Isospin & Electromagnetic Corrections to Weak Matrix Elements

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DIPARTIMENTO DI FISICA

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Mainz Institute for Theoretical Physics

NA62 Kaon Physics Handbook

PLAN OF THE TALK

1) Physics Motivations

2) Lattice Calculations of QED corrections to the hadron Spectrum

4) QED corrections to the hadronic amplitudes

5) $\pi^+ \rightarrow \mu^+ \nu_\mu \ (\gamma)$

6) Conclusion & Outlook

work done in collaboration with N.Carrasco, V.Lubicz, C.T.Sachrajda, F.Sanfillipo, S. Simula, N.Tantalo, C.Tarantino, M.Testa

Physics Motivations: Flavor and New Physics

flavor physics can be used in two ways:

1. “New Physics Reconstruction”

- *an external information on the NP scale is required (i.e. LHC);*
- *precision flavour physics will be necessary to understand the underlying framework;*
- *the main tool are correlations among observables;*
- *needs good theoretical control on uncertainties of both SM and NP contributions;*

2. “Discovery”

- *looks for deviation from the SM whatever the origin is;*
- *needs good theoretical control of the SM contribution only;*
- *in general cannot provide precise information on the NP scale, but a positive result would be a strong evidence that NP is not too far (i.e. in the multi-TeV region);*

The accuracy of lattice calculations of the hadron spectrum (and hence of the quark masses) and of the decay constants and form factors is such that isospin breaking effects cannot be neglected anymore:

FLAG Collaboration, arXiv:1310.8555

$N_f = 2$ $m_{ud} = 3.6(2)$ MeV $m_s = 101(3)$ MeV

$m_s/m_{ud} = 28.1(1.2)$ $\varepsilon = 3\%\text{-}6\%$

$N_f = 2+1$ $m_{ud} = 3.42(6)(7)$ MeV $m_s = 93.8(1.5)(1.9)$ MeV

$m_s/m_{ud} = 27.45(15)(41)$

$f_\pi = 130.2(1.4)$ MeV $f_K = 156.3(0.8)$ MeV $\varepsilon = 0.5\%\text{-}1.1\%$

$f_K/f_\pi = 1.194(5)$ $\varepsilon = 0.4\%$ $F^{K\pi}(0) = 0.967(4)$ $\varepsilon = 0.4\%$

(0.966(3))

FLAG Collaboration

$N_f = 2 + 1$

$f_D = 209.2(3.3)$ MeV $f_{Ds} = 248.6(2.7)$ MeV

ε =1.6 % - 1.1 %

$f_B = 190.5(4.2)$ MeV $f_{Ds} = 227.7(4.5)$ MeV

ε =2.2 % - 2.0 %

Phenomenological relevance of precision physics in the Standard Model and beyond

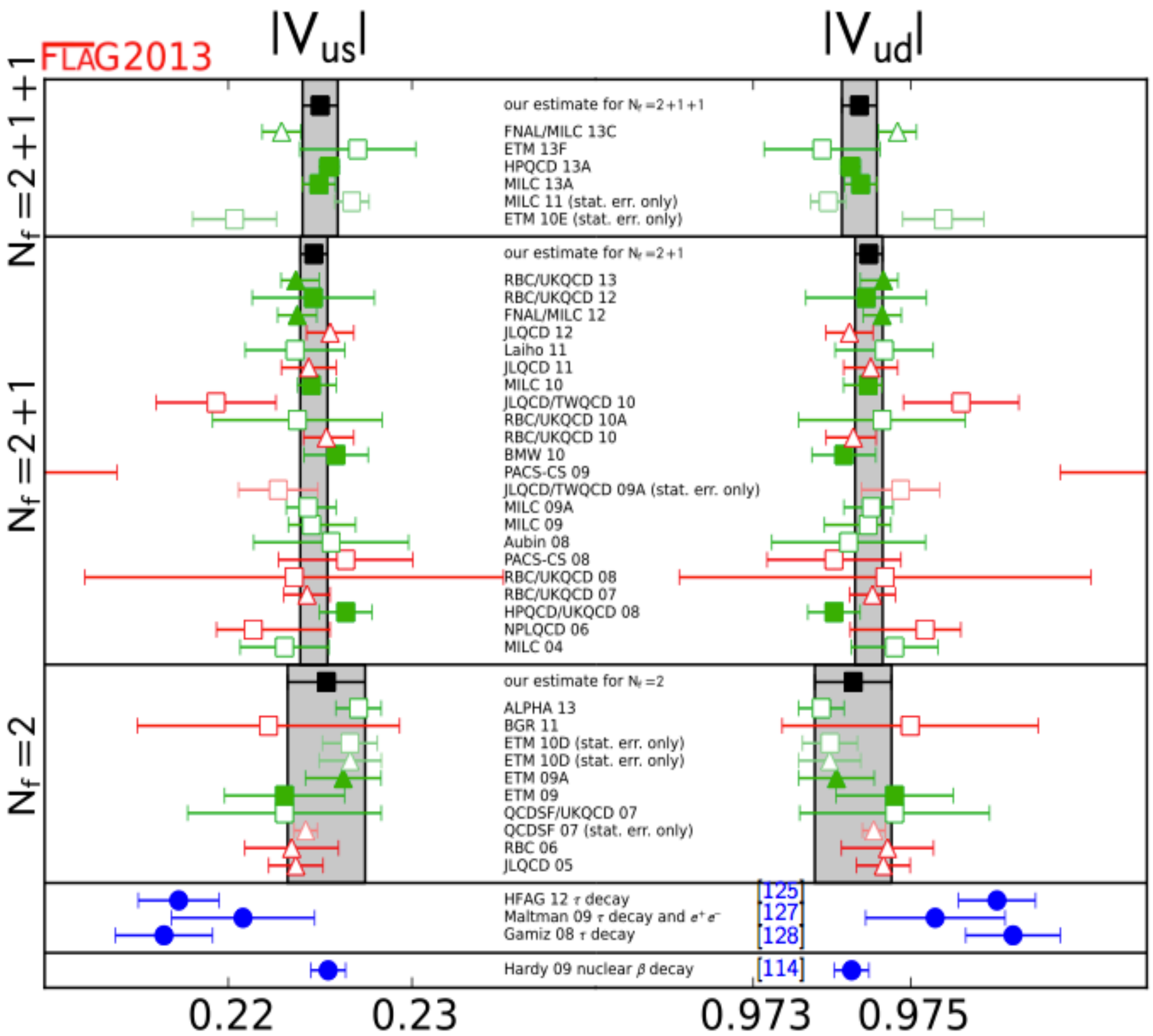
$|V_{us}|F^{K\pi}(0) = 0.2163(5)$ - exp $\varepsilon = 0.2\%$

$|V_{ud}|f_\pi / |V_{us}|f_\pi = 0.2758(5)$ $\varepsilon = 0.2\%$ see discussion below

$|V_{ud}| = 0.97425(22)$ $\varepsilon = 0.02\%$

$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$ in the SM ($|V_{ub}|^2 \approx 1.6\ 10^{-5}$)

FLAG: lattice predictions within the SM



STANDARD MODEL UNITARITY TRIANGLE ANALYSIS (FLAG)

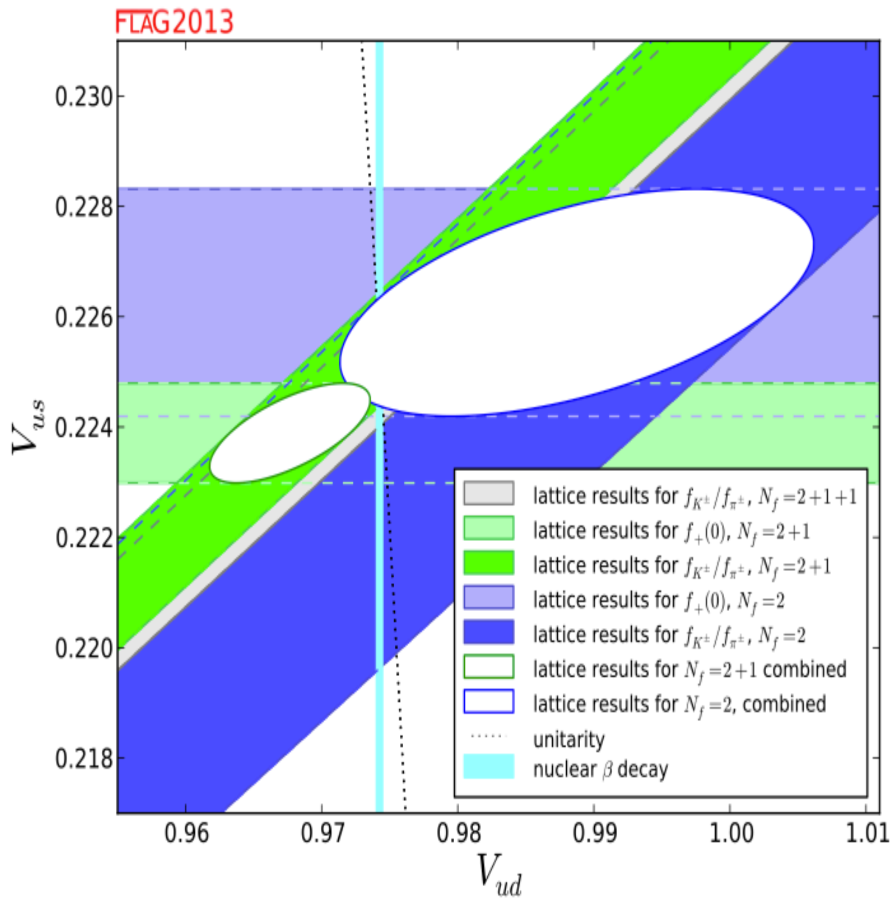
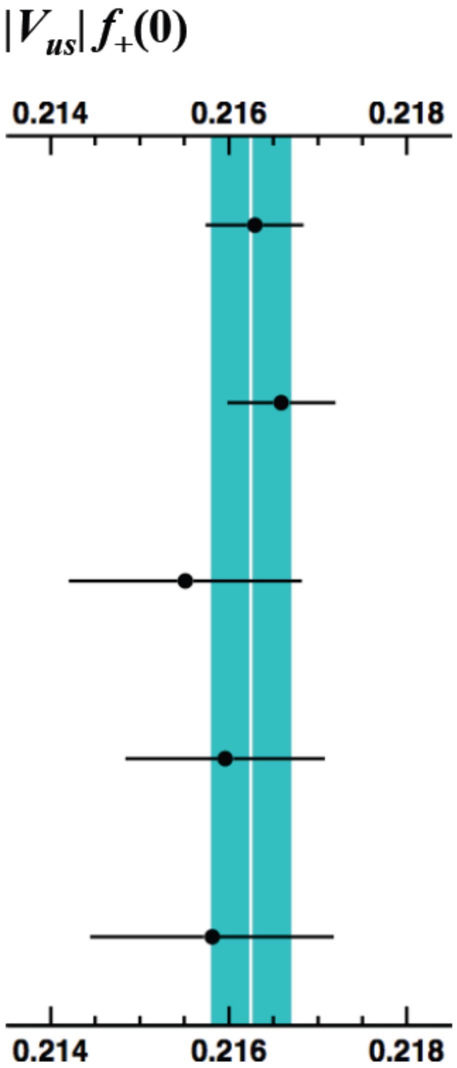


Figure 5: The plot compares the information for $|V_{ud}|$, $|V_{us}|$ obtained on the lattice wit the experimental result extracted from nuclear β transitions. The dotted arc indicates tl correlation between $|V_{ud}|$ and $|V_{us}|$ that follows if the three-flavour CKM-matrix is unitary.

