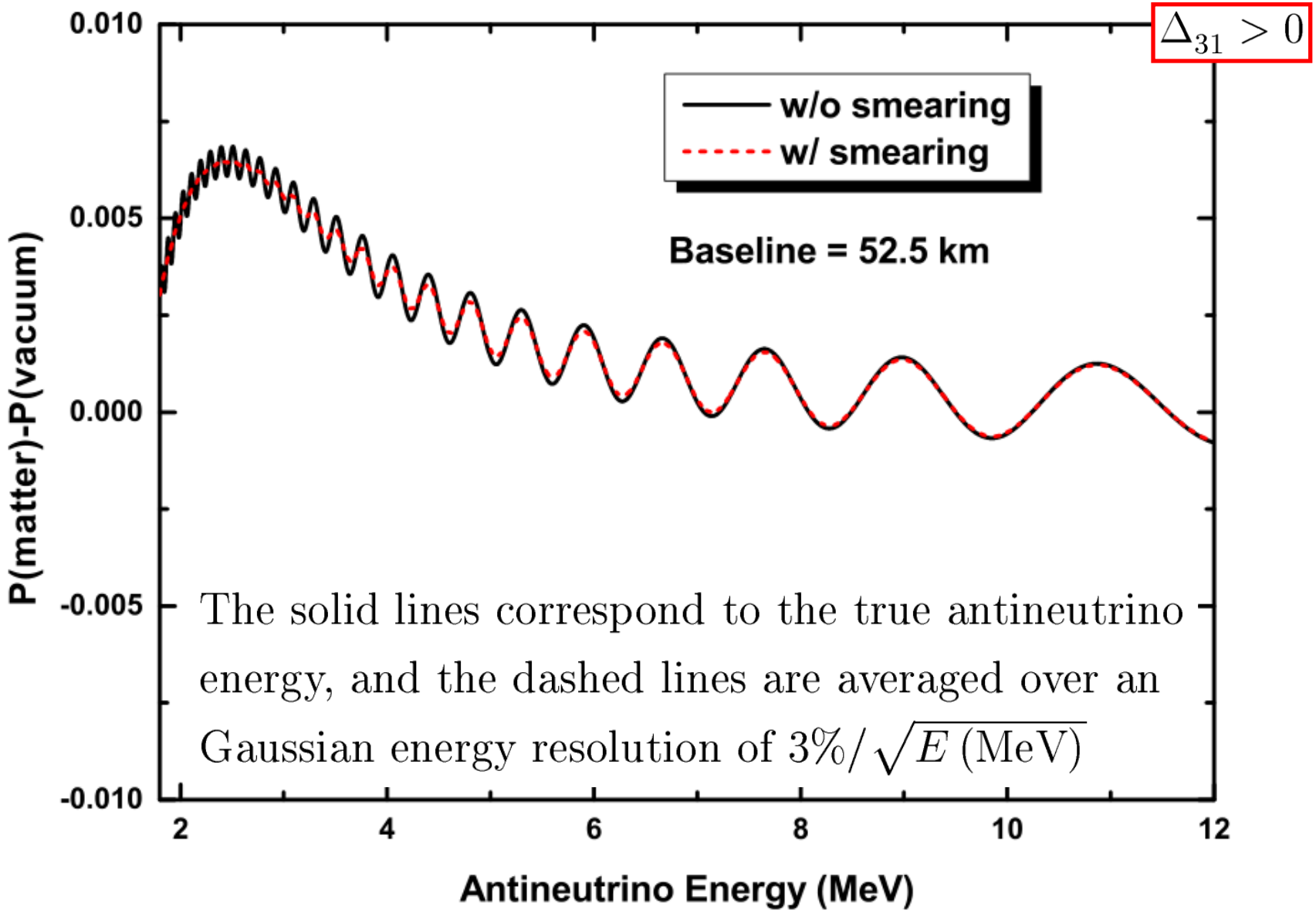
How good approximation (2)

The exact numerical result vs the one from analytical approximations.



