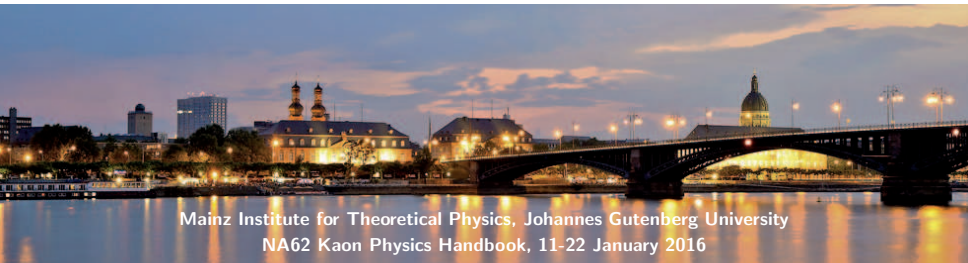




Long and short distances in non-leptonic kaon decays: ϵ'/ϵ

Antonio Pich

IFIC, Univ. Valencia - CSIC

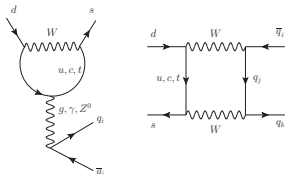


Mainz Institute for Theoretical Physics, Johannes Gutenberg University
NA62 Kaon Physics Handbook, 11-22 January 2016

- ① Theoretical Framework
- ② Octet Enhancement
- ③ CP Violation: ε'/ε

Theoretical Framework

- Sensitivity to Short-Distance Scales:**



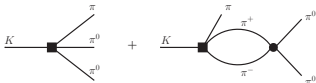
Charm mass prediction

Top quark

GIM cancellation

New Physics ?

- Long-Distance Physics:**



Chiral Dynamics

- Multi-Scale Problem:**

$\log(M/\mu)$

(OPE),

$\log(\mu/m_\pi)$


(χ PT)

M_W

$$\begin{array}{c}
 W, Z, \gamma, g \\
 \tau, \mu, e, \nu_i \\
 t, b, c, s, d, u
 \end{array}$$

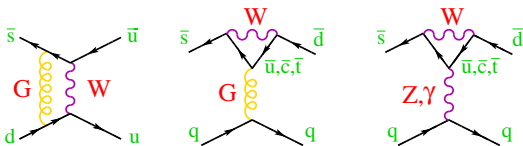
Standard Model

 OPE
 $\lesssim m_c$

$$\begin{array}{c}
 \gamma, g; \mu, e, \nu_i \\
 s, d, u
 \end{array}$$
 $\mathcal{L}_{\text{QCD}}^{(n_f=3)}, \mathcal{L}_{\text{eff}}^{\Delta S=1,2}$
 $N_C \rightarrow \infty$
 M_K

$$\begin{array}{c}
 \gamma; \mu, e, \nu_i \\
 \pi, K, \eta
 \end{array}$$
 χPT

$\Delta S = 1$ TRANSITIONS



$$\mathcal{L}_{\text{eff}}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) Q_i(\mu)$$

$$\begin{aligned}
 Q_1 &= (\bar{s}_\alpha u_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A} & Q_2 &= (\bar{s}u)_{V-A} (\bar{u}d)_{V-A} \\
 Q_{3,5} &= (\bar{s}d)_{V-A} \sum_q (\bar{q}q)_{V\mp A} & Q_4 &= (\bar{s}_\alpha d_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A} \\
 Q_{7,9} &= \frac{3}{2} (\bar{s}d)_{V-A} \sum_q e_q (\bar{q}q)_{V\pm A} & Q_{10} &= \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V-A} \\
 Q_6 &= -8 \sum_q (\bar{s}_L q_R) (\bar{q}_R d_L) & Q_8 &= -12 \sum_q e_q (\bar{s}_L q_R) (\bar{q}_R d_L) \\
 Q_{11,12} &= (\bar{s}d)_{V-A} \sum_\ell (\bar{\ell}\ell)_{V,A} & Q_{13} &= (\bar{s}d)_{V-A} \sum_\nu (\bar{\nu}\nu)_{V-A}
 \end{aligned}$$

- $q > \mu$: $C_i(\mu) = z_i(\mu) - y_i(\mu) (V_{td} V_{ts}^* / V_{ud} V_{us}^*)$

$$O(\alpha_s^n t^n), O(\alpha_s^{n+1} t^n) \quad [t \equiv \log(M/m)]$$

Munich / Rome

- $q < \mu$: $\langle \pi\pi | Q_i(\mu) | K \rangle$? Physics does not depend on μ

CHIRAL PERTURBATION THEORY (χ PT)

- Expansion in powers of p^2/Λ_χ^2 : $\mathcal{A} = \sum_n \mathcal{A}^{(n)}$ ($\Lambda_\chi \sim 4\pi F_\pi \sim 1.2$ GeV)
- Amplitude structure fixed by chiral symmetry

$$\text{SU}(3)_L \otimes \text{SU}(3)_R \rightarrow \text{SU}(3)_V$$

- Short-distance dynamics encoded in Low-Energy Couplings

- $\mathbf{O}(p^2)$ χ PT: Goldstone interactions (π, K, η) $\Phi \equiv \frac{1}{\sqrt{2}} \vec{\lambda} \vec{\varphi}$

$$\mathcal{L}_2^{\Delta S=1} = G_8 F^4 \text{Tr}(\lambda L_\mu L^\mu) + G_{27} F^4 \left(L_{\mu 23} L_{11}^\mu + \frac{2}{3} L_{\mu 21} L_{13}^\mu \right)$$
$$G_R \equiv -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \mathcal{G}_R \quad ; \quad L_\mu = -iU^\dagger D_\mu U \quad ; \quad \lambda \equiv \frac{1}{2} \lambda_{6-i7} \quad ; \quad U \equiv \exp \{ i\sqrt{2} \Phi / F \}$$

