Gravitational Origin of Dark Matter

mostly based on arXiv 1604.08564

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MINI-WORKSHOP ON "BEYOND THE WIMP PARADIGM"
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Approach I

1. invent a model to explain observations
   → many possibilities
2. check if model is consistent, fits into larger framework, has motivations besides cosmology, etc.

Philosophy

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Approach 2

1. construct a consistent model guided by a fundamental question
   ➡ few possibilities
2. check if it can explain (part of) an observational phenomenon

Philosophy

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Consistent Field Theories

Standard Model of Particle Physics & General Relativity

Spin 0: Higgs boson ϕ

Spin 1/2: leptons, quarks ψ^a

Spin 1: gluons, photon, W- & Z-boson A_μ

Spin 2: graviton $g_{\mu\nu}$

Consistent Field Theories

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massive & massless

MASSLESS !

How do we describe massive spin-2 fields ?

Massless + Massive Spin-2 Fields

General Relativity (GR)

Einstein-Hilbert action: $S_{\mathrm{EH}}[g] = M_{\mathrm{P}}^2 \int \mathrm{d}^4x \sqrt{g} \left(R(g) - 2\Lambda \right)$

Einstein's equations: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0$

Around maximally symmetric solutions, $\bar{R}_{\mu\nu} = \Lambda \bar{g}_{\mu\nu}$, the propagating degrees of freedom are those of a massless spin-2 field.

Also nonlinearly GR contains two dynamical degrees of freedom.

General Relativity

=

nonlinear theory

of massless spin-2

Nonlinear Mass Term

... should not contain derivatives nor loose indices.

But if we try to contract the indices of the metric, we get: $g^{\mu\nu} g_{\mu\nu} = 4$

This is not a mass term.

Simplest way out: Introduce second "metric" to contract indices:

$$g^{\mu\nu} f_{\mu\nu} = \mathrm{Tr}\left(g^{-1} f\right) \qquad f^{\mu\nu} g_{\mu\nu} = \mathrm{Tr}\left(f^{-1} g\right)$$

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➡ Massive gravity action is of the form

$$S_{\mathrm{MG}}[g] = \underbrace{S_{\mathrm{EH}}[g]}_{\text{kinetic term}} - \underbrace{\int \mathrm{d}^4x\, V(g, f)}_{\text{mass term}}$$

Nonlinear Mass Term

... should [...] derivatives nor loose indices.

But if we try to contr[...] $^{\mu\nu} g_{\mu\nu} = 4$

This is not a mas[...]

Simplest way o[...]

g^{μ}[...]

Massive gravity[...]

What determines $f_{\mu\nu}$?

Can we make it dynamical ?

$$S_{\rm MG}[g] \;=\; S_{\rm EH}[g] - \int \mathrm{d}^4x\, V(g,f)$$

kinetic term mass term

Bimetric Theory

Isham, Salam & Strathdee (1971)

Nonlinear bimetric action:

$$
\begin{aligned}
S_{\mathrm{b}}[g,f] \;&=\; m_g^2 \int \mathrm{d}^4x \sqrt{g}\,\Big(R(g) - 2\Lambda\Big) \\
&+\; m_f^2 \int \mathrm{d}^4x \sqrt{f}\,\Big(R(f) - 2\tilde{\Lambda}\Big) \;-\; \int \mathrm{d}^4x\, V(g,f)
\end{aligned}
$$

- both metrics are dynamical and treated on equal footing
- should describe massive & massless spin-2 field (5+2 d.o.f.)

This looks nice ...

... but unfortunately the general theory has ghosts!

Ghosts

Ghost = field with negative kinetic energy

$$\mathcal{L} = (\partial_t \phi)^2 \cdots \quad \text{healthy}$$
$$\mathcal{L} = -(\partial_t \phi)^2 \cdots \quad \text{ghost}$$

- consequences: classical instability, negative probabilities at quantum level
  - ➡ must be avoided!

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Modifications of General Relativity tend to be haunted by ghosts.
Modifying gravity is EXTREMELY difficult!

