Radiative corrections in kaon decays

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- Conclusion

For more (radiative and other kaon decay modes), see

V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, J. Portolés, Rev. Mod. Phys. 84, 399 (2012) [arXiv:1107.6001 [hep-ph]]

Introduction

A lot of progress on the experimental side during the last decade or so (ISTRA+ @ IHEP, KTeV @ FNAL, KLOE @ DAΦNE, NA48, NA48/2 @ SPS), and more is to come, e.g. NA62

Illustration with $K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu$

- Geneva-Saclay high-statistics experiment: $3 \cdot 10^4$ events, a_0 at 20%

L. Rosselet et al., Phys. Rev. D 15, 574 (1977)

- BNL-E865: $4 \cdot 10^5$ events

S. Pislak et al., Phys. Rev. 67, 072004 (2003) [Phys. Rev. 81, 119903 (2010)] [hep-ex/0301040]

- NA48/2: $1.1 \cdot 10^6$ events, a_0 at 6%

J. R. Batley et al., Eur. Phys. J. C 70, 635 (2010)

The experimental values of the two S-wave scattering lengths

$$a_0 = 0.222(14) \qquad a_2 = -0.0432(97)$$

compare quite well with the prediction from two-loop chiral perturbation theory

$$a_0 = 0.220(5) \qquad a_2 = -0.0444(10)$$

G. Colangelo, J. Gasser, H. Leutwyler, Nucl. Phys. B 603, 125 (2001)

But taking isospin corrections ($m_u \neq m_d$ and $M_\pi \neq M_{\pi^0}$) into account turns out to be crucial in order to reach this agreement

J. Gasser, PoS KAON , 033 (2008) [arXiv:0710.3048 [hep-ph]]

Precision measurements on kaon decays allow to put constraints on physics beyond the standard model (tests of lepton flavour universality or of CKM unitarity, CP violation, admixture of right-handed currents,...)...

... but also provide information on low-energy strong interactions (e.g. $\pi\pi$ scattering lengths, structure of form factors,...), that allow to test predictions or to determine non-perturbative parameters (low-energy constants) that occur also in other processes

Predictions are often made in a world where $\alpha = 0$ ($M_\pi = M_{\pi^0}$), and even $m_u = m_d$. One has to connect this “theoretician’s paradise” (J. Gasser) to the real world, where $\alpha \neq 0$ ($M_\pi \neq M_{\pi^0}$)

Although it will not always be mentioned explicitly, only infrared finite radiatively-corrected observables will be considered [in particular, amplitudes include emission of one (soft) photon]

Radiative corrections to decay rates are typically at the level of 1% or 2%

$$\Gamma = \Gamma_0 \left[1 + \alpha \frac{\Delta\Gamma}{\Gamma_0}\right] \qquad \alpha \frac{\Delta\Gamma}{\Gamma_0} \sim \pm(1-2)\%$$

Radiative corrections to *differential* decay rates can, locally, be more important, e.g. $\sim \pm 10$%

$$\frac{d^2\Gamma}{dxdy} = \frac{d^2\Gamma_0}{dxdy}\left[1 + \alpha\delta(x,y)\right] \qquad \alpha\delta(x,y) \sim \pm(1-10)\%$$

General framework

At energies well below the electroweak scale, the weak interactions are described by effective lagrangians involving four-fermion operators

- For the $\Delta S = 1$ non-leptonic transitions:

$$\mathcal{L}_{\rm eff}^{\Delta S=1} = -\frac{G_{\rm F}}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) Q_i(\mu)$$

- $C_i(\mu)$ $\longrightarrow$ perturbative QCD corrections from M_W down to $\mu \lesssim m_c$

- For the semi-leptonic transitions:

$$\mathcal{L}_{\rm eff}^{SL} = -\frac{G_{\rm F}}{\sqrt{2}} \left[\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\right] \left\{V_{ud}\left[\bar{u}\gamma^\mu(1-\gamma_5)d\right] + V_{us}\left[\bar{u}\gamma^\mu(1-\gamma_5)s\right]\right\} + {\rm h.\ c.}$$

- No QCD corrections in $\mathcal{L}_{\rm eff}^{SL}$ $\longrightarrow$ factorized form

