SEMILEPTONIC DECAYS

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Overview

Decay	Branching Ratio
$K^+ \to e^+\nu$	$1.582(7)\,10^{-5}$
$K^+ \to \mu^+\nu$	$63.56(11)\%$
$K^+ \to \pi^0 e^+\nu$	$5.07(4)\%$
$K^+ \to \pi^0 \mu^+\nu$	$3.352(33)\%$
$K^+ \to \pi^0\pi^0 e^+\nu$	$2.55(4)\,10^{-5}$
$K^+ \to \pi^+\pi^- e^+\nu$	$4.247(24)\,10^{-5}$
$K^+ \to \pi^+\pi^- \mu^+\nu$	$1.4(9)\,10^{-5}$
$K^+ \to \mu^+\nu\gamma$	$6.2(8)\,10^{-3}$
$K^+ \to e^+\nu\gamma$	$9.4(4)\,10^{-6}$
$K^+ \to \pi^0 e^+\nu\gamma$	$2.56(16)\,10^{-4}$
$K^+ \to \pi^0 \mu^+\nu\gamma$	$1.25(25)\,10^{-5}$
$K^+ \to e^+\nu e^+ e^-$	$2.48(20)\,10^{-8}$
$K^+ \to e^+\nu \mu^+\mu^-$	$1.7(5)\,10^{-8}$

Overview

Decay	Branching Ratio
$K_S \to \pi^\pm e^\mp \nu$	$7.04(8)\ 10^{-4}$
$K_L \to \pi^\pm e^\mp \nu$	$40.55(11)\%$
$K_L \to \pi^\pm \mu^\mp \nu$	$27.04(7)\%$
$K_L \to (\pi\mu)_{\mathrm{atom}}\nu$	$1.05(11)\ 10^{-7}$
$K_L \to \pi^0\pi^\pm e^\mp \nu$	$5.20(11)\ 10^{-5}$
$K_L \to \pi^\pm e^\mp \nu e^+ e^-$	$1.26(4)\ 10^{-5}$
$K_L \to \pi^\pm e^\mp \nu\gamma$	$3.79(6)\ 10^{-3}$
$K_L \to \pi^\pm \mu^\mp \nu\gamma$	$5.65(23)\ 10^{-3}$

Other talks

You already heard a lot about semileptonic decays:

- Lattice: Sachrajda and Martinelli
- Radiative corrections: Knecht
- $\pi\pi$: Colangelo
- Dispersive work on $K_{\ell 4}$: Stoffer
- Dispersive work on rare decays: Stucki
- Estimate of parameters in rare decays: Greynat
- CKM fits: Descotes-Genon
- Mentioned in a few more talks as well

My talk:

- Chiral Perturbation Theory
- What can we learn/test in the various decays
- Some recent work (to show I do have some)

Earlier reviews of mine

- Daϕne physics handbook, Semileptonic Kaon Decays in ChPT, JB, Ecker, Gasser, hep-ph/920820
- 2nd Daϕne physics handbook, Semileptonic Kaon Decays, JB, Colangelo, Ecker, Gasser, hep-ph/9411311
- ...
- KAON07, Radiative and semileptonic decays in ChPT, arXiv:0707.0419

But remember also:
Cirigliano, Ecker, Neufeld, Pich, Portoles, arXiv:1107.6001

Overview

1. Overview
2. Chiral Perturbation Theory
   - Chiral Perturbation Theory
   - A mesonic ChPT program framework
   - Determination of LECs in the continuum
3. Semileptonic decays
   - $K_{\ell 2}$
   - $K_{\ell 2\gamma}$
   - $K \to \ell' \nu \ell^+ \ell^-$
   - $K\pi$ form-factors for $K_{\ell 3}$ and $K \to \pi\nu\bar{\nu}$
   - $K_{\ell 3\gamma}$
   - $K_{\ell 4}$
4. Finite volume
5. Conclusions

Chiral Perturbation Theory

Exploring the consequences of
the chiral symmetry of QCD
and its spontaneous breaking
using effective field theory techniques

Derivation from QCD:
H. Leutwyler,
On The Foundations Of Chiral Perturbation Theory,
Ann. Phys. 235 (1994) 165 [hep-ph/9311274]

For references to lectures see:
http://www.thep.lu.se/~bijnens/chpt/

Chiral Perturbation Theory

A general Effective Field Theory:

- Relevant degrees of freedom
- A powercounting principle (predictivity)
- Has a certain range of validity

Chiral Perturbation Theory:

- Degrees of freedom: Goldstone Bosons from spontaneous breaking of chiral symmetry
- Powercounting: Dimensional counting in momenta/masses
- Breakdown scale: Resonances, so about M_ρ.

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Goldstone Bosons

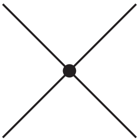
Spontaneous breakdown

- $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
- $SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$
- 8 generators broken $\Longrightarrow$ 8 massless degrees of freedom and interaction vanishes at zero momentum

Goldstone Bosons

Power counting in momenta: Meson loops, Weinberg powercounting

rules

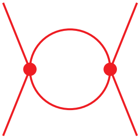


p^2

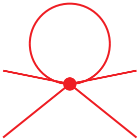
$1/p^2$

$\int d^4p$ $\quad$ p^4

one loop example



$$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$$



$$(p^2)\,(1/p^2)\,p^4 = p^4$$

Chiral Perturbation Theories

- Which chiral symmetry: $SU(N_f)_L \times SU(N_f)_R$, for $N_f = 2, 3, \ldots$ and extensions to (partially) quenched
- Or beyond QCD
- Space-time symmetry: Continuum or broken on the lattice: Wilson, staggered, mixed action
- Volume: Infinite, finite in space, finite T
- Which interactions to include beyond the strong one
- Which particles included as non Goldstone Bosons
- My general belief: if it involves soft pions (or soft K, η) some version of ChPT exists

Lagrangians: Lagrangian structure (mesons, strong)

	2 flavour		3 flavour		PQChPT/N_f flavour	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	52+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966

p^4: Gasser, Leutwyler 84,85

p^6: JB, Colangelo, Ecker 99,00

- L_i LEC = Low Energy Constants = ChPT parameters
- H_i: contact terms: value depends on definition of currents/densities
- Finite volume: no new LECs
- Other effects: (many) new LECs

Mesons: which Lagrangians are known ($n_f = 3$)