- $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2 = 0.9993(5)$ or $1.0000(6)$ *from semileptonic and leptonic respectively*

|Vus| f+(0) from world data: 2012

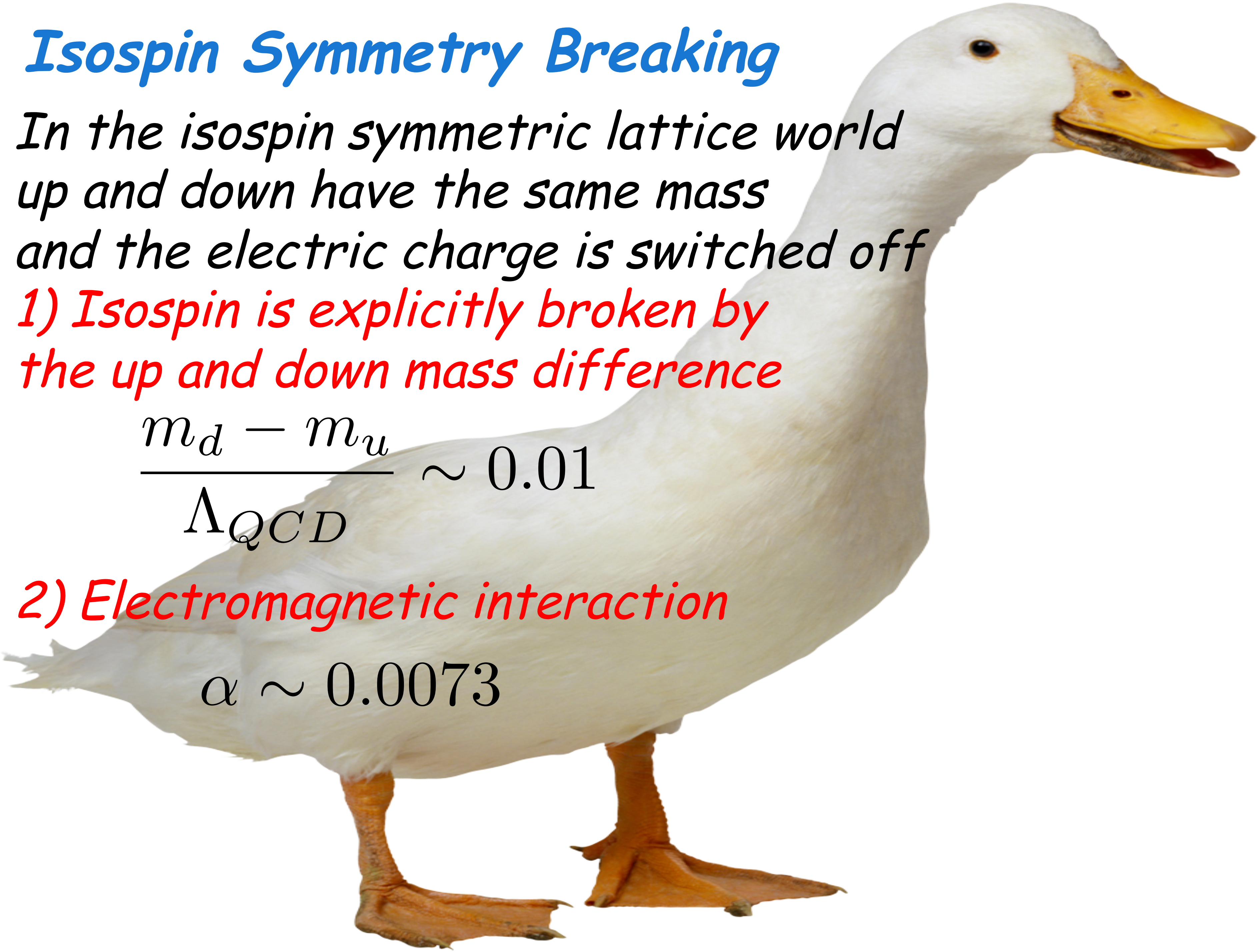


Approx. contrib. to % err from:

		% err	BR	τ	$\delta_{SU,EM}$	Int
K_Le3	**0.2163(5)**	**0.26**	0.09	0.20	0.11	**0.05**
$K_L\mu3$	**0.2166(6)**	**0.28**	0.15	0.18	0.11	**0.06**
K_Se3	**0.2155(13)**	**0.61**	0.60	**0.02**	0.11	**0.05**
$K^\pm e3$	**0.2160(11)**	**0.52**	0.31	0.09	0.41	**0.04**
$K^\pm\mu3$	**0.2158(13)**	**0.63**	0.47	0.08	0.41	**0.06**

Average: $|V_{us}| f_+(0)$ = 0.2163(5) **χ^2/ndf = 0.84/4 (93%)**

Isospin Symmetry Breaking



In the isospin symmetric lattice world
up and down have the same mass
and the electric charge is switched off

1) Isospin is explicitly broken by
the up and down mass difference

$$\frac{m_d - m_u}{\Lambda_{QCD}} \sim 0.01$$

2) Electromagnetic interaction

$$\alpha \sim 0.0073$$

Non-compact lattice QED

- Naively discretised **Maxwell action:**

$$S[A_\mu] = \frac{1}{4} \sum_{\mu,\nu} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2$$

- Pure gauge theory is **free**, it can be solved **exactly**
- Gauge invariance is preserved

QED Corrections to Hadron Masses, or $SU(3)_c \times U(1)$ on the Lattice

QED corrections to the hadron masses only require an ultraviolet cutoff

1) We need a physical condition for any renormalizable coupling to fix the scale i.e. to renormalize the strong (and the electromagnetic) coupling;
2) We must fix the masses of a certain number of hadrons, corresponding to the different flavors, to their physical value;
3) All the other hadron masses are finite and can be predicted
4) Quark masses are determined in your preferred renormalization scheme

QED_{TL} finite-volume effects

- Example — 1-loop QED_{TL} [BMWc, 2014]:

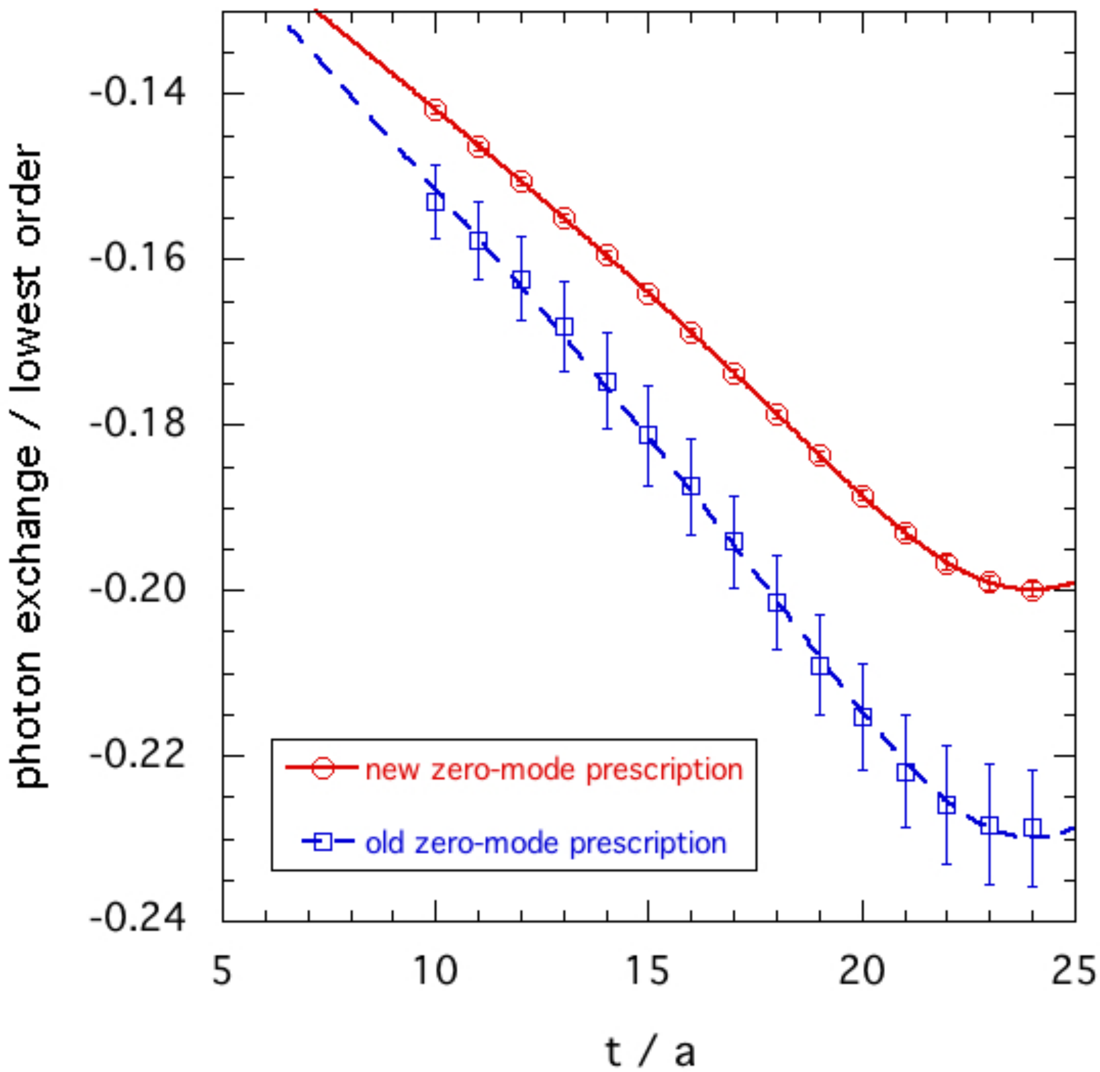
$$m(T,L) \underset{T,L\to+\infty}{\sim} m\left\{1-q^2\alpha\left[\frac{\kappa}{2mL}\left(1+\frac{2}{mL}\left[1-\frac{\pi}{2\kappa}\frac{T}{L}\right]\right)\right.\right.$$
$$\left.\left.-\frac{3\pi}{(mL)^3}\left[1-\frac{\coth(mT)}{2}\right]-\frac{3\pi}{2(mL)^4}\frac{L}{T}\right]\right\}$$

up to exponential corrections, with $\kappa = 2.83729\ldots$

Finite volume effects depend on the regulator of the zero mode, but this is not relevant to the following discussion.

<u>Hadron masses are infrared finite</u>

$$m_{\mathrm{QED_L}}(T,L) \underset{T,L\to+\infty}{=} m\left\{1 - q^2\alpha\left[\frac{\kappa}{2mL}\left(1+\frac{2}{mL}\right) - \frac{3\pi}{(mL)^3}\right]\right\}$$



Full QCD + QED projects

	RBC-UKQCD	PACS-CS	QCDSF-UKQCD	BMWc
arXiv	1006.1311	1205.2961	1311.4554 and Lat. 2014	1406.4088
fermions	DWF	clover	clover	clover
N_f	2+1	1+1+1	1+1+1	1+1+1+1
method	reweighting	reweighting	RHMC	RHMC
$\min(M_\pi)$ (MeV)	420	135	250	195
a (fm)	0.11	0.09	0.08	0.06 — 0.10
$\#a$	1	1	1	4
L (fm)	1.8	2.9	1.9 — 2.6	2.1 — 8.3
$\#L$	1	1	2	11

Portelli @ Lattice 2014 - Calculation at several values of α, then extrapolation/interpolation. not really ``full'' : linear extrapolation to 1/137 without the renormalization of α

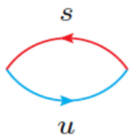
QED & Isospin Corrections to Hadronic Masses: The RM123 approach

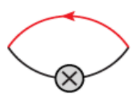
- Identify the isospin breaking term in the action and expand in $\Delta m = (m_d - m_u)/2$

$$S_m = \sum_x \left[m_u \bar{u}u + m_d \bar{d}d \right] = \sum_x \left[\frac{1}{2}(m_u + m_d)(\bar{u}u + \bar{d}d) - \frac{1}{2}(m_d - m_u)\underbrace{(\bar{u}u - \bar{d}d)}\right] = S_0 - \Delta m \hat{S}$$

$$\langle O \rangle = \frac{\int D\phi\, O\, e^{-S_0 + \Delta m\, \hat{S}}}{\int D\phi\, e^{-S_0 + \Delta m\, \hat{S}}} \overset{1st}{\simeq} \frac{\int D\phi\, O\, e^{-S_0} \left(1 + \Delta m\, \hat{S}\right)}{\int D\phi\, e^{-S_0}\left(1 + \Delta m\, \hat{S}\right)} \simeq \frac{\langle O \rangle_0 + \Delta m \langle O\, \hat{S} \rangle_0}{1 + \Delta m \cancel{\langle \hat{S} \rangle_0}} = \langle O \rangle_0 + \Delta m \langle O\, \hat{S} \rangle_0$$

- For the kaon decay constant:

$C_{K^+K^-}(t) = -$ 

$= -$  $-$  $+ \mathcal{O}(\Delta m_{ud})^2$

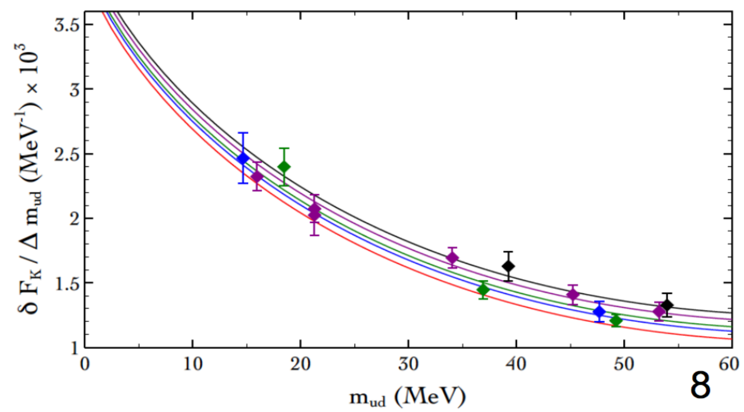
$$\boxed{\delta_{SU(2)} = -0.0080\,(7)}$$

Lattice - Nf=2
RM123 collab. (2012)

