π^0 physics

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NA62 Kaon Physics Handbook, Mainz, 20/1/2016

Outline:

- Introduction
- π^0 basic properties
- processes:
  - $\pi^0 \to \gamma\gamma$
  - $\pi^0 \to e^+e^-$
  - $\pi^0 \to e^+e^-\gamma$ (Dalitz decay)
  - $\pi^0 \to e^+e^-e^+e^-$ (double Dalitz)
  - $\pi^0 \to$ *posit.*γ
  - $\pi^0 \to 4\gamma$
  - $\pi^0 \to \nu\bar{\nu}(\gamma)$
- Summary

Introduction

The study of π^0s within the kaon experiments is well established:

- KTeV at FermiLab using $K_L \to 3\pi^0$ they studied e.g.: $\pi^0 \to e^+e^-$, double Dalitz, $\pi^0 \to \mu e$
- Brookhaven E787/E949 uses $K^+ \to \pi^+\pi^0$ as a source of π^0. They got limits on $\pi^0 \to \gamma X$, $\pi^0 \to \gamma X X'$, $\pi^0 \to \nu\bar{\nu}$ and $\pi^0 \to$ "nothing"

NA62 at CERN with 10^{13} K^+ is the pion factory as well (see Augusto talk):

- 3×10^{12} π^0s
- 1.5×10^{13} π^+s

What shall we do with all that pions?

π^0 history

- conception: Yukawa '35, Kemmer '38
- long birth: Lewis, Oppenheimer, Wouthuysen '48, Carlson, Hooper, King '50, Bjorklund, Crandall, Moyer, York '50 "The existence of a neutral meson is clearly not required at the present stage of the experiments, but is the only one of the above five hypotheses which seems to fit the experimental data."
  Steinberger, Panofsky, Steller '50: "It is clear from these properties that the gamma-rays are the decay products of neutral mesons."
  Ekspong'97: "It was generally felt that the neutral pion marked the end for particle searches."
- two siblings: π^+, π^-, born: Lattes, Muirhead, Occhialini, Powell, '47

π^0's properties

$$I^G\left(J^{PC}\right) = 1^-\left(0^{-+}\right)$$

- mass $m = 134.9766(6)$ MeV
- $m_{\pi^\pm} - m_{\pi^0} = 4.5936(5)$ MeV
- mean life $\tau = (8.4 \pm 0.6) \times 10^{-17}$ s

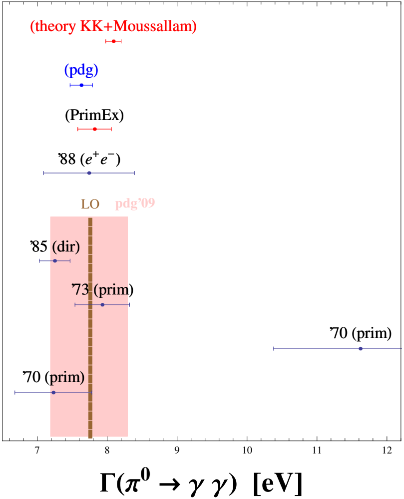
PDG 2015 about π^0

π^0 DECAY MODES

For decay limits to particles which are not established, see the appropriate Search sections (A^0 (axion) and Other Light Boson (X^0) Searches, etc.).

	Mode		Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	2γ		(98.823 ± 0.034) %	S=1.5
Γ_2	$e^+e^-\gamma$		(1.174 ± 0.035) %	S=1.5
Γ_3	γ positronium		$(1.82\pm0.29)\times10^{-9}$	
Γ_4	$e^+e^+e^-e^-$		$(3.34\pm0.16)\times10^{-5}$	
Γ_5	e^+e^-		$(6.46\pm0.33)\times10^{-8}$	
Γ_6	4γ		$<2\times10^{-8}$	CL=90%
Γ_7	$\nu\overline{\nu}$		[a] $<2.7\times10^{-7}$	CL=90%
Γ_8	$\nu_e\overline{\nu}_e$		$<1.7\times10^{-6}$	CL=90%
Γ_9	$\nu_\mu\overline{\nu}_\mu$		$<1.6\times10^{-6}$	CL=90%
Γ_{10}	$\nu_\tau\overline{\nu}_\tau$		$<2.1\times10^{-6}$	CL=90%
Γ_{11}	$\gamma\nu\overline{\nu}$		$<6\times10^{-4}$	CL=90%
Charge conjugation (*C*) or Lepton Family number (*LF*) violating modes				
Γ_{12}	3γ	*C*	$<3.1\times10^{-8}$	CL=90%
Γ_{13}	μ^+e^-	*LF*	$<3.8\times10^{-10}$	CL=90%

π^0 life time



π^0 mean life, PDG history:

1985 $(8.4 \pm 0.6) \times 10^{-17}$ s

...