Loops	$\mathcal{L}_{\text{order}}$	LECs	effects included
$L = 0$	$\mathcal{L}_{p^2}$	2	strong (+ external W, γ)
	$\mathcal{L}_{e^2 p^0}$	1	internal γ
	$\mathcal{L}^{\Delta S=1}_{G_F p^2}$	2	nonleptonic weak
	$\mathcal{L}^{\Delta S=1}_{G_8 e^2 p^0}$	1	nonleptonic weak+internal γ
	$\mathcal{L}^{\text{odd}}_{p^4}$	0	WZW, anomaly
$L \le 1$	$\mathcal{L}_{p^4}$	10	strong (+ external W, γ)
	$\mathcal{L}_{e^2 p^2}$	13	internal γ
	$\mathcal{L}^{\Delta S=1}_{G_8 F p^4}$	22	nonleptonic weak
	$\mathcal{L}^{\Delta S=1}_{G_{27} p^4}$	28	nonleptonic weak
	$\mathcal{L}^{\Delta S=1}_{G_8 e^2 p^2}$	14	nonleptonic weak+internal γ
	$\mathcal{L}^{\text{odd}}_{p^6}$	23	WZW, anomaly
	$\mathcal{L}^{\text{leptons}}_{e^2 p^2}$	5	leptons, internal γ
$L \le 2$	$\mathcal{L}_{p^6}$	90	strong (+ external W, γ)

Chiral Logarithms

The main predictions of ChPT:

- Relates processes with different numbers of pseudoscalars/axial currents
- Chiral logarithms
- includes Isospin and the eightfold way ($SU(3)_V$)
- Unitarity included perturbatively

$$m_\pi^2 = 2B\hat{m} + \left(\frac{2B\hat{m}}{F}\right)^2 \left[\frac{1}{32\pi^2}\log\frac{(2B\hat{m})}{\mu^2} + 2l_3^r(\mu)\right] + \cdots$$

$$M^2 = 2B\hat{m}$$

LECs and μ

$l_3^r(\mu)$

$$\bar{l}_i = \frac{32\pi^2}{\gamma_i}\, l_i^r(\mu) - \log\frac{M_\pi^2}{\mu^2}\,.$$

is independent of the scale μ.

For 3 and more flavours, some of the $\gamma_i = 0$: $L_i^r(\mu)$

Choice of μ :

- m_π, m_K: chiral logs vanish
- pick larger scale
- 1 GeV then $L_5^r(\mu) \approx 0$
  what about large N_c arguments????
- compromise: $\mu = m_\rho = 0.77$ GeV

Expand in what quantities?

- Expansion is in momenta and masses
- But is not unique: relations between masses (Gell-Mann–Okubo) exist
- Express orders in terms of physical masses and quantities (F_π, F_K)?
- Express orders in terms of lowest order masses?
- E.g. $s + t + u = 2m_\pi^2 + 2m_K^2$ in πK scattering
- Note: remaining μ dependence can occur at a given order
- Can make quite some difference in the expansion

I prefer physical masses

- Thresholds correct
- Chiral logs are from physical particles propagating
- but sometimes too many masses so very ambiguous

Program availability

Making the programs more accessible for others to use:

- Two-loop results have very long expressions
- Many not published but available from http://www.thep.lu.se/∼bijnens/chpt/
- Many programs available on request from the authors
- Idea: make a more general framework
- CHIRON:

JB,
"CHIRON: a package for ChPT numerical results at two loops,"
Eur. Phys. J. C **75** (2015) 27 [arXiv:1412.0887]

http://www.thep.lu.se/∼bijnens/chiron/

Program availability: CHIRON

- Present version: 0.54
- Classes to deal with $L_i, C_i,\ L_i^{(n)}, K_i$, standardized in/output, changing the scale,...
- Loop integrals: one-loop and sunsetintegrals
- Included so far (at two-loop order):
  - Masses, decay constants and $\langle \bar{q}q \rangle$ for the three flavour case
  - Masses and decay constants at finite volume in the three flavour case
  - Masses and decay constants in the partially quenched case for three sea quarks
  - Masses and decay constants in the partially quenched case for three sea quarks at finite volume
- A large number of example programs is included
- Manual has already reached 94 pages
- I am continually adding results from my earlier work

LEC determination: (Partial) History/References

- Original determination at p^4: Gasser, Leutwyler, Annals Phys.158 (1984) 142, Nucl. Phys. B250 (1985) 465
- p^6 3 flavour: Amorós, JB, Talavera, Nucl. Phys. B602 (2001) 87 [hep-ph/0101127]
- Review article two-loops: JB, Prog. Part. Nucl. Phys. 58 (2007) 521 [hep-ph/0604043]
- Update of fits + new input: JB, Jemos, Nucl. Phys. B 854 (2012) 631 [arXiv:1103.5945]
- Recent review with more p^6 input: JB, Ecker, Ann. Rev. Nucl. Part. Sci. **64** (2014) 149 [arXiv:1405.6488]
- Review Kaon physics: Cirigliano, Ecker, Neufeld, Pich, Portoles, Rev.Mod.Phys. 84 (2012) 399 [arXiv:1107.6001]
- Lattice: FLAG reports:
  Colangelo et al., Eur.Phys.J. C71 (2011) 1695 [arXiv:1011.4408]
  Aoki et al., Eur. Phys. J. C **74** (2014) 9, 2890 [arXiv:1310.8555]

Three flavour LECs: uncertainties

- $m_K^2, m_\eta^2 \gg m_\pi^2$
- Contributions from p^6 Lagrangian are larger
- Reliance on estimates of the C_i much larger
- Typically: C_i^r: (terms with)
  kinematical dependence $\equiv$ measurable
  quark mass dependence $\equiv$ impossible (without lattice)
      100% correlated with L_i^r
- How suppressed are the $1/N_c$-suppressed terms?
- Are we really testing ChPT or just doing a phenomenological fit?

