$\pi\pi$ scattering length measurement

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Outline

Low energy theorems, chiral expansion

Dispersive methods
- Roy equations
- Chiral symmetry + dispersive methods

What have we learnt?

Concluding remarks

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Low-energy theorem for $\pi\pi$ scattering

Some notation

$$\langle \pi^i \pi^j \text{ out} | \pi^k \pi^l \text{ in} \rangle = \delta^{ij}\delta^{kl} A(s,t,u) + \delta^{ik}\delta^{jl} A(t,u,s) + \delta^{il}\delta^{jk} A(u,t,s)$$

All physical amplitudes can be expressed in terms of $A(s,t,u)$

$$T^{I=0}(s,t,u) = 3A(s,t,u) + A(t,s,u) + A(u,t,s)$$

Low energy theorem

Weinberg 1966

$$A(s,t,u) = \frac{s - M_\pi^2}{F_\pi^2} + \mathcal{O}(p^4) \qquad \Rightarrow \qquad T^{I=0} = \frac{2s - M_\pi^2}{F_\pi^2}$$

S wave projection

(I=0)

$$t_0^0(s) = \frac{2s - M_\pi^2}{32\pi F_\pi^2} \qquad a_0^0 = t_0^0(4M_\pi^2) = \frac{7M_\pi^2}{32\pi F_\pi^2} = 0.16$$

Low-energy theorem for $\pi\pi$ scattering

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S wave projection

(I=2)

$$t_0^2(s) = \frac{2M_\pi^2 - s}{32\pi F_\pi^2} \qquad a_0^2 = t_0^2(4M_\pi^2) = \frac{-M_\pi^2}{16\pi F_\pi^2} = -0.045$$

Higher orders

Higher order corrections are suppressed by $\mathcal{O}(p^2/\Lambda^2)$
$\Lambda \sim 1$ GeV $\Rightarrow$ expected to be a few percent

$$a_0^0 = 0.200 + \mathcal{O}(p^6) \qquad a_0^2 = -0.0445 + \mathcal{O}(p^6)$$

Gasser and Leutwyler (84)

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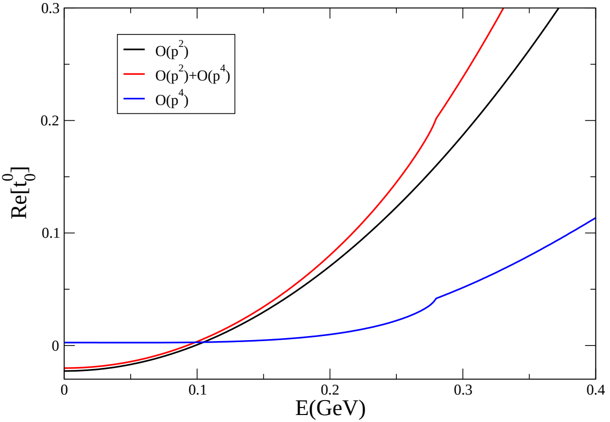
The reason for the rather large correction in a_0^0 is a chiral log

$$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2}\left[1 + \frac{9}{2}\ell_\chi + \dots\right] \qquad a_0^2 = -\frac{M_\pi^2}{16\pi F_\pi^2}\left[1 - \frac{3}{2}\ell_\chi + \dots\right]$$

$$\ell_\chi = \frac{M_\pi^2}{16\pi^2 F_\pi^2} \ln \frac{\mu^2}{M_\pi^2}$$

Gasser and Leutwyler (84)

Higher orders



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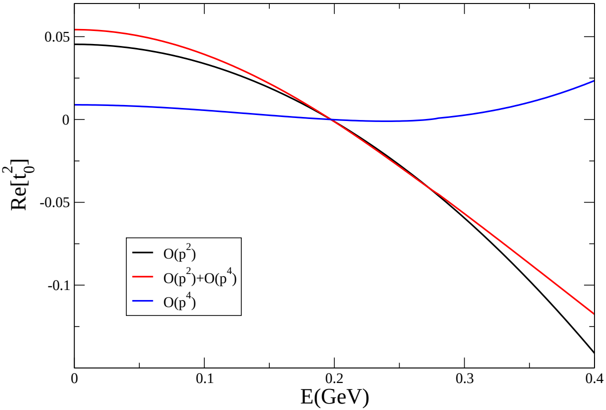