For $\mu << \Lambda_{\rm had} \sim 1 \rm GeV$ (where kaon physics takes place), the relevant degrees of freedom are no longer quarks, but the lightest pseudoscalar mesons that become the Goldstone bosons of the spontaneous breaking of chiral symmetry in the limit of massless light quarks $m_{u,d,s} \to 0$

$\longrightarrow$ construct an effective lagrangian that describes the interactions among these pseudoscalar mesons in a systematic low-energy expansion

S. Weinberg, Physica A 96, 327 (1979)

J. Gasser, H. Leutwyler, Annals Phys. 158, 142 (1984); Nucl. Phys. B 250, 465 (1985)

- strong interactions among mesons at low-energies

$$\mathcal{L}^{\rm str} = \mathcal{L}_2^{\rm str}(2) + \mathcal{L}_4^{\rm str}(10+0) + \mathcal{L}_6^{\rm str}(90+23) + \cdots$$

- $\Delta S = 1$ transitions

$$\mathcal{L}_{\rm eff}^{\Delta S=1} \longrightarrow \mathcal{L}_2^{\Delta S=1}(1+1) + \mathcal{L}_4^{\Delta S=1}(22+28) + \cdots$$

J. A. Cronin, Phys. Rev. 161, 1483 (1967)

J. Kambor, J. H. Missimer, D. Wyler, Nucl. Phys. B 346, 17 (1990)

G. Esposito-Farese, Z. Phys. C 50, 255 (1991)

G. Ecker, J. Kambor, D. Wyler, Nucl. Phys. B 394, 101 (1993)

Adding electromagnetic interactions requires to include the photon as a low-energy degree of freedoms (loops involving virtual photons will produce their own divergences, which require additional low-energy constants)...

$$\mathcal{L}^{\text{str;EM}} = \mathcal{L}_2^{\text{str;EM}}(1) + \mathcal{L}_4^{\text{str;EM}}(13+0) + \cdots$$

$$\mathcal{L}_2^{\text{str;EM}} = e^2 C \langle Q U^\dagger Q U \rangle \qquad \mathcal{L}_4^{\text{str;EM}}(13+0) = \sum_{i=1}^{13} K_i \mathcal{O}_i^{\text{str;EM}}$$

G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B 321, 311 (1989)

R. Urech, Nucl. Phys. B 433, 234 (1995)

H. Neufeld, H. Rupertsberger, Z. Phys. C 71, 131 (1996)

$$\mathcal{L}^{\Delta S=1;EM} = \mathcal{L}_2^{\Delta S=1;EM}(1) + \mathcal{L}_4^{\Delta S=1;EM}(14+?) + \cdots$$

$$\mathcal{L}_2^{\Delta S=1;EM} = e^2 G_8 F_0^6 g_{\text{weak}} \langle \lambda_{23} U^\dagger Q U \rangle \qquad \mathcal{L}_4^{\Delta S=1;EM} = e^2 G_8 F_0^4 \sum_{i=1}^{14} Z_i \mathcal{O}_i^{\Delta S=1;EM}$$

J. Bijnens, M. B. Wise, Phys. Lett. B 137, 245 (1984)

G. Ecker, G. Isidori, Müller, H. Neufeld, A. Pich, Nucl. Phys. 591, 1419 (2000)

... as well as the light leptons (for the description of radiative corrections to semi-leptonic decays)

$$\mathcal{L}^{\text{lept}} = \mathcal{L}_2^{\text{lept}}(0) + \mathcal{L}_4^{\text{lept}}(5) + \cdots \qquad \mathcal{L}_4^{\text{lept}} = \sum_{i=1}^{5} X_i \mathcal{O}_i^{\text{lept}}$$

M. Knecht, H. Neufeld, H. Rupertsberger, P. Talavera, Eur. Phys. J. C 12, 469 (2000)

Determination of low-energy constants

• K_i

- identify the corresponding QCD correlators (two-, three- and four-point functions), convoluted with the free photon propagator