- Loop corrections (χ PT logarithms) unambiguously predicted
- LECs can be determined at $N_C \rightarrow \infty$ (matching)
- $\mathbf{O}(p^2)$ LECs (G_8, G_{27}) can be phenomenologically determined

Nonleptonic Decays

- **Octet Enhancement:** $\frac{A(K \rightarrow \pi\pi)_{I=0}}{A(K \rightarrow \pi\pi)_{I=2}} \approx 22$
 - Short-distance: **gluonic corrections, penguins**
 - Long-distance: **large χ PT corrections (FSI)**
 - Ongoing Lattice efforts

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- **Direct CP Violation:**

$$\eta_{ij} \equiv \frac{A(K_L \rightarrow \pi^i \pi^j)}{A(K_S \rightarrow \pi^i \pi^j)}$$

$$\text{Re}(\epsilon'/\epsilon) = \frac{1}{3} \left(1 - \left| \frac{\eta_{00}}{\eta_{+-}} \right| \right) = (16.8 \pm 1.4) \cdot 10^{-4}$$

NA31, E731, NA48, KTeV

$$\text{Re}(\epsilon'/\epsilon)_{\text{SM}} = (19 \pm 2^{+9}_{-6} \pm 6) \cdot 10^{-4}$$

Pallante-Pich-Scimemi

$K \rightarrow 2\pi$ Isospin Amplitudes

$$A[K^0 \rightarrow \pi^+\pi^-] \equiv A_0 e^{i\chi_0} + \frac{1}{\sqrt{2}} A_2 e^{i\chi_2}$$

$$A[K^0 \rightarrow \pi^0\pi^0] \equiv A_0 e^{i\chi_0} - \sqrt{2} A_2 e^{i\chi_2}$$

$$A[K^+ \rightarrow \pi^+\pi^0] \equiv \frac{3}{2} A_2^+ e^{i\chi_2^+}$$

1) $\Delta I = 1/2$ Rule:

$$\omega \equiv \frac{\text{Re}(A_2)}{\text{Re}(A_0)} \approx \frac{1}{22}$$

2) Strong Final State Interactions:

$$\chi_0 - \chi_2 \approx \delta_0 - \delta_2 \approx 45^\circ$$

$$\varepsilon'_K = \frac{-i}{\sqrt{2}} e^{i(\chi_2 - \chi_0)} \omega \left\{ \frac{\text{Im}(A_0)}{\text{Re}(A_0)} - \frac{\text{Im}(A_2)}{\text{Re}(A_2)} \right\}$$

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$$A[K^+ \rightarrow \pi^+\pi^0] \equiv \frac{3}{2} A_2^+ e^{i\chi_2^+}$$

$$A_0 e^{i\chi_0} = \mathcal{A}_{1/2}$$

$$A_2 e^{i\chi_2} = \mathcal{A}_{3/2} + \mathcal{A}_{5/2}$$

$$A_2^+ e^{i\chi_2^+} = \mathcal{A}_{3/2} - \frac{2}{3} \mathcal{A}_{5/2}$$

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$O(p^2)$ χ PT

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$$\mathcal{A}_{1/2} = \sqrt{2} F_\pi \left(G_8 + \frac{1}{9} G_{27} \right) (M_K^2 - M_\pi^2)$$

$$\mathcal{A}_{3/2} = \frac{10}{9} F_\pi G_{27} (M_K^2 - M_\pi^2)$$

$$\mathcal{A}_{5/2} = 0 ; \quad \delta_0 = \delta_2 = 0$$

$$[\Gamma(K \rightarrow 2\pi) + \delta_I]_{\text{Exp}} \quad \rightarrow \quad |g_8| \approx 5.1 ; \quad |g_{27}| \approx 0.29$$

$$\mathcal{L}_2^{\Delta S=1} = G_8 F^4 \langle \lambda L_\mu L^\mu \rangle + G_{27} F^4 \left(L_{\mu 23} L_{11}^\mu + \frac{2}{3} L_{\mu 21} L_{13}^\mu \right) \\ + e^2 F^6 G_8 g_{ew} \langle \lambda U^\dagger Q U \rangle$$

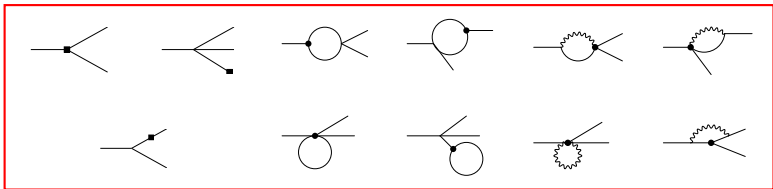
$$\mathcal{A}_{1/2} = \sqrt{2} F_\pi \left\{ G_8 \left[(M_K^2 - M_\pi^2) \left(1 - \frac{2}{3\sqrt{3}} \varepsilon^{(2)} \right) - \frac{2}{3} F_\pi^2 e^2 (g_{ew} + 2Z) \right] \right. \\ \left. + \frac{1}{9} G_{27} (M_K^2 - M_\pi^2) \right\}$$

$$\mathcal{A}_{3/2} = \frac{2}{3} F_\pi \left\{ \left(\frac{5}{3} G_{27} + \frac{2}{\sqrt{3}} \varepsilon^{(2)} G_8 \right) (M_K^2 - M_\pi^2) - F_\pi^2 e^2 G_8 (g_{ew} + 2Z) \right\}$$

$$\mathcal{A}_{5/2} = 0 \quad ; \quad \delta_0 = \delta_2 = 0$$

$$\varepsilon^{(2)} = (\sqrt{3}/4) (m_d - m_u)/(m_s - \hat{m}) \approx 0.011 \quad ; \quad Z \approx (M_{\pi^\pm}^2 - M_{\pi^0}^2)/(2 e^2 F_\pi^2) \approx 0.8$$

