$\pi \to e^+e^-$ decay and searches of New Physics

Pere Masjuan

Johannes Gutenberg-Universität Mainz

(masjuan@kph.uni-mainz.de)

Work done in collaboration with

Pablo Sanchez-Puertas

[hep-ph:1504.07001; hep-ph:1512.09292]

NA62 Kaon Handbook, Mainz, 20 Jan

Outline

- Shopping list at NA62
- Warm up: short walk through the 3σ puzzle
  - Dissection of $\pi^0 \to e^+e^-$ (unitary bound and transition form factor)
- Current situation: Dubna + Prague + Mainz
  - Relation to HLBL of g-2
  - Relation with K_L decays
- New Physics
- *Shopping list* and Conclusions

Preliminary Shopping list

NA62

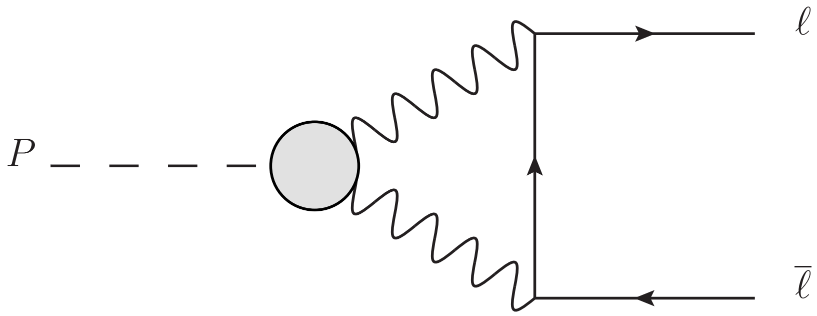
At first, I would suggest

- π^0 Dalitz decays $\pi^0 \to e^+e^-\gamma$ (Inv. mass distr.)
- π^0 Double Dalitz decay $\pi^0 \to e^+e^-\, e^+e^-$ (BR)
- Rare decay $\pi^0 \to e^+e^-$ (BR)
- K Dalitz decay $K_L \to \ell^+\ell^-$ (BR)
- K double Dalitz decay $K_L \to \ell^+\ell^-\gamma$ (Inv. mass distr.)
- Rare decay $K_L \to \ell^+\ell^-\, \ell^+\ell^-$ (BR)

During the talk, I'll try to convince you why

Introduction and Motivation

Experiment



$$\frac{BR(P \to \bar{\ell}\ell)}{BR(P \to \gamma\gamma)} = 2\left(\frac{\alpha m_\ell}{\pi m_P}\right)^2 \beta_\ell(m_P^2)|\mathcal{A}(m_P^2)|^2$$

Introduction and Motivation

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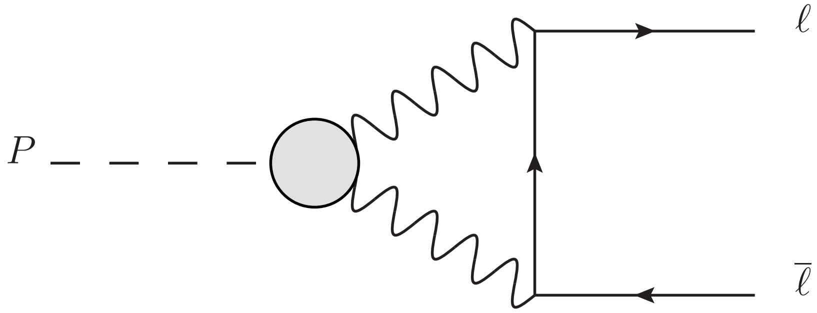


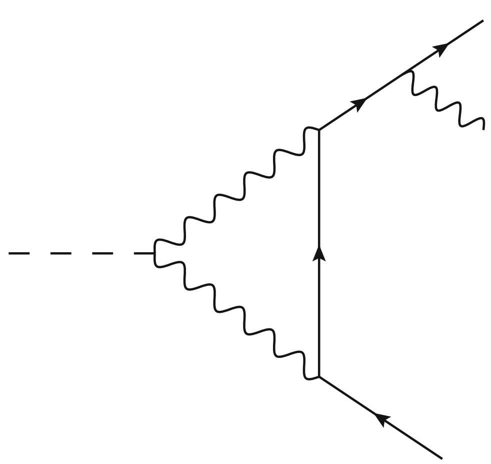
$$\frac{BR(P \to \bar{\ell}\ell)}{BR(P \to \gamma\gamma)} = 2\left(\frac{\alpha m_\ell}{\pi m_P}\right)^2 \beta_\ell(m_P^2)|\mathcal{A}(m_P^2)|^2$$

$$\sim 1.5 \cdot 10^{-10}$$

KTeV '07:

$$BR(\pi^0 \to e^+e^-(\gamma), x > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}$$

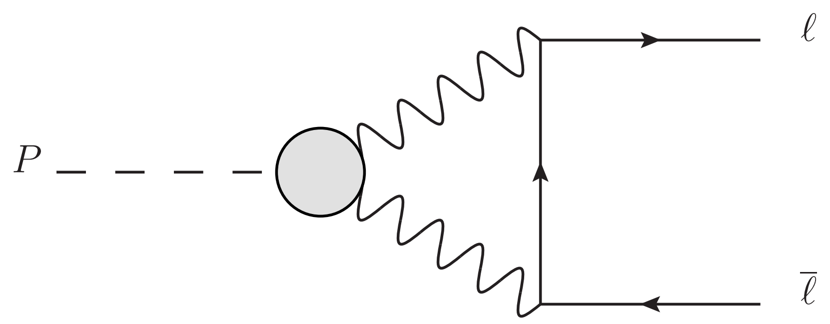
$x \equiv \dfrac{(p+q)^2}{M^2}$ momentum of e+e-, and so to extract BR we want x large, i.e.,no photon



x<0.95 but still is $\pi^0 \to e^+e^-$

Introduction and Motivation

Experiment



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$$\sim 1.5 \cdot 10^{-10}$$

KTeV '07:

$$BR(\pi^0 \to e^+e^-(\gamma), x > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}$$

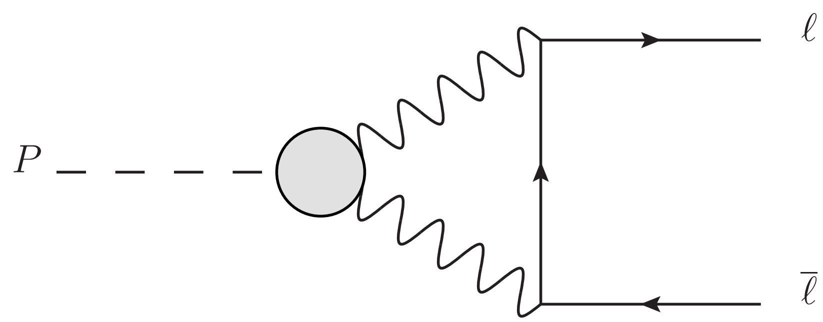
Extrapolation to x=1 + radiative correction + Dalitz decay background

$$\boxed{BR^{w/o\,rad}_{\mathrm{KTeV}}(\pi^0 \to e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}}$$

(dominates de PDG)

Introduction and Motivation

Theory



$$\frac{BR(P \to \bar{\ell}\ell)}{BR(P \to \gamma\gamma)} = 2\left(\frac{\alpha m_\ell}{\pi m_P}\right)^2 \beta_\ell(m_P^2)|\mathcal{A}(m_P^2)|^2$$

The only unknown $\mathcal{A}(m_P^2)$ from loop calculation where the TFF enters.

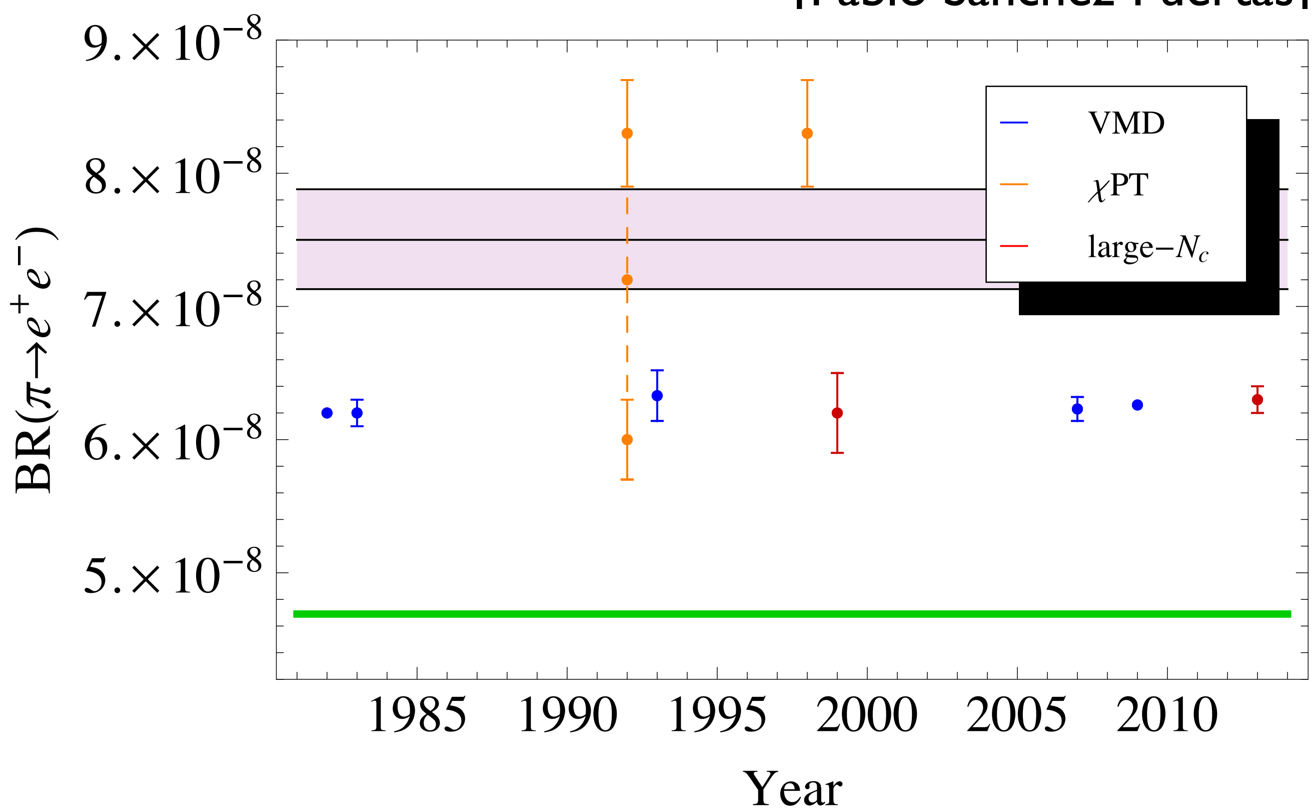
$$\mathcal{A}(q^2) = \frac{2i}{\pi^2}\int d^4k\ \frac{q^2k^2 - (k\cdot q)^2}{k^2(k-q)^2((p-k) - m_\ell^2)}\frac{F_{P\gamma^*\gamma^*}(k^2,(q-k)^2)}{F_{P\gamma\gamma}(0,0)}$$

Introduction and Motivation

Theory

$$\frac{BR(P \to \bar{\ell}\ell)}{BR(P \to \gamma\gamma)} = 2\left(\frac{\alpha m_\ell}{\pi m_P}\right)^2 \beta_\ell(m_P^2)|\mathcal{A}(m_P^2)|^2 = 7.5(5)\cdot 10^{-8}$$

[Pablo Sanchez-Puertas]



[Dorokhov *et al.* '09]

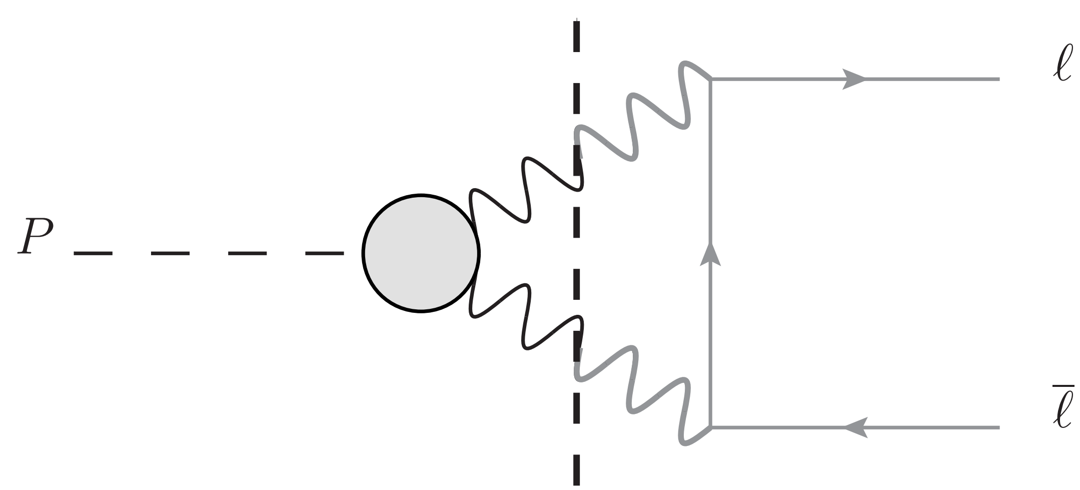
$$BR_{\mathrm{SM}}(\pi^0 \to e^+e^-) = (6.2 \pm 0.1) \times 10^{-8}$$

$\sim 3\sigma$!

