Dispersive Work on Radiative K-decays

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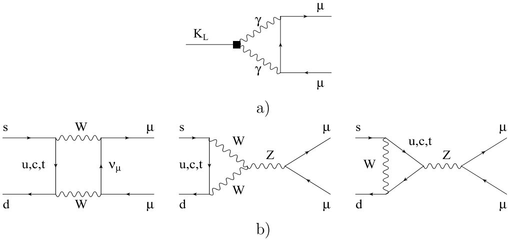
Outline

1. Introduction
2. $K_S \to \gamma^*\gamma^*$
3. Applications
4. Outlook and conclusion

Introduction

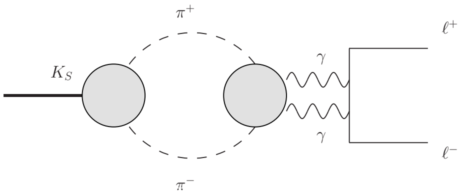
- The rare decays $K_{L,S} \to \ell^+ \ell^-$ are a very usefull source of information on the structure of $\Delta S = 1$ flavor-changing-neutral-current (FCNC) transition.
- Both decays contain long and short distance compoments
- The K_L decays have been measured, the K_S decays not yet
- $B(K_L \to \mu^+ \mu^-) = 6.84 * 10^{-9}$
  $B(K_L \to e^+ e^-) = 9 * 10^{-12}$

$K_L \to \mu^+ \mu^-$



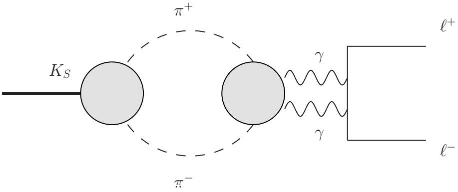
Leading long-distance *a*) and short-distance contributions b)
[Isidori, Unterdorfer 03]

- $K_S \to \ell^- \ell^+$



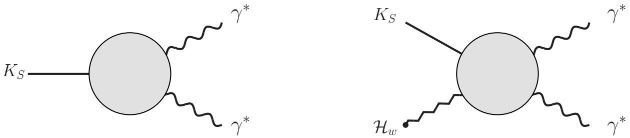
- Not yet observed
- LHCb: $Br(K_S \to \mu^+ \mu^-) < 9 \cdot 10^{-9}$ $[Aaij\ et.\ al. 13]$
- Standart model prediction $Br(K_S \to \mu^+ \mu^-) = 5 \cdot 10^{-12}$ $[Ecker, Pich, 91]$

- The K_S decay contains the CP-violating part of the FCNC $s \to d\ell^+\ell^-$
- This transition is given in the Standart Model (SM) by box and penguin diagrams
- $B(K_S \to \mu^+\mu^-)^{SM}_{short} = 10^{-5} \mid \mathrm{Im}(V^*_{ts} V_{td}) \mid^2 \quad \simeq \mathcal{O}(10^{-13})$
- Measurement of $K_S \to \mu^+\mu^-$
  $\to$ New Physics
  $\to$ Bounds on CP-violating phase of $s \to d\ell^+\ell^-$



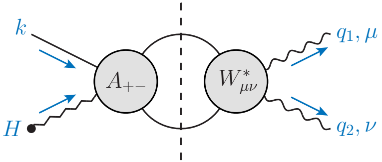
- Lowest order in χPT is the 2 loop diagram. [*Ecker*, *Pich*, 91]
- Next order in χPT involves a lot of unknown LEC's
- Includes the sub amplitude $K_S \to \gamma^*\gamma^*$ with both photons off-shell
- The momenta flowing through the diagram is $s = M_K^2$, which is large enough to produce two on-shell pions. Therefore we expect large corrections due to final state interactions.
- We use dispersive techniques to improve the calculation

$K_S \to \gamma^*\gamma^*$



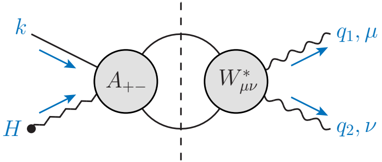
- In a decay kinematics are fixed
- How to formulate a dispersion relation? Integrate over the kaon mass?
- Dispersion relations relate different observable quantities
- Solution is to give the weak Hamiltonian momentum

$$\langle \gamma^*(q_1)\gamma^*(q_2,)|\mathcal{H}_w(H)|K_S(k)\rangle = i(2\pi)^4 e^2 \delta^4(k+H-q_1-q_2)\epsilon_1^{*\mu}\epsilon_2^{*\nu} A_{\mu\nu}$$



$$s = (k+H)^2 = (q_1+q_2)^2$$

Work done in collaboration with
G.Colangelo and L.Tunstall



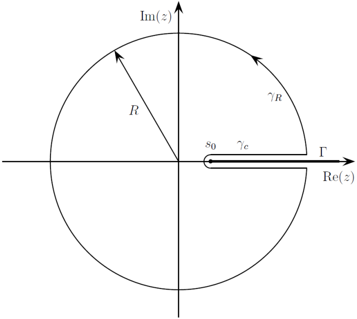
- $\mathcal{H}_w(H)$ is the weak Hamiltonian with $\Delta s = 1$
- The Hamiltonian has to carry momentum H, such that we can formulate a dispersion relation
- Sending $H \to 0$ yields the physical amplitude

