Renormalisation of the energy-momentum tensor on the lattice

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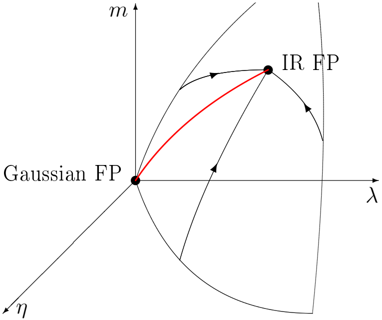
Motivation

- EMT relates to β-function

$$\langle \int d^D x \, T_{\mu\mu} \, \phi(x_1)...\phi(x_n) \rangle$$
$$= - \left(\sum_k \beta_k \frac{\partial}{\partial g_k} + n(\gamma_\phi + d_\phi) \right) \langle \phi(x_1)...\phi(x_n) \rangle$$

- ϕ^4-theory in 3D, $m^2 < 0$: toy model for theories with IR fixed point



Energy-momentum tensor and Ward identity

- Euclidean action

$$S = \int d^D x \, \left(\frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4 \right)$$

- Energy-momentum tensor

$$T_{\mu\rho}(x) = \partial_\mu \phi \partial_\rho \phi - \delta_{\mu\rho} \left(\frac{1}{2} \sum_\lambda \partial_\lambda \phi \partial_\lambda \phi + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4 \right)$$

- Translation Ward identity

$$\langle \, \delta_{x,\rho} P \, \rangle = -\langle \, P \, \partial_\mu \, T_{\mu\rho}(x) \, \rangle$$

- Local operator of translation

$$\delta_{x,\rho} P = \frac{\delta P}{\delta \phi(x)} \, \partial_\rho \phi(x)$$

Translation Ward identity on the lattice

- Lattice action

$$\hat{S} = a^D \sum_n \left(\frac{1}{2} (\hat{\partial}_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4 \right)$$

- Lattice regularisation breaks translation symmetry explicitly

$$\langle \hat{\delta}_{x,\rho} \hat{P} \rangle = -\langle \hat{P} \left(\hat{\partial}_\mu \hat{T}_{\mu\rho} + \hat{R}_\rho \right) \rangle$$

- Renormalised lattice TWI

$$\langle Z_\delta \, \hat{\delta}_{x,\rho} \hat{P} \rangle = -\langle \hat{P} \left(\hat{\partial}_\mu [\hat{T}_{\mu\rho}] + \hat{\bar{R}}_\rho \right) \rangle$$

- Renormalised $\hat{T}_{\mu\rho}(x)$

$$[\hat{T}_{\mu\rho}(x)] = \sum_i c_i \left\{ \hat{T}^{(i)}_{\mu\rho} - \langle \hat{T}^{(i)}_{\mu\rho} \rangle \right\}$$

Renormalisation of the EMT

$$\hat{T}_{\mu\rho}(x) = \hat{\partial}_\mu\phi\hat{\partial}_\rho\phi - \delta_{\mu\rho}\left(\frac{m^2}{2}\phi^2 + \frac{1}{2}\sum_\lambda \hat{\partial}_\lambda\phi\hat{\partial}_\lambda\phi + \frac{\lambda}{4!}\phi^4\right)$$

- Possible mixing: $D \leq 3$, Lorentz, $\phi \to -\phi$, $x \to -x$

$$\hat{\partial}_\mu\phi\hat{\partial}_\rho\phi,\ \ \phi\hat{\partial}_\mu\hat{\partial}_\rho\phi,$$

$$\delta_{\mu\rho}\left(\phi^2,\ \ \phi^4,\ \ \phi^6,\ \ \sum_\lambda \hat{\partial}_\lambda\phi\hat{\partial}_\lambda\phi,\ \ \sum_\lambda \phi\hat{\partial}_\lambda\hat{\partial}_\lambda\phi,\ \ \hat{\partial}_\mu\phi\hat{\partial}_\mu\phi,\ \ \phi\hat{\partial}_\mu\hat{\partial}_\mu\phi\right)$$