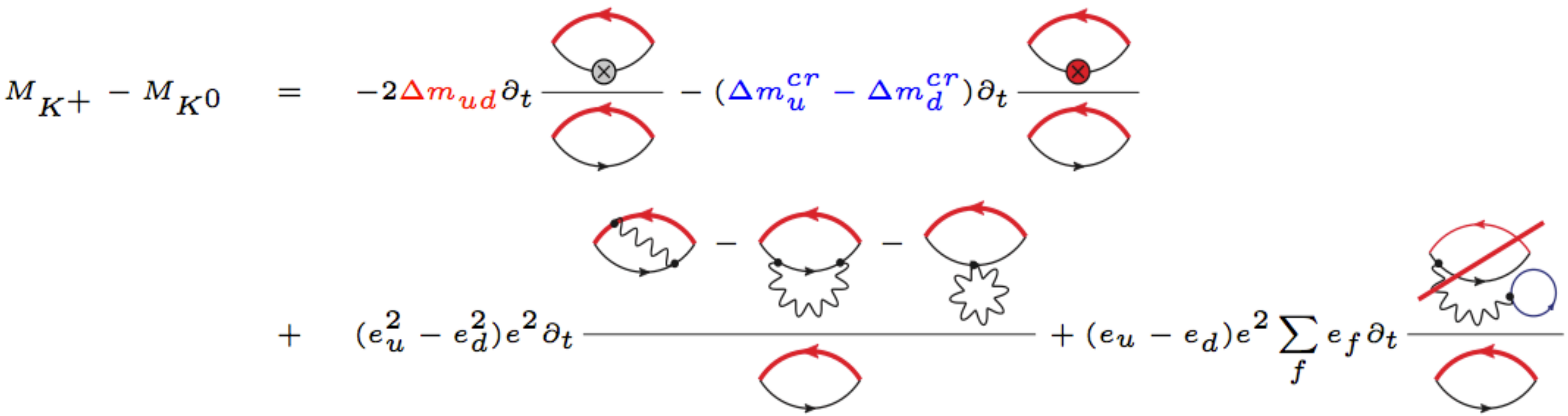
which is ~2.6 σ larger than

$$\boxed{\delta_{SU(2)} = -0.0044\,(12)}$$

ChPT
Cirigliano, Neufeld (2011)



QED & Isospin Corrections to Hadronic Masses: The RM123 approach



Expand the action in the ``small terms'' namely in
α and $(m_u{=}m_d)/\Lambda_{QCD.}$

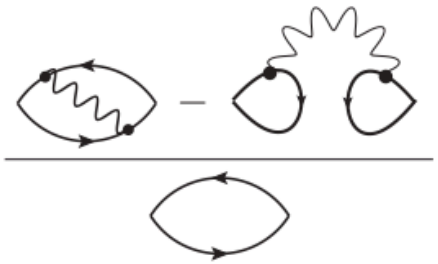
Advantge: We compute the insertion of operators of O(1) and no extrapolation $\alpha \rightarrow 1/137$ is needed;

Disadvantage: Complicated ``disconnected diagrams'' must be computed;

Unavoidable: in electromagnetic corrections to hadronic amplitudes

LIB corrections to pion masses (ii)

$$M_{\pi^+} - M_{\pi^0} \quad = \quad \frac{(e_u - e_d)^2}{2} e^2 \partial_t$$



- there are no contributions proportional to $\hat{m}_d - \hat{m}_u$: the pion mass difference at this order is a pure QED effect
- note: sea quark effects are not neglected, they cancel in the difference!
- the electric charge does not renormalize at this order (a problem that *must* instead be faced at higher orders) and the previous expression is finite,

Some remark on QED Corrections to Hadron Masses

FLAG:

We distinguish the physical mass M_P, $P \in \{\pi^+, \pi^0, K^+, K^0\}$, from the mass $\hat{M}_P$ within QCD alone. The e.m. self-energy is the difference between the two, $M_P^\gamma \equiv M_P - \hat{M}_P$.

however, a world without electromagnetism where we can measure the masses of the mesons and fix the scale and the quark masses does not exist thus

$M_P{}^\gamma$ cannot be a physical quantity and indeed it depends on the convention

It is not clear to me that when comparing the different results these do correspond to the same convention

although useful for a comparison with χpth, $M_P{}^\gamma$ should be abandoned: without QED you only know that the error is of O(α), but you cannot compute it,

with QED the precise determination of error that you would have made depends on the convention, thus who cares?

People who live in glass houses should’nt throw stones

Chi è senza peccato scagli la prima pietra

Even RM123, following the common lore …..

- the value of ε_γ depends upon the renormalization prescription used to separate QED from QCD IB effects

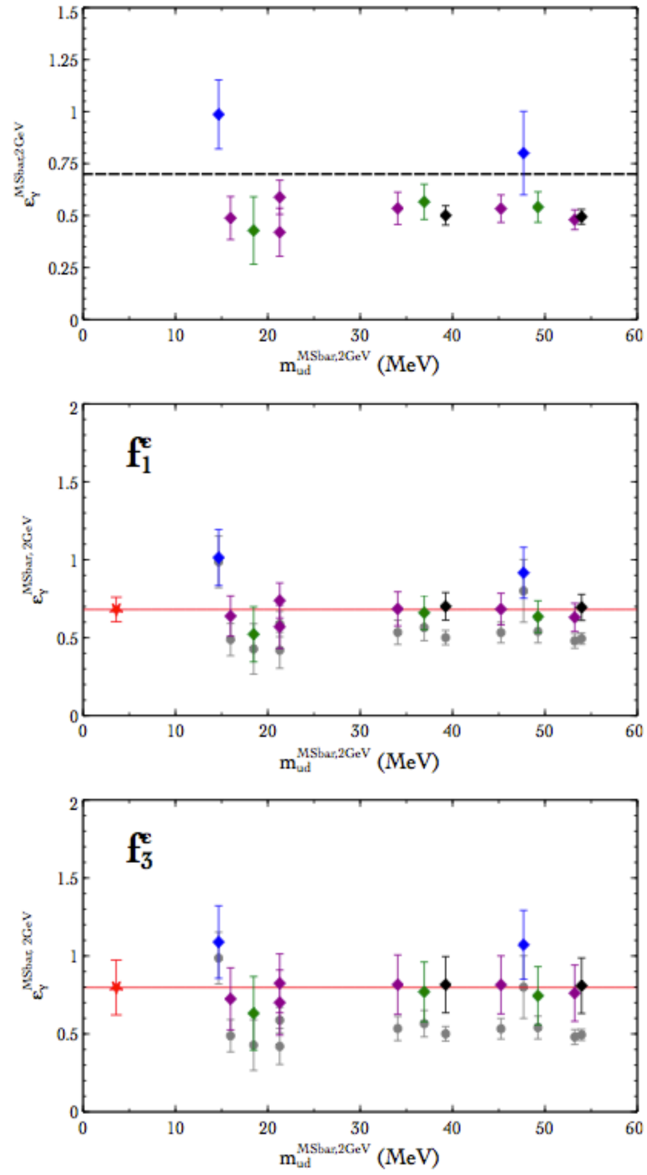
$$\varepsilon_\gamma = \frac{\left[M^2_{K^+} - M^2_{K^0}\right]^{QED} - \left[M^2_{\pi^+} - M^2_{\pi^0}\right]^{QED}}{M^2_{\pi^+} - M^2_{\pi^0}}$$

- it is needed to calculate the light quark masses by starting from QCD ($\hat{m}_u \neq \hat{m}_d$) lattice simulations and using the QCD contribution to the kaon mass splitting as "experimental" input

$$\varepsilon_\gamma = 0.79(18)(18)$$
$$\hat{m}_u/\hat{m}_d = 0.50(2)(3)$$

- note: these results are scale and scheme dependent, $\overline{MS}$ 2 GeV, and *depend* upon the matching prescription used to separate QED from QCD contributions

RM123, Phys.Rev. D87(2013)



N. Tantalo @ CERN 2014

QED (Isospin) Corrections in Hadronic Processes

After the renormalization of the $SU(3)_c \times U(1)$ Lagrangian you still need

1) The renormalization of the operators mediating the physical process of interest (e.g. the Weak effective Hamiltonian). But this is not a novelty;
2) A complex procedure to remove the infrared cutoff because in general the amplitudes, contrary to the masses, are infrared divergent.

A method to solve this problem is presented . This will be done by discussing an explicit example and will allow the discussion of some important theoretical subtelties

How to solve the problem of the infrared divergences discussed through an explicit example

$$\pi \to \ell + \nu_\ell + (\gamma)$$

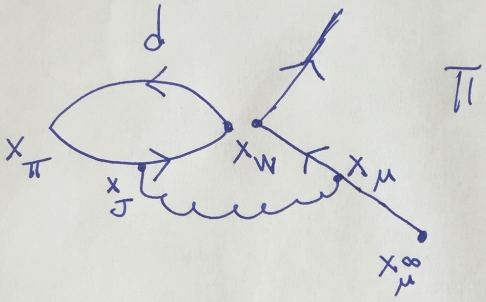
N.Carrasco, V.Lubicz, G.M., C.T.Sachrajda, F.Sanfillipo, N.Tantalo, C.Tarantino, M.Testa

in preparation

NOTE: Chiral Perturbation Theory is NOT Used

1

www.ictp.it



$\pi^+ \to \mu^+ \nu_\mu$

$$- \sum_{\vec{x}_\pi, x_J, x_\mu, \vec{x}_\mu^\infty, \vec{x}_\nu} \operatorname{tr}\left(\Gamma_\pi S_d(x_\pi, x_W)\gamma_L^\rho S_u(x_W, x_J)\gamma^\sigma S_u(x_J, x_\pi)\right)$$

$$\cdot\ \phi_\sigma(x_J) \qquad e^{-i\vec{p}_\nu\cdot\vec{x}_\nu} S(x_\nu, x_W)\gamma_\rho^L S_\mu(x_W, x_\mu)\gamma^\lambda S(x_\mu - x_\mu^\infty)$$

$$\phi_\lambda(x_\mu) \qquad e^{i\vec{p}_\mu\cdot\vec{x}_\mu^\infty}$$

$$\sum_{\vec{x}_\mu^\infty} S(x_\mu - x_\mu^\infty)\, e^{i\vec{p}_\mu\cdot\vec{x}_\mu^\infty} = \int \frac{dq_0}{(2\pi)} \int \frac{d^3q}{(2\pi)^3} e^{iq_0(t_\mu - t_\mu^\infty)}$$

$$e^{i\vec{q}\cdot\vec{x}_\mu} \qquad \frac{e^{i(\vec{p}_\mu - \vec{q}_\mu)\vec{x}_\mu^\infty}}{q_0^2 + \vec{q}^{\,2} + m_\mu^2}$$

$$= \int \frac{dq_0}{2\pi} e^{i\vec{p}_\mu\cdot\vec{x}_\mu} \frac{1}{\left(q_0^2 + \vec{p}_\mu^{\,2} + m_\mu^2\right)} e^{iq_0(t_\mu - t_\mu^\infty)} \to e^{E_\mu t_\mu} \times e^{i\vec{p}_\mu\cdot\vec{x}_\mu} \times \left(\frac{e^{-E_\mu t_\mu^\infty}}{2E_\mu}\right)$$

2

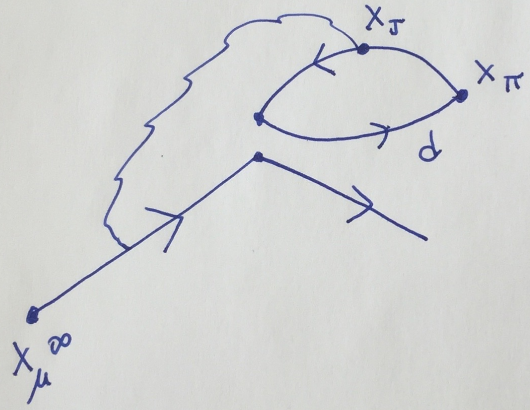
www.ictp.it

$$- \sum_{\vec{x}_\pi, x_J, x_\mu} e^{-i\vec{p}_\mu \cdot \vec{x}_W + E_\nu t_W} \; t_2 \, \Gamma_\pi S_d(x_\pi, x_W) \gamma_L^\rho S_u(x_W, x_J) \gamma^\sigma S_u(x_J, x_\pi)$$

$$\phi_\sigma(x_J) \; e^{i\vec{p}_\mu \cdot \vec{x}_\mu + E_\mu t_\mu}$$

$$\bar{u}(p_\nu = -p_\mu) \gamma_\rho^L S_\mu(x_W - x_\mu) \gamma^\lambda U(p_\mu) \phi_\lambda(x_\mu)$$

3



$$\mu^- \to \pi^- + \nu_\mu$$

$$- \, t_2 \, \Gamma_\pi S_d(x_\pi, x_W) \gamma_L^\rho S_u(x_W, x_J) \gamma^\sigma S_u(x_J, x_\pi) \, e^{i\vec{p}_\nu \cdot (\vec{x}_\pi - \vec{x}_W)} \phi_\sigma(x_J)$$

$$\bar{u}(p_\nu) \gamma_\rho^L S_\mu(x_W - x_\mu) \gamma^\lambda u(p_\mu = p_\mu^0 = m_\mu) \, \phi_\lambda(x_\mu) \, e^{-m_\mu t_\mu}$$

