Short Distance Contributions to Rare and CP Violating Kaon Decays

NA62 Kaon Physics Handbook 2016
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Contents

Why are Kaon Decays so rare?

Before the charm quark: why are the two Branching ratios

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so different in size?

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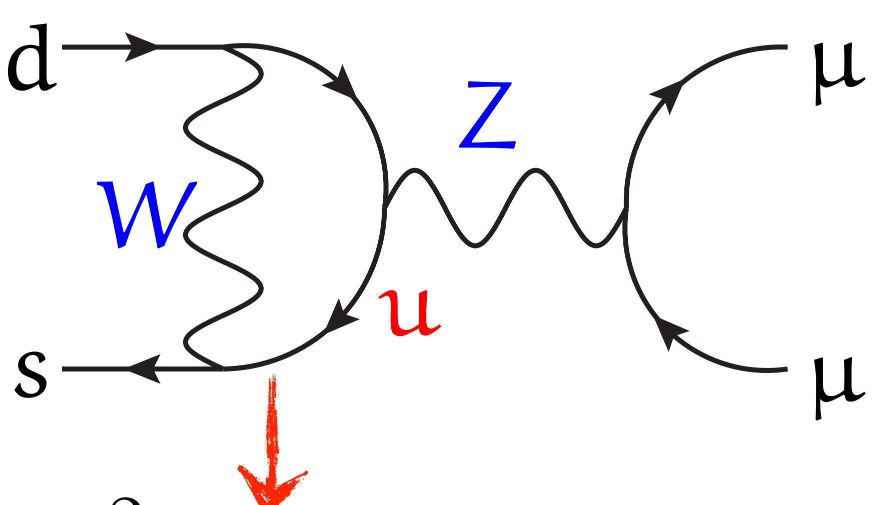
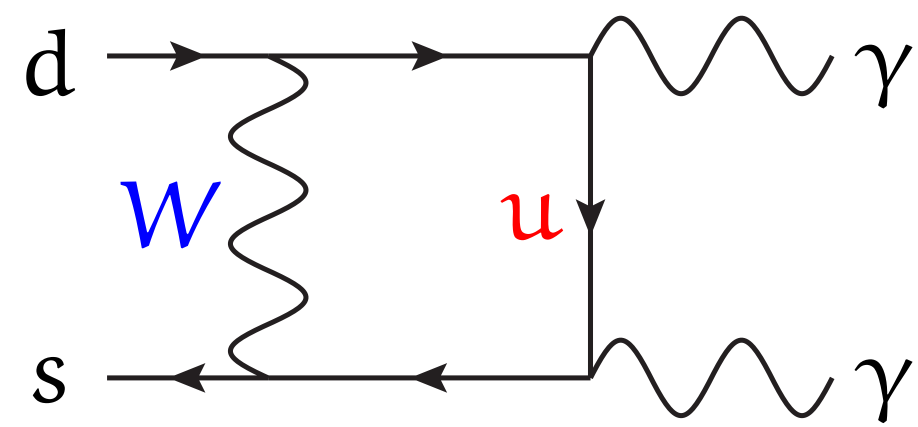
so different in size?

$K_L \to \mu^+ \mu^-$: The 2 μs are in J=0 state → no 1 γ coupling



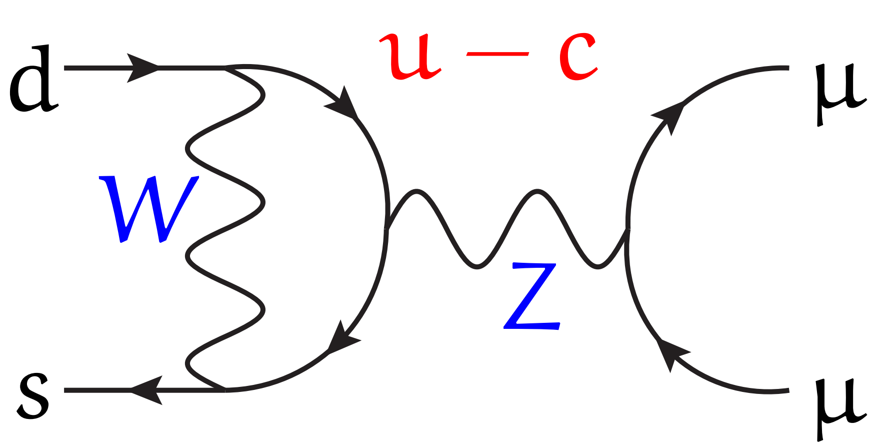
$\mathcal{O}(G_F^2 \Lambda_{\text{Fermi}}) = \mathcal{O}(G_F \alpha)$



$\mathcal{O}(G_F \alpha)$

The GIM Mechanism

GIM: charm quark to suppress neutral currents



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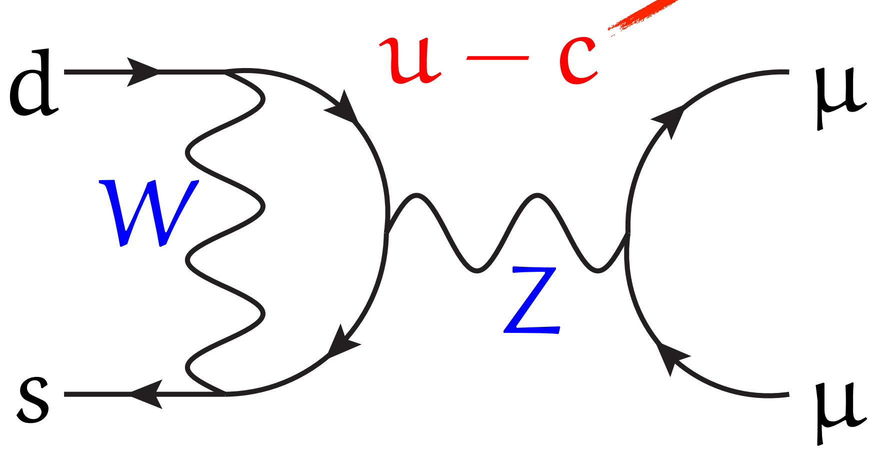

$$\frac{1}{\not{q} - m_u} \rightarrow \frac{1}{\not{q} - m_u} - \frac{1}{\not{q} - m_c}$$

Cutoff is then m_c^2

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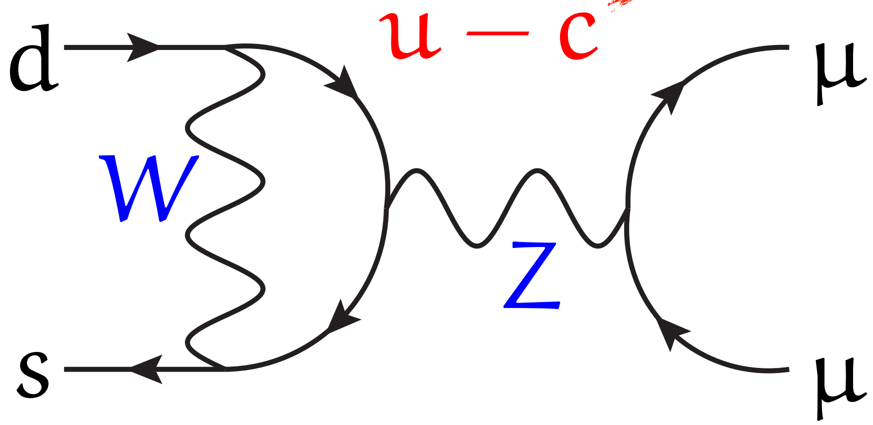

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Cutoff is then m_c^2

Quadratic GIM explains the smallness of $\mathcal{B}r(K_L \to \mu^+\mu^-)$

$\frac{m_c^2}{M_W^2}$ dependence: predict charm quark

Charm Quark Mass

Quadratic GIM suppresses light quark contribution

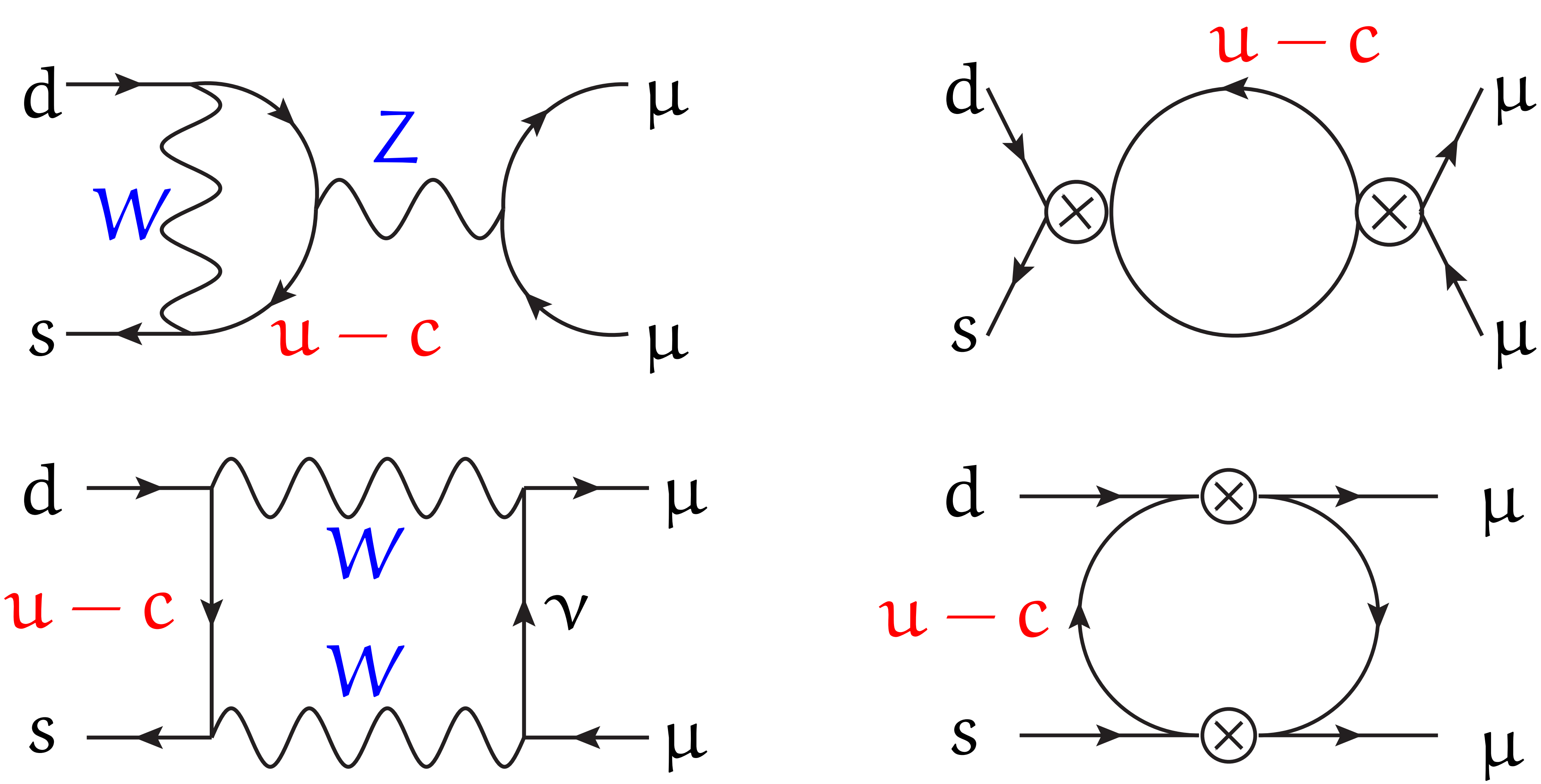
Sensitive to short distances (SD)

Charm Quark Mass

Quadratic GIM suppresses light quark contribution

Sensitive to short distances (SD)

Effective theory language:



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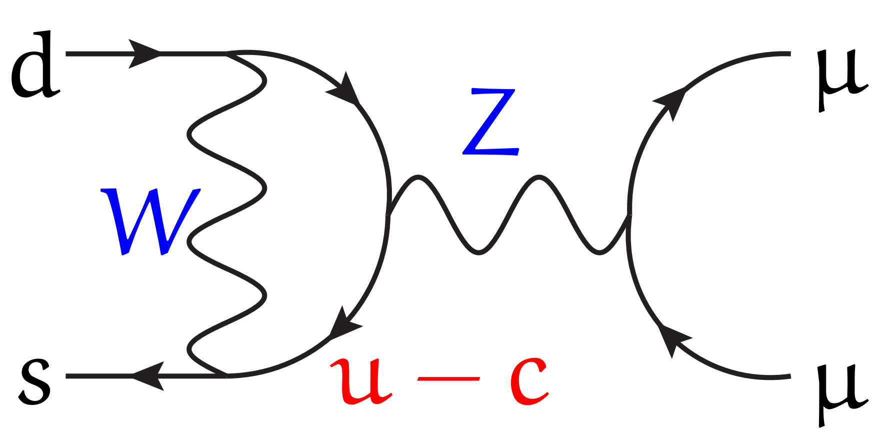