2009 $(8.4 \pm 0.6) \times 10^{-17}$ s

2010 $(8.4 \pm 0.5) \times 10^{-17}$ s

2011 $(8.4 \pm 0.4) \times 10^{-17}$ s

2012 $(8.52 \pm 0.18) \times 10^{-17}$ s ← PrimEx col.

...

today $(8.52 \pm 0.18) \times 10^{-17}$ s

theory: [KK,Moussallam] $(8.04 \pm 0.11) \times 10^{-17}$ s

$\pi^0 \to \gamma\gamma$

- one of the most important processes for theory of particle physics
- π^0 lightest hadron $\Rightarrow$ dominant decay mode $\pi^0 \to \gamma\gamma$ (br=98.82%)
- non-existence of logarithmic correction to the current algebra result at NLO
- connection with the non-renormalization theorem ?
- new experimental activities
- theory – NNLO calculation: [KK,Moussallam'09] $\to$ see next

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$\pi^0 \to \gamma\gamma$: chiral expansion

- in chiral limit exact due to QCD axial anomaly:

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{m_{\pi^0}^3}{64\pi}\left(\frac{\alpha N_C}{3\pi F_\pi}\right)^2 = 7.73\,eV$$

Correction to the current algebra prediction:

- using [Pagels and Zepeda '72] sum rules in [Kitazawa '85]
- NLO corrections are hidden in $F \to F_{\pi^0}$ and $O(p^6)$ LECs [Donoghue, Holstein, Lin '85] [Bijnens, Bramon, Cornet '88]
- in 3-flavour case we can study π^0, η, η' mixing, resulting to [Goity, Bernstein, Holstein '02]:

  $$\Gamma^{\text{NLO}} = 8.1 \pm 0.08\,\text{eV}$$

  in 2-flavour case EM corrections [Ananth., Moussallam '02]:

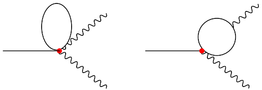
  $$\Gamma^{\text{NLO}} = 8.06 \pm 0.02 \pm 0.06\,\text{eV}$$

- Quite recently another study based on dispersion relations, QCD sum rules, using only the value $\Gamma(\eta \to \gamma\gamma)$ gives [Ioffe, Oganesian '07]:

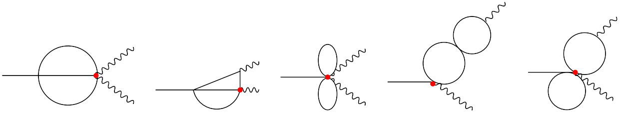
  $$\Gamma^{\text{NLO}} = 7.93 \pm 0.11\,\text{eV}$$

$\pi^0 \to \gamma\gamma$ at NNLO in 2 flavour ChPT: technical part

- NLO: a) One-loop diagrams with one vertex from $\mathcal{L}^{WZ}$, b) tree diagrams with one vertex from $\mathcal{L}^{WZ}$ and one vertex from $O(p^4)$ Lagrangian, c) tree diagrams with one vertex from $O(p^6)$ anomalous-parity sector
- $O(p^6)$ anomalous-parity sector from [Bijnens, Girlanda, Talavera '02]
- representation of chiral field: $U = \sigma + i\frac{\tau.\pi}{F}$, $\sigma = \sqrt{1 - \vec{\pi}^2/F^2}$ (no $\gamma 4\pi$ vertex at LO)
- one-loop

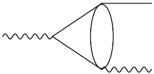


- two-loop



- verification of Z-factor, F_π/F [Bürgi '96], [Bijnens, Colangelo, Ecker, Gasser, Sainio '02]
- double log checked by Weinberg consistency rel. [Colangelo '95]

$\pi^0 \to \gamma\gamma$ technical part: some details on two-loop calculation



$$\mathcal{T} \sim \epsilon^{\mu\nu\rho\sigma} e^1_\alpha\, k^2_\rho\, e^2_\sigma\, p_\lambda$$
$$\times \int \frac{d^d l_1}{i(2\pi)^d} \frac{d^d l_2}{i(2\pi)^d} \frac{l_1^\lambda\, l_1^\mu\, l_2^\alpha\, l_2^\nu}{(l_1^2-M^2)(l_2^2-M^2)[(l_2+k_1)^2-M^2][(l_1+l_2-k_2)^2-M^2]}$$

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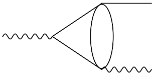

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[Bijnens, Colangelo, Ecker, Gasser, Sainio '97]

