

# Gravitational lenses as a probe of dark matter models

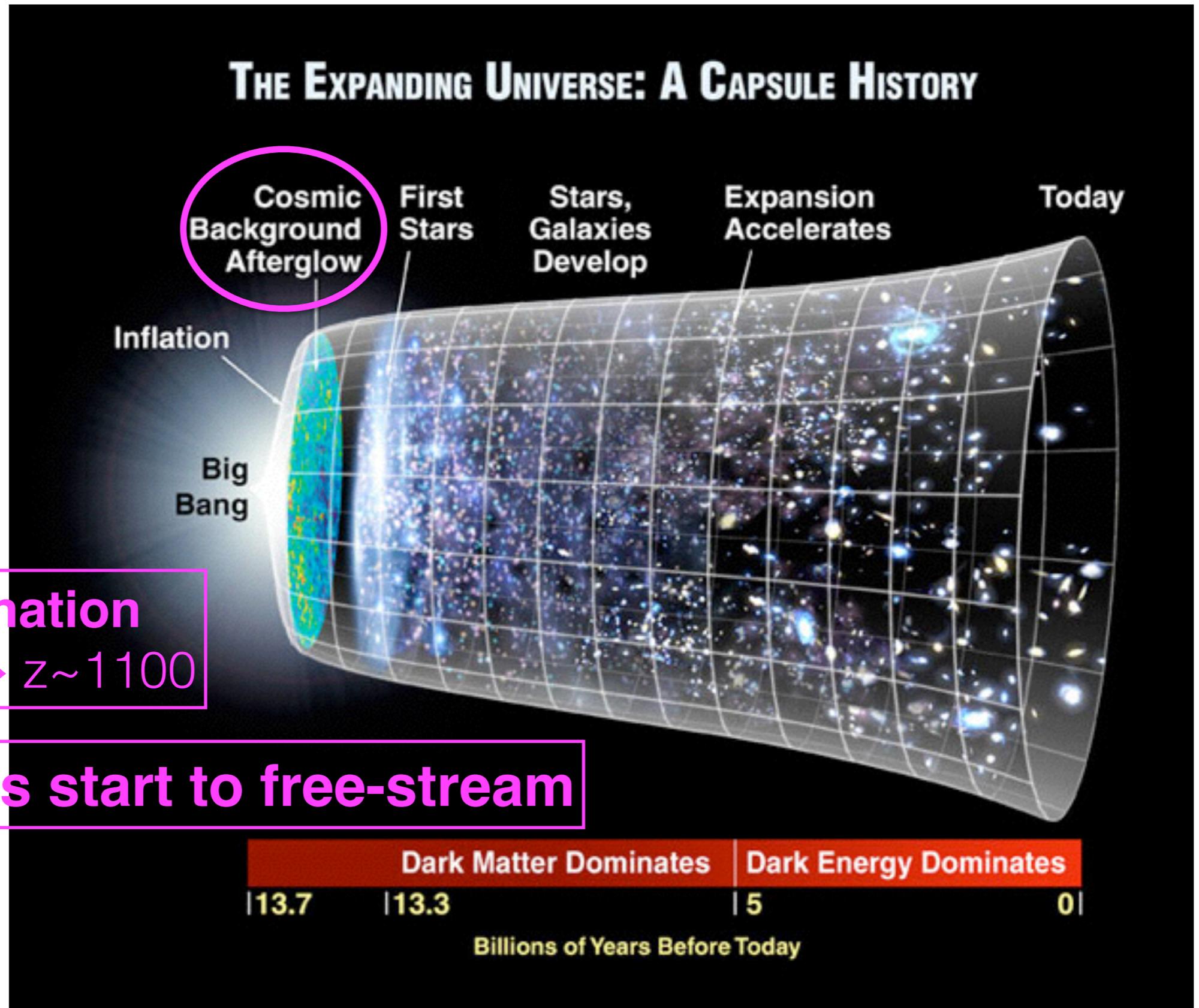
**Ayuki Kamada** (University of California, Riverside)

9 Jun 2016 @ MITP workshop  
“Exploring the Energy Ladder of the Universe”

based on collaborations with  
Akira Harada (The University of Tokyo),  
Kaiki Taro Inoue (Kindai University),  
Ken Osato (The University of Tokyo),  
Toyokazu Sekiguchi (Institute for Basic Science, Korea),  
Masato Shirasaki (National Astronomical Observatory of Japan),  
Tomo Takahashi (Saga University),  
and Naoki Yoshida (Kavli Institute for the Physics and Mathematics of the Universe)

# Brief introduction of structure formation

# History of the Universe

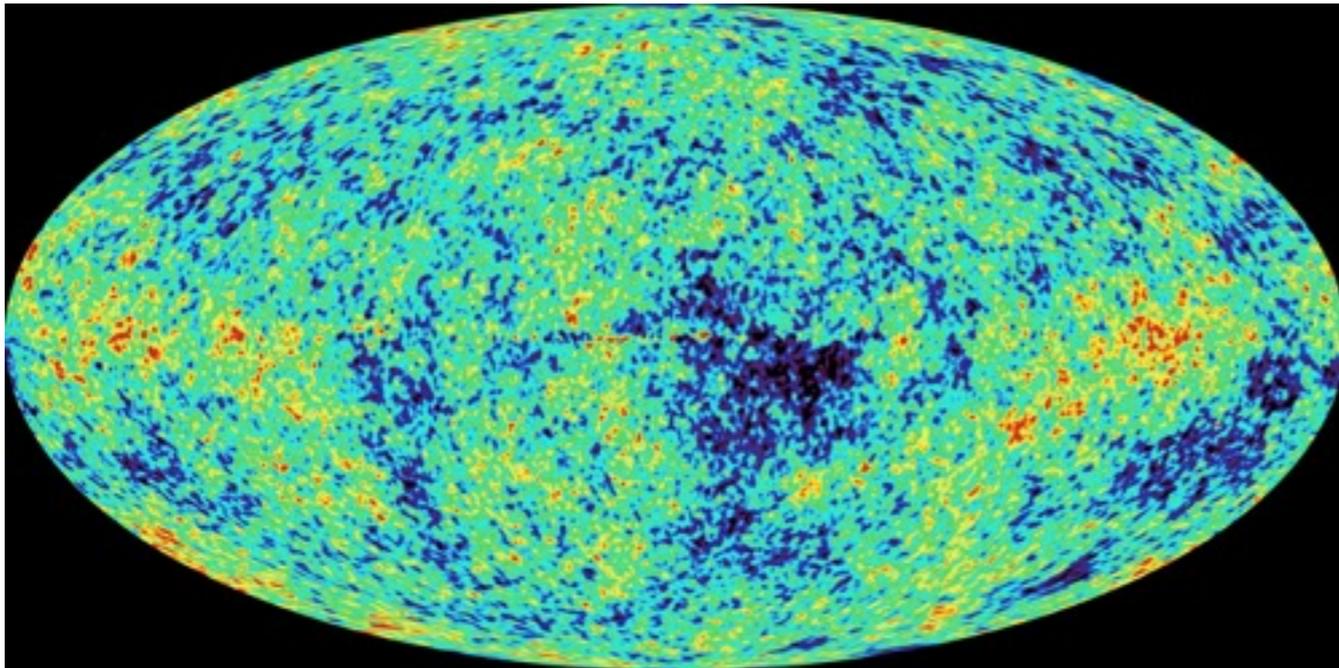


**Recombination**

$3.8 \times 10^5 \text{ yr} \leftrightarrow z \sim 1100$

**Photons start to free-stream**

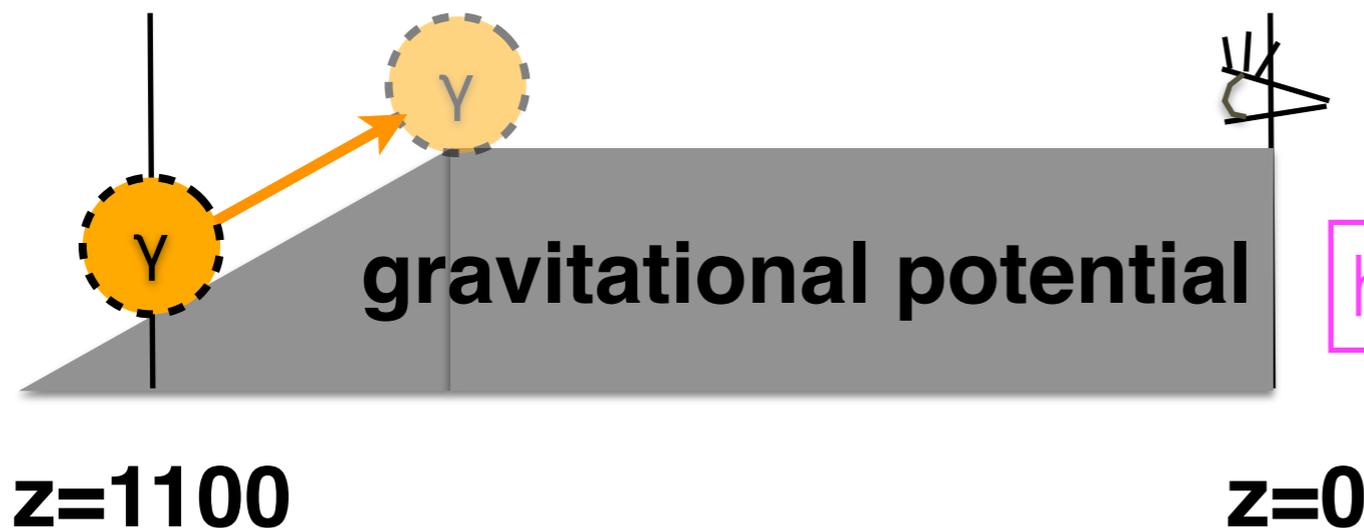
# (Early) Sachs-Wolfe Effect



Cosmic Microwave Background (CMB):  
homogeneous and isotropic universe  
+fluctuations ( $\sim 10^{-5}$ )

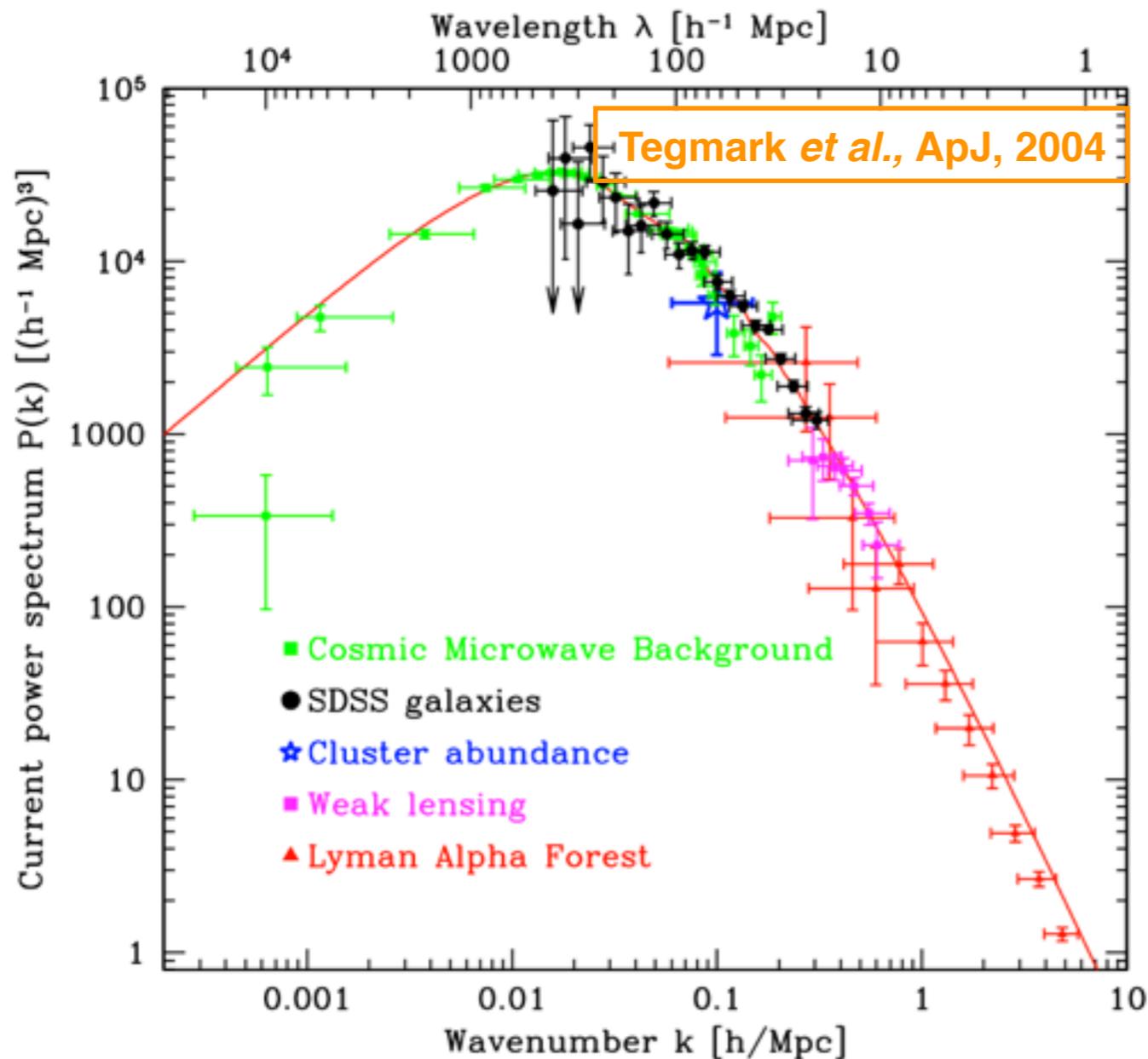
lower temperature spots

lose energy during propagation  
 $\leftrightarrow$  low temperature



higher matter density spots

# (Linear) Matter density fluctuations



matter density fluctuations

$$\delta(\mathbf{x}) \equiv \frac{\delta\rho}{\rho}(\mathbf{x}) = \int \frac{d\mathbf{k}^3}{(2\pi)^3} \delta(\mathbf{k}) e^{i\mathbf{k}\mathbf{x}}$$

$$\langle \delta(\mathbf{k}) \delta^*(\mathbf{k}') \rangle = (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}') P(\mathbf{k})$$

CMB anisotropies

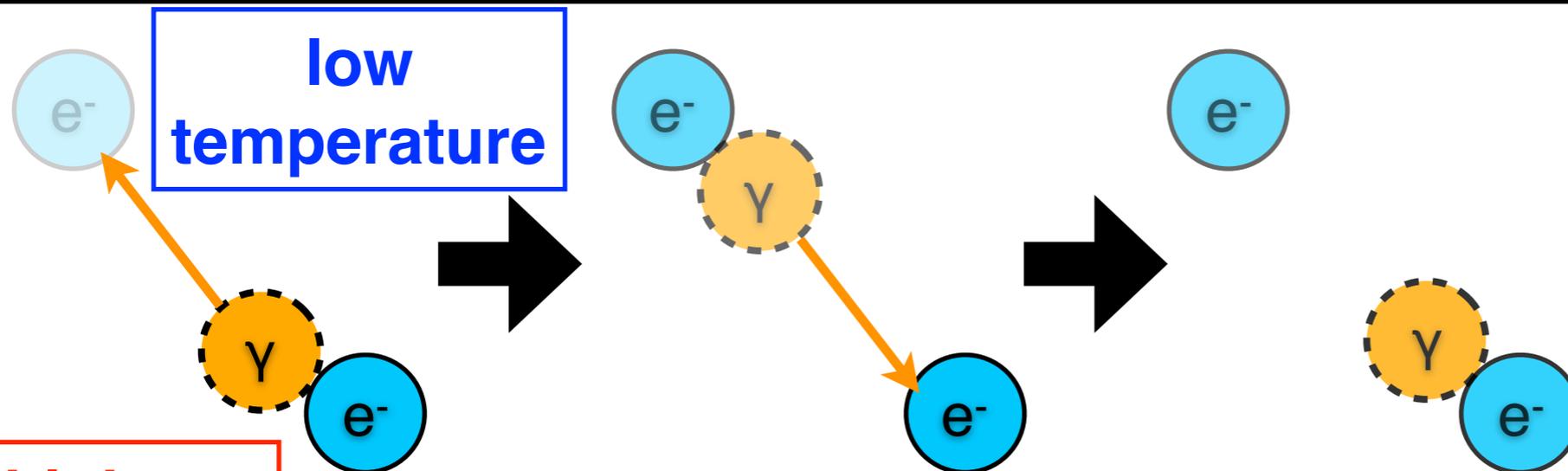
→ matter distribution at 100 Mpc-10 Gpc (cosmic horizon) scales

# Limitation of CMB: Silk damping

Photons  $\gamma$  and baryons  $e^-$   $p^+$  move together along the primordial gravitational potentials

→ **higher photon** ↔ **higher baryon temperature/density spots**

↔ **Finite** mean free path of photons  $\lambda_{fs} \sim \frac{1}{\sigma_T n_e}$  (Thomson scattering)