Dissection of $\pi^0 \to e^+e^-$

As model independent as possible:

Cutcosky rules provides the imaginary part



Dissection of $\pi^0 \rightarrow e^+e^-$

As model independent as possible:

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$$Im\mathcal{A}(q^2) = \frac{\pi}{2\beta_l(q^2)} ln\left(\frac{1-\beta_l(q^2)}{1+\beta_l(q^2)}\right) ; \quad \beta_l(q^2) = \sqrt{1-\frac{4m_l^2}{q^2}}$$

$$q^2 = m_P^2$$

Assuming $|\mathcal{A}|^2 \geq (\mathrm{Im}\mathcal{A})^2$

$$B(\pi^0 \to e^+e^-) \geq B^{\mathrm{unitary}}(\pi^0 \to e^+e^-) = 4.69 \cdot 10^{-8}$$

(doesn't depend on TFF)

Dissection of $\pi^0 \to e^+e^-$

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$$q^2 = m_P^2$$

Use dispersion relations to get the real part

$$Re(\mathcal{A}(q^2)) = \frac{1}{\pi}\int_0^\infty ds \, \frac{Im(\mathcal{A}(s))}{s-q^2}$$

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Use dispersion relations to get the real part

$$Re(\mathcal{A}(q^2)) = \frac{1}{\pi}\int_0^\infty ds\, \underbrace{\cancel{\frac{Im(\mathcal{A}(s))}{s-q^2}}}_{divergent!} \to \mathcal{A}(0) + \frac{q^2}{\pi}\int_0^\infty \frac{ds}{s}\frac{\mathrm{Im}(\mathcal{A}(s))}{s-q^2}$$

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$$q^2 = m_P^2$$

Use dispersion relations to get the real part

$$Re(\mathcal{A}(q^2)) = \mathcal{A}(0) + \frac{1}{\beta_l(q^2)}\left(\frac{\pi^2}{12} + \frac{1}{4} ln^2\left(\frac{1-\beta_l(q^2)}{1+\beta_l(q^2)}\right) + Li_2\left(\frac{1-\beta_l(q^2)}{1+\beta_l(q^2)}\right)\right)$$

$$\mathcal{A}(0) = \chi_P(\mu) - \frac{5}{2} + \frac{3}{2}\ln(m_l^2/\mu^2)$$

Dissection of $\pi^0 \to e^+e^-$

As model independent as possible:

Cutcosky rules provides the imaginary part

$$Im\mathcal{A}(q^2) = \frac{\pi}{2\beta_l(q^2)} ln\left(\frac{1-\beta_l(q^2)}{1+\beta_l(q^2)}\right) ; \quad \beta_l(q^2) = \sqrt{1-\frac{4m_l^2}{q^2}}$$

$$q^2 = m_P^2$$

Use dispersion relations to get the real part

$$Re(\mathcal{A}(m_P^2)) = \left(-\frac{5}{4} + \int_0^\infty dQ^2 \ Kernel(Q^2)\right) + \frac{\pi^2}{12} + ln^2\left(\frac{m_l}{m_P}\right)$$

$\mathcal{O}\left(\frac{m_e}{m_\pi}\right)^2$ + subtraction contains all the information from TFF

Dissection of $\pi^0 \to e^+e^-$

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$$Re(\mathcal{A}(m_P^2)) = \int_0^\infty dQ^2 Kernel(Q^2) + 30.7$$

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$$Im(\mathcal{A}(m_P^2)) \sim 17.5 \qquad Re(\mathcal{A}(m_P^2)) \sim 30.7$$

(Kernel=0)

$$\frac{BR(P \to \bar{\ell}\ell)}{BR(P \to \gamma\gamma)} = 2\left(\frac{\alpha m_\ell}{\pi m_P}\right)^2 \beta_\ell(m_P^2)|\mathcal{A}(m_P^2)|^2 = 19 \cdot 10^{-8}$$

$$\sim 1.5 \cdot 10^{-10}$$

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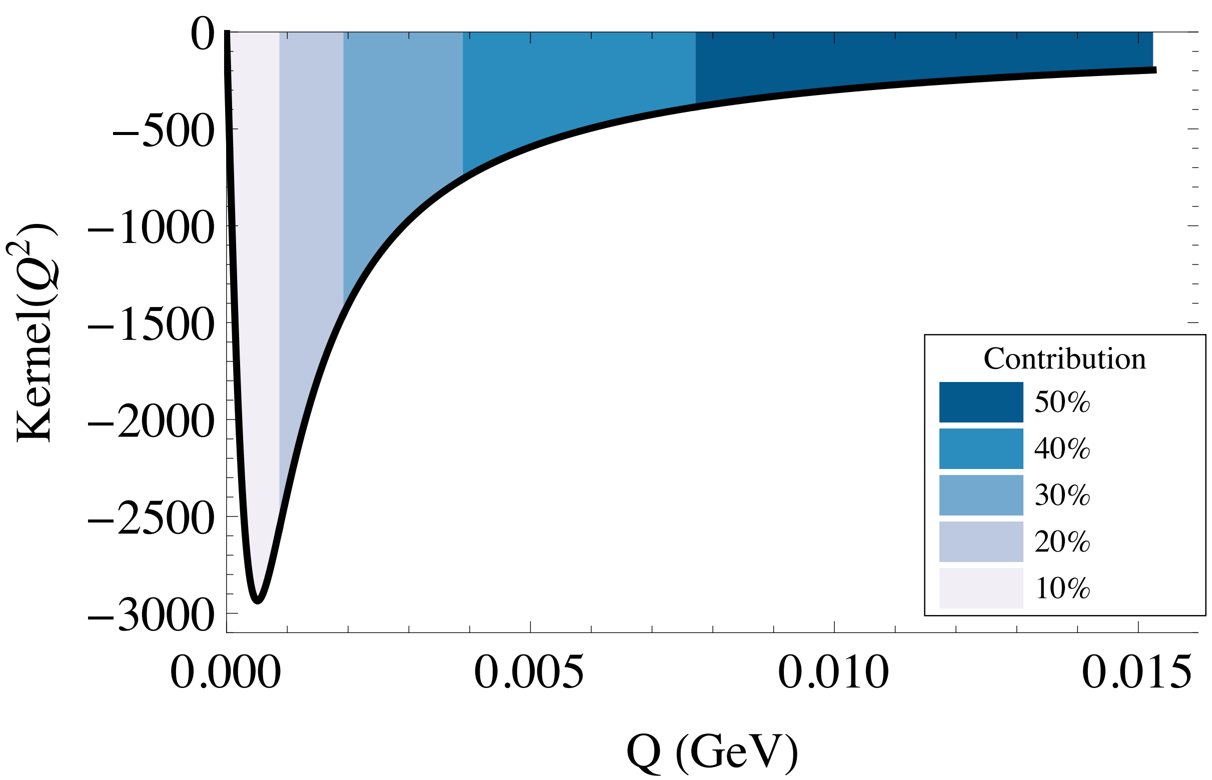
$$\int_0^\infty dQ^2 Kernel(Q^2) \sim -17 \to KTeV \sim 7.5 \cdot 10^{-8}$$

Dissection of $\pi^0 \to e^+e^-$

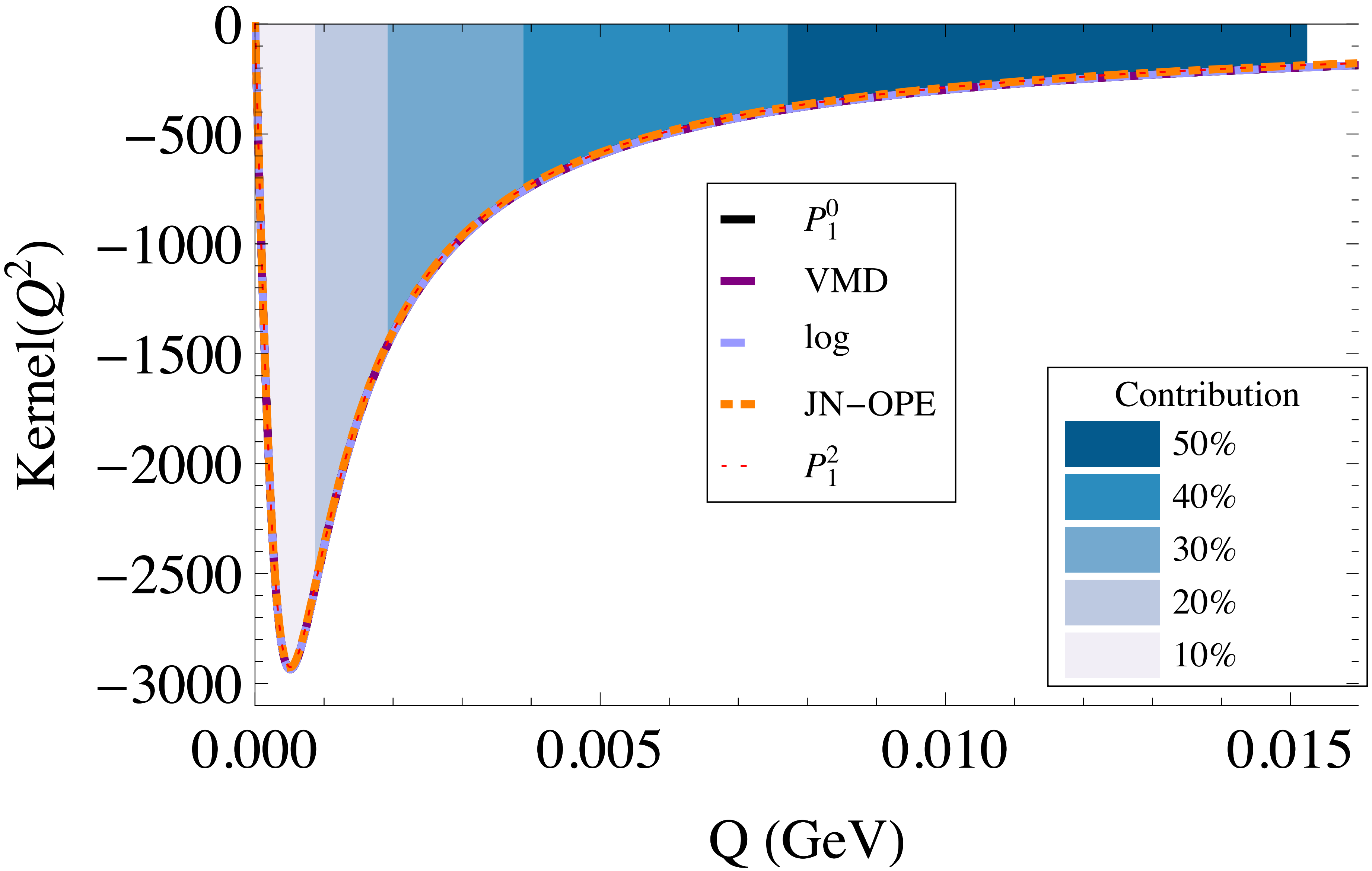
$$Re(\mathcal{A}(m_P^2)) = \int_0^\infty dQ^2 Kernel(Q^2) + 30.7$$



- Its contribution is negative: lowers the BR.
- Peaks at $\sim 2m_e$ and $\langle Q \rangle = 0.09$ GeV.
- Low energies relevant only: slope is enough.

Dissection of π^0→e$^+$e$^-$

$$Re(\mathcal{A}(m_P^2)) = \int_0^\infty dQ^2 Kernel(Q^2) + 30.7$$



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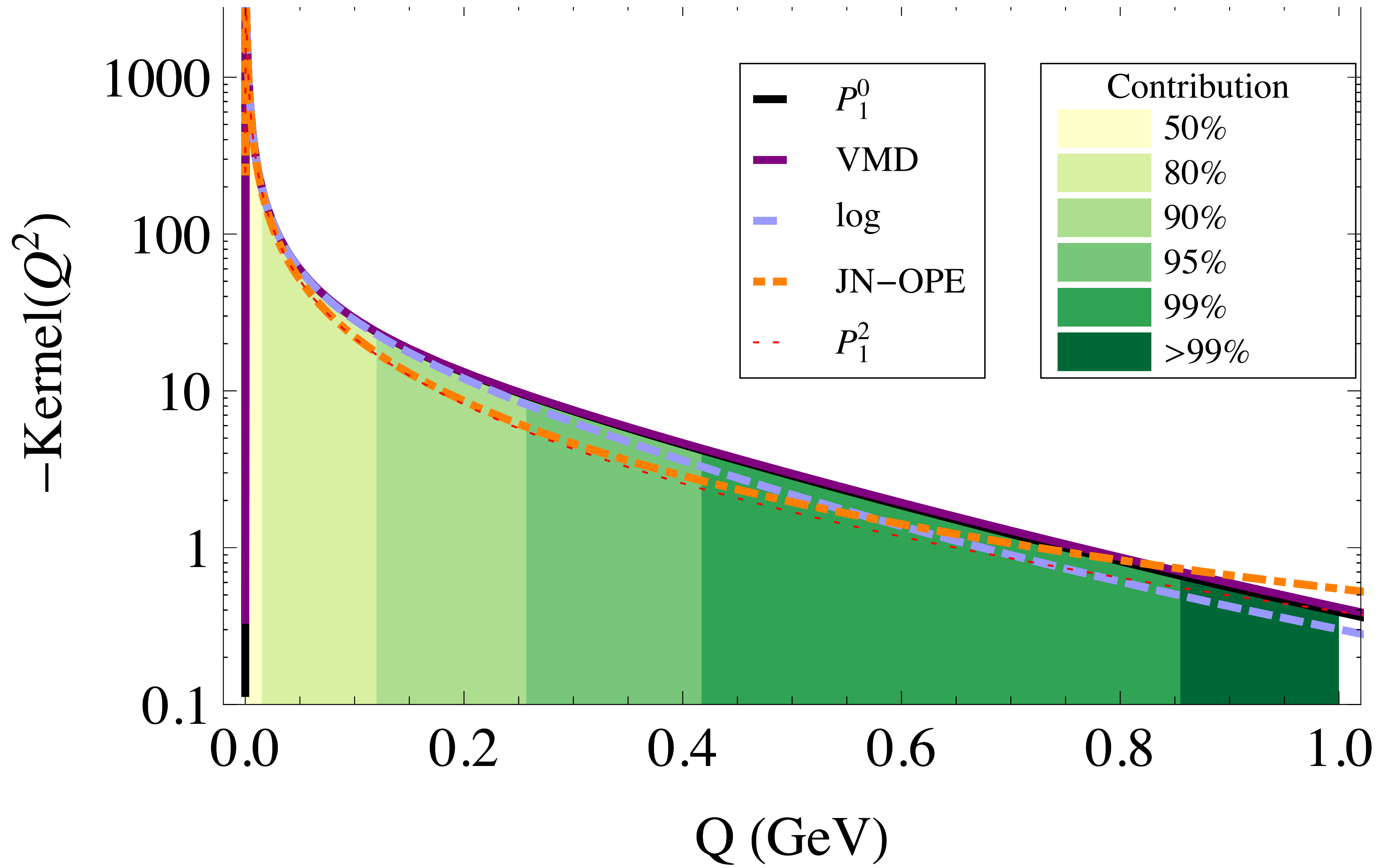

