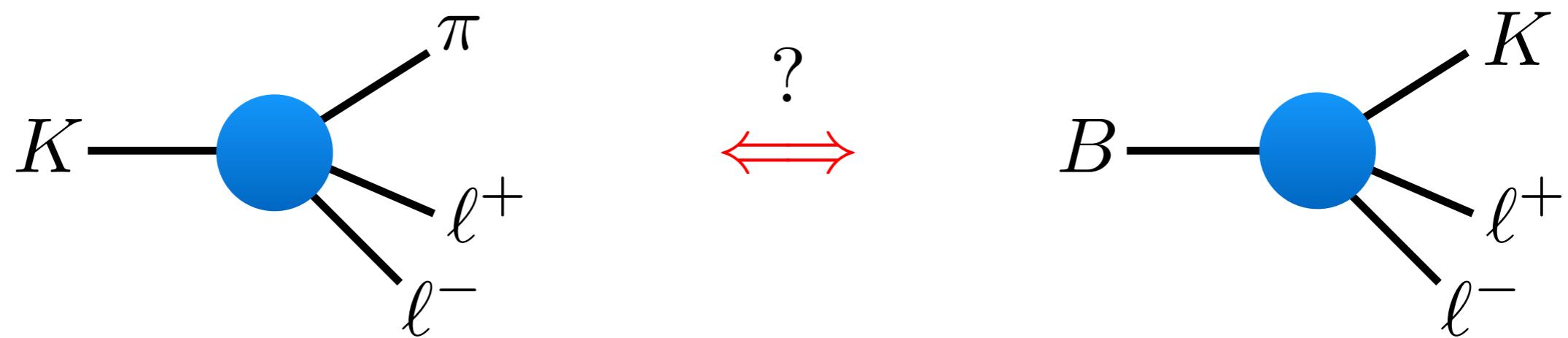


# Lepton Flavour (Universality) Violation in Rare Kaon Decays



In collaboration with A. Crivellin, G. D'Ambrosio & M. Hoferichter [[arXiv:1601.00970](https://arxiv.org/abs/1601.00970)]

$u^b$

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# 0 | Motivation

The Higgs discovery provides the final piece of Standard Model, but... many questions left unanswered



**Theoretical puzzles:** origin of e'weak symmetry breaking, strong CP, scale hierarchies,... [insert your favourite puzzle here]

**Phenomenological puzzles:** dark matter, baryon asymmetry...

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No **direct** signals of physics beyond the Standard Model at Run 1, but several **indirect** hints for new physics in the B-sector...

# 0 | Motivation

## New Physics in semi-leptonic B-decays?

- 2-3 $\sigma$  deviations from SM in  $B \rightarrow K^* \mu^+ \mu^-$   
[Descotes-Genon, Hurth, Matias, Virto (2013)]
- 2.6 $\sigma$  evidence of **L**epton **F**lavour **U**niversality **V**iolation (LFUV):

$$R(K) = \frac{\text{Br}[B \rightarrow K\mu^+\mu^-]}{\text{Br}[B \rightarrow Ke^+e^-]} = 0.745^{+0.090}_{-0.074} \pm 0.036 \quad [\text{LHCb (2014)}]$$

$$R_{\text{SM}}(K) = 1.003 \pm 0.0001 \quad [\text{Bobeth, Hiller, Piranishvili (2007)}]$$

- Combined 3.9 $\sigma$  evidence of LFUV in  $B \rightarrow D^{(*)}\tau\nu_\tau$  decays:

$$\begin{aligned} R(D)_{\text{exp}} &= 0.391 \pm 0.041 \pm 0.028 & R_{\text{SM}}(D) &= 0.297 \pm 0.017 \\ R(D^*)_{\text{exp}} &= 0.322 \pm 0.018 \pm 0.012 & R_{\text{SM}}(D^*) &= 0.252 \pm 0.003 \\ && [\text{HFAG (2015)}] & & [\text{Fajfer, Kamenik, Nisandzic (2012)}] \end{aligned}$$

# 0 | Motivation

## New Physics origin of LFUV?

- $b \rightarrow s$  transitions governed by effective  $\Delta B = 1$  Hamiltonian:

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i^B(\mu) Q_i^B(\mu)$$

- For semi-leptonic decays, need vector and axial-vector operators

$$Q_9^B = \frac{e^2}{32\pi^2} [\bar{s}\gamma^\mu(1-\gamma_5)b] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu\ell]$$

$$Q_{10}^B = \frac{e^2}{32\pi^2} [\bar{s}\gamma^\mu(1-\gamma_5)b] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu\gamma_5\ell]$$