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Roy equations

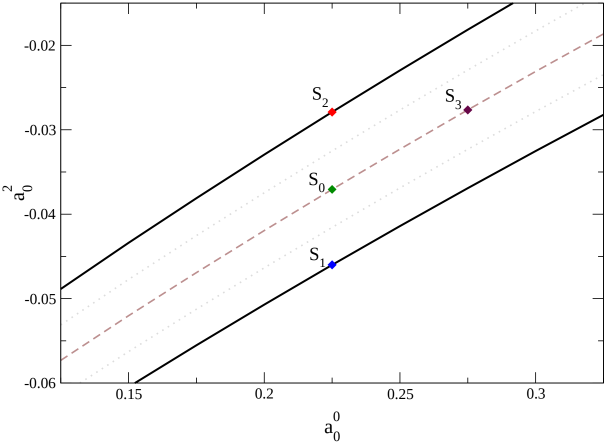
Unitarity effects can be calculated exactly using dispersive methods

Unitarity, analyticity and crossing symmetry $\equiv$ Roy equations

Input: imaginary parts above 0.8 GeV
two subtraction constants, *e.g.* a_0^0 and a_0^2
Output: the full $\pi\pi$ scattering amplitude below 0.8 GeV
Note: if a_0^0, a_0^2 are chosen within the universal band
the solution exist and is unique

Numerical solutions of the Roy equations:
Pennington-Protopopescu, Basdevant-Froggatt-Petersen (70s)
Ananthanarayan, GC, Gasser and Leutwyler (00)
Descotes-Genon, Fuchs, Girlanda and Stern (01)
Garcia-Martin, Kaminski, Pelaez, Ruiz de Elvira, Yndurain (08,11)

Numerical solutions



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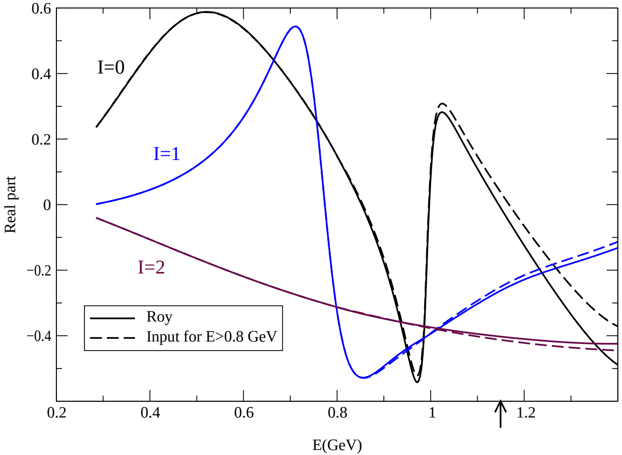

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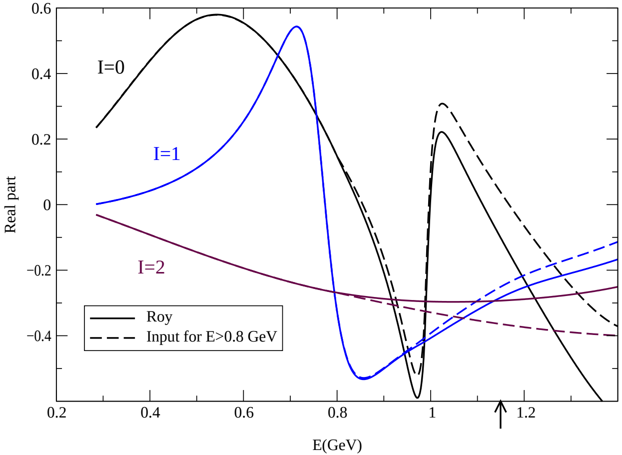