- study their short-distance behaviour

- write spectral sum rules

- saturate with lowest-lying narrow-width resonances

B. Moussallam, Nucl. Phys. B 504, 391 (1997) [hep-ph/9701400]

B. Ananthanarayan, B. Moussallam, JHEP06, 047 (2004) [hep-ph/0405206]

Analogous to the DGMLY sum-rule for C

$$C = -\frac{1}{16\pi^2}\frac{3}{2\pi}\int_0^\infty ds\, s\, \ln\frac{s}{\mu^2}\left[\rho_{VV}(s) - \rho_{AA}^{\pi-\text{polesubt.}}(s)\right]$$

T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low and J. E. Young, Phys. Rev. Lett. 18, 759 (1967)

B. Moussallam, Eur. Phys. J. C 6, 681 (1999) [hep-ph/9804271]

Determination of low-energy constants

• X_i

- two-step matching procedure:

i) compute radiative corrections to $\bar{q}q' \to \ell\nu$ in the SM and in the four-fermion theory

ii) match the radiatively corrected four-fermion theory to the chiral lagrangian, by identifying the QCD correlators (convoluted with the free photon propagator) that describe the X_i's
Saturate the resulting spectral sum rules with lowest-lying resonance states

S. Descotes-Genon, B. Moussallam, Eur. Phys. J. C 42, 403 (2005) [hep-ph/0505077]

• $\mathrm{g}_{\mathrm{weak}}$ and Z_i

Have been estimated in the large-N_c limit

W. A. Bardeen, A. J. Buras, J.-M. Gérard, Nucl. Phys. B 293, 787 (1987)

A. J. Buras, J.-M. Gérard, Phys. Lett. B 192, 156 (1987)

V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, Eur. Phys. J. C 33, 269 (2004)

For instance

$$(g_8 e^2 \mathrm{g}_{\mathrm{weak}})^{\infty} = -\left(\frac{\langle\bar{\psi}\psi\rangle}{F_0^3}\right)^2 \left[3C_8(\mu) + \frac{16}{3}e^2 C_6(\mu)(K_9 - 2K_{10})\right]$$

Determination of low-energy constants

The dependence on the short-distance scale vanishes at leading-order in the large-N_c limit. A scale dependence remains at subleading order in $1/N_c$. The (subleading order) contribution of Q_7 can also be computed,

$$(g_8 e^2 \mathrm{g}_{\mathrm{weak}})^{1/N_c;Q_7} = -\frac{9}{8\pi^2} C_7(\mu) \frac{M_\rho^2}{F_0^2} \left[\ln \frac{\mu^2}{M_\rho^2} + \frac{1}{3} - 2 \ln 2 \right]$$

M. Knecht, S. Peris, E. de Rafael, Phys. Lett. B 457, 227 (1999)

but this does not completely remove the residual scale dependence

Radiative corrections in non-leptonic kaon decay modes

Radiative corrections in $K \to \pi\pi$ amplitudes

parametrizations of the amplitudes

$$
\begin{aligned}
A_{+-} &= A_0 e^{i\chi_0} + \frac{A_2}{\sqrt{2}} e^{i\chi_2} = \bar{A}_0 e^{i\delta_0} + \frac{\bar{A}_2}{\sqrt{2}} e^{i\delta_2} + \Delta A_{+-} \\
A_{00} &= A_0 e^{i\chi_0} - \sqrt{2} A_2 e^{i\chi_2} = \bar{A}_0 e^{i\delta_0} - \sqrt{2} \bar{A}_2 e^{i\delta_2} + \Delta A_{00} \\
A_{+0} &= \frac{3}{3} A_2^+ e^{i\chi_2^+} = \frac{3}{2} \bar{A}_2 e^{i\delta_2} + \Delta A_{+0}
\end{aligned}
$$

Calculation at one loop, including isospin-breaking and radiative corrections

J. Kambor, J. H. Missimer, D. Wyler, Phys. Lett. B 261, 496 (1991)

V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, Eur. Phys. J. C 33, 269 (2004)

V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, Phys. Lett. B 679, 445 (2009)

In the isospin limit, and if CP violating effects are neglected, one has $A_2^+ = A_2$, $\chi_{0,2} = \delta_0^{0,2}(M_K)$ and $\Delta A_x = 0$