Leptonic decays at tree level

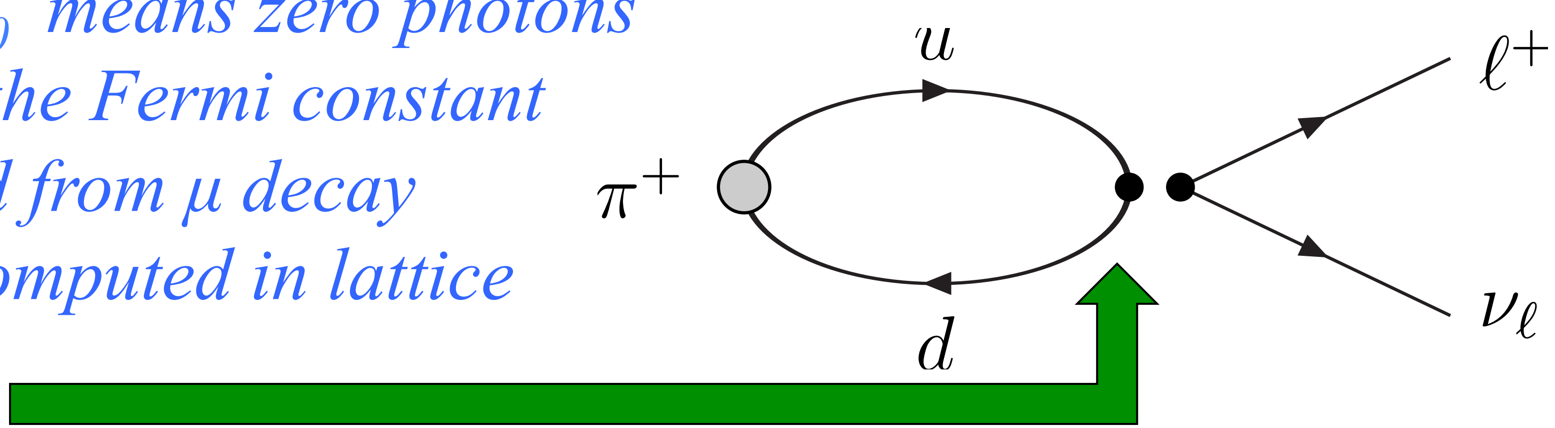
Since the mass of the pion is much lower than M_W we use the effective Hamiltonian

$$\boxed{H_{\mathrm{eff}} = -\frac{G_F}{\sqrt{2}} V_{ud}^{*} (\bar{d}\gamma^{\mu}(1-\gamma^5)u)\,(\bar{\nu}_{\ell}\gamma_{\mu}(1-\gamma^5)\ell)}$$

from which we compute

$$\boxed{\Gamma_0^{\mathrm{tree}}(\pi^+ \to \ell^+\nu_\ell) = \frac{G_F^2\,|V_{ud}|^2 f_\pi^2}{8\pi}\, m_\pi\, m_\ell^2 \left(1-\frac{m_\ell^2}{m_\pi^2}\right)^2}$$

- *0 in Γ_0 means zero photons*
- *G_F is the Fermi constant defined from μ decay*
- *f_π is computed in lattice QCD*



Leptonic decays at O(α) – The ultraviolet matching in the ``W Regularization"

If G_F is the Fermi constant defined at O(α) from μ decay in the standard (convention dependent) way

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left[1 - \frac{8m_e^2}{m_\mu^2}\right]\left[1 + \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right]$$

S.M.Berman, PR 112 (1958) 267; T.Kinoshita and A.Sirlin, PR 113 (1959) 1652

then the effective Hamiltonian in the W-regularization is given by (Sirlin PRD 22 (80) 971)

$$\boxed{H_{\mathrm{eff}} = -\frac{G_F}{\sqrt{2}} V_{ud}^* \left(1 + \frac{\alpha}{\pi}\log\frac{M_Z}{M_W}\right)(\bar{d}\gamma^\mu(1-\gamma^5)u)\,(\bar{\nu}_\ell\gamma_\mu(1-\gamma^5)\ell)}$$

matching the (Wilson) lattice to the W-regularization.

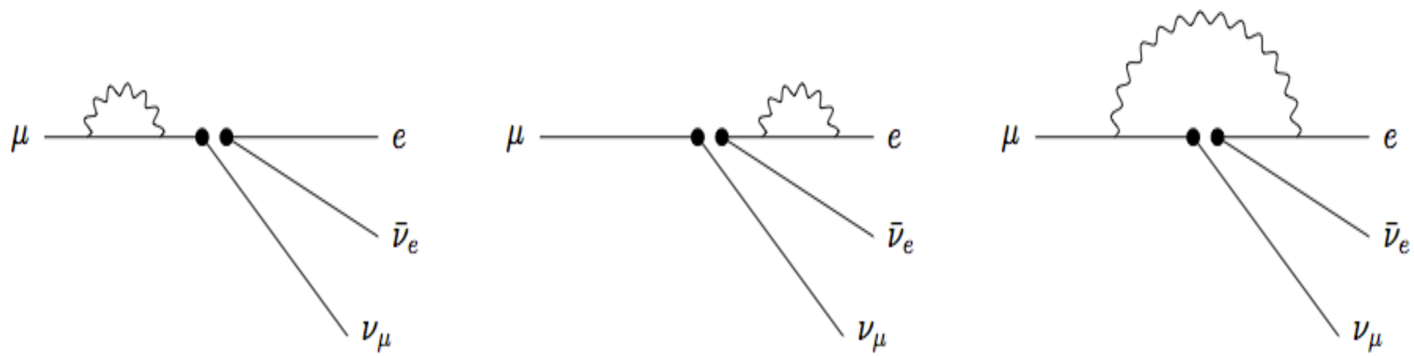
$$O_1^{\mathrm{W-reg}} = \left(1 + \frac{\alpha}{4\pi}\left(2\log a^2 M_W^2 - 15.539\right)\right) O_1^{\mathrm{bare}} + \frac{\alpha}{4\pi}\left(0.536\, O_2^{\mathrm{bare}} + 1.607\, O_3^{\mathrm{bare}} - 3.214\, O_4^{\mathrm{bare}} - 0.804\, O_5^{\mathrm{bare}}\right)$$

W Regularization

The results for the widths are expressed in terms of G_F, the Fermi constant ($G_F = 1.16632(2) \times 10^{-5}\,\mathrm{GeV}^{-2}$). This is obtained from the muon lifetime:

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left[1 - \frac{8m_e^2}{m_\mu^2}\right]\left[1 + \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right]$$

- This expression can be viewed as the definition of G_F. Many EW corrections are absorbed into the definition of G_F; the explicit $O(\alpha)$ corrections come from the following diagrams in the effective theory:



together with the diagrams with a real photon.

- These diagrams are evaluated in the W-regularisation in which the photon propagator is modified by:

A.Sirlin, PRD 22 (1980) 971

$$\frac{1}{k^2} \to \frac{M_W^2}{M_W^2 - k^2}\frac{1}{k^2}. \qquad \left(\frac{1}{k^2} = \frac{1}{k^2 - M_W^2} + \frac{M_W^2}{M_W^2 - k^2}\frac{1}{k^2}\right)$$

W Regularization

$$H_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{ud}^* \left(1 + \frac{\alpha}{\pi}\log\frac{M_Z}{M_W}\right) (\bar{d}\gamma^\mu(1-\gamma^5)u)\,(\bar{\nu}_\ell\gamma_\mu(1-\gamma^5)\ell)$$

matching the (Wilson) lattice to the W-regularization.

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where

$$O_1 = (\bar{d}\gamma^\mu(1-\gamma^5)u)\,(\bar{\nu}_\ell\gamma_\mu(1-\gamma^5)\ell) \qquad O_2 = (\bar{d}\gamma^\mu(1+\gamma^5)u)\,(\bar{\nu}_\ell\gamma_\mu(1-\gamma^5)\ell)$$

$$O_3 = (\bar{d}(1-\gamma^5)u)\,(\bar{\nu}_\ell(1+\gamma^5)\ell) \qquad O_4 = (\bar{d}(1+\gamma^5)u)\,(\bar{\nu}_\ell(1+\gamma^5)\ell)$$

$$O_5 = (\bar{d}\sigma^{\mu\nu}(1+\gamma^5)u)\,(\bar{\nu}_\ell\sigma_{\mu\nu}(1+\gamma^5)\ell)\,.$$

Rate at O(α)

$$\Gamma(\Delta E) = \Gamma_0 + \Gamma_1(\Delta E)$$

$|V_{ud}|$

where $$\Gamma(\Delta E) = \int_0^{\Delta E} dE_\gamma \frac{d\Gamma}{dE_\gamma}$$

contrary to the hadron masses

at O(α) both Γ_0 and $\Gamma_1(\Delta E)$ are

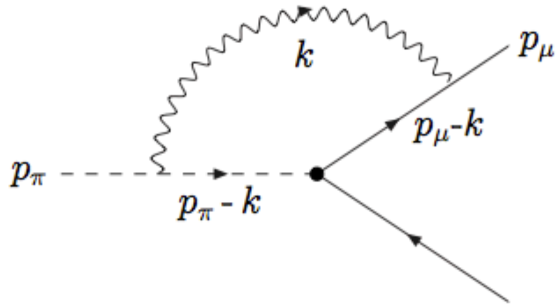
INFRARED DIVERGENT

although the divergence cancel in the sum

F. Bloch, A. Nordsieck Phys.Rev. 52 (1937) T.D. Lee, M. Nauenberg Phys.Rev. 133 (1964)

and the infinite volume limit cannot be separately taken

Courtesy of C. Sachrajda



$$
\begin{aligned}
I &\sim \int_{\text{small } k} d^4k \, \frac{1}{(k^2+i\epsilon)((p_\mu-k)^2-m_\mu^2+i\epsilon)((p_\pi-k)^2-m_\pi^2+i\epsilon)} \\
&\sim \int_{\text{small } k} d^4k \, \frac{1}{k^2(-2p_\mu\cdot k)(-2p_\pi\cdot k)} \\
&\sim \int_{\text{small } k} d^4k \, \frac{1}{k^4} \qquad \Rightarrow \text{infrared divergence}\,.
\end{aligned}
$$

- This leads to a contribution to Γ_0 of

$$
\Gamma_0^{\pi\mu} = \Gamma_0^{\text{tree}} \frac{\alpha}{4\pi}\left(\frac{2(1+r_\mu^2)}{1-r_\mu^2}\log r_\mu^2 \log\left(\frac{m_\pi^2}{m_\gamma^2}\right)+\cdots\right),
$$

where the photon mass, m_γ, is introduced to regulate the infrared divergences and $r_\mu = m_\mu/m_\pi$.

Rate at O(α)

$$\Gamma(\Delta E) = \Gamma_0 + \Gamma_1(\Delta E)$$

$|V_{ud}|$

where
$$\Gamma(\Delta E) = \int_0^{\Delta E} dE_\gamma \frac{d\Gamma}{dE_\gamma}$$

contrary to the hadron masses

at O(α) both Γ_0 and $\Gamma_1(\Delta E)$ are

INFRARED DIVERGENT

although the divergence cancel in the sum

F. Bloch, A. Nordsieck Phys.Rev. 52 (1937) T.D. Lee, M. Nauenberg Phys.Rev. 133 (1964)

and the infinite volume limit cannot be separately taken

At this stage we propose to compute $\Gamma_1(\Delta E)$ in perturbation theory @ values of ΔE corresponding to photons which are sufficiently soft for the point-like approximation of the pion to be valid

$(\Delta E \ll \Lambda_{QCD} \approx 4\pi f_\pi)$

but hard enough with respect to the experimental resolution.

A value of O(10-20 MeV) seems to be appropriate both theoretically and experimentally.

F. Ambrosino et al., KLOE Collaboration,
PLB 632 (2006) 76; EPJC 64 (2009) 627; 65 (2010) 703(E);
J. Bijnens, G. Ecker, J. Gasser, NPB 396 (1993) 81; V.Cirigliano, I.Rosell, JHEP 0710 (2007) 005

In the future, as techniques and resources improve, it may be better to compute $\Gamma_1(\Delta E)$ nonperturbatively over a larger range of photon energies

(about the analytical continuation to the Euclidean see later)

NOTE: we do not use chiral perturbation theory !!