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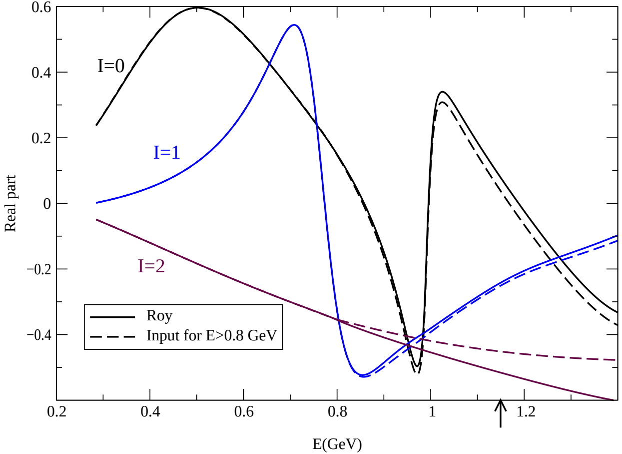
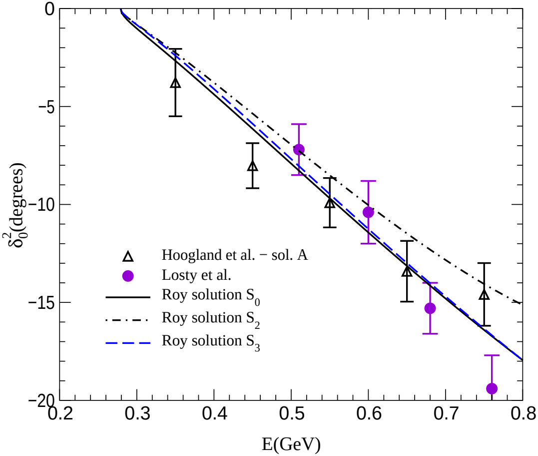


Numerical solutions

Example of a fit to data



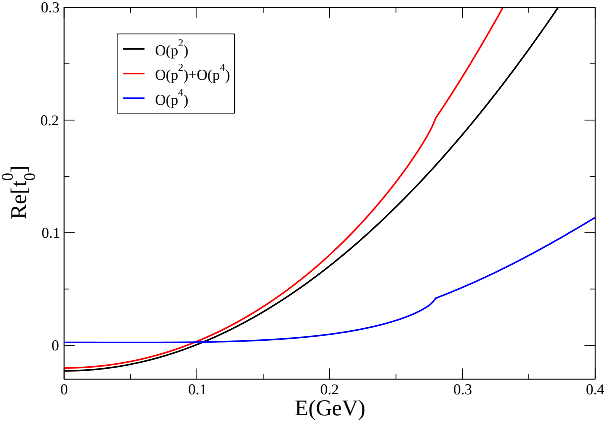
Combining CHPT and dispersive methods

In CHPT the two subtraction constants are predicted

Subtracting the amplitude at threshold (a_0^0, a_0^2) is not mandatory

The freedom in the choice of the subtraction point
can be exploited to use the chiral expansion
where it converges best, *i.e.* below threshold

Combining CHPT and dispersive methods



Combining CHPT and dispersive methods

The convergence of the series at threshold is greatly improved if CHPT is used only below threshold

CHPT at threshold

$$\begin{array}{rcccc} a_0^0 & = & 0.159 \to & 0.200 \to & 0.216 \\ 10 \cdot a_0^2 & = & -0.454 \to & -0.445 \to & -0.445 \\ & & p^2 & p^4 & p^6 \end{array}$$

GC, Gasser and Leutwyler (01)

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\end{array}
$$

CHPT below threshold + Roy

$$
\begin{array}{rcccc}
a_0^0 & = & 0.197 \rightarrow & 0.2195 \rightarrow & 0.220 \\
10 \cdot a_0^2 & = & -0.402 \rightarrow & -0.446 \rightarrow & -0.444
\end{array}
$$

GC, Gasser and Leutwyler (01)

Roy+ChPT: final results

GC, Gasser and Leutwyler (01)