- $A_{\mu\nu} = \Sigma_{i=1}^{5} \, T_{\mu\nu}^{i} \, A_i$
- To get these Lorentz structures, one writes down all possible terms with indices μ, ν which are built of (q_1, q_2, k) $\rightarrow$10 terms
- Impose the ward-identities $q_1^{\mu} A_{\mu\nu} = q_2^{\nu} A_{\mu\nu} = 0$
- Then one has to make sure that there are no kinematic zeros and poles in the Lorentz structures $T_{\mu\nu}^{i}$
- Finally one has five scalar functions A_i which are suitable for dispersion relations
- The limit $H \rightarrow 0$ reduces the set to two structures
  $T_{\mu\nu}^{1} = q_1 \cdot q_2 g_{\mu\nu} - q_{2,\mu} q_{1,\nu}$
  $T_{\mu\nu}^{2} = q_1^2 q_2^2 g_{\mu\nu} + q_1 \cdot q_2 q_{1,\mu} q_{2,\nu} - q_1^2 q_{2,\mu} q_{2,\nu} - q_2^2 q_{1,\mu} q_{1,\nu}$

Dispersion relation

Cauchy's theorem:

$$f(z) = \frac{1}{2\pi i} \oint \frac{f(z')}{z' - z} dz'$$



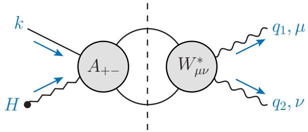
$$f(s) = \frac{1}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im} f(s')}{s' - s - i\epsilon} ds$$

- In cases when $f(s)$ does not fall down fast enough for $s \to \infty$ one uses subtractions

$$f(s) = f(s_0) + \frac{s - s_0}{\pi} \int_{4M_\pi^2}^{\Lambda^2} \frac{\mathrm{Im} f(s')}{(s' - s_0)(s' - s - i\epsilon)} ds$$

The dispersion relation yields the full amplitude, provided one knows the imaginnary part. The latter is related by unitarity to subamplitudes.

$$\begin{aligned}
\mathrm{Im} A_{\mu\nu} &= \frac{1}{2} \int d\phi_2 \, A_{+-}(s) \, W^*_{\mu\nu}(s, q_1^2, q_2^2) \\
A_{+-} &= \langle \pi^+ \pi^- | \mathcal{H}_w | K_S \rangle \\
\epsilon^\mu \epsilon^\nu W_{\mu\nu} &= \langle \gamma^* \gamma^* | \pi^+ \pi^- \rangle
\end{aligned}$$



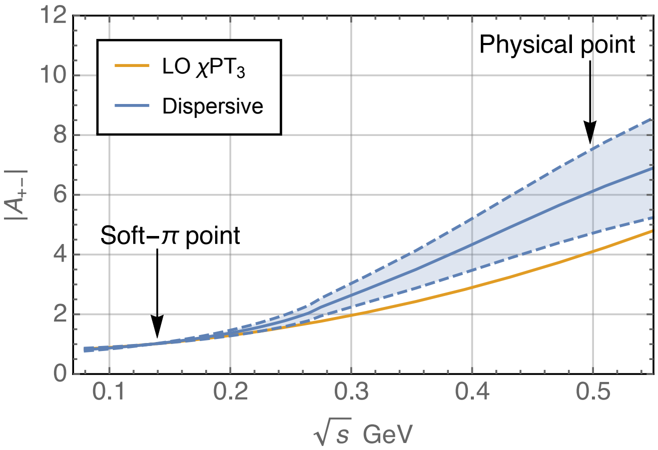
- The amplitude $K \to \pi\pi$ was calculated as a scattering process with dispersion relations
  [Büchler et. al.,01],[Mercolli, 12]
- Since it is a scattering proces it makes sense to do a partial wave expansion
  $\langle \pi^+\pi^- | \mathcal{H}_w | K_S \rangle = A_{+-}(s,t,u) = \Sigma_\ell a_\ell^0(s)\, P_\ell(z)$
- They found that only the s-wave is important and all the higher order waves can be neglected