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Modifications of General Relativity tend to be haunted by ghosts.
Modifying gravity is EXTREMELY difficult!

Boulware & Deser (1972):
Nonlinear massive gravity always has a ghost!

“

Massive gravity stinks.
If you want to modify gravity,
try something else...

Quote from lecture notes by Kurt Hinterbichler, 2010
(now turned into a very nice review!)

The Ghost-Free Theory

Development

Creminelli, Nicolis, Papucci, Trincherini (2005):
attempt to construct ghost-free candidate theory; fails only because of unfortunate sign mistake

de Rham, Gabadadze, Tolley (2010):
construction of candidate theory for massive gravity in flat reference frame; ghost-free in “decoupling limit”

Hassan, Rosen, ASM, von Strauss (2011/12):
proof of absence of ghost in fully nonlinear theory

Hassan & Rosen (2011):
generalisation to ghost-free bimetric theory

 - free bimetric theory

Hassan & Rosen (2011)

$$S_{\mathrm{b}}[g,f] = m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) + m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) - \int \mathrm{d}^4x\, V(g,f)$$

$$\boxed{V(g,f) = m^4 \sqrt{g} \sum_{n=0}^{4} \beta_n\, e_n\left(\sqrt{g^{-1}f}\right) = m^4 \sqrt{f} \sum_{n=0}^{4} \beta_{4-n}\, e_n\left(\sqrt{f^{-1}g}\right)}$$

$$e_1(S) = \mathrm{Tr}[S] \qquad e_2(S) = \tfrac{1}{2}\left((\mathrm{Tr}[S])^2 - \mathrm{Tr}[S^2]\right)$$

$$e_3(S) = \tfrac{1}{6}\left((\mathrm{Tr}[S])^3 - 3\,\mathrm{Tr}[S^2]\mathrm{Tr}[S] + 2\,\mathrm{Tr}[S^3]\right)$$

 - free bimetric theory

Hassan & Rosen (2011)

$$
\begin{aligned}
S_{\mathrm{b}}[g,f] &= m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) \\
&+ m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) - \int \mathrm{d}^4x\, V(g,f)
\end{aligned}
$$

$$
\boxed{V(g,f) = m^4 \sqrt{g} \sum_{n=0}^{4} \beta_n\, e_n\left(\sqrt{g^{-1}f}\right) = m^4 \sqrt{f} \sum_{n=0}^{4} \beta_{4-n}\, e_n\left(\sqrt{f^{-1}g}\right)}
$$

more details: ASM, Mikael von Strauss; 1512.00021

What is the physical content of ghost-free bimetric theory ?

Mass spectrum

Hassan, ASM, von Strauss (2012)

Proportional solutions: $\bar{f}_{\mu\nu} = c^2 \bar{g}_{\mu\nu}$ with $c =$ const.

➡ Maximally symmetric backgrounds with $R_{\mu\nu}(\bar{g}) = \Lambda_g \bar{g}_{\mu\nu}$

Perturbations: $g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$ $\quad f_{\mu\nu} = c^2 \bar{g}_{\mu\nu} + \delta f_{\mu\nu}$

Can be diagonalised into mass eigenstates $(\alpha \equiv m_f / m_g)$:

$$\delta G_{\mu\nu} \propto \delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu}$$ massless (2 d.o.f.)

$$\delta M_{\mu\nu} \propto \delta f_{\mu\nu} - c^2 \delta g_{\mu\nu}$$ massive (5 d.o.f.)

with Fierz-Pauli mass $m_{\mathrm{FP}} = m_{\mathrm{FP}}(\alpha, \beta_n, c)$

Ghost-free bimetric theory

=

nonlinear theory

of massless & massive spin-2

Deviations from General Relativity

What is the physical metric ?

How does matter couple
to the tensor fields ?