O [p⁴, (m_u - m_d) p², e²p⁰, e²p²] χPT



- **Nonleptonic weak Lagrangian:** $\mathcal{O}(G_F p^4)$

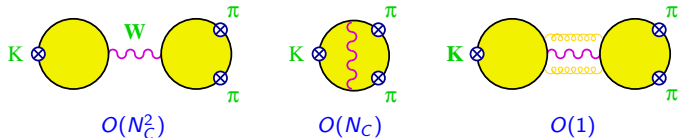
$$\mathcal{L}_{\text{weak}}^{(4)} = \sum_i G_8 N_i F^2 O_i^8 + \sum_i G_{27} D_i F^2 O_i^{27} + \text{h.c.}$$

- **Electroweak Lagrangian:** $\mathcal{O}(G_F e^2 p^{0,2})$

$$\mathcal{L}_{\text{EW}} = e^2 F^6 G_8 g_{\text{ew}} \text{Tr}(\lambda U^\dagger Q U) + e^2 \sum_i G_8 Z_i F^4 O_i^{\text{EW}} + \text{h.c.}$$

- $\mathcal{O}(e^2 p^{0,2})$ **Electromagnetic** + $\mathcal{O}(p^4)$ **Strong:** Z, K_i, L_i

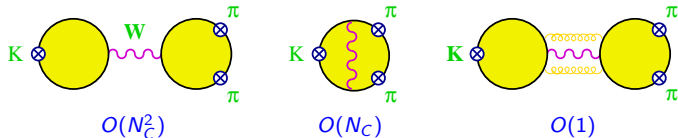
Weak Currents Factorize at Large N_c



$$A[K^0 \rightarrow \pi^0 \pi^0] = 0 \quad \rightarrow \quad A_0 = \sqrt{2} A_2$$

No $\Delta I = \frac{1}{2}$ enhancement at leading order in $1/N_c$

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No $\Delta I = \frac{1}{2}$ enhancement at leading order in $1/N_C$

- Multiscale problem:

OPE

$$\frac{1}{N_C} \log \left(\frac{M_W}{\mu} \right) \sim \frac{1}{3} \times 4$$

Short-distance logarithms must be summed

- Large χ PT logarithms:

FSI

$$\frac{1}{N_C} \log \left(\frac{\mu}{M_\pi} \right) \sim \frac{1}{3} \times 2$$

Infrared logarithms must also be included

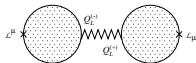
$$[\delta_I \sim O(1/N_C), \delta_0 - \delta_2 \approx 45^\circ]$$

Dynamical understanding of the $\Delta I = 1/2$ rule

AP – E. de Rafael, PL B374 (1996) 186

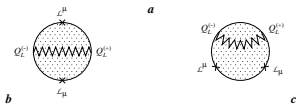
$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} F^4 \left[a \text{Tr}(Q_L^{(-)} L_\mu) \text{Tr}(Q_L^{(+)} L^\mu) + b \text{Tr}(Q_L^{(-)} L_\mu Q_L^{(+)} L^\mu) + c \text{Tr}(Q_L^{(-)} Q_L^{(+)} L_\mu L^\mu) \right]$$

$\mathcal{O}(N_c^2)$



$$Q_L^{(+)} = \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad Q_L^{(-)} = Q_L^{(+)\dagger}$$

$\mathcal{O}(N_c)$



$$g_8 = \frac{3}{5}(a + b) - b + c$$

$$g_{27} = \frac{3}{5}(a + b)$$

$$a = 1 + \mathcal{O}\left(\frac{1}{N_c^2}\right) \quad ; \quad c = \text{Re}C_4 - 16 L_5 \text{Re}C_6(\mu^2) \left[\frac{\langle \bar{\psi}\psi \rangle}{f_\pi^3} \right]^2 + \mathcal{O}\left(\frac{1}{N_c^2}\right) \simeq 0.3 \pm 0.2$$

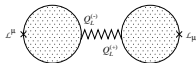
$$|g_{27}| \simeq 0.29 \quad \Rightarrow \quad b \simeq -0.52 + \mathcal{O}\left(\frac{1}{N_c^2}\right) \quad \Rightarrow \quad g_8 \simeq 1.1 + \mathcal{O}\left(\frac{1}{N_c^2}\right)$$

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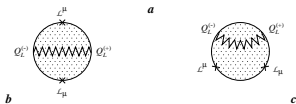
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$b < 0$ predicted through explicit calculations

AP–E. de Rafael, NP B358 (1991) 311

Bardeen-Buras-Gerard, Bijnens-Prades, Bertolini et al

Confirmed through inclusive QCD analysis

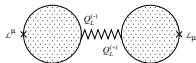
M. Jamin–AP, NP B425 (1994) 15

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AP – E. de Rafael, PL B374 (1996) 186

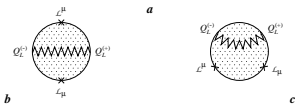
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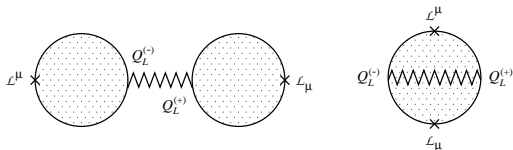
Confirmed recently by lattice calculations