- $A_{+-}(s) = c_0\, \Omega_0^0(s) \left(1 + F[X] \frac{s}{M_K^2}\right)$
- With the Omnes function

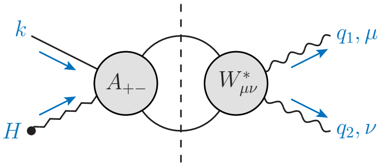
$$\Omega_0^0(s) = exp\left(\frac{s}{\pi} \int_{4M\pi^2}^{\Lambda^2} \frac{\delta_0^0(s')}{s'(s'-s)} ds'\right)$$

- $\delta_0^0(s')$ are the phase shifts of of $\pi\pi$ scattering.
- $A_{+-}^{\chi PT} = \frac{G_8 F_\pi}{4}(M_K^2 + 3s - 4M_\pi^2)$
- $F[X]$ estimates the errors of higher chiral orders and the value of X is typically veried between ± 0.3

$$F[X] = \frac{3M_K^2(1+X)}{M_K^2 - M_\pi^2(4+3X)}$$



- $-\frac{1}{2F_\pi} A(K \to \pi) = A_{+-}(M_\pi^2, M_K^2, M_\pi^2) + \mathcal{O}(M_\pi^2)$



$$
\begin{aligned}
\mathrm{Im} A_{\mu\nu} &= \frac{1}{2} \int d\phi_2 \, A_{+-}(s) \, W^*_{\mu\nu}(s, q_1^2, q_2^2) \\
A_{+-} &= \langle \pi^+ \pi^- | \mathcal{H}_w | K_S \rangle \\
\epsilon^\mu \epsilon^\nu W_{\mu\nu} &= \langle \gamma^* \gamma^* | \pi^+ \pi^- \rangle
\end{aligned}
$$

- The amplitude $\pi\pi \to \gamma\gamma$ can be decomposed into five lorentz structures times scalar functions $W_{\mu\nu} = \Sigma_{i=1}^{5} T^i_{\mu\nu} W_i$
- The integration over the two body phase space contains angular integrations, which projects the Amplitude onto helicity partial waves

$$h^0_{++}(s, q_1^2, q_2^2) = \frac{1}{2} \int_{-1}^{1} dcos\theta \, \epsilon_+^\mu \epsilon_+^\nu W_{\mu\nu}(s, q_1^2, q_2^2) \, .$$

- We need the s-waves

$$\begin{aligned} W_1 &= \frac{2}{\lambda(s, q_1^2, q_2^2)} \left[2\sqrt{q_1^2 q_2^2} h^0_{00}(s) - (s - q_1^2 - q_2^2) h^0_{++}(s) \right] , \\ W_2 &= \frac{2}{\lambda(s, q_1^2, q_2^2)} \left[-\frac{s - q_1^2 - q_2^2}{\sqrt{q_1^2 q_2^2} h^0_{00}(s) + 2 h^0_{++}(s)} \right] , \\ W_3 &= W_4 = W_5 = 0 \, . \end{aligned}$$

$$
\begin{aligned}
\mathrm{Im} A_{\mu\nu}(s) &= \frac{1}{16\pi}\sqrt{1-\frac{4M_\pi^2}{s}} A_{+-}(s)\left(T^1_{\mu\nu} W_1^* + T^2_{\mu\nu} W_2^*\right) \\
&= T^1_{\mu\nu}\mathrm{Im} A_1 + T^2_{\mu\nu}\mathrm{Im} A_2
\end{aligned}
$$

- A once subtracted dispersion relation yields the full amplitude

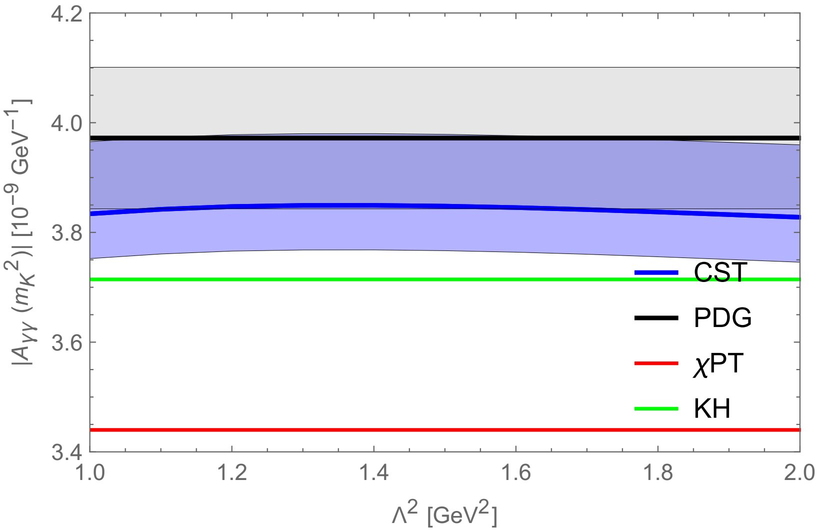
$$
A_i = a_i + \frac{s-s_0}{\pi}\int_{4M_\pi^2}^{\Lambda^2} ds' \frac{\mathrm{Im} A_i(s')}{(s'-s_0)(s'-s)}
$$