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Testing if ChPT works: relations

Yes: JB, Jemos, Eur.Phys.J. C64 (2009) 273-282 [arXiv:0906.3118]

Systematic search for relations between observables that do not depend on the C_i^r

Included:

- m_M^2 and F_M for π, K, η.
- 11 $\pi\pi$ threshold parameters
- 14 πK threshold parameters
- 6 $\eta \to 3\pi$ decay parameters,
- 10 observables in $K_{\ell 4}$
- 18 in the scalar formfactors
- 11 in the vectorformfactors
- Total: 76

We found 35 relations

Relations at NNLO: summary

- We did numerics for $\pi\pi$ (7), πK (5) and $K_{\ell 4}$ (1)
  13 relations
- $\pi\pi$: similar quality in two and three flavour ChPT
  The two involving a_3^- significantly did not work well
- πK: relation involving a_3^- not OK
  one more has very large NNLO corrections
- The relation with $K_{\ell 4}$ also did not work: related to that ChPT has trouble with curvature in $K_{\ell 4}$
- Conclusion: Three flavour ChPT "sort of" works

Fits: inputs

Amorós, JB, Talavera, Nucl. Phys. B602 (2001) 87 [hep-ph/0101127] (ABT01)

JB, Jemos, Nucl. Phys. B 854 (2012) 631 [arXiv:1103.5945] (BJ12)

JB, Ecker, arXiv:1405.6488, Ann. Rev. Nucl. Part. Sci .64 (2014) 149-174 (BE14)

- $M_\pi, M_K, M_\eta, F_\pi, F_K/F_\pi$
- $\langle r^2\rangle^\pi_S$, c^π_S slope and curvature of F_S
- $\pi\pi$ and πK scattering lengths a^0_0, a^2_0, $a^{1/2}_0$ and $a^{3/2}_0$.
- Value and slope of F and G in $K_{\ell 4}$
- $\frac{m_s}{\hat{m}} = 27.5$ (lattice)
- $\bar{l}_1, \ldots, \bar{l}_4$
- more variation with C_i^r, a penalty for a large p^6 contribution to the masses
- 17+3 inputs and 8 L_i^r+34 C_i^r to fit

Main fit

	ABT01	BJ12	L_4^r free	BE14
	old data			
$10^3 L_1^r$	0.39(12)	0.88(09)	0.64(06)	0.53(06)
$10^3 L_2^r$	0.73(12)	0.61(20)	0.59(04)	0.81(04)
$10^3 L_3^r$	$-2.34(37)$	$-3.04(43)$	$-2.80(20)$	$-3.07(20)$
$10^3 L_4^r$	$\equiv 0$	0.75(75)	0.76(18)	$\equiv 0.3$
$10^3 L_5^r$	0.97(11)	0.58(13)	0.50(07)	1.01(06)
$10^3 L_6^r$	$\equiv 0$	0.29(8)	0.49(25)	0.14(05)
$10^3 L_7^r$	$-0.30(15$	$-0.11(15)$	$-0.19(08)$	$-0.34(09)$
$10^3 L_8^r$	0.60(20)	0.18(18)	0.17(11)	0.47(10)
χ^2	0.26	1.28	0.48	1.04
dof	1	4	?	?
F_0 [MeV]	87	65	64	71

$? = (17+3) - (8+34)$

Main fit: Comments

- All values of the C_i^r we settled on are "reasonable"
- Leaving L_4^r free ends up with $L_4^r \approx 0.76$
- keeping L_4^r small: also L_6^r and $2L_1^r - L_2^r$ small (large N_c relations)
- Compatible with lattice determinations
- Not too bad with resonance saturation both for L_i^r and C_i^r, including from the scalars
- decent convergence (but enforced for masses)
- Many prejudices went in: large N_c, resonance model, quark model estimates,...

Some results of this fit

$$
\begin{aligned}
\text{Mass:} & \\
m_\pi^2/m_{\pi phys}^2 &= 1.055(p^2) - 0.005(p^4) - 0.050(p^6)\,, \\
m_K^2/m_{Kphys}^2 &= 1.112(p^2) - 0.069(p^4) - 0.043(p^6)\,, \\
m_\eta^2/m_{\eta phys}^2 &= 1.197(p^2) - 0.214(p^4) + 0.017(p^6)\,, \\
\text{Decay constants:} & \\
F_\pi/F_0 &= 1.000(p^2) + 0.208(p^4) + 0.088(p^6)\,, \\
F_K/F_\pi &= 1.000(p^2) + 0.176(p^4) + 0.023(p^6)\,. \\
\text{Scattering:} & \\
a_0^0 &= 0.160(p^2) + 0.044(p^4) + 0.012(p^6)\,, \\
a_0^{1/2} &= 0.142(p^2) + 0.031(p^4) + 0.051(p^6)\,.
\end{aligned}
$$

$K_{\ell 2}$

- $K^+ \to \mu^+ \nu$: determining $F_K |V_{us}|$
- $K^+ \to e^+ \nu$: Lepton universality, NA62/1 or NA48/3
- ChPT known to two loops: JB, Amoros, Talavera, hep-ph/9907264, with $m_u - m_d$ hep-ph/0101127
- Radiative corrections: talk by Knecht

$K_{\ell 2\gamma}$

- $K^+ \to \mu^+\nu\gamma$: determining $F_K|V_{us}|$ (radiative corrections to $K^+\mu^+\nu$)
- $K^+ \to e^+\nu\gamma$ and $K^+ \to e^+\nu\gamma$: Bremssstrahlung and structure dependent parts
- Structure dependent parts: Vector and axial vector form factor and the former is related to $\pi^0\gamma^*\gamma$
- ChPT one-loop: JB, Gasser, Ecker, 1993
- ChPT two-loop: Geng, Ho, Wu 2004

$K_{\ell 2\gamma}$

$$K^+(p) \to l^+(p_l)\nu_l(p_\nu)\gamma(q) \qquad [K_{l2\gamma}]$$

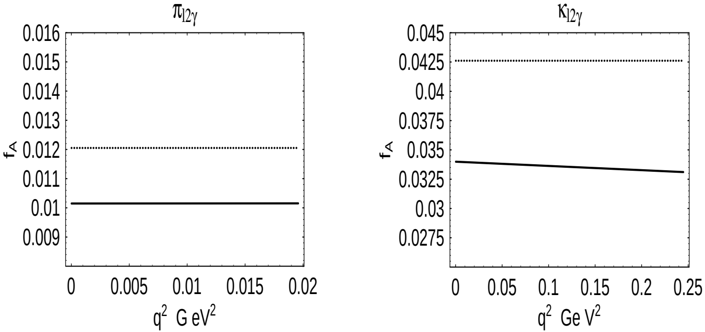
$$\begin{aligned}
T &= -iG_F e V_{us}^\star \epsilon_\mu^\star \{F_K L^\mu - H^{\mu\nu} l_\nu\} \\
L^\mu &= m_l \bar{u}(p_\nu)(1+\gamma_5)\left(\frac{2p^\mu}{2pq} - \frac{2p_l^\mu + \not{q}\gamma^\mu}{2p_l q}\right) v(p_l) \\
l^\mu &= \bar{u}(p_\nu)\gamma^\mu(1-\gamma_5)v(p_l) \\
H^{\mu\nu} &= iV(W^2)\epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta - A(W^2)(qWg^{\mu\nu} - W^\mu q^\nu) \\
W^\mu &= (p-q)^\mu = (p_l + p_\nu)^\mu .
\end{aligned}$$