RBC-UKQCD, PRL 110 (2013) 15, 152001

PRD 91 (2015) 7, 074502

“A qualitative picture towards the understanding of the underlying physics begins to emerge”

AP – E. de Rafael, PL B374 (1996) 186



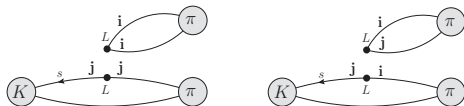
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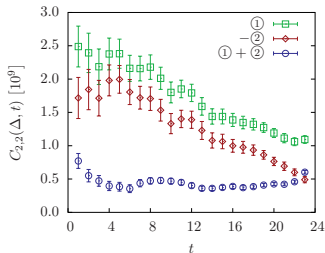
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“Emerging understanding of the $\Delta I = 1/2$ Rule from Lattice QCD”

RBC-UKQCD, PRL 110 (2013) 15, 152001

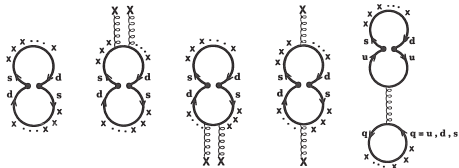


$$b \approx -0.7 a$$



Effective Action Model: Bosonization in Gluonic Background

AP-E. de Rafael, NP B358 (1991) 311



$$\Delta = \frac{1}{N_C} \left[1 - \frac{N_C}{2} \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{16\pi^2 f_\pi^4} + \mathcal{O}(\alpha_s^2 N_C^2) \right] < 0$$

$$g_{27} \approx \frac{3}{5} C_+(\mu^2) \left\{ 1 + \Delta + \mathcal{O}(1/N_C^2) \right\}$$

$$g_8 \approx \frac{1}{2} C_-(\mu^2) \left\{ 1 - \Delta + \mathcal{O}(1/N_C^2) \right\} + \frac{1}{10} C_+(\mu^2) \left\{ 1 + \Delta + \mathcal{O}(1/N_C^2) \right\} + c$$

$$c = C_4(\mu^2) - 16 C_6(\mu^2) L_5 \left[\frac{\langle \bar{\psi}\psi \rangle}{f_\pi^3} \right]^2 + \mathcal{O}(1/N_C^2)$$

$$b = \frac{1}{2} C_+(\mu^2) \left\{ 1 + \Delta + \mathcal{O}(1/N_C^2) \right\} - \frac{1}{2} C_-(\mu^2) \left\{ 1 - \Delta + \mathcal{O}(1/N_C^2) \right\} < 0$$

$$\mu \sim m_c, \quad \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \sim 330 \text{ MeV}^4 \quad \longrightarrow \quad b \sim -0.6$$

Two-point Functions

AP–E. de Rafael, NP B358 (1991) 311, PL B374 (1996) 186

M. Jamin–AP, NP B425 (1994) 15

$$\Psi^{\Delta S=1,2}(q^2) \equiv i \int d^4x e^{iq \cdot x} \langle 0 | T \left(\mathcal{H}_{\text{eff}}^{\Delta S=1,2}(x), \mathcal{H}_{\text{eff}}^{\Delta S=1,2}(0)^\dagger \right) | 0 \rangle = \sum_{ij} C_i C_j^* \Psi_{ij}(q^2)$$



$$\frac{1}{\pi} \text{Im} \Psi_{\pm\pm}(t) = \theta(t) \frac{2}{45} N_c^2 \left(1 \pm \frac{1}{N_c} \right) \frac{t^4}{(4\pi)^6} \alpha_s(t)^{-2a_{\pm}} C_{\pm}^2(M_W^2) \left[1 + \frac{3}{4} \frac{\alpha_s(t) N_c}{\pi} \mathcal{K}_{\pm} \right]$$

$$a_{\pm} = \pm \frac{9}{11N_c} \frac{1 \mp 1/N_c}{1 - 6/11N_c}$$

$$\mathcal{K}_+ = 1 - \frac{30587}{3630} \frac{1}{N_c} + \frac{164936}{19965} \frac{1}{N_c^2} - \frac{51591}{14641} \frac{1}{N_c^3} + \frac{440193}{322102} \frac{1}{N_c^4} + \dots = -\frac{3649}{3645}$$

$$\mathcal{K}_- = 1 + \frac{30587}{3630} \frac{1}{N_c} + \frac{169706}{19965} \frac{1}{N_c^2} + \frac{70335}{14641} \frac{1}{N_c^3} + \frac{1810209}{322102} \frac{1}{N_c^4} + \dots = +\frac{18278}{3645}$$

A large $\log(M_1/M_2)$ compensates a $1/N_C$ suppression

① Short-distance: $\frac{1}{N_C} \log(M_W/\mu)$

Bardeen-Buras-Gerard

$$\rightarrow \begin{cases} g_8^\infty = 1.13 \pm 0.05_\mu \pm 0.08_{L_5} \pm 0.05_{m_s} \\ g_{27}^\infty = 0.46 \pm 0.01_\mu \end{cases}$$

Cirigliano et al, Pallante et al

② Long-distance (χ PT): $\frac{1}{N_C} \log(\mu/m_\pi)$

Kambor et al, Pallante et al

$$\begin{aligned} g_8^{\text{LO}} = 5.0 & \quad \rightarrow \quad g_8^{\text{NLO}} = 3.6 \\ g_{27}^{\text{LO}} = 0.285 & \quad \rightarrow \quad g_{27}^{\text{NLO}} = 0.286 \end{aligned}$$

