Mainz, 20th January 2016

$K^+ \to \pi^+\pi^0 ee$ and related decays

Giancarlo D'Ambrosio

(INFN- Sezione Napoli)

NA62 Kaon Physics Handbook

Outline

- Weak counterterm structures, Ni's, VMD?
- $K \to 3\pi$, VMD?
- $K^+ \to \pi^+\pi^0\gamma$, NA48/2 results
- $K^+ \to \pi^+\pi^0 ee$, NA48/2 results
- $K_{S,L} \to \pi^0 ee$

Understanding the weak counterterms

- The strong chiral lagrangian well understood, properties of QCD
- status of weak chiral lagrangian

Vector Meson Dominance in the strong sector

Ecker, Gasser, de Rafael, Pich

L_i	L_i expts	V	A	Total (Scalar incl.)	Total QCD rel. incl.
L_1	0.4 ± 0.3	0,6	0	0,6	0,9
L_2	1.4 ± 0.3	1,2	0	1,2	1,8
L_3	-3.5 ± 1.1	-3,6	0	-3,0	-4,9
L_4	-0.3 ± 0.5	0	0	0	0
L_5	1.4 ± 0.5	0	0	1,4	1,4
L_6	-0.2 ± 0.3	0	0	0	0
L_7	-0.4 ± 0.2	0	0	-0,3	-0,3
L_8	0.9 ± 0.3	0	0	0,9	0,9
L_9	6.9 ± 0.7	6,9	0	6,9	7,3
L_{10}	-5.5 ± 0.7	-10	4	-6,0	-5,5

QCD inspired relations relations

$$F_V = 2G_V = \sqrt{2} f_\pi$$

$$F_A = f_\pi$$

$$M_A = \sqrt{2} M_V$$

KSFR: $G_V = \sqrt{2}\ F_\pi$ determined by dominance of pion, V,A to recover QCD short distance constraints

$$L_1^V = \frac{L_2^V}{2} = -\frac{L_3^V}{6} = \frac{G_V^2}{8M_V^2}, \qquad L_9^V = \frac{F_V G_V}{2M_V^2}, \qquad L_{10}^{V+A} = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}$$

QCD inspired relations relations

$$L_1^V = L_2^V/2 = -L_3^V/6 = L_9^V/8 = -L_{10}^{V+A}/6 = f_\pi^2/(16M_V^2)$$

Weak interaction

The symmetry of the short distance hamiltonian $-\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* C_- (\overline{s}_L \gamma^\mu u_L)(\overline{u}_L \gamma_\mu d_L)$

described in CHPT

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1} + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K \to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+ \to \pi^+ \gamma\gamma, K \to \pi l^+ l^-} + \cdots$$

VMD not as successful, in particular for K-3pi, where in principle large VMD important

Not only a bookkeeping but predictive already

π	2π	3π	N_i
$\pi^+\gamma^*$	$\pi^+\pi^0\gamma^*$		$N^r_{14} - N^r_{15}$
$\pi^0\gamma^*\ (S)$	$\pi^0\pi^0\gamma^*\ (L)$		$2N^r_{14} + N^r_{15}$
$\pi^+\gamma\gamma$	$\pi^+\pi^0\gamma\gamma$		$N_{14} - N_{15} - 2N_{18}$
	$\pi^+\pi^-\gamma\gamma\ (S)$		”
	$\pi^+\pi^0\gamma$	$\pi^+\pi^+\pi^-\gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$
	$\pi^+\pi^-\gamma\ (S)$	$\pi^+\pi^0\pi^0\gamma$	”
		$\pi^+\pi^-\pi^0\gamma\ (L)$	”
		$\pi^+\pi^-\pi^0\gamma\ (S)$	$7(N^r_{14} - N^r_{16}) + 5(N^r_{15} + N_{17})$
	$\pi^+\pi^-\gamma^*\ (L)$		$N^r_{14} - N^r_{15} - 3(N^r_{16} - N_{17})$
	$\pi^+\pi^-\gamma^*\ (S)$		$N^r_{14} - N^r_{15} - 3(N^r_{16} + N_{17})$
	$\pi^+\pi^0\gamma^*$		$N^r_{14} + 2N^r_{15} - 3(N^r_{16} - N_{17})$
	$\pi^+\pi^-\gamma\ (L)$	$\pi^+\pi^-\pi^0\gamma\ (S)$	$N_{29} + N_{31}$
		$\pi^+\pi^+\pi^-\gamma$	”
	$\pi^+\pi^0\gamma$	$\pi^+\pi^0\pi^0\gamma$	$3N_{29} - N_{30}$
		$\pi^+\pi^-\pi^0\gamma\ (S)$	$5N_{29} - N_{30} + 2N_{31}$
		$\pi^+\pi^-\pi^0\gamma\ (L)$	$6N_{28} + 3N_{29} - 5N_{30}$

$$K^+ \to \pi^+ l^+ l^-$$
$$K_S \to \pi^0 l^+ l^-$$

Vectors and axials

Counterterm combination	Processes	VMD weak coupling
$N^r_{14} - N^r_{15}$	$K^+ \to \pi^+\gamma^*$ $K^+ \to \pi^+\pi^0\gamma^*$	$-0.020\,\eta_V \ + \ 0.004\,\eta_A$
$2N^r_{14} + N^r_{15}$	$K_S \to \pi^0\gamma^*$	$0.08\,\eta_V$
$N_{14} - N_{15} - 2N_{18}$	$K^+ \to \pi^+\gamma\gamma$ $K^+ \to \pi^+\pi^0\gamma\gamma$	$-0.01\,\eta_A$
$N_{14} - N_{15} - N_{16} - N_{17}$	$K^+ \to \pi^+\pi^0\gamma$ $K_S \to \pi^+\pi^-\gamma$	$-\,0.010\,\eta_A$
$N^r_{14} - N^r_{15} - 3(N^r_{16} - N_{17})$	$K_L \to \pi^+\pi^-\gamma^*$	$-0.004\,\eta_V \ + \ 0.018\,\eta_A$
$N^r_{14} - N^r_{15} - 3(N^r_{16} + N_{17})$	$K_S \to \pi^+\pi^-\gamma^*$	$0.05\,\eta_V \ - \ 0.04\,\eta_A$
$N^r_{14} + 2N^r_{15} - 3(N^r_{16} - N_{17})$	$K^+ \to \pi^+\pi^0\gamma^*$	$0.12\,\eta_V \ + \ 0.01\,\eta_A$
$N_{29} + N_{31}$	$K_L \to \pi^+\pi^-\gamma$	$0.005\,\eta_V \ + 0.003\,\eta_A$
$3N_{29} - N_{30}$	$K^+ \to \pi^+\pi^0\gamma$	$-0.005\,\eta_V \ - \ 0.003\,\eta_A$

Observation hidden by other effects: different analysis maybe useful (Kaon charge radius)
NA48 has a good chance

K->2 pi/3pi fit

Kambor Missimer Wyler, '90s

$$\mathcal{M}(K_L \to \pi^+\pi^-\pi^0) = \alpha_1 - \beta_1 u + (\zeta_1 + \xi_1)u^2 + \frac{1}{3}(\zeta_1 - \xi_1)v^2$$

$$\mathcal{M}(K_L \to \pi^0\pi^0\pi^0) = -3\alpha_1 - \zeta_1(3u^2 + v^2) \ ,$$

$$\mathcal{M}(K^+ \to \pi^+\pi^+\pi^-) = 2\alpha_1 + \beta_1 u + (2\zeta_1 - \xi_1)u^2 + \frac{1}{3}(2\zeta_1 + \xi_1)v^2 \ ,$$

$$\mathcal{M}(K^+ \to \pi^+\pi^0\pi^0) = -\alpha_1 + \beta_1 u - (\zeta_1 + \xi_1)u^2 - \frac{1}{3}(\zeta_1 - \xi_1)v^2 \ ,$$

$$\alpha_1 = \alpha_1^{(0)} - \frac{2g_8}{27 f_K f_\pi} m_K^4 \{(k_1 - k_2) + 24\mathcal{L}_1\} \ ,$$

$$\beta_1 = \beta_1^{(0)} - \frac{g_8}{9 f_K f_\pi} m_\pi^2 m_K^2 \{(k_3 - 2k_1) - 24\mathcal{L}_2\} \ ,$$

$$\zeta_1 = -\frac{g_8}{6 f_K f_\pi} m_\pi^4 \{k_2 - 24\mathcal{L}_1\} \ ,$$

$$\xi_1 = -\frac{g_8}{6 f_K f_\pi} m_\pi^4 \{k_3 - 24\mathcal{L}_2\} \ ,$$

Table 1

The values of the amplitudes in eqs. (4) and (5) obtained from fits to experiment are shown in the first two columns. Our value of $\delta_2-\delta_0$ is obtained from $K\to 2\pi$ decays alone, while some additional constraints were used in ref. [8]. The $K\to 3\pi$ amplitudes $\alpha_1, ..., \xi'_3$ are in units of 10^{-8}. The results of lowest and next-to-lowest order chiral perturbation theory are displayed in the two columns to the right.