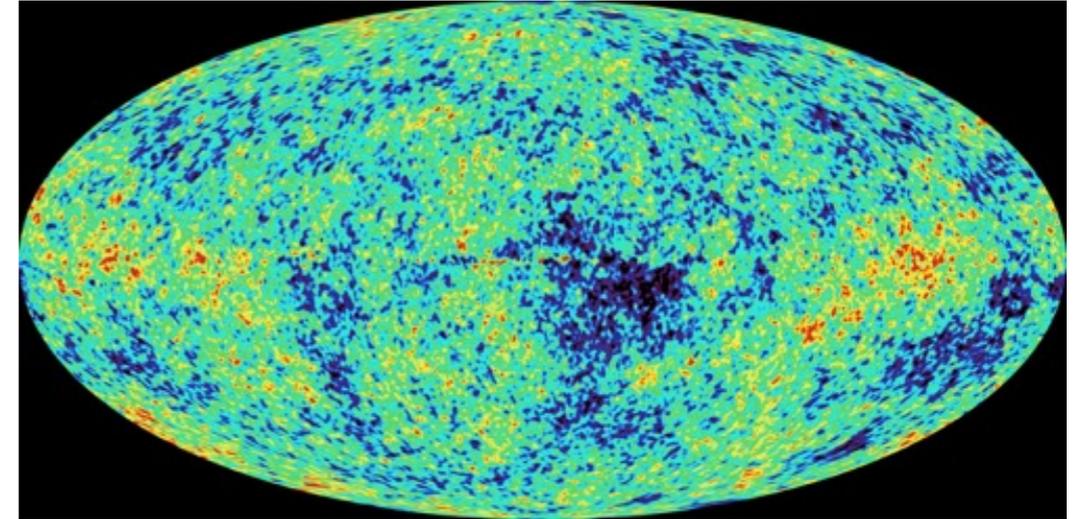
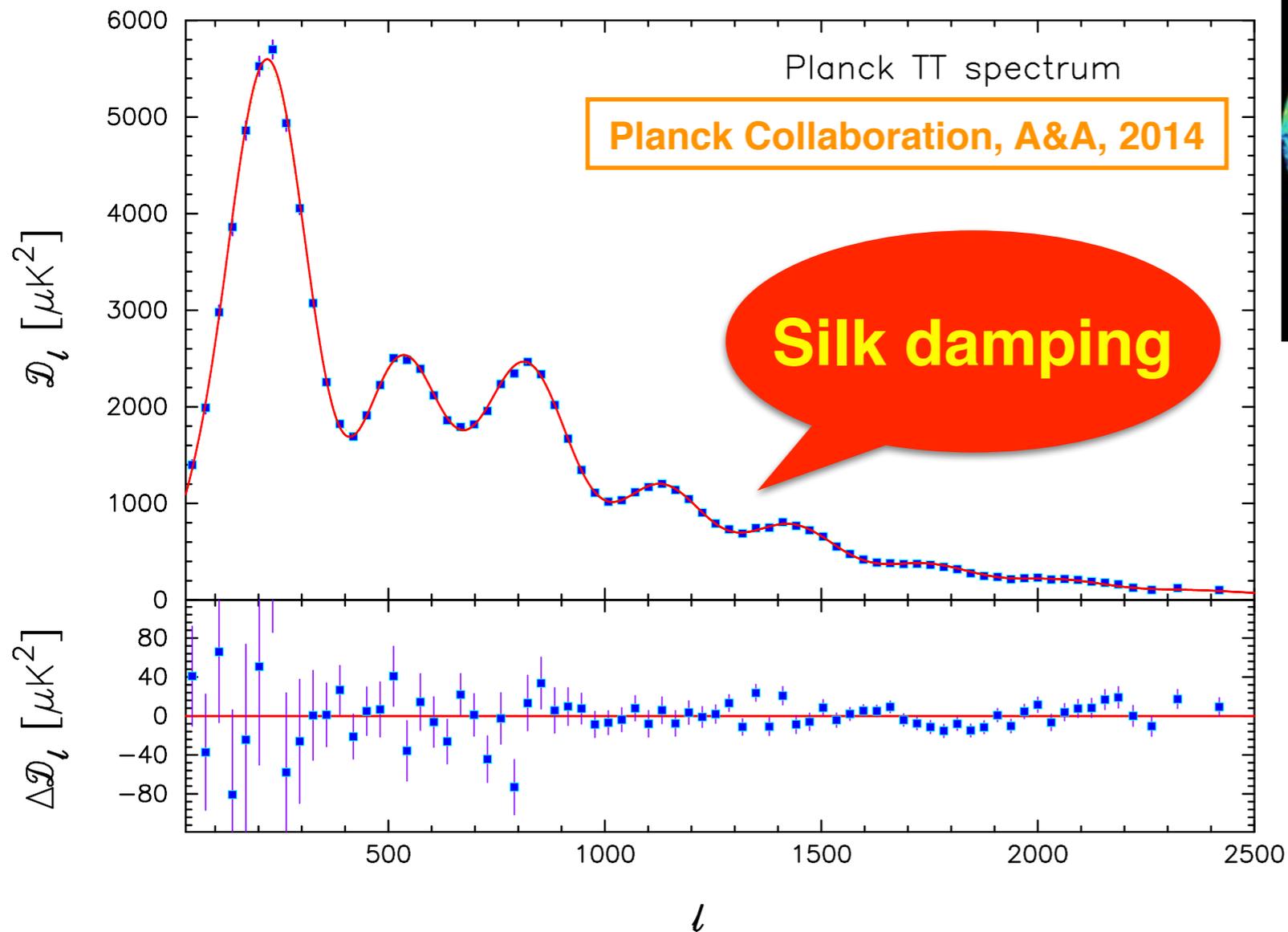
**high temperature**

**Different spots communicate each other**  
→ **Initial fluctuations damped**

**Diffusion length-Mean free path: Random walk**

$$\lambda_{diff} \sim \lambda_{fs} \sqrt{\#scat} \quad \#scat \sim \frac{ct}{\lambda_{fs}}$$

# Success of $\Lambda$ CDM model in CMB



Parameter	Planck	
	Best fit	68% limits
$\Omega_b h^2$ . . . . .	0.022068	$0.02207 \pm 0.00033$
$\Omega_c h^2$ . . . . .	0.12029	$0.1196 \pm 0.0031$
$100\theta_{MC}$ . . . . .	1.04122	$1.04132 \pm 0.00068$
$\tau$ . . . . .	0.0925	$0.097 \pm 0.038$
$n_s$ . . . . .	0.9624	$0.9616 \pm 0.0094$
$\ln(10^{10} A_s)$ . . . . .	3.098	$3.103 \pm 0.072$

**CMB observations of undamped modes ( $\ell < 2500$ ) simultaneously determine six parameters of  $\Lambda$ CDM model and are well-reproduced**

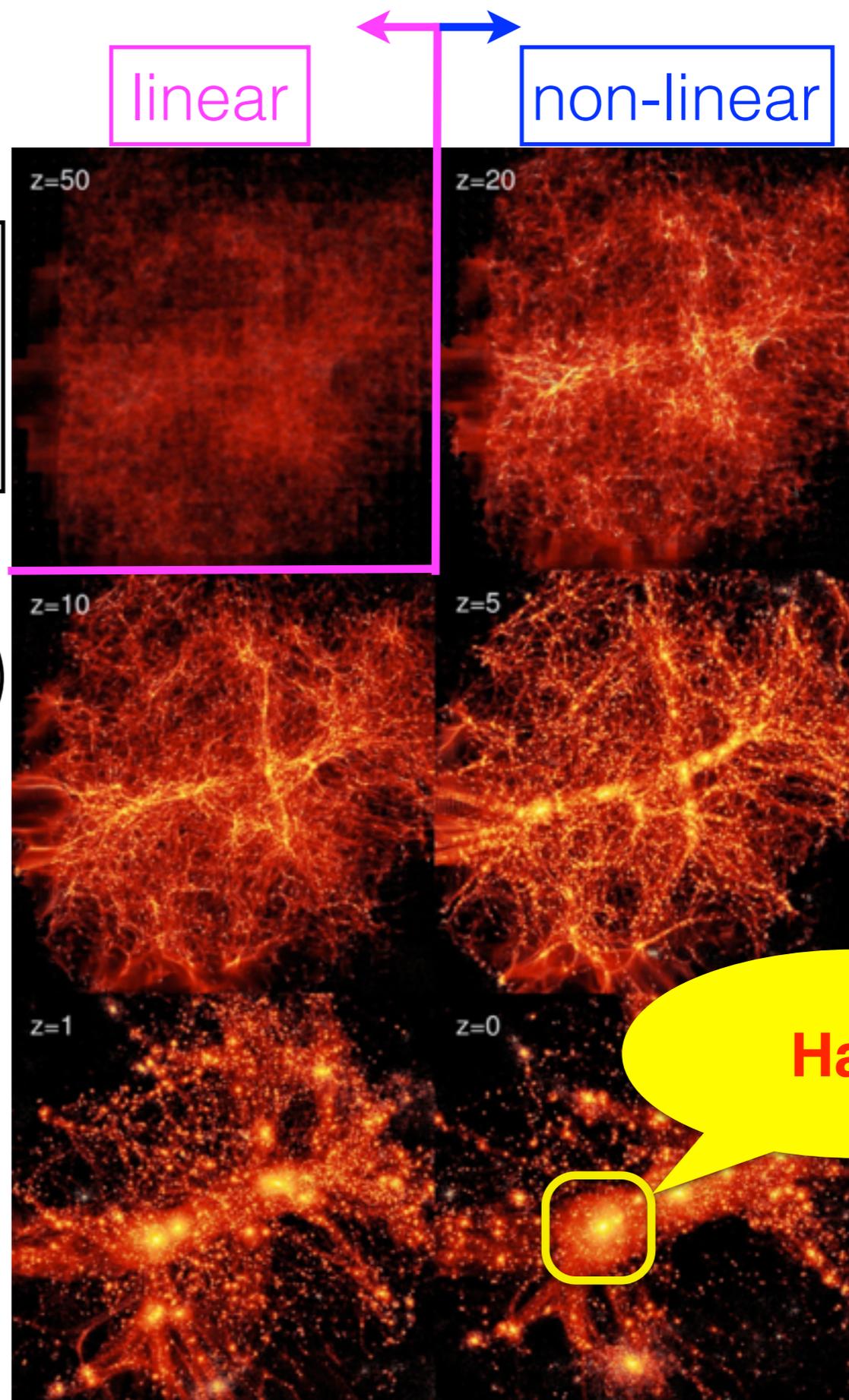
# Non-linear evolution of matter distribution

Under the matter-domination, density fluctuations grow such that  $\delta \propto a$

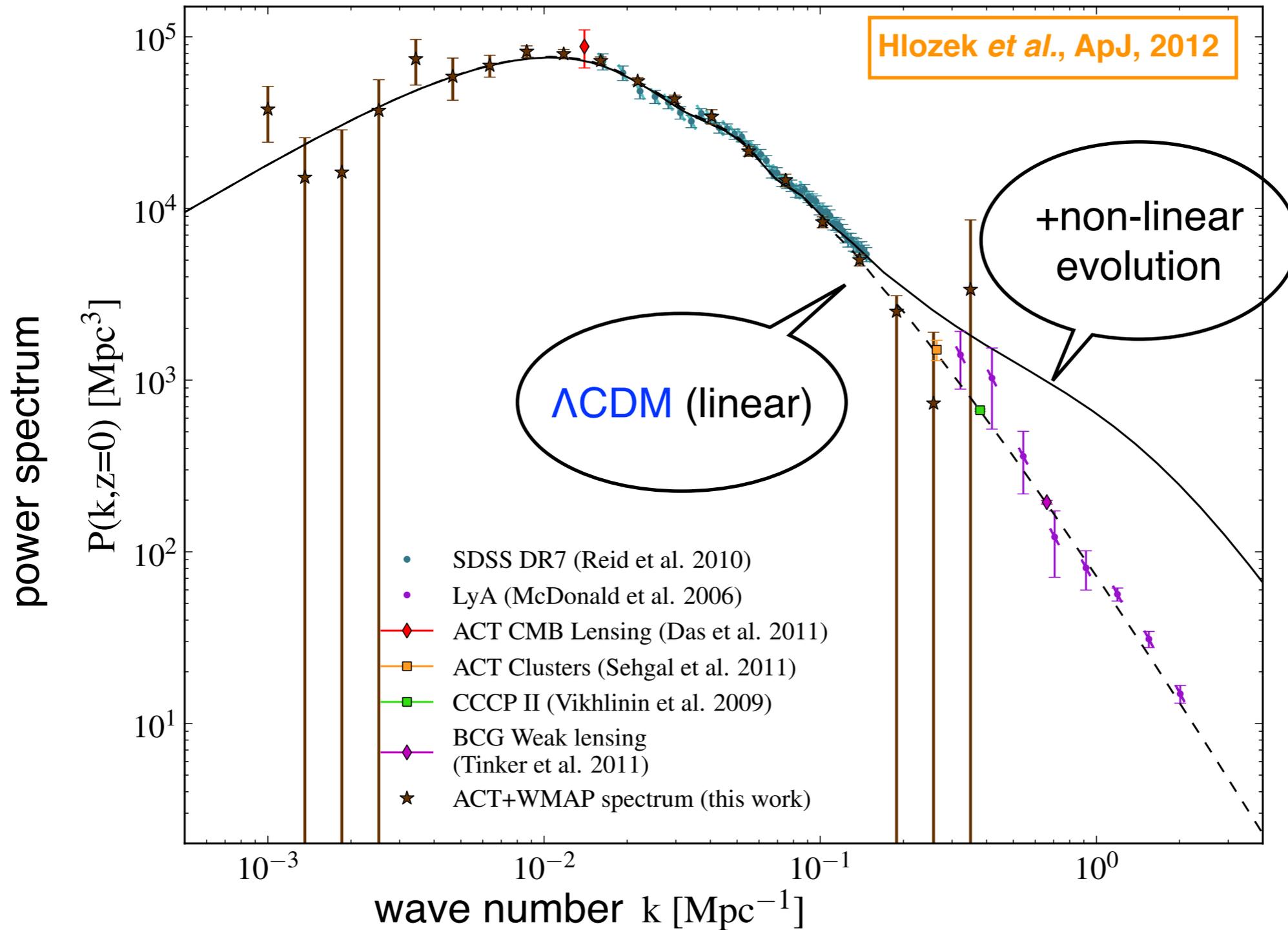


*N*-body simulation  
- Lagrange picture

Get non-linear ( $\delta > 1$ ) and start to form halos, which host luminous objects like galaxies



# Large scale structure of the Universe

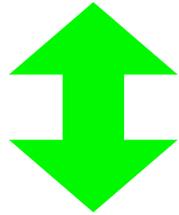


$\Lambda$ CDM model reproduces  
large scale ( $> \text{Mpc}$ ) structure of the Universe well

# Cold Dark Matter?

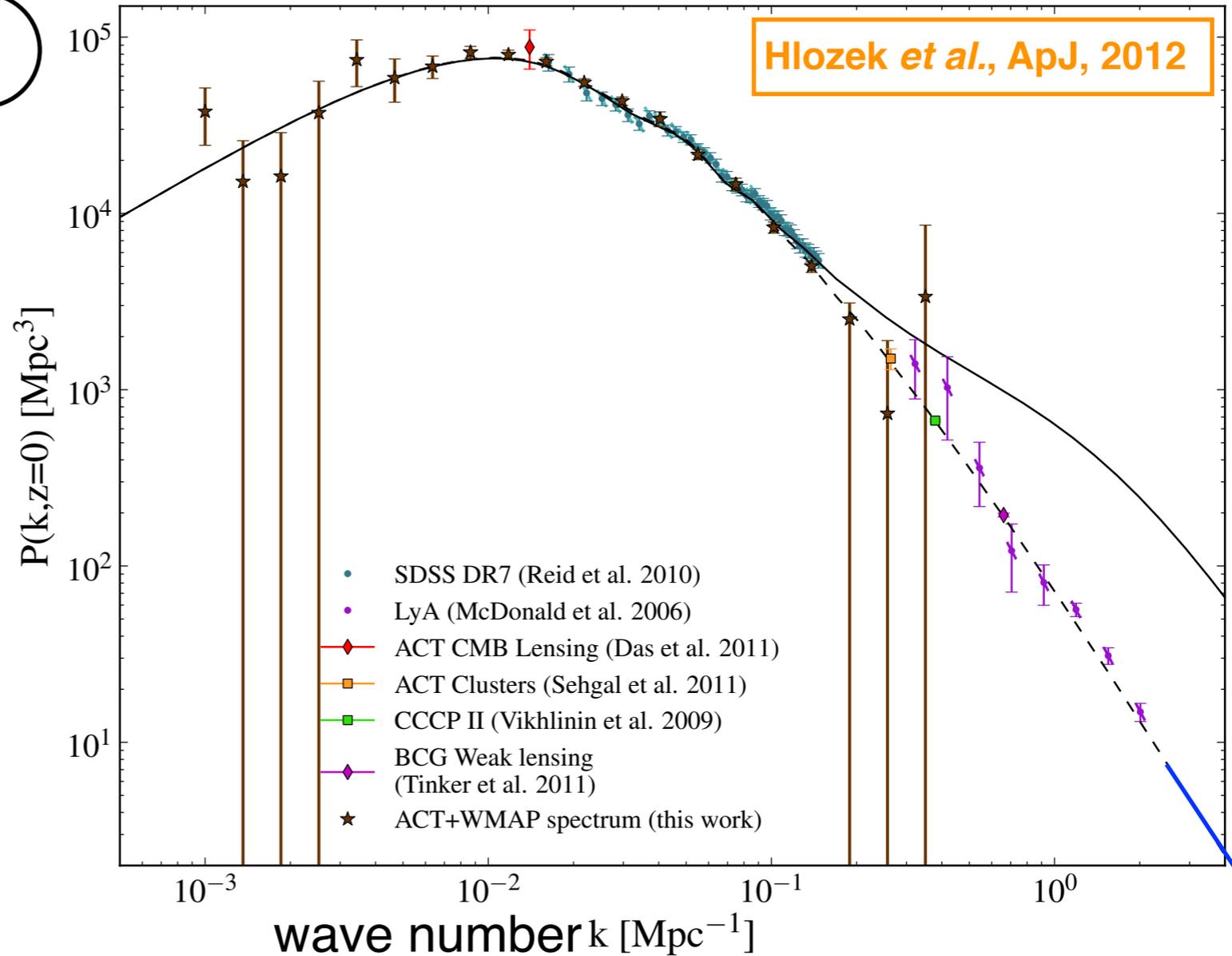
hypothetical

cold dark matter:  
 null thermal velocity  
 only gravitationally  
 interacting



particle physics DM  
 candidates:  
 finite (sizable) thermal  
 velocity  
 interacting in many ways

power spectrum



(Small scale) matter density fluctuations  
 (deviations from  $\Lambda$ CDM) contain imprints of the nature of DM