L_μ: IB or inner Bremsstrahlung part

V and A: SD or structure dependent part, starts at p^4

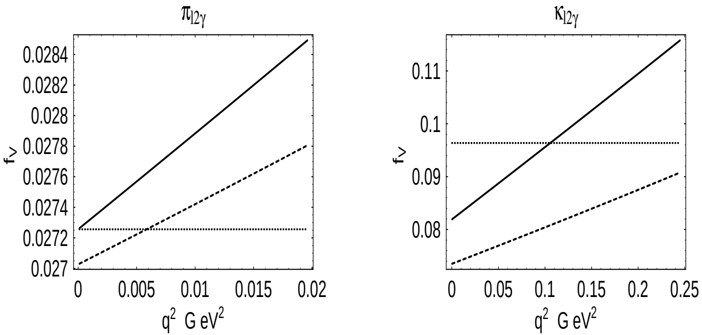
V: anomaly at p^4, known to p^6: Ametller,JB,Bramon,Cornet 1993

A: p^4 JB,Ecker,Gasser 1993, p^6 Geng,Ho, Wu 2004



From Geng,Ho, Wu 2004

$K_{\ell 2\gamma}$



From Geng,Ho, Wu 2004

dotted: p^4

solid p^6 C_i^W from VMD, dashed p^6 C_i^W from CQM

$K \to \ell'\nu\ell^+\ell^-$

- Same formfactors as previous decay but now depend on both $m^2_{\ell'\nu}$ and $m^2_{l^+l_-}$.
- $V(m^2_{\ell'\nu}, m^2_{l^+l_-})$: anomalous part: related by ChPT to the $\pi^0 \to \gamma^*\gamma^*$ physics and thus the same questions
- $A(m^2_{\ell'\nu}, m^2_{l^+l_-})$: allows to study mixed axial and vector terms. But need precision away from constant form-factors

Definition of $K\pi$ form-factors

Needed for $K^+_{\ell 3}$, $K^0_{\ell 3}$, $K^{+,0} \to \pi^{+,0}\nu\bar\nu$
We have four transitions:

$$\langle \pi^0(p')|\bar s\gamma_\mu u|K^+(p)\rangle = \frac{1}{\sqrt{2}}\left[(p+p')f_+^{K^+\pi^0} + (p-p')f_+^{K^+\pi^0}\right]$$
$$\langle \pi^-(p')|\bar s\gamma_\mu u|K^0(p)\rangle = \left[(p+p')f_+^{K^+\pi^0} + (p-p')f_+^{K^+\pi^0}\right]$$
$$\langle \pi^+(p')|\bar s\gamma_\mu d|K^+(p)\rangle = \left[(p+p')f_+^{K^+\pi^0} + (p-p')f_+^{K^+\pi^0}\right]$$
$$\langle \pi^0(p')|\bar s\gamma_\mu d|K^0(p)\rangle = \frac{-1}{\sqrt{2}}\left[(p+p')f_+^{K^+\pi^0} + (p-p')f_+^{K^+\pi^0}\right]$$

- Scalar formfactor: $f_0^{K^i\pi^i} = f_+^{K^i\pi^i} + \frac{(p-p')^2}{m^2_{K^i}-m^2_{\pi^i}} f_-^{K^i\pi^i}$
- In the isospin limit: all cases have the same form-factors
- Behrends-Sirlin-Ademollo-Gatto:
  $f_{+,0} = 1 + a(m_s - \hat m)^2 + \cdots$

Measurements

- Both neutral and charged decay
- f_+ and f_0
- $f_+(t) = f_+(0)\left(1 + \lambda_+ t + \lambda'_+ t^2 + \cdots\right)$
- $f_0(t) = f_+(0)\left(1 + \lambda_0 t + \cdots\right)$
- Alternatively use dispersive parametrizations
- KLOE,NA48,ISTRA,KTeV: large number of recent measurements
- Correlations in the form-factor measurements very important
- f_+: VMD and $SU(3)$ breaking
- f_0: Scalar meson dominance? (or dispersive better?)

Isospin breaking: general results

To first order: insert $\frac{1}{2}(m_u - m_d)\left(\bar{u}u - \bar{d}d\right)$ once (JB, Ghorbani, 0711.0148)

$$\begin{aligned}
f_k^{K^+\pi^0} &= f_k^A(t) + \delta f_k^B(t) + \cdots \\
f_k^{K^0\pi^-} &= f_k^A(t) - \delta f_k^D(t) + \cdots \\
f_k^{K^+\pi^+} &= f_k^A(t) + \delta f_k^D(t) + \cdots \\
f_k^{K^0\pi^0} &= f_k^A(t) - \delta f_k^B(t) + \cdots
\end{aligned}$$

$\delta = m_u - m_d$, $t = (p - p')^2$
Valid for $k = +, -, 0$ and for scalar current matrix elements

- $f_k^{K^+\pi^0}(t) - f_k^{K^0\pi^-}(t) - f_k^{K^+\pi^+}(t) + f_k^{K^0\pi^0}(t) = \mathcal{O}(\delta^2)$
- $r(t) = \dfrac{f_k^{K^+\pi^0}(t) f_k^{K^0\pi^0}(t)}{f_k^{K^0\pi^-}(t) f_k^{K^+\pi^+}(t)} = 1 + \mathcal{O}(\delta^2)$

Chiral Perturbation Theory

- p^2: $f_+ = 1$, $f_- = 0$ (current algebra)
- p^4 : $f_+(0)$ Leutwyler-Roos 1984
- p^4: Gasser-Leutwyler 1985 and isospin corrections for the weak decays
- Radiative corrections: talk by Knecht
- p^4 and radiative corrections for rare decays: Mescia-Smith, arXiv:0705.2025
- p^6 isospin limit: JB, Talavera, hep-ph/0303103 (see also Post, Schilcher, hep-ph/0112352)
- p^6 isospin breaking: JB, Ghorbani, arXiv:0711.0148
- Numbers for $K_{\ell 3}$: see Kastner, Neufeld, arXiv:0805.2222