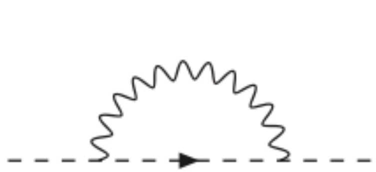
MASTER FORMULA for the rate at O(α)

$$\Gamma(\Delta E) = \lim_{V\to\infty} (\Gamma_0 - \Gamma_0^{pt}) + \lim_{V\to\infty} (\Gamma_0^{pt} + \Gamma_1(\Delta E))$$

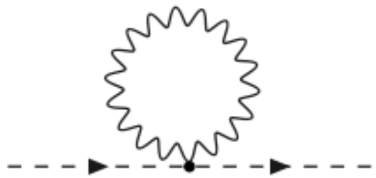
pt = point-like & perturbative

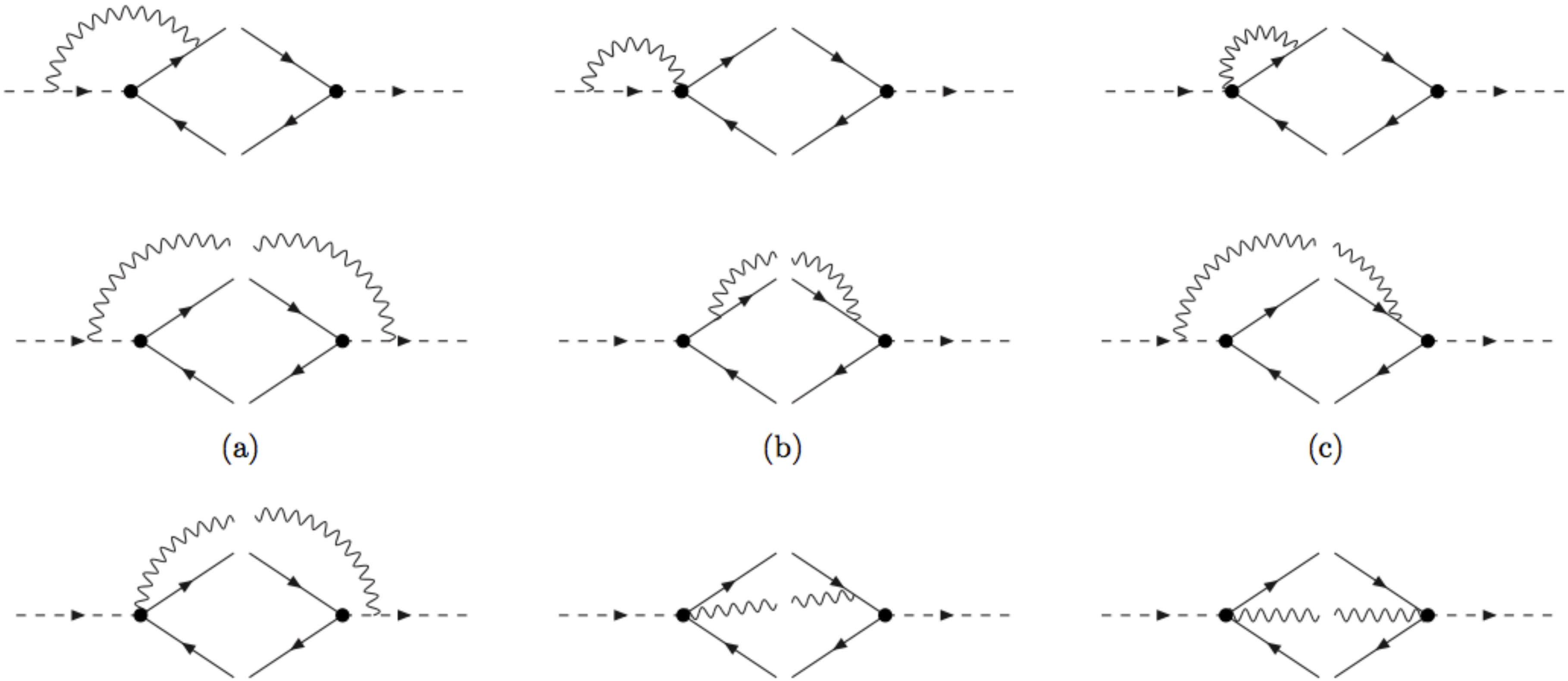
- the infrared divergences in Γ_0 and Γ_0^{pt} are exactly the same and cancel in the difference
- $\Gamma(\Delta E) = \Gamma_0^{pt} + \Gamma_1(\Delta E)$ is infrared finite since is a physical, well defined quantity *F. Bloch, A. Nordsieck Phys.Rev. 52 (1937) T.D. Lee, M. Nauenberg Phys.Rev. 133 (1964)*
- the infrared divergences in $\Delta\Gamma_0 (L) = \Gamma_0 - \Gamma_0^{pt}$ and $\Gamma(\Delta E) = \Gamma_0^{pt} + \Gamma_1(\Delta E)$ cancel separately hence they can be regulated with different infrared cutoff
- Γ_0 and Γ_0^{pt} are also ultraviolet finite

We now discuss the two terms, $\Delta\Gamma_0 (L)$ and $\Gamma(\Delta E)$



and





(a) (b) (c)

(d) (e) (f)

Leptonic decays at O(α) – Perturbative Calculation of $\Gamma(\Delta E) = \Gamma_0^{pt} + \Gamma_1(\Delta E)$

U.V. & Infrared finite but contains $\log(M_W)$ & $\log(\Delta E)$

$$\Gamma(\Delta E) = \Gamma_0^{\rm tree} \times \Bigg(1 + \frac{\alpha}{4\pi}\Bigg\{3\log\left(\frac{m_\pi^2}{M_W^2}\right) + \log\left(r_\ell^2\right) - 4\log(r_E^2) + \frac{2-10r_\ell^2}{1-r_\ell^2}\log(r_\ell^2)$$
$$-2\frac{1+r_\ell^2}{1-r_\ell^2}\log(r_E^2)\log(r_\ell^2) - 4\frac{1+r_\ell^2}{1-r_\ell^2}\,{\rm Li}_2(1-r_\ell^2) - 3$$
$$+\Big[\frac{3+r_E^2-6r_\ell^2+4r_E(-1+r_\ell^2)}{(1-r_\ell^2)^2}\log(1-r_E) + \frac{r_E(4-r_E-4r_\ell^2)}{(1-r_\ell^2)^2}\log(r_\ell^2)$$
$$-\frac{r_E(-22+3r_E+28r_\ell^2)}{2(1-r_\ell^2)^2} - 4\frac{1+r_\ell^2}{1-r_\ell^2}\,{\rm Li}_2(r_E)\Big]\Bigg\}\Bigg)$$

We think that this is a new result;
$\Gamma(\Delta E_1)$ *T.Kinoshita, PRL 2 (1959) 477*

$$\boxed{r_E = \frac{2\Delta E}{m_\pi} \qquad r_\ell = \frac{m_\ell}{m_\pi}}$$

Leptonic decays at O(α) – Perturbative Calculation of $\Gamma(\Delta E) = \Gamma_0^{pt} + \Gamma_1(\Delta E)$

- The total rate is readily computed by setting r_E to its maximum value, namely $r_E = 1 - r_\ell^2$, giving

$$\Gamma^{pt} = \Gamma_0^{tree} \times \left\{ 1 + \frac{\alpha}{4\pi} \left(3\log\left(\frac{m_\pi^2}{M_W^2}\right) - 8\log(1-r_\ell^2) - \frac{3r_\ell^4}{(1-r_\ell^2)^2}\log(r_\ell^2) - 8\frac{1+r_\ell^2}{1-r_\ell^2}\mathrm{Li}_2(1-r_\ell^2) + \frac{13-19r_\ell^2}{2(1-r_\ell^2)} + \frac{6-14r_\ell^2-4(1+r_\ell^2)\log(1-r_\ell^2)}{1-r_\ell^2}\log(r_\ell^2) \right) \right\}.$$

- This result agrees with the well known results in literature providing an important check of our calculation.

Structure dependent contributions to the O(α) perturbative calculation of $\Gamma_1(\Delta E)$

1) For sufficiently small values of $\Delta E(/\Lambda_{QCD})$ the structure dependent contributions to $\Gamma_1(\Delta E)$ can be neglected

2) How big are they for experimentally accessible values of ΔE ? We can have an estimate from chiral perturbation theory (although not all LEC are available)

J.Bijnens, G.Ecker and J.Gasser, hep-ph/9209261, J.Bijnens, G.Colangelo, G.Ecker and J.Gasser, hep-ph/9411311. V. Cirigliano and I. Rosell, arXiv:0707.3439 [hep-ph]], L. Ametller, J. Bijnens, A. Bramon and F. Cornet, hep-ph/9302219.

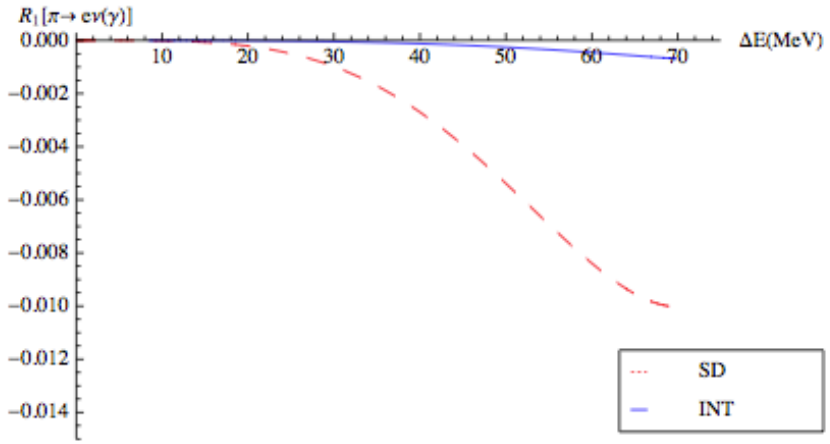
$$R_1^A(\Delta E) = \frac{\Gamma_1^A(\Delta E)}{\Gamma_0^{\alpha,\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E)}\,, \qquad A = \{\mathrm{SD,INT}\}\,,$$

The structure dependent contributions to perturbative calculation of $\Gamma_1(\Delta E)$: the decay into an electron is the worse case !

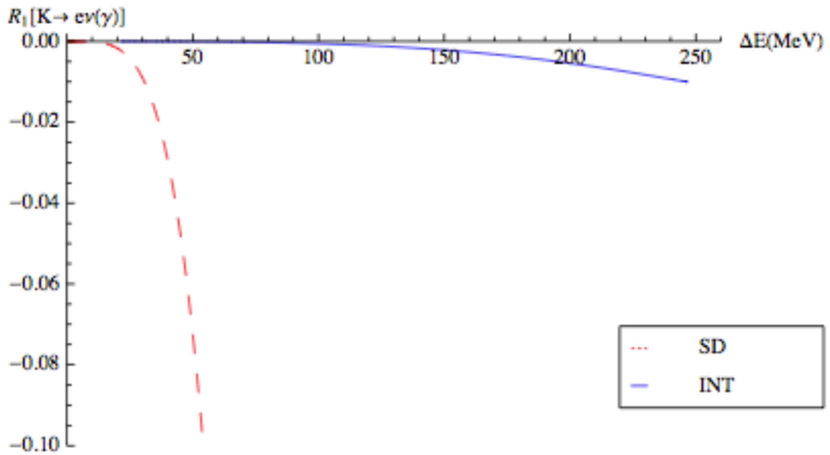
In the case of the decay in a muon the effect is of the $O(10^{-3}\text{-}10^{-7})$

In the case of B mesons, due to the small scale represented by m_{B*} - m_B, it is likely that it will be necessary to perform a full non-perturbative calculation of the real emission

Pion



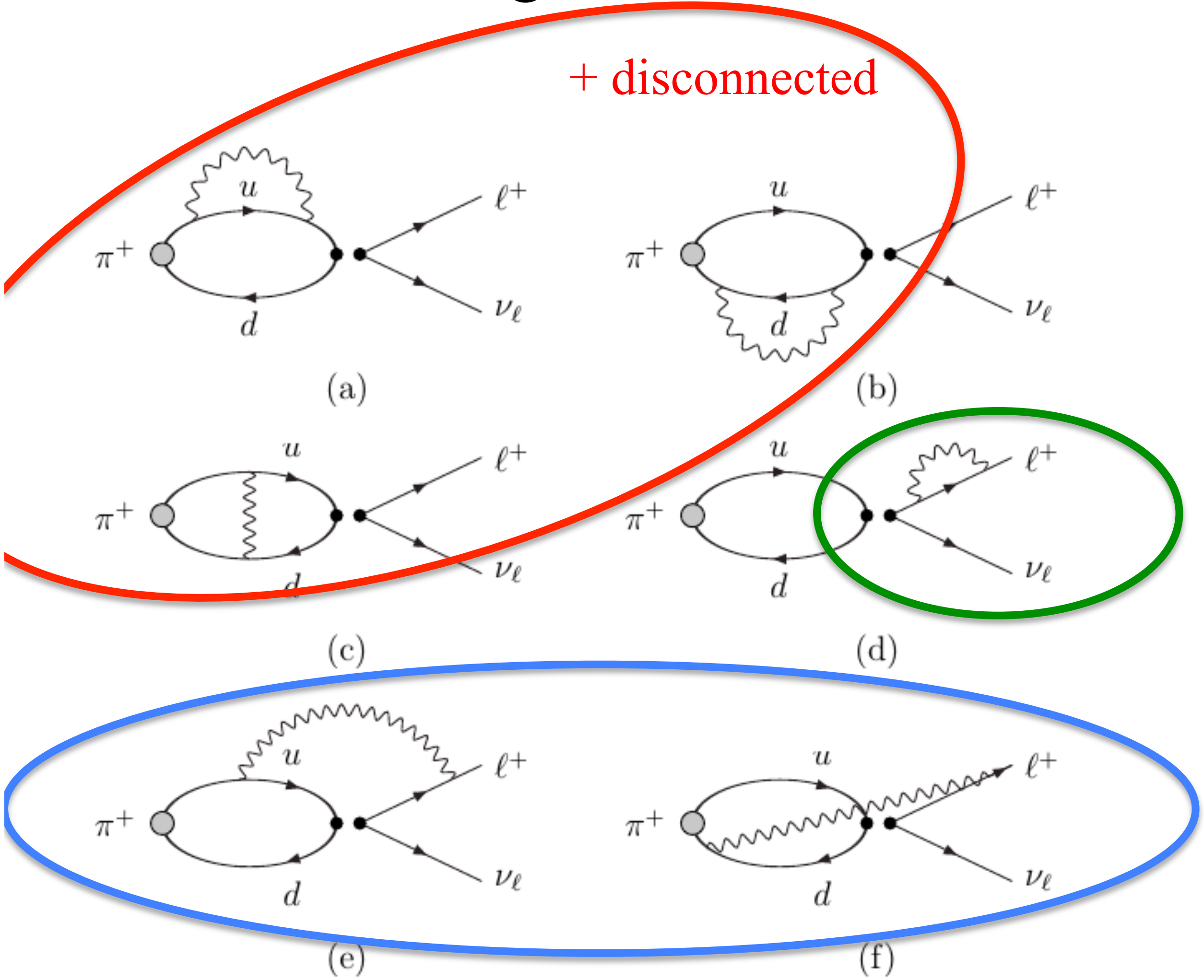
Kaon



D. Becirevic, B. Haas and E. Kou, arXiv:0907.1845 [hep-ph]

Leptonic decays at O(α) – The first term of the Master Formula $\Delta\Gamma(L) = \Gamma_0 - \Gamma_0^{pt}$