# Notes on use of large scale structure

## Observation

- Indirect measurement -
- only luminous objects (baryons) can be detected
  - After structure formation (non-linear evolution),  
baryons may not follow DM distribution
  - Difficult to detect halos that host faint objects

## Theory

- Non-linear evolution - perturbation theory breaks down
  - time/resource-consuming  $N$ -body simulations (and detailed comparison with analytic approach)
- co-evolution of DM halos and baryon processes (supernova explosions, galaxy formation...)
- **State-of-art hydrodynamic simulations and elaborate modeling of baryon processes**

# Advantage of gravitational lenses

## Observation

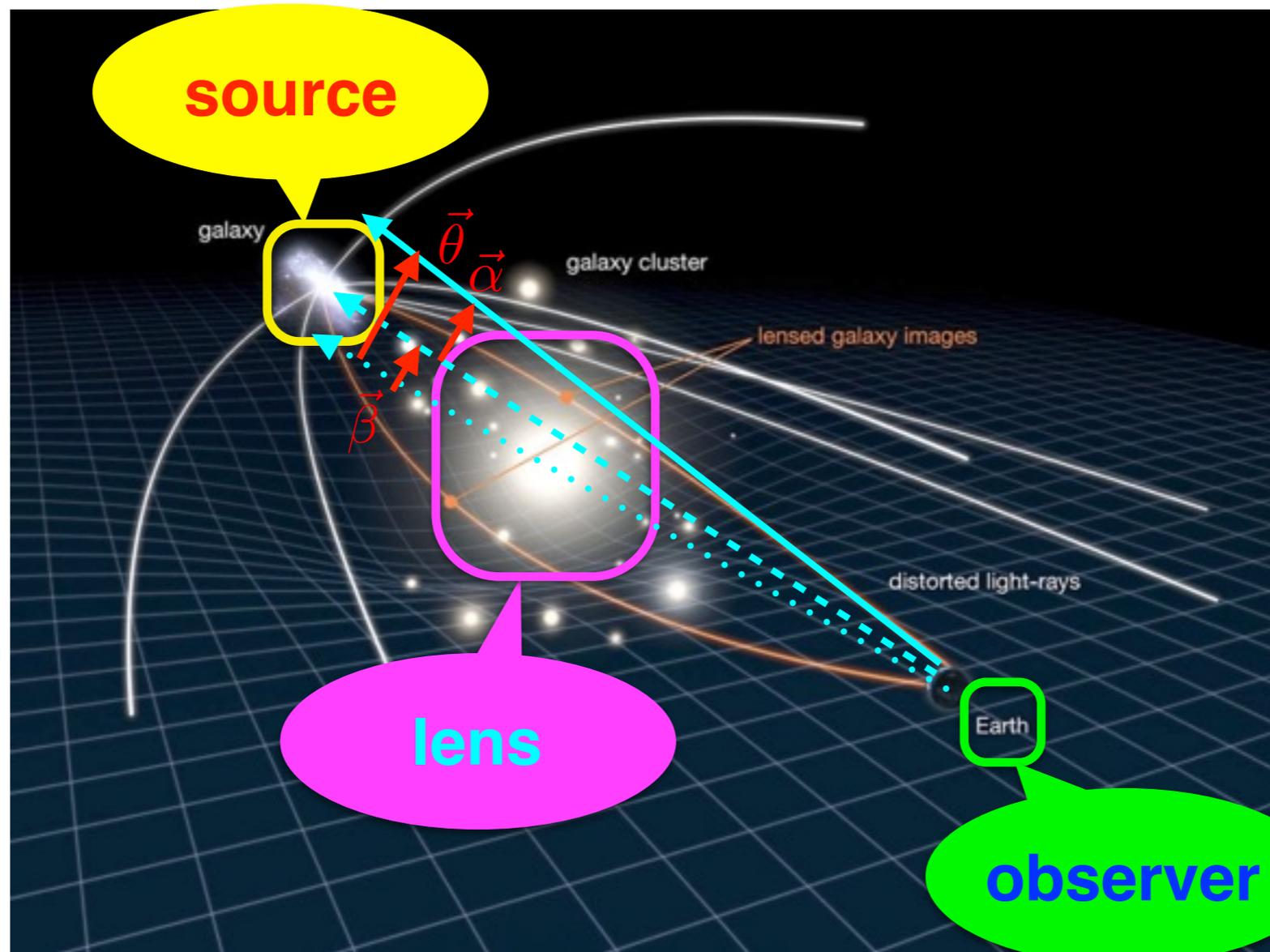
Direct measurement -

**gravitational potentials: matter (DM+baryons) distribution**

## Theory

- Non-linear evolution - perturbation theory breaks down
  - time/resource-consuming  $N$ -body simulations (and detailed comparison with analytic approach)
- co-evolution of DM halos and baryon processes (supernova explosions, galaxy formation...)
  - **state-of-art hydrodynamic simulations and elaborate modeling of baryon processes**

# Gravitational lens I



Position distortion (Born approximation)

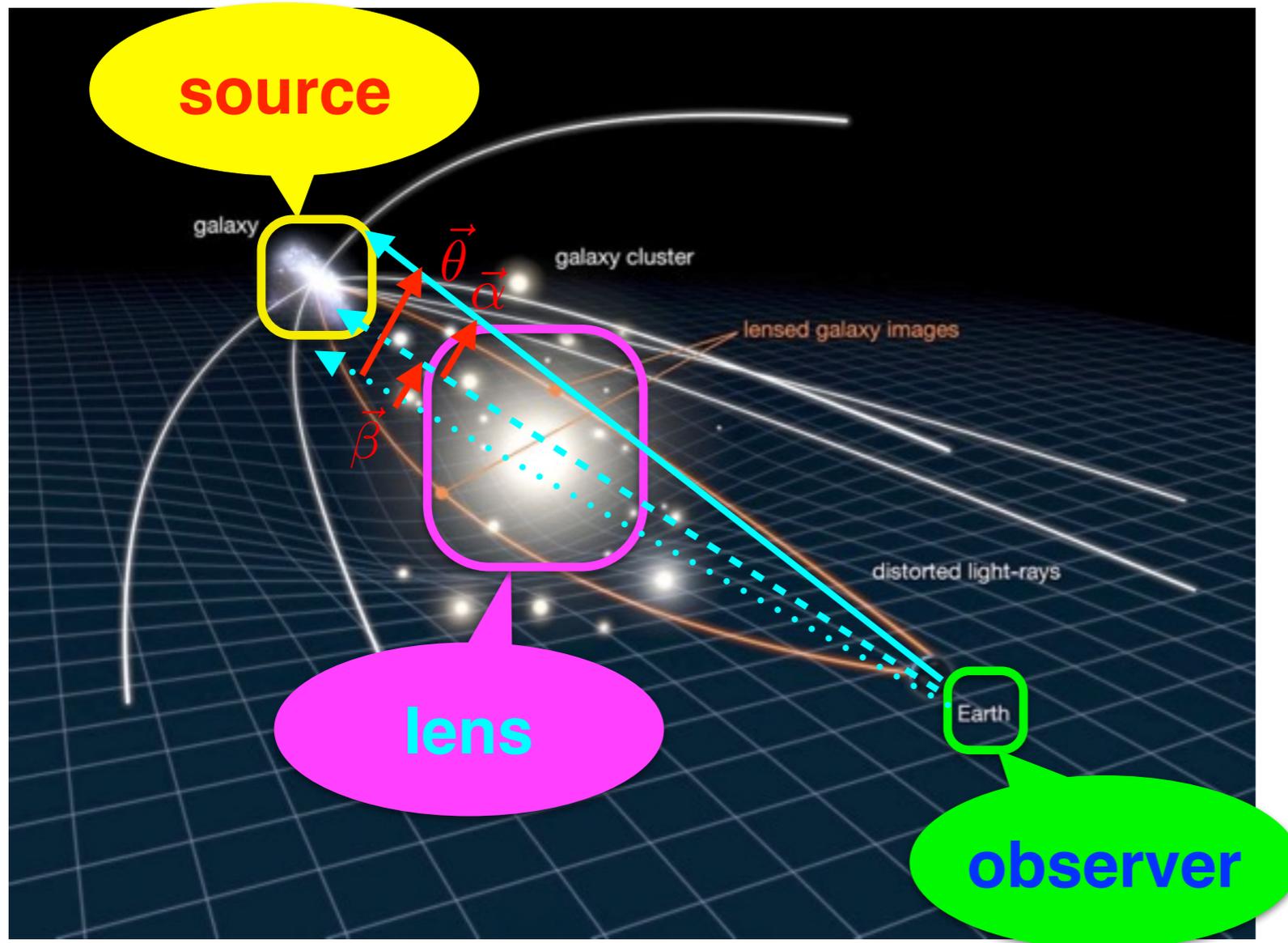
$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta}) \quad \vec{\alpha} = \partial_{\vec{\theta}} \psi$$

$$\psi(\theta) = \frac{D_{ds}}{D_d D_s} \frac{2}{c^2} \int \Phi(D_d \vec{\theta}, z) dz$$

$\vec{\beta}$ : original position  
 $\vec{\theta}$ : apparent position  
 D: angular distance

1st derivative of  
 gravitational potential

# Gravitational lens II



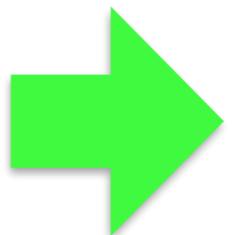
2nd derivative of gravitational potential

$\kappa$  : convergence  
 $\gamma$  : shear

Specific intensity: conserved  
 $dE/dt/dA/(\cos\theta d\Omega)/dv$   
Flux: magnified  
 $dE/dt/dA/dv$

Image Distortion

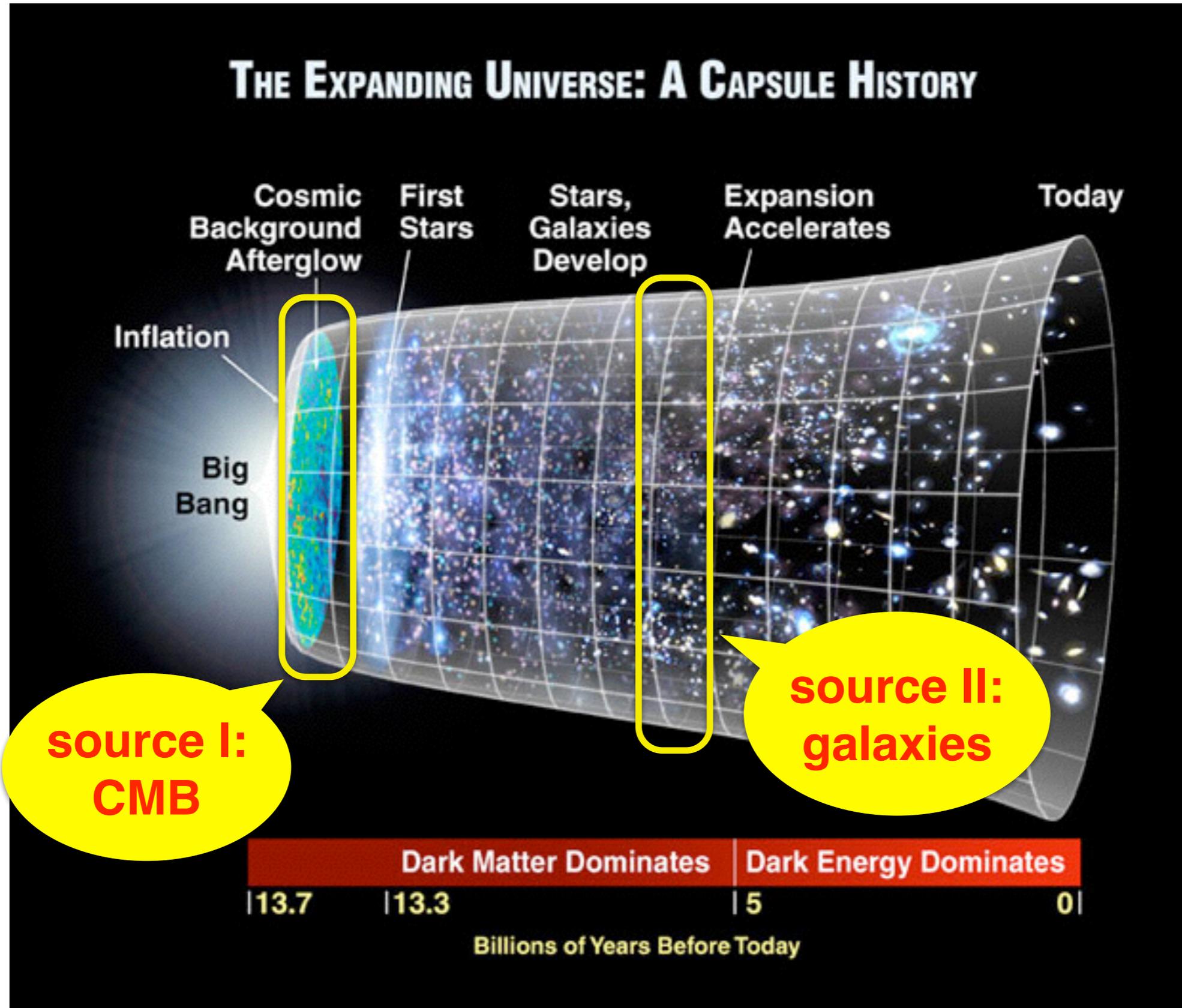
$$A_{ij} = \frac{\partial \beta^i}{\partial \theta^j} \equiv \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}$$