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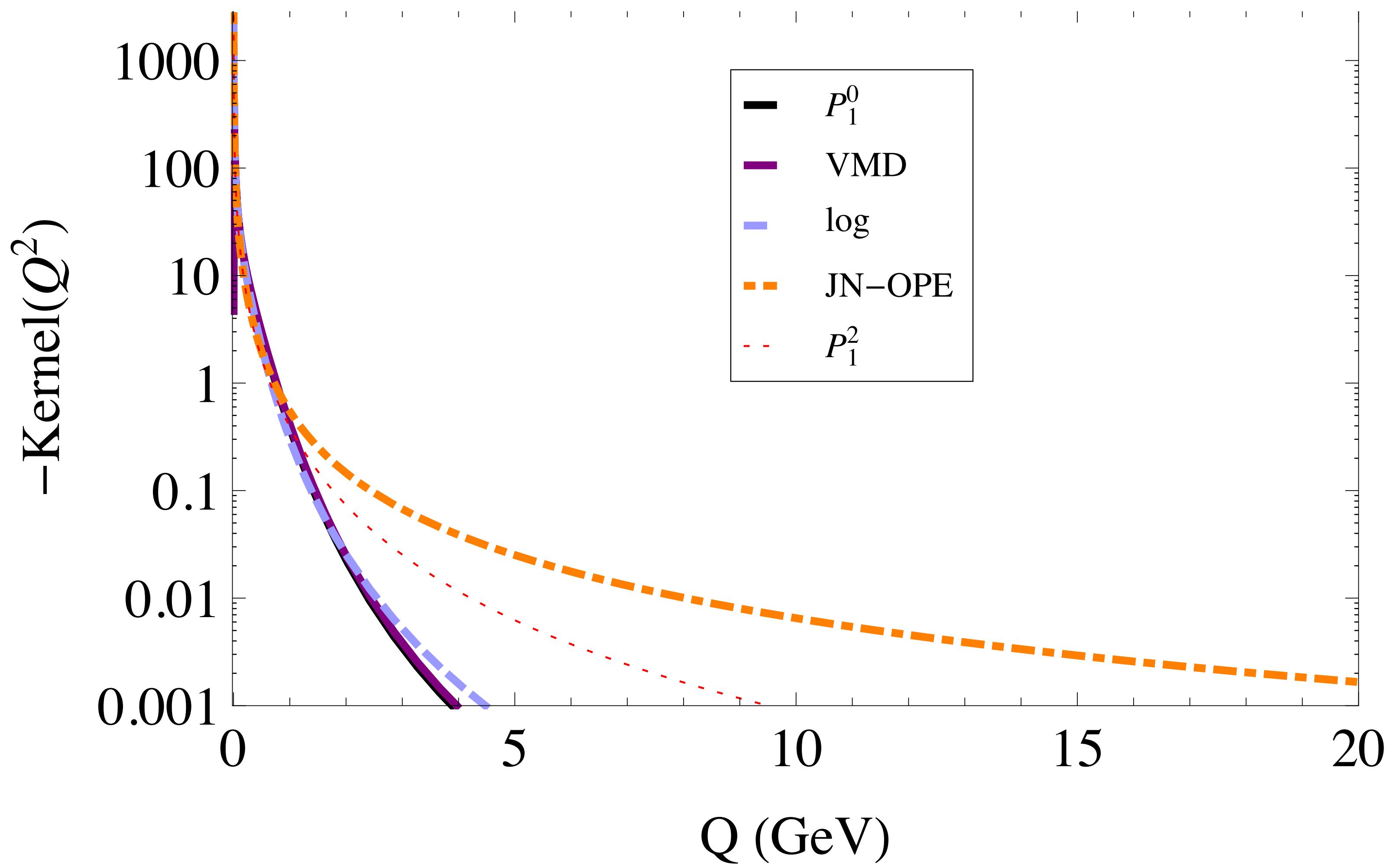
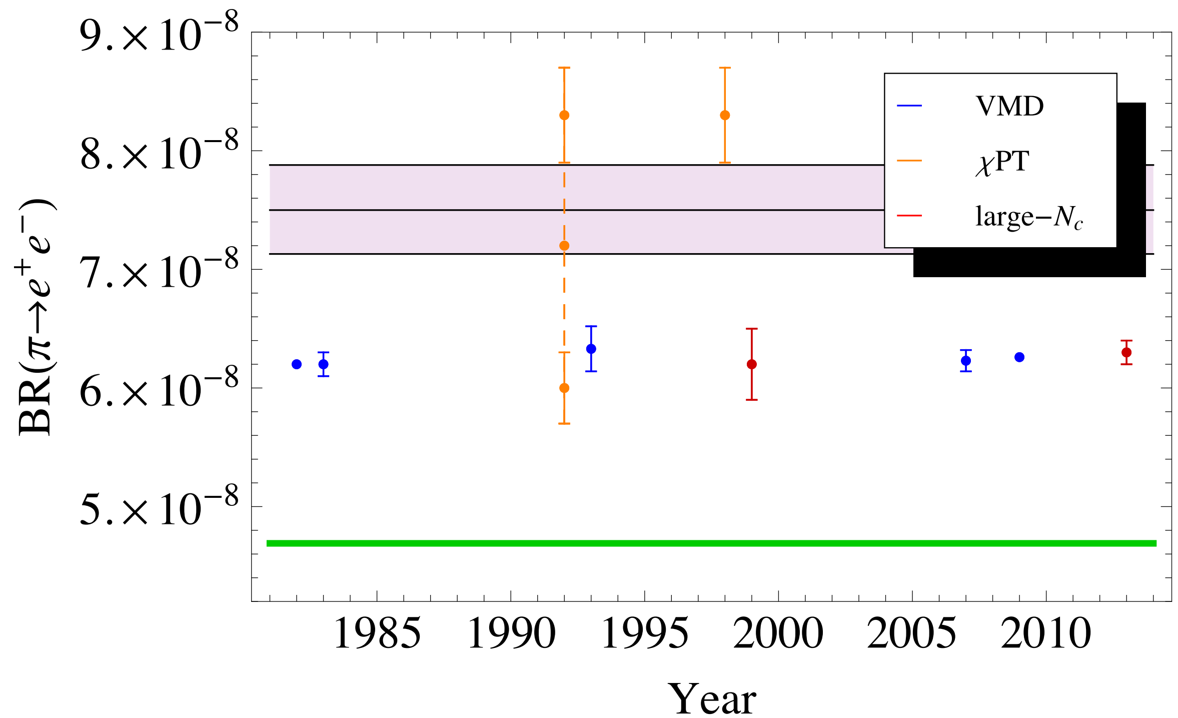


Dissection of $\pi^0 \to e^+e^-$

$$Re(\mathcal{A}(m_P^2)) = \int_0^\infty dQ^2 Kernel(Q^2) + 30.7$$

all in all, old the models give the same value

$$\int_0^\infty dQ^2 Kernel(Q^2) \sim -20 \to BR \sim 6.3 \cdot 10^{-8}$$



[Babu and Ma, '82]
[Bergstrom *et al*, '83]
[Savage *et al*, '92]
[Ametller *et al*, '93]
[Gomez Dumm and Pich, '98]
[Knecht *et al*, '99]
[Dorokhov *et al*, '07'09]
[PM, Sanchez-Puertas '15]

Current situation

Dubna+Prague+Mainz(?)

- Ways to improve from theory side:
  - Dubna (Dorokhov, Ivanov,...): Include all kind of corrections m_e/m_π, m_e/Λ (which also means not using DR)
  - Prague (Novotny, Kampf, Husek...): Improve on radiative corrections
  - Mainz (Masjuan, Sanchez-Puertas...): Improve on the implementation of the TFF
    - Consider New Physics contributions

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Dubna contribution: corrections m_e/m_π, m_e/Λ

Dorokhov and Ivanov, '07

$$\mathcal{O}\left(\frac{m_e}{m_\pi}\right)^2$$

Used VMD to confront KTeV measurement
(also compare different models for TFF)

$$F_{\pi\gamma^*\gamma^*}(Q^2, Q^2) = F\pi\gamma\gamma(0,0)\frac{1}{1+Q^2/Q_0^2}$$

with Q_0 from a monopole fit to CLEO+CELLO data

Dubna contribution: corrections m_e/m_π, m_e/Λ

Dorokhov and Ivanov, '08

$$\mathcal{O}\left(\frac{m_e}{\Lambda}\right)^2 \qquad \mathcal{O}\left(\frac{m_e}{\Lambda}\log\frac{m_e}{\Lambda}\right)^2$$

Λ
the cut-off
or
VMD "mass"

Dorokhov, Ivanov and Kovalenko '09

$$\mathcal{O}\left(\frac{m_\pi}{\Lambda}\right)^2 \qquad \mathcal{O}\left(\frac{m_e}{m_\pi}\right)^2$$

Resummation of power corrections using Mellin-Barnes techniques.
Conclusion: corrections negligible!

$$\boxed{BR_{\mathrm{SM}}(\pi^0 \to e^+e^-) = (6.2 \pm 0.1) \times 10^{-8} \qquad \sim 3\sigma}$$

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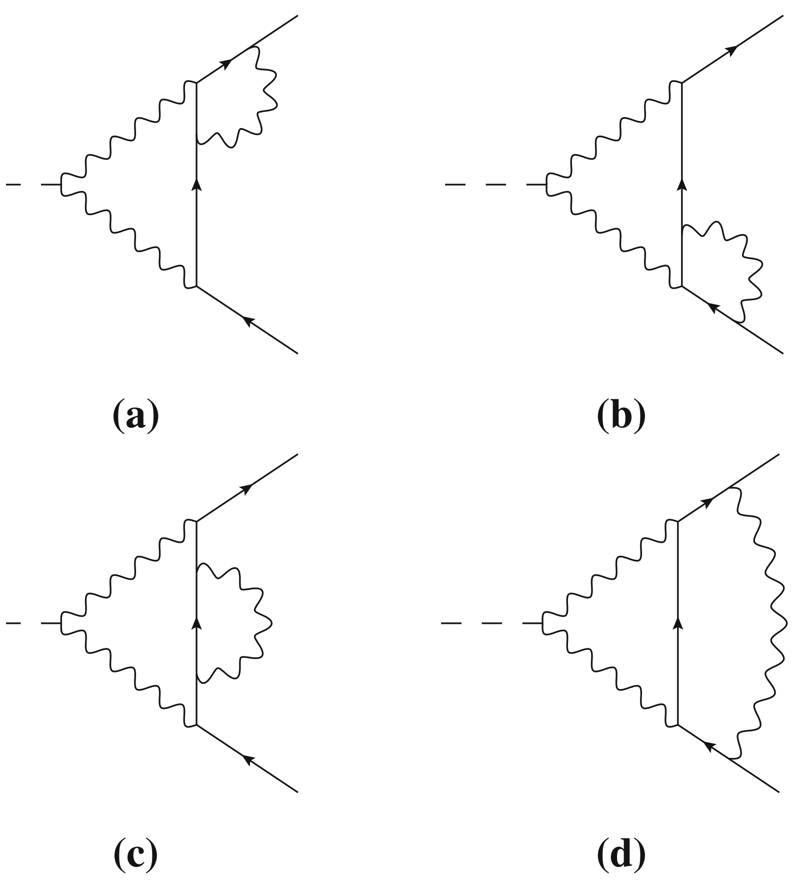
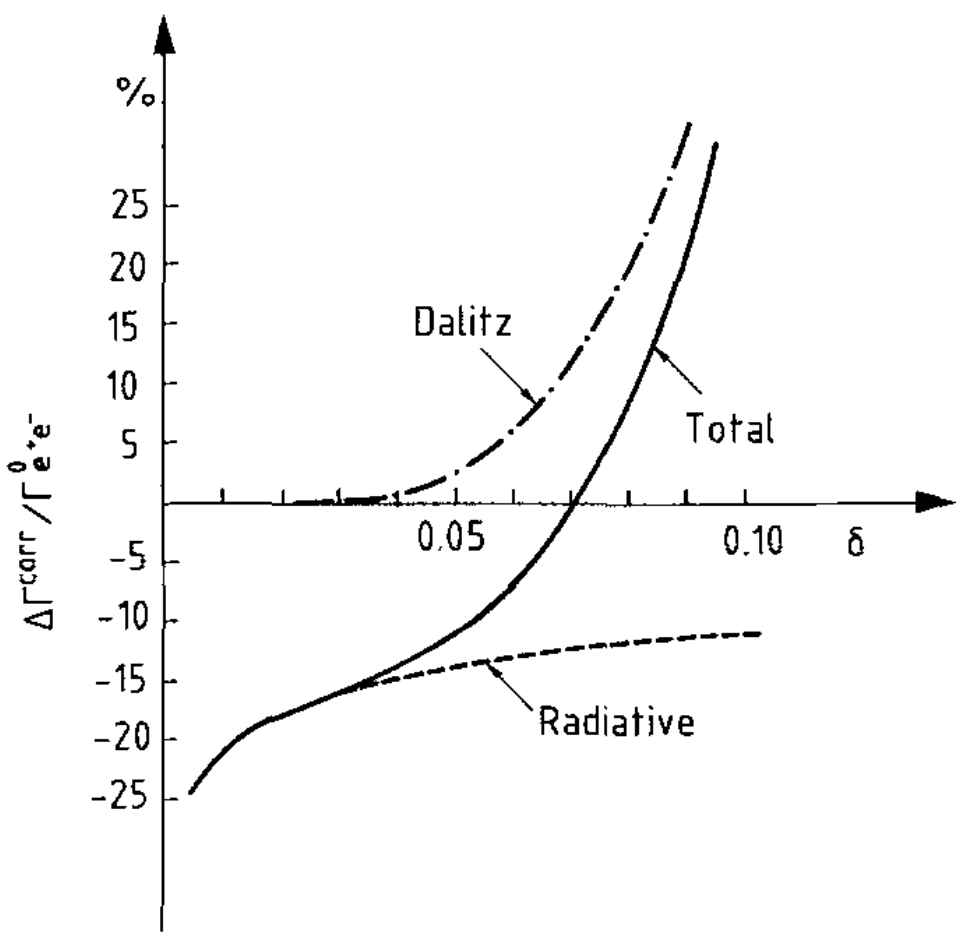
Prague contribution: Radiative corrections