Scattering lengths

$$\begin{aligned} a_0^0 &= 0.220 \pm 0.001 + 0.009\Delta\ell_4 - 0.002\Delta\ell_3 \\ 10 \cdot a_0^2 &= -0.444 \pm 0.003 - 0.01\Delta\ell_4 - 0.004\Delta\ell_3 \end{aligned}$$

where $\bar{\ell}_4 = 4.4 + \Delta\ell_4 \qquad \bar{\ell}_3 = 2.9 + \Delta\ell_3$

Adding errors in quadrature $[\Delta\ell_4 = 0.2,\ \Delta\ell_3 = 2.4]$

$$\begin{aligned} a_0^0 &= 0.220 \pm 0.005 \\ 10 \cdot a_0^2 &= -0.444 \pm 0.01 \\ a_0^0 - a_0^2 &= 0.265 \pm 0.004 \end{aligned}$$

Roy+ChPT: final results

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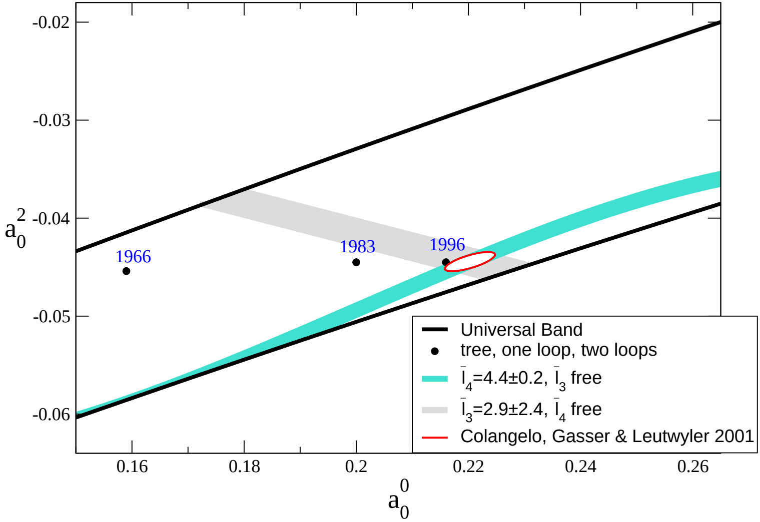
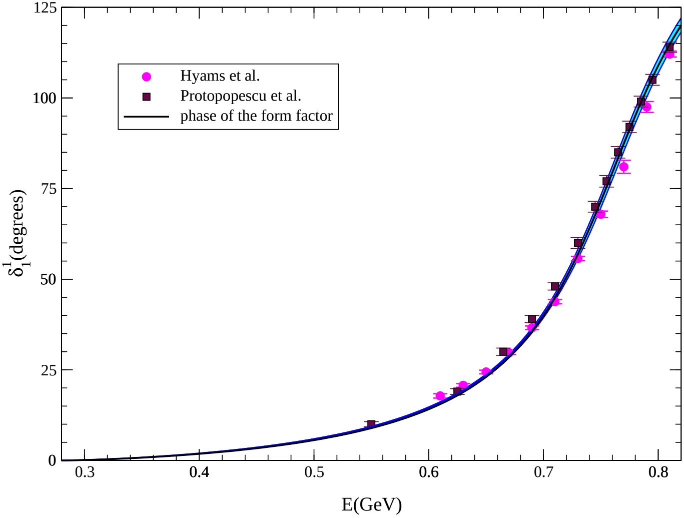


Final result for the phase shifts

Phase shifts:



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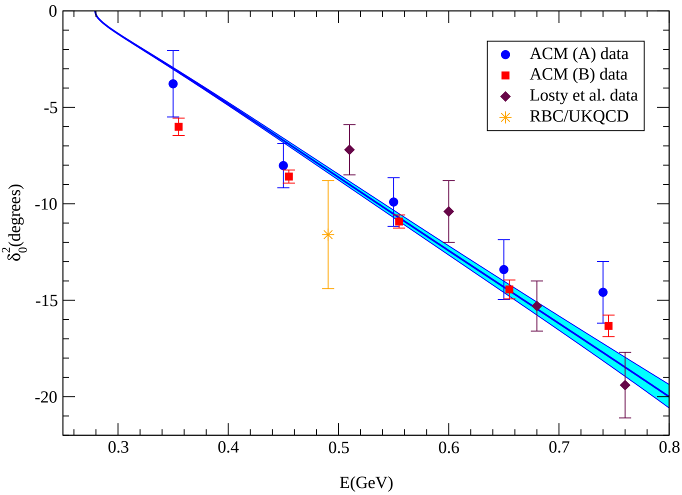