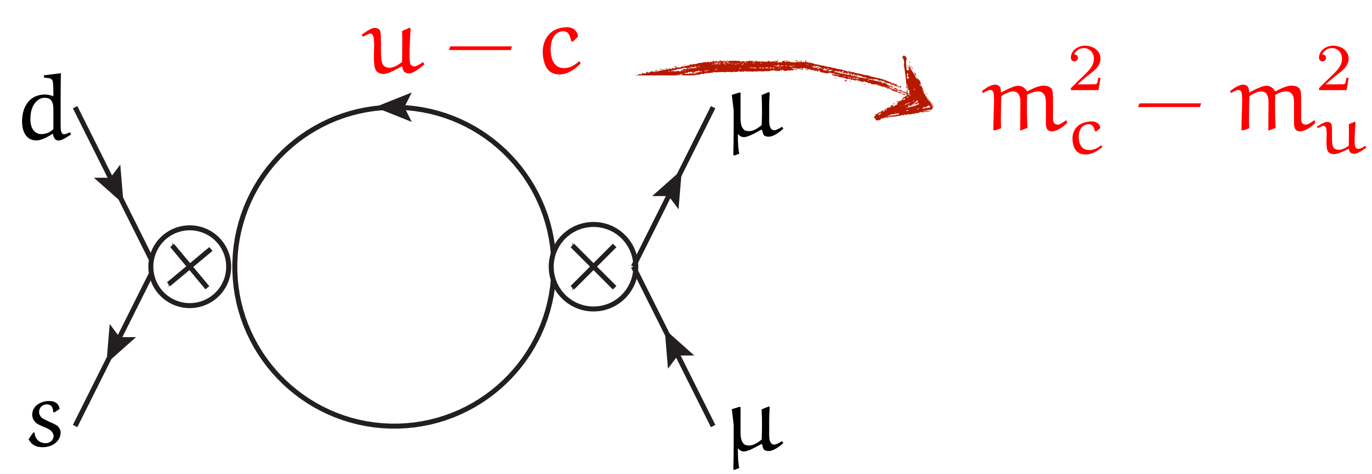

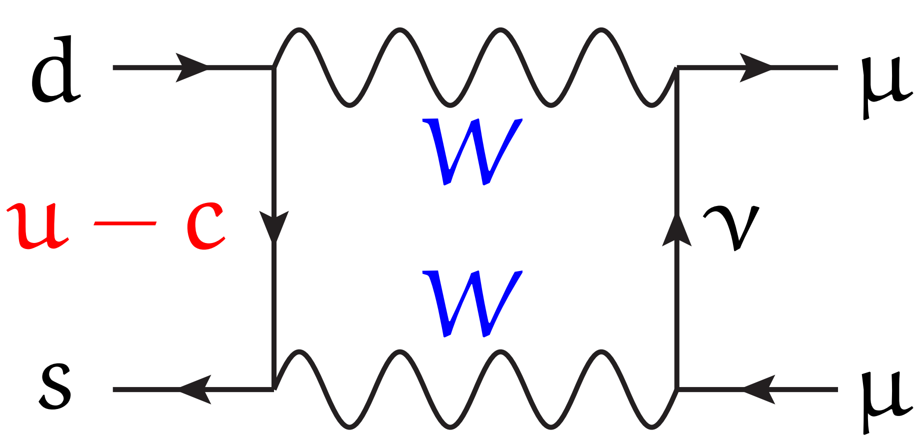

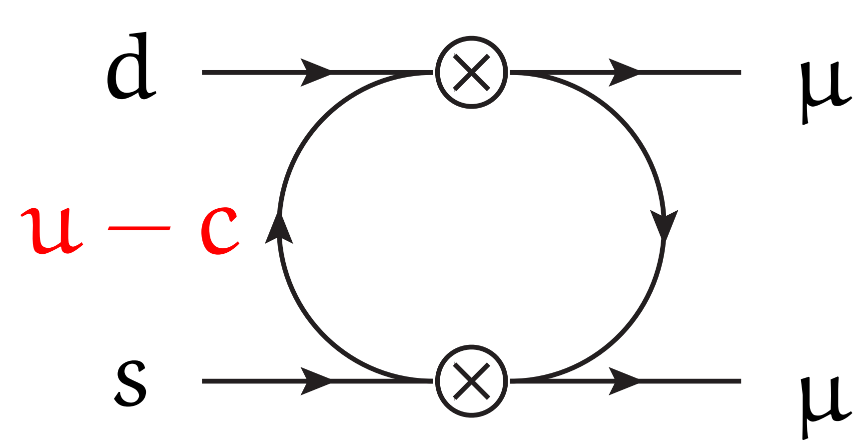

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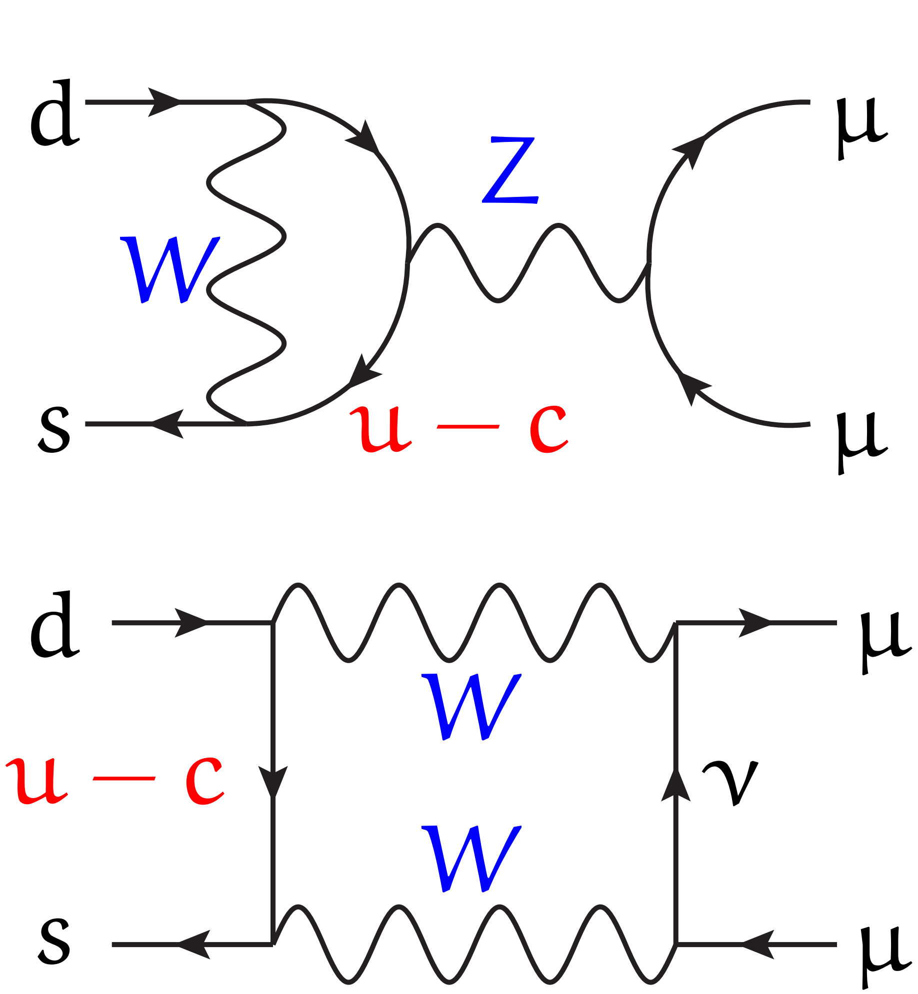

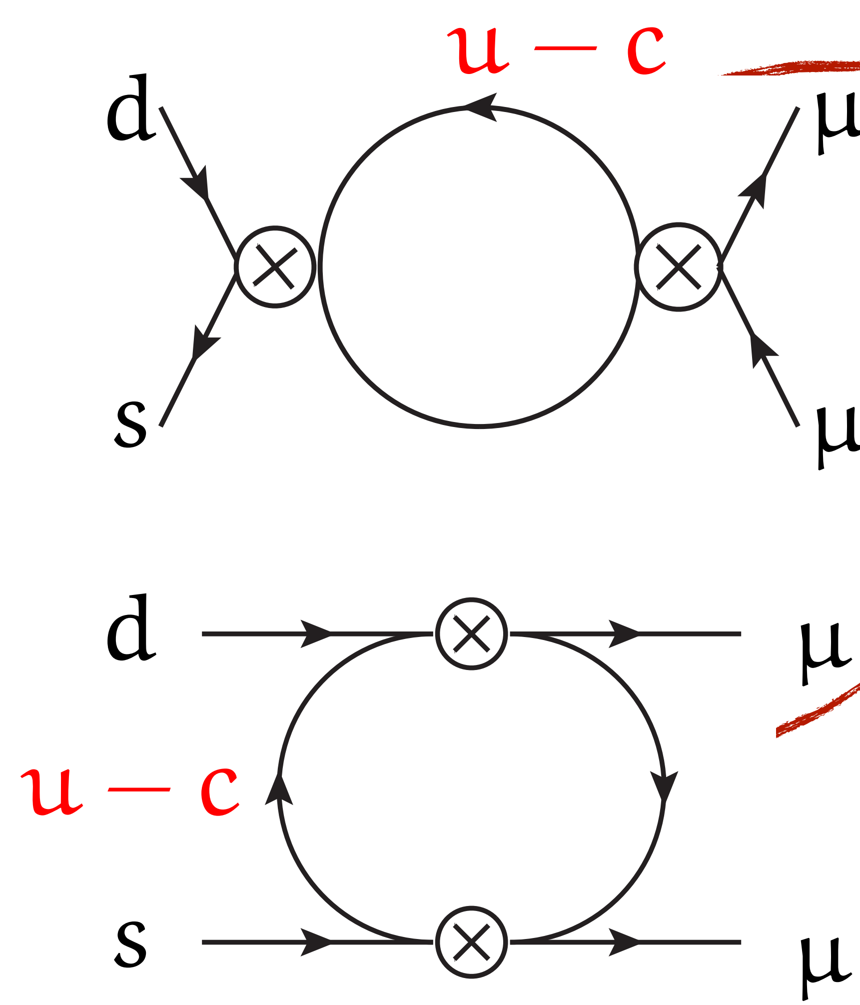

$m_c^2 - m_u^2$

$m_c^2 G_F^2 \log \frac{m_c}{M_W}$

Charm Quark Mass

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Sensitive to short distances (SD)

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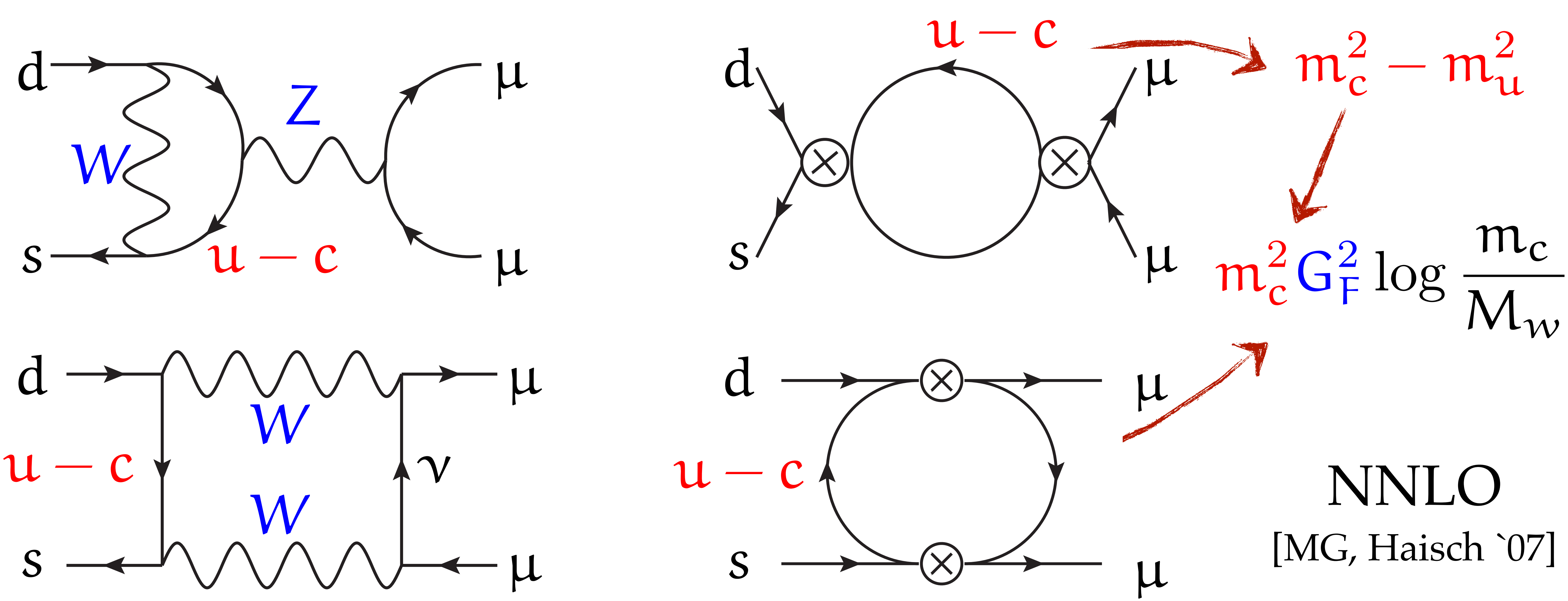
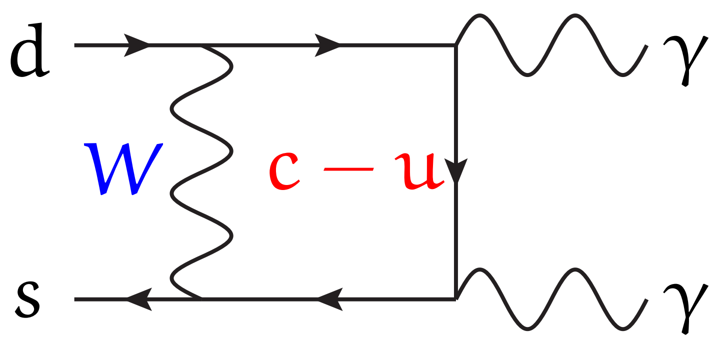
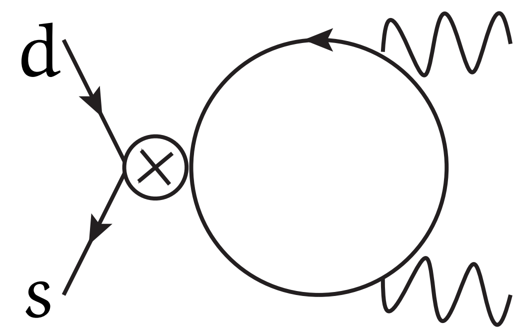


Contributions to $K_L \to \mu^+ \mu^-$

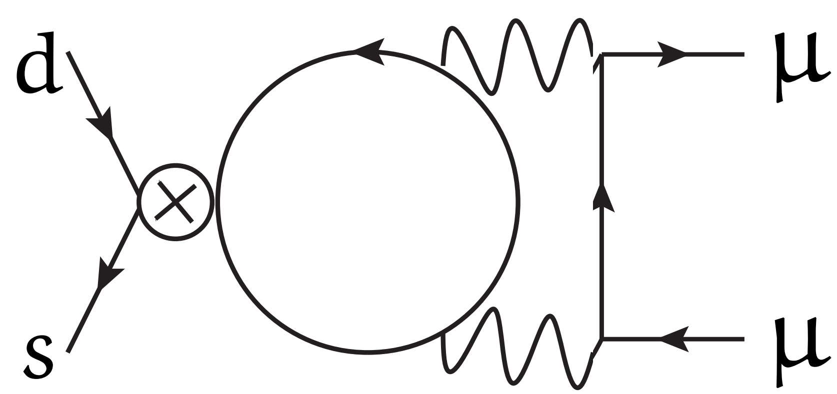
No quadratic suppression for $K_L \to \gamma\gamma$





$$G_F \log \frac{\Lambda_{QCD}}{m_c}$$

(same for photon penguin)



couplings to γ spoil short distance sensitivity

Suppressed Light Quark Contribution

$\text{Log}(\Lambda_{QCD}/m_{c,u})$ from coupling to final state electrons

$\Rightarrow K \to \pi\, \bar{\upsilon}\, \upsilon$ should have a clean theory prediction