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$$J(t) = \int \frac{d^d l_1}{i(2\pi)^d} \frac{l_1^\lambda \, l_1^\mu}{(l_1^2 - M^2)((l_1+t)^2 - M^2)} \qquad \to \qquad \frac{g_{\lambda\mu}}{4(2w+3)} \int_{4M^2}^\infty \frac{[d\sigma]}{\sigma - t^2}(4M^2 - \sigma)$$

$$\Rightarrow R^{\alpha\nu} = \frac{1}{4(2w+3)} \int \frac{d^d l_2}{i(2\pi)^d} \frac{l_2^\alpha \, l_2^\nu}{(l_2^2 - M^2)((l_2+k_1)^2 - M^2)} \int_{4M^2}^\infty \frac{[d\sigma](4M^2 - \sigma)}{\sigma - t^2}$$

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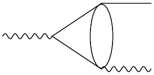

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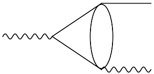
Divergences can be separated by Taylor expansion around $\sigma = \infty$

$$\int_{4M^2}^\infty [d\sigma](4M^2 - \sigma)\sigma^l = -\frac{2(2w+3)}{(4\pi)^{2+\omega}} \Gamma(-2-w-l) \frac{\Gamma(-l)}{\Gamma(-2l)} (M^2)^{w+2+l}, \qquad \text{conv. for } l < -2$$

$\pi^0 \to \gamma\gamma$ technical part: some details on two-loop calculation



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so far one would obtain:

$$R = \frac{3581}{8064} + \frac{\pi^2}{24} + R_1 + R_2\,, \qquad \text{with} \qquad R_1 = -\frac{1}{84}\int_4^\infty ds \sqrt{\frac{(s-4)^3}{s}}(\log(s) pol_1 + pol_2)$$

and R_2 can be expressed as double integral, where one integral comes from

$$I = \frac{M^6}{4}\int dx dy dz \int \frac{d^d l}{i(2\pi)^d} \frac{60\, x^3 y^2 z^3 (1-z)^4\, l^2}{[A_z - x(1-y)B_z - l^2]^6} \quad \text{with} \quad A_z = z\sigma + (1-z)M^2, \quad B_z = z(1-z)M^2$$

n.b. possible expansion in $B_z/A_z \leq 1/9$

$\pi^0 \to \gamma\gamma$ at NNLO, result

$$A_{NNLO} = \frac{e^2}{F_\pi}\Big\{ \frac{1}{4\pi^2}$$
$$+ \frac{16}{3} m_\pi^2(-4c_3^{Wr} - 4c_7^{Wr} + c_{11}^{Wr}) + \frac{64}{9}B(m_d - m_u)(5c_3^{Wr} + c_7^{Wr} + 2c_8^{Wr})$$
$$- \frac{M^4}{24\pi^2F^4}\Big(\frac{1}{16\pi^2}L_\pi\Big)^2 + \frac{M^4}{16\pi^2F^4}L_\pi\Big[\frac{3}{256\pi^4} + \frac{32F^2}{3}\big(2c_2^{Wr} + 4c_3^{Wr} + 2c_6^{Wr} + 4c_7^{Wr} - c_{11}^{Wr}\big)\Big]$$
$$+ \frac{32M^2B(m_d - m_u)}{48\pi^2F^4}L_\pi\left[-6c_2^{Wr} - 11c_3^{Wr} + 6c_4^{Wr} - 12c_5^{Wr} - c_7^{Wr} - 2c_8^{Wr}\right]$$
$$+ \frac{M^4}{F^4}\lambda_+ + \frac{M^2B(m_d - m_u)}{F^4}\lambda_- + \frac{B^2(m_d - m_u)^2}{F^4}\lambda_{--}\Big\}$$

$$\lambda_+ = \frac{1}{\pi^2}\left[-\frac{2}{3}d_+^{Wr}(\mu) - 8c_6^r - \frac{1}{4}(l_4^r)^2 + \frac{1}{512\pi^4}\left(-\frac{983}{288} - \frac{4}{3}\zeta(3) + 3\sqrt{3}\,\mathrm{Cl}_2(\pi/3)\right)\right]$$
$$+ \frac{16}{3}F^2\left[8l_3^r(c_3^{Wr} + c_7^{Wr}) + l_4^r(-4c_3^{Wr} - 4c_7^{Wr} + c_{11}^{Wr})\right]$$
$$\lambda_- = \frac{64}{9}\left[d_-^{Wr}(\mu) + F^2 l_4^r\,(5c_3^{Wr} + c_7^{Wr} + 2c_8^{Wr})\right]$$
$$\lambda_{--} = d_{--}^{Wr}(\mu) - 128F^2 l_7(c_3^{Wr} + c_7^{Wr})\ .$$

- 4 LECs in 2 combinations of NLO
- additional 4 LECs in 3 combinations of NNLO

Is it at all possible to make some reliable prediction?