Radiative corrections in $K \to \pi\pi$ amplitudes

Using the experimental experimental decay rates and $\delta_0^0(M_K) = (39.2 \pm 1.5)^{\mathrm{o}}$, one extracts

$$\delta_0^0(M_K) - \delta_0^2(M_K) = (52.5 \pm 0.8_{\mathrm{th}} \pm 2.8_{\mathrm{th}})^{\mathrm{o}}$$

in reasonable agreement with the value

$$\delta_0^0(M_K) - \delta_0^2(M_K) = (47.4 \pm 0.4)^{\mathrm{o}}$$

obtained from Roy equations and K_{e4} data, given the theoretical uncertainties

Radiative corrections in $K \to \pi\pi\pi$

Important experimental feature: cusp at $M_{00} = 2M_\pi$ in the invariant mass distribution of the two neutral pions of $K \to \pi\pi^0\pi^0$

First observed by NA48/2 in a sample of $2.3 \cdot 10^7$ $K^\pm \to \pi^\pm\pi^0\pi^0$

J. R. Batley et al., Phys. Lett. B 633, 176 (2006)

Correctly interpreted as a rescattering effect $\pi^+\pi^- \to \pi^0\pi^0$ ($M_\pi \neq M_{\pi^0}$), corresponding to the combination $a_0 - a_2$ of S-wave scattering lengths

N. Cabibbo, Phys. Rev. Lett. 93, 121801 (2004)

But simple phenomenological parametrizations

N. Cabibbo and G. Isidori, JHEP0503, 021 (2005) [hep-ph/0502130]

E. Gamiz, J. Prades and I. Scimemi, Eur. Phys. J. C 50, 405 (2007) [hep-ph/0602023]

or one-loop ChPT calculations including isospin breaking

J. Bijnens, F. Borg, Nucl Phys. B 697, 319 (2004); Eur. Phys. J. C **39**, 347 (2005); Eur. Phys. J. C 40, 383 (2005)

either do not give the correct analyticity properties or do not give a sufficiently accurate description of the cusp

Radiative corrections in $K \to \pi\pi\pi$

Better description obtained by combining a non relativistic EFT framework

$$|\mathbf{p}|/M_\pi \sim \mathcal{O}(\epsilon)$$

and a *systematic* expansion in powers of the scattering lengths, including orders ϵ^2, $a\epsilon^3$, $a^2\epsilon^2$

G. Colangelo, J. Gasser, B. Kubis and A. Rusetsky, Phys. Lett. B 638, 187 (2006) [hep-ph/0604084]

J. Gasser, B. Kubis and A. Rusetsky, Nucl. Phys. B 850, 96 (2011) arXiv:1103.4273 [hep-ph]

Radiative corrections were also included

M. Bissegger, A. Fuhrer, J. Gasser, B. Kubis and A. Rusetsky, Nucl. Phys. B 806, 178 (2009) arXiv:0807.0515 [hep-ph]

$$\longrightarrow a_0 - a_2 = 0.2571 \pm 0.0056$$

J. R. Batley et al, Eur. Phys. J. C 64, 589 (2009)

Later also observed by KTeV in a sample of $6.8 \cdot 10^7$ $K_L \to \pi^0\pi^0\pi^0$ events but the rescattering effect is quite smaller

$$a_0 - a_2 = 0.215 \pm 0.031$$

E. Abouzaid et al., Phys. Rev. D 78, 032009 (2008)

Radiative corrections in semi-leptonic kaon decay modes

$M_\pi \neq M_{\pi^0}$ effects in the phases of $K_{\ell 4}$ form factors

Standard angular analysis of the K_{e4}^{+-} form factors provides information on low-energy $\pi\pi$ scattering (Watson's theorem) through the phase difference

$$[\delta_S(s) - \delta_P(s)]_{\text{exp}}$$

N. Cabibbo, A. Maksymowicz, Phys. Rev. B 137, 438 (1965); Erratum-ibid 168, 1926 (1968)

F.A. Berends, A. Donnachie, G.C. Oades, Phys. Rev. 171, 1457(1968)

measurable in the interference of the F^{+-} and G^{+-} form factors.