CP violation is absent in 2 generation Standard Model

$\Rightarrow$ CP violating decays should exhibit increased short distance sensitive

Top quark

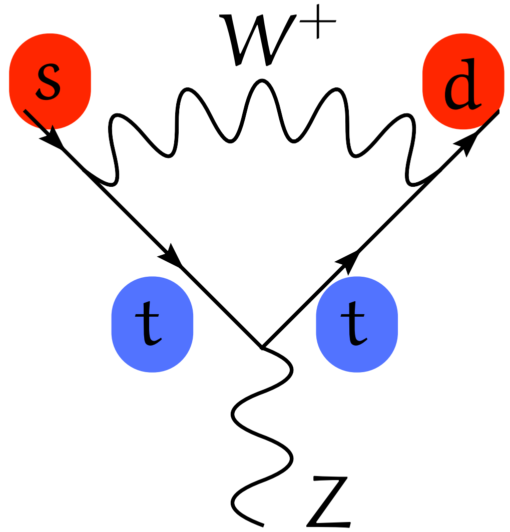
m_c^2/M_W^2 suppression

$\rightarrow$ top-quark dominates

$K \rightarrow \pi\, \bar{\nu}\, \nu$

$$V_{ij} = \mathcal{O}\begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \boxed{\lambda^3 \quad \lambda^2} & & 1 \end{pmatrix}$$

$$\lambda = \mathcal{O}(0.2)$$



Leading $\frac{m_t^2}{M_{G^\pm}^2}$ contribution from the goldstone diagram

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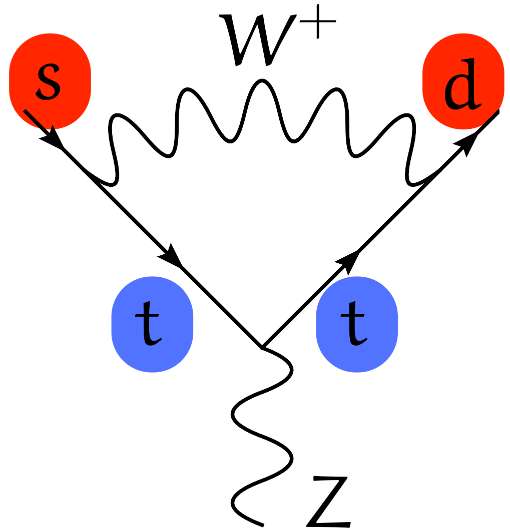

Leading $\dfrac{m_t^2}{M_{G^\pm}^2}$ contribution from the goldstone diagram

FCNCs which are dominated by top-quark loops:

$b \longrightarrow s:$	$b \longrightarrow d:$	$s \longrightarrow d:$
$\lvert V_{tb}^* V_{ts}\rvert \propto \lambda^2$	$\lvert V_{tb}^* V_{td}\rvert \propto \lambda^3$	$\lvert V_{ts}^* V_{td}\rvert \propto \lambda^5$

are extremely suppressed (λ^5) for Kaon decays

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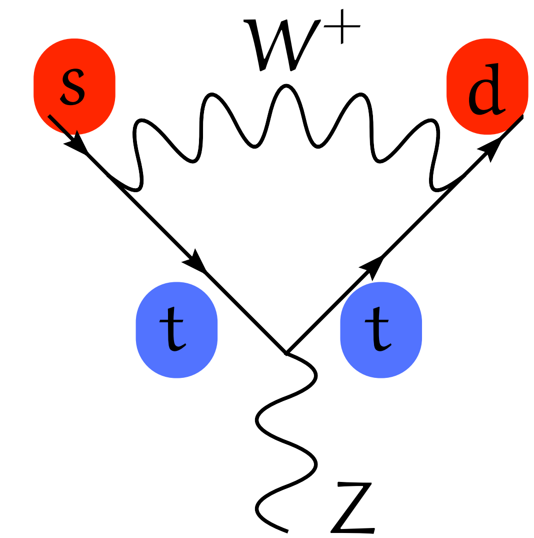

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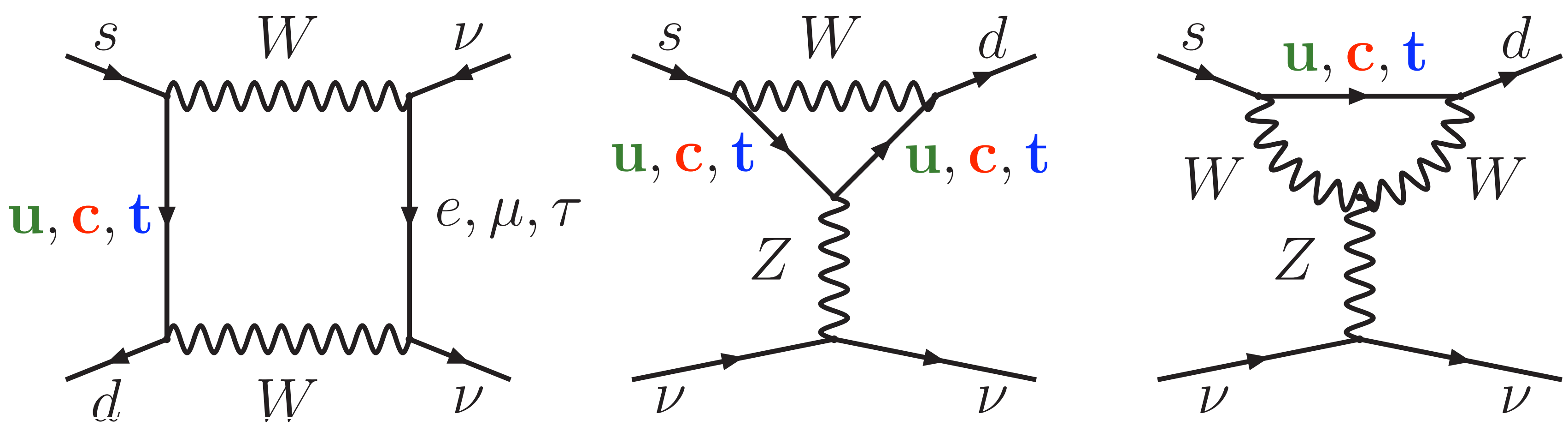
With 10% accuracy Kaons are sensitive to O(100) TeV

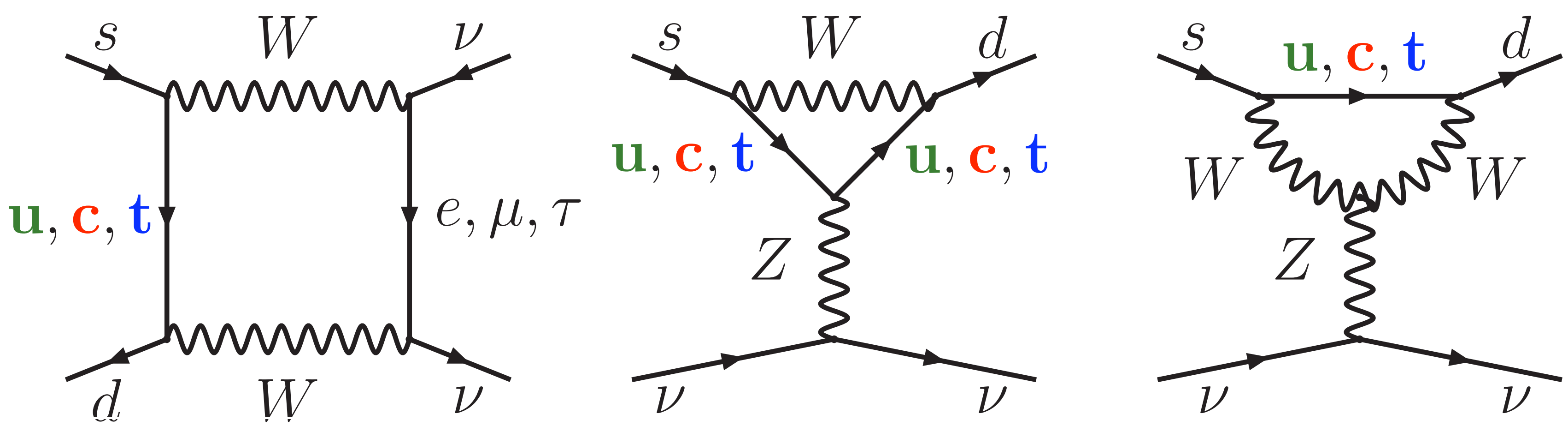
$K^+ \to \pi^+ \bar{\upsilon}\, \upsilon$ at M_W



$$x_i = \frac{m_i^2}{M_W^2}$$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td} (F(x_t) - F(x_u)) + V_{cs}^* V_{cd} (F(x_c) - F(x_u))$$

$K^+ \to \pi^+ \bar{\upsilon}\, \upsilon$ at M_W



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Quadratic GIM: $\boxed{\lambda^5 \frac{m_t^2}{M_W^2}}$

Matching (NLO +EW):

[Misiak, Urban; Buras, Buchalla;
Brod, MG, Stamou`11]

$$Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$$

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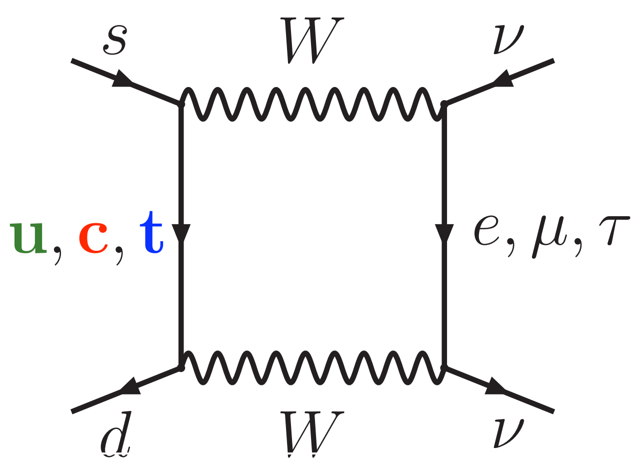

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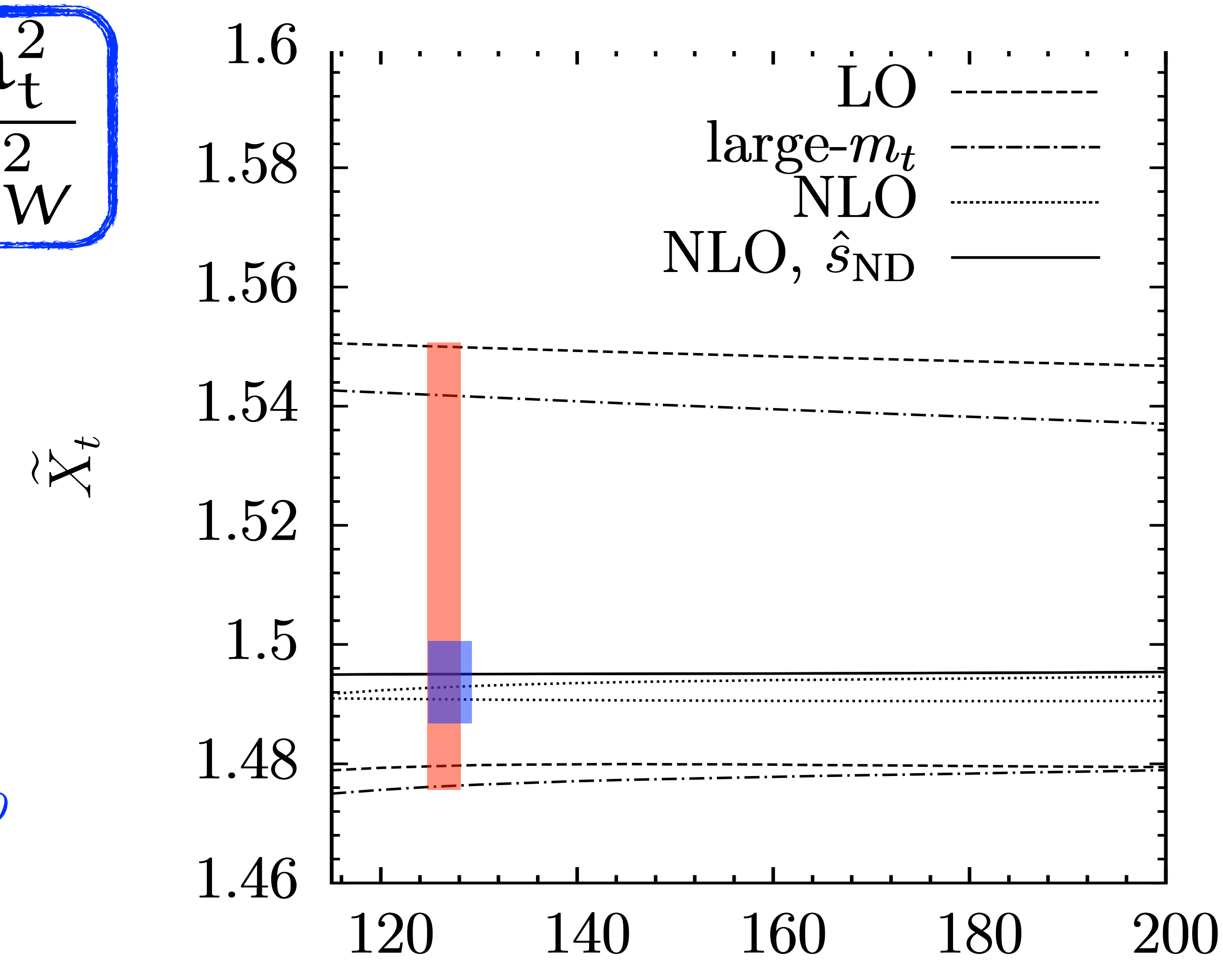
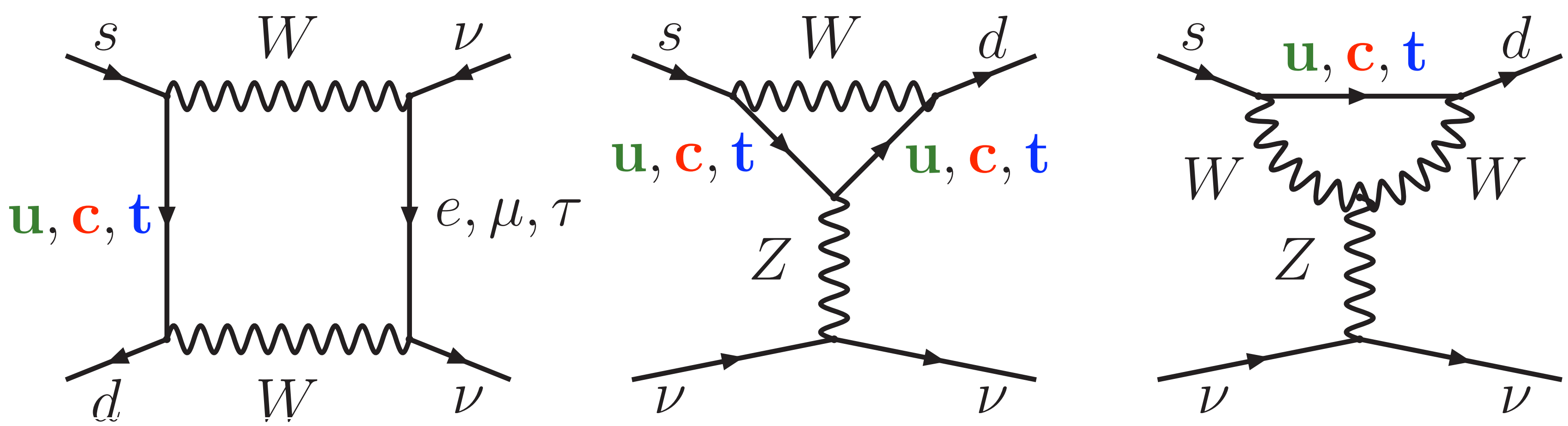
After 2011 uncertainty at 1%