	Devlin and Dickey	Our fit	Lowest order	Order p^4
$a_{1/2}$ [keV]	0.4687 ± 0.0006	0.4699 ± 0.0012	0.4698	0.4698
$a_{3/2}$ [keV]	0.0210 ± 0.0001	0.0211 ± 0.0001	0.0211	0.0211
$\delta_2-\delta_0$ (deg)	-45.6 ± 5	-61.5 ± 4	0	-29
α_1	91.46 ± 0.24	91.71 ± 0.32	74.0	91.8
α_3	-7.14 ± 0.36	-7.36 ± 0.47	-4.1	-7.6
β_1	-25.83 ± 0.41	-25.68 ± 0.27	-16.5	-25.6
β_3	-2.48 ± 0.48	-2.43 ± 0.41	-1.0	-2.5
γ_3	2.51 ± 0.36	2.26 ± 0.23	1.8	2.5
ζ_1	-0.37 ± 0.11	-0.47 ± 0.15	-	-0.6
ζ_3	-	-0.21 ± 0.08	-	-0.02
ξ_1	-1.25 ± 0.12	-1.51 ± 0.30	-	-1.5
ξ_3	-	-0.12 ± 0.17	-	-0.05
ξ'_3	-	-0.21 ± 0.51	-	-0.08
χ^2/DOF	12.8/3	10.3/2	4121/5	37/13

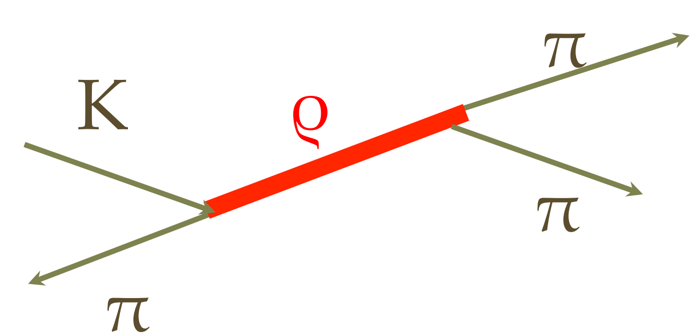
Vector meson dominance in $K \to 3\pi$

$$\mathcal{M}(K_L \to \pi^+\pi^-\pi^0) = \alpha_1 - \beta_1 u + (\zeta_1 + \xi_1)u^2 + \frac{1}{3}(\zeta_1 - \xi_1)v^2$$
$$\mathcal{M}(K_L \to \pi^0\pi^0\pi^0) = -3\alpha_1 - \zeta_1(3u^2 + v^2) \ ,$$
$$\mathcal{M}(K^+ \to \pi^+\pi^+\pi^-) = 2\alpha_1 + \beta_1 u + (2\zeta_1 - \xi_1)u^2 + \frac{1}{3}(2\zeta_1 + \xi_1)v^2 \ ,$$
$$\mathcal{M}(K^+ \to \pi^+\pi^0\pi^0) = -\alpha_1 + \beta_1 u - (\zeta_1 + \xi_1)u^2 - \frac{1}{3}(\zeta_1 - \xi_1)v^2 \ ,$$

$$\alpha_1 = \alpha_1^{(0)} - \frac{2g_8}{27 f_K f_\pi} m_K^4 \{(k_1 - k_2) + 24\mathcal{L}_1\} \ ,$$
$$\beta_1 = \beta_1^{(0)} - \frac{g_8}{9 f_K f_\pi} m_\pi^2 m_K^2 \{(k_3 - 2k_1) - 24\mathcal{L}_2\} \ ,$$
$$\zeta_1 = -\frac{g_8}{6 f_K f_\pi} m_\pi^4 \{k_2 - 24\mathcal{L}_1\} \ ,$$
$$\xi_1 = -\frac{g_8}{6 f_K f_\pi} m_\pi^4 \{k_3 - 24\mathcal{L}_2\} \ ,$$

$$u = \frac{s_3 - s_0}{m_\pi^2} \ , \qquad v = \frac{s_1 - s_2}{m_\pi^2} \ , \qquad s_i = (p_K - p_{\pi_i})^2 \ , \qquad s_0 = \frac{1}{3}\sum_{i=1}^{3} s_i \ .$$

Angular momentum decomposition
β_1 should be dominated by
ρ exchange



$$k_3 = 3(N_1 + N_2 - N_3)$$

It has VMD

Isidori, Pugliese
Ecker Kambor Wyler

We measure the slope, let's check theory predictions

In factorization

$$k_3/24 = 3(N_1 + N_2 - N_3)/24 = L_3 + 3/4L_9$$

using $L_i^{\rm VMD}$

TH VMD 0

$$\frac{k_3}{24} = \left(L_3 + \frac{3}{4}L_9\right) \overset{\rm expt}{\sim 1.7}$$

in units 10^{-3}

using $L_i^{\rm exp}$

5D 1.7

using $L_i^{\rm holo}$

departures from KSFR

Luigi Cappiello, Oscar Cata and G.D.

$$K(p_K) \to \pi(p_1)\pi(p_2)\gamma(q)$$

- Lorentz + gauge invariance $\Rightarrow$ Electric (E) and Magnetic(M) amplitude

$$A(K \to \pi\pi\gamma) = F^{\mu\nu}\left[E\partial_\mu K\partial_\nu\pi + M\varepsilon_{\mu\nu\rho\sigma}\partial^\rho K\partial^\sigma\pi\right]$$

- Unpolarizated photons

$$\frac{d^2\Gamma}{dz_1 dz_2} \sim |E|^2 + |M|^2$$

$$|E^2| = |E_{IB}|^2 + 2Re(E^*_{IB}E_D) + |E_D|^2$$

$$\downarrow$$

Low Theorem $\Rightarrow E_{IB} \sim \frac{1}{E^*_\gamma}$ + c $\qquad E_D$, M chiral tests

We need FIGHT DE/IB$\sim 10^{-3}$

	IB	DE_{exp}	
$K_S \to \pi^+\pi^-\gamma$	10^{-3}	$< 9 \cdot 10^{-5}$	$E1$
$K^+ \to \pi^+\pi^0\gamma$	10^{-4} $(\Delta I = \frac{3}{2})$	$(0.599 \pm 0.037)10^{-5}$ NA48/2	$M1$,$E1$
$K_L \to \pi^+\pi^-\gamma$	10^{-5} (CPV)	$(2.92 \pm 0.07)10^{-5}$ KTeVnew	$M1$, VMD

CPV is only from IB K_L (also measured in $K_L \to \pi^+\pi^-e^+e^-$)

BUT IB suppressed in K^+ and K_L.

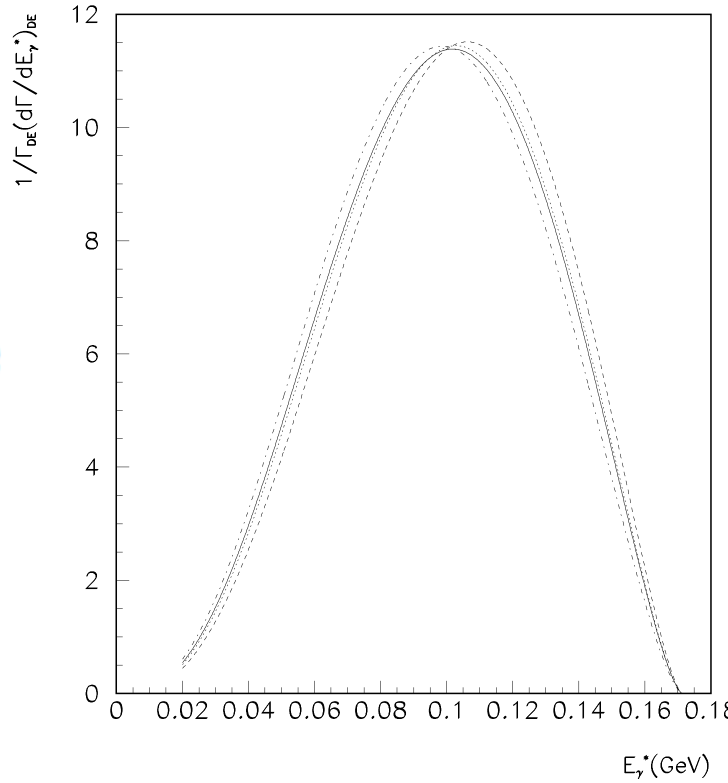
$K_L \to \pi^+\pi^-\gamma$

M1 transtions clearly measured KTeV (00) with large slope

form factor measured

$$\mathcal{F} = 1 + \frac{a}{1 - \frac{m_k^2}{m_\rho^2} + \frac{2 m_K E_\gamma^*}{m_\rho^2}}$$

E_γ^* photon energy



KTeV:

- a$= -1.243 \pm 0.057$
-

	linear slope	quadratic slope	$\mathcal{F}$
χ^2/DOF	43.2/27	37.6/26	38.8/27

$\Rightarrow$ Large VMD: $\rho-$pole

a determined by anomalous Ni's

Weak magnetic p^4 CT's

Table 5

Vector and axial-vectors contribution to the N_i coefficients of the W_i octet operators, in the basis of Ref. [17] relevant to radiative anomalous non-leptonic kaon decays at $O(G_F)$. The hypothesis of factorization is only used to relate ω_1^R with ω_2^R