Magnification

$$\mu = \frac{1}{\det A} = \frac{1}{(1 - \kappa)^2 - \gamma_1^2 - \gamma_2^2}$$

# Sources in this talk



# Constraints on light gravitino mass

based on

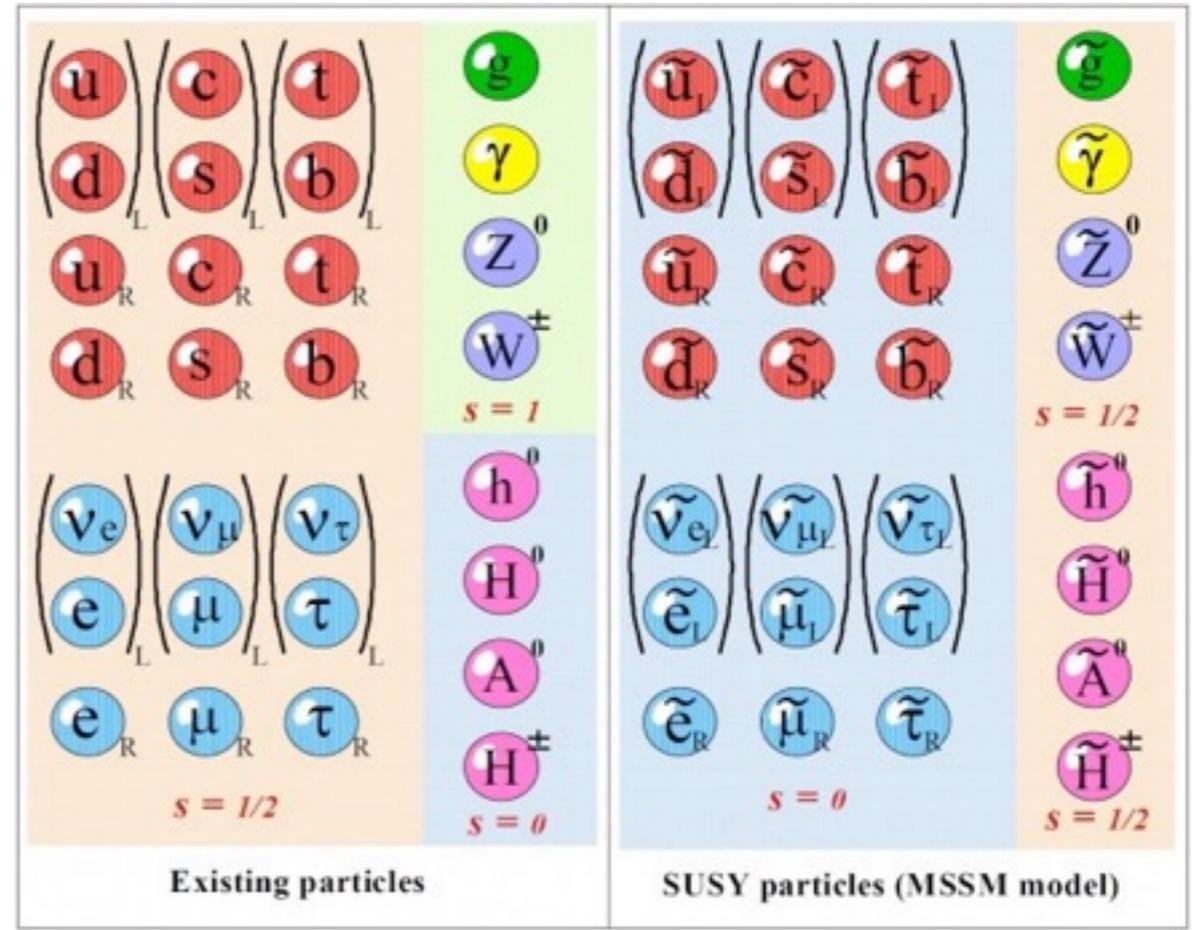
AK, M. Shirasaki, and N. Yoshida, JHEP, 2014

K. Osato, T. Sekiguchi, M. Shirasaki, AK, N. Yoshida, JCAP, 2016

# Supersymmetry (SUSY)

**Nontrivially extended Poincaré symmetry - Boson (spin integer) ↔ Fermion (spin half-integer)**

graviton (spin 2)  
↔ gravitino (spin 3/2)



supersymmetric extension of Standard Model: MSSM

→ achieve grand unification

→ solve the hierarchy problem

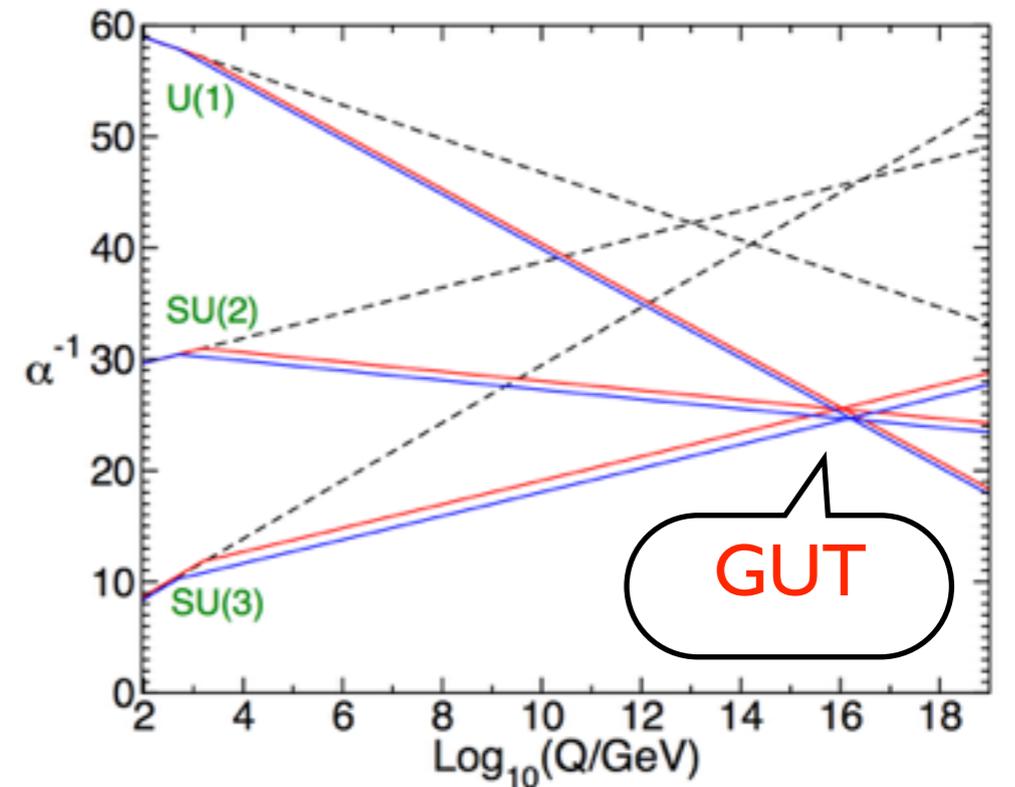
Martin, arXiv:hep-ph/9709356

$$m_{h^0}^2 = m_{h^0,0}^2 + \Delta(m_{h^0}^2)$$

model prediction  
(126 GeV)<sup>2</sup>

model parameter

quantum correction  
~(10<sup>17</sup> GeV)<sup>2</sup>



# SUSY phenomenology

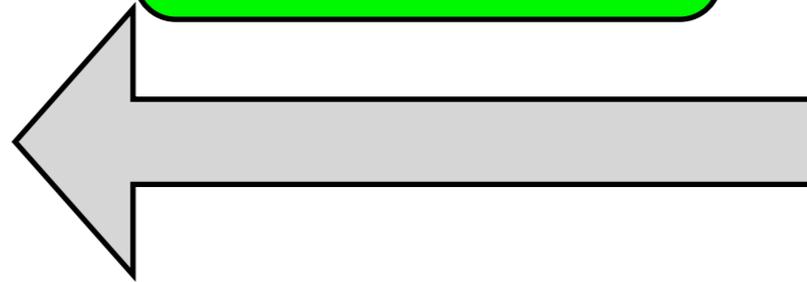
SUSY should be broken in the Visible sector

SUSY is originally broken in the Hidden sector

Mediation  
(Interaction)

MSSM sector  
(Visible sector)

SUSY-breaking  
sector  
(Hidden sector)



Gauge-mediated SUSY breaking (GMSB)

Messenger fields - SM gauge interaction  
- coupling to SUSY breaking field

naturally ensures that **flavor-blindness**  
of SUSY breaking in MSSM

# (minimal) GMSB mass spectrum

superpotential of minimal model

$$W = (\lambda S + M_{\text{mess}}) \sum_{n=1}^{N_5} \Phi_n \bar{\Phi}_n$$

sfermion mass squared

$$m_{\phi_i}^2 = 2\Lambda^2 N_5 \sum_{a=1}^3 C_a(i) \left( \frac{g_a^2}{16\pi^2} \right)^2 f(x)$$

gaugino mass

$$M_a = \frac{g_a^2}{16\pi^2} \Lambda N_5 g(x)$$

gravitino mass

$$m_{3/2} = \frac{F}{\sqrt{3}M_{\text{pl}}}$$

$\Phi_n$  : messenger superfields

$N_5$  : # of messengers

$M_{\text{mess}}$  : messenger mass

$S$  ( $\langle S \rangle = \theta^2 F$ )

: SUSY-breaking  
(goldstino) field

$F$  : SUSY-breaking scale

$\Lambda$  ( $= \lambda F / M_{\text{mess}}$ )

: MSSM SUSY-  
breaking scale

$x = \Lambda / M_{\text{mess}} \leq 1$

$$f(x) = \frac{1+x}{x^2} \left[ \ln(1+x) - 2\text{Li}_2\left(\frac{x}{1+x}\right) + \frac{1}{2}\text{Li}_2\left(\frac{2x}{1+x}\right) \right] + (x \rightarrow -x)$$

$$g(x) = \frac{1}{x^2} (1+x) \ln(1+x) + (x \rightarrow -x)$$

# (minimal) GMSB mass spectrum

superpotential of minimal model

$$W = (\lambda S + M_{\text{mess}}) \sum_{n=1}^{N_5} \Phi_n \bar{\Phi}_n$$

$\Phi_n$  : messenger superfields

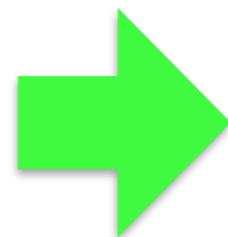
$N_5$  : # of messengers

sfermion mass squared

distinctive feature of GMSB

$$m_\phi = \mathcal{O}(1-100) \text{ TeV}$$

$$M = \mathcal{O}(1-100) \text{ TeV}$$



$$m_{3/2} = \mathcal{O}(1-100) \text{ eV}$$

$$M_{\text{mess}} \sim \Lambda = \mathcal{O}(10^{3-4}) \text{ TeV} (\ll M_{\text{pl}})$$

gravitino mass

$$m_{3/2} = \frac{F}{\sqrt{3}M_{\text{pl}}}$$

breaking scale

$$x = \Lambda/M_{\text{mess}} \leq 1$$

$$f(x) = \frac{1+x}{x^2} \left[ \ln(1+x) - 2\text{Li}_2\left(\frac{x}{1+x}\right) + \frac{1}{2}\text{Li}_2\left(\frac{2x}{1+x}\right) \right] + (x \rightarrow -x)$$

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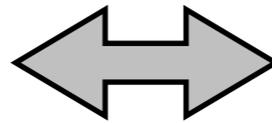
# Light gravitino in cosmology

## Thermal history

Produced and thermalized  
just after the reheating of the Universe

$$T_{\text{dec}3/2} = \mathcal{O}(10\text{--}100) \text{ GeV}$$

at the freeze-out of  
the other SUSY particles

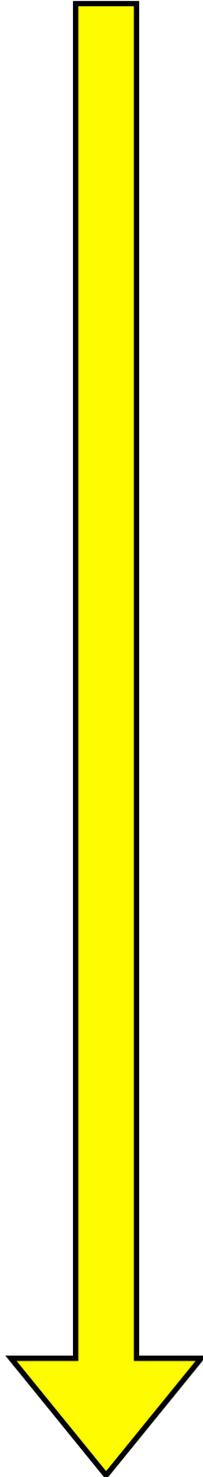


Decouple from the thermal background and  
start to free-stream with speed of light

$$(1 + z_{\text{nr}3/2}) \simeq 1 \times 10^4 (m_{3/2}/1 \text{ eV}) (g_{*s3/2}/90)^{1/3}$$

Contribute to the mass density  
of the Universe

Light gravitino  
as “hot”  
component

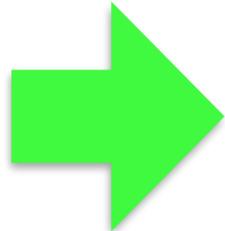


# Relic gravitino

Present mass density of the gravitino

$$\Omega_{3/2} h^2 = 0.13 \left( \frac{g_{3/2}}{2} \right) \left( \frac{m_{3/2}}{100 \text{ eV}} \right) \left( \frac{g_{*s3/2}}{90} \right)^{-1}$$

$$\Omega_{3/2} / \Omega_{\text{DM}}^{\text{CMB obs}} = \mathcal{O}(1-10)\% \quad \text{for } m_{3/2} = \mathcal{O}(1-10) \text{ eV}$$



Assume another cold and stable particle exist

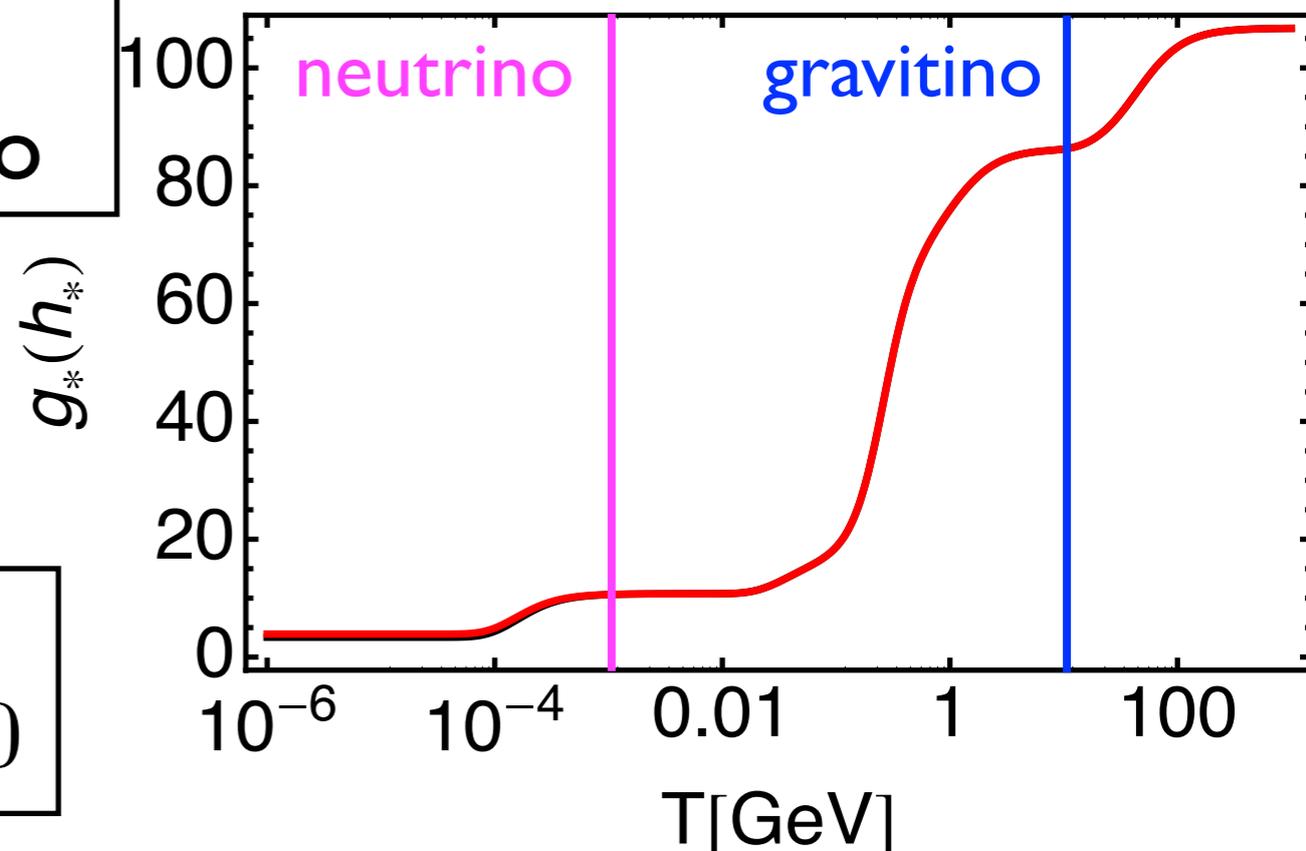
$g_{*s3/2}$ : effective degrees of freedom for the entropy density at the decoupling of the gravitino