$K^+ \to \pi^+ \bar{\nu}\nu$ at M_W



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Operator
Mixing (RGE)

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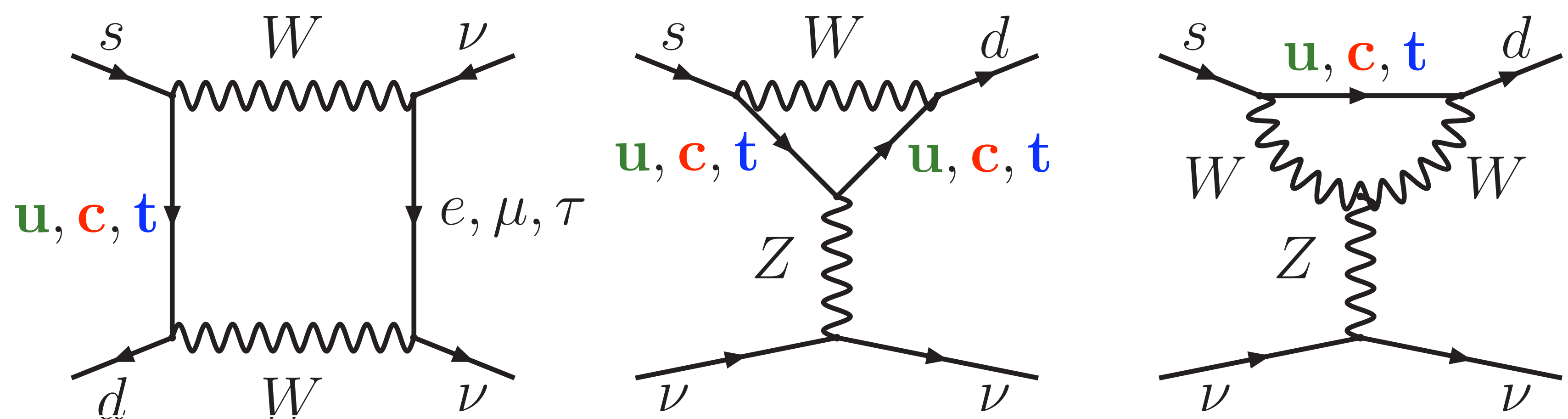

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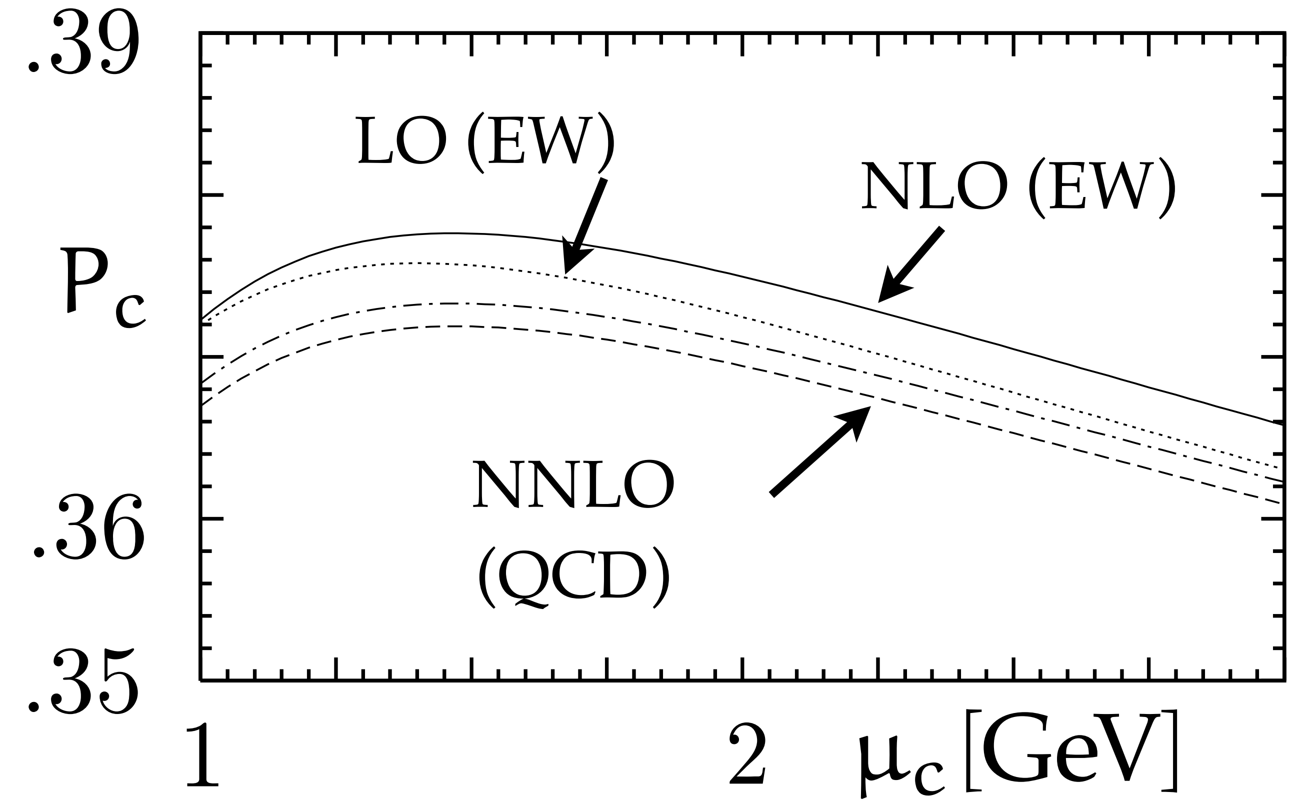
Matrix element from K_{l3} decays
(Isospin symmetry: $K^+ \to \pi^0 e^+ \upsilon$)
[Mescia, Smith]

$K^+ \to \pi^+ \bar{\upsilon}\, \upsilon$ from M_W to m_c

P_c: charm quark contribution to $K^+ \to \pi^+ \bar{\upsilon}\, \upsilon$ (30% to BR)

Series converges very well (NNLO:10%→2.5% uncertainty)

NNLO+EW [Buras, MG, Haisch, Nierste; Brod MG]



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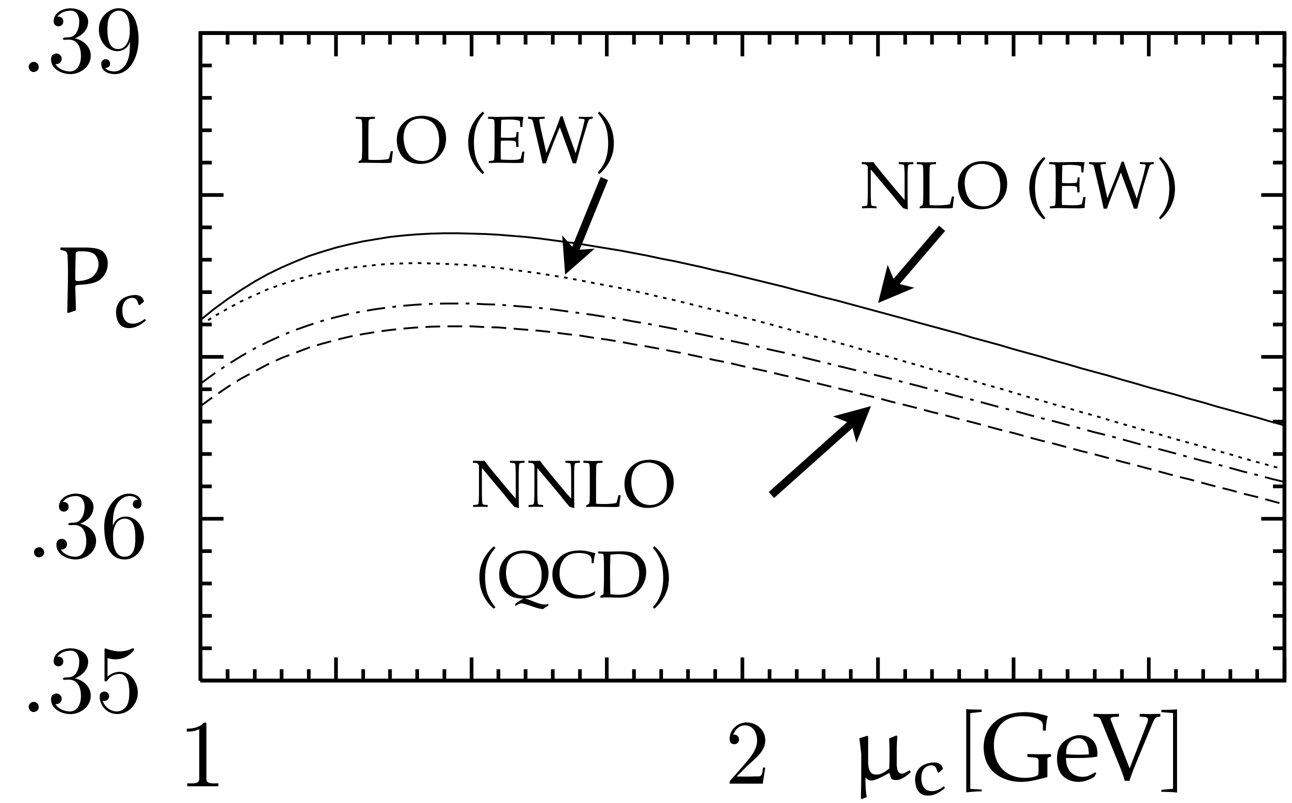

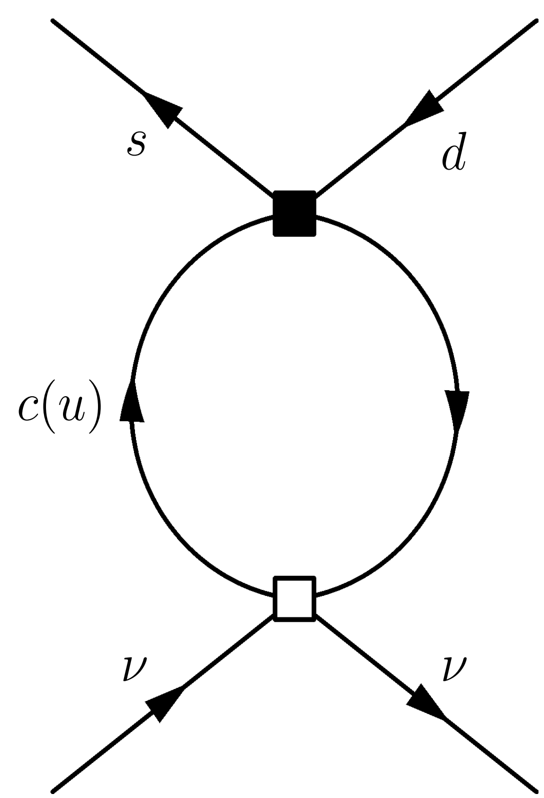

No GIM below the charm quark mass scale

higher dimensional operators UV scale dependent

One loop ChiPT calculation approximately cancels this scale dependence $\quad \delta P_{c,u} = 0.04 \pm 0.02$

[Isidori, Mescia, Smith `05]