Cirigliano et al

③ Isospin Violation: $g_{27}^{\text{NLO}} = 0.297$

Cirigliano et al

Phenomenological $K \rightarrow \pi\pi$ Fit

Cirigliano-Ecker-Neufeld-Pich

	LO-IC	LO-IB	NLO-IC	NLO-IB
$\text{Re } g_8$	4.96	4.99	3.62 ± 0.28	3.61 ± 0.28
$\text{Re } g_{27}$	0.285	0.253	0.286 ± 0.029	0.297 ± 0.029
$\chi_0 - \chi_2$	47.5°	47.8°	$(47.5 \pm 0.9)^\circ$	$(51.3 \pm 0.8)^\circ$

IC $\equiv [m_u - m_d = \alpha = 0]$; IB $\equiv [m_u - m_d \neq 0, \alpha \neq 0]$

Isospin Limit: $[\delta_0 - \delta_2]_{K \rightarrow \pi\pi} = (52.5 \pm 0.8_{\text{exp}} \pm 2.8_{\text{th}})^\circ$

$\pi\pi \rightarrow \pi\pi$: $\delta_0 - \delta_2 = (47.7 \pm 1.5)^\circ$

Colangelo-Gasser-Leutwyler '01

Isospin Breaking in ϵ'/ϵ

$$\epsilon'_K \sim \omega_+ \left\{ \frac{\text{Im } A_0^{(0)}}{\text{Re } A_0^{(0)}} (1 + \Delta_0 + f_{5/2}) - \frac{\text{Im } A_2}{\text{Re } A_2^{(0)}} \right\}$$

$$\sim \omega_+ \left\{ \frac{\text{Im } A_0^{(0)}}{\text{Re } A_0^{(0)}} (1 - \Omega_{\text{eff}}) - \frac{\text{Im } A_2^{\text{emp}}}{\text{Re } A_2^{(0)}} \right\}$$

$$\omega \equiv \frac{\text{Re } A_2}{\text{Re } A_0} = \omega_+ (1 + f_{5/2}) \quad ; \quad \omega_+ \equiv \frac{\text{Re } A_2^+}{\text{Re } A_0} \quad , \quad \Omega_{IB} = \frac{\text{Re } A_0^{(0)}}{\text{Re } A_2^{(0)}} \cdot \frac{\text{Im } A_2^{\text{non-emp}}}{\text{Im } A_0^{(0)}}$$

Cirigliano-Ecker-Neufeld-Pich

$\times 10^{-2}$	$\alpha = 0$		$\alpha \neq 0$	
	LO	NLO	LO	NLO
Ω_{IB}	11.7	15.9 ± 4.5	18.0 ± 6.5	22.7 ± 7.6
Δ_0	-0.004	-0.41 ± 0.05	8.7 ± 3.0	8.4 ± 3.6
$f_{5/2}$	0	0	0	8.3 ± 2.4
Ω_{eff}	11.7	16.3 ± 4.5	9.3 ± 5.8	6.0 ± 7.7

$$\Omega_{\text{eff}} = 0.06 \pm 0.08$$

$$\equiv \Omega_{IB} - \Delta_0 - f_{5/2}$$

$$\Omega_{IB}^{\pi^0 \eta} = 0.16 \pm 0.03$$

Anatomy of ε'/ε calculation

$$\frac{\varepsilon'_K}{\varepsilon_K} \sim \left[\frac{105 \text{ MeV}}{m_s(2 \text{ GeV})} \right]^2 \left\{ B_6^{(1/2)} (1 - \Omega_{\text{eff}}) - 0.4 B_8^{(3/2)} \right\}$$

- ① $O(p^4)$ χ PT Loops: **Large correction** (FSI) Pallante-Pich-Scimemi
- ② $O(p^4)$ LECs fixed at $N_C \rightarrow \infty$: **Small correction**
- ③ Isospin Breaking $O[(m_u - m_d)p^2, e^2 p^2]$: **Sizeable corrections**
 $\Omega_{\text{eff}} = 0.06 \pm 0.08$ Cirigliano-Ecker-Neufeld-Pich
- ④ $O(p^4)$ LECs $[\text{Re}(g_8), \text{Re}(g_{27})]$ and phase-shifts fitted to data
- ⑤ $m_s(2 \text{ GeV}) = 110 \pm 20 \text{ MeV}$ (quark condensate)

$$N_C \rightarrow \infty$$

$$g_8 = \left(\frac{3}{5} C_2 - \frac{2}{5} C_1 + C_4 \right) - 16 L_5 \left(\frac{\langle \bar{q} q \rangle(\mu)}{F_\pi^3} \right)^2 C_6(\mu)$$

$$g_{27} = \frac{3}{5} (C_2 + C_1)$$

$$e^2 g_8 g_{ew} = -3 \left(\frac{\langle \bar{q} q \rangle(\mu)}{F_\pi^3} \right)^2 \left[C_8(\mu) + \frac{16}{9} C_6(\mu) e^2 (K_9 - 2 K_{10}) \right]$$

$$\frac{\langle \bar{q} q \rangle(\mu)}{F_\pi^3} = \frac{M_{K^0}^2}{(m_s + m_d)(\mu) F_\pi} \left\{ 1 - \frac{8M_{K^0}^2}{F_\pi^2} (2L_8 - L_5) + \frac{4M_{\pi^0}^2}{F_\pi^2} L_5 \right\}$$

- Equivalent to standard calculations of B_i
- μ dependence only captured for $Q_{6,8}$