$$g_{*s3/2} \simeq 90 \quad \text{E. Pierpaoli et al., PRD 1998}$$

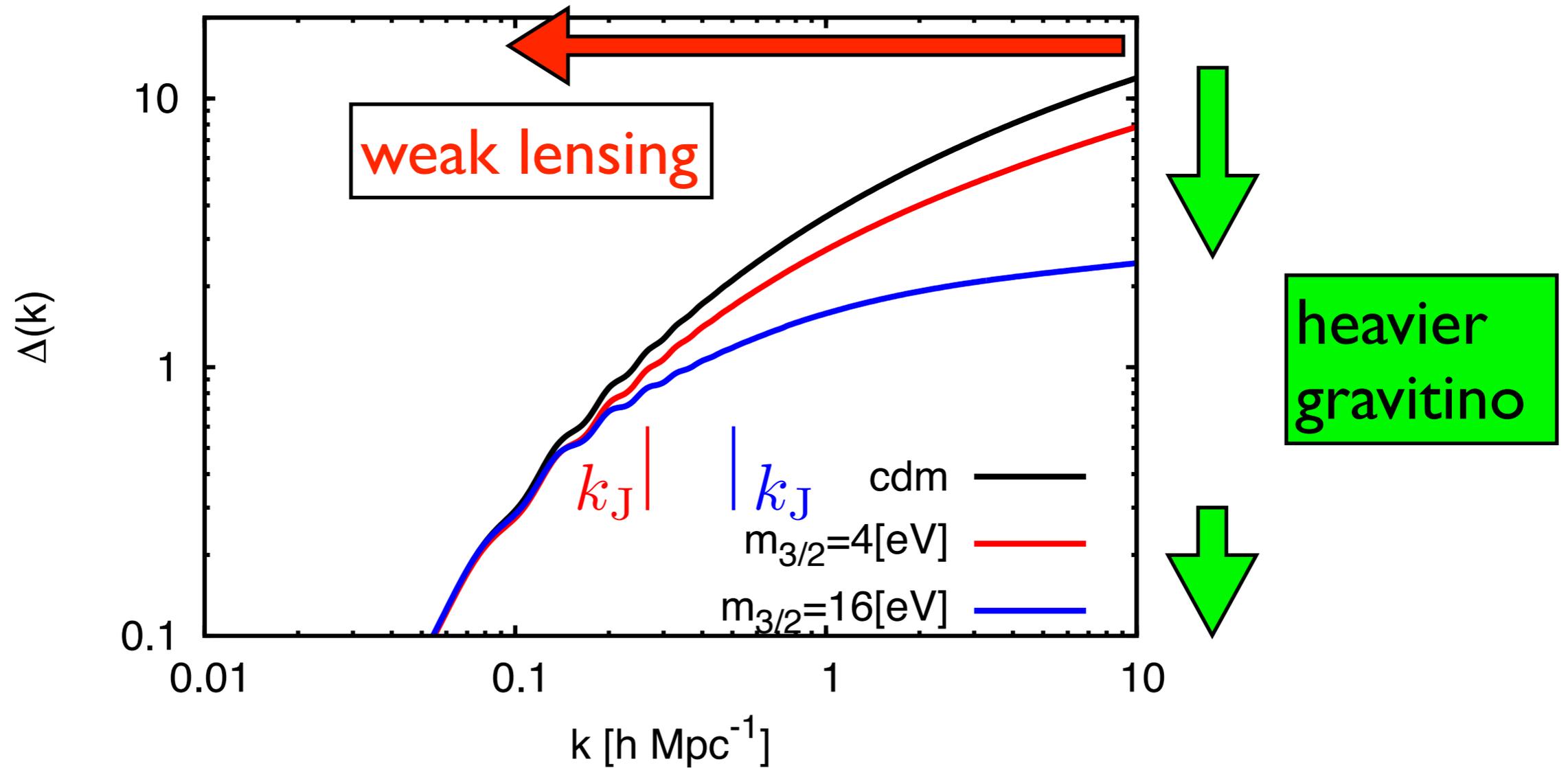
for  $m_{3/2} = \mathcal{O}(1-10) \text{ eV}$

Different from the neutrino

$$g_{*s\nu} \simeq 43/4 \quad \longleftrightarrow \quad g_{*s3/2} \simeq 90$$



# Linear matter power spectra w/ light gravitino



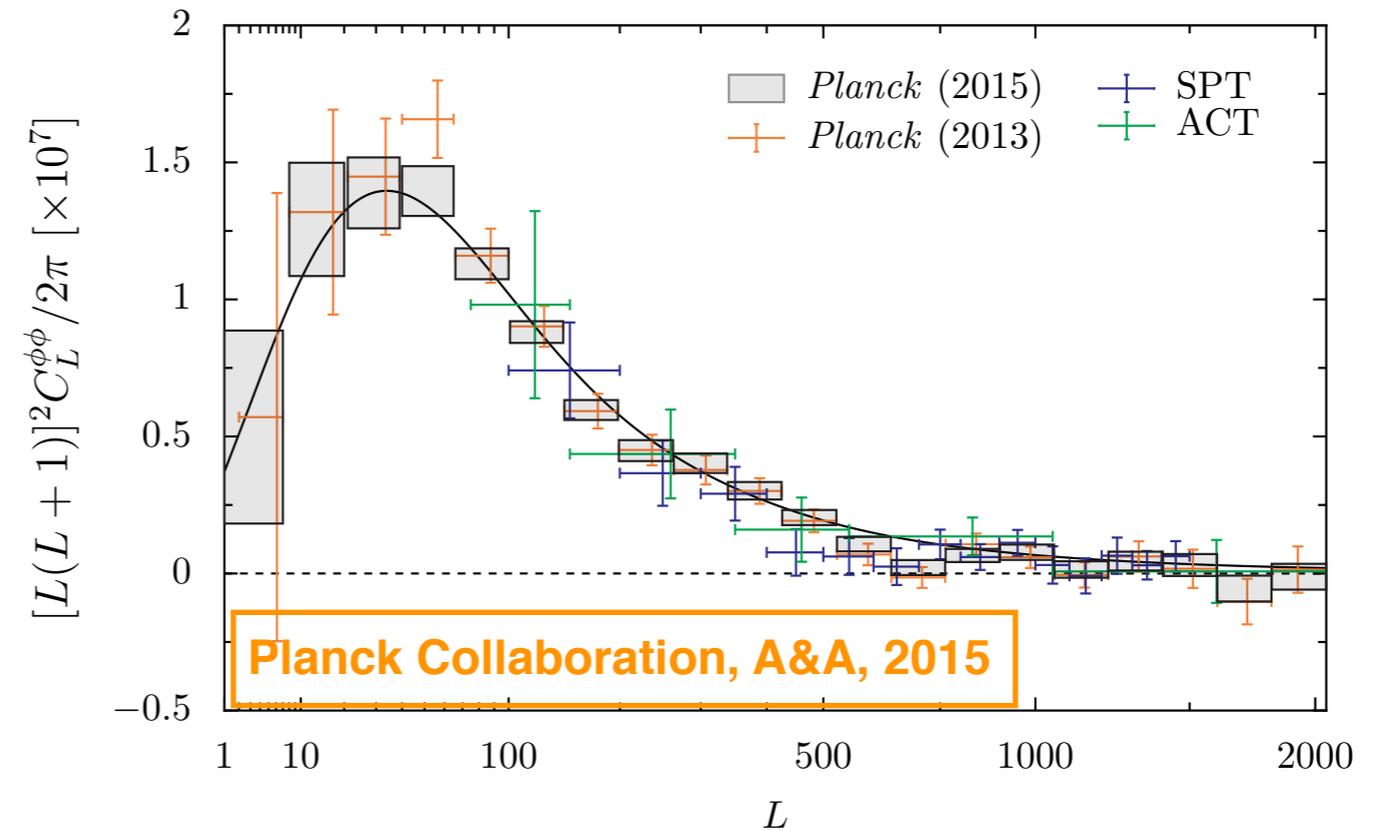
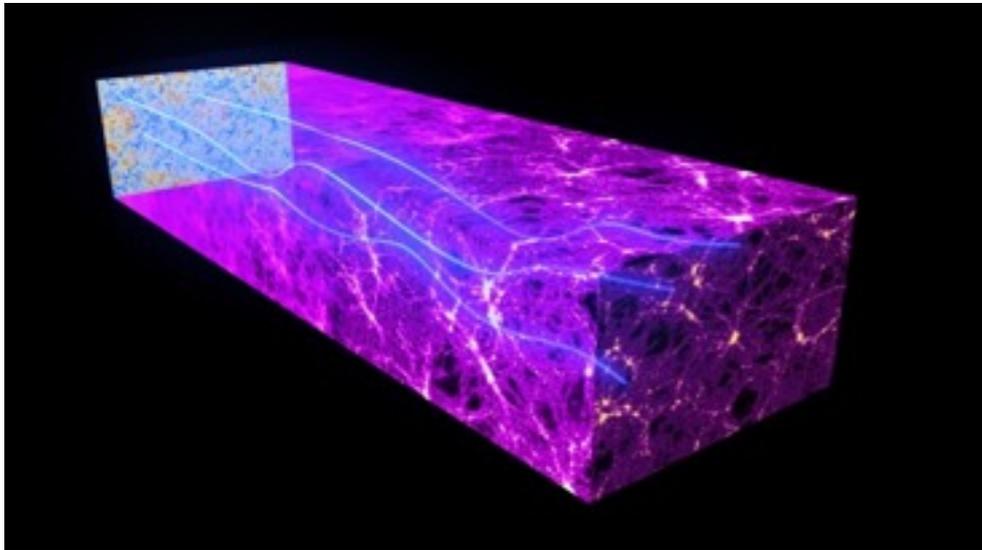
**cutoff scale:**  $k_J = a \sqrt{\frac{4\pi G \rho_M}{\langle v^2 \rangle}} \Big|_{a=a_{\text{eq}}} \simeq 0.86 \text{ Mpc}^{-1} \left(\frac{g_{3/2}}{2}\right)^{-1/2} \left(\frac{m_{3/2}}{100 \text{ eV}}\right)^{1/2} \left(\frac{g_{*s3/2}}{90}\right)^{5/6}$

Upper bound on the gravitino mass in cosmology  
 $\Leftrightarrow$  Lower bound in the LHC

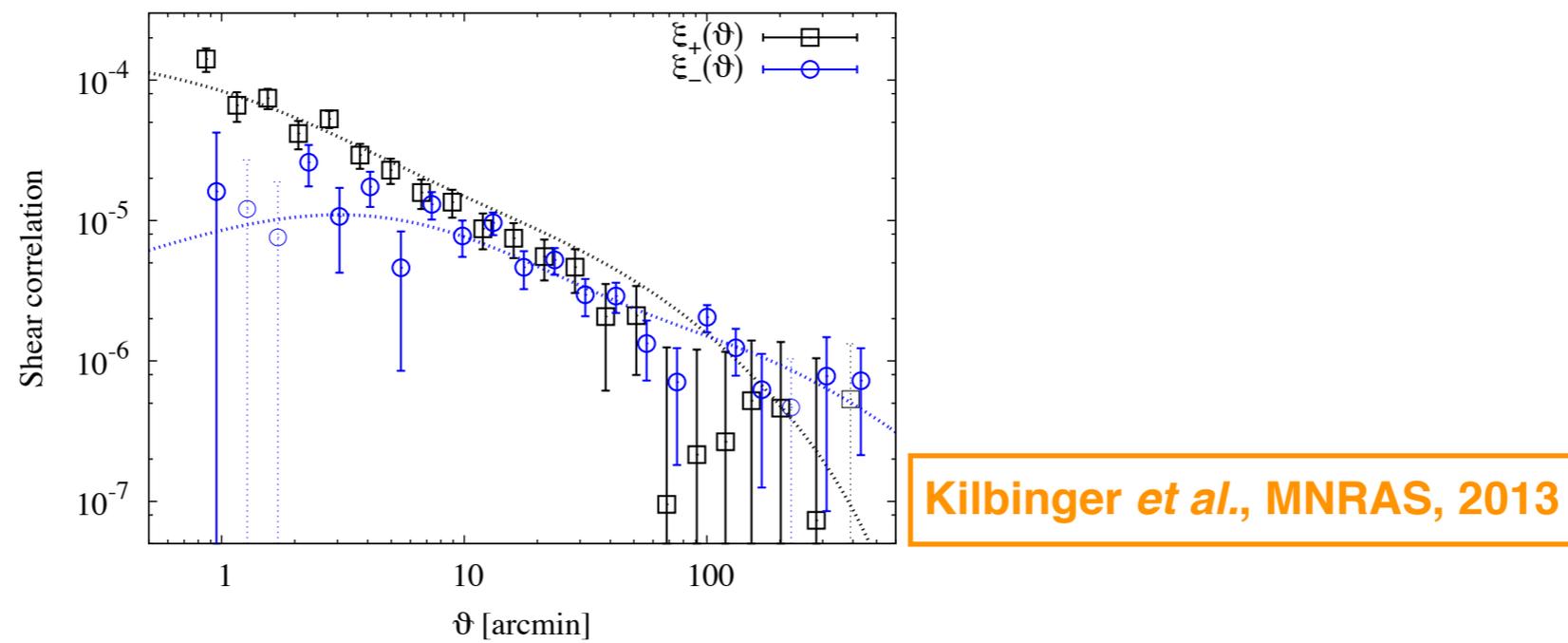
# Cosmological data

## Planck TT,TE,EE+lowP - constraint $\Lambda$ CDM model parameters

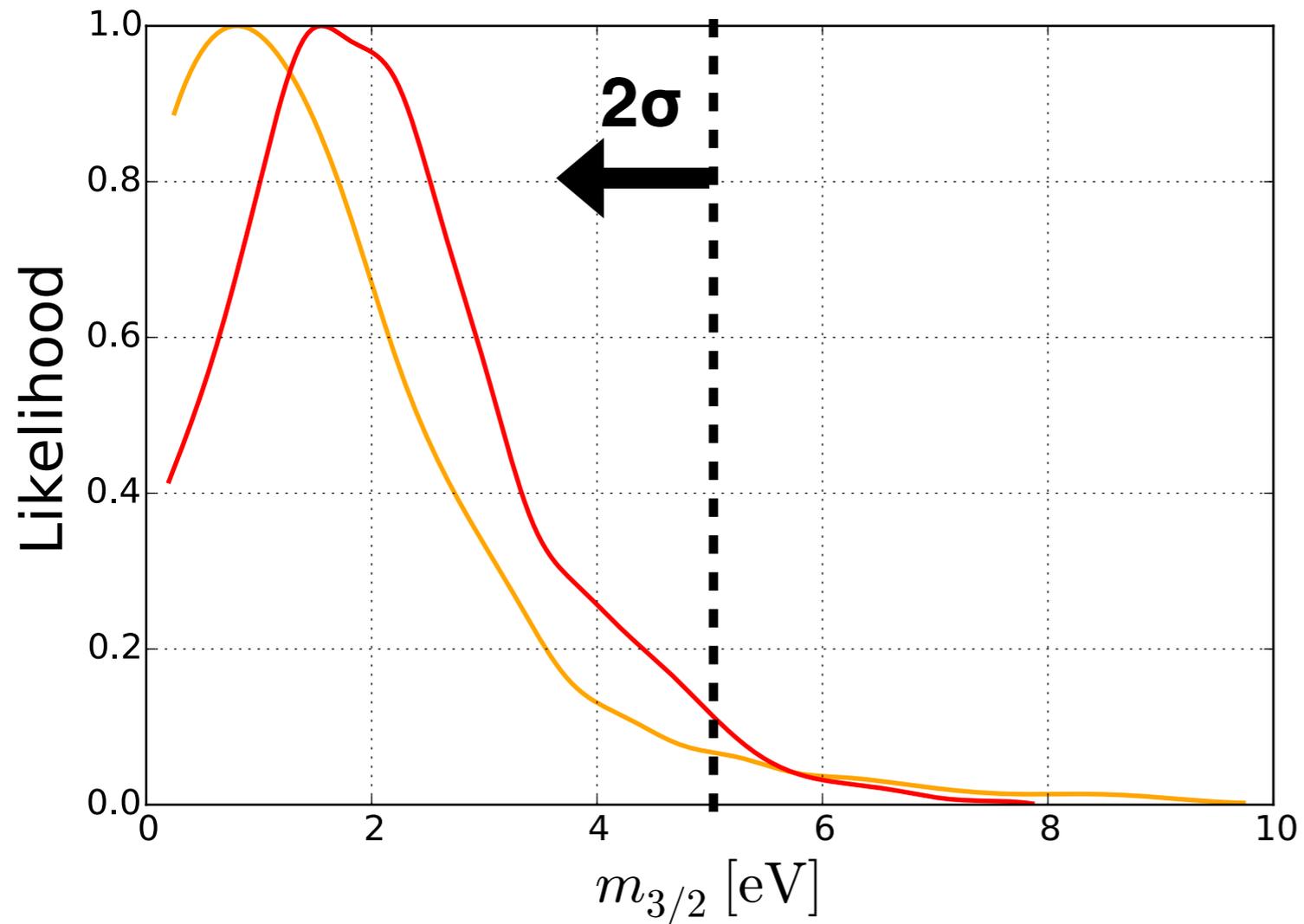
### Planck lensing



## Canada France Hawaii Lensing Survey (CFHTLenS) + BOSS



# Results



— Planck + lensing (gravitino)  
 — Planck + lensing + CFHTLenS + BOSS (gravitino)

marginalized over the  
 other parameters

$m_{3/2} \lesssim 5$  eV is consistent with the lens data at 95% CL

# Higgs mass in MSSM

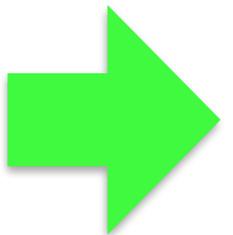
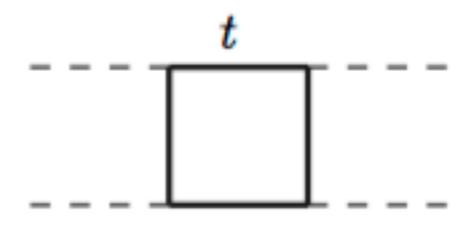
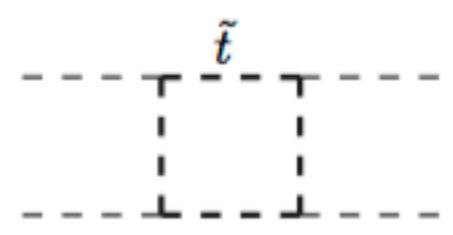
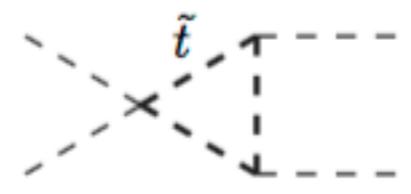
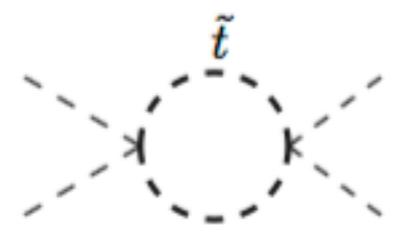
$$m_{h^0}^2 = m_{h^0,0}^2 + \Delta(m_{h^0}^2)$$

model prediction  
(126 GeV)<sup>2</sup>

< m<sub>Z</sub><sup>2</sup>  
(91 GeV)<sup>2</sup>

v : Higgs VEV  
m<sub>t</sub> : top quark mass  
m<sub>τ̃</sub> : stop mass

quantum correction  
~ m<sub>t</sub><sup>4</sup> / (16π<sup>2</sup>v<sup>2</sup>) ln(m<sub>τ̃</sub>/m<sub>t</sub>)  
~ ln(m<sub>3/2</sub>)



gravitino mass as large as  
m<sub>3/2</sub> = O(100) eV (m<sub>τ̃</sub> = O(10) TeV)  
is needed in MSSM of perturbative GMSB (λ < 1)