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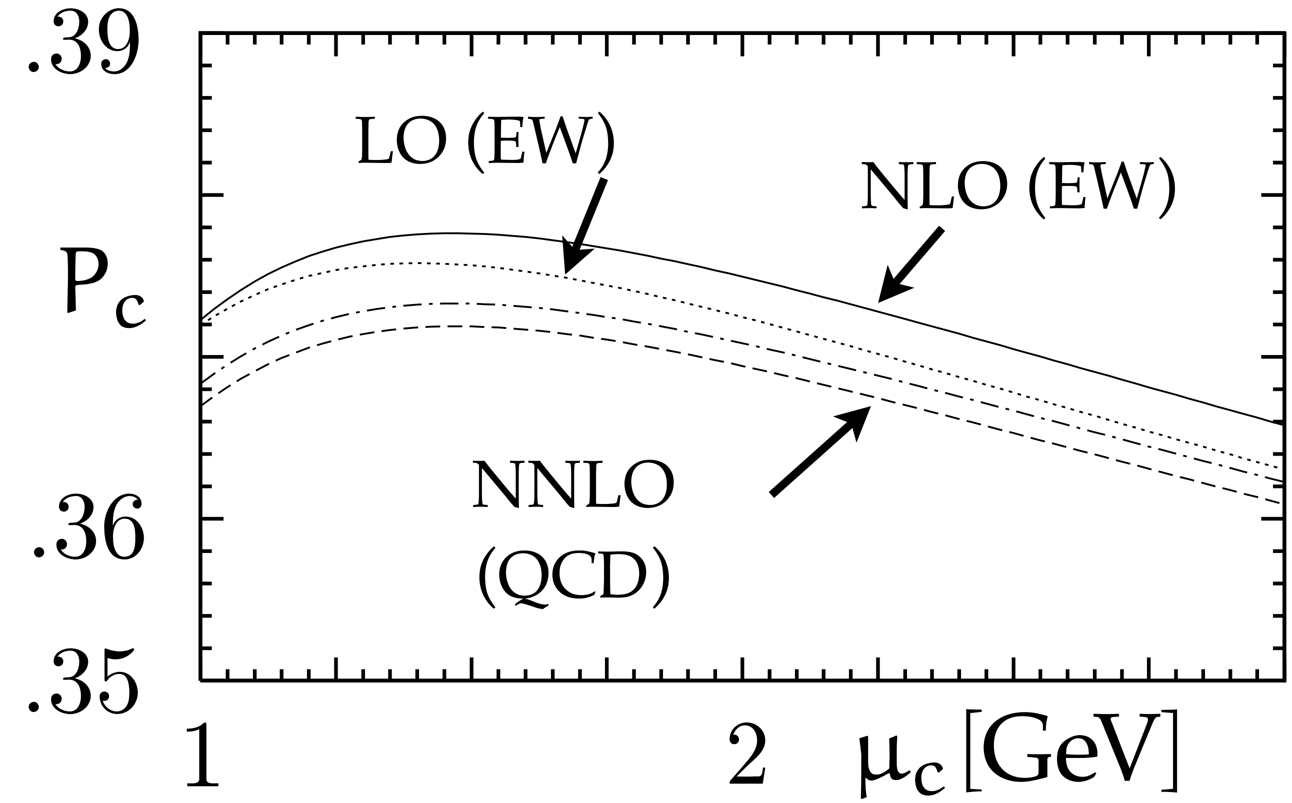

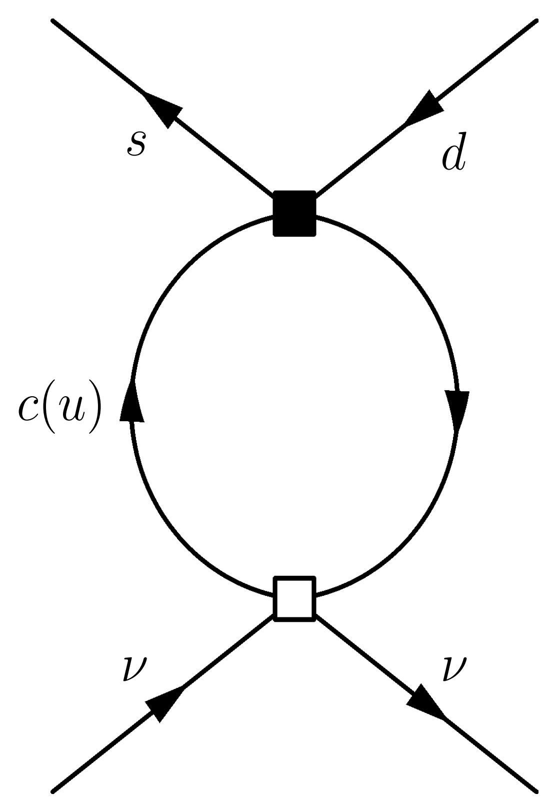

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[Isidori, Mescia, Smith `05]

Could be calculated on the lattice

[Isidori, Martinelli, Turchetti `06] [Christ, Fang, Portelli, Sachrajda `15]

$K \to \pi \bar{\upsilon} \upsilon$: Error Budget

$BR^{th}(K^+ \to \pi^+ \bar{\upsilon}\upsilon) = 7.8(8)(3) \cdot 10^{-11}$

$BR^{exp}(K^+ \to \pi^+ \bar{\upsilon}\upsilon) = 17(11) \cdot 10^{-11}$

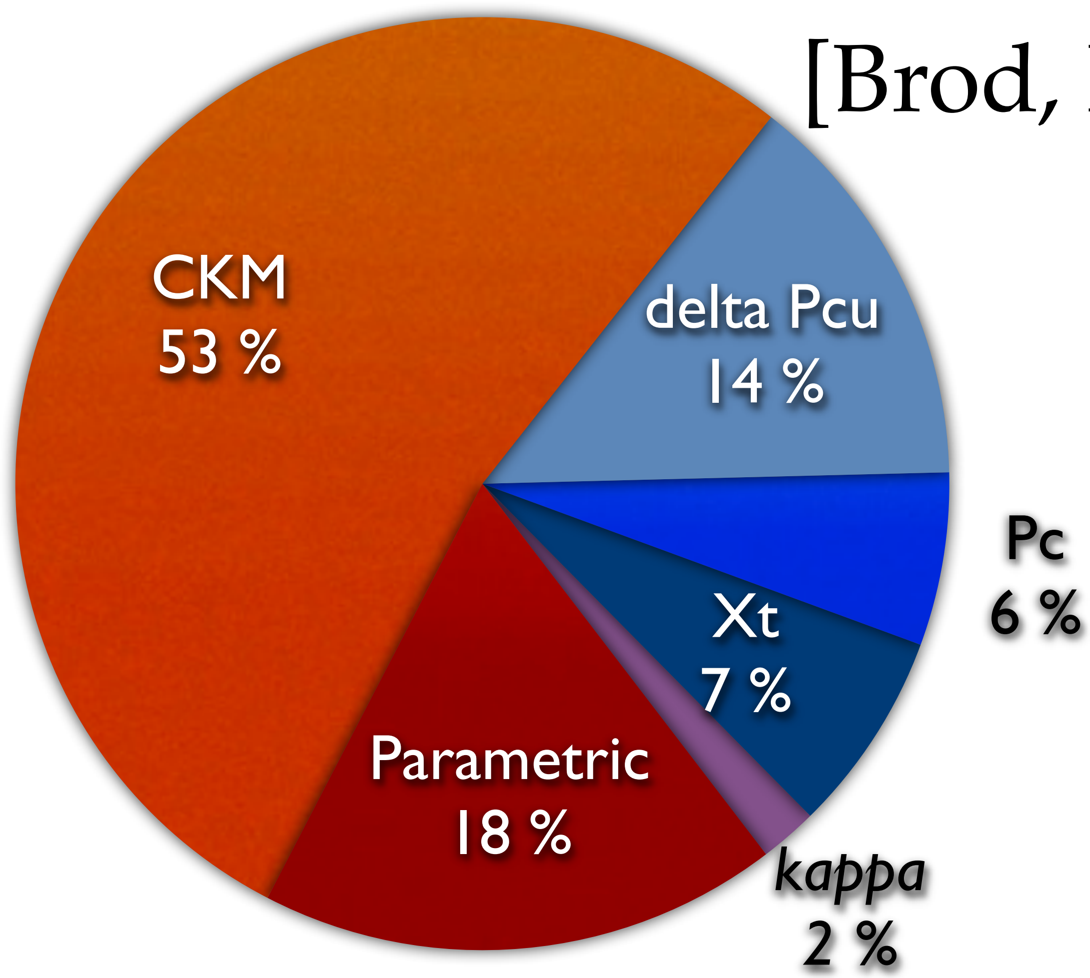
[E787, E949 ´08] NA62 $\to$ 10% accuracy

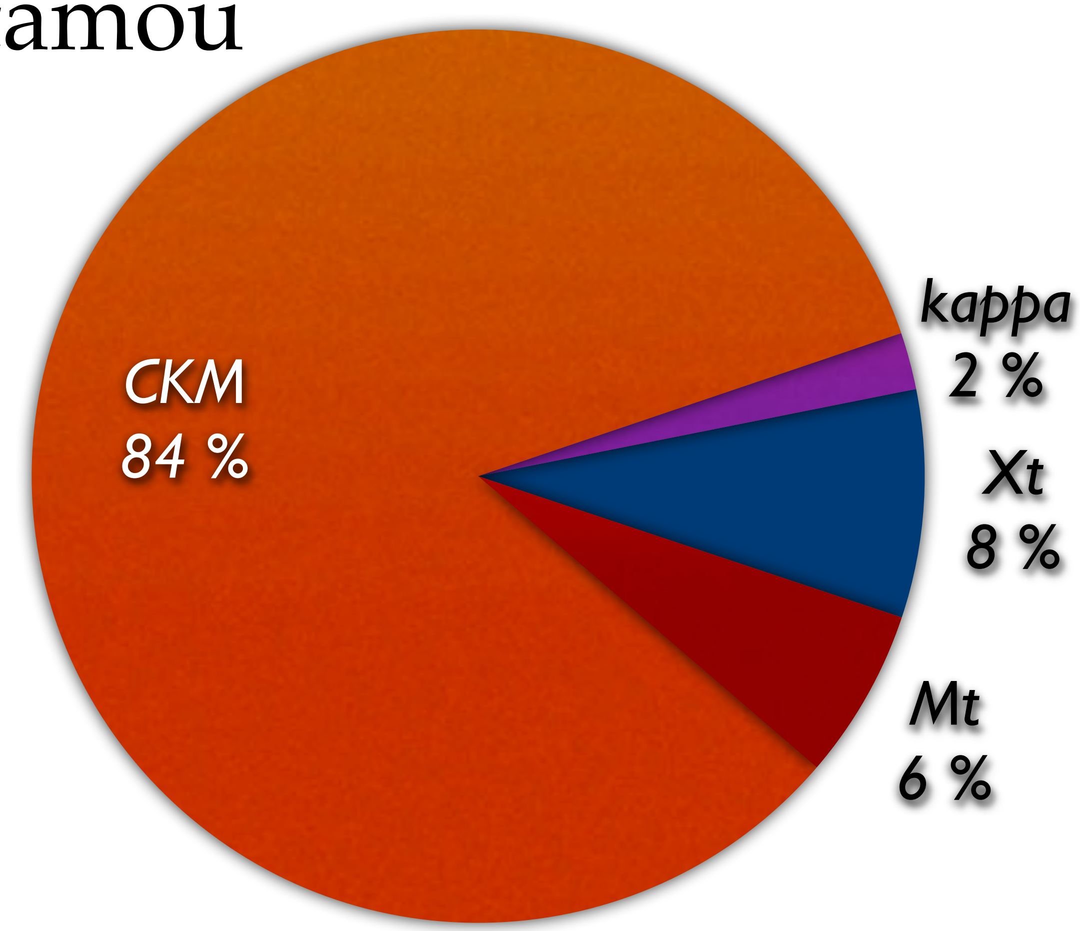
$BR^{th}(K_L \to \pi^0 \bar{\upsilon}\upsilon) = 2.43(39)(6) \cdot 10^{-11}$

$BR^{exp}(K^+ \to \pi^+ \bar{\upsilon}\upsilon) < 6.7 \cdot 10^{-8}$

[E391a ´08]

[Brod, MG, Stamou \`2011]





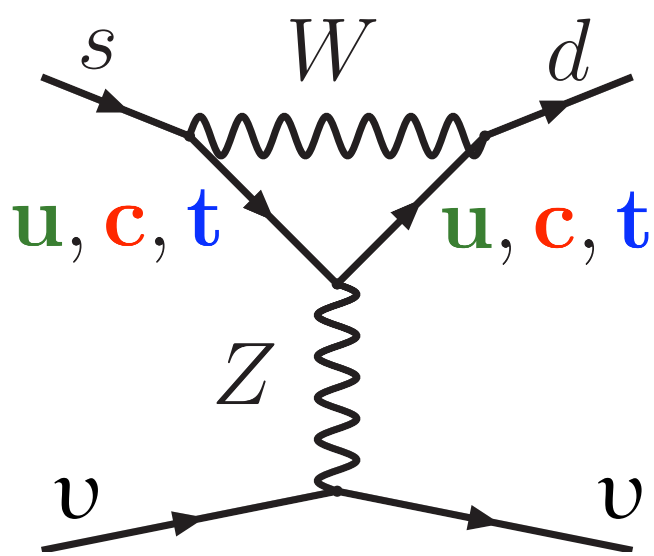
$BR^+ = 8.4(6) \cdot 10^{-11}$ (CKM tree)

$BR_L = 3.4(6) \cdot 10^{-11}$ (CKM tree)

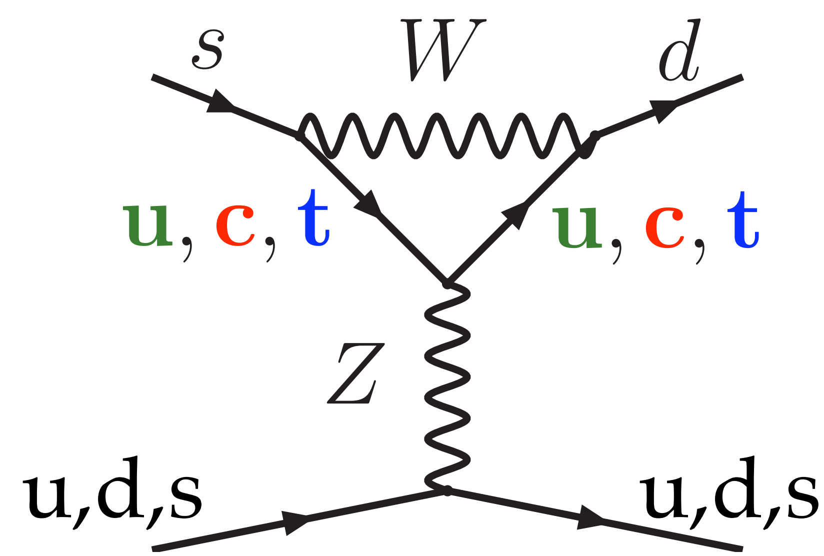
[Buras et.al. \`15]

CP Violation

$K_L \to \pi\, \bar{\upsilon}\, \upsilon$ might be correlated to CP violation in ε_K and $\varepsilon' / \varepsilon$



$\longleftrightarrow$



$\mathrm{Re}(\varepsilon' / \varepsilon) \simeq \varepsilon' / \varepsilon$ measures CP violation in the $K \to \pi\, \pi$ decay

$K \to \pi\, \pi$ decay amplitude receives contribution from

QCD Penguins and Electroweak Penguins

K^0 Meson Mixing

Schrödinger type equation for meson mixing

$$i\frac{d}{dt}\begin{pmatrix}|K^0(t)\rangle \\ |\overline{K}^0(t)\rangle\end{pmatrix} = \left[\begin{pmatrix}M_{11} & M_{12} \\ M_{12}^* & M_{11}\end{pmatrix} - \frac{i}{2}\begin{pmatrix}\Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{11}\end{pmatrix}\right]\begin{pmatrix}|K^0(t)\rangle \\ |\overline{K}^0(t)\rangle\end{pmatrix}$$