N_i	W_i	Vectors	Axial-vectors	Expressions using $\omega_1^R = \sqrt{2}(m_R^2/m_R^2) f_R \eta_R,\ \omega_2^R = -\omega_1^R$ Vectors	Axial-vectors
28	$i\varepsilon_{\mu\nu\rho\sigma}\langle\Delta u^\mu\rangle\langle u^\nu u^\rho u^\sigma\rangle$	$-\frac{F_\pi^2}{m_V^2}\theta_V\omega_2^V$	–	$\sqrt{2} f_V \theta_V \eta_V$	–
29	$\varepsilon_{\mu\nu\rho\sigma}\langle\Delta\left[f_+^{\rho\sigma} - f_-^{\rho\sigma} u^\mu u^\nu\right]\rangle$	$\frac{F_\pi^2}{m_V^2}\frac{h_V}{2}\omega_1^V$	$-\frac{F_\pi^2}{m_A^2}\frac{h_A}{2}\omega_1^A$	$\frac{1}{\sqrt{2}} f_V h_V \eta_V$	$-\frac{1}{\sqrt{2}} f_A h_A \eta_A$
30	$\varepsilon_{\mu\nu\rho\sigma}\langle\Delta u^\mu\rangle\langle f_+^{\rho\sigma} u^\nu\rangle$	$-2\frac{F_\pi^2}{m_V^2} h_V\omega_2^V$	–	$2\sqrt{2} f_V h_V \eta_V$	–
31	$\varepsilon_{\mu\nu\rho\sigma}\langle\Delta u^\mu\rangle\langle f_-^{\rho\sigma} u^\nu\rangle$	–	$-2\frac{F_\pi^2}{m_A^2} h_A\omega_2^A$	–	$2\sqrt{2} f_A h_A \eta_A$

- All these terms can be generated from Q_- and WZW term   Cheng; Bijnens, Ecker, Pich

- Also VMD contributions   GD Portoles; GD Gao

$$K^+ \to \pi^+\pi^0\gamma$$

$$A(K \to \pi\pi\gamma) = F^{\mu\nu}\left[E\partial_\mu K\partial_\nu\pi + M\varepsilon_{\mu\nu\rho\sigma}\partial^\rho K\partial^\sigma\pi\right]$$

$E1$ and $M1$ are measured with Dalitz plot

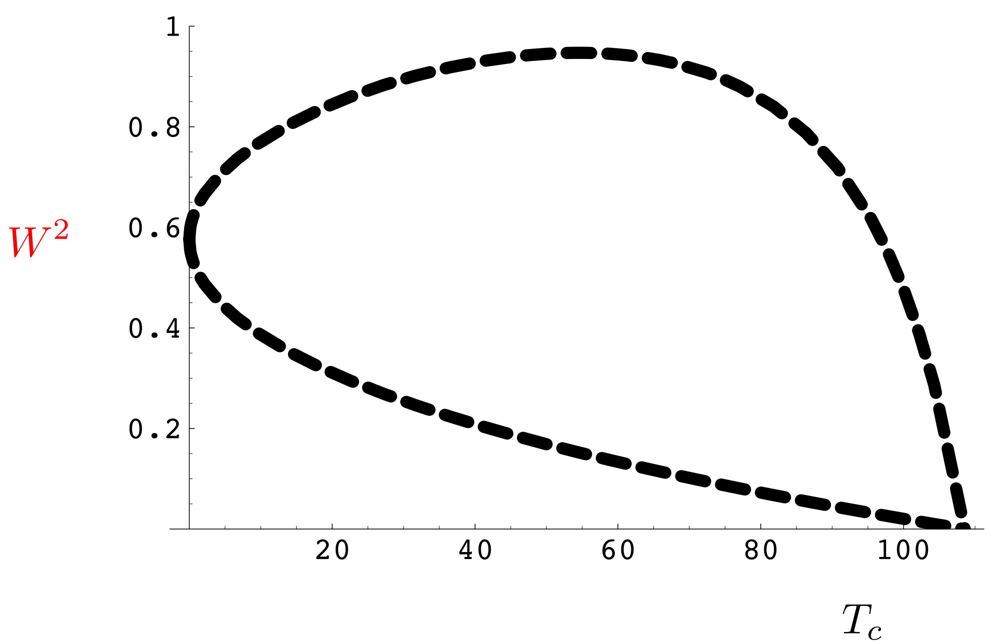
$$\frac{\partial^2\Gamma}{\partial T_c^*\partial W^2} = \frac{\partial^2\Gamma_{IB}}{\partial T_c^*\partial W^2}\left[1 + \frac{m_{\pi^+}^2}{m_K}2Re\left(\frac{E1}{eA}\right)W^2 + \frac{m_{\pi^+}^4}{m_K^2}\left(\left|\frac{E1}{eA}\right|^2 + \left|\frac{M1}{eA}\right|^2\right)W^4\right]$$

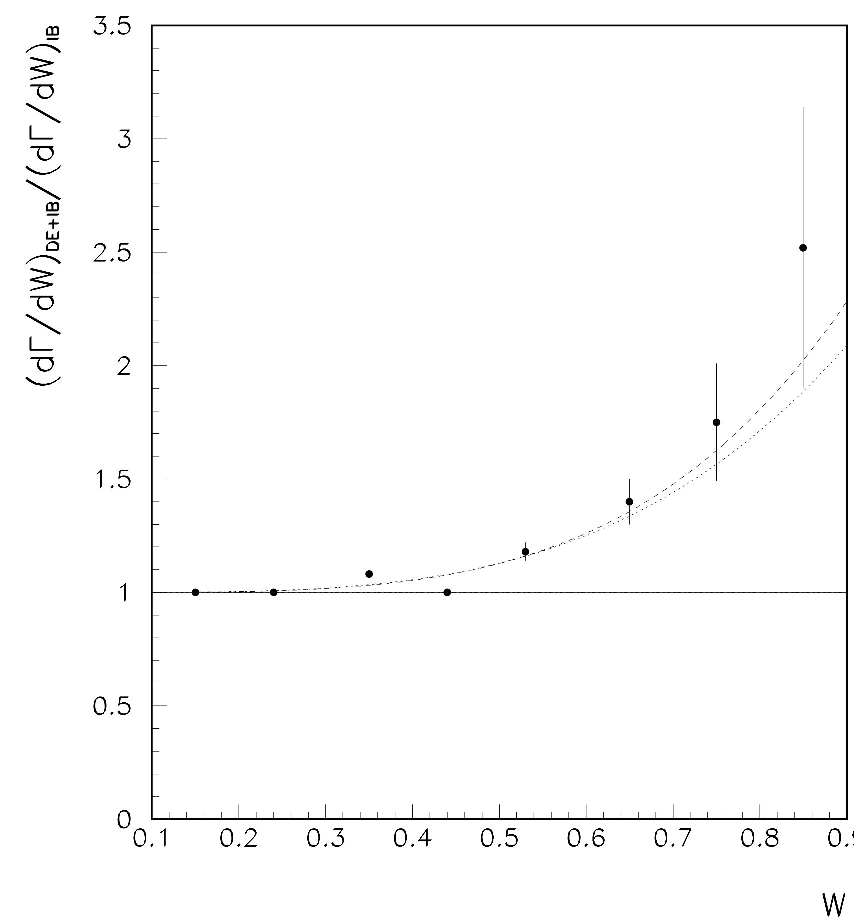
$$W^2 = (q\cdot p_K)(q\cdot p_+)/(m_\pi^2 m_K^2)$$

$$A = A(K^+ \to \pi^+\pi^0)$$

Departure from IB

$W^2 = (q \cdot p_K)(q \cdot p_+)/(m_\pi^2 m_K^2)$

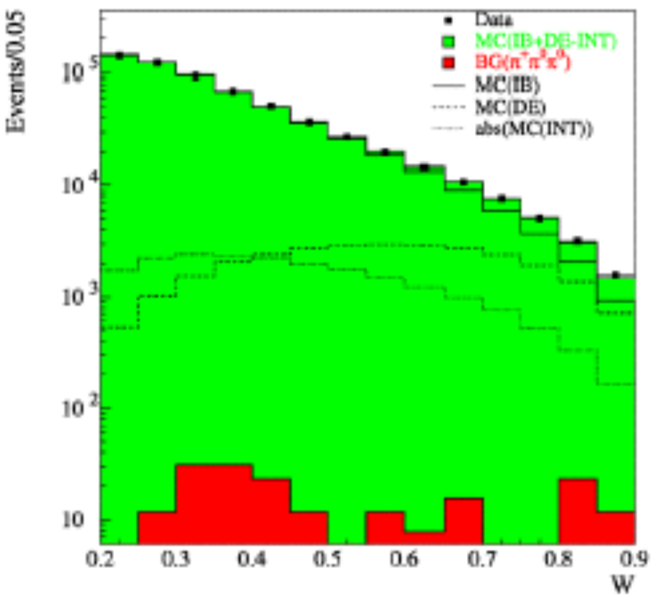




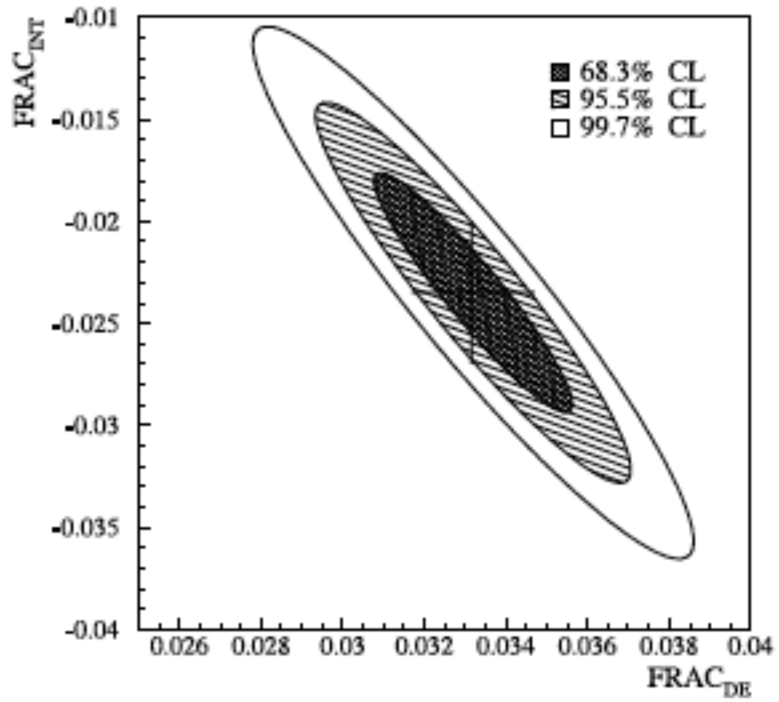
$$\frac{\partial^2 \Gamma}{\partial T_c^* \partial W^2} = \frac{\partial^2 \Gamma_{IB}}{\partial T_c^* \partial W^2} \left[1 + \frac{m_{\pi^+}^2}{m_K} 2Re(\frac{E1}{eA}) W^2 + \frac{m_{\pi^+}^4}{m_K^2} \left(|\frac{E1}{eA}|^2 + |\frac{M1}{eA}|^2 \right) W^4 \right]$$