Comparison with solutions of the Roy equations

$$[\delta_S(s) - \delta_P(s)]_{\text{exp}} = f_{\text{Roy}}(s; a_0^0, a_0^2)$$

allows to extract the values of the $\pi\pi$ S-wave scattering lengths in the isospin channels $I = 0, 2$

$f_{\text{Roy}}(s; a_0^2, a_0^2)$ follows from:

- dispersion relations (analyticity, unitarity, crossing, Froissard bound)
- $\pi\pi$ data at energies $\sqrt{s} \geq 1$ GeV
- isospin symmetry

S.M. Roy, Phys. Lett. B 36, 353 (1971)

Solutions can be constructed for $(a_0^0, a_0^2) \in$ Universal Band

B. Ananthanarayan, G. Colangelo, J. Gasser, H. Leutwyler, Phys. Rep. 353, 207 (2001)

$M_\pi \neq M_{\pi^0}$ effects in the phases of $K_{\ell 4}$ form factors

Once radiative corrections have been taken care of, it is still important to take isospin-breaking corrections due to $M_\pi \neq M_{\pi^0}$ into account before analysing data

J. Gasser, PoS KAON, 033 (2008)

Evaluation of IB corrections in ChPT

G. Colangelo, J. Gasser, A. Rusetsky, Eur. Phys. J. C 59, 777 (2009)

$$\longrightarrow a_0^0 = 0.2220(128)_{\text{stat}}(50)_{\text{syst}}\,(37)_{\text{th}} \qquad a_0^2 = -0.0432(86)_{\text{stat}}(34)_{\text{syst}}(28)_{\text{th}}$$

However, IB corrections were evaluated at fixed values of the scattering lengths

$$[\delta_S(s) - \delta_P(s)]_{\text{exp}} = f_{\text{Roy}}(s; a_0^0, a_0^2) + \delta f_{\text{IB}}(s; (a_0^0)^{\text{LO}}_{\text{ChPT}}, (a_0^2)^{\text{LO}}_{\text{ChPT}})$$

Drawback shared by other studies devoted to isospin breaking in ChPT (QCD+QED)

V. Cuplov, PhD thesis (2004); V. Cuplov, A. Nehme, hep-ph/0311274

A. Nehme, Nucl. Phys. B 682, 289 (2004)

P. Stoffer, Eur. Phys. J. C 74, 2749 (2004)

Is it possible to obtain

$$[\delta_S(s) - \delta_P(s)]_{\text{exp}} = f_{\text{Roy}}(s; a_0^0, a_0^2) + \delta f_{\text{IB}}(s; a_0^0, a_0^2) \quad ?$$

What is the quantitative effect in the determination of the scattering lengths?

$M_\pi \neq M_{\pi^0}$ effects in the phases of $K_{\ell 4}$ form factors

Adapt the approach (“reconstruction theorem”) described in

[J. Stern, H. Sazdjian, N. H. Fuchs, Phys. Rev. D 47, 3814 (1993), arXiv:hep-ph/9301244]

for the $\pi\pi$ scattering amplitude, and implemented in

[M. Knecht, B. Moussallam, J. Stern, N.H. Fuchs, Nucl. Phys. B 457, 513 (1995), arXiv:hep-ph/9507319]

Rests on very general principle

a) Relativistic invariance

b) Analyticity, unitarity, crossing

c) Chiral counting

Note: isospin symmetry not required

$\longrightarrow \delta f_{\text{IB}}(s; a_0^0, a_0^2)$ worked out at NLO

S. Descotes-Genon, M. K., Eur. Phys. J. C 72, 1962 (2012) [arXiv:1202.5886 [hep-ph]]

V. Bernard, S. Descotes-Genon, M. K., Eur. Phys. J. C 73, 2478 (2013) [arXiv:1305.3843 [hep-ph]]

Re-analysis of NA48/2 data

$$a_0^0 = 0.221 \pm 0.018 \qquad a_0^2 = -0.0453 \pm 0.0106$$

to be compared to

$$a_0^0 = 0.2220(128)_{\text{stat}}(50)_{\text{syst}}\,(37)_{\text{th}} \qquad a_0^2 = -0.0432(86)_{\text{stat}}(34)_{\text{syst}}(28)_{\text{th}}$$

Radiative corrections to the K_{e4}^{00} decay rate

NA48/2: $\sim 65\,100\ K_{e4}^{00}$ events

J.R. Batley et al., JHEP 1408, 159 (2014)

In the isospin limit, one form factor is common to K_{e4}^{+-} and K_{e4}^{00} ($F^{+-} = F^{00}$).