- Each of the two terms is U.V. finite but contains $\log(M_W)$
- Infrared divergences cancel in the difference

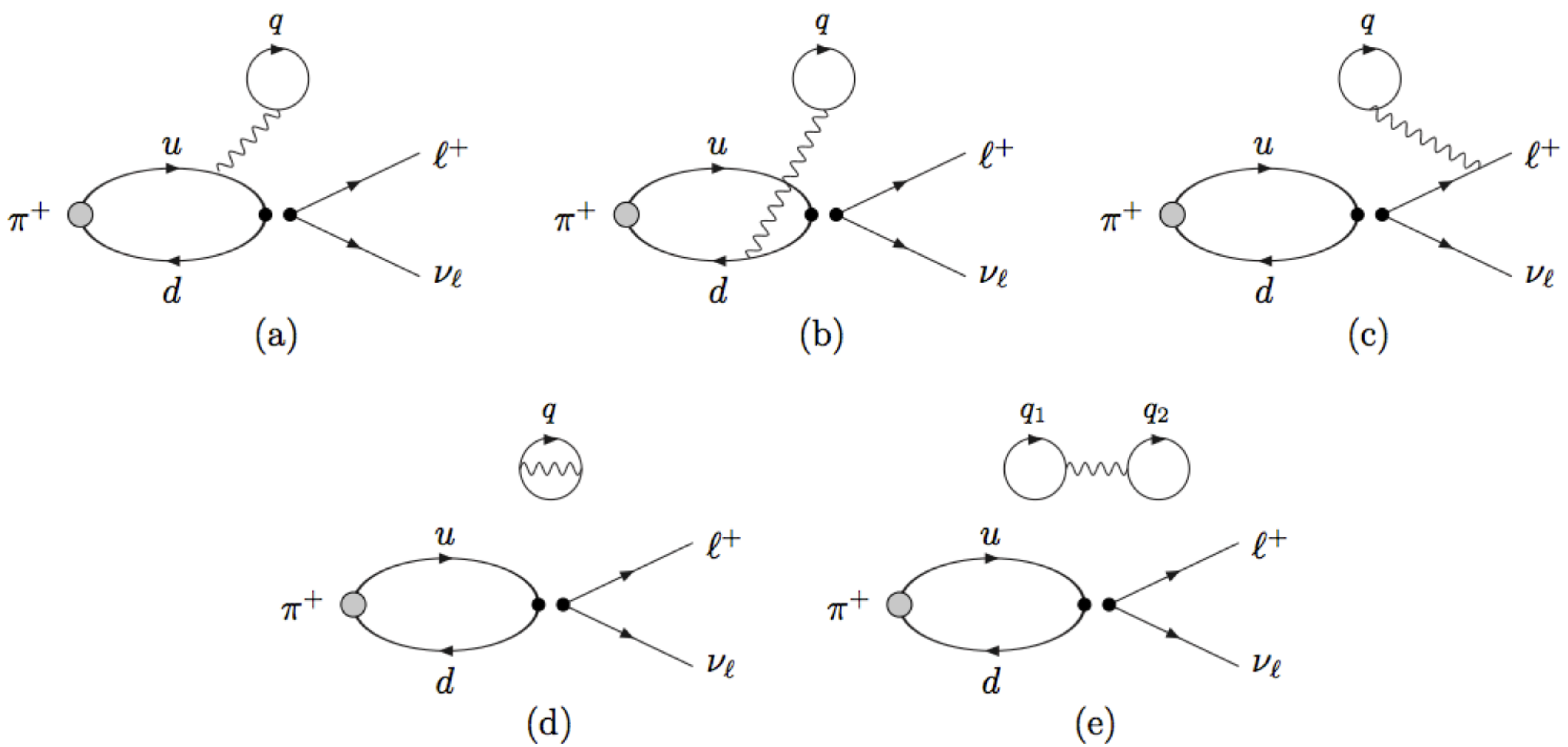


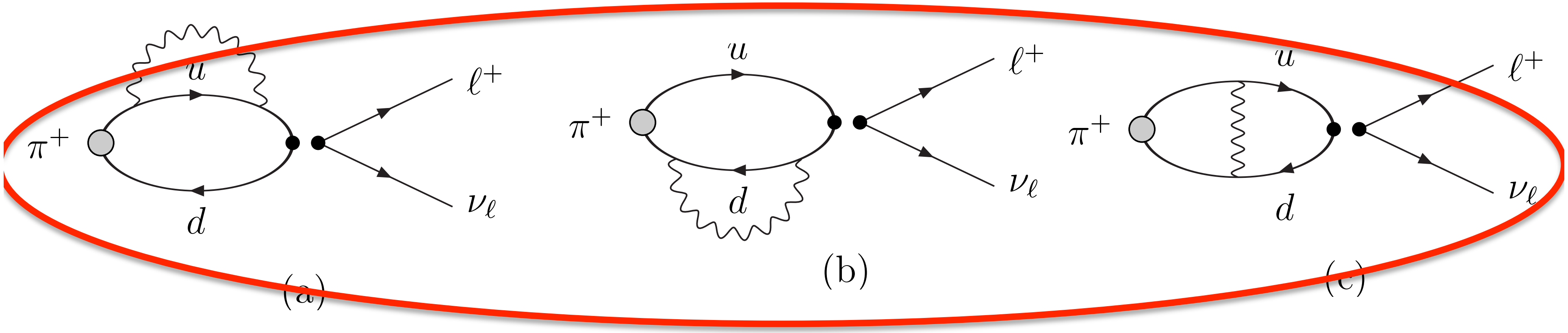
at this order we can take the difference of the amplitudes

Can be computed as discussed in arXiv: 1303.4896, Phys.Rev. D87(2013)

NOT by including the electromagnetic field in the action

DISCONNECTED DIAGRAMS





The relevant correlation function is (the lepton leg is trivial)

$$C_1(t) = \frac{1}{2}\int d^3\mathbf{x}\, d^4x_1\, d^4x_2\ \langle 0|T\{J^\nu_W(0)\, j^\mu(x_1) j_\mu(x_2)\phi^\dagger(\mathbf{x},t)\}\,|0\rangle\ \Delta(x_1,x_2)$$

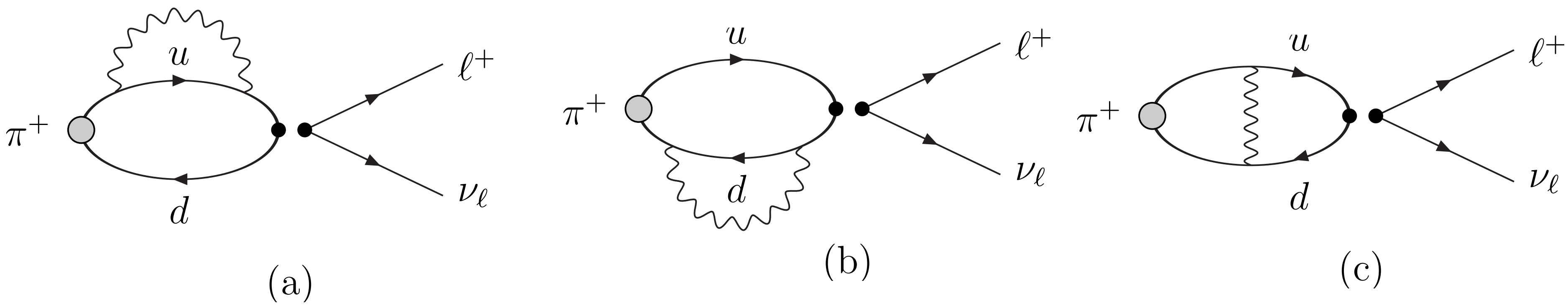
weak V-A current

electromagnetic current

$$j_\mu(x) = \sum_f Q_f\, \bar{f}(x)\gamma_\mu f(x)$$

this is the same set of diagrams used to compute the electromagnetic corrections to the pion (hadron) mass

(the lepton leg is completely irrelevant)



Combining $C_1(t)$ with the lowest order correlator

$$C_0(t) + C_1(t) \simeq \frac{e^{-m_\pi t}}{2m_\pi} Z^\phi \langle 0 \,|\, J_W^0(0) \,|\, \pi^+ \rangle$$

where the $O(\alpha)$ corrections are included; by writing

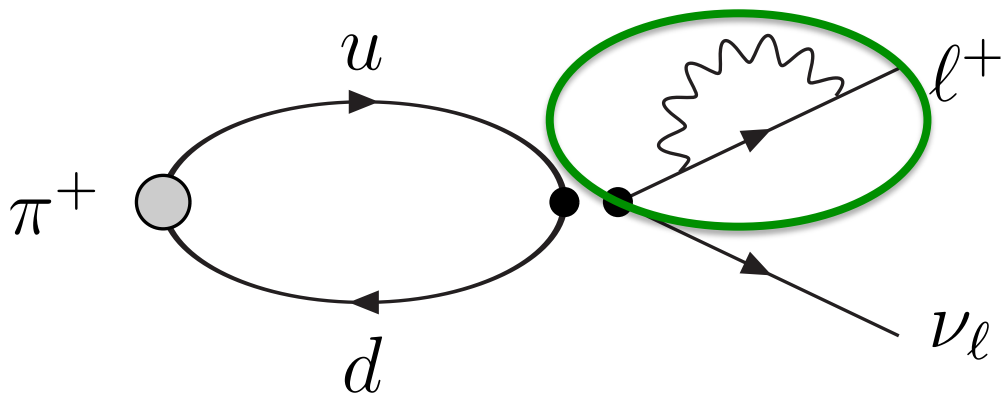
$$e^{-m_\pi t} \simeq e^{-m_\pi^0 t} (1 - \delta m_\pi \, t)$$

δm_π is infrared finite and gauge invariant

Z^ϕ and the matrix element of the axial current *however* are infrared divergent and cannot be interpreted as a correction to f_π

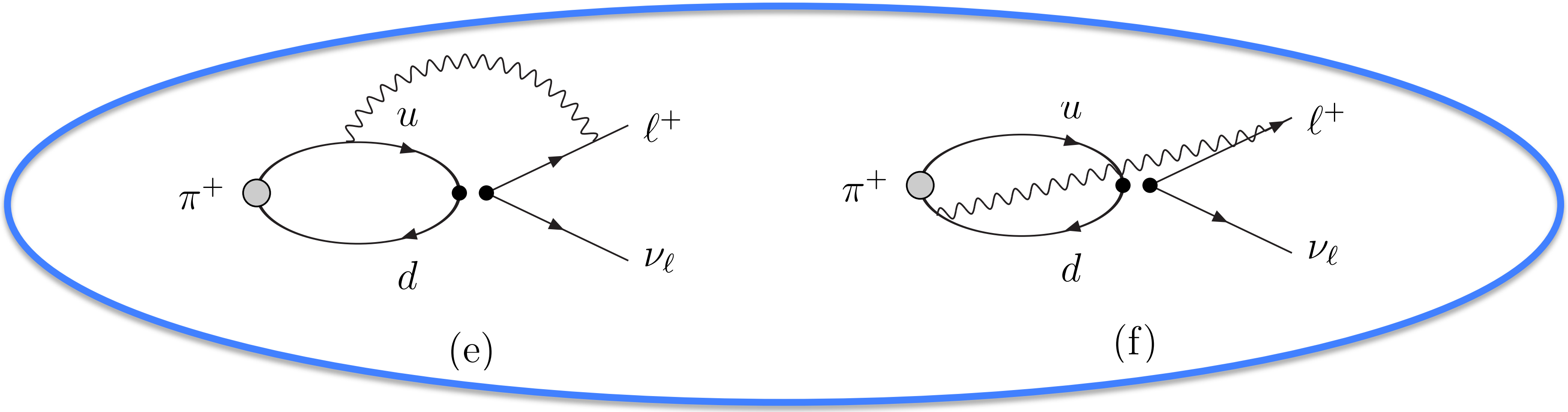
This diagram is an easy case: its contribution to $\Delta\Gamma(L) = \Gamma_0 - \Gamma_0^{pt}$ can be readily obtained in perturbation theory.