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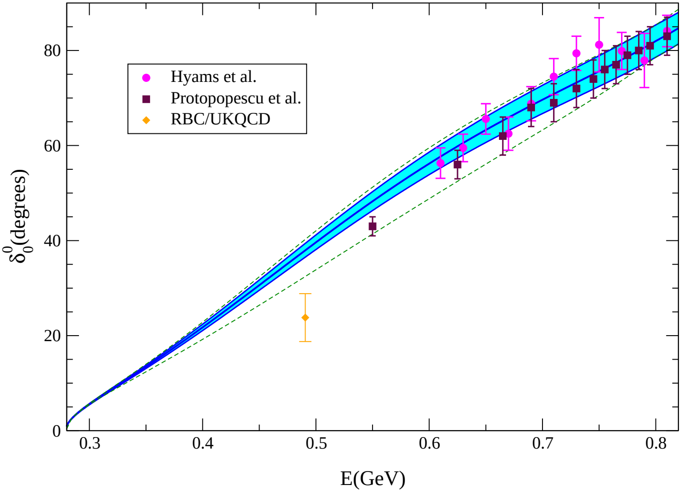

Final result for the phase shifts

Determination by RBC/UKQCD:

$$\begin{aligned}\delta_0^0(MK) &= 23.8(4.9)(1.2)\\ \delta_0^2(MK) &= -11.6(2.5)(1.2)\end{aligned}$$

Our determination (at $M_k = 0.4976$ GeV)

$$\begin{aligned}\delta_0^0(MK) &= 39.2(1.5)\\ \delta_0^2(MK) &= -8.5(0.15)\end{aligned}$$

or (at $M_k = 0.4906$ GeV value used by RBC/UKQCD):

$$\begin{aligned}\delta_0^0(MK) &= 38.0(1.3)\\ \delta_0^2(MK) &= -8.3(0.15)\end{aligned}$$

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Sensitivity to the quark condensate

The constant $\bar{\ell}_3$ appears in the chiral expansion of the pion mass

$$M_\pi^2 = 2B\hat{m}\left[1 + \frac{2B\hat{m}}{16\pi F_\pi^2}\bar{\ell}_3 + \mathcal{O}(\hat{m}^2)\right]$$

$$\hat{m} = \frac{m_u + m_d}{2} \qquad B = -\frac{1}{F^2}\langle 0|\bar{q}q|0\rangle$$

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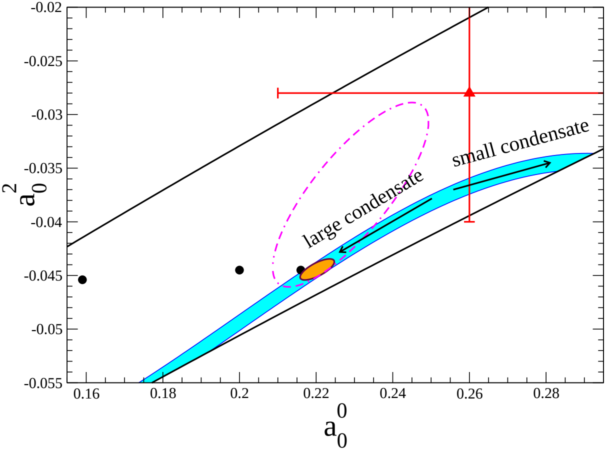
Its size tells us what fraction of the pion mass is given by the Gell-Mann–Oakes–Renner term