$f_0(t)$ and $f_+(0)$

JB, Talavera, hep-ph/0303103 Main Result:

$$
\begin{aligned}
f_0(t) \quad = \quad & 1 - \frac{8}{F_\pi^4}\left(C_{12}^r + C_{34}^r\right)\left(m_K^2 - m_\pi^2\right)^2 \\
& +8\frac{t}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right)\left(m_K^2 + m_\pi^2\right) + \frac{t}{m_K^2 - m_\pi^2}\left(F_K/F_\pi - 1\right) \\
& -\frac{8}{F_\pi^4}t^2 C_{12}^r + \overline{\Delta}(t) + \Delta(0)\,.
\end{aligned}
$$

$\overline{\Delta}(t)$ and $\Delta(0)$ contain NO C_i^r and only depend on the L_i^r at order p^6
$\Longrightarrow$
All needed parameters can be determined experimentally

Now update input with the new results:

$$\Delta(0) = -0.02276\,(p^4) + 0.01140\,(p^6 \text{ pure loop}) + 0.0504\,(p^6 L_i^r)$$

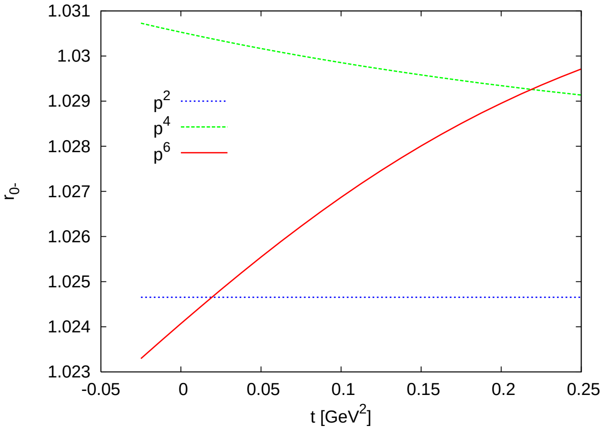
Update on $K_{\ell 3}$

- Take Bijnens-Talavera 2003 result but update for BE14 parameters
- $f_+^{K^0\pi^-}(0) = 1 - 0.02276 - 0.00754 = 0.970 \pm 0.008$
- in good agreement with the latest lattice numbers (Juettner, lattice 2015, preliminary FLAG
  2+1: 0.9677(37)
  2+1+1: 0.9704(32)
- Note original JB-Talavera: 0.976(10)
- FLAG1: 0.956(08)
- Jamin, Oller, Pich, hep-ph/0401080: 0.978(09)

$K_{\ell 3}$: Callan-Treiman

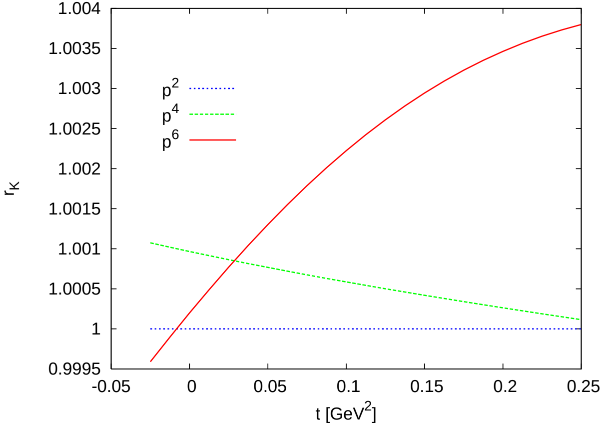
- Callan-Treiman: $f_0(m_K^2 - m_\pi^2) = \frac{F_K}{F_\pi} + \mathcal{O}(m_\pi^2)$
  $SU(2)$ current algebra
  must hold to all orders in $SU(3)$ ChPT
- Define $\Delta_{CT} = f_0(m_K^2 - m_\pi^2) = \frac{F_K}{F_\pi}$
- p^4 Gasser, Leutwyler 1985: $\Delta_{CT} = -3.5\,10^{-3}$
- p^6 Using JB, Talavera, 2003 $\Delta_{CT} = -6.2\,10^{-3}$
- p^6 isospinbreaking
  JB, Ghorbani, 2007
  $\Delta_{CT}^{K^+\pi^0} = 15.1\,10^{-3}$
  $\Delta_{CT}^{K^0\pi^-} = -5.6\,10^{-3}$
  $\Delta_{CT}^{K^+\pi^+} = -9.4\,10^{-3}$
  $\Delta_{CT}^{K^0\pi^0} = -26.4\,10^{-3}$
- Add p^6 contribution from C_i:
  $\Delta_{CT}^{C_i^r} = \frac{16}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right) m_\pi^2 \left(m_K^2 - m_\pi^2\right)$
  $\Delta_{CT}^{C_i^r} = 1.3\,10^{-3}$

r_{0-}



$r_{0-} = f_+^{K^+\pi^0} / f_+^{K^0\pi^-}$

r_K



$r_{0-} = f_+^{K^+\pi^+} / f_+^{K^0\pi^-}$

$K_{\ell 3\gamma}$ or $K \to \pi \ell \nu \gamma$

p^2: Fearing, Fischbach, Smith 1970 IB only

p^4: JB, Ecker, Gasser, 1993

p^6: Axial form-factors fully known

p^6: Vector form-factors: approximately known

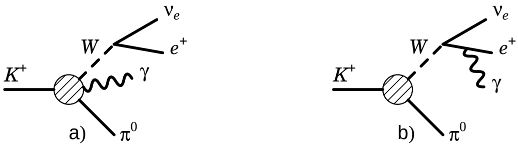
Gasser,Kubis,Paver,Verbeni hep-ph/0412130: $K_{Le\nu\gamma}$

Müller,Kubis,Meißner hep-ph/0607151: T-odd correlations

Kubis,Müller,Gasser,Schmid hep-ph/0611366: $K^+_{e\nu\gamma}$

Approximately known: structure functions smooth
cuts: p-wave or far away: approximate by polynomials