Anomalous Dimension Matrix

$$\gamma_s^{(0)} = \begin{pmatrix} -\frac{3}{N_c^2} & \frac{3}{N_c} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{N_c} & -\frac{3}{N_c^2} & -\frac{1}{3N_c^2} & \frac{1}{3N_c} & -\frac{1}{3N_c^2} & \frac{1}{3N_c} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{11}{3N_c^2} & \frac{11}{3N_c} & -\frac{2}{3N_c^2} & \frac{2}{3N_c} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{N_c} - \frac{n_f}{3N_c^2} & \frac{n_f}{3N_c} - \frac{3}{N_c^2} & -\frac{n_f}{3N_c^2} & \frac{n_f}{3N_c} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{3}{N_c^2} & -\frac{3}{N_c} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{n_f}{3N_c^2} & \frac{n_f}{3N_c} & -\frac{n_f}{3N_c^2} & -3 + \frac{n_f}{3N_c} + \frac{3}{N_c^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{N_c^2} & -\frac{3}{N_c} & 0 & 0 \\ 0 & 0 & \frac{-n_u + \frac{n_d}{2}}{3N_c^2} & \frac{n_u - \frac{n_d}{2}}{3N_c} & \frac{-n_u + \frac{n_d}{2}}{3N_c^2} & \frac{n_u - \frac{n_d}{2}}{3N_c} & 0 & -3 + \frac{3}{N_c^2} & 0 & 0 \\ 0 & 0 & \frac{1}{3N_c^2} & -\frac{1}{3N_c} & \frac{1}{3N_c^2} & -\frac{1}{3N_c} & 0 & 0 & -\frac{3}{N_c^2} & 0 \\ 0 & 0 & \frac{-n_u + \frac{n_d}{2}}{3N_c^2} & \frac{n_u - \frac{n_d}{2}}{3N_c} & \frac{-n_u + \frac{n_d}{2}}{3N_c^2} & \frac{n_u - \frac{n_d}{2}}{3N_c} & 0 & 0 & 0 & -\frac{3}{N_c^2} \end{pmatrix}$$

Only γ_{66} and γ_{88} survive the large- N_c limit

$$\mathcal{A}_n^{(\mathbf{x})} = \mathbf{a}_n^{(\mathbf{x})} \left[\mathbf{1} + \Delta_L \mathcal{A}_n^{(\mathbf{x})} + \Delta_C \mathcal{A}_n^{(\mathbf{x})} \right]$$

1) $O(p^4)$ χ PT Loops: Large correction [NLO in $1/N_C$]

$$\Delta_L \mathcal{A}_{1/2}^{(8)} = 0.27 \pm 0.05 + 0.47 i \quad ;$$

Pallante-Pich-Scimemi

$$\Delta_L \mathcal{A}_{1/2}^{(27)} = 1.02 \pm 0.60 + 0.47 i \quad ; \quad \Delta_L \mathcal{A}_{3/2}^{(27)} = -0.04 \pm 0.05 - 0.21 i$$

$$\Delta_L \mathcal{A}_{1/2}^{(g)} = 0.27 \pm 0.05 + 0.47 i \quad ; \quad \Delta_L \mathcal{A}_{3/2}^{(g)} = -0.50 \pm 0.20 - 0.21 i$$

$$\mathcal{A}_n^{(X)} = \mathbf{a}_n^{(X)} \left[\mathbf{1} + \Delta_L \mathcal{A}_n^{(X)} + \Delta_C \mathcal{A}_n^{(X)} \right]$$

1) $O(p^4)$ χ PT Loops: Large correction [NLO in $1/N_C$]

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2) Local $O(p^4)$ couplings fixed at $N_C \rightarrow \infty$ \rightarrow $\Delta_C \mathcal{A}_n^{(X)}$

Small correction to $O(p^2)$ results

3) Isospin Breaking: $O[(m_u - m_d)p^2, e^2 p^2]$ Sizeable corrections

Cirigliano-Ecker-Neufeld-Pich

4) $\text{Re}(g_8), \text{Re}(g_{27}), \chi_0 - \chi_2$ fitted to data

$$\frac{\varepsilon'_K}{\varepsilon_K} \sim \left[\frac{105 \text{ MeV}}{m_s(2 \text{ GeV})} \right]^2 \left\{ B_6^{(1/2)} (1 - \Omega_{\text{eff}}) - 0.4 B_8^{(3/2)} \right\}$$

Delicate Cancellation. Strong Sensitivity to:

- m_s (quark condensate) $m_s(2 \text{ GeV}) = 110 \pm 20 \text{ MeV}$
- Isospin Breaking ($m_u \neq m_d, \alpha$) $\Omega_{\text{eff}} = 0.06 \pm 0.08$
- Penguin Matrix Elements

Cirigliano-Ecker-Neufeld-Pich

χ PT Loops (FSI): $B_{6,\infty}^{(1/2)} \times (1.35 \pm 0.05)$; $B_{8,\infty}^{(3/2)} \times (0.54 \pm 0.20)$

$$\frac{\varepsilon'_K}{\varepsilon_K} \sim \left[\frac{105 \text{ MeV}}{m_s(2 \text{ GeV})} \right]^2 \left\{ B_6^{(1/2)} (1 - \Omega_{\text{eff}}) - 0.4 B_8^{(3/2)} \right\}$$

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Pallante-Pich-Scimemi '01: (updated '04)

$$\text{Re}(\varepsilon'/\varepsilon) = \left(19 \pm 2_{\mu} \pm 6_{-6 m_s} \pm 6_{1/N_C} \right) \times 10^{-4}$$

Experimental world average: $\text{Re}(\varepsilon'/\varepsilon) = (16.8 \pm 1.4) \times 10^{-4}$