The recipe is simply to redefine the operator $O_1^{W\text{-}reg}$ and compute f_π in the numerical simulation



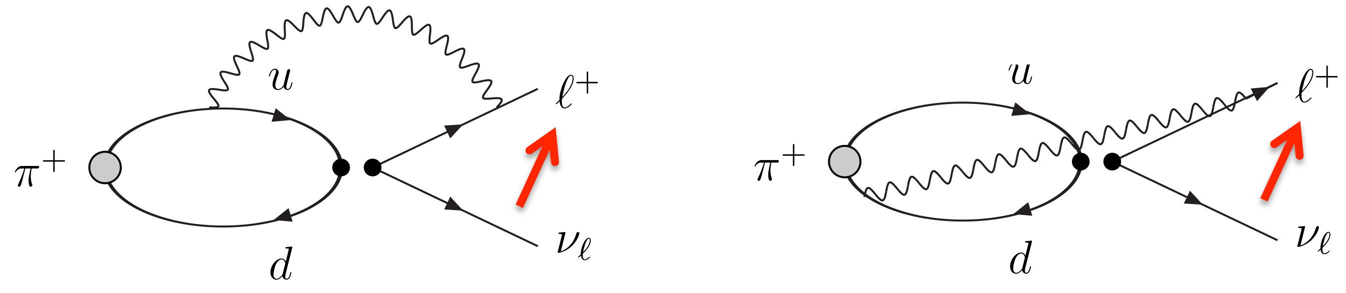
(d)

NASTY DIAGRAMS



- *Certainly these diagrams are not simply a generalization of the evaluation of f_π ; they are also infrared divergent)*
- *We have to isolate the finite volume ground state (necessity of a mass gap – Minkowski $\longleftrightarrow$ Euclidean continuation J. Gasser and G.R.S. Zarnauskas, Phys. Lett. B 693 (2010) 122)*
- *Finite volume effects, expected of the $O(1/L\, \Lambda_{QCD})$ after the cancellation of the infrared divergence, should be investigated in a numerical simulation.*

Calculation of the `nasty' diagrams in a lattice simulation



(e) (f)

The starting point is the Minkowski Green function

$$\int d^4x_1 d^4x_2 < 0|\, T(j_\mu(x_1) J_W^\nu(0))\, |\pi > \; iD_F(x_1-x_2)\{\bar{u}(p_{\nu_\ell})\gamma^\nu(1-\gamma^5)(iS_F(x_2))\gamma^\mu v(p_\ell)\}e^{ip_\ell\cdot x_2}$$

from which we can compute the on-shell amplitude

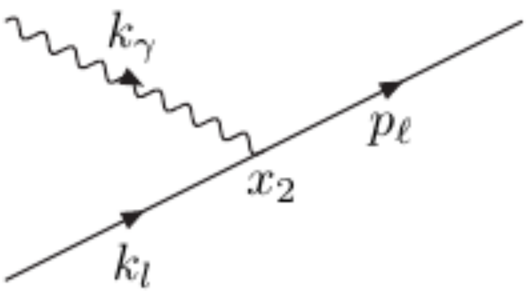
$$\bar{u}_\alpha(p_{\nu_\ell})(\bar{M}_1)_{\alpha\beta}v_\beta(p_\ell) = \quad -i\lim_{k_0\to m_\pi}({k_0}^2-m_\pi^2)\int d^4x_1 d^4x_2\, d^4x\, e^{-ik^0y^0} < 0|T(j_\mu(x_1)J_W^\nu(0)\pi(x))|0 >$$
$$\times iD_F(x_1-x_2)\{\bar{u}(p_{\nu_\ell})\gamma_\nu(1-\gamma^5)(iS_F(x_2))\gamma^\mu v(p_\ell)\}e^{ip_\ell\cdot x_2}$$

which in the Euclidean simulation becomes

$$\bar{C}_1(t)_{\alpha\beta} = \quad \int d^3\mathbf{x}\, d^4x_1\, d^4x_2\, \langle 0|T\{J_W^\nu(0)\, j_\mu(x_1)\phi^\dagger(\mathbf{x},t)\}|\,0\rangle\, \Delta(x_1-x_2)$$
$$\times\left(\gamma_\nu(1-\gamma^5)S(x_2)\gamma^\mu\right)_{\alpha\beta} e^{E_\ell\, t_2} e^{-i\mathbf{p}_\ell\cdot\mathbf{x}_2}$$

A few technical but non trivial IMPORTANT slides: *the continuation from Minkowski to Euclidean*

we need to ensure that the t_2 integration up to ∞ converges in spite of the factor $e^{E_l\, t_2}$ where $E_l = \sqrt{m_l^2 + p_l^2}$ is the energy of the outgoing charged lepton



1) Momentum conservation: since we integrate over x_2
$p_l = k_l + k_\gamma$

2) The integrations over the energies k_{4l} and $k_{4\gamma}$ lead to the exponential factor $e^{-(\omega_l + \omega_\gamma - E_l)\, t_2}$ where $\omega_l = \sqrt{m_l^2 + k_l^2}$, $\omega_\gamma = \sqrt{m_\gamma^2 + k_\gamma^2}$, and m_γ is the mass of the photon introduced as an infra-red cut-off.

A few technical but non trivial IMPORTANT slides:
the continuation from Minkowski to Euclidean

3) … but $(\omega_l+\omega_\gamma) \geq \sqrt{(m_l+m_\gamma)^2 + p_l^2} > E_l = \sqrt{m_l^2 + p_l^2}$

thus the argument of the exponent $e^{-(\omega_l + \omega_\gamma - E_l)\, t_2}$ is negative for every term appearing in the sum over the intermediate states and the integral over t_2 converges

4) note that the integration over t_2 is also convergent if we set $m_\gamma=0$ but remove photon zero mode in finite volume. In this case $(\omega_l+\omega_\gamma) > E_l+[1-(p_l/E_l)]\ (k_\gamma)_{min}$

- necessity of a mass gap
- absence of a lighter intermediate state

under these conditions

$$\bar{C}_1(t)_{\alpha\beta} \simeq Z_0^{\phi}\, \frac{e^{-m_\pi^0|t|}}{2m_\pi^0}\, (\bar{M}_1)_{\alpha\beta}$$

and the contribution to the amplitude from these diagrams is given by

$$\bar{u}_\alpha(p_{\nu_\ell})(\bar{M}_1)_{\alpha\beta} v_\beta(p_\ell)$$

$$\Gamma(\Delta E) = \boxed{\lim_{V\to\infty}\left(\Gamma_0 - \Gamma_0^{pt}\right)} + \lim_{V\to\infty}\left(\Gamma_0^{pt} + \Gamma_1^{pt}(\Delta E)\right)$$

- $\Gamma_0^{pt}(L)$ is calculated in perturbation theory with a pointlike pion

- UV divergences are regularized with the W-regularization
- IR divergences are regularized by the finite volume (same of $\Gamma_0(L)$)

- For the pion self energy, the result is:

Preliminary

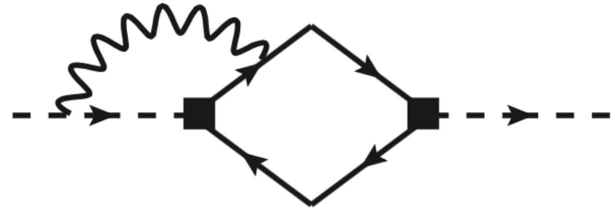
$$\frac{1}{\pi}\left(\frac{2\pi}{L}\right)^3 \sum_{\vec{q}} \left\{ \frac{1}{\left(M_W^4 - 4m_\pi^2 E_{W,\vec{q}}^2\right)^2} \left[16 m_\pi^4 \left(\frac{\vec{q}^{\,2}}{E_{W,\vec{q}}} + \frac{M_W^2}{E_{W,\vec{q}}} + \frac{M_W^2}{E_{\pi,\vec{q}}} \right) + M_W^4 \left(\frac{4\vec{q}^{\,2}}{E_{W,\vec{q}}} - \frac{4\vec{q}^{\,2}}{E_{\pi,\vec{q}}} + \frac{M_W^2}{E_{W,\vec{q}}} + \frac{M_W^2}{E_{\pi,\vec{q}}} \right) \right.\right.$$

$$\left.\left. -4 M_W^2 m_\pi^2 \left(\frac{3\vec{q}^{\,2}}{E_{W,\vec{q}}} - \frac{3\vec{q}^{\,2}}{E_{\pi,\vec{q}}} + \frac{2M_W^2}{E_{W,\vec{q}}} + \frac{2M_W^2}{E_{\pi,\vec{q}}} \right) \right] - (M_W \to 0) \right\} \qquad \boxed{\vec{q} = \frac{2\pi}{L}(n_x, n_y, n_z)} \qquad E_{X,\vec{q}} = \sqrt{M_X^2 + \vec{q}^{\,2}}$$

Courtesy by V. Lubicz

Γ_0^{pt} *The nasty diagram*

sum vs integral under study



$$
\left(\frac{2\pi}{L}\right)^3 \sum_{\vec{q}} \frac{1}{\pi}
$$

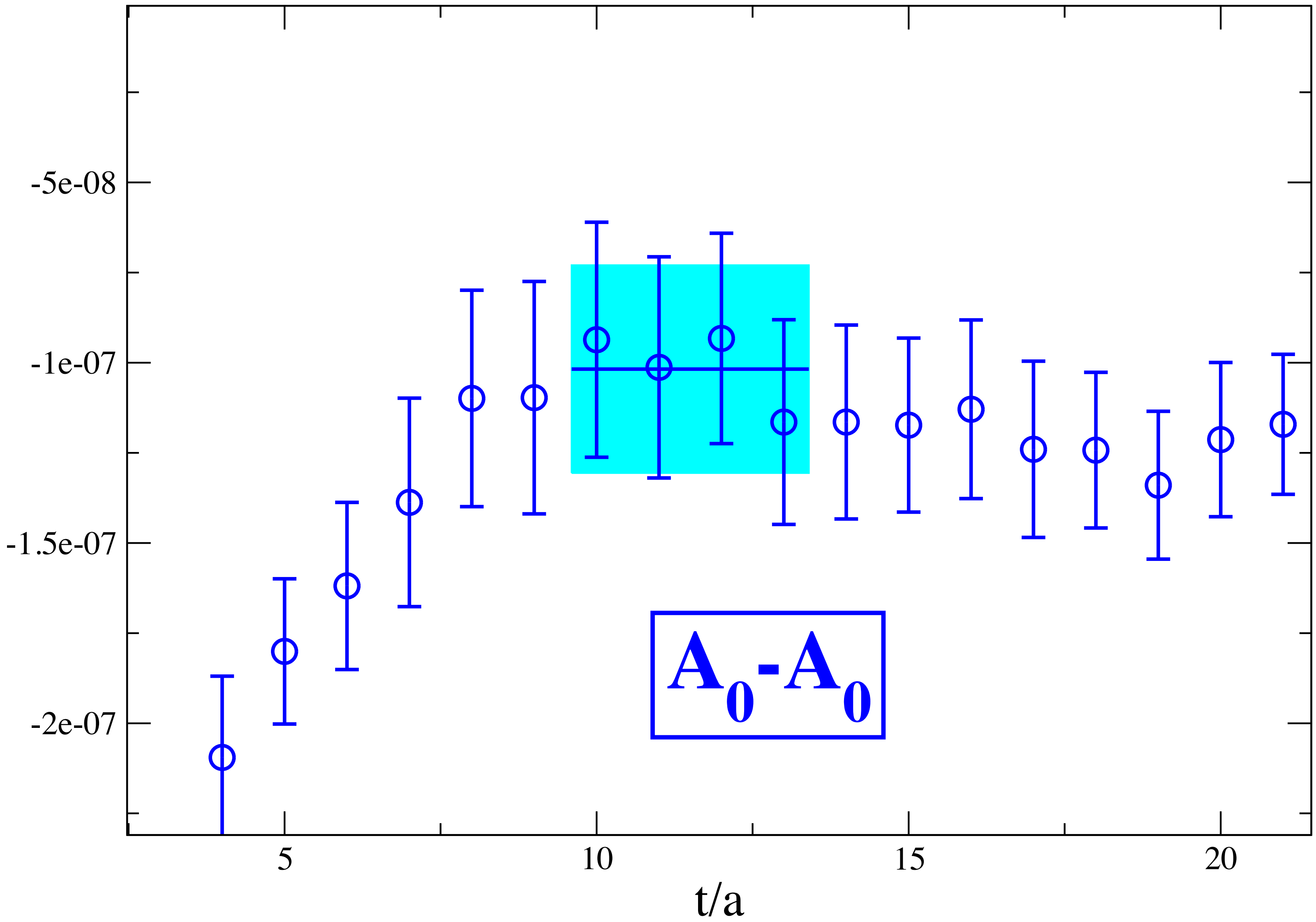
$$
\left(\frac{1}{m_W{}^2|\vec{q}\cdot\vec{q}|^{1/2}} - \frac{2m^2}{(m^2-m_\mu{}^2)m_W{}^2|\vec{q}\cdot\vec{q}|^{1/2}} + \frac{2m_\mu{}^2}{(m^2-m_\mu{}^2)m_W{}^2|\vec{q}\cdot\vec{q}|^{1/2}} + \frac{4|\vec{q}\cdot\vec{q}|^{1/2}}{m_W{}^4} - \frac{4m^2|\vec{q}\cdot\vec{q}|^{1/2}}{(m^2-m_\mu{}^2)m_W{}^4} + \right.
$$