Before Prague:

Bergstrom '83: approach (soft-photon+cut-off) to two-loop QED radiative correction + Dalitz decay interference





$\delta = 1 - x$

Correction: $\sim -13\%$

Prague contribution: Radiative corrections

Vasko, Novotny '11 + Husek, Kampf, Novotny'14

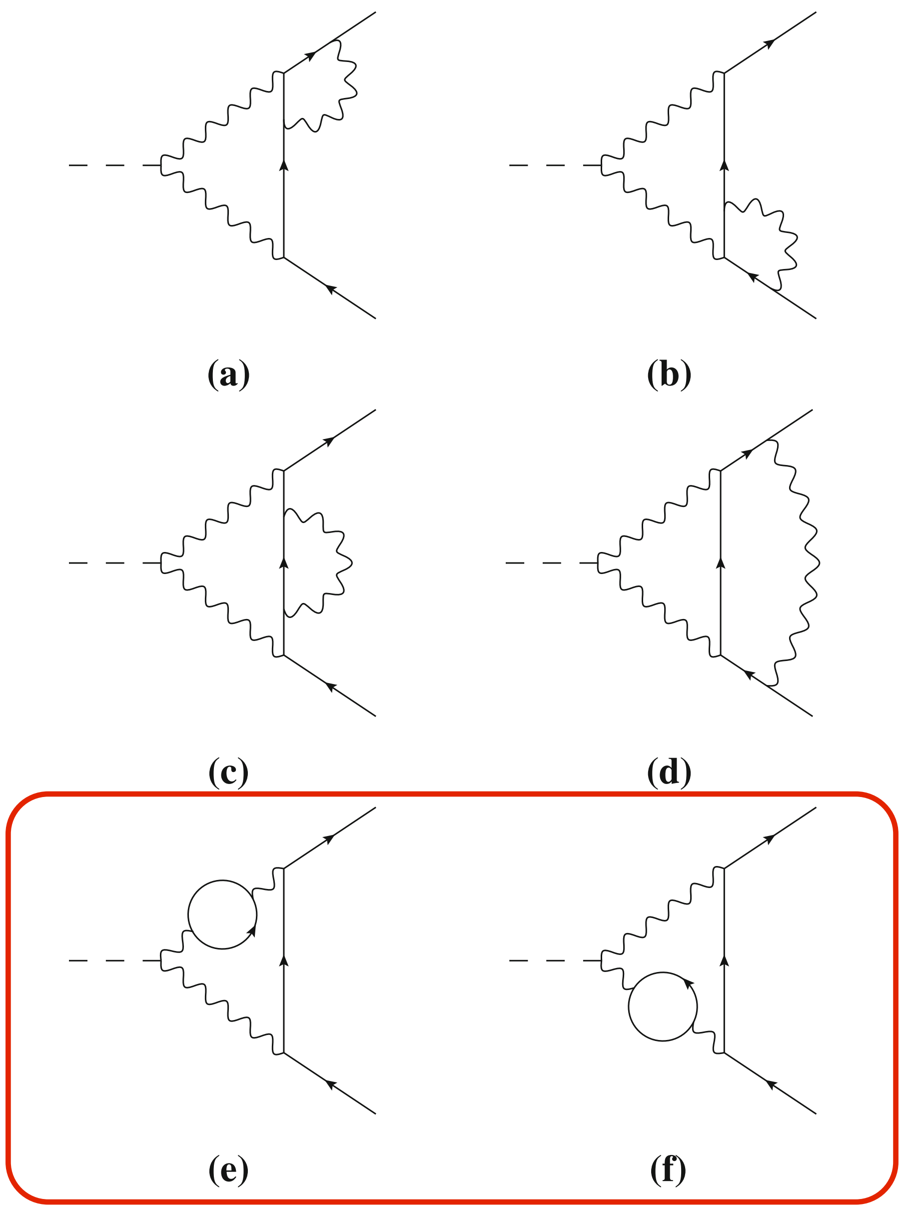


Fig. 2 Two-loop virtual radiative corrections for $\pi^0 \to e^+e^-$ process

Include more diagrams which are subleading but numerically important

Prague contribution: Radiative corrections

Vasko, Novotny '11 + Husek, Kampf, Novotny'14

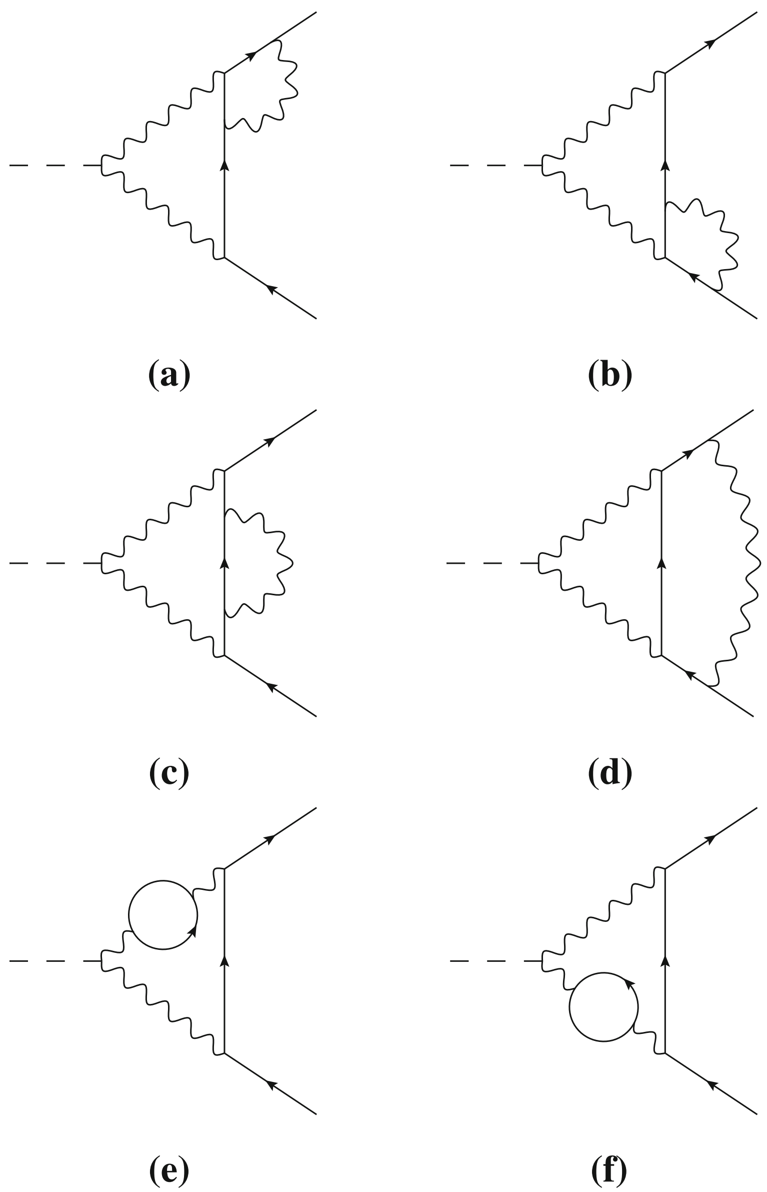
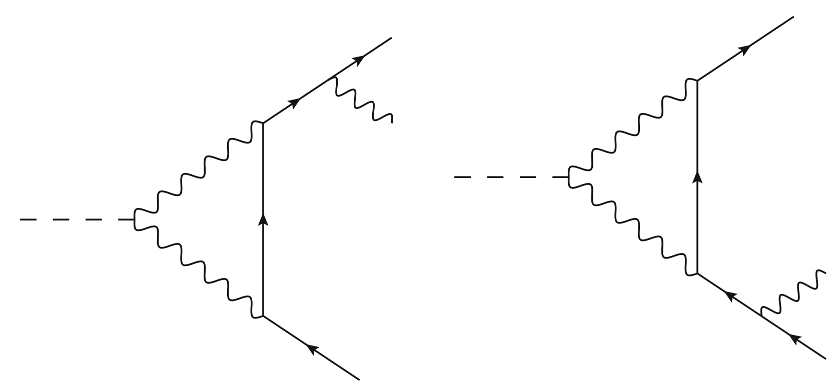


Fig. 2 Two-loop virtual radiative corrections for $\pi^0 \to e^+e^-$ process

Calculate the Bremsshtralung in the soft-photon limit



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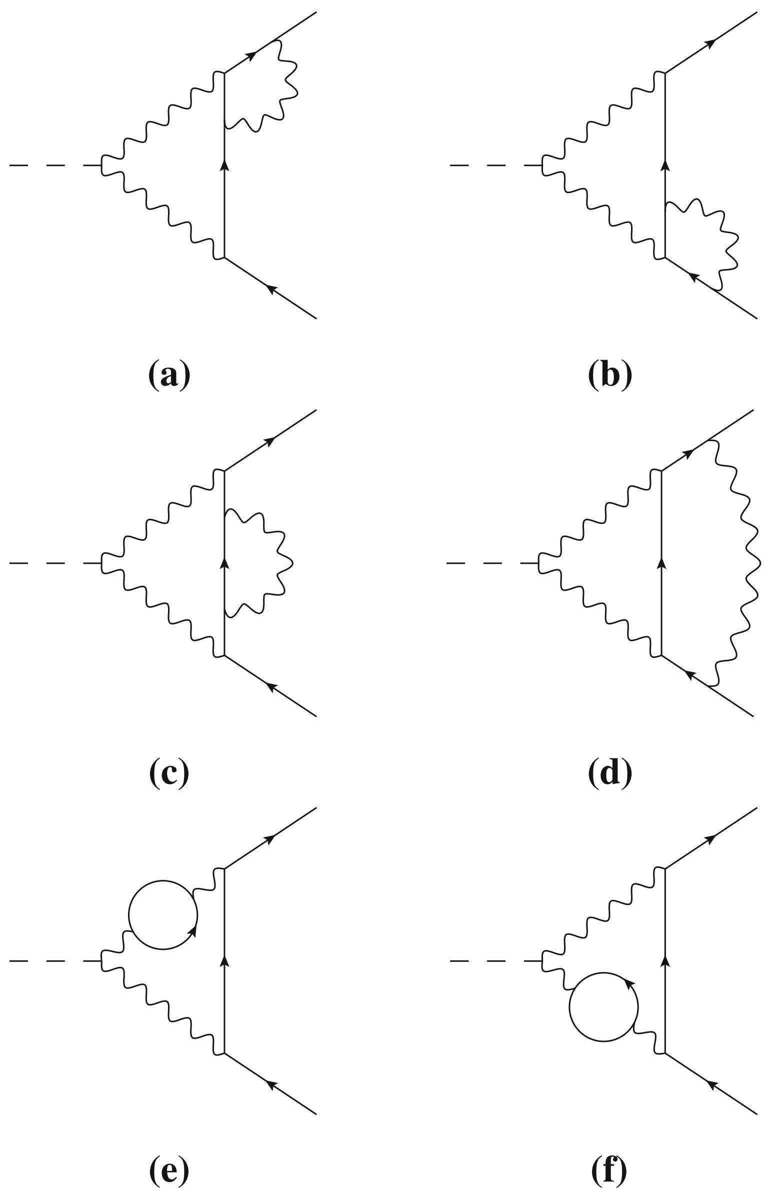
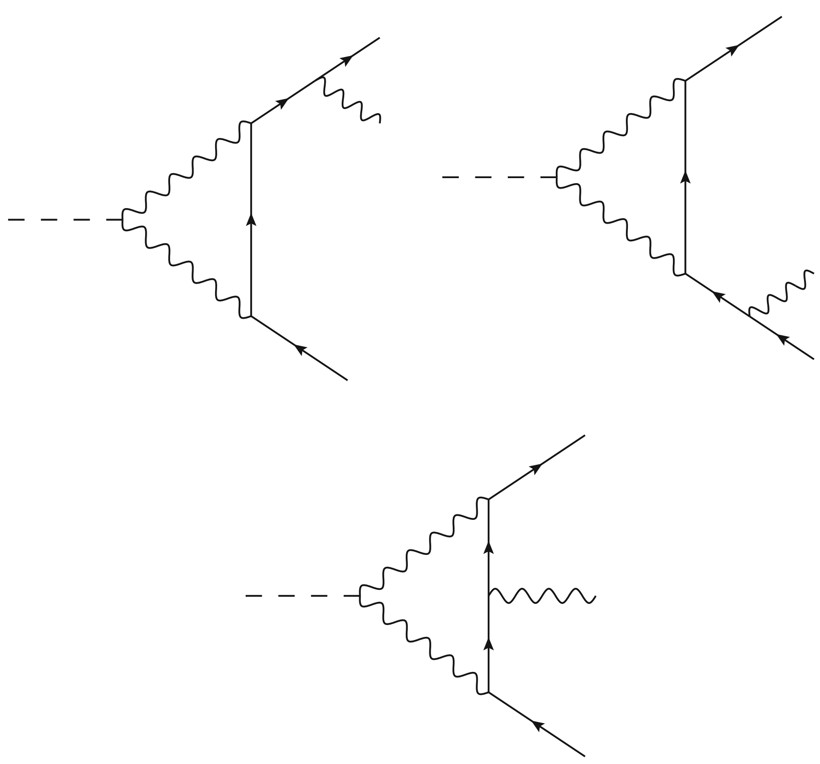