Challenge: Control of subleading $1/N_C$ corrections to χ PT couplings

Electroweak Penguins contribute at $\mathcal{O}(p^0)$ ($m_q, p \rightarrow 0$)

$$e^2 g_8 g_{ew} F^6 = 6 C_7(\mu) \langle \mathcal{O}_1(\mu) \rangle - 12 C_8(\mu) \langle \mathcal{O}_2(\mu) \rangle \xrightarrow{N_c \rightarrow \infty} -\frac{1}{3} C_8(\mu) \langle \bar{q}q(\mu) \rangle^2$$
$$\langle \mathcal{O}_1(\mu) \rangle \equiv \langle 0 | (s_L \gamma^\mu d_L) (\bar{d}_R \gamma_\mu s_R) | 0 \rangle \quad ; \quad \langle \mathcal{O}_2(\mu) \rangle \equiv \langle 0 | (s_L s_R) (\bar{d}_R d_L) | 0 \rangle$$

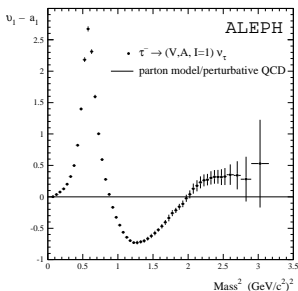
Electroweak Penguins contribute at $O(p^0)$ ($m_q, p \rightarrow 0$)

$$e^2 g_8 g_{ew} F^6 = 6 C_7(\mu) \langle \mathcal{O}_1(\mu) \rangle - 12 C_8(\mu) \langle \mathcal{O}_2(\mu) \rangle \xrightarrow{N_c \rightarrow \infty} -\frac{1}{3} C_8(\mu) \langle \bar{q}q(\mu) \rangle^2$$

$$\langle \mathcal{O}_1(\mu) \rangle \equiv \langle 0 | (s_L \gamma^\mu d_L) (\bar{d}_R \gamma_\mu s_R) | 0 \rangle \quad ; \quad \langle \mathcal{O}_2(\mu) \rangle \equiv \langle 0 | (s_L s_R) (\bar{d}_R d_L) | 0 \rangle$$

These $D=6$ vacuum condensates appear in the left-right correlator:

$$\Pi_{LR}^{\mu\nu}(q) \equiv 2i \int d^4x e^{iqx} \langle 0 | T(L^\mu(x), R^\nu(0)^\dagger) | 0 \rangle \equiv (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_{LR}(-q^2) = \frac{1}{2} \Pi_{VV-AA}^{\mu\nu}(q)$$



$$\Pi_{LR}(Q^2) = \underbrace{\frac{1}{\pi} \int_0^\infty dt \frac{\text{Im} \Pi_{LR}(t)}{t + Q^2}}_{\text{Data}} = \underbrace{\frac{1}{2} \sum_{n=1}^\infty \frac{\langle \tilde{\mathcal{O}}_{2n+4} \rangle}{(Q^2)^{n+2}}}_{\text{QCD OPE}}$$

$$\lim_{Q^2 \rightarrow \infty} -Q^6 \Pi_{LR}(Q^2) = 4\pi\alpha_s \left[4 \langle \mathcal{O}_2 \rangle + \frac{2}{N_C} \langle \mathcal{O}_1 \rangle \right] + O(\alpha_s^2)$$

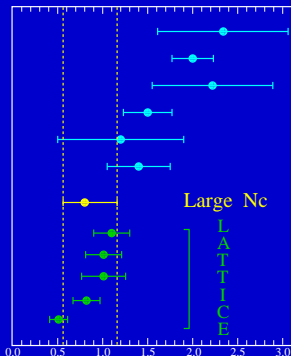
$$\langle \mathcal{O}_1 \rangle = -\frac{i}{2} g_{\mu\nu} \int \frac{d^D q}{(2\pi)^D} \Pi_{LR}^{\mu\nu}(q) \sim \int_0^\infty dQ^2 Q^D \Pi_{LR}(Q^2)$$

Electroweak Penguins contribute at $O(p^0)$ ($m_q, p \rightarrow 0$)

$$e^2 g_8 g_{ew} F^6 = 6 C_7(\mu) \langle \mathcal{O}_1(\mu) \rangle - 12 C_8(\mu) \langle \mathcal{O}_2(\mu) \rangle \xrightarrow{N_c \rightarrow \infty} -\frac{1}{3} C_8(\mu) \langle \bar{q}q(\mu) \rangle^2$$

$$\langle \mathcal{O}_1(\mu) \rangle \equiv \langle 0 | (s_L \gamma^\mu d_L) (\bar{d}_R \gamma_\mu s_R) | 0 \rangle \quad ; \quad \langle \mathcal{O}_2(\mu) \rangle \equiv \langle 0 | (s_L s_R) (\bar{d}_R d_L) | 0 \rangle$$

M_8 (GeV³)



KPdR 01
 FGdR 04
 CDGM 01
 CDGM 03
 BGP 01
 Narison 01

$$M_8 \equiv \langle (2\pi)_{I=2} | Q_8(\mu_0) | K^0 \rangle |_{m_q=p=0}$$

$$= \frac{8}{F^3} \langle \mathcal{O}_2(\mu_0) \rangle$$

$\mu_0 = 2$ GeV

$$M_8 \xrightarrow{N_c \rightarrow \infty} \frac{2}{F^3} \langle \bar{q}q(\mu_0) \rangle^2$$