$K_{\ell 3\gamma}$



Remainder is from Kubis et al. 2006

$$
\begin{aligned}
&T(K^+_{e3\gamma}) = \\
&\frac{G_F}{\sqrt{2}}\, e\, V_{us}^*\, \epsilon^\mu(q)^* \Big[\left(V_{\mu\nu} - A_{\mu\nu}\right) \bar{u}(p_\nu)\, \gamma^\nu\, (1-\gamma_5)\, v(p_e) \\
&+ \frac{F_\nu}{2 p_e q}\, \bar{u}(p_\nu)\, \gamma^\nu\, (1-\gamma_5) \left(m_e - \not{p}_e - \not{q}\right) \gamma_\mu\, v(p_e) \Big]
\end{aligned}
$$

$K_{\ell 3\gamma}$

$$A_{\mu\nu} = \frac{i}{\sqrt{2}}\left[\epsilon_{\mu\nu\rho\sigma}\left(A_1\, p'^{\rho} q^{\sigma} + A_2\, q^{\rho} W^{\sigma}\right) + \epsilon_{\mu\lambda\rho\sigma}\, p'^{\lambda} q^{\rho} W^{\sigma}\left(\frac{A_3}{M_K^2 - W^2}\, W_\nu + A_4\, p'_\nu\right)\right],$$

$$V_{\mu\nu} = V_{\mu\nu}^{IB} + V_{\mu\nu}^{SD}$$

$V_{\mu\nu}^{SD}$ has again 4 structure function V_i

$V_{\mu\nu}^{IB}$: IB part, mainly determined by Low's theorem and from the $K_{\ell 3}$ form-factors

$$R\left(E_\gamma^{\mathrm{cut}},\, \theta_{e\gamma}^{\mathrm{cut}}\right) = \frac{\Gamma\left(K_{e3\gamma}^{\pm}, E_\gamma^* > E_\gamma^{\mathrm{cut}}, \theta_{e\gamma}^* > \theta_{e\gamma}^{\mathrm{cut}}\right)}{\Gamma\left(K_{e3}^{\pm}\right)},$$

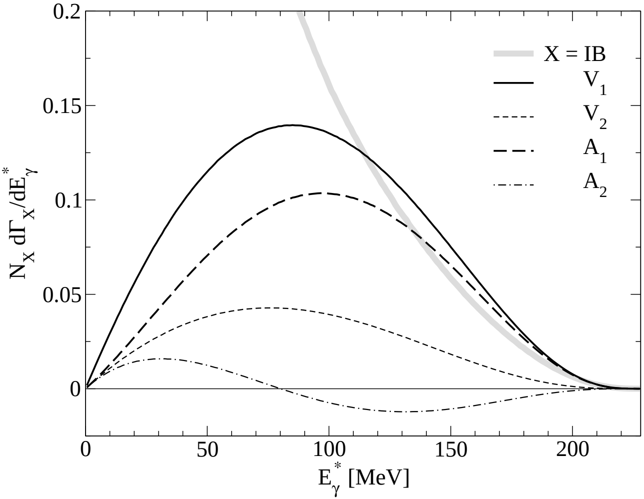
Many uncertainties drop out

$$R\left(\bar{\lambda}_+, \bar{\lambda}''_+\right) = R(1,0)\left\{1 + c_1\ (\bar{\lambda}_+ - 1) + c_2\ (\bar{\lambda}_+ - 1)^2 + c_3\,\bar{\lambda}''_+ + \ldots\right\}$$

R^{IB} accordingly (with expansion coefficients c_i^{IB})

E_γ^{cut}	$\theta_{e\gamma}^{\mathrm{cut}}$	$R^{\mathrm{IB}} \cdot 10^2$	$R \cdot 10^2$	$c_1 \cdot 10^3$	$c_2 \cdot 10^4$	$c_3 \cdot 10^4$
30 MeV	20°	0.640	0.633 ± 0.002	12.5 ± 0.4	-5.4 ± 0.3	16.9 ± 0.4
30 MeV	10°	0.925	0.918 ± 0.002	11.1 ± 0.3	-4.7 ± 0.2	15.0 ± 0.3
10 MeV	20°	1.211	1.204 ± 0.002	7.5 ± 0.2	-3.2 ± 0.2	10.1 ± 0.2
10 MeV	10°	1.792	1.785 ± 0.002	6.7 ± 0.2	-2.8 ± 0.1	9.0 ± 0.1
10 MeV	$26^\circ - 53^\circ$	0.554	0.553 ± 0.001	5.7 ± 0.1	-2.4 ± 0.1	7.5 ± 0.1

$K_{\ell 3\gamma}$



$$\frac{d\Gamma}{dE^*_\gamma} = \frac{d\Gamma_{\rm IB}}{dE^*_\gamma} + \sum_{i=1}^{4}\left(\langle V_i\rangle \frac{d\Gamma_{V_i}}{dE^*_\gamma} + \langle A_i\rangle \frac{d\Gamma_{A_i}}{dE^*_\gamma}\right) + \mathcal{O}\left(|T^{\rm SD}|^2,\, \Delta V_i,\, \Delta A_i\right)$$

$K_{\ell 4}$

$$
\begin{aligned}
K^+(p) &\to \pi^+(p_+)\pi^-(p_-)\ell^+(p_\ell)\nu_\ell(p_\nu)\,,\\
K^+(p) &\to \pi^0(p_+)\pi^0(p_-)\ell^+(p_\ell)\nu_\ell(p_\nu)\,,\\
K^0(p) &\to \pi^-(p_+)\pi^0(p_-)\ell^+(p_\ell)\nu_\ell(p_\nu)\,.
\end{aligned}
$$

Kinematical variables for hadronic system: t, u, s_π, s_ℓ

$$
T^{+-} = \frac{G_F}{\sqrt{2}} V_{us}^* \overline{u}(p_\nu)\gamma_\mu(1-\gamma_5)v(p_\ell)(V^\mu - A^\mu)\,,
$$

$$
\begin{aligned}
V_\mu &= -\frac{H}{m_K^3}\epsilon_{\mu\nu\rho\sigma}(p_\ell+p_\nu)^\nu(p_+ + p_-)^\rho(p_+ - p_-)^\sigma\,,\\
A_\mu &= -\frac{i}{m_K}[(p_+ + p_-)_\mu\, F + (p_+ - p_-)_\mu\, G + (p_\ell + p_\nu)_\mu\, R]\,.
\end{aligned}
$$

$K_{\ell 4}$

$$T^{+-} = \frac{T^{-0}}{\sqrt{2}} + T^{00}$$

T^{-0} is anti-symmetric under $t \leftrightarrow u$
T^{00} is symmetric.