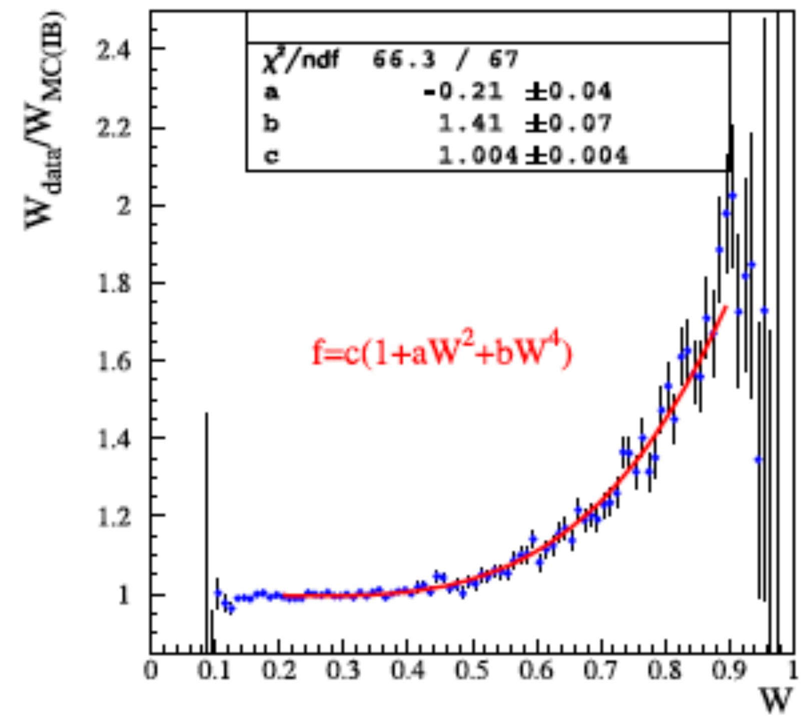
IB from Low theorem

NA48/2 , 600 K candidates



(a)





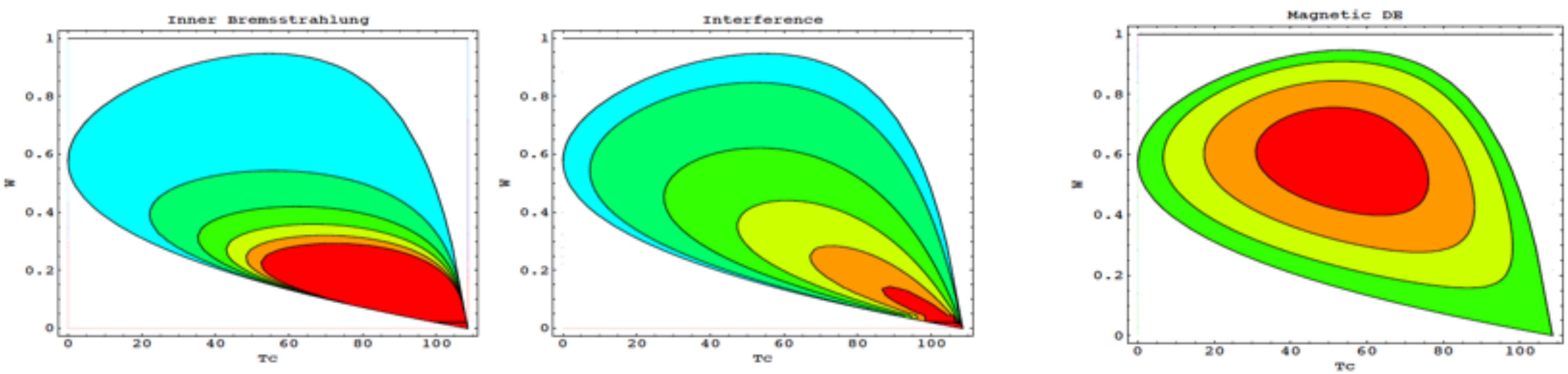
$NA48$	$T_c^* \in [0, 80]\, MeV$
$Frac(DE) =$	$(3.32 \pm 0.15 \pm 0.14) \times 10^{-2}$
$Frac(INT) =$	$(-2.35 \pm 0.35 \pm 0.39) \times 10^{-2}$

Frac(DE) ratio to IB

Frac(INT) ratio to IB

first experiment IB from theory

Dalitz plot



$$\frac{\partial^2\Gamma}{\partial T_c^*\partial W^2} = \frac{\partial^2\Gamma_{IB}}{\partial T_c^*\partial W^2}\left[1 + \frac{m_{\pi^+}^2}{m_K}2Re(\frac{E1}{eA})W^2 + \frac{m_{\pi^+}^4}{m_K^2}\left(|\frac{E1}{eA}|^2 + |\frac{M1}{eA}|^2\right)W^4\right]$$

$K^+ \to \pi^+\pi^0 ee$ Kinematics

arXiv:hep-ph/9411311 v1 16 Nov 1994

SEMILEPTONIC KAON DECAYS

J. Bijnens [1], G. Colangelo [2,3], G. Ecker [4] and J. Gasser [2]

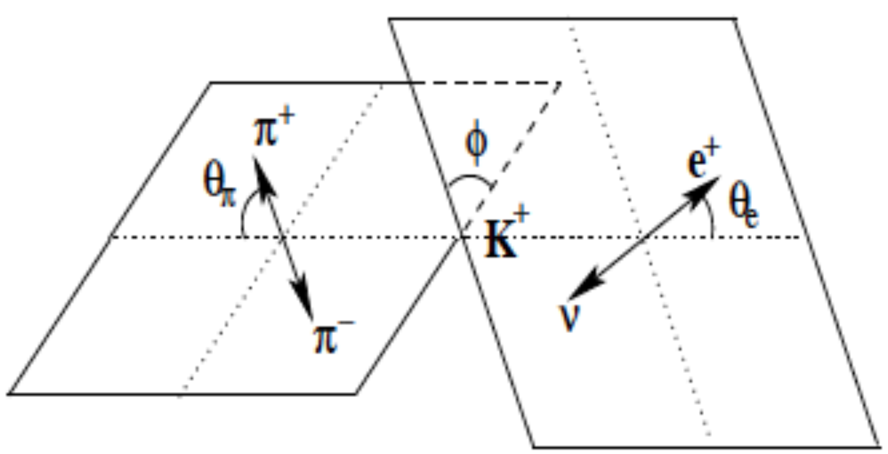
[1] Nordita, Blegdamsvej 17, DK–2000 Copenhagen
[2] Inst. Theor. Physik, Univ. Bern, Sidlerstrasse 5, CH–3012 Bern
[3] Dipartimento di Fisica, Università di Roma II - "Tor Vergata"
Via della Ricerca Scientifica 1, I–00173 Roma

K_{l4} and $\pi\pi$ strong phases $\delta_I^l(s)$

Cabibbo Maksymowicz

$$\frac{G_F}{\sqrt{2}} V_{us}\ \bar{e}\gamma^\mu(1-\gamma^5)\nu\ H_\mu(p_1,p_2,q)$$

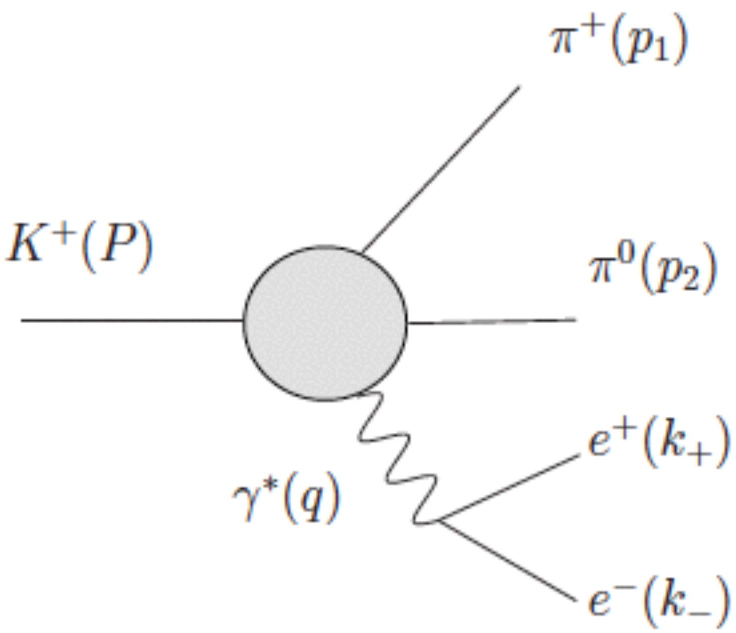
$$H^\mu = F_1 p_1^\mu + F_2 p_2^\mu + F_3\ \varepsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\alpha} q_\beta. \qquad F_i(s) = f_i(s) e^{i\delta_0^0(s)} + ..$$