Applications $K_S \to \gamma\gamma$

- $q_1^2 = q_2^2 = 0\,, \quad \Gamma = \frac{M_K^3}{32\pi}|4\pi\alpha A_1|^2$
- How do we fix the subtraction constant a_1?
- Use χPT below threshold, where the rescattering effects are small
- $A_{\gamma\gamma}^{\chi PT}(s) = \frac{\alpha}{2\sqrt{2}\pi} G_8 F_\pi (M_K^2 + 3s - 4M_\pi^2) F(s, M_\pi^2) - [M_\pi^2 \to M_K^2]$
- $F(s, M^2)$ is a loop function
- Solve numerically for $A_{\gamma\gamma}^{\chi PT}(s_0) = 0$
  $\to s_0 = -0.098\,[Gev^2]$
- Match leading order χPT and dispersive representation at s_0
  $\to a_1 = 0$

- $\mathrm{Im} A_1(s) = \frac{-1}{16\pi s}\sqrt{1 - \frac{4M_\pi^2}{s}}\Omega(s)c_0 \left(1 + F[X]\frac{s}{M_K^2}\right)|h^0_{++}(s)|^*$
- $A_1(s) = \frac{s-s_0}{\pi}\int_{4M_\pi^2}^{\Lambda^2} ds' \frac{\mathrm{Im} A_1(s')}{(s'-s_0)(s'-s)}$
- The missing ingridiant is the helicity partial wave $h^0_{++}(s)$
- This function can be obtained by an Mushkhelishvili-Omnes analysis of the experimental data of $(e^+e^- \to \gamma^*\gamma^* \to \pi^+\pi^-)$. [*Garcia*, *Moussallam*, 10]

Cut-off dependence of $A_{\gamma\gamma}(K_S \to \gamma\gamma)$ and variation of $X = \pm 0.3$

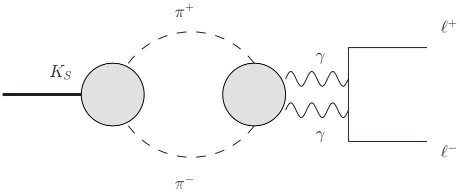


$\times[10^{-9}\ \text{GeV}^{-1}]$	CST ($\Lambda = \sqrt{2}$ GeV)	χPT_3	PDG
Re $A_{\gamma\gamma}$	$-2.93^{+0.11}_{-0.17}$	-1.68	–
Im $A_{\gamma\gamma}$	2.46	2.97	–
$\lvert A_{\gamma\gamma}\rvert$	$3.83^{+0.13}_{-0.08}$	3.41	3.97 ± 0.13
Br $[10^{-6}]$	$2.45^{+0.16}_{-0.11}$	2.0	2.63 ± 0.17

- $K_S \to \gamma\ell^+\ell^-$
- Need as input $h^0_{++}(s, q_1^2, 0)$ [*Garcia*, *Moussallam*, 10]
- Work in progress
- Not measured yet. χPT predicts

$$\frac{B(K_S \to \gamma e^+ e^-)}{B(K_S \to \gamma\gamma)} = 1.6{\cdot}10^{-2}\,, \quad \frac{B(K_S \to \gamma\mu^+\mu^-)}{B(K_S \to \gamma\gamma)} = 3.75{\cdot}10^{-4}$$

- $K_S \to \ell^+\ell^-$
- Need to know $h^0_{++}(s, q_1^2, q_2^2)$.
- This helicity partial wave also enters in the calculation of the light by light contribution in $g-2$. People are working on it.



Outlook and conclusion

- Formula for $K_S \to \gamma^* \gamma^*$
- On-shell prediction consistent with experimental value and improves the χPT value
- Uncertainties ($X = \pm 0.3$) can be reduced by better knowledge of some LEC's. Lattice?
- Calculation based on dispersion relations and χPT.
- Apply the formalism to $K_S \to \ell^+ \ell^-$, need h^0_{++} for arbitrary photon momenta q_1, q_2
- $Br(K_S \to \mu^+ \mu^-)_{exp} = 9 \cdot 10^{-9}$
  $Br(K_S \to \mu^+ \mu^-)_{\chi PT} = 5 \cdot 10^{-12}$