Diagonalise

$$|K_S\rangle = p|K^0\rangle + q|\overline{K}^0\rangle$$
$$|K_L\rangle = p|K^0\rangle - q|\overline{K}^0\rangle$$

M_{12} from $\Delta_s = 2$ Box $\longleftrightarrow$ Electroweak process

$\Gamma_{12} \longleftrightarrow \Delta\Gamma$ maximal and $\Delta I = 1/2$ saturates $\Gamma_{12} = A_0 \overline{A}_0$

CP violation in Kaons

CP violation in mixing, interference & decay → non-zero

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | K_L^0 \rangle}{\langle \pi^+\pi^- | K_S^0 \rangle} \qquad \eta_{00} = \frac{\langle \pi^0\pi^0 | K_L^0 \rangle}{\langle \pi^0\pi^0 | K_S^0 \rangle}$$

Only CP violation in mixing (Re ϵ_K), interference of mixing and decay (Im ϵ_K, Im ϵ') and direct CP violation (Re ϵ')

$$\epsilon_K = (\eta_{00} + 2\eta_{+-})/3 \qquad \epsilon' = (\eta_{+-} - \eta_{00})/3$$

$$\epsilon_K \simeq \frac{\langle (\pi\pi)_{I=0} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle} \qquad \epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12}^K)}{\Delta M_K} + \xi \right)$$

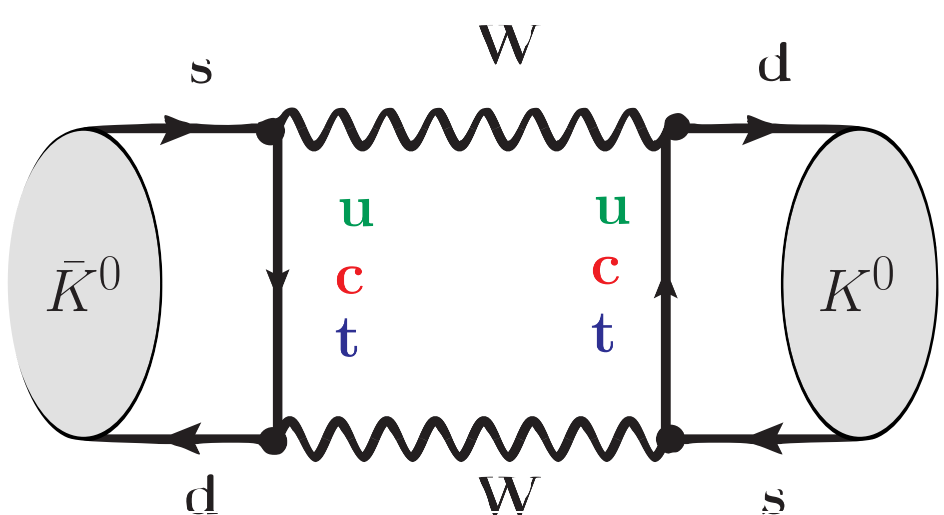
from experiment

small

ε_K: CP violation in Kaon Mixing

$$2M_K M_{12} = \langle K^0 | H^{|\Delta S|=2} | \bar{K}^0 \rangle - \frac{i}{2} \int d^4x \, \langle K^0 | H^{|\Delta S|=1}(x) \, H^{|\Delta S|=1}(0) | \bar{K}^0 \rangle$$

dispersive part



Local Interaction:

$$\tilde{Q} = (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma^\mu d_L)$$

Lattice: $\langle K^0 | \tilde{Q} | \bar{K}^0 \rangle$

(+75(1)%): $\lambda_t \lambda_t \, m_t^2 / M_W^2 +$ Only known at NLO

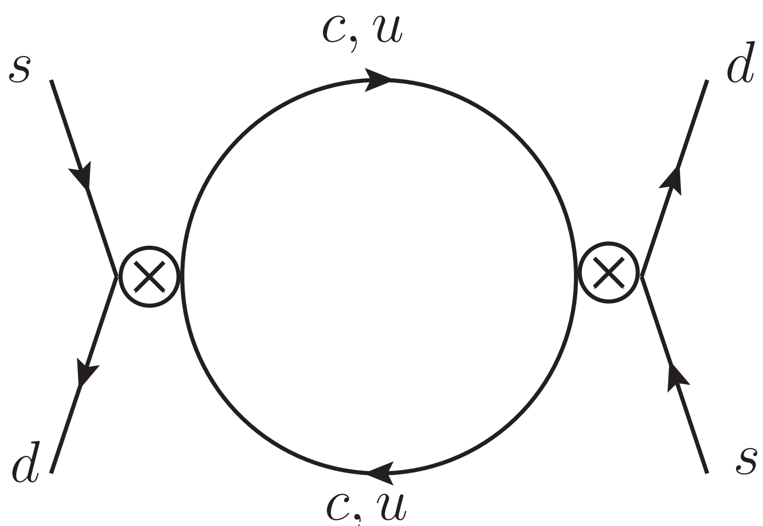
(+40(6)%): $\lambda_c \lambda_t \, m_c^2 / M_W^2 \log(m_c^2 / M_W^2) +$

η_{ct}: 3-loop RGE, 2-loop Matching [Brod, MG `10]

(-15(6)%): $\lambda_c \lambda_c \, m_c^2 / M_W^2$

η_{cc}: 3-loop RGE, 3-loop Matching [Brod, MG `12] NNLO

Long Distance ϵ_K



$$\int d^4x d^4y \, \langle K^0 | \, T\{H(x) \, H(y)\} \, | \bar{K}^0 \rangle$$

Integrate over $t_A < t_{x,y} < t_B$
[Christ et. al.]

Study for ΔM_K [Bai et.al. `14] and ideas for ϵ_K

Use $\lambda_u \lambda_t$ instead of $\lambda_c \lambda_t$

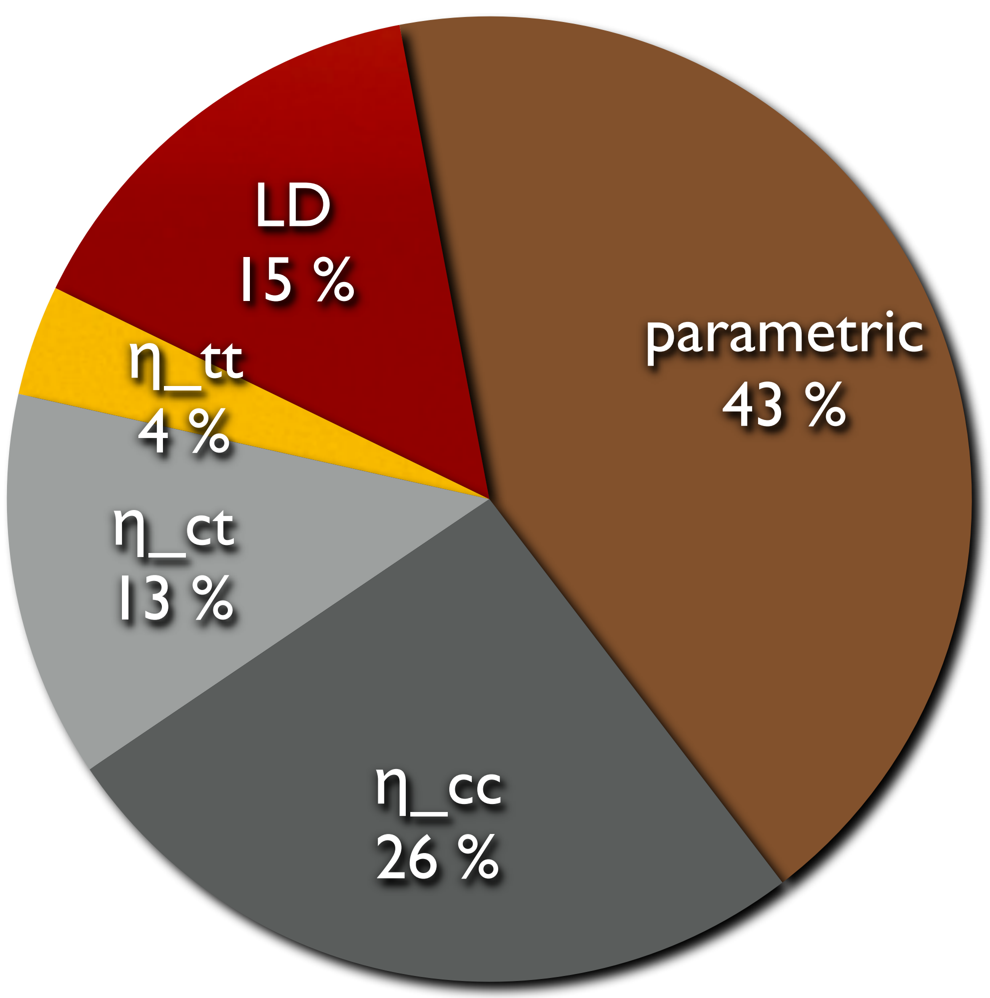
$\lambda_u \lambda_u$ finite after GIM & charm – renormalise $\Delta S=1$ Operator

$\lambda_u \lambda_t$ log divergent – renormalise $\Delta S=1$ & $\Delta S=2$ Operator, i.e. match Lattice to continuum perturbation theory.

Residual Theory Uncertainty

After Lattice QCD & NNLO progress: η_{cc} dominant uncertainty

ε_K is very important for phenomenology:
Future improvements are expected from Lattice QCD and interplay with perturbative QCD



[Brod et.al. `12]

$$|\epsilon_K| = 1.81(28) \cdot 10^{-3}$$

$$\stackrel{\text{exp.}}{=} 2.23(1) \cdot 10^{-3}$$

V_{cb} dominates parametric uncertainty

uncertainty in B_K sub-leading

CP violation in Kaons

CP violation in mixing, interference & decay → non-zero

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | K_L^0 \rangle}{\langle \pi^+\pi^- | K_S^0 \rangle} \qquad \eta_{00} = \frac{\langle \pi^0\pi^0 | K_L^0 \rangle}{\langle \pi^0\pi^0 | K_S^0 \rangle}$$

Only CP violation in mixing (Re ϵ_K), interference of mixing and decay (Im ϵ_K, Im ϵ') and direct CP violation (Re ϵ')

$$\epsilon_K = (\eta_{00} + 2\eta_{+-})/3 \qquad \epsilon' = (\eta_{+-} - \eta_{00})/3$$

Using: $\lambda_{ij} = \frac{q}{p}\frac{\langle \pi^i\pi^j | \bar{K}^0 \rangle}{\langle \pi^i\pi^j | K^0 \rangle}$ and $|1 - \lambda_{ij}| \ll 1$

$$\boxed{\epsilon' \approx \frac{1}{6}(\lambda_{00} - \lambda_{+-})} + \frac{1}{12}(\lambda_{00} - \lambda_{+-})(2 - \lambda_{00} - \lambda_{+-}) + \ldots$$

Formula for ε' / ε

a_0, a_2 & a_2^+ from experiment

[Cirigliano, et.al. `11]

a_0 & a_2: isospin amplitudes for isospin conservation

$$\langle \pi^0 \pi^0 | K^0 \rangle = a_0 e^{i\chi_0} + a_2 e^{i\chi_2}/\sqrt{2}$$

$$\langle \pi^+ \pi^- | K^0 \rangle = a_0 e^{i\chi_0} - a_2 e^{i\chi_2}\sqrt{2}$$

$$\langle \pi^+ \pi^0 | K^+ \rangle = 3 a_2^+ e^{i\chi_2^+}/2$$

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$$\langle \pi^+\pi^0|K^+\rangle = 3a_2^+ e^{i\chi_2^+}/2$$

Current theory gives us only: $A_I = \langle (\pi\pi)_I|\mathcal{H}_{\text{eff}}|K\rangle$

Normalise to K^+ decay (ω_+, a) and ε_K , expand in A_2/A_0 and CP violation:

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expand in A_2/ A_0 and CP violation:

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) \simeq \frac{\epsilon'}{\epsilon} = -\,\frac{\omega_+}{\sqrt{2}\,|\epsilon_K|}\left[\,\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\,(1-\hat{\Omega}_{\text{eff}}) - \frac{1}{a}\,\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\,\right]$$

[Buras, MG, Jäger, Jamin `15]

Adjusted to keep electroweak
penguins in Im A_0 [Cirigliano, et.al. `11]

Computation of A_0 & A_2

Currently we use the effective Hamiltonian below the charm:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} \big(z_i(\mu) + \tau \; y_i(\mu) \big) \, Q_i(\mu) \,, \qquad \tau \equiv - \frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*}$$

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current-current

$$Q_{1,2/\pm} = (\bar{s}_i u_j)_{V-A}\, (\bar{u}_k d_l)_{V-A}$$

QCD & electroweak penguins

$$Q_{3,\ldots,6} = (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s} (\bar{q}_k q_l)_{V\pm A}$$

$$Q_{7,\ldots,10} = (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_k q_l)_{V\pm A}$$

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We have z_i & y_i at NLO [Buras et.al., Ciuchini et. al. `92 `93]

And now also a Lattice QCD calculation of: $\langle (\pi\pi)_I | Q_i | K \rangle = \langle Q_i \rangle_I$ by RBC-UKQCD [Blum et. al., Bai et. al. `15]

Compute Im A_I/Re A_I

We need an expression for Im A_0/Re A_0 and Im A_2/Re A_2

$$\mathrm{Re}A_0 = \frac{G_F}{\sqrt{2}}\,V_{ud}V^*_{us}\left(z_+\langle Q_+\rangle_0 + z_-\langle Q_-\rangle_0\right)\,, \quad \mathrm{Re}A_2 = \frac{G_F}{\sqrt{2}}\,V_{ud}V^*_{us}\,z_+\langle Q_+\rangle_2$$

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Fierz relations for (V-A)x(V-A) give, e.g.: $\langle Q_4\rangle_0 = \langle Q_3\rangle_0 + 2\langle Q_-\rangle_0$

$$\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)_{V-A} = \mathrm{Im}\tau\,\frac{2y_4}{(1+q)z_-} + \mathcal{O}(p_3)$$

Is only a function of Wilson coefficients and of the ratio

$$q = (z_+(\mu)\langle Q_+(\mu)\rangle_0)/(z_-(\mu)\langle Q_-(\mu)\rangle_0)$$

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Expression with $p_3 = \langle Q_3\rangle_0 / \langle Q_4\rangle_0$ and EW penguins given in