Matter coupling

Yamashita, de Felice, Tanaka;
de Rham, Heisenberg, Ribeiro (2015)

Only one metric can couple to matter!

$$S_{gf} = m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) \quad + m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f)$$
$$- m^4 \int \mathrm{d}^4x \sqrt{g} \sum_{n=0}^{4} \beta_n e_n \left(\sqrt{g^{-1}f}\right)$$
$$+ \int \mathrm{d}^4x \sqrt{g}\, \mathcal{L}_{\mathrm{matter}}(g, \phi)$$

- only coupling that does not re-introduce the ghost
- $g_{\mu\nu}$ is gravitational metric

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- only coupling that does not re-introduce the ghost
- $g_{\mu\nu}$ is gravitational metric

The gravitational metric is not massless !!

Physical Interpretation

Baccetti, Martin-Moruno, Visser (2012);
Hassan, ASM, von Strauss (2012/14);
Akrami, Hassan, Koennig, ASM, Solomon (2015)

$\boxed{\text{Bimetric theory = General Relativity (GR) + corrections}}$

Recall: $\delta g_{\mu\nu} \propto \delta G_{\mu\nu} - \alpha^2 \delta M_{\mu\nu}$

Assume that $\alpha = m_f/m_g$ is small (i.e. weak gravity!)

- the gravitational metric is almost massless
- the massive spin-2 field interacts only weakly with matter

Physical Interpretation

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$\alpha \to 0$ is the General Relativity limit of bimetric theory

Ghost-free bimetric theory

=

General Relativity +
additional (heavy?) tensor field

Spin-2 Dark Matter

Dark Matter

Babichev, Marzola, Raidal, ASM, Urban, Veermäe, von Strauss (2016); Aoki, Mukohyama (2016)

Redefining the nonlinear fields gives: $G \sim \bar{g} + \delta G \qquad M \sim \delta M$

$$\mathcal{L} \sim \sqrt{G}\left[\underbrace{m_{\mathrm{Pl}}^2 R(G)}_{\text{Gravity}} + \underbrace{K(G, \nabla\nabla M) + V(G, M)}_{\text{Dark Matter}} + \underbrace{\mathcal{L}_{\mathrm{m}}(G, \alpha M, \phi)}_{\text{Baryonic Matter}}\right]$$

In the General Relativity (GR) limit of bimetric theory, $\alpha \to 0$:

massive spin-2 field decouples from matter, interacts only with gravity.

A large spin-2 mass further suppresses deviations from GR.

Consistency checks

Our spin-2 dark matter is part of gravity (!) and...

- ... gravitates just like baryonic matter
- ... does not decay into gravitons and its decay rate into Standard Model fields is sufficiently small: $\Gamma(\delta M \to XX) \sim \frac{\alpha^2 m_{\mathrm{FP}}^3}{m_{\mathrm{Pl}}^2}$

  ➡ automatically stable
- ... has interactions with baryonic matter which are naturally suppressed by the Planck scale
- ... can be produced thermally for a mass of 1 - 10^8 TeV