- crucial to measure $\sin\delta \Longrightarrow$ interf F_3
- Look angular plane asymmetry

$K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^- e^+ e^-$

Sehgal et al; Savage,Wise et al



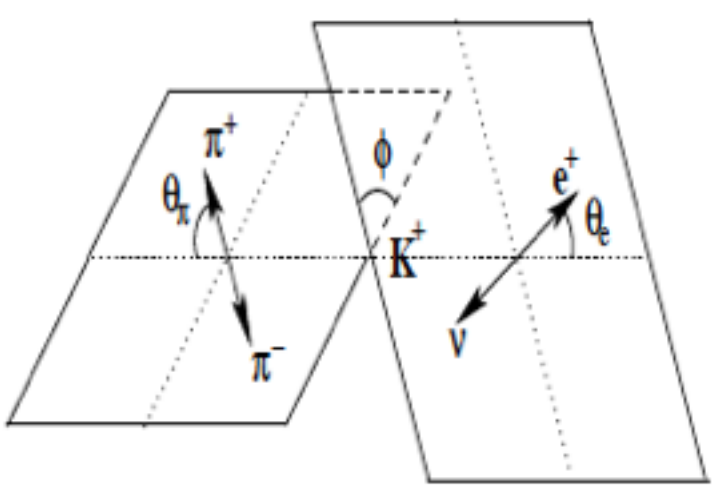
- $\mathcal{M}_{LD} = \frac{e}{q^2}\, \bar{e}\gamma^\mu(1-\gamma^5)\; e\; H_\mu$
- $H^\mu = F_1 p_1^\mu + F_2 p_2^\mu + F_3\; \varepsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\alpha} q_\beta$
- $F_{1,2} \sim E \quad F_3 \sim M$

- Interference $E \quad M$ novel compared to $K_L \to \pi^+\pi^-\gamma$
- $E \quad M$ known from $K_L \to \pi^+\pi^-\gamma$ (IB and DE)

$$K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^-e^+e^-$$

$$\frac{d^5\Gamma}{dE_\gamma^* dT_c^* dq^2 d\cos\theta_\ell d\phi} = \mathcal{A}_1 + \mathcal{A}_2 \sin^2\theta_\ell + \mathcal{A}_3 \sin^2\theta_\ell \cos^2\phi$$
$$+\mathcal{A}_4 \sin 2\theta_\ell \cos\phi + \mathcal{A}_5 \sin\theta_\ell \cos\phi + \mathcal{A}_6 \cos\theta_\ell$$
$$+\mathcal{A}_7 \sin\theta_\ell \sin\phi + \mathcal{A}_8 \sin 2\theta_\ell \sin\phi + \mathcal{A}_9 \sin^2\theta_\ell \sin 2\phi$$

- $\mathcal{A}_{1,..,4}$ IB, $\mathcal{A}_{8,9}$, CPV B-M interf.
- $\dfrac{\Re(E_B\ M^*)}{|E_B|^2 + |M|^2}$ is maximal,
- $\mathcal{A}_{5,6,7}$ interf. axial leptonic current , $\mathcal{A}_P$ SD

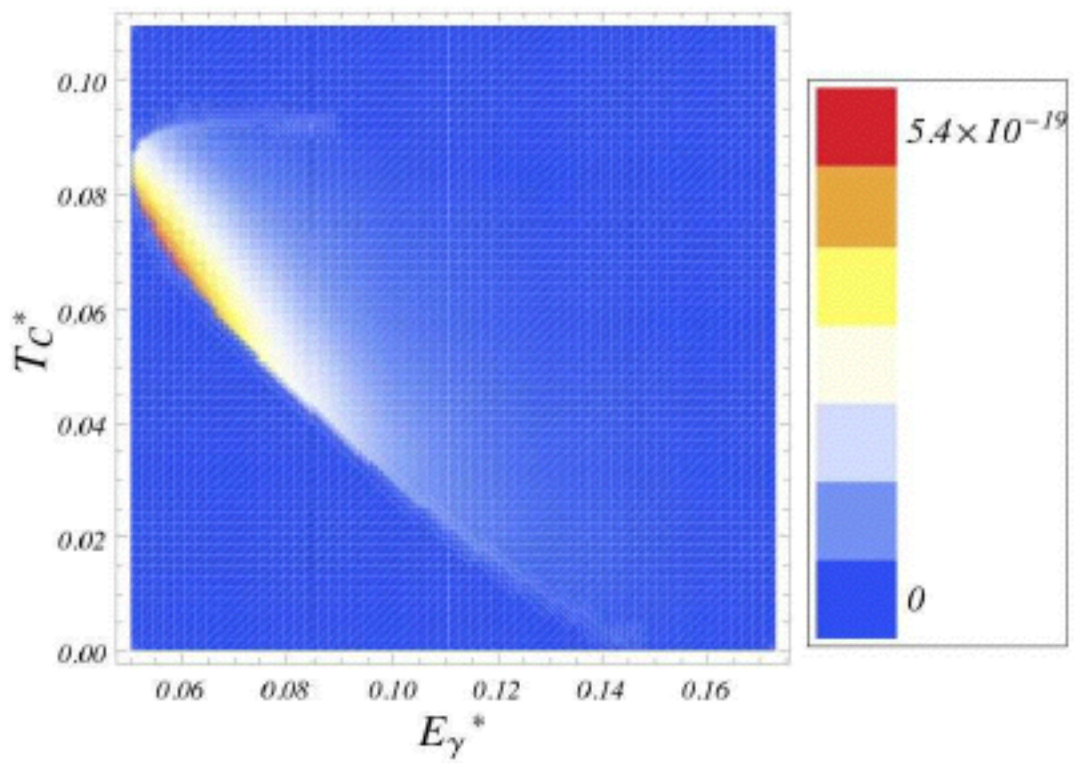


Marco Sozzi's question

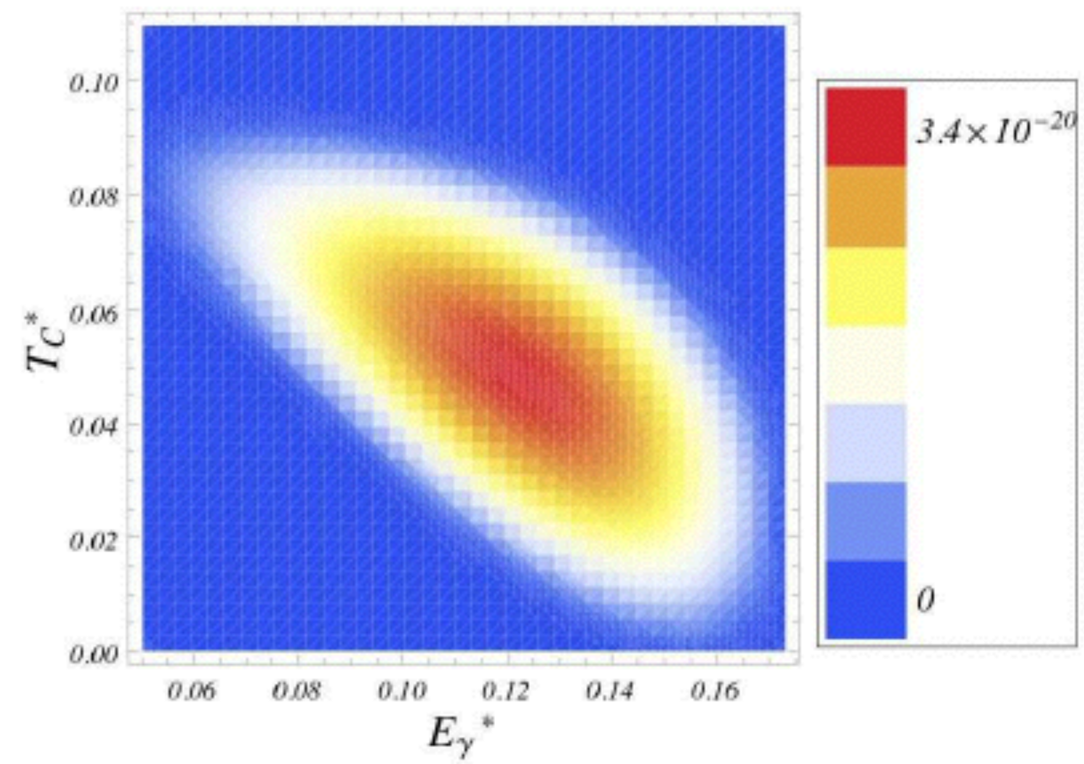
$K^+ \to \pi^+\pi^0\gamma^* \to \pi^+\pi^0 e^+ e^-$ Cappiello, Cata,G.D. and Gao,

- the asymm. , $\frac{\Re(E_B\, M^*)}{|E_B|^2 + |M|^2}$, not as lucky $E_B >> M$:
- $B(K^+)_{IB} \sim 3.3 \times 10^{-6} \;\sim\; 50\; B(K^+)_M$
- Short distance info without having simultaneously K^+ and K^-, asymm. in phase space, (P-violation) interesting! No ϵ-contamination
- interesting Dalitz plots (at fixed q^2) to disentangle M from E_B
- at $q^2 = 50\mathrm{MeV}$ IB only 10 times larger than DE

q_c (MeV)	B $[10^{-8}]$	B/M	B/E	B/BE	B/BM
$2m_l$	418.27	71	4405	128	208
55	5.62	12	118	38	44
100	0.67	8	30	71	36
180	0.003	12	5	-19	44