Numerical illustration (1)

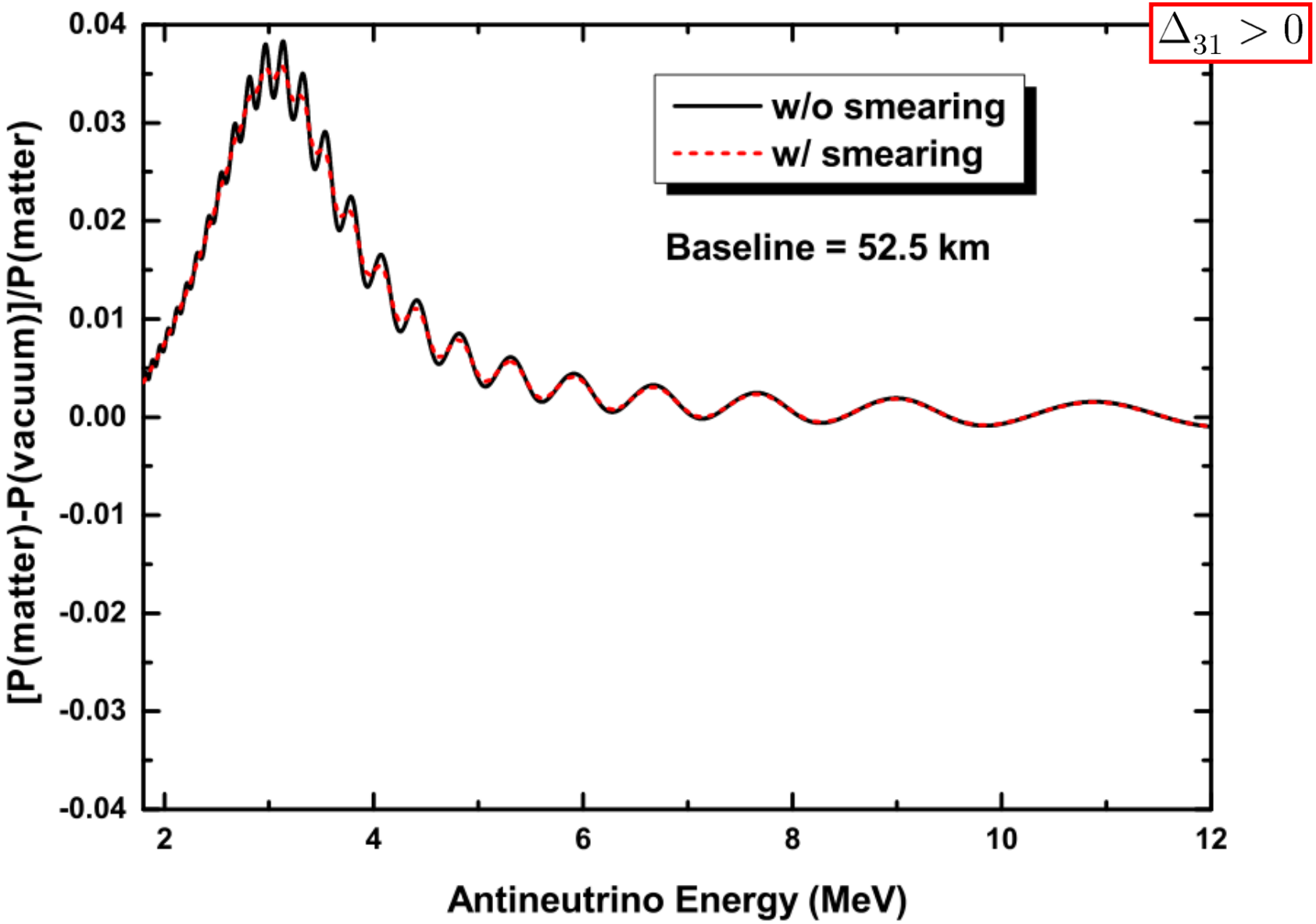
The absolute difference between probabilities in matter and vacuum:



The solid lines correspond to the true antineutrino energy, and the dashed lines are averaged over an Gaussian energy resolution of $3\%/\sqrt{E\,(\mathrm{MeV})}$

Numerical illustration (2)

The relative difference between probabilities in matter and vacuum:



Sensitivity (1)

The JUNO experiment's nominal setup & simulation results have been described in detail in the Yellow Book (1507.05613):