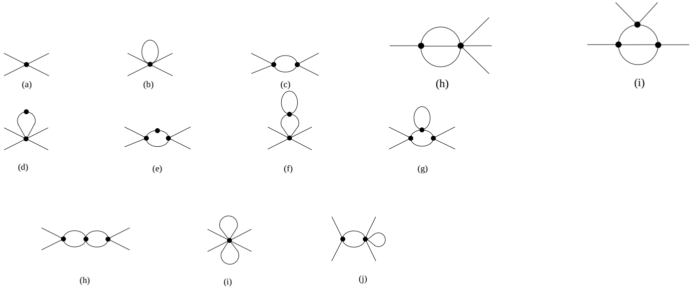
Lowest order: Weinberg: $F = G = \frac{m_K}{\sqrt{2}F_\pi}$,

Order p^4: JB 1990, Riggenbach et al. 1991

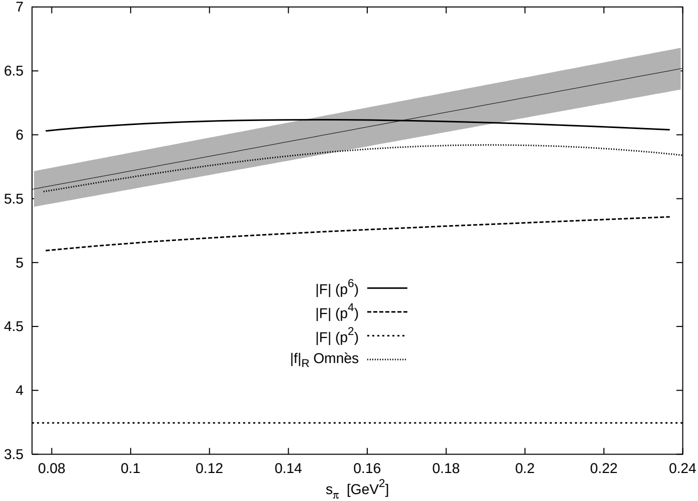
Could fit data with reasonable corrections:
Determine L_i^r $i = 1, 2, 3$

Dispersive estimate of p^6 corrections:
JB, Colangelo, Gasser 1994

Parametrization for experiment: Amoros,JB, 1999
Full p^6 calculation: Amoros, JB, Talavera 2000
Ametller,JB,Bramon,Cornet 1993 (H only)
Isospin breaking at p^4: Nehme et al.
Isospin breaking and radiative corrections: Colangelo, Gasser, Rusetsky
Radiative corrections: Stoffer
Dispersive (most recent): Stoffer

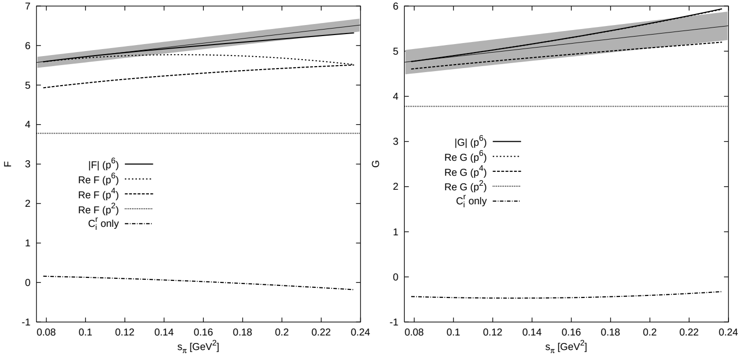


$K_{\ell 4}$



7
6.5
6
5.5
5
4.5
4
3.5
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.22
0.24
s_π [GeV^2]
|F| (p^6)
|F| (p^4)
|F| (p^2)
|f|$_R$ Omnès

$K_{\ell 4}$



7
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F
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0.12
0.14
0.16
0.18
0.2
0.22
0.24
s_π [GeV^2]
|F| (p^6)
Re F (p^6)
Re F (p^4)
Re F (p^2)
C_i^r only
6
5
4
3
2
1
0
-1
G
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.22
0.24
s_π [GeV^2]
|G| (p^6)
Re G (p^6)
Re G (p^4)
Re G (p^2)
C_i^r only

Finite volume

- Lattice QCD calculates at different quark masses, volumes boundary conditions,. . .
- A general result by Lüscher: relate finite volume effects to scattering (1986)
- Chiral Perturbation Theory is also useful for this
- Start: Gasser and Leutwyler, Phys. Lett. B184 (1987) 83, Nucl. Phys. B 307 (1988) 763
  M_π, F_π, $\langle \bar{q} q \rangle$ one-loop equal mass case
- I will stay with ChPT and the p regime ($M_\pi L >> 1$)
- $1/m_\pi = 1.4$ fm
  may need to go beyond leading $e^{-m_\pi L}$ terms
- Convergence of ChPT is given by $1/m_\rho \approx 0.25$ fm

Finite volume: selection of ChPT results

- masses and decay constants for π, K, η one-loop
  Becirevic, Villadoro, Phys. Rev. D 69 (2004) 054010
- M_π at 2-loops (2-flavour)
  Colangelo, Haefeli, Nucl.Phys. B744 (2006) 14 [hep-lat/0602017]
- $\langle \bar{q}q \rangle$ at 2 loops (3-flavour)
  JB, Ghorbani, Phys. Lett. B636 (2006) 51 [hep-lat/0602019]
- Twisted mass at one-loop
  Colangelo, Wenger, Wu, Phys.Rev. D82 (2010) 034502 [arXiv:1003.0847]
- Twisted boundary conditions
  Sachrajda, Villadoro, Phys. Lett. B 609 (2005) 73 [hep-lat/0411033]
- This talk:
  - Twisted boundary conditions and some funny effects
  - Some results on masses 3-flavours at two loop order

Twisted boundary conditions

- On a lattice at finite volume $p^i = 2\pi n^i / L$: very few momenta directly accessible
- Put a constraint on certain quark fields in some directions: $q(x^i + L) = e^{i\theta^i_q} q(x^i)$
- Then momenta are $p^i = \theta^i / L + 2\pi n^i / L$. Allows to map out momentum space on the lattice much better Bedaque,...
- But:
  - Box: Rotation invariance $\to$ cubic invariance
  - Twisting: reduces symmetry further