IB

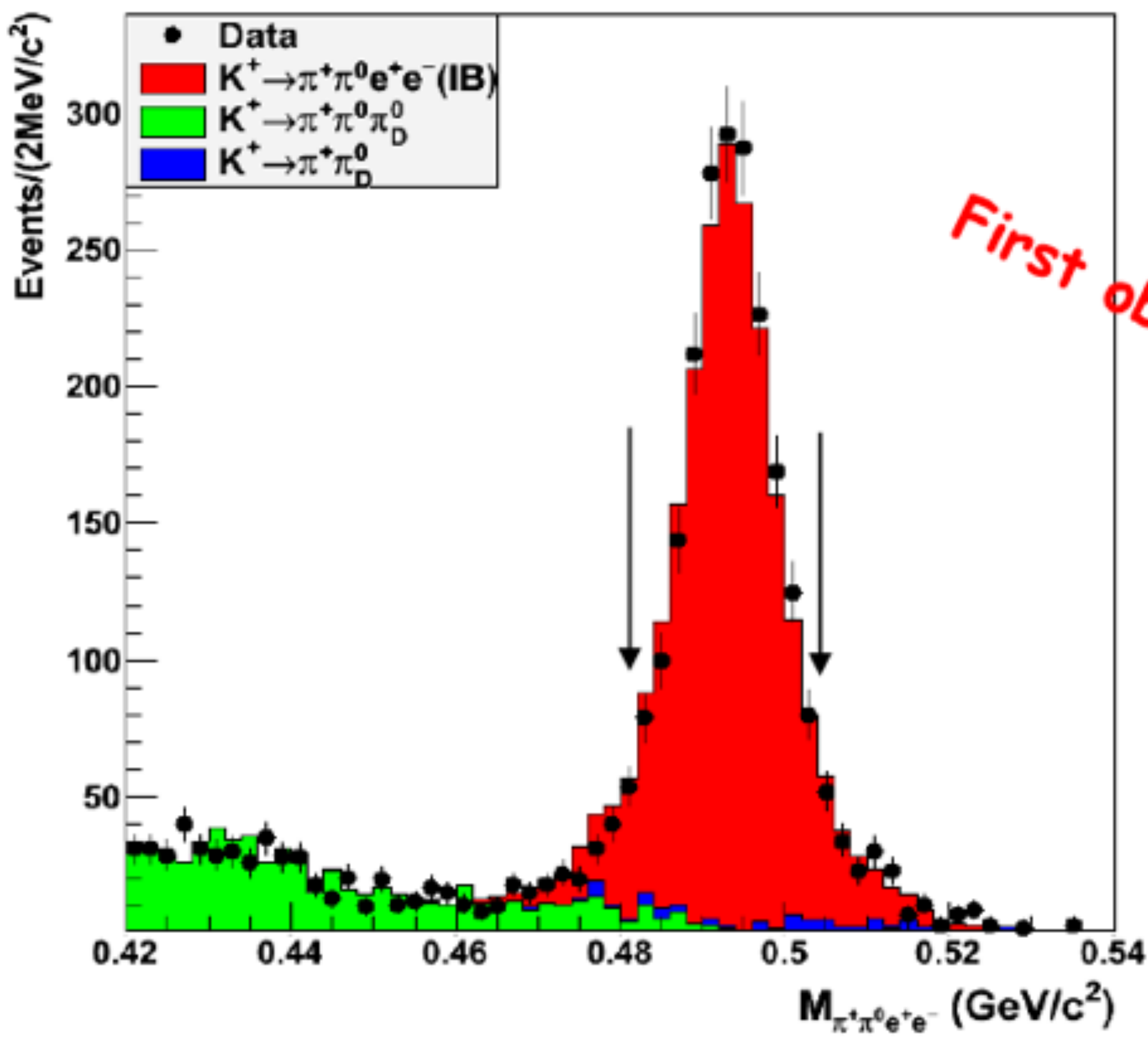


DE

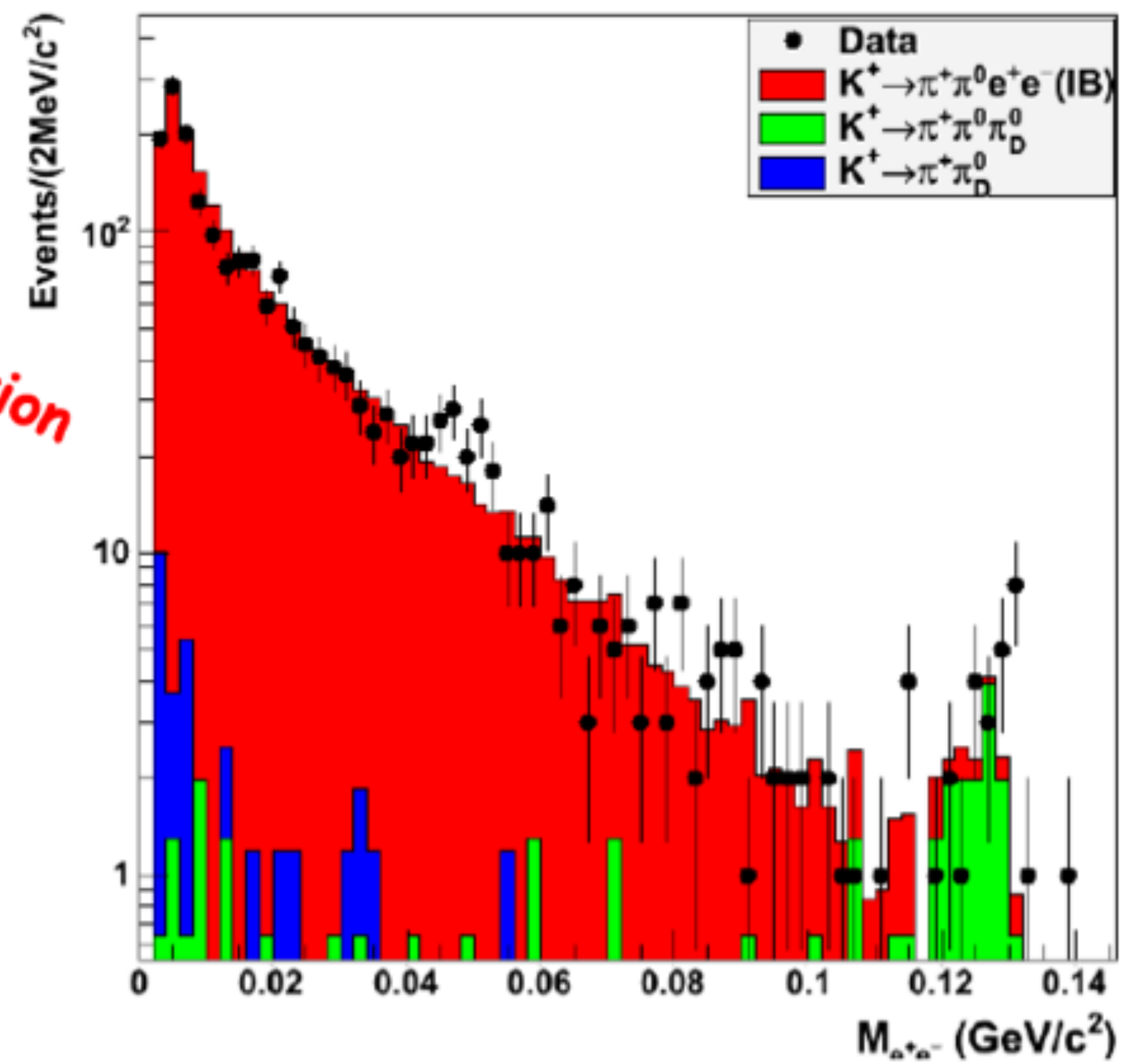
$K^{\pm} \to \pi^{\pm}\pi^0 e^- e^+$ Data samples and background estimates