Fig. 2 Two-loop virtual radiative corrections for $\pi^0 \to e^+e^-$ process

Calculate the Bremsshtralung without the soft-photon limit



Prague contribution: Radiative corrections

Vasko, Novotny '11 + Husek, Kampf, Novotny'14

$$\frac{\mathrm{BR}(\pi^0 \to e^+e^-(\gamma), x > 0.95)}{\mathrm{BR}(\pi^0 \to \gamma\gamma)} =$$

$$\frac{\Gamma(\pi^0 \to e^+e^-)}{\Gamma(\pi^0 \to \gamma\gamma)} \left[1 + \delta^{(2)}(0.95) + \Delta^{BS}(0.95) + \delta^{D}(0.95)\right]$$

$\delta^{(2)}(0.95) \equiv \delta^{\text{virt.}} + \delta^{\text{BS}}_{\text{soft}}(0.95)$ complete QED two-loop corr. including soft-photon BS

$\Delta^{\text{BS}}(x^{\text{cut}}) \equiv \delta^{\text{BS}}(x^{\text{cut}}) - \delta^{\text{BS}}_{\text{soft}}(x^{\text{cut}})$ soft-photon correction

$\delta^{D}(0.95)$ Dalitz decay background (omitted in KTeV)

Prague contribution: Radiative corrections

Vasko, Novotny '11 + Husek, Kampf, Novotny'14

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$$\frac{\Gamma(\pi^0 \to e^+e^-)}{\Gamma(\pi^0 \to \gamma\gamma)} [1 + \delta^{(2)}(0.95) + \Delta^{BS}(0.95) + \delta^{D}(0.95)]$$

$$\delta^{(2)}(0.95) \equiv \delta^{\mathrm{virt.}} + \delta^{\mathrm{BS}}_{\mathrm{soft}}(0.95) = (-5.8 \pm 0.2)\,\% \qquad \mathrm{vs} \quad \sim -13\%$$

$$\Delta^{\mathrm{BS}}(0.95) = (0.30 \pm 0.01)\,\% \qquad \delta^{D}(0.95) = \frac{1.75 \times 10^{-15}}{[\Gamma^{\mathrm{LO}}(\pi^0 \to e^+e^-)/\mathrm{MeV}]}$$

$$\boxed{BR^{w/o\,rad}_{\text{"KTeV"}}(\pi^0 \to e^+e^-) = (6.87 \pm 0.36) \times 10^{-8}}$$

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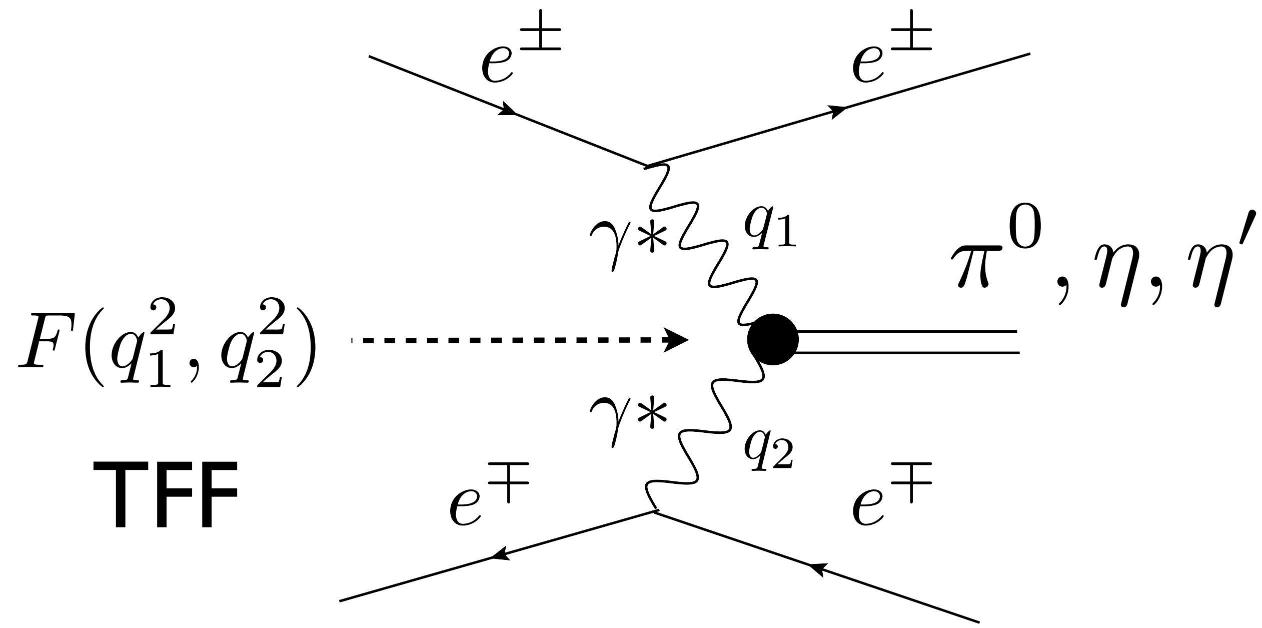
Mainz contribution: TFF parameterization

Use data from
the Transition Form Factor
for numerical integral

$$F_{P\gamma^*\gamma^*}(m_P^2, q_1^2, q_2^2)$$

double-tag method



Remember: only low-energy region is needed

Mainz contribution: TFF parameterization

~~Use data from the Transition Form Factor for numerical integral~~

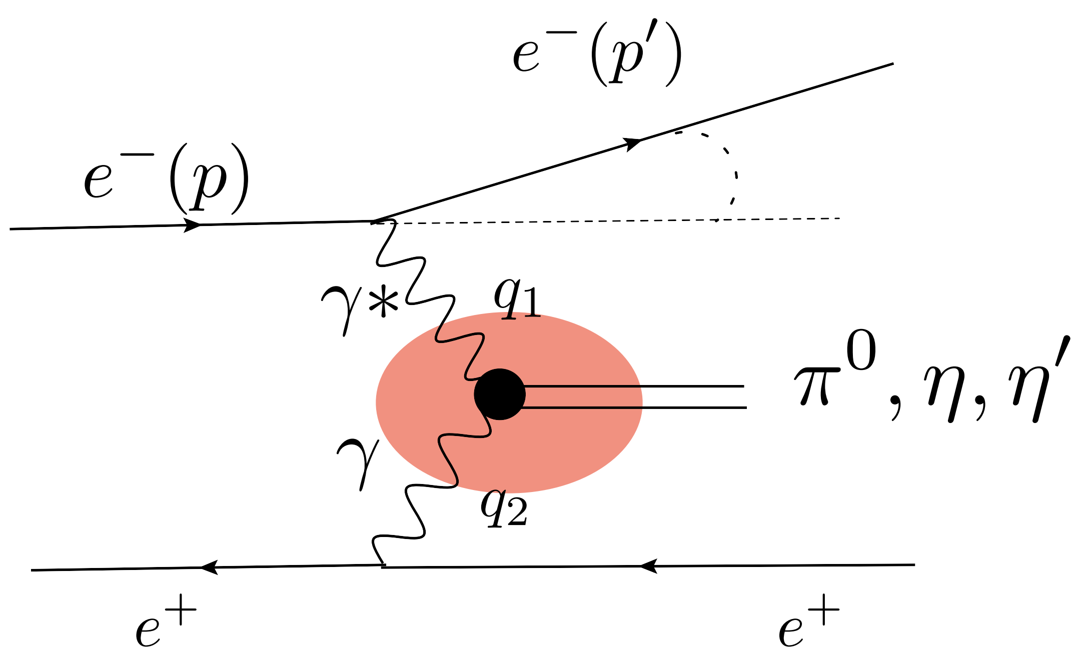
$$\cancel{F_{P\gamma^*\gamma^*}(m_P^2, q_1^2, q_2^2)}$$

double-tag method

Use data from the Transition Form Factor to constrain your hadronic model

$$\boxed{F_{P\gamma^*\gamma}(m_P^2, q_1^2, 0)}$$

single-tag method



Mainz contribution: TFF parameterization

~~Use data from the Transition Form Factor for numerical integral~~

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double-tag method

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$$\boxed{F_{P\gamma^*\gamma}(m_P^2, q_1^2, 0)}$$

single-tag method

How??

Nice synergy between experiment and theory

Doubly virtual π⁰-TFF

[P.M., P. Sanchez-Puertas, '15]

For $BR_{SM}(\pi^0 \to e^+e^-)$ we need $F_{\pi^0\gamma^*\gamma^*}(Q^2, Q^2)$

Proposal: bivariate PA Chisholm '73

$$P_M^N(Q_1^2, Q_2^2) = \frac{T_N(Q_1^2, Q_2^2)}{R_M(Q_1^2, Q_2^2)} = a_0 + a_1(Q_1^2 + Q_2^2) + a_{1,1}Q_1^2Q_2^2 + a_2(Q_1^4 + Q_2^4) + \cdots$$

$$P_1^0(Q_1^2, Q_2^2) = \frac{a_0}{1 + a_1(Q_1^2 + Q_2^2) + (2a_1^2 - a_{1,1})Q_1^2Q_2^2}$$

$$P_2^1(Q_1^2, Q_2^2) = \frac{a_0 + a_1(Q_1^2 + Q_2^2) + a_{1,1}Q_1^2Q_2^2}{1 + b_1(Q_1^2 + Q_2^2) + b_{1,1}Q_1^2Q_2^2 + b_2(Q_1^4 + Q_2^4) + b_{2,1}(Q_1^4Q_2^2 + Q_1^2Q_2^4) + b_{2,2}Q_1^4Q_2^4}$$

[convergence pattern]

Doubly virtual π^0-TFF

Chisholm '73

Proposal: bivariate PA

$$P_1^0(Q_1^2, Q_2^2) = \frac{a_0}{1 + a_1(Q_1^2 + Q_2^2) + (2a_1^2 - a_{1,1})Q_1^2 Q_2^2}$$

a_1 from accurate study of space-like data [PM. '12]

$a_{1,1}$ from a systematic fit to doubly virtual SL data

Doubly virtual π^0-TFF

Chisholm '73

Proposal: bivariate PA

$$P_1^0(Q_1^2, Q_2^2) = \frac{a_0}{1 + a_1(Q_1^2 + Q_2^2) + (2a_1^2 - a_{1,1})Q_1^2 Q_2^2}$$

a_1 from accurate study of space-like data [PM. '12]

$a_{1,1}$ from a systematic fit to doubly virtual SL data

OPE indicates: $\lim_{Q^2\to\infty} P_1^0(Q^2, Q^2) \sim Q^{-2}$ i.e., $a_{1,1} = 2a_1^2$

Doubly virtual π^0-TFF

Chisholm '73

Proposal: bivariate PA

$$P_1^0(Q_1^2, Q_2^2) = \frac{a_0}{1 + a_1(Q_1^2 + Q_2^2) + (2a_1^2 - a_{1,1})Q_1^2 Q_2^2}$$

a_1 from accurate study of space-like data [PM. '12]

$a_{1,1}$ from a systematic fit to doubly virtual SL data

ChPT indicates: $P_1^0(Q_1^2, Q_2^2) = P_1^0(Q_1^2) \times P_1^0(Q_2^2)$ i.e., $a_{1,1} = a_1^2$