Ajaib et al., PLB, 2012

$$W = (\lambda S + M_{\text{mess}}) \sum_{n=1}^{N_5} \Phi_n \bar{\Phi}_n$$

# Implications

**Some modification** is needed for GMSB models to reproduce simultaneously the observed Higgs mass and gravitational lens measurements

- extension of the Higgs sector: NMSSM

Yanagida *et al.*, JHEP, 2012

- non-perturbative GMSB

-- hidden baryons as good candidate for cold and stable particle

- non-standard cosmological history

-- low reheating temperature  $T_{RH} \lesssim m_{NLSP}/20$

-- entropy production after the gravitino decoupling

# Small scale crisis vs Quadruple lens system

based on

A. Harada, **AK**, JCAP, 2016

**AK**, K. T. Inoue, T. Takahashi, arXiv:1604.01489

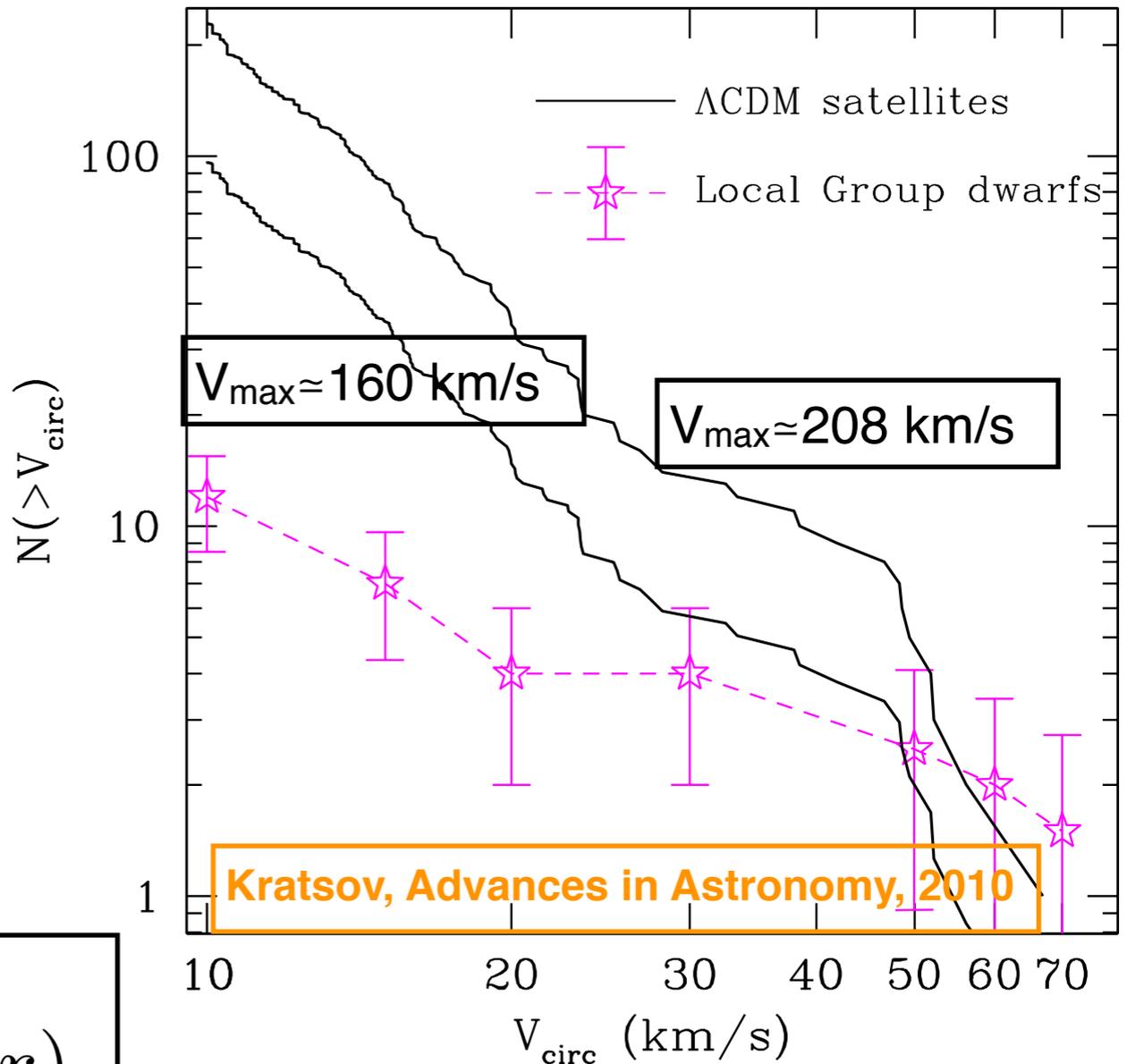
# Small scale crisis I

When  $N$ -body simulations in  $\Lambda$ CDM model and observations are compared, problems appear at (sub-)galactic scales:  
**small scale crisis**

## missing satellite problem

$N$ -body (DM-only) simulations in  $\Lambda$ CDM model  $\rightarrow$   
 Milky Way-size halos host  $O(10)$  times larger number of subhalos than that of observed dwarf spheroidal galaxies

cumulative number of subhalos



circular velocity

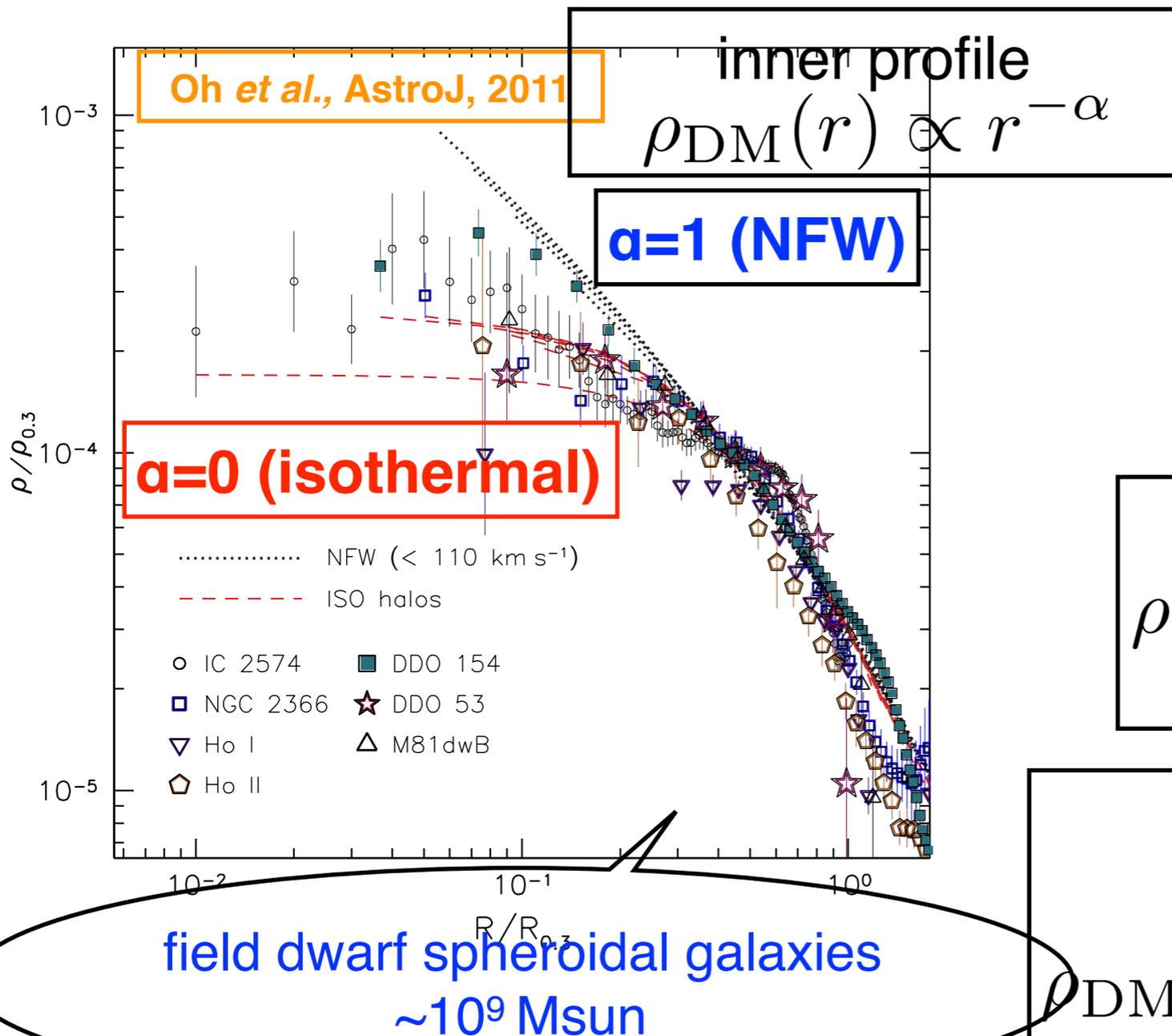
$$V_{\text{circ}}^2(r) = \frac{GM(<r)}{r}$$

maximal circular velocity of subhalo

# Small scale crisis II

## cusp vs core problem

$N$ -body (DM-only) simulations in  $\Lambda$ CDM model  $\rightarrow$   
**Universal DM profile independent of halo size: NFW profile**



Observations infer **core** profile  
 in the inner region rather than  
**cuspy** NFW profile

NFW profile

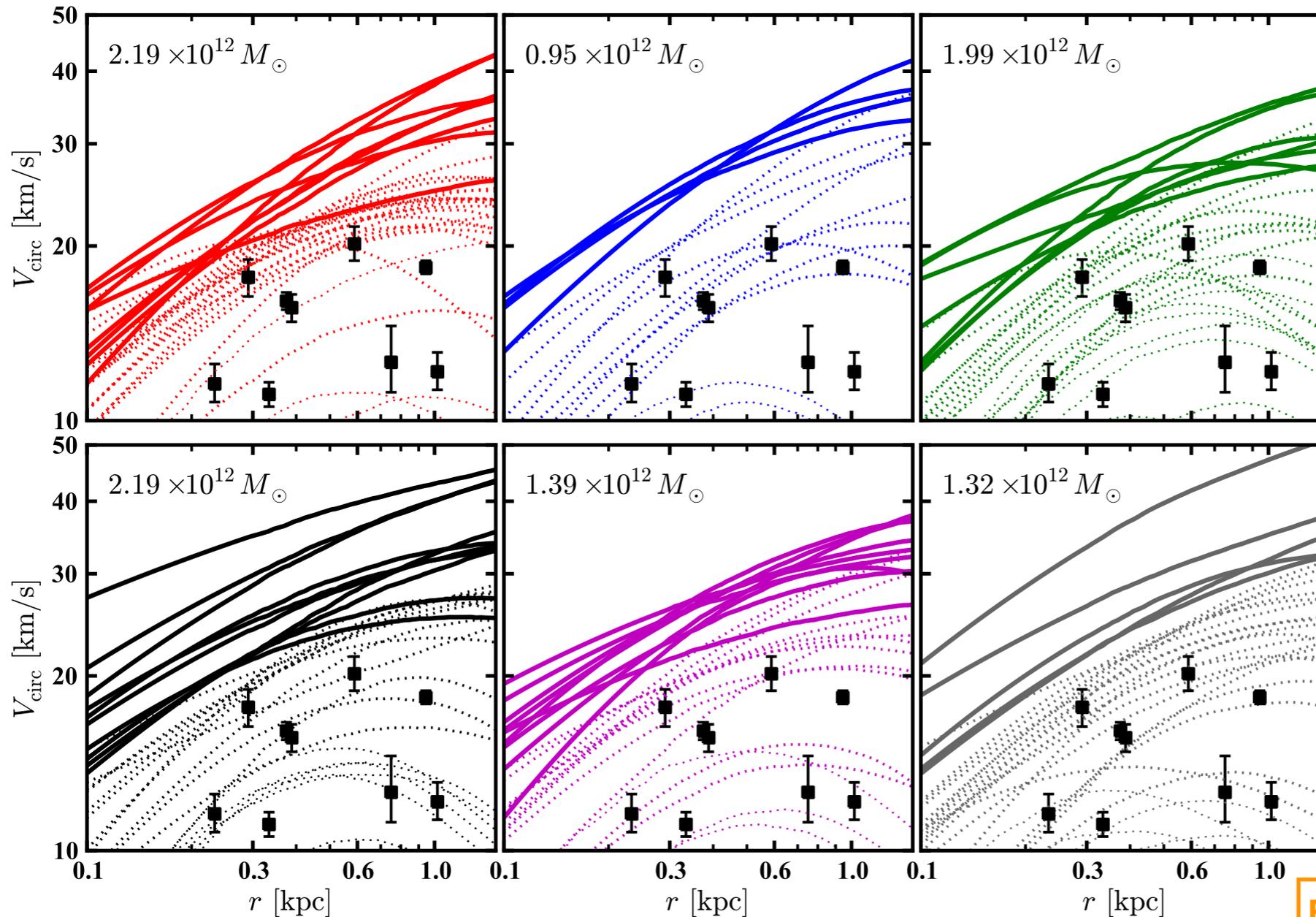
$$\rho_{\text{DM}}(r) = \frac{\rho_s}{r/r_s (1 + r/r_s)^2}$$

isothermal profile

$$\rho_{\text{DM}}(r) = \rho_{\text{DM}}^0 \begin{cases} 1 & (r \ll r_0) \\ (r_0/r)^2 & (r \gg r_0) \end{cases}$$

# Small scale crisis (III)

**too big to fail problem**

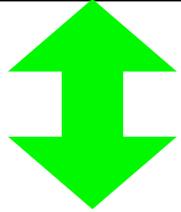


Boylan-Kolchin *et al.*, MNRAS, 2011

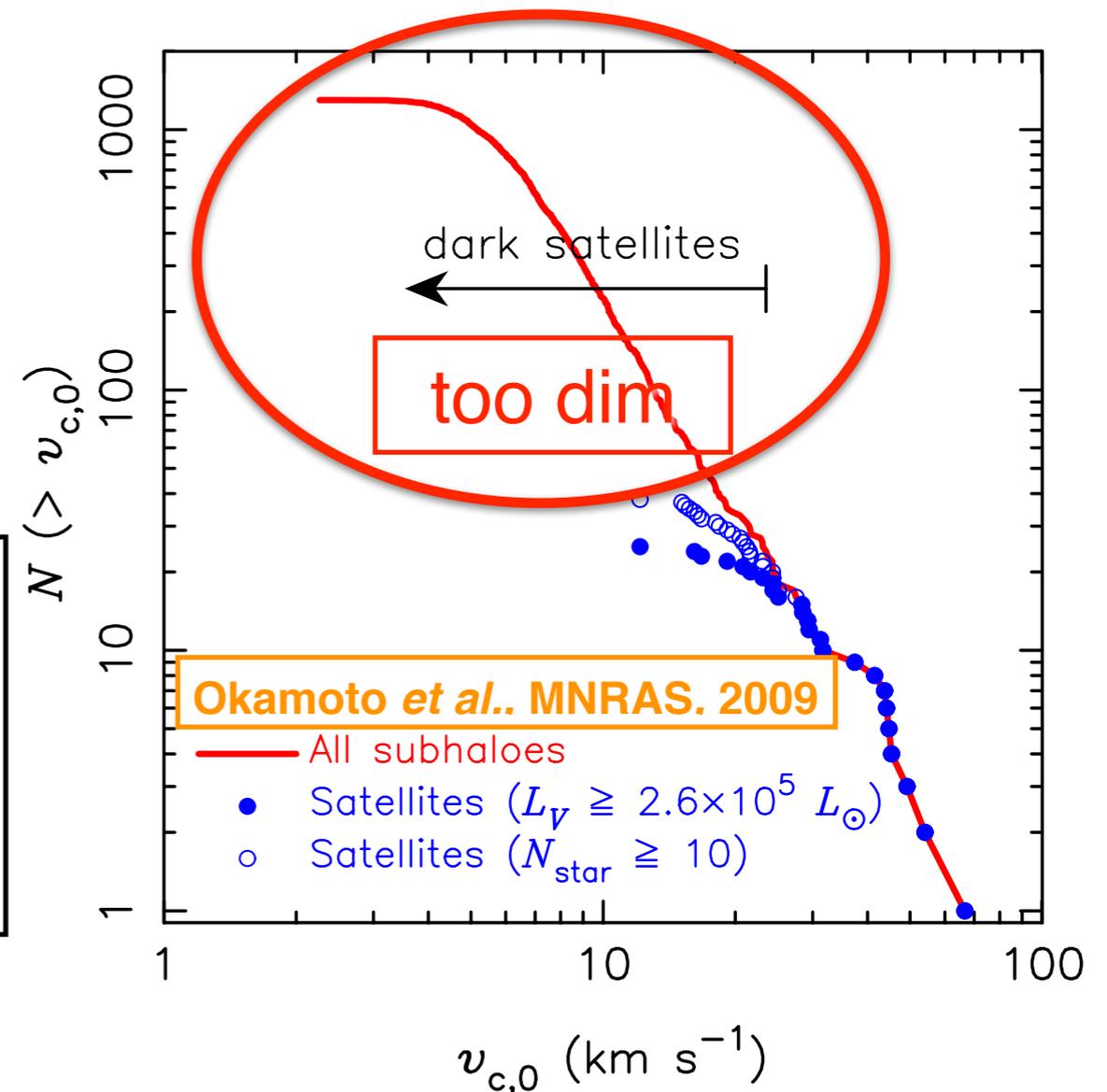
$N$ -body (DM-only) simulations in  $\Lambda$ CDM model  $\rightarrow$   
 $\sim 10$  subhalos with deepest potential wells in Milky Way-size halos do  
 not have observed counterparts (dwarf spheroidal galaxies)