$$M_{\mathrm{GMOR}}^2 \equiv 2B\hat{m}$$

Sensitivity to the quark condensate

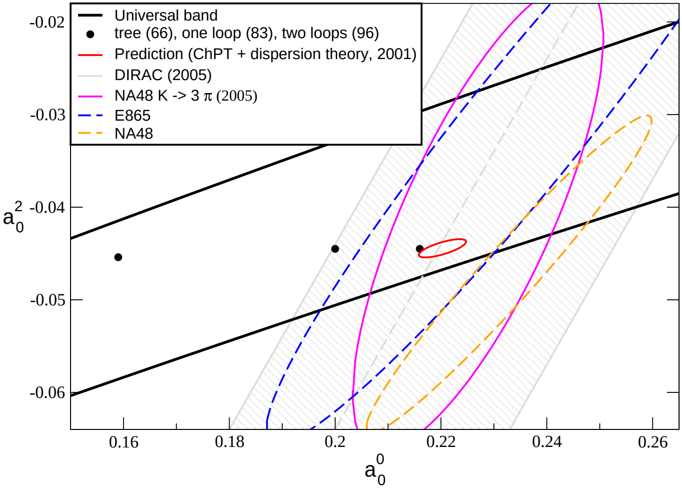


The E865 data on $K_{\ell 4}$ imply that

GC, Gasser and Leutwyler PRL (01)

$$M_{\mathrm{GMOR}} > 94\% M_\pi$$

Experimental tests



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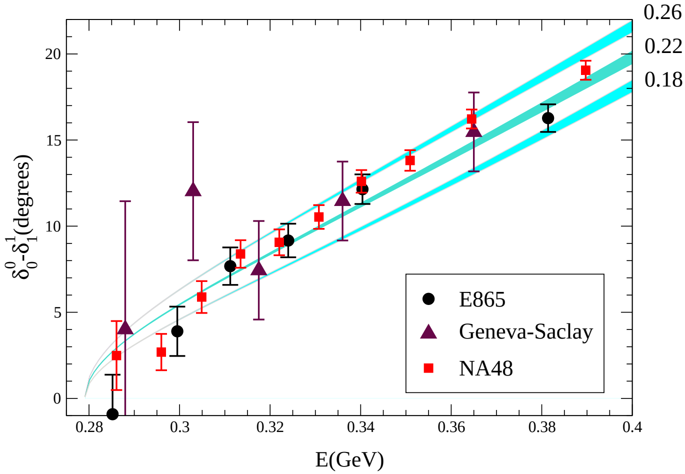

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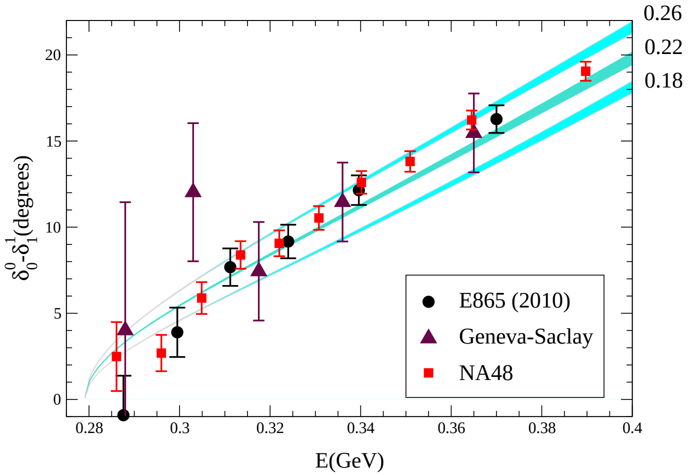

E865 corrected their data

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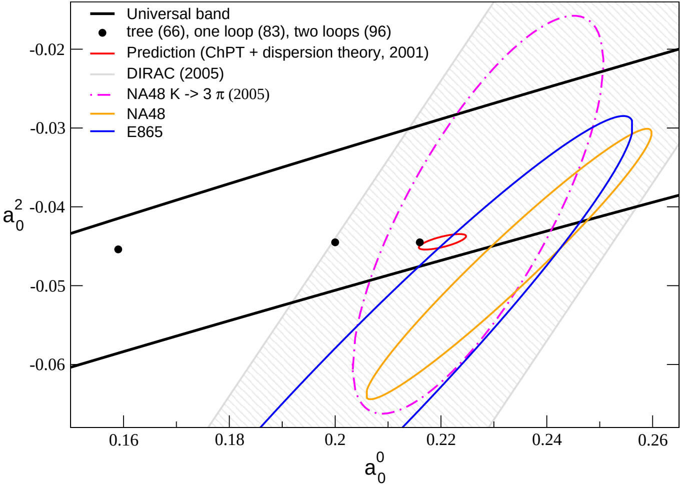