$$
\frac{4m_\mu{}^2|\vec{q}\cdot\vec{q}|^{1/2}}{(m^2-m_\mu{}^2)m_W{}^4} + \frac{m^2\sqrt{\frac{1}{m^2+\vec{q}\cdot\vec{q}}}}{2(m^2-m_\mu{}^2)\vec{q}\cdot\vec{q}} + \frac{m_\mu{}^2\sqrt{\frac{1}{m^2+\vec{q}\cdot\vec{q}}}}{2(m^2-m_\mu{}^2)\vec{q}\cdot\vec{q}} - \frac{m_\mu{}^2\sqrt{\frac{1}{m_\mu{}^2+\vec{q}\cdot\vec{q}}}}{(m^2-m_\mu{}^2)\vec{q}\cdot\vec{q}} - \frac{3\sqrt{\frac{1}{m_W{}^2+\vec{q}\cdot\vec{q}}}}{m_W{}^2} -
$$

$$
\frac{4\vec{q}\cdot\vec{q}\sqrt{\frac{1}{m_W{}^2+\vec{q}\cdot\vec{q}}}}{m_W{}^4} + \frac{4m^2\sqrt{m_W{}^2+\vec{q}\cdot\vec{q}}}{(m^2-m_\mu{}^2)m_W{}^4} - \frac{4m_\mu{}^2\sqrt{m_W{}^2+\vec{q}\cdot\vec{q}}}{(m^2-m_\mu{}^2)m_W{}^4} + \frac{m^2\left(\frac{1}{m_\gamma{}^2+\vec{q}\cdot\vec{q}}\right)^{3/2}\mathrm{Log}[r_\mu{}^2]}{2(m^2-m_\mu{}^2)} +
$$

$$
\frac{m_\mu{}^2\left(\frac{1}{m_\gamma{}^2+\vec{q}\cdot\vec{q}}\right)^{3/2}\mathrm{Log}[r_\mu{}^2]}{2(m^2-m_\mu{}^2)} - \frac{m^2\mathrm{Log}\left[\frac{1+|\vec{q}\cdot\vec{q}|^{1/2}\sqrt{\frac{1}{m^2+\vec{q}\cdot\vec{q}}}}{1-|\vec{q}\cdot\vec{q}|^{1/2}\sqrt{\frac{1}{m^2+\vec{q}\cdot\vec{q}}}}\right]}{2(m^2-m_\mu{}^2)\left(|\vec{q}\cdot\vec{q}|^{1/2}\right)^3} - \frac{m_\mu{}^2\mathrm{Log}\left[\frac{1+|\vec{q}\cdot\vec{q}|^{1/2}\sqrt{\frac{1}{m^2+\vec{q}\cdot\vec{q}}}}{1-|\vec{q}\cdot\vec{q}|^{1/2}\sqrt{\frac{1}{m^2+\vec{q}\cdot\vec{q}}}}\right]}{2(m^2-m_\mu{}^2)\left(|\vec{q}\cdot\vec{q}|^{1/2}\right)^3} +
$$

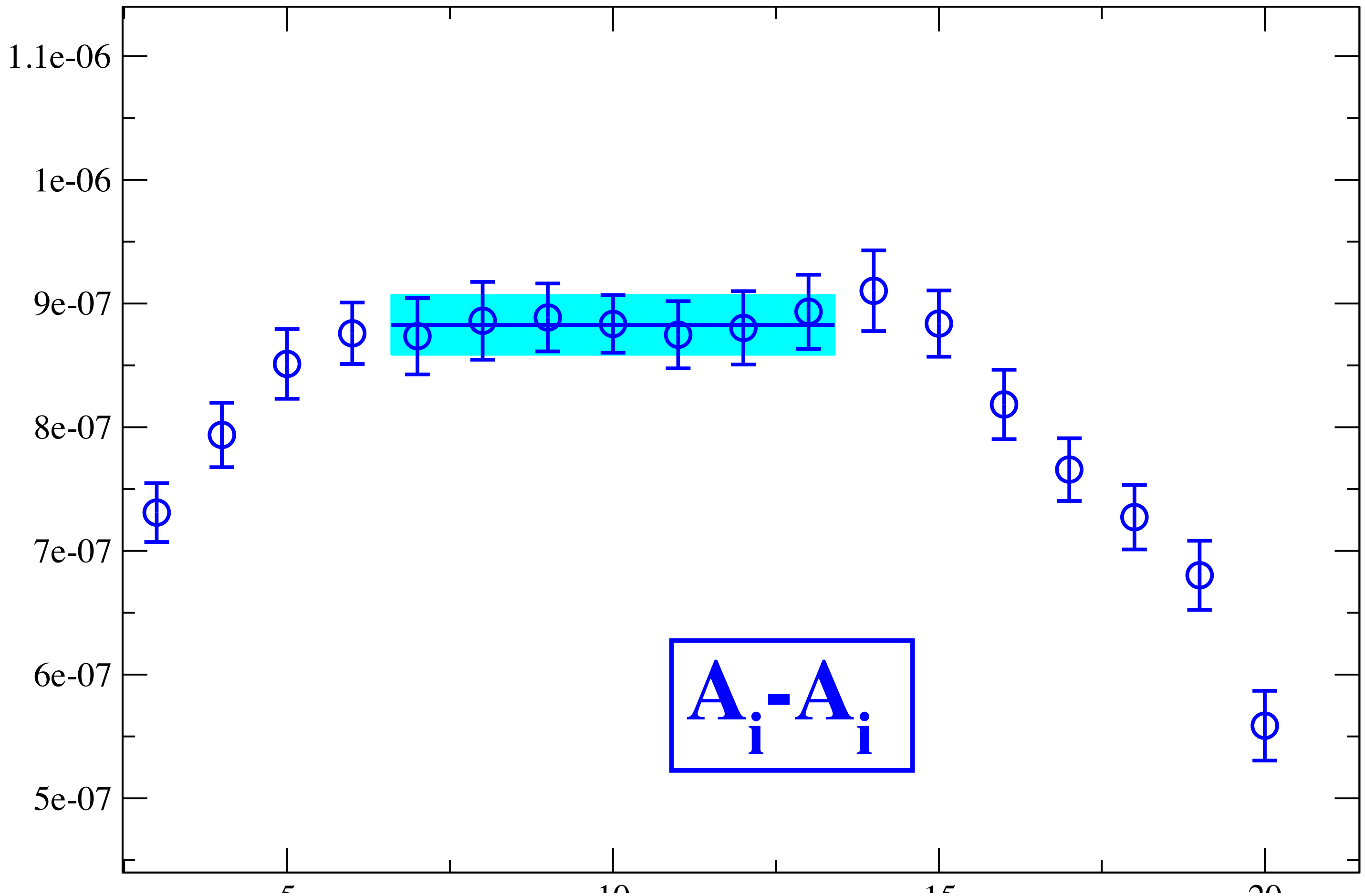
$$
\left. \frac{m^2\mathrm{Log}\left[\frac{1+|\vec{q}\cdot\vec{q}|^{1/2}\sqrt{\frac{1}{m_\mu{}^2+\vec{q}\cdot\vec{q}}}}{1-|\vec{q}\cdot\vec{q}|^{1/2}\sqrt{\frac{1}{m_\mu{}^2+\vec{q}\cdot\vec{q}}}}\right]}{2(m^2-m_\mu{}^2)\left(|\vec{q}\cdot\vec{q}|^{1/2}\right)^3} + \frac{m_\mu{}^2\mathrm{Log}\left[\frac{1+|\vec{q}\cdot\vec{q}|^{1/2}\sqrt{\frac{1}{m_\mu{}^2+\vec{q}\cdot\vec{q}}}}{1-|\vec{q}\cdot\vec{q}|^{1/2}\sqrt{\frac{1}{m_\mu{}^2+\vec{q}\cdot\vec{q}}}}\right]}{2(m^2-m_\mu{}^2)\left(|\vec{q}\cdot\vec{q}|^{1/2}\right)^3} \right)
$$

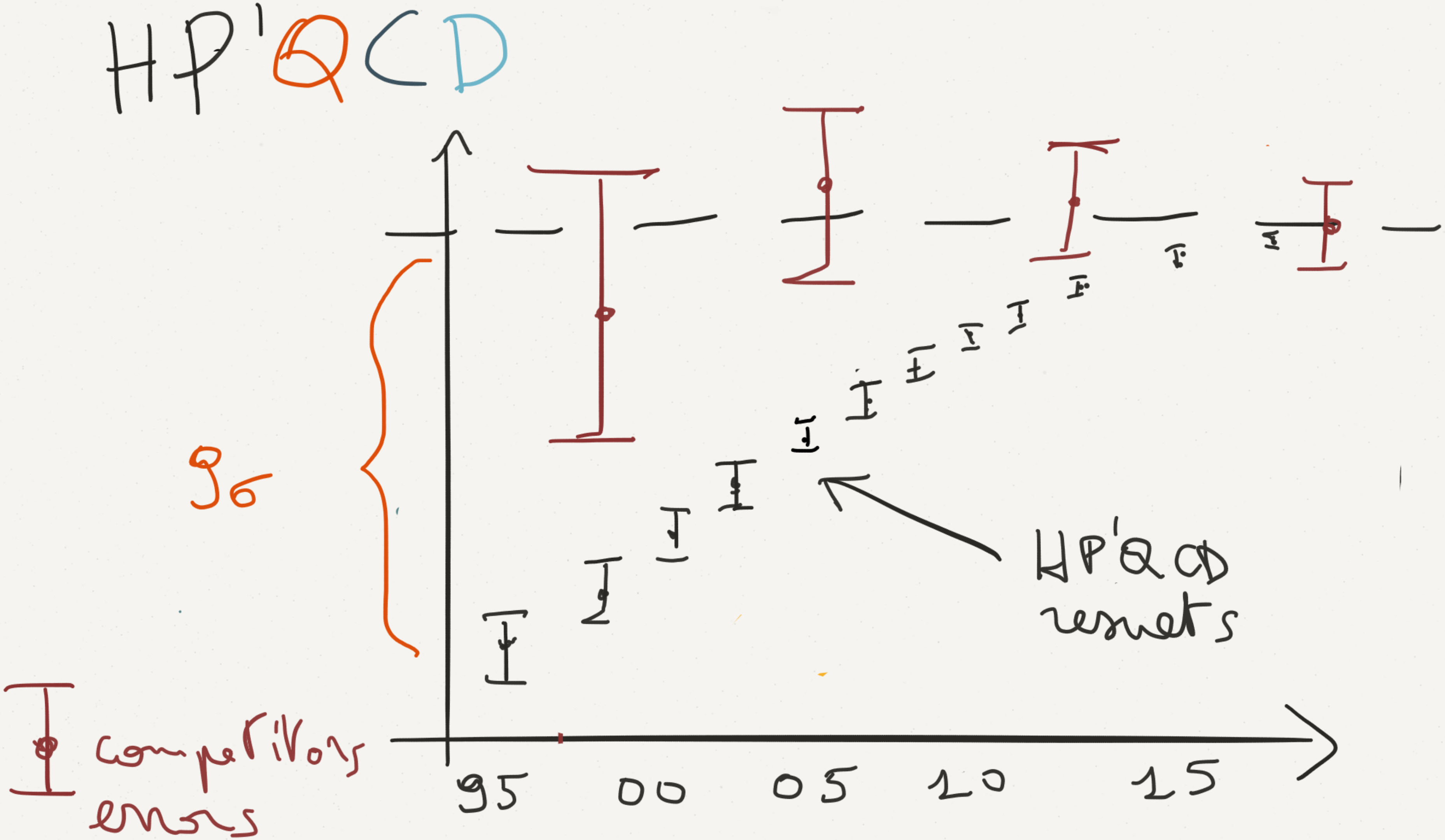


-5e-08
-1e-07
-1.5e-07
-2e-07
5
10
15
20
t/a
A_0-A_0

The quality of the results is quite good even with a modest statistics

$24^3 \times 48$ lattice with $a = 0.086$ fm, $m_\pi \simeq$ 475 MeV, 240 configs





HP'QCD = Hyperbolic Precision QCD

(see also Cappuccino Collaboration)

To conclude

- We have presented a method to compute QED corrections to hadronic processes;
- For these quantities the presence of infrared divergences in the intermediate stages of the calculation make the procedure much more complicated than in the case of the hadronic spectrum;
- In order to obtain the physical answer virtual corrections and real photon emissions must be combined together;
- It is not sufficient to add the electromagnetic interaction to the quark action, because separate explicit real and virtual emission diagrams must be evaluated for any given process;
- We have discussed a specific case, namely the radiative corrections to the leptonic decay of charged pseudoscalar mesons. The method can e however be extended to many other cases like for example to semileptonic decays.

To conclude

- The condition for the applicability of our strategy is that there is a mass gap between the decaying particle and the intermediate states generated by the emission of the photon, and that none of these states is lighter than the initial hadron.
- In the calculation of electromagnetic corrections a general issue is finite size effects. In this respect our method reduces to compute infrared finite, gauge invariant quantities for which we do expect finite size corrections which are comparable to those encountered for the spectrum. This expectation will be checked in forthcoming numerical studies, and eventually studied theoretically in chiral perturbation theory.
- The implementation of our method, although challenging, is within reach of the present lattice technology. The accuracy necessary to make the results phenomenologically interesting is not exceedingly high since the effect that we want to predict is, in general, of the order of a few percent.

erc

UTfit

THANKS FOR YOUR ATTENTION

mitp Mainz Institute for Theoretical Physics

INFN Istituto Nazionale di Fisica Nucleare