This can be tested with the available data:

$$\begin{aligned} |V_{us}|f_s[K_{e4}^{+-}] &= 1.285 \pm 0.001_{\rm stat} \pm 0.004_{\rm syst} \pm 0.005_{\rm ext} \\ (1+\delta_{EM})|V_{us}|f_s[K_{e4}^{00}] &= 1.369 \pm 0.003_{\rm stat} \pm 0.006_{\rm syst} \pm 0.009_{\rm ext} \end{aligned}$$

$$\longrightarrow \quad (1+\delta_{EM})\frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]} = 1.065 \pm 0.010$$

δ_{EM} not known (apart from B. Morel, Quoc-Hung Do, Nuovo Cim. A 46, 253 (1978))

Radiative corrections to the K_{e4}^{00} decay rate

At lowest-order in ChPT

$$\frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]} = \left(1 + \frac{3}{2R}\right) \sim 1.040 \qquad R \equiv \frac{m_s - m_{ud}}{m_d - m_u} \sim 36$$

V. Cuplov, PhD thesis (2004); V. Cuplov, A. Nehme, hep-ph/0311274

A. Nehme, Nucl. Phys. B 682, 289 (2004)

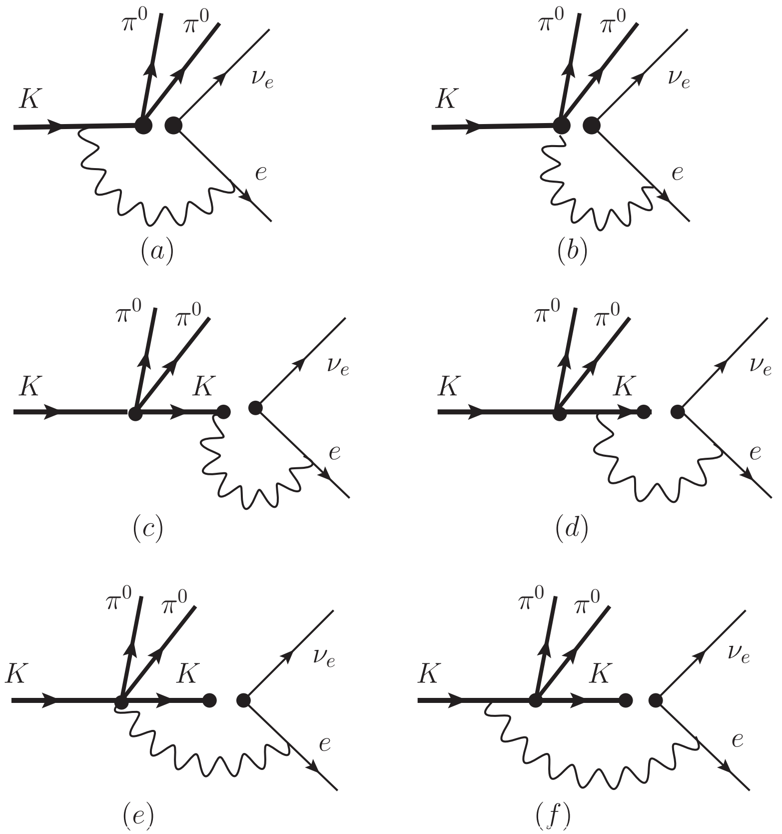
$\longrightarrow \delta_{EM}$ has to explain $\sim 1/3$ of the effect

Asymmetric treatment of the NA48/2 data as far as radiative corrections are concerned:

- K_{e4}^{+-} $\longrightarrow$ Sommerfeld-Gamow-Sakharov factors and PHOTOS for photon emission + w.f. factors of QED, treating the mesons as pointlike
- K_{e4}^{00} $\longrightarrow$ no radiative corrections applied (S-G-S factors not relevant)

Main issue: radiative corrections have been applied to K_{e4}^{+-} data. Computation of δ_{EM} should be carried out within the same framework as used there in order to make comparison meaningful

Size of δ_{EM}? $\longrightarrow$ what does PHOTOS contain ?