$\pi^0 \to \gamma\gamma$: modified counting

- Use of $SU(3)$ phenomenology via $c_i^{Wr} \leftrightarrow C_i^{Wr}$ connection (based on [Gasser, Haefeli, Ivanov, Schmid '07,'08])

$$c_i^{Wr} = \frac{\alpha_i}{m_s} + \left(\beta_i + \gamma_{ij} C_j^{Wr} + \delta_i \ln \frac{B_0 m_s}{\mu^2}\right) + O(m_s)$$

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- implementation of modified counting

$$m_u, m_d \sim O(p^2) \quad \text{and} \quad m_s \sim O(p)$$

Result:

$$A_{NNLO}^{mod} = \frac{e^2}{F_\pi}\left\{ \frac{1}{4\pi^2} - \frac{64}{3} m_\pi^2 C_7^{Wr} + \frac{1}{16\pi^2}\frac{m_d - m_u}{m_s}\left[1 - \frac{3}{2}\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right] \right.$$
$$+ 32B(m_d - m_u)\left[\frac{4}{3} C_7^{Wr} + 4 C_8^{Wr}\left(1 - 3\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right)\right.$$
$$\left.\left. - \frac{1}{16\pi^2 F_\pi^2}\left(3L_7^r + L_8^r - \frac{1}{512\pi^2}(L_K + \frac{2}{3} L_\eta)\right)\right] - \frac{1}{24\pi^2}\left(\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right)^2 \right\}$$

Phenomenology

- F_π from [Marciano, Sirlin '93] π_{l2} decay:

$$\Gamma = \frac{G_F^2}{4\pi}|V_{ud}F_\pi|^2 m_l^2 m_\pi (1-z^2)^2\left(1+\frac{2\alpha}{\pi}\ln\frac{m_Z}{m_\rho}\right)(1+C_1+\ldots)$$

  - C_1 estimate via ChPT
    - [Knecht,Neufeld,Rupertsberger,Talavera '00]

$$C_1 = \frac{Z}{4}\left(3+2\ln\frac{m_\pi^2}{m_\rho^2}+\ln\frac{m_K^2}{m_\rho^2}\right)-\frac{1}{2}+f(K_i^r,X_i^r)$$

    - [Ananth.,Moussallam '02] [Descotes, Moussallam '06]

$$C_1 = -2.56 \pm 0.50$$

  - V_{ud} [Towner,Hardy '08]

$$V_{ud} = 0.97418(26)$$

$\Rightarrow$ $F_\pi = 92.22 \pm 0.07$ MeV

[rem.: *of course if SM is correct. See the end of presentation*]

$\pi \to \gamma\gamma$: Phenomenology

- $F_\pi = 92.22 \pm 0.07$ MeV

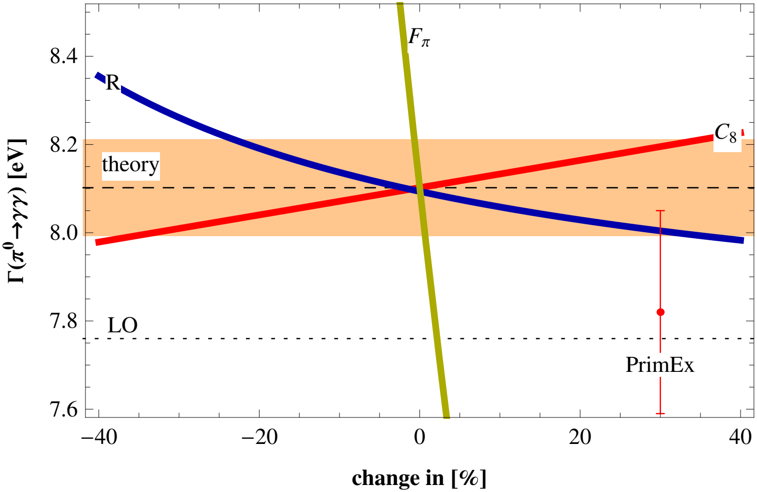
  using quark mass ratio (from lattice), pseudo-scalar meson masses, R from $\eta \to 3\pi$ (ChPT: [Bijnens,Ghorbani '07])
- $\frac{m_d - m_u}{m_s} = (2.29 \pm 0.23)\, 10^{-2}$
- $B(m_d - m_u) = (0.32 \pm 0.03)\, M_{\pi^0}^2$
- $3L_7 + L_8^r(\mu) = (0.10 \pm 0.06)\, 10^{-3} \quad (\mu = M_\eta)$ (from pseudo-scalar meson masses formula [Gasser, Leutwyler '85])
- $C_7^W = 0$ (more precisely $C_7^W \ll C_8^W$, motivated by simple resonance saturation)
- $C_8^W = (0.58 \pm 0.2) 10^{-3}$GeV^{-2} (from $\eta \to 2\gamma$)

result $\Gamma_{\pi^0 \to 2\gamma} = (8.09 \pm 0.11)$eV

[or $\tau_{\pi^0} = (8.04 \pm 0.11) \times 10^{-17}$ s]