- **20 kt liquid scintillator detector**
- **energy resolution 3%/sqrt{E (MeV)}**
- **total thermal power 36 GWth**
- **power-weighted baseline 52.5 km**
- **detection efficiency 80% (assumption)**
- **nominal running time 6 years**

$$p = \{\Delta_{21}, \Delta_*, \theta_{12}, \theta_{13}\}$$

$$\eta \equiv A(\rho)/A(\rho = 2.6\ \mathrm{g/cm^3})$$

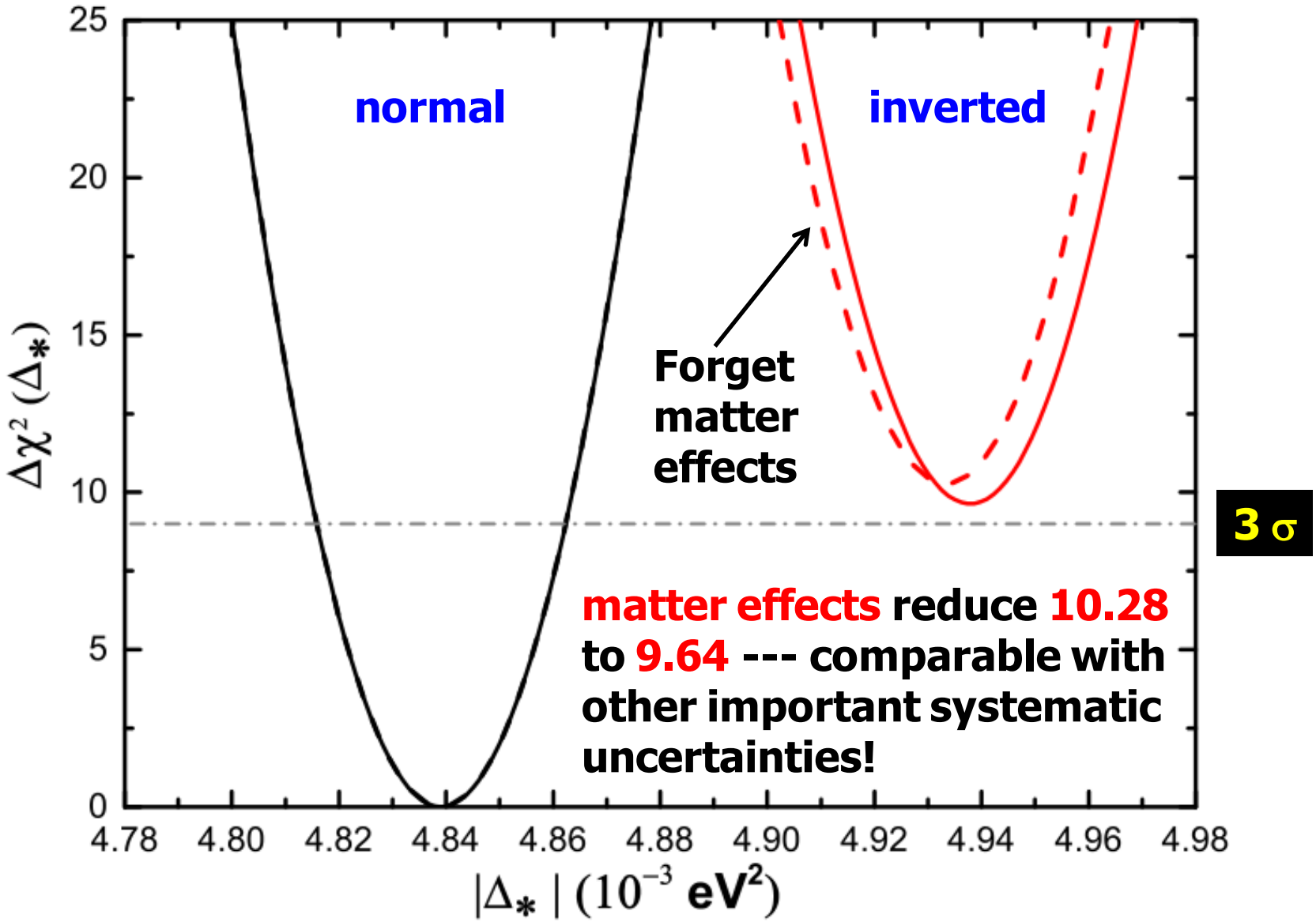
The standard chi-square function:

$$\chi^2 = \sum_{i=1}^{N_{\rm bin}} \frac{\left[M_i(p^M,\eta) - T_i(p^T,\eta)\left(1 + \sum_k \alpha_{ik}\epsilon_k\right)\right]^2}{M_i(p^M,\eta)} + \sum_k \frac{\epsilon_k^2}{\sigma_k^2}$$

Nominal systematic uncertainties: correlated reactor rate uncertainty ~2%; uncorrelated rate uncertainty ~0.8%; energy-uncorrelated bin-to-bin reactor flux spectrum uncertainty ~1%; and detector-related uncertainty ~1% (see the Yellow Book for details).