- Perturbative analysis shows that all divergencies are $\propto \phi^2$

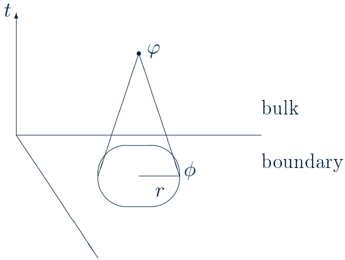
$$[\hat{T}_{\mu\rho}] = \frac{c_1}{2}\hat{\partial}_\mu\phi\hat{\partial}_\rho\phi + \delta_{\mu\rho}\left(\frac{c_2}{2}\phi^2 + \frac{c_3}{2}\sum_\lambda \hat{\partial}_\lambda\phi\hat{\partial}_\lambda\phi + \frac{c_4}{4!}\phi^4\right)$$

$$c_2 = -m^2 + \text{divergent term}$$

Wilson flow - gradient flow on the lattice

- Flow equation [*Monahan,Orginos 2014*]

$$\partial_t \varphi(t,x) = \hat{\partial}^2 \varphi(t,x), \qquad \varphi(t,x)|_{t=0} = \phi(x)$$



- Smoothing effect, radius $r = \sqrt{2Dt}$

Renormalisation of the EMT using the Wilson flow

- Renormalised TWI

$$\langle\, Z_\delta\ \hat{\delta}_{x,\rho}\hat{P}\,\rangle = -\langle\, \hat{P}\left(\hat{\partial}_\mu[\hat{T}_{\mu\rho}] + \hat{\hat{R}}_\rho\right)\rangle$$

- Renormalisation condition [*Del Debbio,Patella,Rago 2013*]
    - Choose probe $\hat{P}_t$: function of fields at $t > 0$, then:
    - Coefficients c_i can be tuned such that EMT is finite
    - $\hat{\hat{R}}_\rho \to 0$

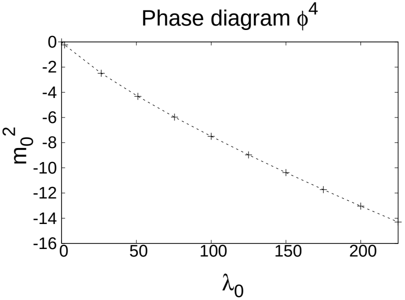
$$Z_\delta\,\langle\, \hat{\delta}_{x,\rho}\hat{P}_t\,\rangle = -\sum_i c_i\,\langle\, \hat{P}_t\ \hat{\partial}_\mu\,\hat{T}^{(i)}_{\mu\rho}(x)\,\rangle$$

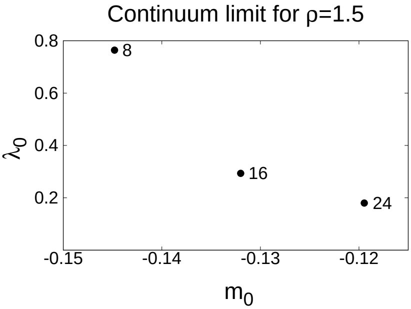
- Determine Z_δ separately
- System of 4 equations with 4 different operators $P_t^{(k)}$

$$Z_\delta\ V^{(k)} = -\sum_i c_i\ M^{(k,i)}$$

Where to look

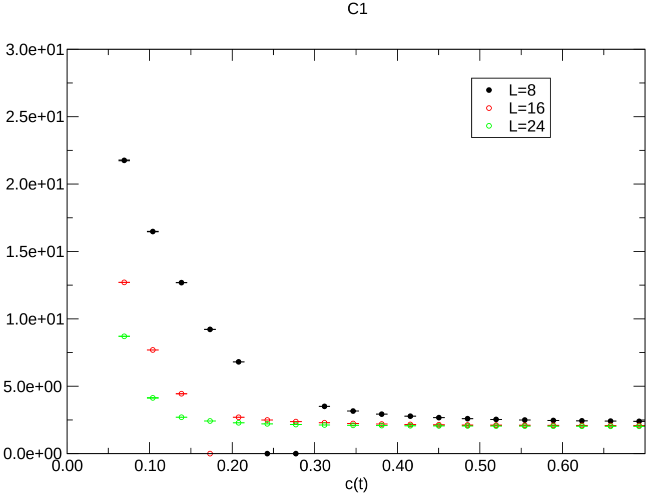
- Interested in staying close to critical line, $m_0^2 < 0$
- Line of constant physics defined by $\rho = \lambda_R / m_R$