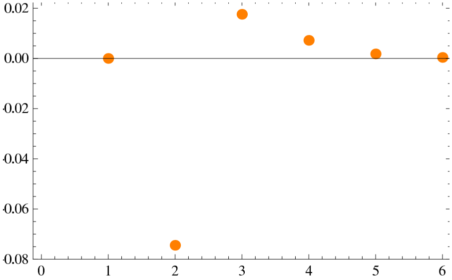
$\pi \to \gamma\gamma$: Phenomenology

dependence on parameters $C_8^W\ \left[\frac{5\times10^{-6}}{F_\pi^2}\right]$, $R\ [41.3]$ and $F_\pi\ [92.22\,\text{MeV}]$:



$\pi^0 \to \gamma\gamma$: leading logs (for details see [Bijnens,KK,Lanz'12])

Leading logarithm contribution of individual orders in percent of the leading order:



Adler-Lee-Treiman-Zee-Terentev theorem on triangle and box anomaly

$$F^{3\pi}(0,0,0) = \frac{1}{eF_\pi^2} F_{\pi\gamma\gamma}(0,0)$$

is valid up to 2-loop order for LL beyond the soft-photon limit

$\pi^0 \to e^+e^-$

- first studied by [S. Drell '59]
- radiative corrections: [L.Bergström '83]
- most recent experiment: KTeV E799-II [Abouzaid'07]
- radiative corrections play important role

$\pi^0 \to e^+e^-$

KTeV's measurement:

$$\frac{\Gamma(\pi^0 \to e^+e^-,\, x > 0.95)}{\Gamma(\pi^0 \to e^+e^-\gamma,\, x > 0.232)} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}\,.$$

by extrapolating the Dalitz branching ratio to the full range of x

$$B(\pi^0 \to e^+e^-(\gamma),\, x > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}\,.$$

Extrapolating the radiative tail using Bergström:

$$B^{\text{no-rad}}_{\text{KTeV}}(\pi^0 \to e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}\,.$$

Theoretical prediction [Dorokhov, Ivanov '07, '10]

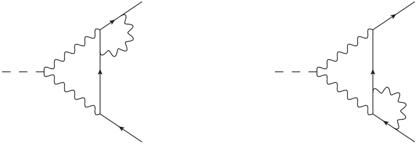
$$B^{\text{no-rad}}_{\text{SM}}(\pi^0 \to e^+e^-) = (6.23 \pm 0.09) \times 10^{-8}\,. \qquad (1)$$

3.3 $\sigma \Rightarrow$ New physics?

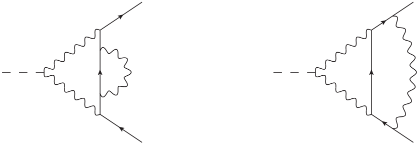
In any case, radiative corrections play an important role in the analysis

$\pi^0 \to e^+e^-$

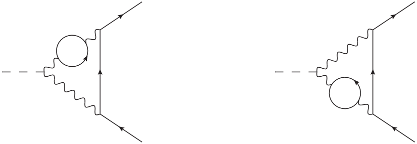
Radiative corrections $\to$ two-loop graphs



(a) (b)



(c) (d)



(e) (f)

$\pi^0 \to e^+e^-$

- two-loop contributions, together with Bremsstrahlung (= Dalitz) [Dorokhov et al. '08], [Vasko,Novotny '11], [Husek,KK,Novotny'14]
- counter-term chiral Lagrangian for $\pi^0 l\bar{l}$ [Savage et al'92]
- modelled using the resonances [Knecht '99]

$$\chi^{(r)}_{\mathsf{LMD}}(M_\rho) = 2.2 \pm 0.9$$

- rem.: different models possible, see e.g. next talk of Pere Masjuan, for $\chi = 2.76(23)$
- KTeV implies [Husek,KK,Novotny'14]

$$\chi^{(r)}_{\mathsf{KTeV}}(M_\rho) = 4.5 \pm 1.0$$

- original discrepancy down to 2 σ level
- note: weak contributions mediated via $\pi^0 \to Z^* \to e^+e^-$ three orders of magnitude smaller than EM

$\pi^0 \to e^+e^-$: possibility at NA48/NA62

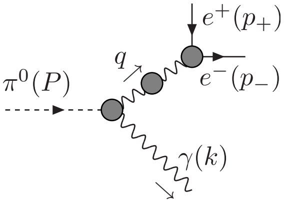
- I am grateful to Evgueni Goudzovski for the following summary
- NA48/2: 330 decays
- relatively large (but well controlled) background ~ 800 events
- rough estimate of error: 15% (c.f. with KTeV precision 5.2%)
- more optimistic situation at NA62
- one can expect larger sample, and smaller background

Dalitz decay

[K.K., Knecht, Novotný '06]

History

- First calculated by [Dalitz '51].
- Radiative corrections studied by [Joseph '60], [Lautrup, Smith'71], [Mikaelian, Smith'72]
- and during the 1980s by Tupper, Grose, Samuel, Lambin, Pestieau, Roberts...