Moriond 2015

Milena Misheva NA48/2



First observation

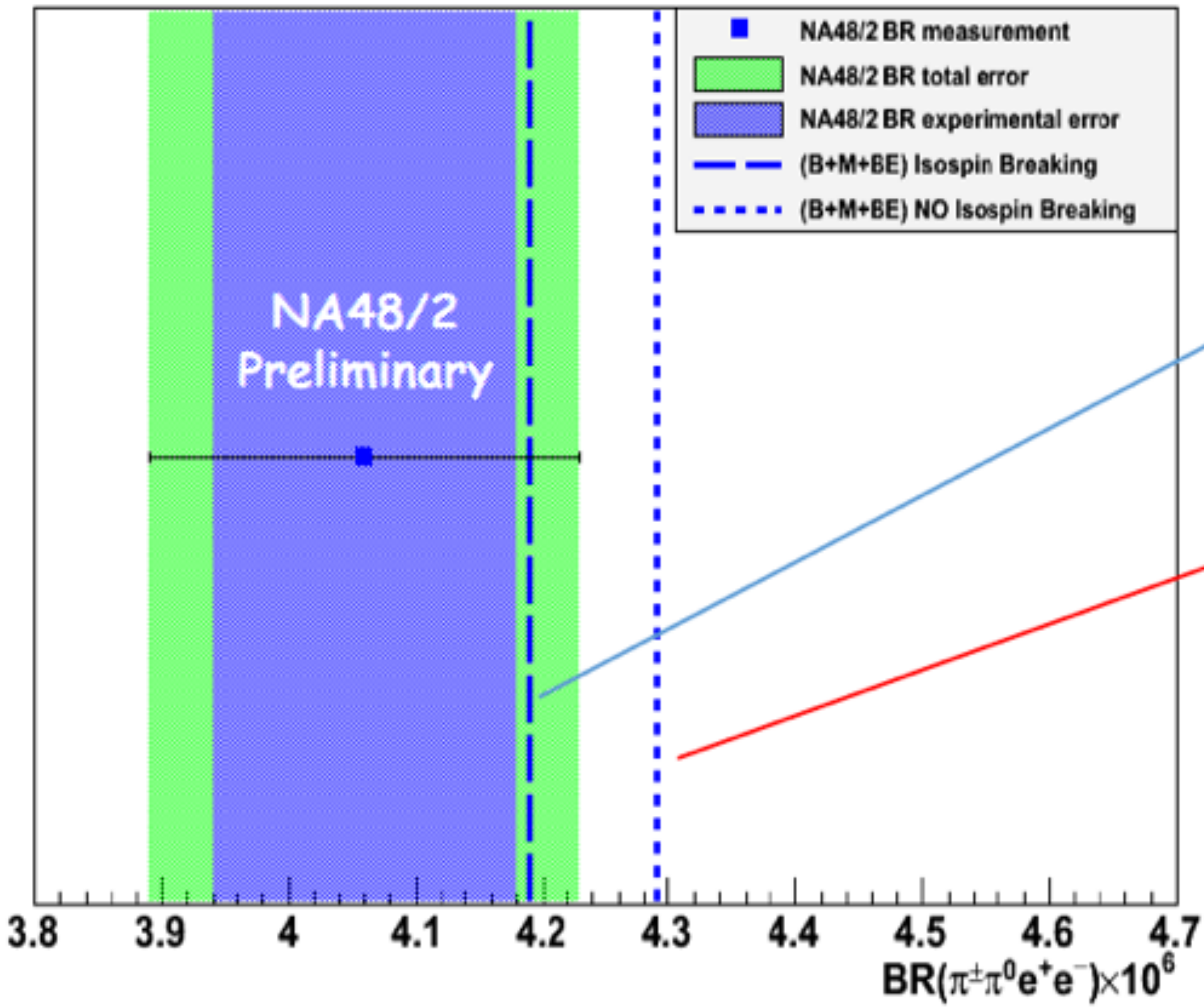


- 1916 -total number of $K^{\pm} \to \pi^{\pm}\pi^0 e^- e^+$ candidates
- Total background (~3%)
  - $K^{\pm} \to \pi^{\pm}\pi^0\pi^0_{e\text{-}e\text{+}\gamma}$ (30 ± 5.5)events
  - $K^{\pm} \to \pi^{\pm}\pi^0_{e\text{-}e\text{+}\gamma}(\gamma)$ (26 ± 5.1)events

- Background suppression
  - $K^{\pm} \to \pi^{\pm}\pi^0 e^- e^+ \gamma$ ($M^2_{\pi\pi} > 0.120$ (GeV/c²)²)
  - $K^{\pm} \to \pi^{\pm} e^- e^+ \gamma (\gamma)$ ($|M_{ee\gamma} - M_{\pi 0}$ PDG$| > 7$ MeV)
- 1860 genuine $K^{\pm} \to \pi^{\pm}\pi^0 e^- e^+$ events

Preliminary result of $BR(K^{\pm} \to \pi^{\pm}\pi^{0}e^{-}e^{+})$

Moriond 2015
NA48/2 Misheva



L. Cappiello, O. Cata, G. D'Ambrosio, Dao Neng-Gao,

Eur. Phys. J. C 72:1872 (2012) :

Isospin breaking (private communication)

$BR(K^{\pm} \to \pi^{\pm}\pi^{0}e^{-}e^{+})^{Theory} = 4.19 \cdot 10^{-6}$

No isospin breaking (published)

$BR(K^{\pm} \to \pi^{\pm}\pi^{0}e^{-}e^{+})^{Theory} = 4.29 \cdot 10^{-6}$

No radiative corrections in the theoretical predictions!

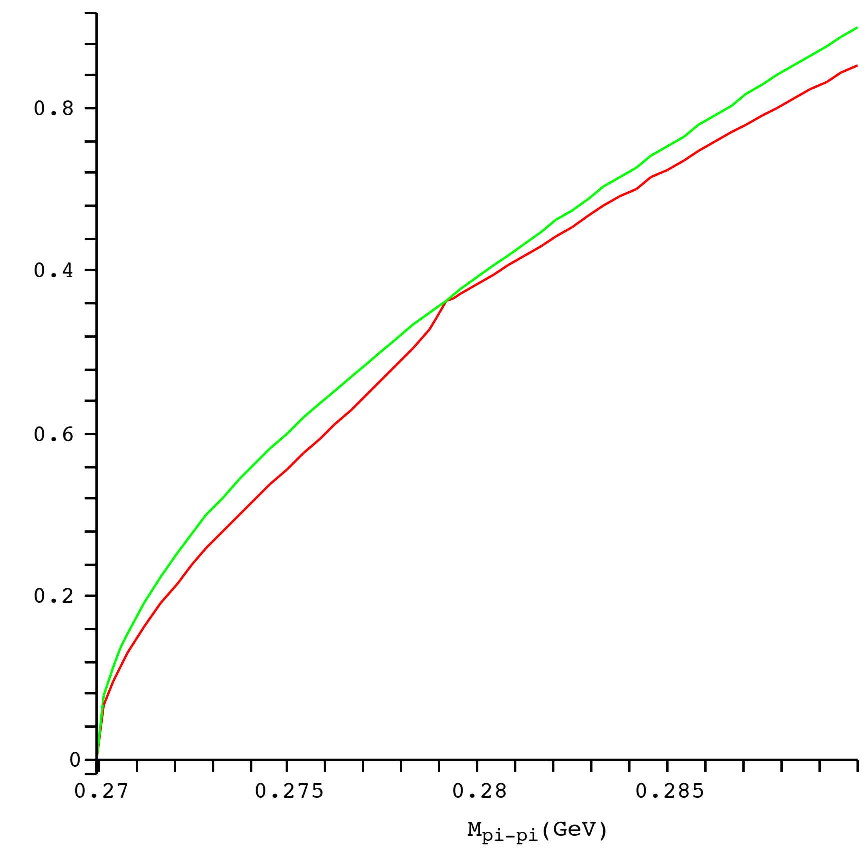
Rad. corr. is taken into account in the experimental result via Photos implementation in the MC simulaton.

NA48/2
2003 data

$$BR(K^{\pm} \to \pi^{\pm}\pi^{0}e^{-}e^{+})_{total} = (4.06 \pm 0.12_{exp} \pm 0.13_{ext}) \times 10^{-6}$$

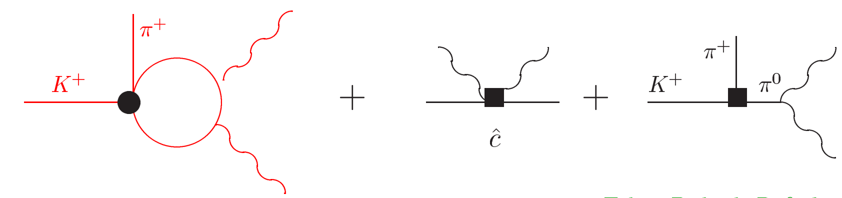
Cusp effect in K->3pi

- in 2002 Mannelli at CERN discusses that their incredible energy resolution may lead to pionium discovery in K+ ->pi+ pi0 pi0
- But the plot (expt red curve) on the right was not yet understood



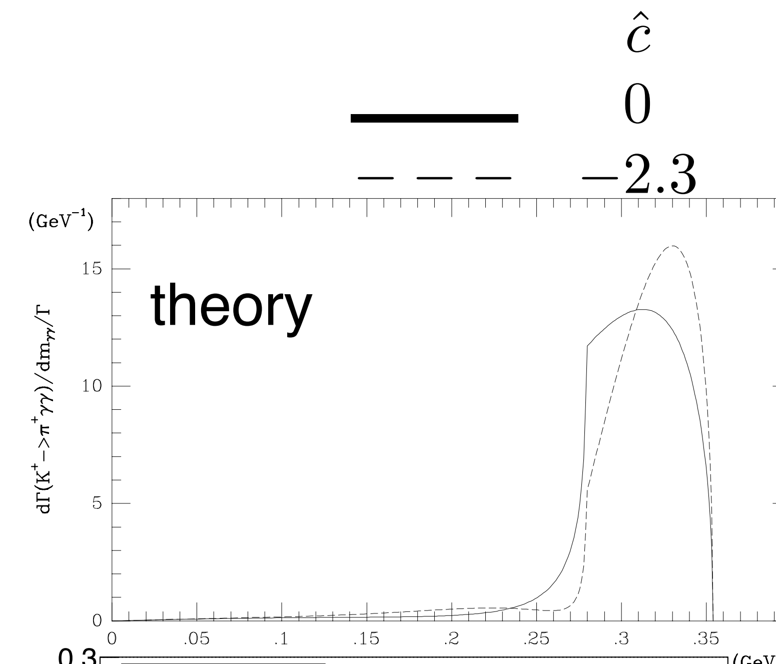
$K^+ \to \pi^+\gamma\gamma$ NA48/2 + NA62 ('14)