[Bijnens, Kampf, Lanz '12]

Doubly virtual π^0-TFF

Chisholm '73

Proposal: bivariate PA

$$P_1^0(Q_1^2, Q_2^2) = \frac{a_0}{1 + a_1(Q_1^2 + Q_2^2) + (2a_1^2 - a_{1,1})Q_1^2 Q_2^2}$$

a_1 from accurate study of space-like data [PM. '12]

$$0 \leq a_{1,1} \leq 2a_1^2$$

$$BR_{SM}^{PA}(\pi^0 \to e^+e^-) = (6.22 - 6.43)(4) \times 10^{-8}$$

statistics+theoretical error

$$BR_{SM}^{Z}(\pi^0 \to Z^* \to e^+e^-) = -0.02 \times 10^{-8}$$

Doubly virtual π⁰-TFF

Chisholm '73

Proposal: bivariate PA

$$P_2^1(Q_1^2,Q_2^2) = \frac{a_0 + a_1(Q_1^2+Q_2^2) + a_{1,1}Q_1^2Q_2^2}{1 + b_1(Q_1^2+Q_2^2) + b_{1,1}Q_1^2Q_2^2 + b_2(Q_1^4+Q_2^4) + b_{2,1}(Q_1^4Q_2^2+Q_1^2Q_2^4) + b_{2,2}Q_1^4Q_2^4}$$

a_1, b_1, b_2 from accurate study of space-like data [PM.'12]

$b_{2,2}=0$ $\quad \lim_{Q^2\to\infty} P_1^0(Q^2,Q^2) \sim Q^{-2}$

$b_{1,1}, b_{2,1}$ $\quad \lim_{Q^2\to\infty} P_2^1(Q^2,Q^2) \sim \frac{2F_\pi}{3}\left(\frac{1}{Q^2} - \frac{8\delta^2}{9Q^4} + \mathcal{O}(Q^{-6})\right)$ [Novikov et al '84]

$a_{1,1}$ free (condition: no poles in space like) $1.92a_1^2 \leq a_{1,1} \leq 2a_1^2$

$$BR_{SM}^{P_2^1}(\pi^0 \to e^+e^-) = (6.23 - 6.24)(4) \times 10^{-8}$$

statistics+theoretical error

(Z included)

Doubly virtual π^0-TFF

Chisholm '73

Proposal: bivariate PA

Systematic error $P_1^0(Q_1^2,Q_2^2)$ vs $P_2^1(Q_1^2,Q_2^2)$

$$BR_{SM}^{P_2^1}(\pi^0 \to e^+e^-) = 6.23(4)(2) \times 10^{-8}$$

(Z included)

statistics+theoretical error

method checked for different models
+ to shrink the window: data (data-driven approach)

Doubly virtual π^0-TFF

Chisholm '73

Proposal: bivariate PA

New SM prediction:

statistics+theoretical error

$$\boxed{BR^{P^1_2}_{SM}(\pi^0 \to e^+e^-) = 6.23(4)(2) \times 10^{-8}}$$

(Z included)

- More precise (50% error reduction)
- Data driven
- Systematic error
- Improved loop integral

$$\chi(\mu) = 2.5(2)$$

[Dorokhov *et al.* '09]

$$BR_{\rm SM}(\pi^0 \to e^+e^-) = (6.2 \pm 0.1) \times 10^{-8}$$

Doubly virtual π^0-TFF

$$BR^{w/o\,rad}_{\text{"KTeV"}}(\pi^0 \to e^+e^-) = (6.87 \pm 0.36) \times 10^{-8}$$

$$\Updownarrow \qquad \sim 2\sigma$$

$$BR^{P^1_2}_{SM}(\pi^0 \to e^+e^-) = 6.23(4)(2) \times 10^{-8}$$

(with published KTeV ~3.2σ)

Doubly virtual π^0-TFF

Can we still match the KTeV value?

$$P_2^1(Q_1^2,Q_2^2) = \frac{a_0 + a_1(Q_1^2+Q_2^2) + a_{1,1}Q_1^2Q_2^2}{1 + b_1(Q_1^2+Q_2^2) + b_{1,1}Q_1^2Q_2^2 + b_2(Q_1^4+Q_2^4) + b_{2,1}(Q_1^4Q_2^2+Q_1^2Q_2^4) + b_{2,2}Q_1^4Q_2^4}$$

a_1, b_1, b_2 from accurate study of space-like data [PM.'12]

$b_{2,2}=0$ $\quad \lim_{Q^2\to\infty} P_1^0(Q^2,Q^2) \sim Q^{-2}$

$b_{1,1}, b_{2,1}$ $\quad \lim_{Q^2\to\infty} P_2^1(Q^2,Q^2) \sim \frac{2F_\pi}{3}\left(\frac{1}{Q^2} - \frac{8\delta^2}{9Q^4} + \mathcal{O}(Q^{-6})\right)$

$a_{1,1}$ free to fix KTeV (condition: no poles in space like)

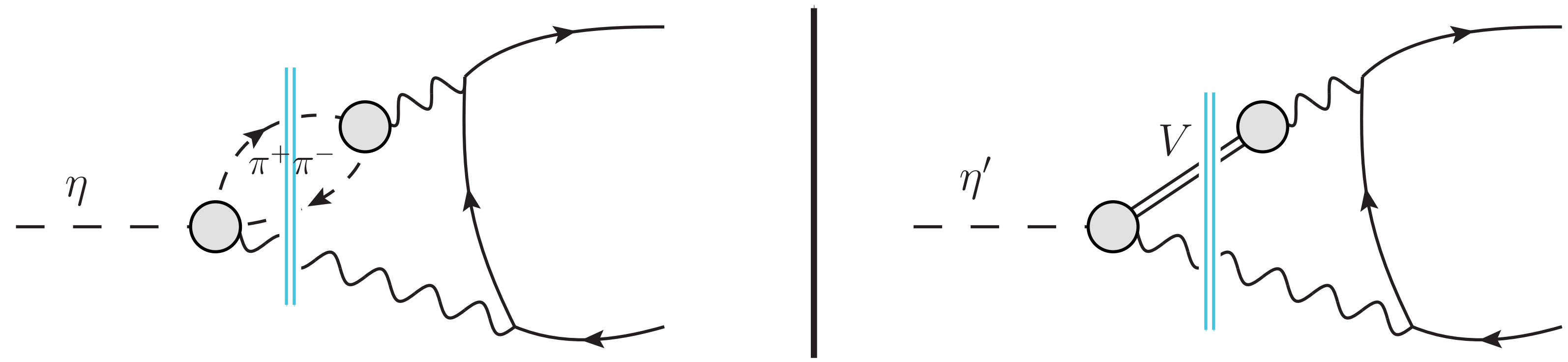
$$BR_{SM}^{P_2^1}(\pi^0 \to e^+e^-) \equiv 7.48(38)\times 10^{-8} \to \delta^2 \sim 20\mathrm{GeV}^2$$

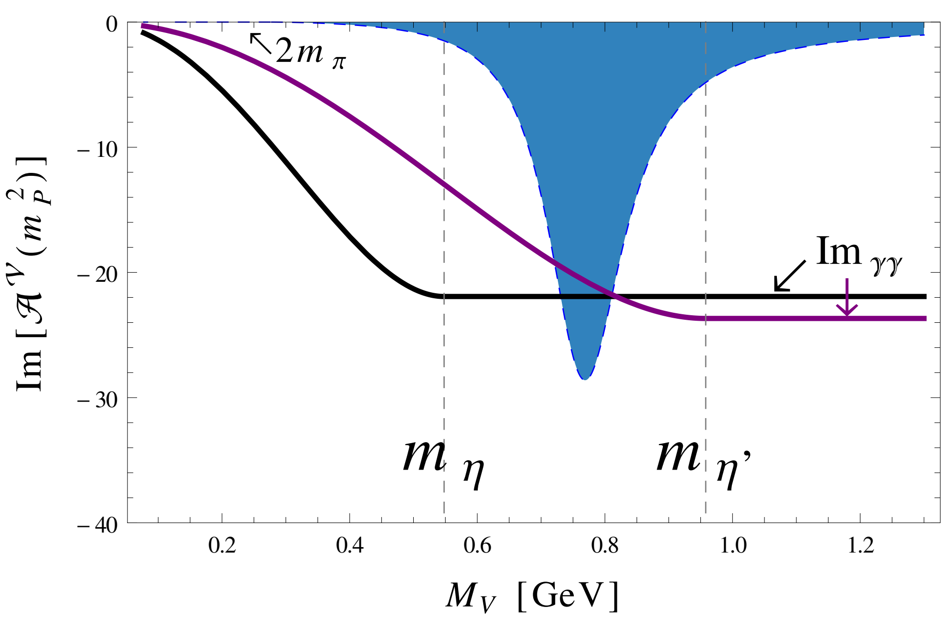
$\boxed{\text{KTeV = very slowly converging OPE}}$ $\quad \delta^2 = 0.2\mathrm{GeV}^2$ [Novikov et al '84]

Doubly virtual η,η'-TFF

[PM, P. Sanchez-Puertas '15]

$$\eta, \eta' \to \ell^+ \ell^-$$





Our predictions

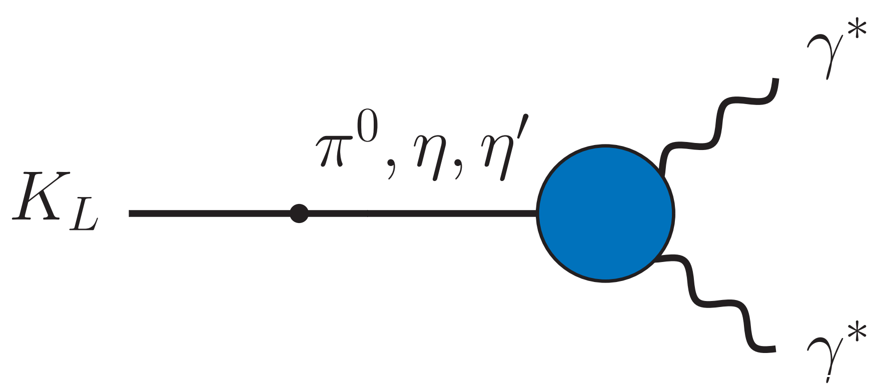
Process	BR
$\eta \to e^+e^-$	$(5.31 \div 5.44)(3)(2)(1) \times 10^{-9}$
$\eta \to \mu^+\mu^-$	$(4.72 \div 4.52)(2)(3)(4) \times 10^{-6}$
$\eta' \to e^+e^-$	$(1.82 \div 1.87)(7)(2)(16) \times 10^{-10}$
$\eta' \to \mu^+\mu^-$	$(1.36 \div 1.49)(5)(3)(25) \times 10^{-7}$

Experimental measurements or bounds

$\eta \to e^+e^-$	$\eta \to \mu^+\mu^-$	$\eta' \to e^+e^-$	$\eta' \to \mu^+\mu^-$
$\leq 2.3 \times 10^{-6}$ [16]	$5.8(8) \times 10^{-6}$ [17]	$\leq 5.6 \times 10^{-9}$ [18, 19]	–

Doubly virtual K_L-TFF

Prediction for $K_L \to \ell^+\ell^-$ (see talk by L. Tunstall)



[Gomez Dumm and Pich, '98]
[Knecht *et al*, '99]
[Isidori, Unterdorfer '03]

Normalized to $K_L \to \gamma\gamma$

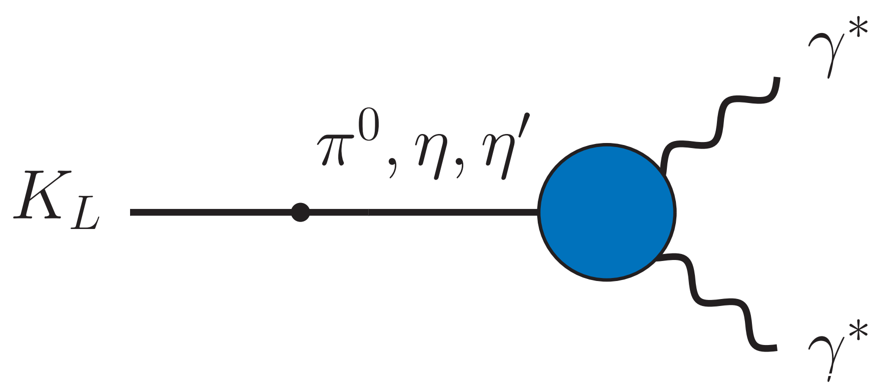
$$Re(\mathcal{A}(q^2)) = \mathcal{A}(0) + \frac{1}{\beta_l(q^2)}\left(\frac{\pi^2}{12} + \frac{1}{4} ln^2\left(\frac{1-\beta_l(q^2)}{1+\beta_l(q^2)}\right) + Li_2\left(\frac{1-\beta_l(q^2)}{1+\beta_l(q^2)}\right)\right)$$

$$\mathcal{A}(0) = \chi_P(\mu) - \frac{5}{2} + \frac{3}{2}\ln(m_l^2/\mu^2)$$

all hadronic information (TFF, m_P...)