[Buras, MG, Jäger & Jamin \`15]

Prediction for ε' / ε

I=2 Similarly for (V-A)x(V-A):

I=0 (V-A)x(V-A) I=2 (V-A)x(V-A)

$$\frac{\varepsilon'}{\varepsilon} = 10^{-4}\left[\frac{\mathrm{Im}\lambda_{\rm t}}{1.4\cdot 10^{-4}}\right]\left[a\left(1-\hat{\Omega}_{\rm eff}\right)\left(-4.1(8)+24.7\,B_6^{(1/2)}\right)+1.2(1)-10.4\,B_8^{(3/2)}\right]$$

(V-A)x(V+A) Matrix elements B_6=0.57(19) and B_8=0.76(5) from Lattice QCD [Blum et. al., Bai et. al. `15]

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$$\left(\frac{\epsilon'}{\epsilon}\right)_{\mathrm{SM}} = 1.9(4.5)\times 10^{-4}$$

$$\left(\frac{\epsilon'}{\epsilon}\right)_{\mathrm{exp}} = 16.6(2.3)\times 10^{-4}$$

2.9 σ difference

quantity	error on ε'/ε
$B_6^{(1/2)}$	4.1
NNLO	1.6
$\hat{\Omega}_{\mathrm{eff}}$	0.7
p_3	0.6
$B_8^{(3/2)}$	0.5
p_5	0.4
$m_s(m_c)$	0.3
$m_t(m_t)$	0.3

NLO vs NNLO

Theory prediction only at NLO at the moment

Convergence at m_c is not clear – should calculate next order

Long term use Lattice QCD

Status of ε' / ε NNLO

Energy	Fields	Order
μ_W	g,γ,W,Z,h, u,d,s,c,b,t	NNLO Q_1-Q_6 & Q_{8g} i) NNLO EW Penguins (traditional Basis) ii)
RGE	γ,g,u,d,s,c,b	NNLO Q_1-Q_6 & Q_{8g} iii)
μ_b	γ,g,u,d,s,c,b	NNLO Q_1-Q_6 iv)
RGE	γ,g,u,d,s,c	NNLO Q_1-Q_6 & Q_{8g} iii)
μ_c	γ,g,u,d,s,c	NLO Q_1-Q_{10} v)
RGE	γ,g,u,d,s	NNLO Q_1-Q_6 & Q8g iii)
$M_{Lattice}$	g,u,d,s	NLO Q_1-Q_{10} (traditional Basis) vi)

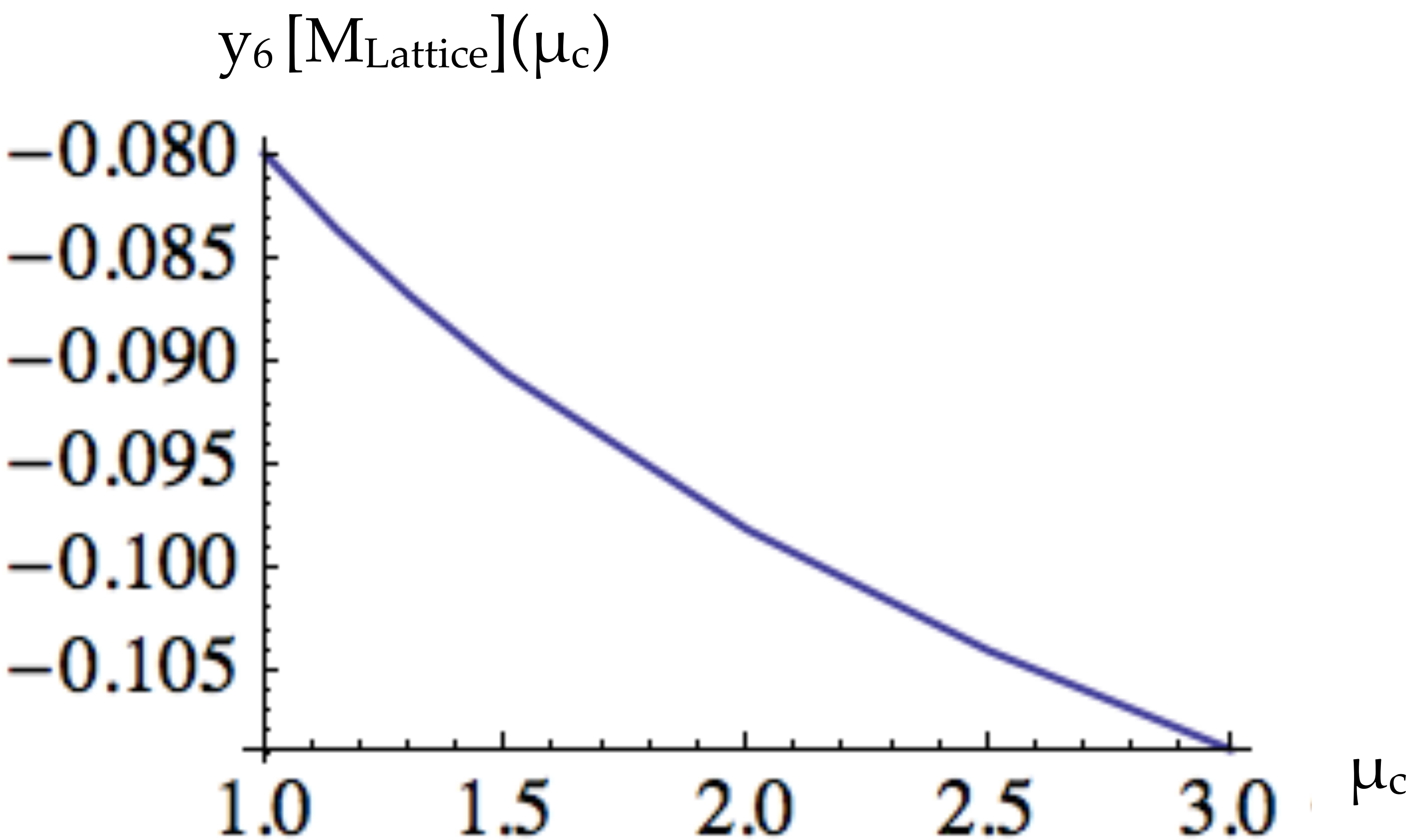
i) [Misiak, Bobeth, Urban]
ii) [Gambino,Buras, Haisch]
iii)[Gorbahn, Haisch]
iv)[Gorbahn, Brod]
v) [Buras, Jamin, Lautenbacher]
vi)[Blum et. al., Bai et. al. '15]

y_6 ($M_{Lattice}$) at NLO

The (V-A) (V+A) × y_6 : largest contribution to Im(A_0) and $\varepsilon' / \varepsilon$

How well do we know y_6 at the scale of the Lattice matrix elements $M_{Lattice}$?

Integrating out m_c results in strong μ_c dependence.



For $\alpha_s(M_Z)$ =0.1185 GeV

Plot the residual μ_c dependence:

1, RGE: $y_6(\mu_b) \rightarrow y_6(\mu_c)$

2, Match $y_6^{(f=4)}(\mu_c) \rightarrow y_6^{(f=3)}(\mu_c)$

3, RGE: $y_6(\mu_c) \rightarrow y_6(M_{Lattice})$

The scale dependence in 1 & 3 is canceled by the log(μ_c) in 2.

NNLO Operator Basis

The traditional basis requires the calculation of traces with **γ5** .

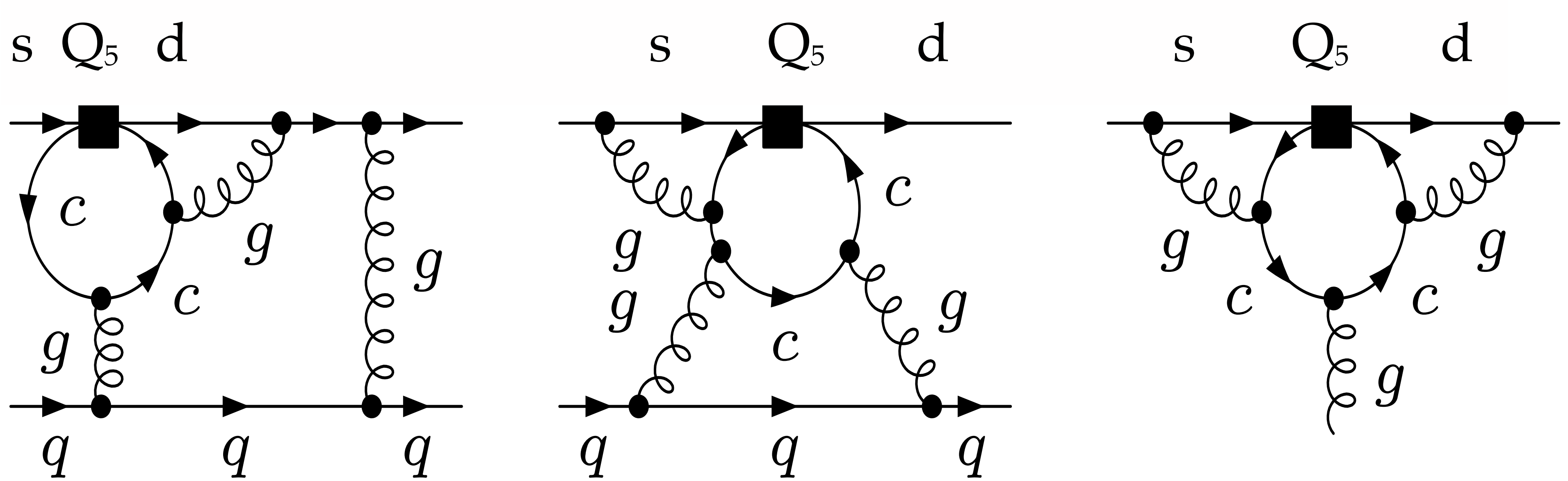
$$\mathcal{O}_{5,6} = (\bar{s}_i d_j)_{\text{V-A}} \sum_{u,d,s} (\bar{q}_k q_l)_{\text{V+A}}$$

Issues with the treatment of the γ_5 in D dimensions

Higher order calculations can be significantly simplified if we use a different operator basis.

$$\mathcal{O}^{m}_{5,6} = (\bar{s}_i \gamma_\mu \gamma_\nu \gamma_\rho P_L d_j)_{\text{V-A}} \sum_{u,d,s} (\bar{q}_k \gamma^\mu \gamma^\nu \gamma^\rho q_l)$$

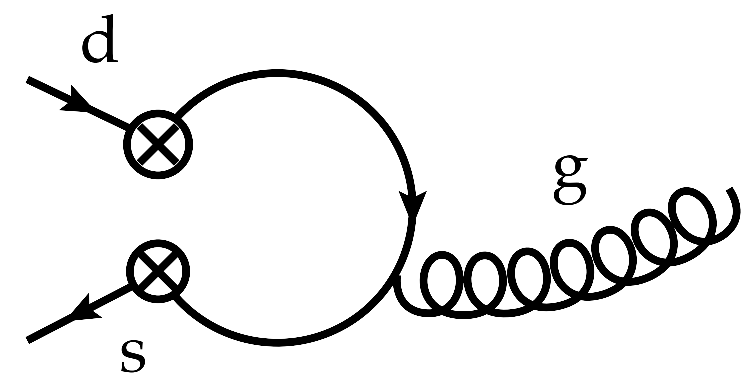
No trace of γ_5



Charm Matching NLO

O_1 & O_2 have the largest Wilson Coefficients.

Only one type of $s \to d$ gluon diagram for O_1 & O_2



We perform an off-shell matching:

expanding in external momentum $O(k^2)$

$$\mathcal{O}_{31} = \frac{1}{g}\bar{s}_L\gamma^\mu T^a b_L D^\nu G^a_{\mu\nu} + \mathcal{O}_4$$

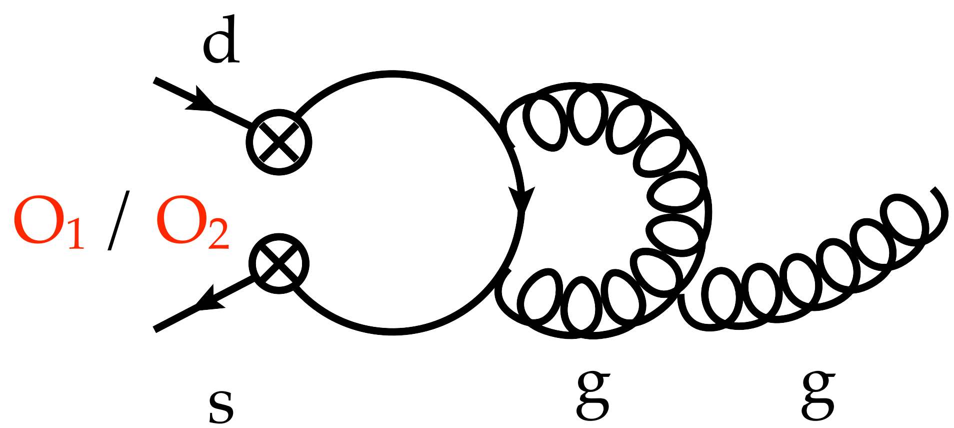
$$\mathcal{O}_4 = (\bar{s}_L\gamma^\mu T^a b_L)\sum_q(\bar{q}\gamma^\mu T^a q)$$

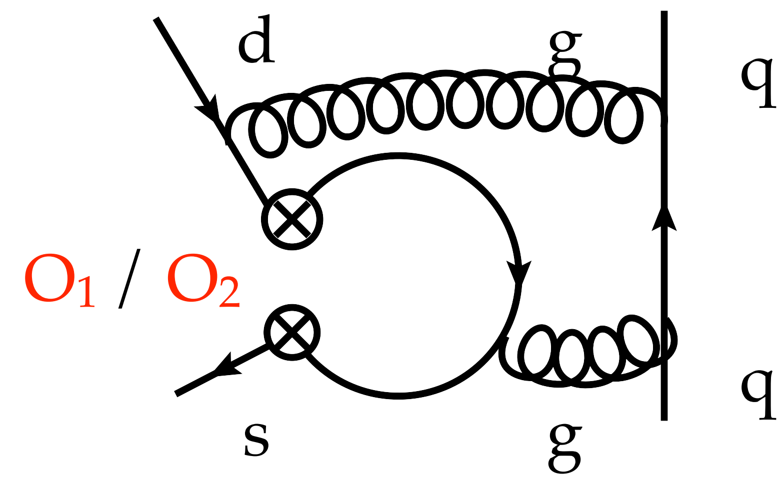
There are no one-light-particle-irreducible diagrams for $s \to d\,\bar{u}\,u$.