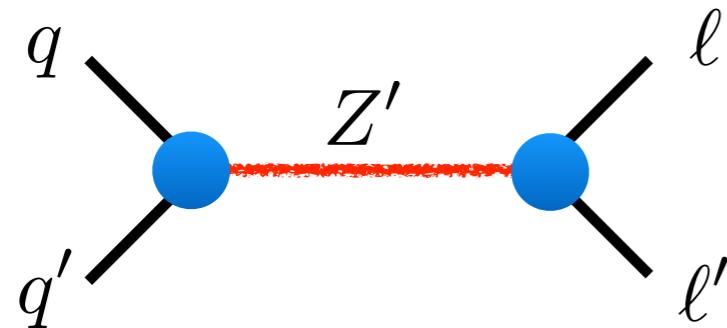
- Explanation of B-anomalies requires  $C_{9,10}^{\text{NP}} \sim O(1)$   
[Descotes-Genon, Hofer, Matias, Virto (2015)]

# 0 I Motivation

## New Physics origin of LFUV?

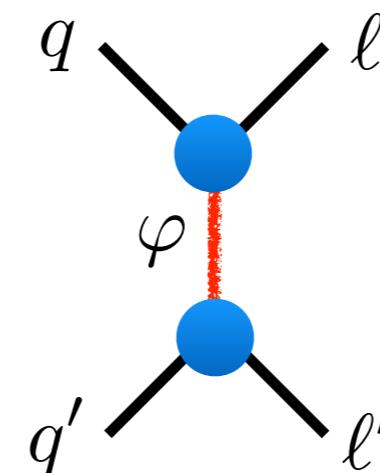
- Many models, but much of the focus on those which can generate current-current interactions  $\sim (\bar{s}\gamma_\alpha P_L b)(\bar{\mu}\gamma^\alpha \mu)$

$Z'$  bosons



[Crivellin, D'Ambrosio, Heeck (2015);  
Glashow, Lane, Guadagnoli (2014);  
Buras, De Fazio, Girrbach (2013)  
Buras, Girrbach (2013)...]

Leptoquarks  $\varphi$



[Calibbi, Crivellin, Ota;  
Alonso, Grinstein, Camalich;  
Bauer, Neubert (2015)...]

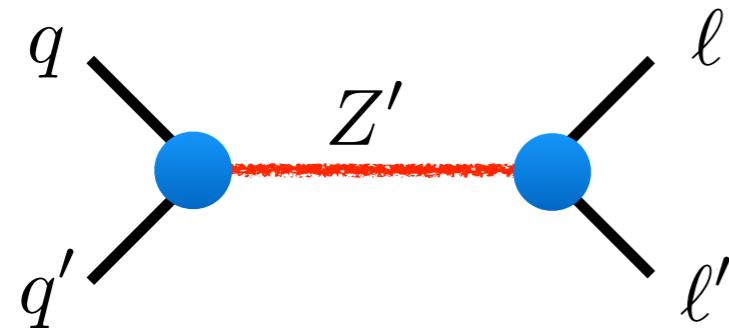
- Same type of NP is (in principle) correlated with kaon sector
- E.g.  $Z'$  interactions can produce effects in  $\epsilon'/\epsilon$  [Buras, De Fazio (2015); Buras (2016)]

# 0 I Motivation

## New Physics origin of LFUV?

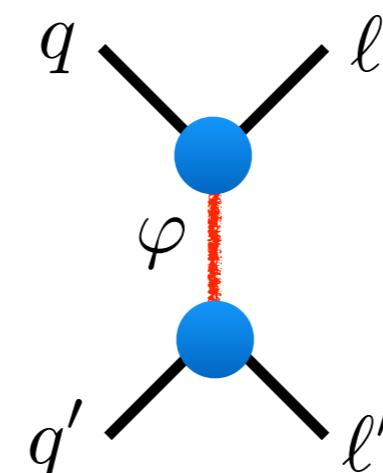
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**Can rare kaon decays test NP explanations for the B-anomalies?**

# 1 | Constraining LFUV and LFV at NA62?

Consider **analogous processes** to those relevant for B-anomalies:

- Key mode for **LFUV**:  $K^\pm \rightarrow \pi^\pm \ell^+ \ell^-$ ,  $\ell = \mu$  or  $e$
- Spectrum measured, but PDG average of  $\mu^+ \mu^-$  mode dominated by E787 measurement (scale factor = 2.6):

$$\text{Br}[K^+ \rightarrow \pi^+ e^+ e^-] = (3.00 \pm 0.09) \times 10^{-7}$$

$$\text{Br}[K^+ \rightarrow \pi^+ \mu^+ \mu^-] = (9.4 \pm 0.6) \times 10^{-8}$$

- Also have **neutral** decays:  $K_{L,S} \rightarrow \pi^0 \ell^+ \ell^-$

$$\left. \begin{array}{l} \text{Br}[K_S \rightarrow \pi^0 e^+ e^-] = 3.0^{+1.5}_{-1.2} \times 10^{-9} \\ \text{Br}[K_S \rightarrow \pi^0 \mu^+ \mu^-] = 2.9^{+1.5}_{-1.2} \times 10^{-9} \end{array} \right\} \quad \begin{array}{l} \text{No spectrum,} \\ \text{prospects for LHCb?} \end{array}$$

$$\left. \begin{array}{l} \text{Br}[K_L \rightarrow \pi^0 e^+ e^-] < 2.8 \times 10^{-10} \\ \text{Br}[K_L \rightarrow \pi^0 \mu^+ \mu^-] < 3.8 \times 10^{-10} \end{array} \right\} \quad \begin{array}{l} \text{Future measurement} \\ \text{at KOTO or NA}\textcolor{red}{XX}\text{?} \end{array}$$

# 1 | Constraining LFUV and LFV at NA62?

- Also have pure leptonic modes:

$$\text{Br}[K_L \rightarrow \mu^+ \mu^-] = (6.84 \pm 0.11) \times 10^{-6}$$

$$\text{Br}[K_L \rightarrow e^+ e^-] = 9_{-4}^{+6} \times 10^{-12}$$

(For dispersive analysis of  $K_S \rightarrow \ell^+ \ell^-$  see R. Stucki's [talk](#))

- Key modes for **LFV**:

$$\text{Br}[K^+ \rightarrow \pi^+ \mu^+ e^-] < 1.3 \times 10^{-11} \quad (\text{E865, E777})$$

$$\text{Br}[K^+ \rightarrow \pi^+ \mu^- e^+] < 5.2 \times 10^{-10} \quad (\text{E865})$$

NA62 projection  $\implies \text{Br}[K^+ \rightarrow \pi^+ \mu^+ e^-] < 0.7 \times 10^{-12}$

- Compare with  $\text{Br}[K_L \rightarrow \mu^\pm e^\mp] < 4.7 \times 10^{-12}$  (E871)

## 2 | Weak interactions at low energies

Consider low energy scales  $\mu \ll m_{t,b,c}$  and decouple heavy quarks

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) Q_i(\mu) \quad (\Delta S = 1)$$

- Wilson coefficients decomposed as follows

$$C_i(\mu) = z_i(\mu) + \tau y_i(\mu), \quad \tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*}$$

- Consider analogous semi-leptonic operators to B-sector

$$Q_9^B \quad \longleftrightarrow \quad Q_{7V} = [\bar{s}\gamma^\mu(1-\gamma_5)d] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu\ell]$$

$$Q_{10}^B \quad \longleftrightarrow \quad Q_{7A} = [\bar{s}\gamma^\mu(1-\gamma_5)d] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu\gamma_5\ell]$$

## 2 | Weak interactions at low energies

- Non-perturbative methods required for evaluation of matrix elements like  $\langle \gamma^* \pi | \mathcal{L}_{\text{eff}} | K \rangle$  [Lattice prospects covered in C. Sachrajda's talk]
- In  $\chi\text{PT}_3$  : amplitudes calculated via asymptotic series

$$\mathcal{A} = \left\{ \mathcal{A}_{\text{LO}} + \mathcal{A}_{\text{NLO}} + \mathcal{A}_{\text{NNLO}} + \dots \right\}$$

- Powers of  $O(m_K)$  momentum and  $m_{u,d,s} = O(m_K^2)$
- Expected rate of convergence  $|\mathcal{A}_{\text{NLO}}/\mathcal{A}_{\text{LO}}| \sim 0.3$
- In  $0^{++}$  channel, corrections large due to final-state interactions  
[Pion scattering and “marriage” to dispersion theory covered in G. Colangelo's talk]

## 2 | Weak interactions at low energies

- In leading order with  $U = U(\pi, K, \eta)$  :

$$\mathcal{L}_{\text{weak}}^{\Delta S=1} = g_8 Q_8(U \partial U^\dagger) + g_{27} Q_{27}(U \partial U^\dagger) + \text{h.c}$$