Sensitivity (2)

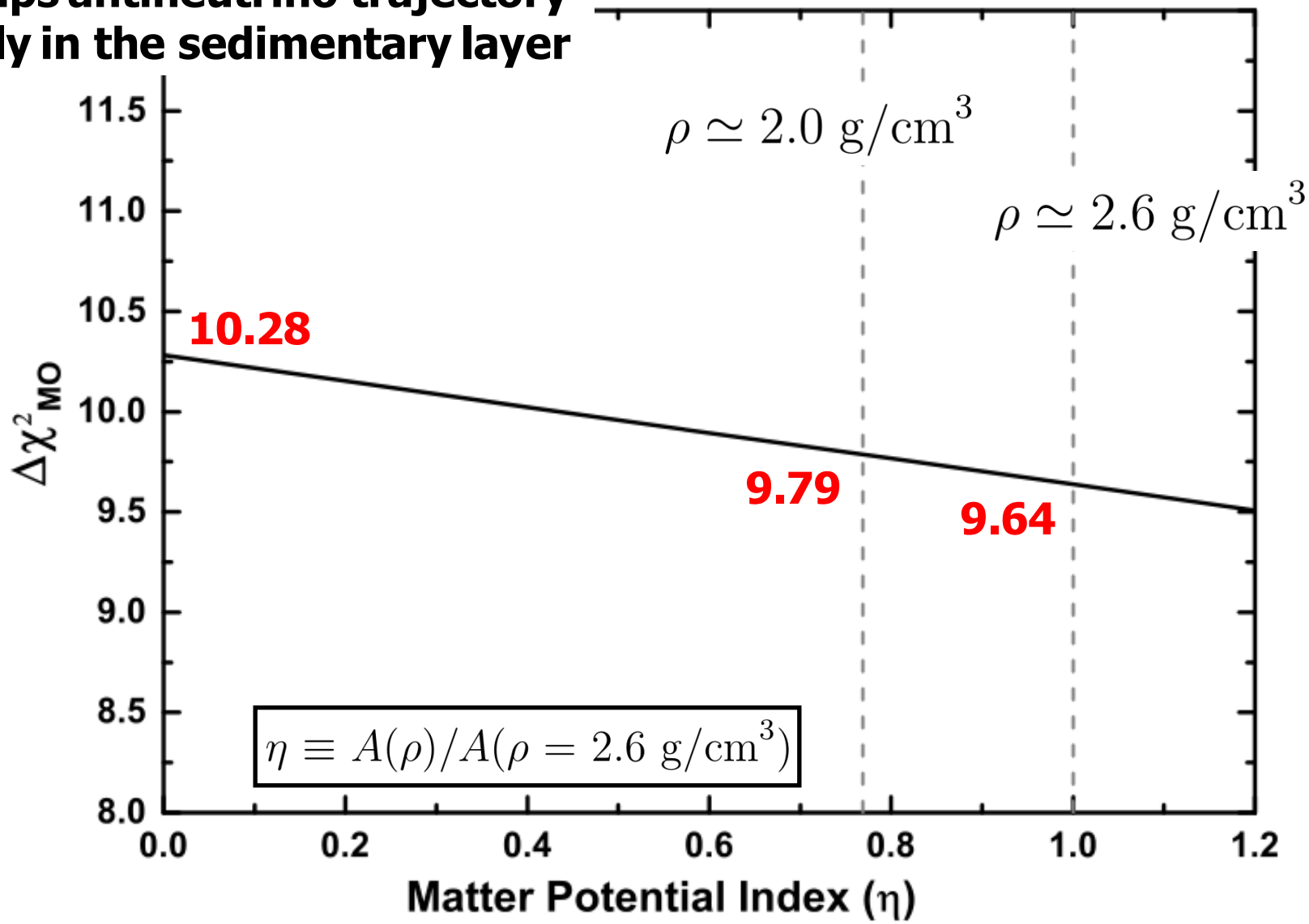
The neutrino mass ordering sensitivity $\Delta\chi^2_{\rm MO} = \left|\chi^2_{\rm min}({\rm NMO}) - \chi^2_{\rm min}({\rm IMO})\right|$



Sensitivity (3)

The neutrino mass ordering sensitivity to the matter index η as below.