Doubly virtual K$_L$-TFF

Prediction for $K_L \to \ell^+\ell^-$



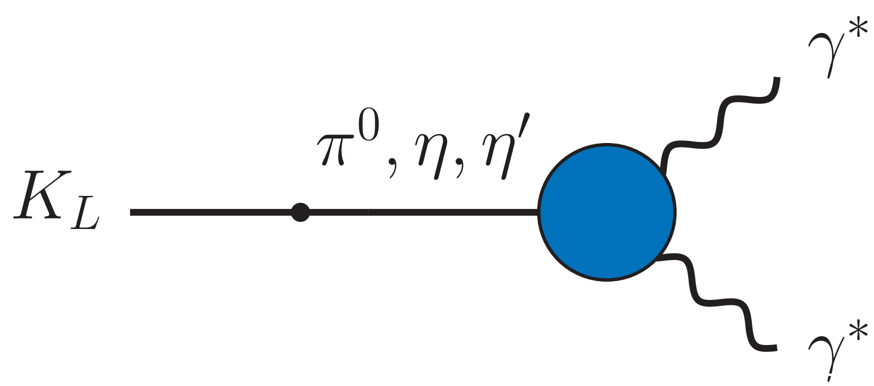
$$\mathcal{A}(0) = \chi_P(\mu) - \frac{5}{4} + \frac{3}{2}\ln(m_l^2/\mu^2)$$

- To calculate $\chi_K(\mu)$ we need $K_L \to \ell^+\ell^-\gamma$ + Canterbury

$\Rightarrow$ Can obtain $\chi_K(\mu)$ at LO, NLO, NNLO,...

Doubly virtual K$_L$-TFF

Prediction for $K_L \to \ell^+\ell^-$



$$\mathcal{A}(0) = \chi_P(\mu) - \frac{5}{4} + \frac{3}{2}\ln(m_l^2/\mu^2)$$

- To calculate $\chi_K(\mu)$ we need $K_L \to \ell^+\ell^-\gamma$ + Canterbury

$\Rightarrow$ Can obtain $\chi_K(\mu)$ at LO, NLO, NNLO,...

(see talk by L. Tunstall)

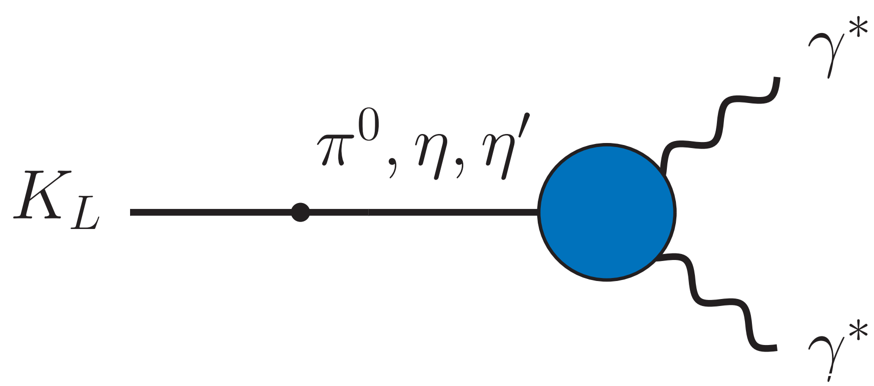
- Even better, $\chi_K^{\ell}(\mu)$ and then $\chi_K^{\mu}(\mu) - \chi_K^{e}(\mu)$

$\Lambda_P \neq \Lambda_{P'}$

$$\boxed{\chi_P^{\mu}(\mu) - \chi_P^{e}(\mu) \simeq \frac{m_P^2}{3\Lambda^2}\left(1 + \frac{m_P^2}{4\Lambda^2}\right)\ln\left(\frac{m_\mu^2}{m_e^2}\right) + \frac{10 m_\mu^2}{3\Lambda^2}\ln\left(\frac{m_\mu^2}{\Lambda^2}\right)}$$

Doubly virtual K_L-TFF

Prediction for $K_L \to \ell^+ \ell^-$



$$\mathcal{A}(0) = \chi_P(\mu) - \frac{5}{4} + \frac{3}{2}\ln(m_l^2/\mu^2)$$

(see talk by L. Tunstall)

$$\boxed{\chi_P^{\mu}(\mu) - \chi_P^{e}(\mu) \simeq \frac{m_P^2}{3\Lambda^2}\left(1 + \frac{m_P^2}{4\Lambda^2}\right)\ln\left(\frac{m_\mu^2}{m_e^2}\right) + \frac{10 m_\mu^2}{3\Lambda^2}\ln\left(\frac{m_\mu^2}{\Lambda^2}\right)}$$

$$\Lambda_P \neq \Lambda_{P'}$$

$$\Rightarrow \quad \chi_P^{\mu}(\mu) - \chi_P^{e}(\mu) \sim 2 - 3 \qquad \forall P$$

Naive New Physics contributions

General Lagrangian (after Fierz)

$$\mathcal{L} = \frac{g}{4m_W} \sum_f m_A c_f^A \left(\overline{f}\gamma_\mu A^\mu \gamma_5 f\right) + 2m_f c_f^{\mathcal{P}} \left(\overline{f} i\gamma_5 f\right) \mathcal{P}$$

$$i\mathcal{M} = \frac{igc_\ell^A m_A}{4m_W} [\overline{u}_{p,s}\gamma_\mu\gamma_5 v_{p',s'}] \frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_A^2}\right)}{m_P^2 - m_A^2} \frac{igm_A}{4m_W} \overbrace{\sum_q \langle 0| \, c_q^A \overline{q}\gamma^\mu\gamma_5 q \, |P(q)\rangle}^{\langle 0|J_{\mu 5}^{\mathrm{NP}}|P(q)\rangle},$$

$$i\mathcal{M} = \frac{igc_\ell^{\mathcal{P}}}{2m_W} m_\ell [\overline{u}_{p,s} i\gamma_5 v_{p',s'}] \frac{i}{q^2 - m_{\mathcal{P}}^2} \frac{ig}{2m_W} \overbrace{\sum_q \langle 0| \, c_q^{\mathcal{P}} m_q \overline{q} i\gamma_5 q \, |P(q)\rangle}^{\langle 0|\mathcal{P}^{\mathrm{NP}}|P(q)\rangle},$$

Naive New Physics contributions

$$\frac{\mathrm{BR}(\pi^0 \to e^+e^-)}{\mathrm{BR}(\pi^0 \to \gamma\gamma)} = 2\left(\frac{\alpha m_e}{\pi m_\pi}\right)^2 \beta_e \left| \mathcal{A}(q^2) + \frac{\sqrt{2} F_\pi G_F}{4\alpha^2 F_{\pi\gamma\gamma}} \times f^{A(P)} \right|^2$$

$$f^A = c_e^A (c_u^A - c_d^A) \qquad f^P = \frac{1}{4} c_e^P (c_u^P - c_d^P) \frac{m_\pi^2}{m_\pi^2 - m_P^2} \qquad c \sim \mathcal{O}\left(\frac{g}{g_{SU(2)_L}}\right)$$

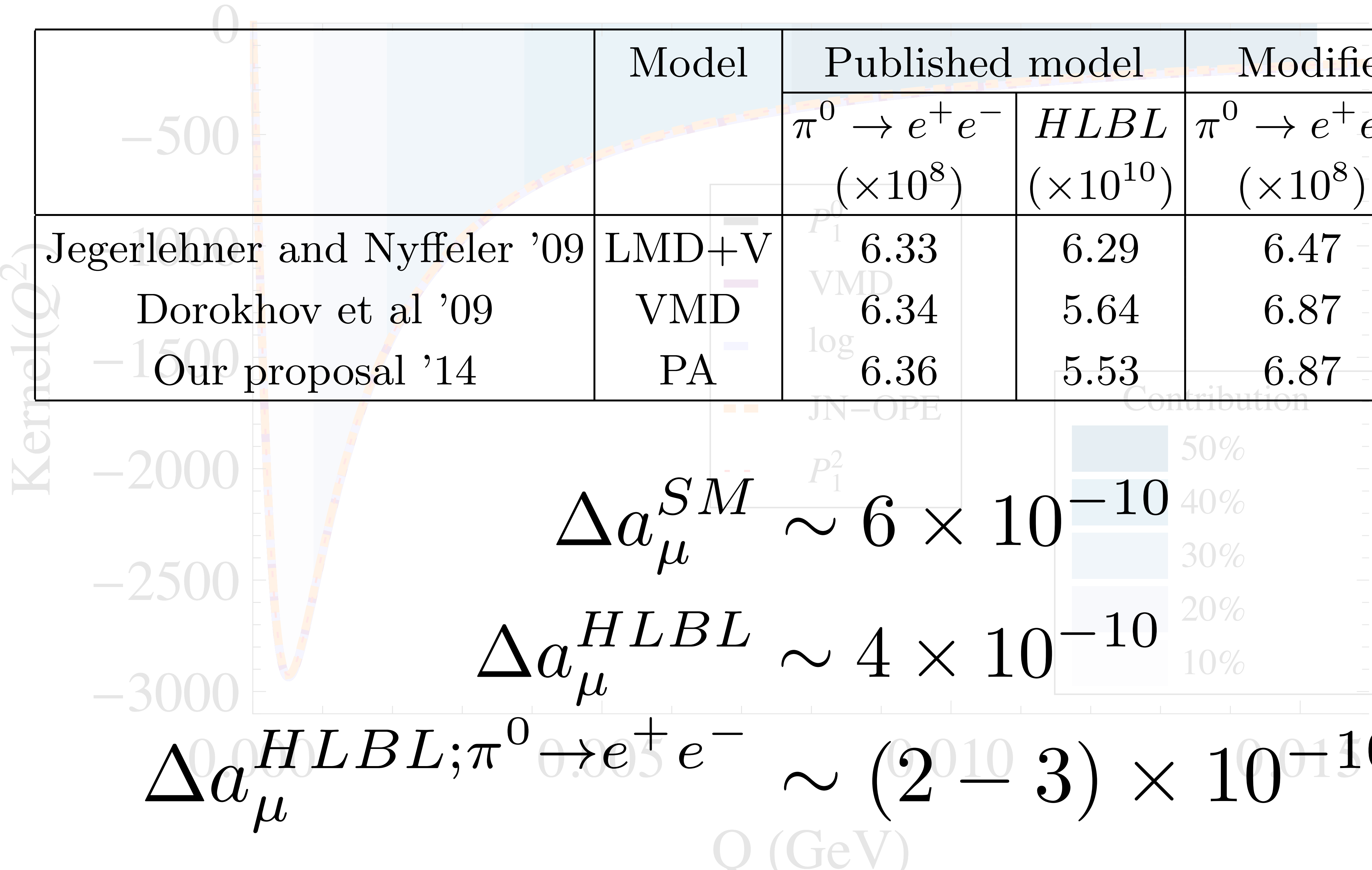
$$\frac{\mathrm{BR}(\pi^0 \to e^+e^-)}{\mathrm{BR}(\pi^0 \to \gamma\gamma)} = \mathrm{SM}(1 + \epsilon_{Z,NP})$$

Z contribution (Arnellos, Marciano, Parsa '82) $\quad c_u^Z = -c_{d,e}^Z = 1 \quad \epsilon_Z \sim 0.3\%$

Our estimate based on existing exp. constrains: $\quad \epsilon_{NP} \sim 0.3\%$
[Marciano et al. '12,'14; Kahn et al '08]

negligible!

Impact of $\pi^0 \to e^+e^-$ on HLBL



	Model	Published model		Modified model	
		$\pi^0 \to e^+e^-$ $(\times 10^8)$	$HLBL$ $(\times 10^{10})$	$\pi^0 \to e^+e^-$ $(\times 10^8)$	$HLBL$ $(\times 10^{10})$
Jegerlehner and Nyffeler '09	LMD+V	6.33	6.29	6.47	5.22
Dorokhov et al '09	VMD	6.34	5.64	6.87	2.44
Our proposal '14	PA	6.36	5.53	6.87	2.85

$$\Delta a_\mu^{SM} \sim 6 \times 10^{-10}$$

$$\Delta a_\mu^{HLBL} \sim 4 \times 10^{-10}$$

$$\Delta a_\mu^{HLBL;\pi^0 \to e^+e^-} \sim (2-3) \times 10^{-10}$$

+ similar effect for the η decay!

Preliminary Shopping list
NA62

At first, I would suggest

- π^0 Dalitz decays $\pi^0 \to e^+e^-\gamma$ (Inv. mass distr.)
- π^0 Double Dalitz decay $\pi^0 \to e^+e^-\, e^+e^-$ (BR)
- Rare decay $\pi^0 \to e^+e^-$ (BR)
- K Dalitz decay $K_L \to \ell^+\ell^-$ (BR)
- K double Dalitz decay $K_L \to \ell^+\ell^-\gamma$ (Inv. mass distr.)
- Rare decay $K_L \to \ell^+\ell^-\, \ell^+\ell^-$ (BR)

During the talk, I'll try to convince you why

Conclusions

- $\pi^0 \to e^+e^-$ is an interesting process
  - hadronic effects are important at all energies
  - but the scale is at the electron mass

- Standard approaches fail to reproduce the KTeV experimental measurement
  - something to be understood: corrections known, radiative known, TFF-data driven, no NP, ...?