# Possible solution I

Above Discussions are based on  $N$ -body (DM-only) simulations in  $\Lambda$ CDM model



Gravitational potentials are shallower at smaller scales  $\rightarrow$   
Baryonic heating and cooling processes may be important

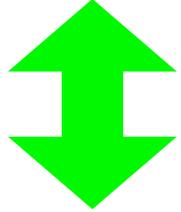


## Baryonic processes

- **heating from ionizing photons** - ionizing photons emitted and spread around reionization of the Universe heat and evaporate gases
- **mass loss by supernova explosions** - supernova explosions blow gases from inner region  $\rightarrow$  DM redistribute along shallower potential

## Possible solution II

Above Discussions are based on  
*N*-body (DM-only) simulations in  $\Lambda$ CDM model

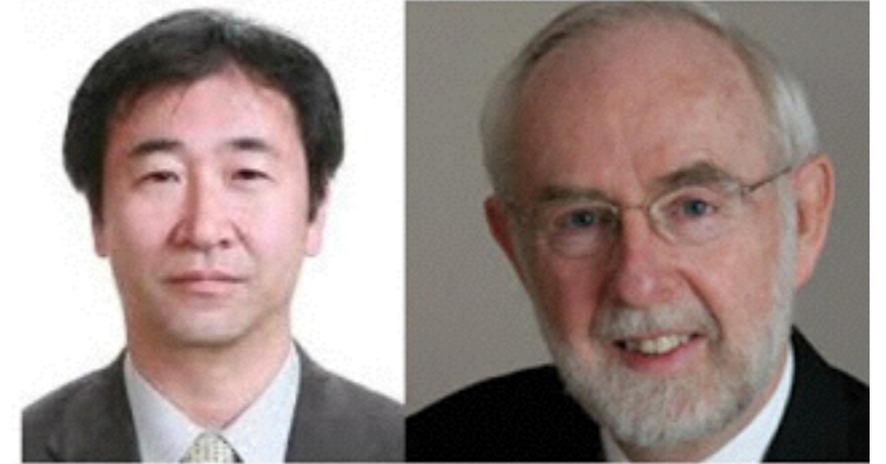


**alternative models ↔ nature of DM**

- **warmness** - thermal velocities induce pressure of DM fluid and prevent gravitational growth (Jeans analysis)
- **interactions with SM particles** - DM fluid couples to photons/neutrinos (moving with speed of light) in a direct/indirect manner.
- **self-interaction** - induced heat transfer of DM fluid heats DM particles in inner region and flatten inner profile

# Warmness of DM: sterile neutrino

right-handed neutrino  
inferred by observed **neutrino oscillation**  
- heavier than active ones  
through the see-saw mechanism



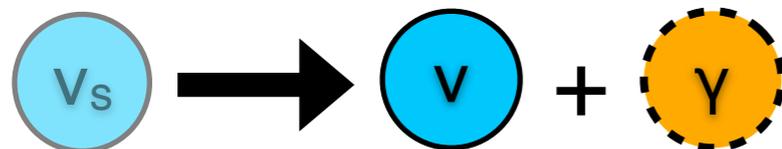
$m_s$  : mass

$\theta$  : mixing angle

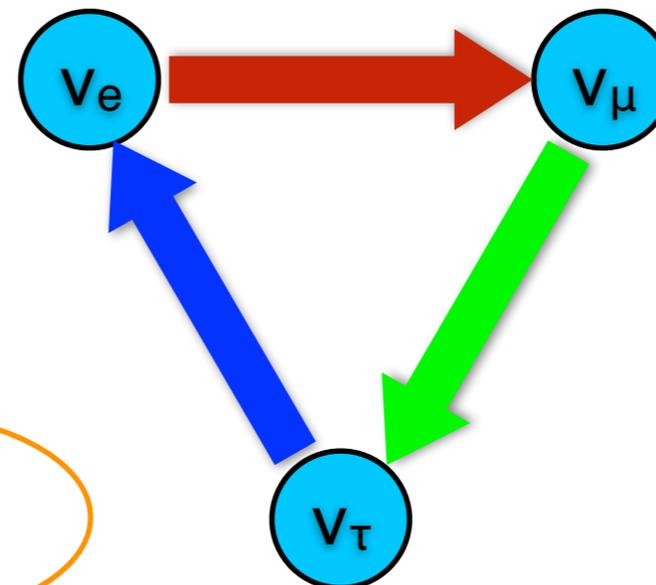
$m_s = 7 \text{ keV}$

$\sin^2(2\theta) \sim 10^{-9}$

radiative decay  $\tau \sim 10^{27} \text{ s}$



X-ray



Nobel Prize 2015  
Takaaki Kajita  
and  
Arthur B McDonald

**anomalous line of 3.5 keV ?**

**not confirmed**

E. Bulbul *et al.*, ApJ, 2014

A. Boyarsky *et al.*, PRL, 2014

T. Tamura *et al.*, PASJ, 2015

N. Sekiya *et al.*, PASJ, 2015

XMM-Newton, Chandra

Suzaku

May sterile neutrino DM reconcile **missing satellite problem** and **anomalous X-ray line** simultaneously?

# Sterile neutrino as mixed dark matter

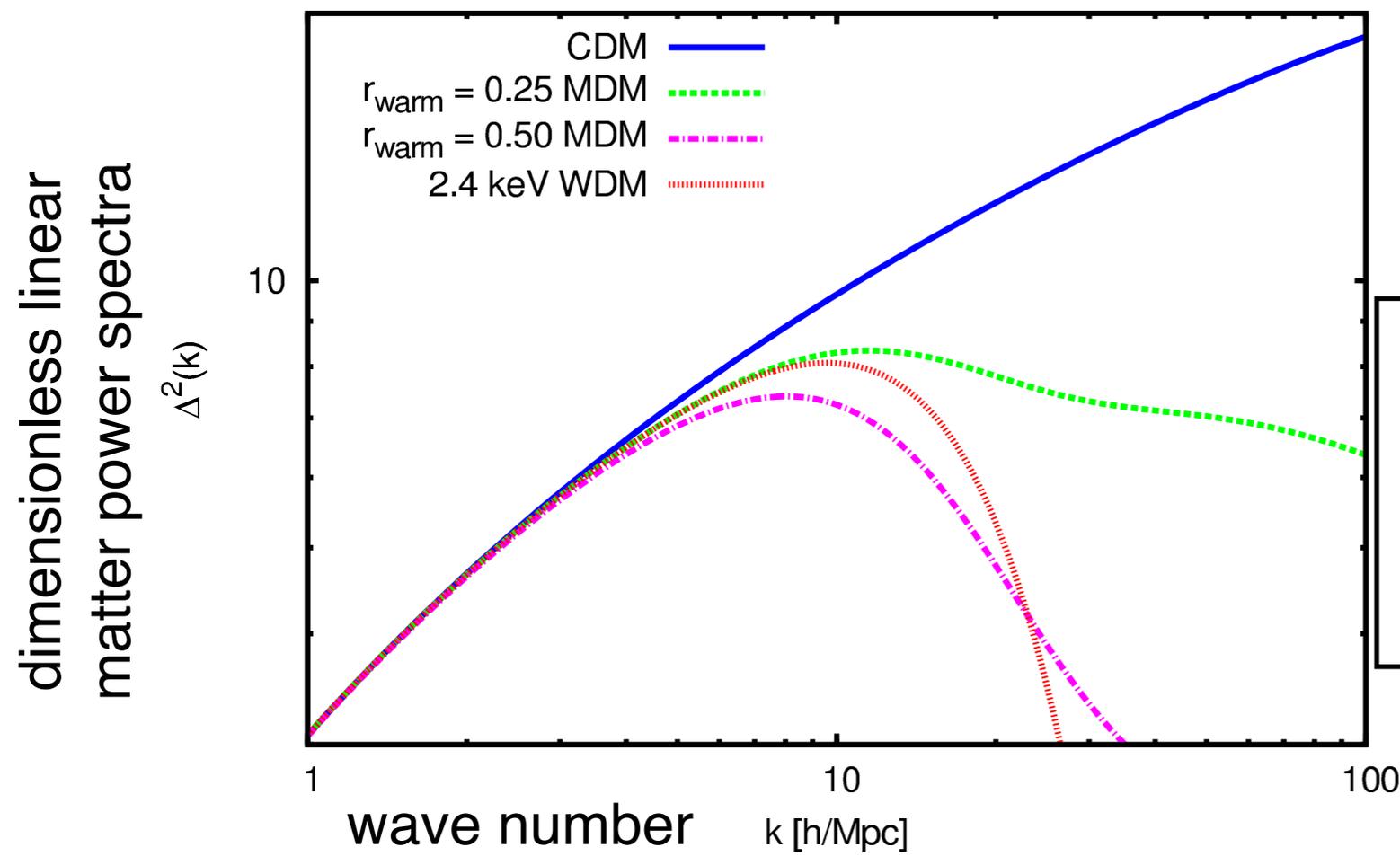
sterile neutrino radiative decay as an origin of 3.5 keV anomaly

Abazajian, PRL, 2014

large lepton asymmetry  $n_L/n_\gamma \sim 10^{-4}$  (c.f.  $n_B/n_\gamma \sim 10^{-10}$ )

Kamada *et al.*, JCAP, 2016

sterile neutrino (WDM) relic accounts for 20-60% of DM mass density (rest: CDM)  $\rightarrow$  CDM+WDM=MDM



Linear matter power is suppressed at galactic scales, while the suppression is weaker than in pure WDM models

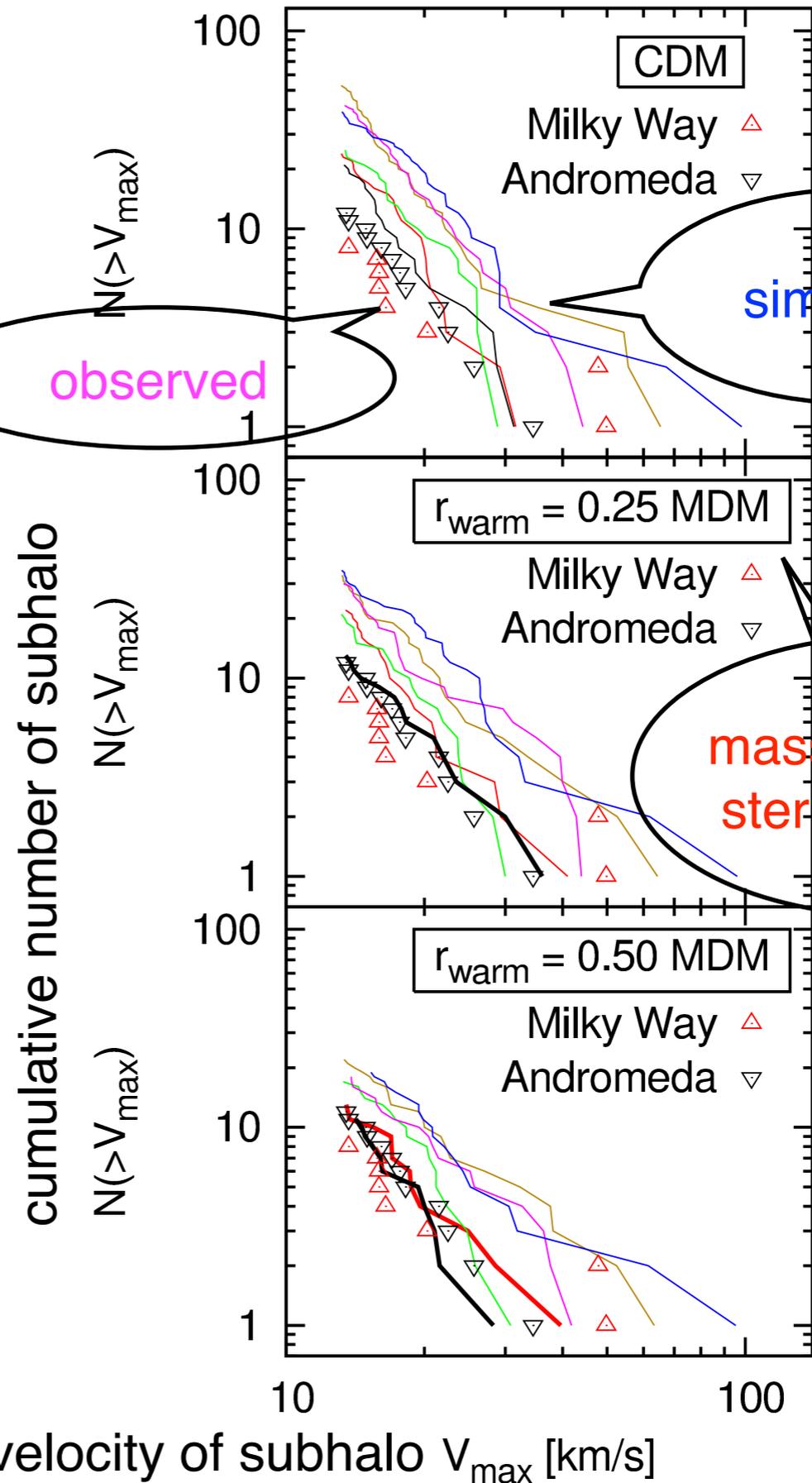
# Missing satellite problem in MDM models

cumulative velocity function  
of subhalos

Too many subhalos in CDM model

1 out of 6 halos may reproduce  
the number of observed  
dwarf spheroidal galaxies