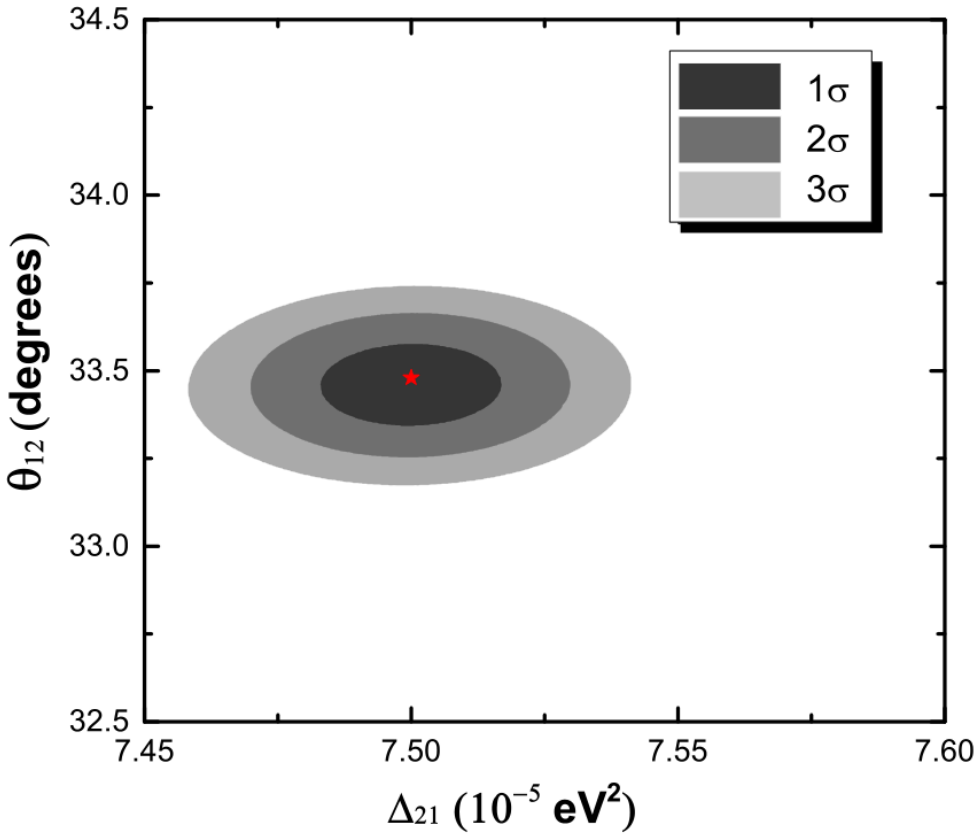
Perhaps antineutrino trajectory largely in the sedimentary layer



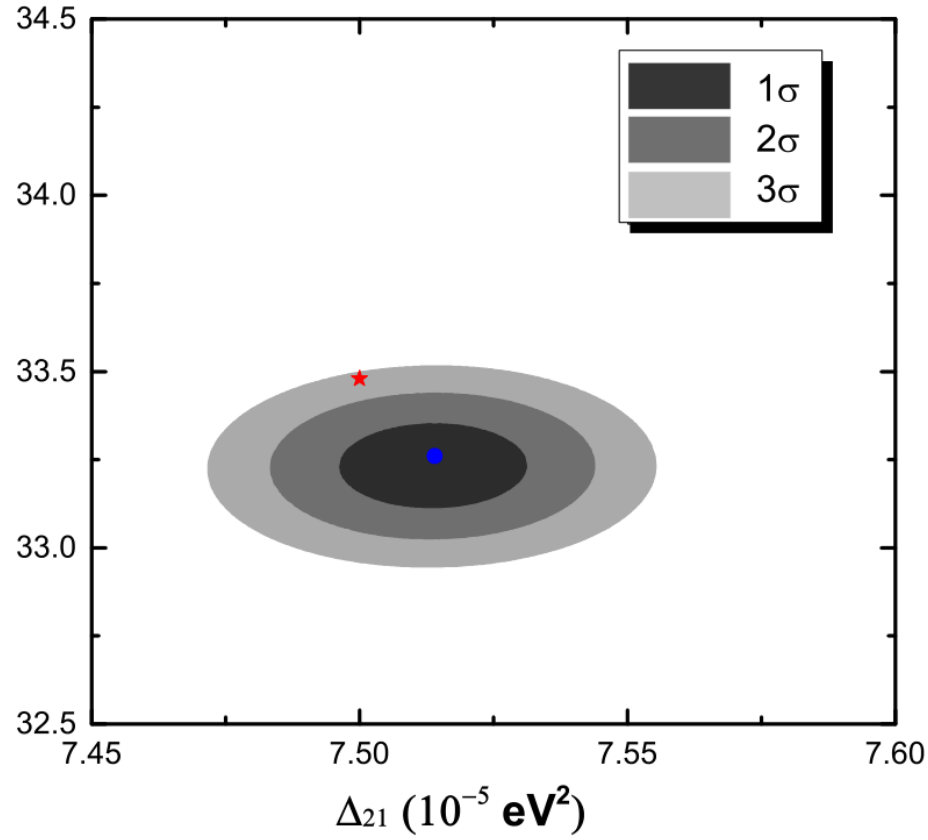
Sensitivity (4)