$$x = m_{ee}^2/M_\pi^2, \quad y = \frac{E_+ - E_-}{E_\gamma}\bigg|_{\pi^0 \to 0}$$

NLO studied via $\delta(x,y)$ and $\delta(x)$:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}x\mathrm{d}y} = \delta(x,y)\,\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x\mathrm{d}y}, \quad \frac{\mathrm{d}\Gamma}{\mathrm{d}x} = \delta(x)\,\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x}.$$

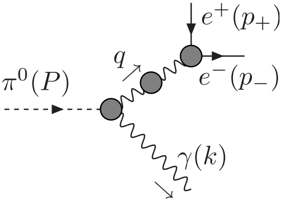
with (point-like pion)

$$\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x\mathrm{d}y} = \frac{\alpha^3}{(4\pi)^4}\frac{M_{\pi^0}}{F_\pi^2}\frac{(1-x)^3}{x^2}\,[M_{\pi^0}^2 x(1+y^2) + 4m^2],$$

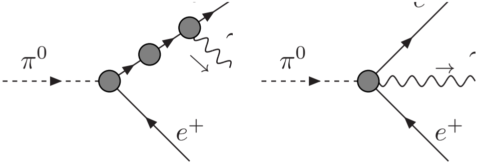
$$\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x} = \frac{\alpha^3}{(4\pi)^4}\frac{8}{3}\frac{M_{\pi^0}}{F_\pi^2}\frac{(1-x)^3}{x^2}\,(xM_{\pi^0}^2 + 2m^2).$$

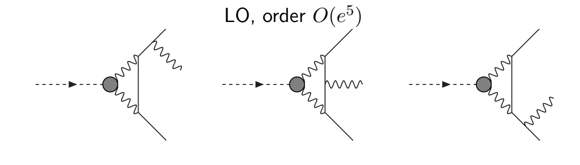
Dalitz decay: Anatomy of the amplitude

- one-photon reducible graphs: electron-positron pair is produced by a single photon (Dalitz pair)

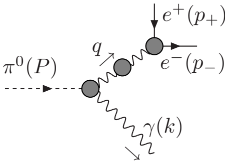


- one-photon irreducible graphs





Dalitz decay: slope parameter



$$\Gamma_\mu^{1\gamma R}(p_+,p_-,k) = \mathrm{i}e^2 \varepsilon_\mu{}^{\nu\alpha\beta} q_\alpha k_\beta \, \mathcal{F}_{\pi^0\gamma\gamma^*}(q^2) \, \mathrm{i}D^T_{\nu\rho}(q)(-\mathrm{i}e)\Lambda^\rho$$

$\mathcal{F}_{\pi^0\gamma\gamma^*}(q^2)$ is related to the doubly off-shell form factor $\mathcal{A}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$

$$\int d^4x \, e^{il\cdot x} \langle 0|T(j^\mu(x) j^\nu(0)|\pi^0(P)\rangle = -\mathrm{i}\varepsilon^{\mu\nu\alpha\beta} l_\alpha P_\beta \, \mathcal{A}_{\pi^0\gamma^*\gamma^*}(l^2, (P-l)^2)$$

One can define a slope parameter a_π

$$\mathcal{F}_{\pi^0\gamma\gamma^*}(q^2) \; = \; \mathcal{F}_{\pi^0\gamma\gamma^*}(0) \left[1 \, + \, a_\pi \, \frac{q^2}{M_{\pi^0}^2} \, + \cdots \right],$$

$$\frac{\mathrm{d}\Gamma^{exp}}{\mathrm{d}x} - \delta_{QED}(x) \frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x} = \frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x} [1 + 2x \, a_\pi].$$

Dalitz decay: Low's theorem

Soft limit due to the Low's theorem, naively:

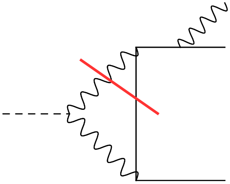
$$\mathcal{M}_{\pi^0\to e^+e^-\gamma} = \mathcal{M}^{\mathsf{Low}} + O(k)$$

$$\mathcal{M}^{\mathsf{Low}} = (s^{(0)} + s^{(1)})P_{\pi^0 e^- e^+}$$

or equivalently (the LO is of the order $O(k)$)

$$\delta^{1\gamma IR}(x,y) = \delta^{\mathsf{Low}}(x,y) + O(1)$$

However, one should be careful. Due to the non-analyticity (the branch-cut of the intermediate $e\gamma$ state starts at m^2)



one should expect the logarithms i.e. →

Dalitz decay: Low's theorem

Soft limit due to the Low's theorem, properly:

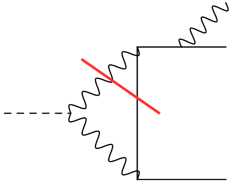
$$\mathcal{M}_{\pi^0 \to e^+ e^- \gamma} = \mathcal{M}^{\mathsf{Low}} + O(k \ln k) + O(k)$$

$$\mathcal{M}^{\mathsf{Low}} = (s^{(0)} + s^{(1)}) P_{\pi^0 e^- e^+}$$

or equivalently (the LO is of the order $O(k)$)

$$\delta^{1\gamma IR}(x,y) = \delta^{\mathsf{Low}}(x,y) + O(\ln(1-x)) + O(1)$$

However, one should be careful. Due to the non-analyticity (the branch-cut of the intermediate $e\gamma$ state starts at m^2)



one should expect the logarithms

for details see [KK,Knecht,Novotny '06]

Dalitz decay: summary of [KK,Knecht,Novotny'06] & [Husek,KK,Novotny'15]

- Our works provide a detailed analysis of NLO radiative corrections to the Dalitz decay amplitude.
- The off-shell pion-photon transition form factor was included:
  this requires a treatment of non perturbative strong interaction effects
- The one-photon irreducible contributions, which had been usually neglected, were included.
  We have shown that, although these contributions are negligible as far as the corrections to the total decay rate are concerned, they are however sizeable in regions of the Dalitz plot which are relevant for the determination of the slope parameter a_π of the pion-photon transition form factor.
- Our prediction for the slope parameter $a_\pi = 0.029 \pm 0.005$ is in good agreement with the determinations obtained from the (model dependent) extrapolation of the CELLO and CLEO data.
  Unfortunately, the experimental error bars on the latest values of a_π extracted from the Dalitz decay are still too large
- used in NA48 analysis for the search of dark photon [1504.00607]

Dalitz decay: one last remark

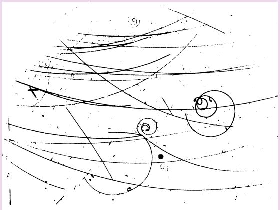
- important for normalization in rare pion and kaon decays
- PDG prediction was stable for 30y (LAMPF '81).
- reason for the change: ALEPH '08 (in principle also KTeV).
- Theoretical prediction is stable and well-understood
- shift in the central value of Dalitz would have an impact in other measurements, eg.

$$K_L \to e^+e^-\gamma = (9.13{\pm}0.26){\times}10^{-6}\Big|_{\text{KTeV+pdg}} \to (8.70{\pm}0.13){\times}10^{-6}\Big|_{\text{KTeV+KTeV}}$$

Double Dalitz decay

History

- determination of parity of pion via $\pi^0 \to \gamma\gamma$[Yang '50], experimentally difficult
- using internal conversion [Kroll, Wada '55]
- First measurement (and todays PDG number) [Samios et al. '62], hydrogen bubble chamber: 8×10^6 π^0-decays on approx 800 thousand pictures $\to$ 200 double-dalitzs (10t. dalitzs)

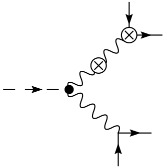


  - $B(\pi^0 \to e^+e^-e^+e^-) = (3.18 \pm 0.3) \times 10^{-5}$
  - π^0 is pseudoscalar (only 3.6σ significance)
- [Miyazaki and E. Takasugi '73] adding the effect of lepton exchange to Kroll-Wada
- new study: [Barker et al. '03] (some disagreement with previous)
- new measurement: KTeV '08
  - confirmation of negative π^0 parity
  - first searches for parity & CPT violation
  - $B(\pi^0 \to e^+e^-e^+e^-) = (3.46 \pm 0.19) \times 10^{-5}$

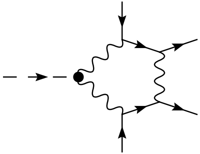
Double Dalitz decay

collaboration with M. Knecht, J. Novotný

It seems natural to convert the on-shell photon to the other Dalitz pair and obtain immediately Double Dalitz decay. This is true for LO:



However, for higher orders we have new topologies [Barker et al. '03]:



We are recalculating these results and try to put them together with our parameters introduced in the context of $\pi^0 \to e^+e^-\gamma$.