- Its impact in the HLBL cannot be forgotten, it might be one of the largest uncertainties if the puzzle persists back-up

Our proposal: use Padé Approximants

[P.M.'12; R. Escribano, P.M., P. Sanchez-Puertas, '13]

We need low-energy region (data driven) + high-energy tail
we don't want a model rather a method providing systematics

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We need low-energy region (data driven) + high-energy tail
we don't want a model rather a method providing systematics

$$F_{P\gamma*\gamma}(Q^2,0) = a_0^P \left(1 + b_P \frac{Q^2}{m_P^2} + c_P \frac{Q^4}{m_P^4} + \dots\right)$$

$\Gamma_{P\to\gamma\gamma}$ slope curvature

We have published space-like data for $Q^2 F_{P\gamma*\gamma}(Q^2,0)$

$$Q^2 F_{P\gamma*\gamma}(Q^2,0) = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \dots$$

$$P_M^N(Q^2) = \frac{T_N(Q^2)}{R_M(Q^2)} = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \dots + \mathcal{O}((Q^2)^{N+M+1})$$

Our proposal: use Padé Approximants

[P.M.'12; R. Escribano, P.M., P. Sanchez-Puertas, '13]

We need low-energy region (data driven) + high-energy tail
we don't want a model rather a method providing systematics

$$F_{P\gamma*\gamma}(Q^2,0) = \underbrace{a_0^P}_{\Gamma_{P\to\gamma\gamma}}\left(1 + \underbrace{b_P}_{\text{slope}}\frac{Q^2}{m_P^2} + \underbrace{c_P}_{\text{curvature}}\frac{Q^4}{m_P^4} + \ldots\right)$$

We have published space-like data for $Q^2 F_{P\gamma*\gamma}(Q^2,0)$

$$Q^2 F_{P\gamma*\gamma}(Q^2,0) = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \ldots$$

$$P_1^1(Q^2) = \frac{a_0 Q^2}{1 - a_1 Q^2} \longrightarrow \begin{array}{l} P_1^N(Q^2) = P_1^1(Q^2), P_1^2(Q^2), P_1^3(Q^2), \ldots \\ P_N^N(Q^2) = P_1^1(Q^2), P_2^2(Q^2), P_3^3(Q^2), \ldots \end{array}$$

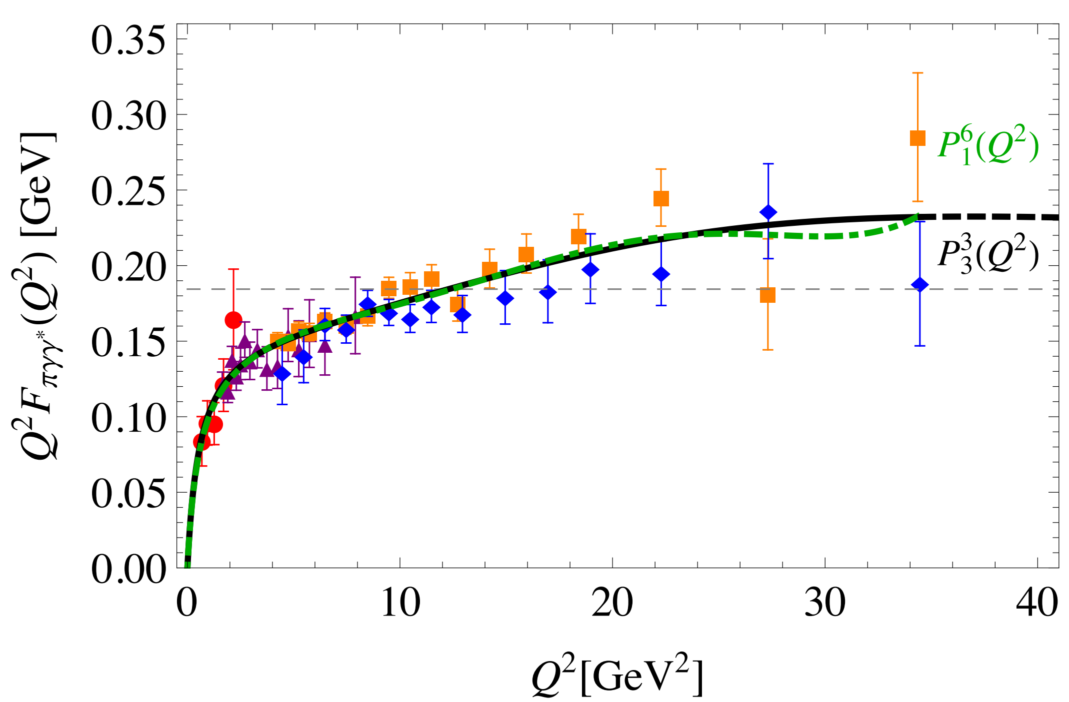
sequence of approximations, i.e., <u>theoretical error</u>

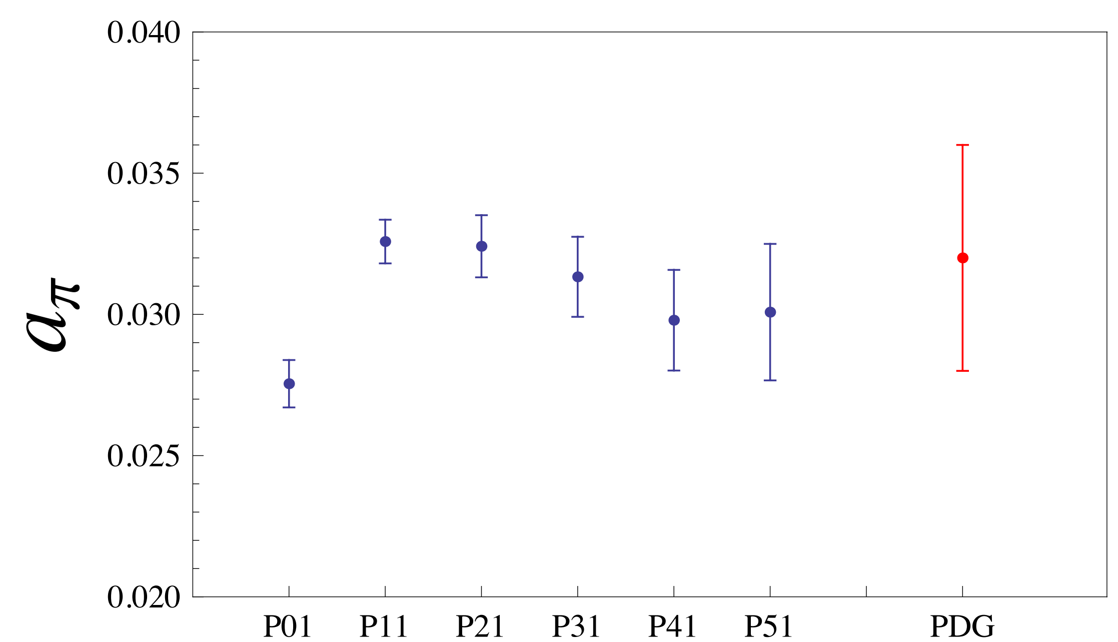
Our proposal: use Padé Approximants

[P.M.'12; R. Escribano, P.M., P. Sanchez-Puertas, '13]

Fit to Space-like data: CELLO'91, CLEO'98, BABAR'09 and Belle'12

$P_1^N(Q^2)$ up to N=5 [P.M, '12]

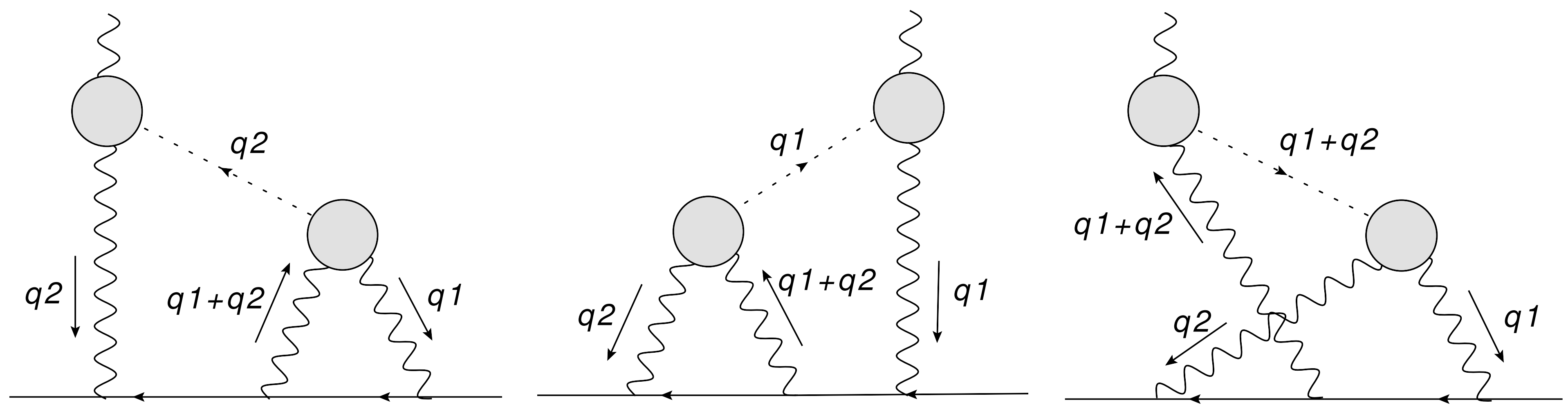




$P_N^N(Q^2)$ up to N=3

Accurate description of the low-energy region making full use of available experimental data

Dissection of the HLBL contribution



$$a_\mu^{LbL;P} = -e^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1+q_2)^2 [(p+q_1)^2 - m^2][(p-q_2)^2 - m^2]}$$

$$\times \left(\frac{F_{P^*\gamma^*\gamma^*}(q_2^2, q_1^2, (q_1+q_2)^2) F_{P^*\gamma^*\gamma^*}(q_2^2, q_2^2, 0)}{q_2^2 - M_P^2} T_1(q_1, q_2; p) \right.$$

Use data from
the Transition Form Factor

$$\left. + \frac{F_{P^*\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2) F_{P^*\gamma^*\gamma^*}((q_1+q_2)^2, (q_1+q_2)^2, 0)}{(q_1+q_2)^2 - M_P^2} T_2(q_1, q_2; p) \right)$$

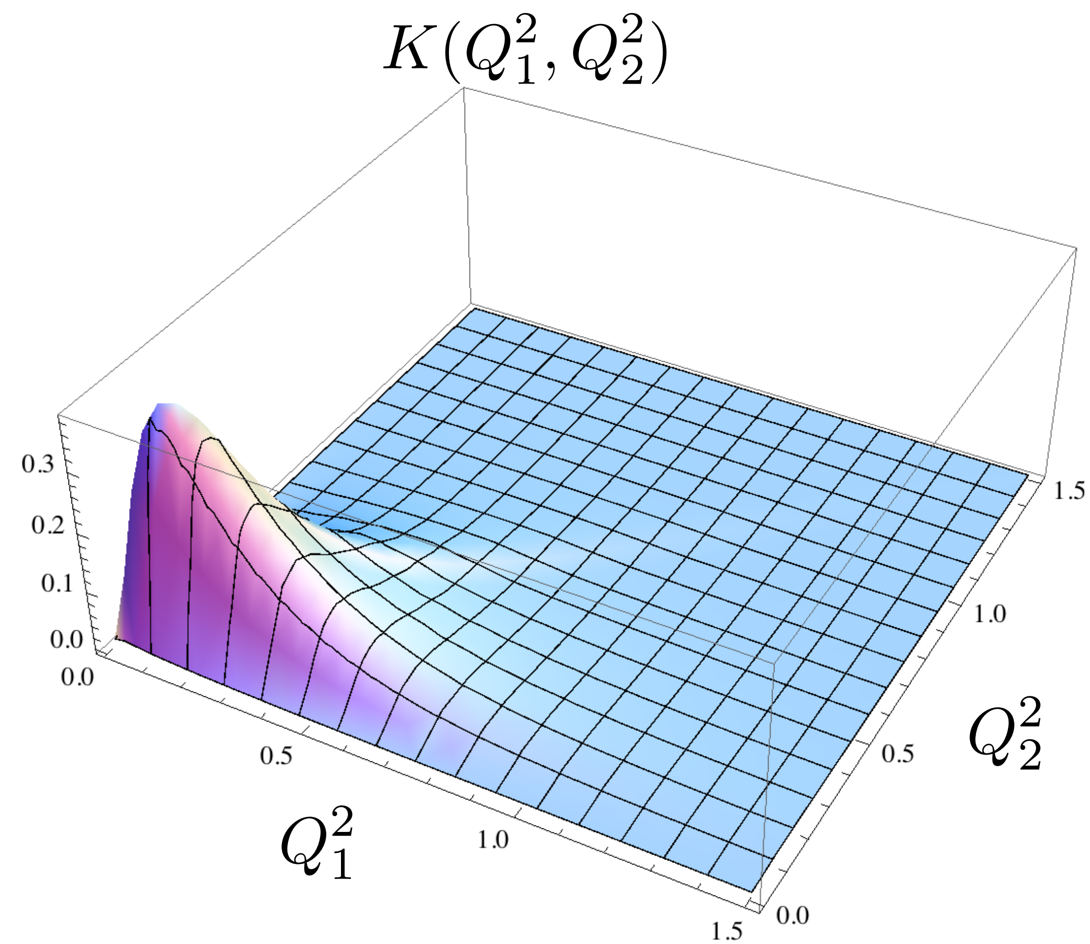
Dissection of the HLBL contribution

- Extraction of meson TFF and HLBL
  - Using CLEO, CELLO, BaBar and Belle to obtain the TFF Low-energy Constants, constrain hadronic model and estimation of π^0-HLBL

$$a_\mu^{LbyL;\pi^0} = e^6 \int \frac{d^4Q_1}{(2\pi)^4} \int \frac{d^4Q_2}{(2\pi)^4} K(Q_1^2, Q_2^2)$$

Using $F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2) \sim P_1^0(Q_1^2, Q_2^2)$

(main energy range from 0 to 1 GeV^2)



The role of doubly virtual TFF data