Terrestrial matter effects on the antineutrino oscillation parameters .

including matter effects



excluding matter effects



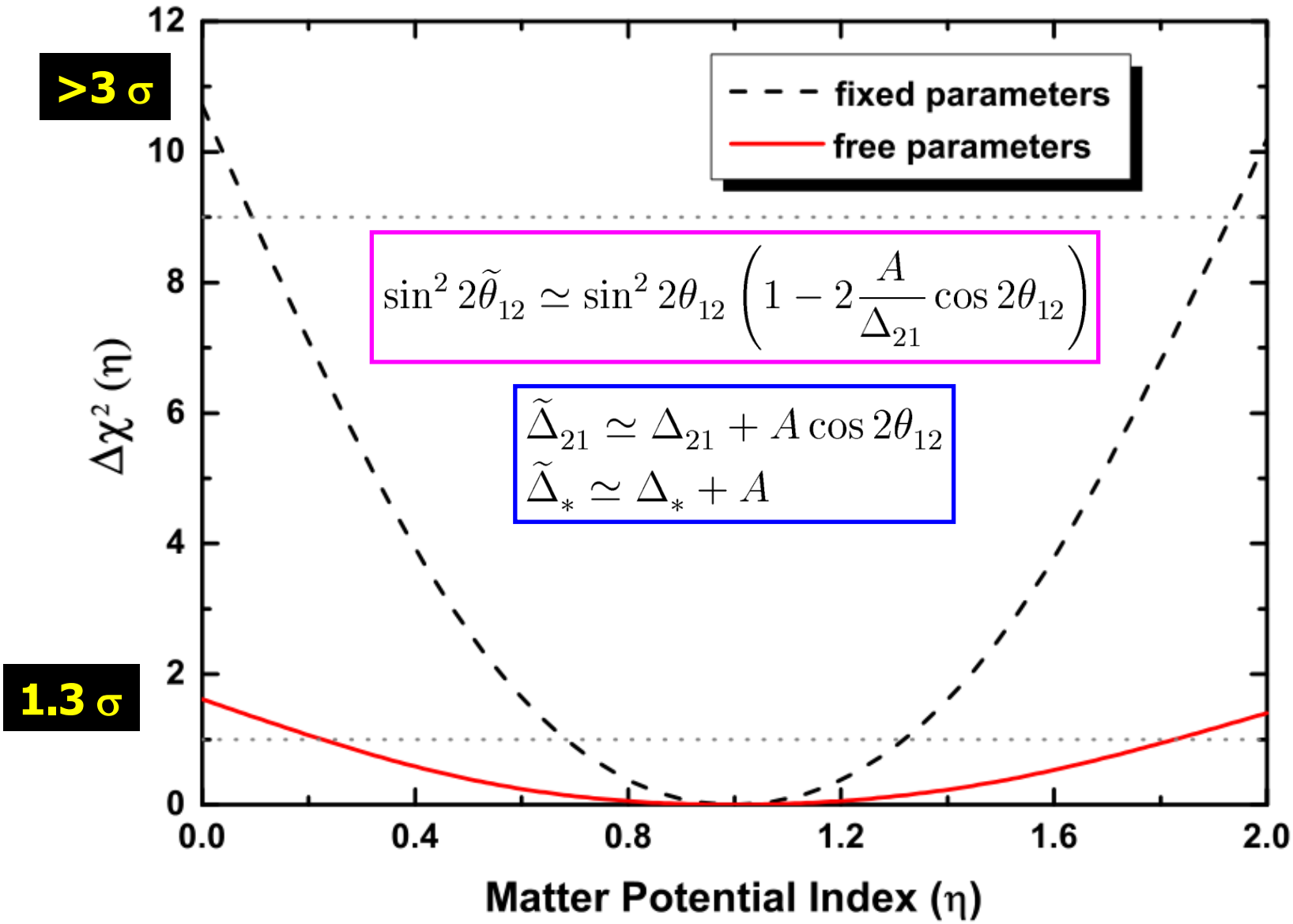
red star: true values

blue dot: best-ft values

Δ_{21} and θ_{12} deviate around 0.8σ and 2.4σ from their true values, respectively.