Detection

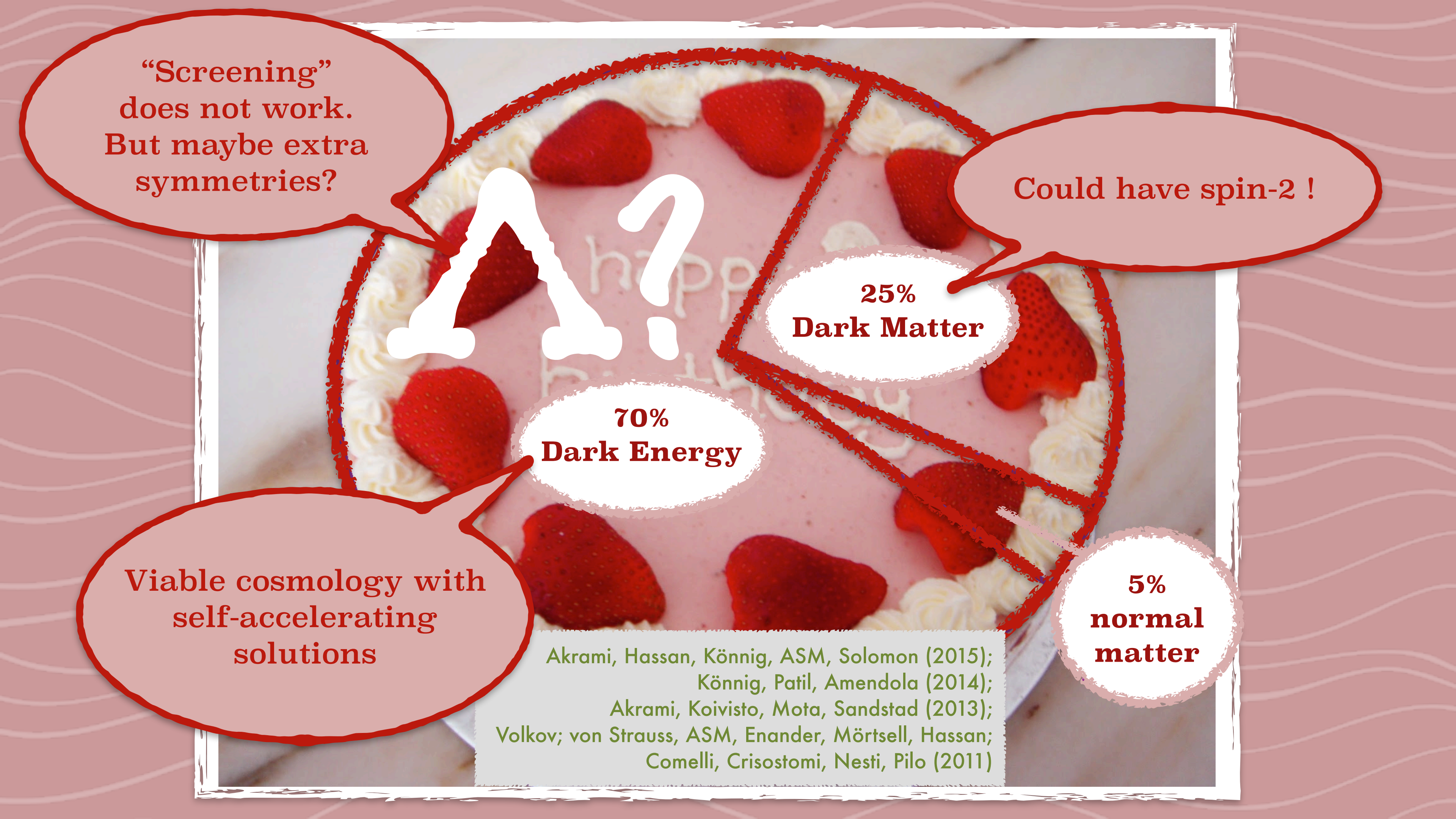
- not observable in current indirect and direct detection experiments
- massive spin-2 field may gravitate differently in curved backgrounds
  - ➡ non-standard behaviour of dark matter around massive objects ?
- dark matter self-interactions: could be observable in cluster collisions and in power spectrum
- correlations with gravitational waves   Aoki, Mukohyama (2016)

Lessons learned

Ghost-free bimetric theory…

- is one of the few known consistent modifications of General Relativity
- can be interpreted as gravity in the presence of an extra spin-2 field
- contains an interesting dark matter candidate whose coupling to baryonic matter is suppressed by the Planck scale

Λ?
happy birthday
25%
Dark Matter
70%
Dark Energy
5%
normal
matter


"Screening" does not work. But maybe extra symmetries?
Λ?
Could have spin-2 !
25% Dark Matter
70% Dark Energy
Viable cosmology with self-accelerating solutions
5% normal matter
Akrami, Hassan, König, ASM, Solomon (2015);
König, Patil, Amendola (2014);
Akrami, Koivisto, Mota, Sandstad (2013);
Volkov; von Strauss, ASM, Enander, Mörtsell, Hassan;
Comelli, Crisostomi, Nesti, Pilo (2011)