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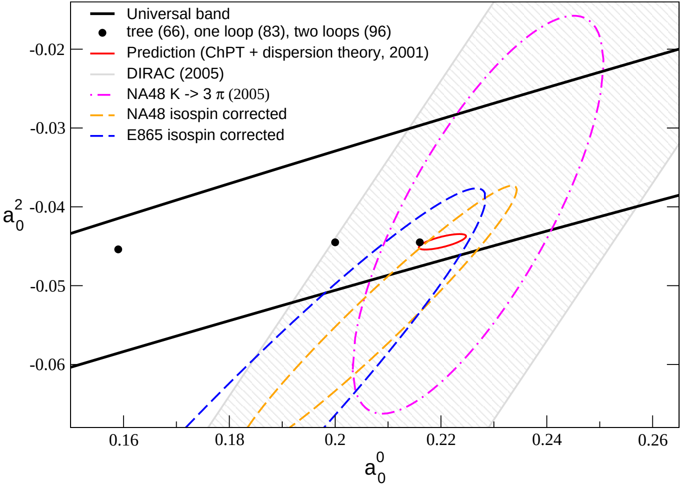

isospin breaking corrections recently calculated for K_{e4} are essential at this level of precision GC, Gasser, Rusetsky (09)

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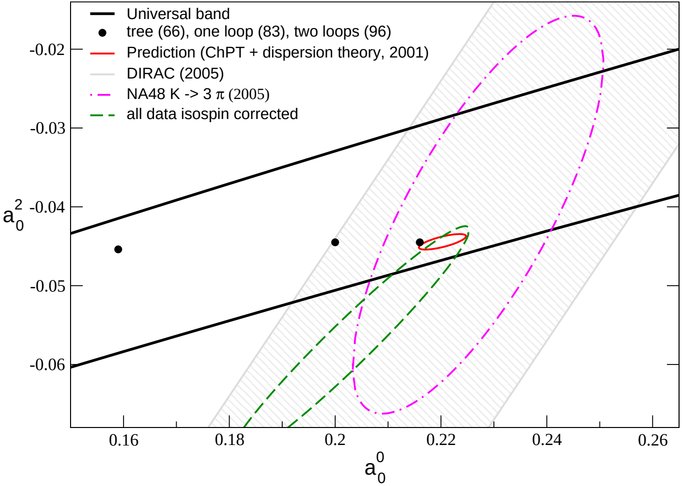

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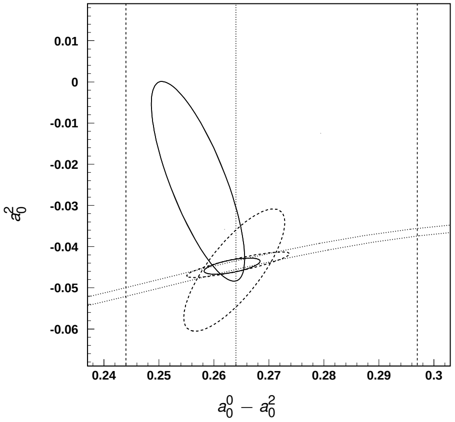