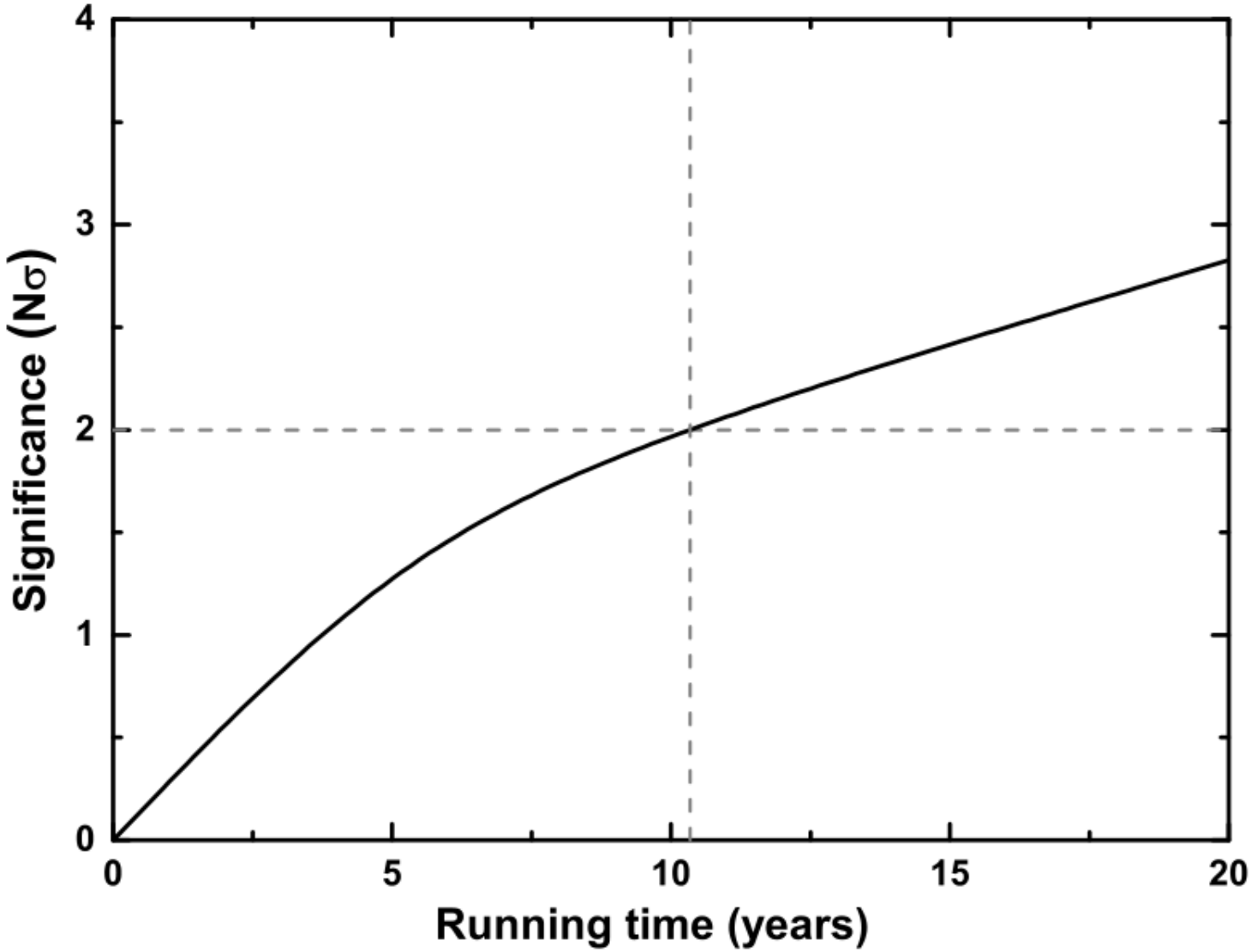
Sensitivity (5)

Sensitivity of terrestrial matter effects with the JUNO nominal setup.