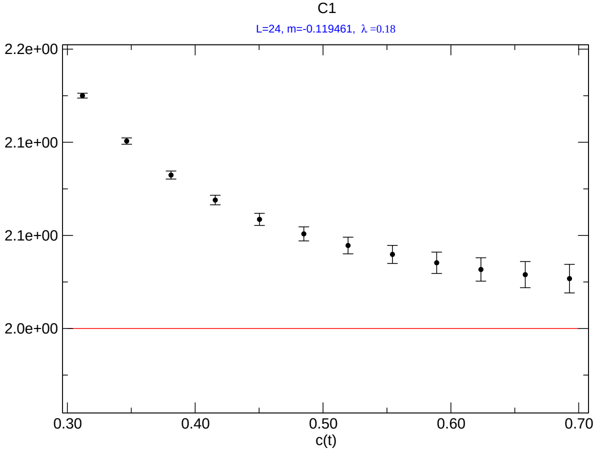
Results - c_1

- Plateau more pronounced for finer lattices
- $c(t) = \sqrt{6t}/L$



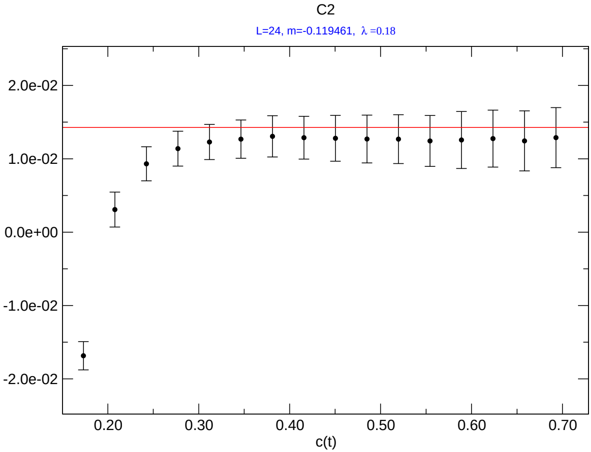
Results - c_1

- $c_1 \ \frac{1}{2}\hat{\partial}_\mu\phi\hat{\partial}_\rho\phi$
- Expected value: $c_1 = 2$



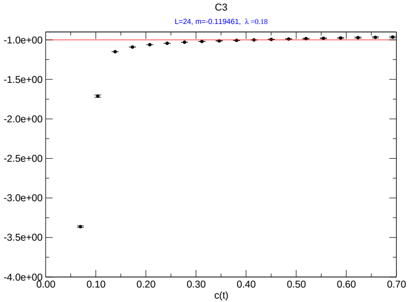
Results - c_2

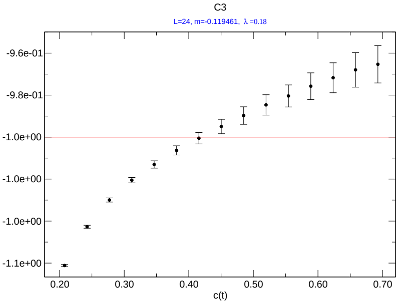
- $c_2\,\frac{1}{2}\phi^2\delta_{\mu\rho}$
- Expected value: $c_2 = -m_0^2 + \text{small corr.} = 0.0143 + \text{small corr.}$