Non factorizable radiative corrections

Besides w.f. factors of QED, only diagram (a) is considered in a PHOTOS-like treatment of radiative corrections [diagrams (b), (c), and (d) vanish for $m_e \to 0$]

Adding the diagrams for the emission of a soft photon, one obtains

$$\Gamma^{\text{tot}} = \Gamma(K_{e4}^{00}) + \bar{\Gamma}^{\text{soft}}(K_{e4\gamma}^{00}) = \Gamma_0(K_{e4}^{00}) \times (1 + 2\delta_{EM})$$

with $\delta_{EM} = 0.018 \longrightarrow \frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]} = 1.065 \pm 0.010 - 0.018 \sim \left(1 + \frac{3}{2R}\right)$

V. Bernard, S. Descotes-Genon, M. K., Eur. Phys. J. C 75, 145 (2015) [arXiv:1501.07102 [hep-ph]]

Radiative corrections to the K_{e4}^{00} decay rate

what does PHOTOS for K_{e4}^{+-} contain?
(apart from generation of emission of photons from charged legs)

Q. Xu, Z. Was, Chin. Phys. 34, 889 (2010)
Y.M. Bystritskiy, S.R. Gevorkyan, E.A. Kuraev, Eur. Phys. J. C 64, 47 (2009). arXiv:0906.0516 [hep-ph]

- wave-function renormalization of external charged legs
- vertex diagram (a) [with contribution from form factor R neglected, although it is not suppressed by m_e in this case]

$\longrightarrow$ dependence on UV cut-off, reabsorbed into Fermi constant
(but will be different in K_{e4}^{00}...)

The pion β decay

$\pi^\pm \to \pi^0 e^\pm \nu$

Possible source of information on $|V_{ud}|$

Many advantages:

- pure vector transition (like super-allowed Fermi transitions, but in contrast to neutron β decay)

- no problem with nuclear transition matrix elements in evaluation of radiative corrections (like neutron β decay, but in contrast to super-allowed Fermi transitions)

- protected from first-order isospin breaking corrections (Ademollo-Gatto-Sirlin theorem)

- can be evaluated in two-flavour low-energy EFT: $f_+^{\pi_\beta}(0) = 1.0046(5)$

V. Cirigliano, M. Knecht, H. Neufeld, H. Pichl, Eur. Phys. J. C 27, 255 (2003) [hep-ph/0209226]

$\longrightarrow$ cleanest way to extract V_{ud}

Serious drawback: $\Gamma_{\pi_\beta}/\Gamma_{\mathrm{tot}} \sim 1 \cdot 10^{-8}$

PIBETA exp. at PSI: $\Gamma_{\pi_\beta}/\Gamma_{\mathrm{tot}} = [1.036 \pm 0.004_{\mathrm{stat}} \pm 0.004_{syst} \pm 0.003_{\pi_{e2}}] \cdot 10^{-8}$

$\sim 10^6 \pi^+/\mathrm{sec}, 6.4 \cdot 10^4$ events $\longrightarrow V_{ud}^{\mathrm{PIBETA}} = 0.9728(30)$

D. Pocanić et al. (PIBETA Coll.), Phys. Rev. Lett. 93, 181803 (2004) [hep-ex/0312030]

Conclusions

High precision reached by the data concerning non-leptonic and semi-leptonic decay modes of the kaons has made the treatment of isospin-breaking effects ($m_u \neq m_d$ and $\alpha \neq 0$) unavoidable

A lot of activity has been going on, extending the scope of the low-energy EFT in order to meet this necessity (inclusion of photons, leptons). Only a tiny fraction of the numerous applications has been mentioned here

The issue of additional low-energy constants has been dealt with in a rather satisfactory manner (progress on estimates of the Z_i's would be welcome, though)

The effects due to $M_\pi \neq M_{\pi^0}$ are important (especially for $K \to \pi\pi\pi$ and for K_{e4}). ChPT at NLO is no longer sufficient.
$\longrightarrow$ This issue can be dealt with through more elaborated/adapted approaches, like NREFT, dispersive representations,...

Watch out for possible biases if the radiative corrections to form factors and/or decay distributions are given for fixed values of the parameters one actually wants to extract from data
$\longrightarrow$ Not the case for $\delta_0(s) - \delta_1(s)$ extracted from K_{e4}

Treatment of radiative corrections in K_{e4} rather rudimentary and does not match the quality of the data $\longrightarrow$ Improvements should be possible