- At NLO, many (unknown) low energy constants

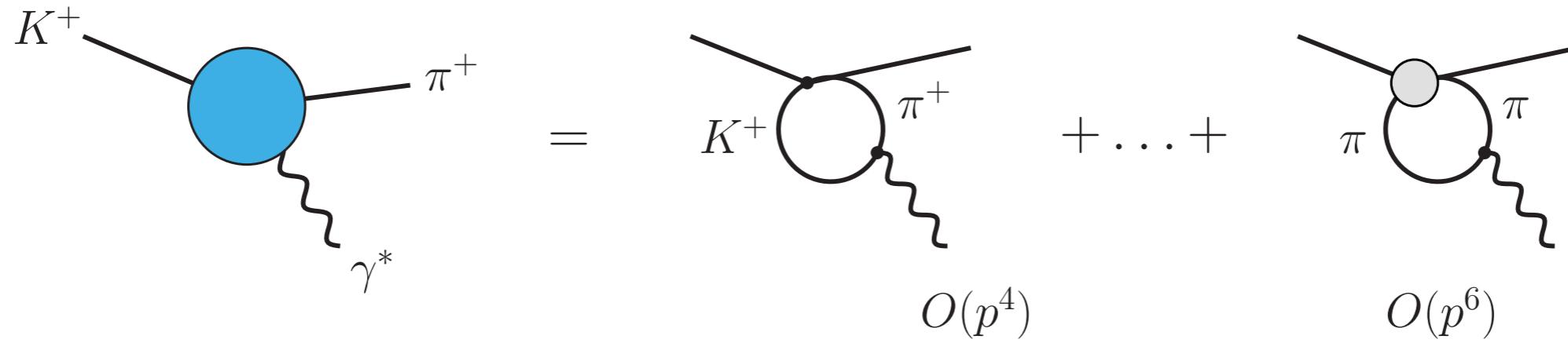
$$\mathcal{L}_{\text{weak}}^{\text{NLO}} = \sum_i N_i \mathcal{O}_i$$

- $\Delta I = 1/2$  rule  $\Rightarrow$  “octet dominance”  $|g_8/g_{27}| \approx 22$
- Disentangle contact terms from final-states interactions?
- A lattice measurement of the  $K \rightarrow \pi$  amplitude **on-shell** is free from re-scattering effects: [Crewther (1985); Crewther, Tunstall (2015)]

$$\langle \pi | [F_5, \mathcal{H}_{\text{weak}}] | K \rangle, \quad \text{non-zero momentum transfer } q_\mu = O(m_K)$$

### 3 | LFUV and $K^\pm \rightarrow \pi^\pm \ell^+ \ell^-$

- Amplitude dominated by  $K^\pm \rightarrow \pi^\pm \gamma^*$  transition



[Ecker, Pich, de Rafael (87)]

[D'Ambrosio, Ecker,  
Isidori, Portoles (98)]

- Chiral dynamics contained in vector form factor:

$$V_+(z) = a_+ + b_+ z + V_+^{\pi\pi}(z), \quad z = q^2/m_K^2$$

- Probe LECs in  $a_+$  and  $b_+$  via spectrum:  $\frac{d\Gamma}{dz} \propto |V_+(z)|^2$

### 3 | LFUV and $K^\pm \rightarrow \pi^\pm \ell^+ \ell^-$

- At lowest order  $\chi\text{PT}_3$ :

$$a_+ = \frac{g_8}{G_F} (1/3 - w_+) , \quad b_+ = -\frac{g_8}{G_F} \frac{1}{60}$$

- Curse of the LECs

$$w_+ = \frac{64\pi^2}{3} [N_{14}^r(\mu) - N_{15}^r(\mu) + 3L_9^r(\mu)] + \frac{1}{3} \ln \frac{\mu^2}{m_K m_\pi}$$


- Estimates based on VMD  $\Rightarrow b_+/a_+ = m_K^2/m_\rho^2 \simeq 0.4$   
[\[D'Ambrosio, Ecker, Isidori, Portoles 98\]](#)
- Can we disentangle long- and short-distance effects?

$$Q_{7V} = [\bar{s}\gamma^\mu(1-\gamma_5)d] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu\ell] \quad \iff \quad a_+^{\text{NP}} = \frac{2\pi\sqrt{2}}{\alpha} V_{ud} V_{us}^* C_{7V}^{\text{NP}}$$