$\pi^0 \to Ps\ \gamma$

- decay of pion to positronium and photon, very similar to the Dalitz decay (same particle content)
- Due to binding energy, threshold for Ps is below e^+e^-
- QED+non relativistic bound state formalism Nemenov '72:

$$\frac{\Gamma(\pi^0 \to Ps + \gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} = \frac{\alpha^4}{2} \sum_{n=1}^{\infty} \frac{1}{n^3} \Big(1 + O(\frac{M_{Ps}^2}{m_\pi^2}, \alpha)\Big) \approx \frac{\alpha^4}{2} \zeta(3)$$
$$\approx 1.7 \times 10^{-9}$$

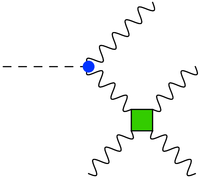
(the sum over all radial excitations of the S-wave $J = 1$ positronium state)

- radiative correction calculated and small Vysotskii '79
- experiment Serpukhov '89 (=pdg)

$$\rho_\pi = (1.84 \pm 0.29) \times 10^{-9}$$

$\pi^0 \to 4\gamma$

- important background for $\pi^0 \to 3\gamma$
- interesting probe of the light-by-light scattering



- theoretical estimate: Bratkovskaya,Kuraev, Silagadze '95

$$\text{Br}(\pi^0 \to 4\gamma) = (2.6 \pm 0.1) \times 10^{-11}$$

- 3 orders below experimental limit, LAMPF '88

$$\text{Br}_{pdg}(\pi^0 \to 4\gamma) < 2 \times 10^{-8}$$

$\pi^0 \to \nu\bar{\nu}$

- strictly speaking, from the beginning *must* be discussed beyond SM
- theoretical calculation, see EW contribution to $\pi^0 \to e^+e^-$

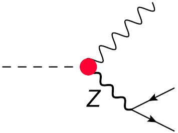
$$\mathcal{A} = \sqrt{2} G_F F_\pi m_\nu \bar{u}\gamma_5 v$$

$$\Rightarrow \quad \mathrm{Br}_{\nu\bar{\nu}} = \Big(\frac{4\pi F_\pi^2 G_F}{\alpha}\Big)^2 \Big(\frac{m_\nu}{M_\pi}\Big)^2 \sqrt{1 - \frac{4m_\nu^2}{M_\pi^2}}$$

- note the maximum for the ratio $m_\nu = M_\pi/\sqrt{6}$
- pdg limit on (tau) $m_\nu < 18.2\,\mathrm{MeV}$ leads to $\mathrm{Br}_{\nu\bar{\nu}} < 5 \times 10^{-10}$
- cosmology limits push it much lower
- exp.limit (E949/pdg): $\sim 10^{-7}$
- possible impact of $\pi^0 \to \nu\bar{\nu}(\gamma)$ in stellar cooling processes of the type $\gamma\gamma \to \nu\bar{\nu}(\gamma)$

$\pi^0 \to \nu\bar{\nu}\gamma$

- "Wolfram" process [Wolfram'76], see however [Arnelos,Marciano,Parsa'82]
- without helicity supression
- represents weak radiative decay



- $\text{Br}_{\nu\bar{\nu}\gamma} \approx 10^{-18}$
- possible test of τ neutrino magnetic moment [Grasso, Lusignoli'92]
- exp. limit (E787) on $\pi^0 \to \gamma +$ "nothing" Br $\lesssim 10^{-4}$

π^0 and new physics

- for more details on new physics $\to$ see next talk
- one crucial ingredient for chiral dynamics: F_π
- F_π from π_{l2} based on SM; deviation from standard $V-A$ leads to an effective $\hat{F}_\pi$ [Bernard,Oertel,Passemar,Stern '08]

$$F_\pi^2 = \hat{F}_\pi^2(1+\epsilon), \quad \text{with} \qquad \epsilon \sim V_R^{ud}/V_L^{ud}$$

- connection between F_π and F_{π^0} tiny [KK,Moussallam '09]

$$\frac{F_{\pi^+}}{F_{\pi^0}}\Big|_{QCD} - 1 = \frac{B^2(m_d-m_u)^2}{F_\pi^4}\left[-16\,c_9^r(\mu) - \frac{l_7}{16\pi^2}\left(1+\log\frac{m_\pi^2}{\mu^2}\right)\right]$$
$$\simeq 0.7\times 10^{-4}\ .$$

- $\Rightarrow$ one can thus use $\pi^0\to\gamma\gamma$ for determination of F_π:

$$F_\pi \approx F_{\pi^0} = 93.85 \pm 1.3(\text{exp.}) \pm 0.6(\text{theory})\,\text{MeV} = 93.85 \pm 1.4\,\text{MeV}$$

- n.b. $\hat{F}_\pi = 92.22(7) \Rightarrow \epsilon \approx 3-4\%$ 1σ significance for right-handed currents

Summary

Radiative and quantum corrections play an essential role for the π^0 processes. Shortly discussed:

- $\pi^0 \to \gamma\gamma$
- $\pi^0 \to e^+e^-$
- $\pi^0 \to e^+e^-\gamma$ (Dalitz decay)
- $\pi^0 \to e^+e^-e^+e^-$ (double Dalitz)
- $\pi^0 \to$ *posit.*γ
- $\pi^0 \to 4\gamma$
- $\pi^0 \to \nu\bar{\nu}(\gamma)$

Interesting possibility for NA62: combination of π^0 and π^+ decays for various scenarios beyond SM

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Thank you.