Figure from NA48/2 Eur.Phys.J.C64:589,2009

Lattice determinations of $\bar{\ell}_{3,4}$

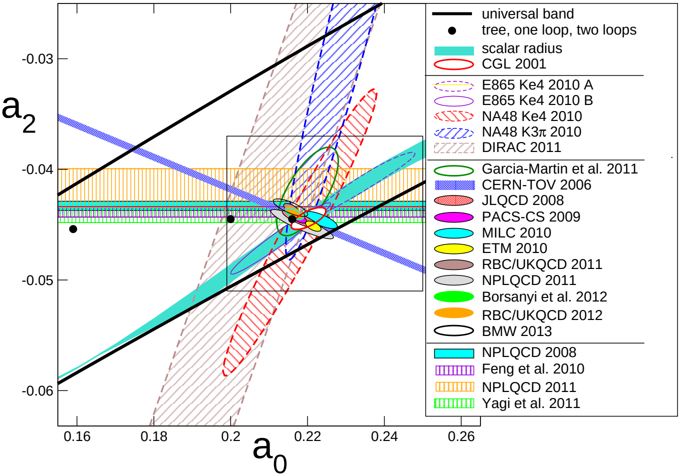

Figure courtesy of H. Leutwyler

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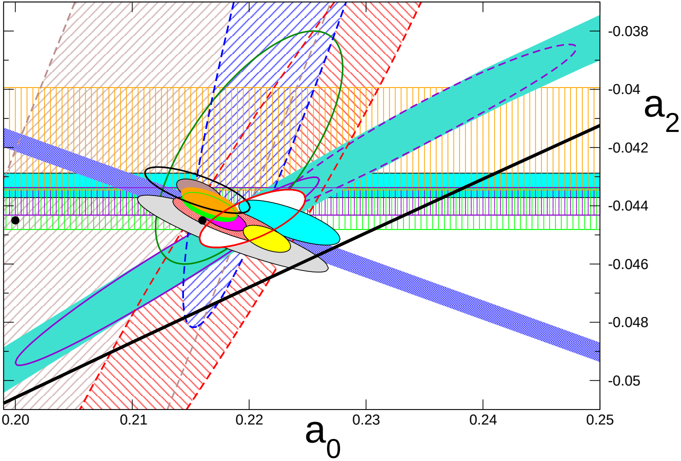

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Remarks on a possible new measurement of a_0^0

- the burning question about the mechanism of chiral symmetry breaking has been answered experimentally by past measurements of a_0^0
- the precision of the theoretical prediction for a_0^0 has *de facto* already been improved by lattice determinations of $\bar{\ell}_3$
- increasing the precision of the experimental measurement of a_0^0 will require a better handling of radiative corrections ($\rightarrow$ talks by P. Stoffer and M. Knecht)
- this could lead to a precise determination of $\bar{\ell}_3$, *i.e.* of the quadratic dependence of M_π on $\hat{m}$
- in itself this is a remarkable achievement, but not a qualitative change in our knowledge of nonperturbative QCD

Remarks on the relevance of the measurement of a_0^0

- the burning question about the mechanism of chiral symmetry breaking has been answered experimentally by past measurements of a_0^0
- an accurate measurement of the S wave scattering lengths implies also a precise knowledge of the $\pi\pi$ phase shifts below $\sim$ 1 GeV
- which makes a dispersive treatment of several other low-energy matrix elements – in particular K decays – meaningful and potentially very precise
- *e.g.* $K_{\ell 4}$, $K \to \pi\pi$, $K \to 3\pi$, $K_S \to \gamma^{(*)}\gamma^{(*)}$, $K_S \to \ell^+\ell^-$, $K \to \pi\gamma\gamma$, $K \to \pi\ell^+\ell^-$, ΔM_K, ϵ_K
  ($\to$ talks by P. Stoffer and R. Stucki)

Dispersion relations: basics I

- analyticity properties of Green's functions (and form factors and scattering amplitudes) can be rigorously established
- the absence of singularities for complex (unphysical) values of kinematic variables[1] follows from causality
- the presence of singularities is related to dynamical phenomena (exchange of particles) and can be understood in terms of the underlying dynamics
- analytic functions are determined by their singularities: dispersion relations provide an explicit representation of this mathematical property
- QFTs satisfy these properties automatically.
  Weinberg: QFT emerges by imposing analyticity and unitarity (and other properties)

[1] Exceptions known: anomalous thresholds.

Dispersion relations: basics II

- dispersion relations are exact
- their usefulness is directly related to our knowledge of the singularities of the function of interest
- depending on where one wants to calculate the function, some singularities (or regions thereof) may be more important than others: approximation schemes may be successfully applied
- singularities at infinity = subtraction constants, if present are essential input
- use of dispersion relations in combination with QFT calculations (whether perturbative or not) is always possible