Sensitivity (6)

To establish matter effects and rule out the vacuum oscillation, a near detector is needed. About 10-year running may reach 2 σ significance



Summary

♣ **The neutrino mass ordering remains an open issue that depends on an answer from the experimental side.**

♣ **But, I bet 50 Euro on the normal mass hierarchy, and it will be greater if the mass spectrum is nearly degenerate.**

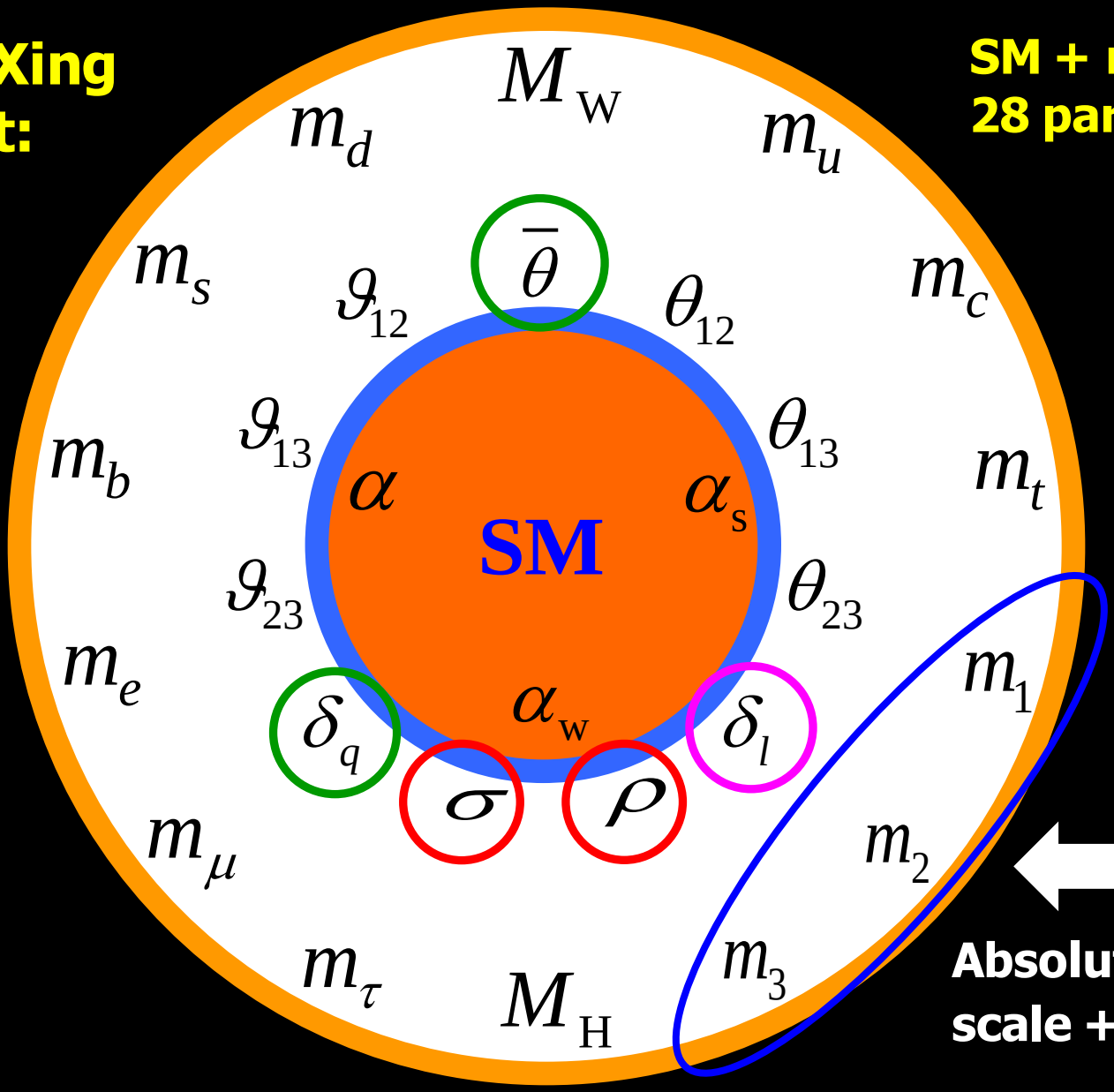
Maury Goodman argues that an intelligent design would prefer the inverted hierarchy.

Einstein: Subtle is the Lord, but malicious He is not (1921).

♣ **JUNO (or RENO-50) has a chance to resolve this issue in a roughly matter-free way. However, we have shown that matter effects are not as small as originally imagined. It's therefore necessary to take into account them in the data analysis in the near future.**

Outlook: Success in flavors is still a long way off

Fritzsch-Xing pizza plot:



SM + massive ν's
28 parameters

Absolute ν mass scale + hierarchy

Martinus Veltman: Anyway, we go on until we go wrong