No evanescent operators are generated at one-loop.

NNLO Matching

There are Q_1 & Q_2 have the largest Wilson Coefficients.



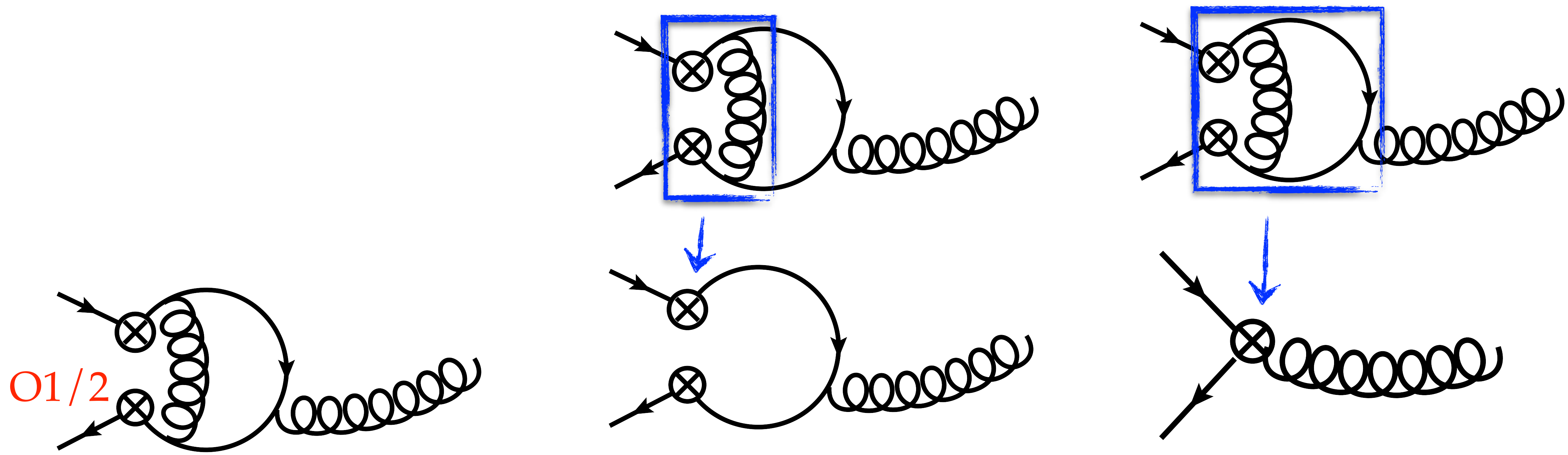


The calculation produces several types of structures,

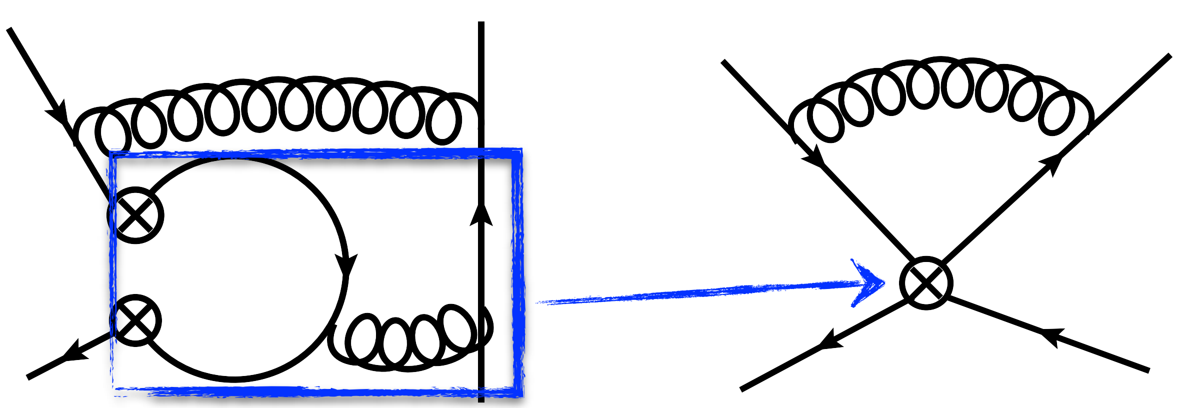
$$(\bar{s}_i \gamma^\mu P_L T^a_{ij} d_j) G^a_\mu k_1^2 \quad (\bar{s}_i \gamma_\nu T^a_{ij} P_L d_j) G^a_\mu k_1^\mu k_2^\nu \quad \ldots$$

– more than operators.

Renormalisation f=4



Our procedure: Full (f=4) theory is still divergent after renormalisation.

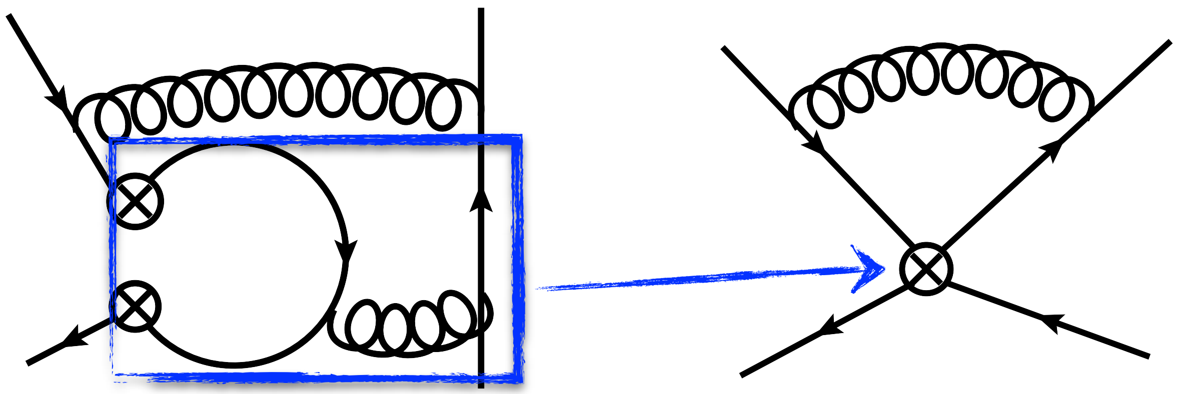


Counterterm matrix element

vanishing for $m_s = m_d = m_u = 0$

Renormalisation f=3

Vanishing f=4 matrix element



Counterterm matrix element

vanishing for $m_s = m_d = m_u = 0$

Will be canceled in f=3 theory by
One-loop matching coefficient × one-loop operator mixing

$A_{full} = A_{eff}$ results then in finite threshold corrections for

Additional Check: All results can be projected onto the Physical and EOM vanishing Operator Basis.
Note: Evanescent Operators only contribute in f=4 theory at NNLO

Future improvements?

RBC-UKQCD will reduce the statistical uncertainty.

While 1/N [Buras Gerard `15] consistent with RBC-UKQCD, we still need an independent Lattice calculation.

Perturbative NNLO calculation is currently performed to hopefully reduce theory uncertainty.
First numerics – considering only NNLO matching contributions of O_1/O_2 – suggest that perturbation theory seems to be OK.
TODO: NNLO continuum matching

Long term: Lattice treatment of isospin violation and computation above charm scale.

Perturbative BSM Calculations

Effective theory give model independent results,
but different operators contribute to
$K \to \pi \bar{\upsilon} \upsilon$ and $K \to \pi \pi$ – and other observables.

It might be interesting to have results for rare decays
as functions of a minimal set

of masses and coupling constants

and still arrive at a renormalisable result?
(In the SM calculation we e.g. need $M_W = M_Z \cos(\theta_w)$)

Toy example: Only Extra Vectors

Toy example: consider theories with arbitrary number of $W^{\pm}$

Equivalence to spontaneously broken theories allows us R_{ξ}-gauge fix the Vector Bosons

and use STI to fix Goldstone-Boson interactions

$$\mathcal{L}_3 = \sum_{f_1 f_2 v_1 \sigma} g^{\sigma}_{v_1 \bar{f}_1 f_2} V_{v_1,\mu} \bar{f_1} \gamma^{\mu} P_{\sigma} f_2 + \sum_{v_1 v_2 v_3} g_{v_1 v_2 v_3} \left[V_1, V_2, V_3\right]$$

$$\left[V_1, V_2, V_3\right] = \frac{i}{6}\left(V_{1,\mu} V_{2,\nu}\, \partial^{[\mu} V_3^{\nu]} + V_{3,\mu} V_{1,\nu}\, \partial^{[\mu} V_2^{\nu]} + V_{2,\mu} V_{3,\nu}\, \partial^{[\mu} V_1^{\nu]}\right)$$

In the Standard Model $g^{L}_{W^+ \bar{u}_j d_k} = \frac{e}{s_w \sqrt{2}} V_{jk}$

Renormalisation

STIs lead to the following constraints on the couplings:

$$g^{\sigma}_{\nu_1^+ \bar{t} s} g^{\sigma}_{Z\bar{t}t} \to \sum_{\nu_2} g_{Z\nu_1^+\nu_2^-} g^{\sigma}_{\nu_2^+\bar{t}s} + g^{\sigma}_{\nu_1^+\bar{t}s} g^{\sigma}_{Z\bar{s}s}$$

plus the one $\propto m_t$:

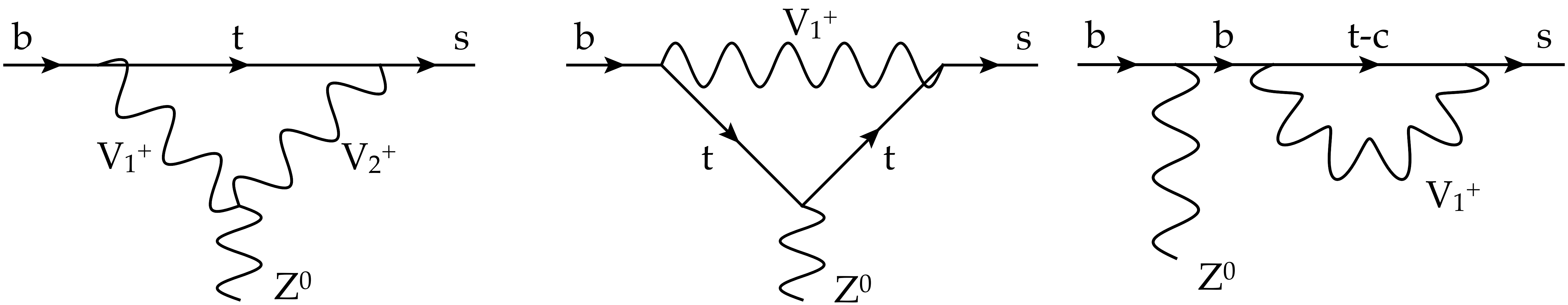
$$g^{\sigma}_{\nu_1^+ \bar{t} s} g^{\bar{\sigma}}_{Z\bar{t}t} \to \sum_{\nu_2} \frac{M^2_{\nu_1} - M^2_Z}{2M^2_{\nu_2}} g_{Z\nu_1^+\nu_2^-} g^{\sigma}_{\nu_2^+\bar{t}s} + \tfrac{1}{2} g^{\sigma}_{\nu_1^+\bar{t}s} \left(g^{\sigma}_{Z\bar{s}s} + g^{\sigma}_{Z\bar{t}t} \right)$$

Which results in the (renormalisation) condition

$$g^{\sigma}_{\nu_1^+ \bar{t} s} g^{\bar{\sigma}}_{Z\bar{t}t} \to \sum_{\nu_2} \frac{M^2_{\nu_1} + M^2_{\nu_2} - M^2_Z}{2M^2_{\nu_2}} g_{Z\nu_1^+\nu_2^-} g^{\sigma}_{\nu_2^+\bar{t}s} + g^{\sigma}_{\nu_1^+\bar{t}s} g^{\sigma}_{Z\bar{s}s}$$

generalisation of the $M_W = M_Z \cos(\theta_w)$ relation

Full 1-loop result



Applying the derived constraints on the full result yields

$$\sum_{\nu_1\nu_2} g_{Z\nu_1^+\nu_2^-}\, g^{L}_{\nu_1^-\bar{b}t}\, g^{L}_{\nu_2^+\bar{t}s}\, F_1(m_t, M_{\nu_1}, M_{\nu_2}) + \sum_{\nu_1} g^{L}_{Z\bar{s}s}\, g^{L}_{\nu_1^-\bar{b}t}\, g^{L}_{\nu_2^+\bar{t}s}\, F_0(m_t, M_{\nu_1})$$

Z-coupling to top-quark eliminated

Just like in the Standard Model we have a result in terms of a fewer number of couplings and finite loop functions F_1 and F_2

Extended to arbitrary perturbative model

Right handed Z penguin with
additional W's and charged scalars:

$$\sum_{s_i s_j} f_1(m_t, M_{s_i}, M_{s_j}) \left\{ g_{Z s_i^- s_j^+} + \delta_{ij} \left(g^R_{Z\bar{d}d} - g^R_{Z\bar{t}t} \right) \right\} y^L_{s_i^+ \bar{s} t} y^R_{s_j^- \bar{t} d} +$$

$$\sum_{W_i W_j} f_2(m_t, M_{W_j}, M_{W_i}) g^R_{Z\bar{t}t} g^R_{W_j^+ \bar{s} t} g^R_{W_i^- \bar{t} d}$$

$$\sum_{s_i W_j} f_3(m_t, M_{s_i}, M_{W_j}) g_{Z W_j^+ s_i^-} y^R_{s_i^+ \bar{s} t} g^R_{W_j^- \bar{t} d}$$

$$\sum_{s_i W_j} f_4(m_t, M_{s_i}, M_{W_j}) g_{Z W_j^- s_i^+} g^R_{W_j^+ \bar{s} t} y^R_{s_i^- \bar{t} d}$$

[Brod, Gorbahn in progress]

Outlook

There has been a continuous improvement in theory (Lattice + perturbation theory) and experiment.
And there is still work do be done, e.g.
continuum and perturbative matching at NNLO

This leads to an increased new physics sensitivity.

Independent confirmation of $K \to \pi\,\pi$ matrix elements on the Lattice would be exciting.

The measurements of $K \to \pi\,\bar{\upsilon}\,\upsilon$ and
the theory improvements in ε' & ε will
provide new information on short distance physics.