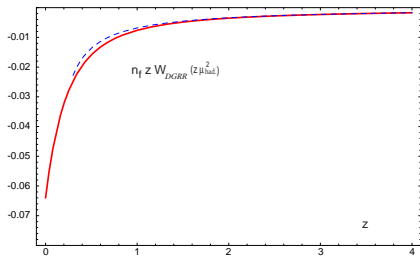
$$\approx \frac{2M_K^4 F^3}{(m_s + m_q)^2(\mu_0) F_\pi^2}$$

n_f/N_C Correction to QCD PENGUIN $(m_q \rightarrow 0)$

Hambye-Peris-de Rafael '03

$$\text{Im}(g_8) \doteq \text{Im}[C_6(\mu)] \left\{ -16L_5 \left(\frac{\langle \bar{q}q \rangle}{F^3} \right)^2 + \frac{8n_f}{16\pi^2 F^4} \int_0^\infty dQ^2 Q^{D-2} \mathcal{W}_{DGRR}(Q^2) \right\}$$

$$\left(\frac{q^\alpha q^\beta}{q^2} - g^{\alpha\beta} \right) \mathcal{W}_{DGRR}(-q^2) = \int d\Omega_q d^4x d^4y d^4z e^{iqx} \langle T [(\bar{s}_L q_R)(x) (\bar{q}_R d_L)(0) (\bar{d}_R \gamma_\alpha u_R)(y) (\bar{u}_R \gamma^\alpha s_R)(z)] \rangle_{\text{con}}$$



Available theoretical information: (poor)

$$\lim_{Q^2 \rightarrow \infty} Q^2 \mathcal{W}_{DGRR}(Q^2) = -\frac{F^4 \pi \alpha_s}{6Q^2} \left[1 - 16L_5 \left(\frac{\langle \bar{q}q \rangle}{F^3} \right)^2 \right]$$

$$\lim_{Q^2 \rightarrow 0} Q^2 \mathcal{W}_{DGRR}(Q^2) = \left(\frac{\langle \bar{q}q \rangle}{F^2} \right)^2 \left\{ \frac{F^2}{8Q^2} - \left(L_5 - \frac{5}{2} L_3 \right) \right\}$$

Big enhancement (~ 3) claimed

Infrared instability from pion pole: $\int_0^\infty dQ^2 \frac{Q^{-\epsilon}}{Q^2 + m_\pi^2} \sim \frac{2}{\epsilon m_\pi^\epsilon}$

Large non-factorizable contribution claimed before

Bardeen et al, Bijnens-Prades

Recent Lattice Results

Isospin limit:

RBC-UKQCD 1505.07863, 1502.00263

$$\sqrt{\frac{3}{2}} \operatorname{Re} A_2 = (1.50 \pm 0.04 \pm 0.14) \cdot 10^{-8} \text{ GeV} \quad \text{exp : } 1.482(2) \cdot 10^{-8} \text{ GeV}$$

$$\sqrt{\frac{3}{2}} \operatorname{Im} A_2 = -(6.99 \pm 0.20 \pm 0.84) \cdot 10^{-13} \text{ GeV}$$

$$\sqrt{\frac{3}{2}} \operatorname{Re} A_0 = (4.66 \pm 1.00 \pm 1.21) \cdot 10^{-7} \text{ GeV} \quad \text{exp : } 3.112(1) \cdot 10^{-7} \text{ GeV}$$

$$\sqrt{\frac{3}{2}} \operatorname{Im} A_0 = -(1.90 \pm 1.23 \pm 1.04) \cdot 10^{-11} \text{ GeV}$$

$$\operatorname{Re}(\varepsilon'/\varepsilon) = (1.38 \pm 5.15 \pm 4.43) \cdot 10^{-4} \quad \text{exp : } (16.8 \pm 1.4) \cdot 10^{-4}$$

$$\delta_0 = (23.8 \pm 4.9 \pm 1.2)^\circ \quad \text{exp : } (39.2 \pm 1.5)^\circ$$

$$\delta_2 = -(11.6 \pm 2.5 \pm 1.2)^\circ \quad \text{exp : } -(8.5 \pm 1.5)^\circ$$


Modelling (some) non-factorizable $1/N_C$ corrections

Buras-Gérard, 1507.06326

$$B_6^{(1/2)} = 1 - \frac{3}{2} \left[\frac{F_\pi}{F_K - F_\pi} \right] \frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2} \ln\left(1 + \frac{\Lambda^2}{\tilde{m}_6^2}\right) = 1 - 0.66 \ln\left(1 + \frac{\Lambda^2}{\tilde{m}_6^2}\right)$$

$$B_8^{(1/2)} = 1 + \frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2} \ln\left(1 + \frac{\Lambda^2}{\tilde{m}_8^2}\right) = 1 + 0.08 \ln\left(1 + \frac{\Lambda^2}{\tilde{m}_8^2}\right)$$

$$B_8^{(3/2)} = 1 - 2 \frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2} \ln\left(1 + \frac{\Lambda^2}{\tilde{m}_8^2}\right) = 1 - 0.17 \ln\left(1 + \frac{\Lambda^2}{\tilde{m}_8^2}\right)$$

 $B_6^{(1/2)} \leq B_8^{(3/2)} < 1$

- FSI ($1/N_C$) not included
- Part of 1-loop χ PT corrections (?)
- Difficult to account in a matching calculation

Summary

Kaons continue providing important physics information:

- Interesting interplay of short and long-distances
- Sensitive to heavy mass scales. **New Physics?**
- Superb probe of flavour dynamics and *CP*
- Excellent testing ground of χ PT dynamics

Increased sensitivities at ongoing experiments ($K \rightarrow \pi \nu \bar{\nu}$)

Theoretical challenge: precise control of QCD effects