Results - c_3

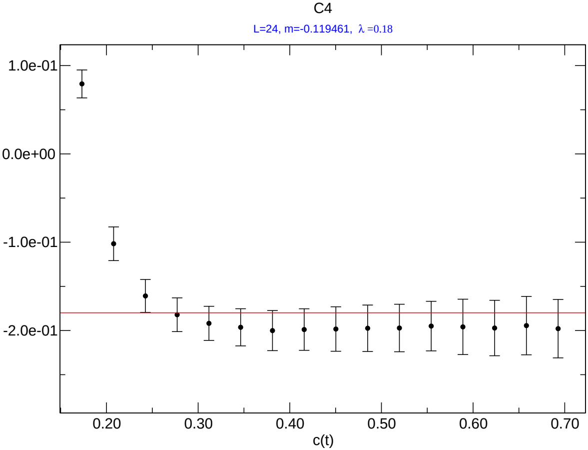
- $c_3 \, \frac{1}{2} \sum_\lambda \hat{\partial}_\lambda \phi \hat{\partial}_\lambda \phi \delta_{\mu\rho}$
- Expected value: $c_3 = -1$





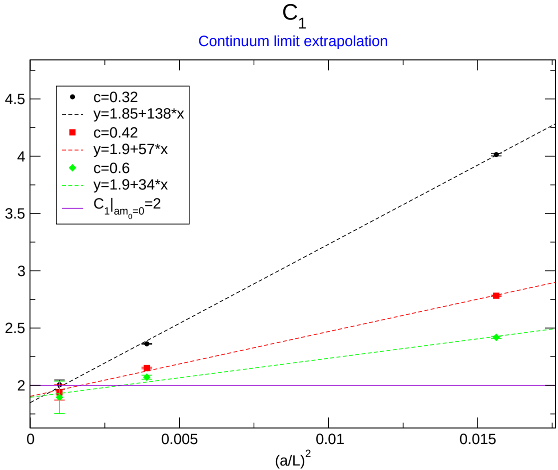
Results - c_4

- $c_4 \, \frac{1}{4!}\phi^4\delta_{\mu\rho}$
- Expected value: $c_4 = -\lambda_0 = -0.18$



Outlook: Continuum limit

- Test: continuum limit of c_1 at different flow times t in free theory



Summary

- We are able to find the coefficients of the renormalised EMT on the lattice at finite a
- The Wilson flow provides a new way to implement Ward identities free from contact terms
- We seem to be able to reproduce correct continuum limit
- Left to do: improve techniques

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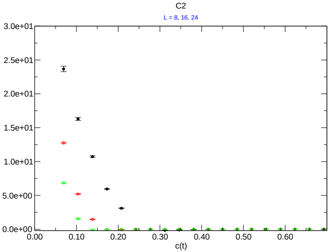
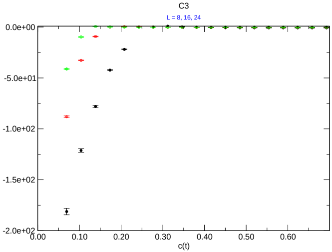
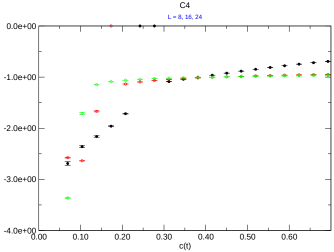
Thank you!

C2, C3, C4


C2
L = 8, 16, 24
3.0e+01
2.5e+01
2.0e+01
1.5e+01
1.0e+01
5.0e+00
0.0e+00
0.00
0.10
0.20
0.30
0.40
0.50
0.60
c(t)


C3
L = 8, 16, 24
0.0e+00
-5.0e+01
-1.0e+02
-1.5e+02
-2.0e+02
0.00
0.10
0.20
0.30
0.40
0.50
0.60
c(t)


C4
L = 8, 16, 24
0.0e+00
-1.0e+00
-2.0e+00
-3.0e+00
-4.0e+00
0.00
0.10
0.20
0.30
0.40
0.50
0.60
c(t)