  Consequences:
  - $m^2(\vec{p}^2) = E^2 - \vec{p}^2$ is not constant
  - There are typically more form-factors
  - In general: quantities depend on more (all) components of the momenta
  - Charge conjugation involves a change in momentum

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Twisted boundary conditions: Two-point function

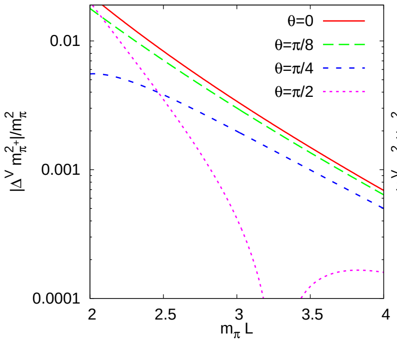
JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]

- $\displaystyle\int_V \frac{d^d k}{(2\pi)^d}\frac{k_\mu}{k^2-m^2} \neq 0$
- $\langle \bar{u}\gamma^\mu u\rangle \neq 0$
- $j_\mu^{\pi^+} = \bar{d}\gamma_\mu u$
  satisfies $\partial^\mu \left\langle T(j_\mu^{\pi^+}(x) j_\nu^{\pi^-}(0))\right\rangle = \delta^{(4)}(x)\left\langle \bar{d}\gamma_\nu d - \bar{u}\gamma_\nu u\right\rangle$
- $\displaystyle \Pi^a_{\mu\nu}(q) \equiv i\int d^4x e^{iq\cdot x}\left\langle T(j^a_\mu(x) j^{a\dagger}_\nu(0))\right\rangle$
  Satisfies WT identity. $q^\mu \Pi^{\pi^+}_{\mu\nu} = \left\langle \bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right\rangle$
- ChPT at one-loop satisfies this
  see also Aubin et al, Phys.Rev. D88 (2013) 7, 074505 [arXiv:1307.4701]

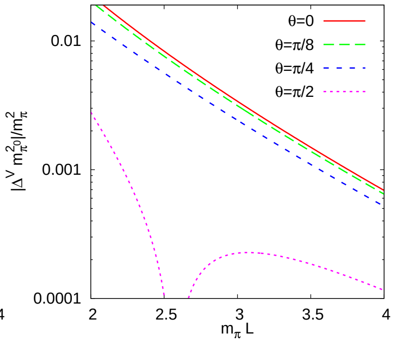
Twisted boundary conditions: volume correction masses

JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]

$m_\pi L = 2$, $\vec{\theta}_u = (\theta, 0, 0)$, $\vec{\theta}_d = \vec{\theta}_s = 0$



Charged pion mass

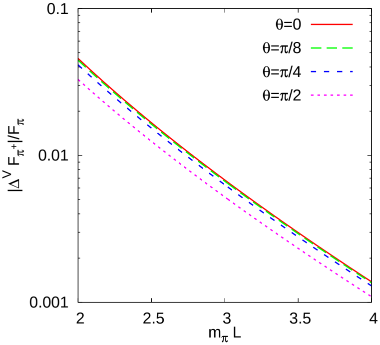


Neutral pion mass

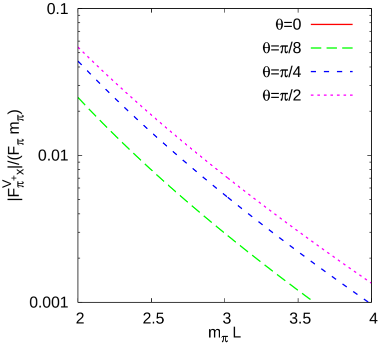
$\Delta^V X = X^V - X^\infty$ (dip is going through zero)

Volume correction decay constants: F_{π^+}

- JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]
- $\langle 0 | A^M_\mu | M(p) \rangle = i\sqrt{2} F_M p_\mu + i\sqrt{2} F^V_{M\mu}$
- Extra terms are needed for Ward identities



relative for F_π



Extra for $\mu = x$

Volume correction electromagnetic formfactor

- JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]
  earlier two-flavour work:
  Bunton, Jiang, Tiburzi, Phys.Rev. D74 (2006) 034514 [hep-lat/0607001]
- $\langle M'(p')|j_\mu|M(p)\rangle = f_\mu = f_+(p_\mu + p'_\mu) + f_- q_\mu + h_\mu$
- Extra terms are again needed for Ward identities
- Note that masses have finite volume corrections
  - q^2 for fixed $\vec{p}$ and $\vec{p}'$ has corrections
    small effect
  - This also affects the ward identities, e.g.
    $q^\mu f_\mu = (p^2 - p'^2) f_+ + q^2 f_- + q^\mu h_\mu = 0$
    is satisfied but all effects should be considered

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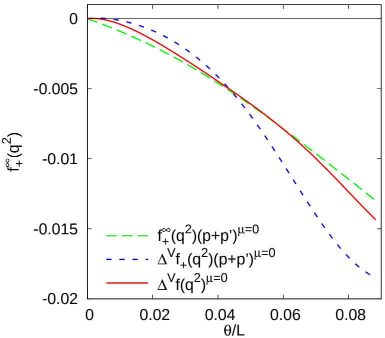
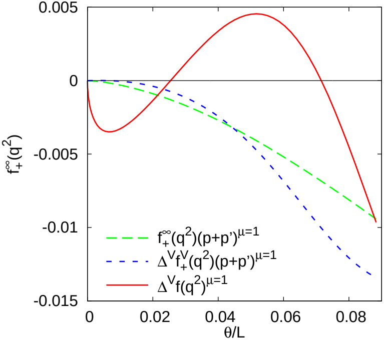
Volume correction electromagnetic formfactor

- $f_\mu = -\frac{1}{\sqrt{2}}\langle \pi^0(p')|\bar{d}\gamma_\mu u|\pi^+(p)\rangle$
  $= \left(1 + f_+^\infty + \Delta^V f_+\right)(p+p')_\mu + \Delta^V f_- q_\mu + \Delta^V h_\mu$
- Pure loop plotted: $f_+^\infty(p+p'), \Delta^V f_+(p+p')$ and $\Delta^V f_\mu$



$\mu = t$



$\mu = x$

Finite volume corrections large, different for different μ

Conclusions

- Short introduction to ChPT
- A very fast overview of all semileptonic modes and what they are useful for
- Looking forward on possible improvements from NA62
- Lattice: finite volume now limiting factor for $f_+(0)$ (MILC), relevant ChPT calculation in progress