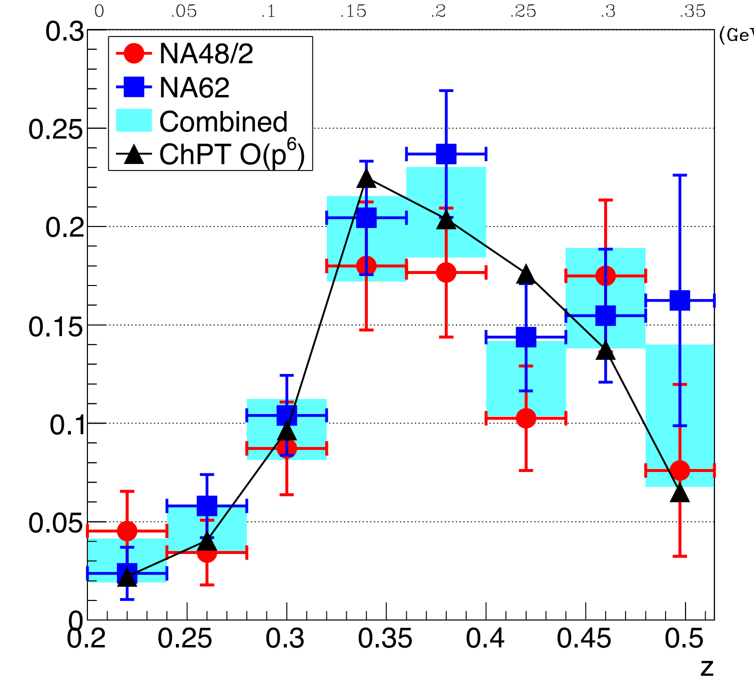
Auxiliary channel useful to assess the CP conserving contribution to $K_L \to \pi^0 ee$



Ecker, Pich, de Rafael

Final 381 evts NA48/2 + NA62
during a 3-day special NA48/2 run in
2004 and a 3-month NA62 run in 2007





$$B = (1.003 \pm 0.051_{\text{stat}} \pm 0.024_{\text{syst}}) \cdot 10^{-6}$$

$$\hat{c} = 1.86 \pm 0.26$$

$\hat{c}$ Ni's combination

quantity	Ref. [18]	Ref. [6]	Our fit	p^2	$\tilde{K}_i = 0$
$\|A_0\|$	0.4687 ± 0.0006	0.4699 ± 0.0012	0.4622 ± 0.0014	input	input
$\|A_2\|$	0.0210 ± 0.0001	0.0211 ± 0.0001	0.0212 ± 0.0001	input	input
$\delta_2 - \delta_0$	$(-45.6 \pm 5)^o$	$(-61.5 \pm 4)^o$	$(-58.2 \pm 4)^o$	—	—
α_1	91.4 ± 0.24	91.71 ± 0.32	93.16 ± 0.36	74.0(73.5)	59.4
α_3	-7.14 ± 0.36	-7.36 ± 0.47	-6.72 ± 0.46	−4.8(4.8)	−6.5
β_1	-25.83 ± 0.41	-25.68 ± 0.27	-27.06 ± 0.43	−17.7(16.2)	−21.9
β_3	-2.48 ± 0.48	-2.43 ± 0.41	-2.22 ± 0.47	−1.2(1.1)	−1.0
g_3	2.51 ± 0.36	2.26 ± 0.23	2.95 ± 0.32	2.3(2.1)	2.5
ζ_1	-0.37 ± 0.11	-0.47 ± 0.15	-0.40 ± 0.19	—	0.26
ζ_3	—	-0.21 ± 0.08	-0.09 ± 0.10	—	−0.01
ξ_1	-1.25 ± 0.12	-1.51 ± 0.30	-1.83 ± 0.30	—	−0.46
ξ_3	—	-0.12 ± 0.17	-0.17 ± 0.16	—	−0.01
ξ'_3	—	-0.21 ± 0.51	-0.56 ± 0.42	—	−0.06
χ^2/DOF	12.8/3	10.3/2	5.4/5	—	—

Bijnens et al fit

Measurement	BNL E787 [5]	NA48/2 [7]	NA48/2 [6] and present analysis
Decay mode	$K^+_{\pi\gamma\gamma}$	$K^\pm_{\pi\gamma ee}$	$K^\pm_{\pi\gamma\gamma}$
$G_8 m_K^2 \times 10^6$	2.24	2.210	2.202
$\alpha_1 \times 10^8$	91.71	91.7	93.16
$\alpha_3 \times 10^8$	−7.36	−7.4	−6.72
$\beta_1 \times 10^8$	−25.68	−25.7	−27.06
$\beta_3 \times 10^8$	−2.43	−2.4	−2.22
$\gamma_3 \times 10^8$	2.26	2.3	2.95
$\zeta_1 \times 10^8$	−0.47	−0.5	−0.40
$\xi_1 \times 10^8$	−1.51	−1.5	−1.83
η_i $(i = 1; 2; 3)$	0	0	0

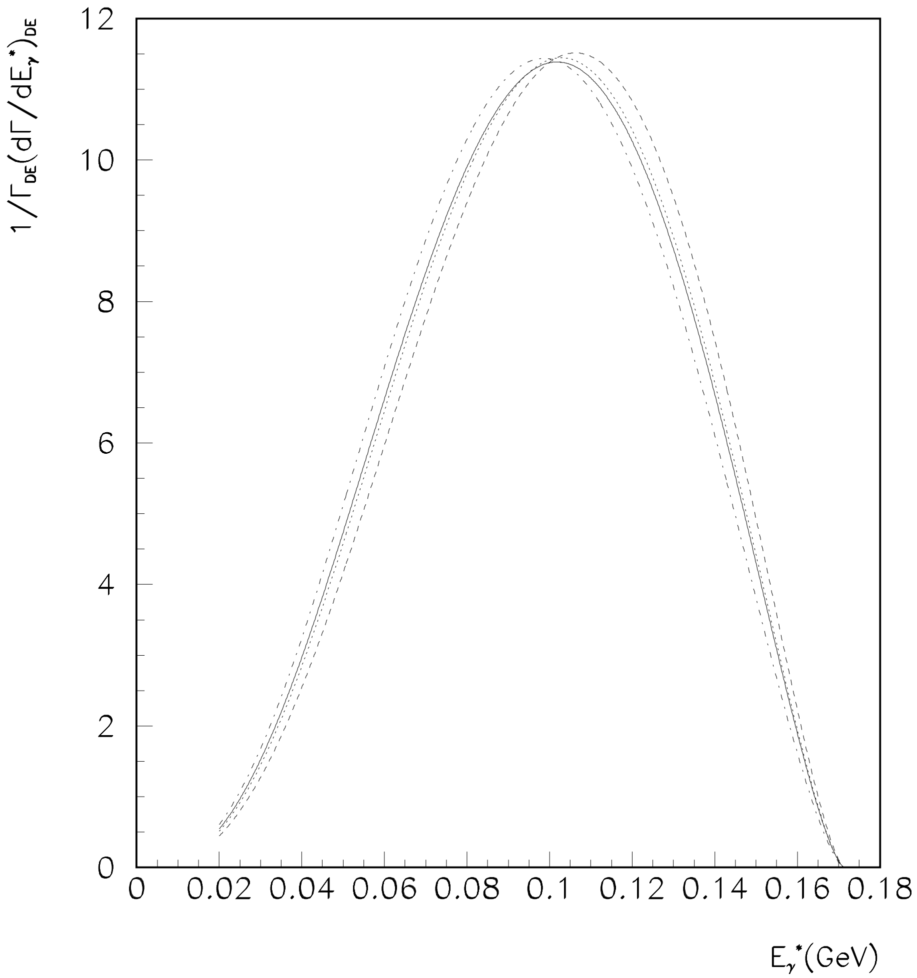
Conclusion

- Talking about a kaon experiment we have always to expect unexpected accuracy and consequently brilliant measurement
- We , theorists, will do the best now, but the future maybe even more optimistic

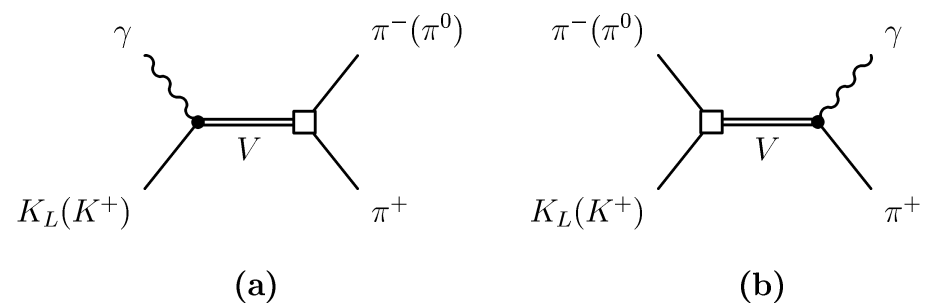
$K^+ \to \pi^+\gamma\gamma$ NA62 sensitivity

$$A(K \to 3\pi) = a + b\,Y + c\,Y^2 + d\,X^2$$

This decay $K^+ \to \pi^+\gamma\gamma$: The error obtained in the form factor ($\hat{c}$) is dominated by the expt K-> 3pi error in the quadratic slope !



12
10
8
6
4
2
0
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
1/Γ_DE(dΓ/dE_γ^*)_DE
E_γ^*(GeV)



γ
π^-(π^0)
V
K_L(K^+)
π^+
(a)
π^-(π^0)
γ
V
K_L(K^+)
π^+
(b)