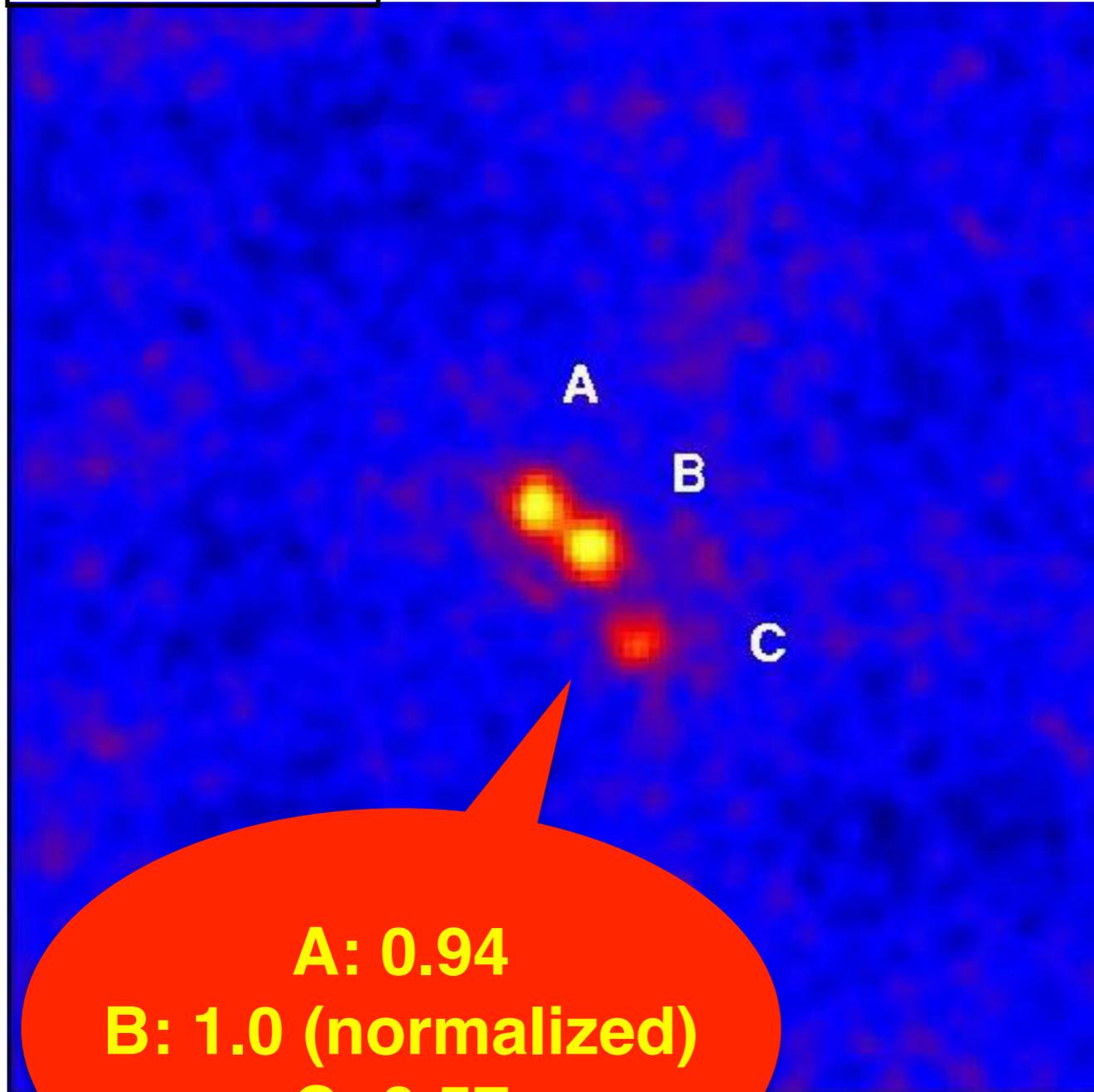
2 out of 6 halos appear concordant



# Anomalous Flux Ratio

**B1422+231**

M. Chiba *et al.*, ApJ, 2005



**A: 0.94**  
**B: 1.0 (normalized)**  
**C: 0.57**

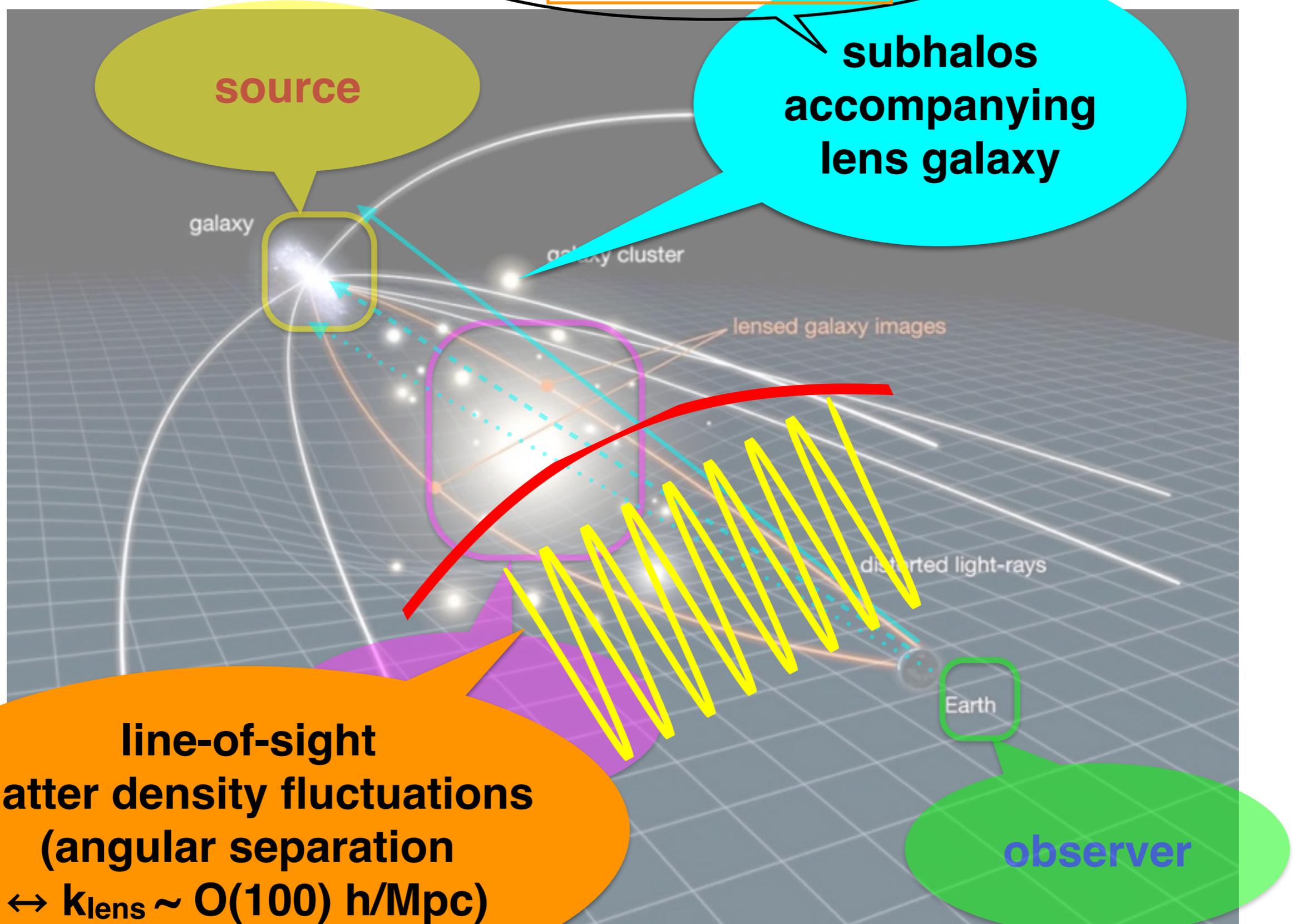
Singular isothermal profile  
 for lens galaxy  
 provides a good-fit  
 for lens positions

1st derivative of  
 gravitational potential

2nd derivative of  
 gravitational potential

Derived flux ratio  
 $(A+C)/B \approx 1$  is not  
 concordant with observed  
 value  $(A+C)/B \approx 1.5$   
 ( $2\sigma$  or more)

# Perturbers



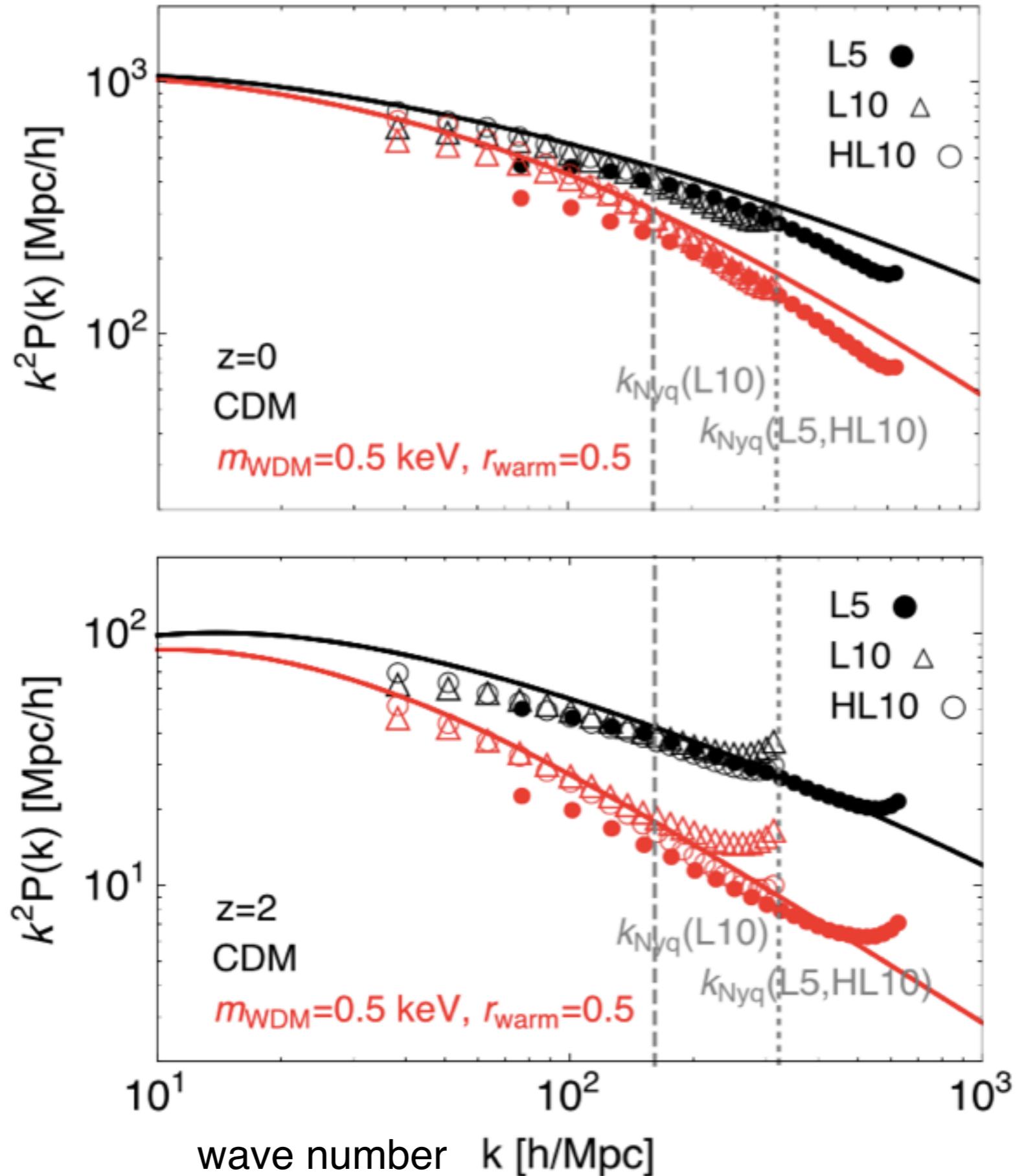
# Line-of-sight matter density fluctuations

$N$ -body simulation  
to measure  
**non-linear**  
matter power spectra



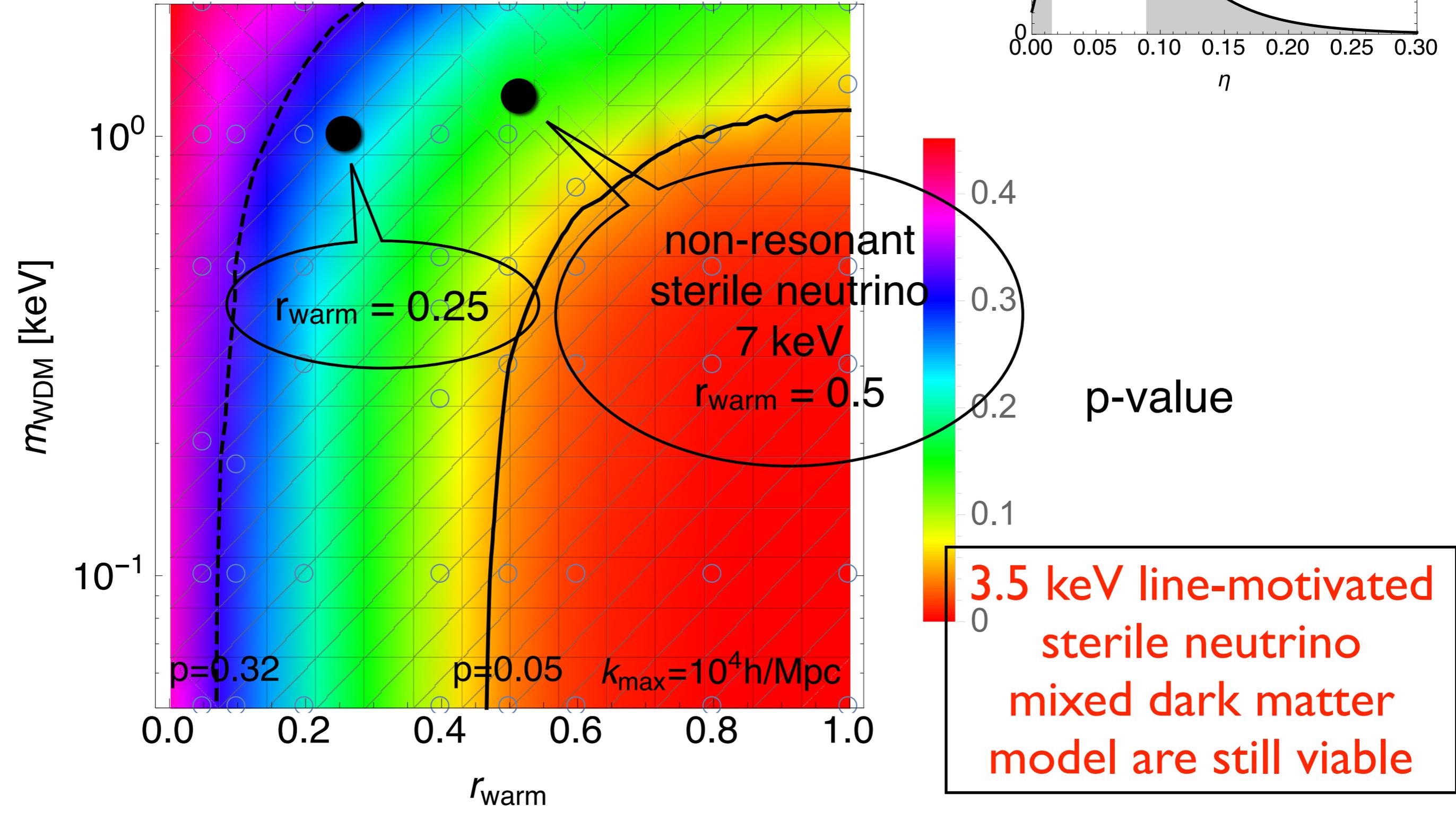
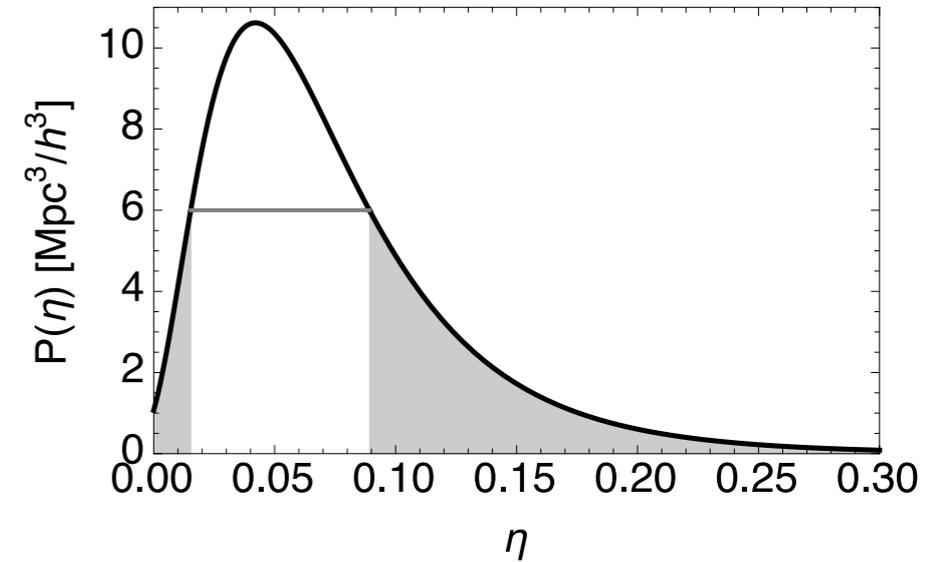
perturbed magnification  
 $\eta$ : a convolution of  
non-linear matter spectra  
with some Kernel

non-linear matter power spectra



# Likelihood

p-value: probability of finding a sample that is more unlikely than the observed value



# Conclusions and Prospects

# Conclusions

- Through the small-scale matter distribution, we can explore the **existence of long-lived particles** produced in the early Universe (being careful of baryonic processes)

- **Gravitational lens** is a powerful tool to probe the (relatively) small-scale clustering property of the Universe

- To be compatible with the very light gravitino and the Higgs mass, **GMSB is needed to have an extended Higgs sector or some non-standard thermal history of the Universe**

- 3.5 keV-line motivated (sterile neutrino) mixed DM model can reproduce simultaneously the small number of the observed dwarf spheroidal galaxies and anomalous flux in QSO quadrupole lens systems

# Prospects

- Weak lens surveys are now on-going (DES, HSC-Wide, ...) and **higher-quality data will be available soon**

- **SMG lens samples are expected to be found** in on-going ALMA data

- In any case, more precise understanding of structure formation (non-linear evolution) in non- $\Lambda$ CDM models is indispensable

-- need a help of analytic approach