### 3 | LFUV and $K^\pm \rightarrow \pi^\pm \ell^+ \ell^-$

- **Observe:** in SM  $a_+^{\ell\ell}$  same in both modes, so difference (if any) must be due to NP

$$C_{7V}^{\mu\mu} - C_{7V}^{ee} = \alpha \frac{a_+^{\mu\mu} - a_+^{ee}}{2\pi\sqrt{2}V_{ud}V_{us}^*}$$

- Fits to E865 and NA48/2 spectra

$$a_+^{ee} = -0.584 \pm 0.008 \quad a_+^{\mu\mu} = -0.575 \pm 0.039$$

- Can correlate with B-sector coefficients via **Minimal Flavour Violation (MFV)** hypothesis:

$$C_9^{B,\mu\mu} - C_9^{B,ee} = -\frac{a_+^{\mu\mu} - a_+^{ee}}{\sqrt{2}V_{td}V_{ts}^*} \approx -19 \pm 79 \quad \iff \quad C_9^{B,\text{NP}} = O(1)$$

- Determination of  $a_+^{ee} - a_+^{\mu\mu}$  requires improvement of  $O(50 - 100)$

## 4 | LFUV and $K_S \rightarrow \pi^0 \ell^+ \ell^-$

- Analysis similar to  $K^\pm \rightarrow \pi^\pm \ell^+ \ell^-$  with analogous vector FF:

$$V_S(z) = a_S + b_S z$$

- No spectrum measurements, so use VMD and fit  $a_S^{\ell\ell}$  from rates

$$|a_S^{ee}| = 1.06_{-0.21}^{+0.26}, \quad |a_S^{\mu\mu}| = 1.54_{-0.32}^{+0.40}$$

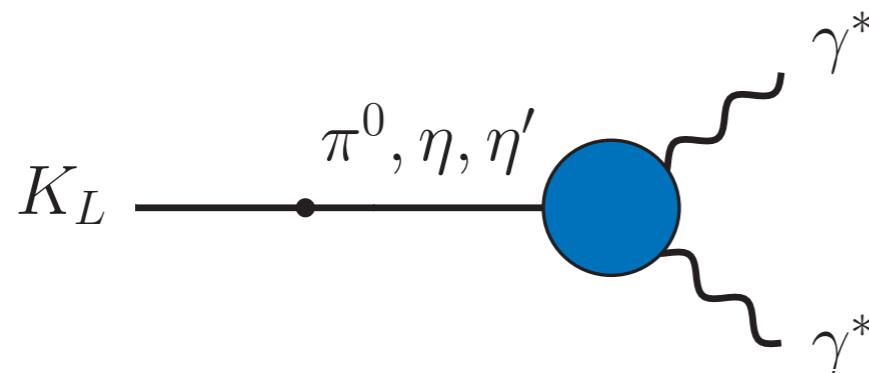
- Sign of  $a_S$  key input in indirect CP violating component of  $K_L$  decay

$$\begin{aligned} \text{Br}[K_L \rightarrow \pi^0 e^+ e^-] &\Big|_{\text{CPV}} \\ &= 10^{-12} \left[ 14.8 |a_S|^2 \pm 6.2 |a_S| \left( \frac{\Im \lambda_t}{10^{-4}} \right) + 2.5 \left( \frac{\Im \lambda_t}{10^{-4}} \right)^2 \right], \end{aligned}$$

$$\begin{aligned} \text{Br}[K_L \rightarrow \pi^0 \mu^+ \mu^-] &\Big|_{\text{CPV}} \\ &= 10^{-12} \left[ 3.5 |a_S|^2 \pm 1.5 |a_S| \left( \frac{\Im \lambda_t}{10^{-4}} \right) + 1.1 \left( \frac{\Im \lambda_t}{10^{-4}} \right)^2 \right] \end{aligned}$$

## 5 | LFUV and $K_L \rightarrow \ell^+ \ell^-$

- Probes axial-vector interactions
- At lowest order, transition mediated via pseudo scalar poles



[Gomez Dumm, Pich (98);  
Knecht, Peris, Perrottet, de Rafael (99);  
Isidori, Unterdorfer (03)]

- Dispersive component of amplitude (normalised to  $K_L \rightarrow \gamma\gamma$ ):

$$F_{\ell, \text{disp}} = \frac{1}{4\beta_\ell} \log^2 \left( \frac{1 - \beta_\ell}{1 + \beta_\ell} \right) + \frac{1}{\beta_\ell} \text{Li}_2 \left( \frac{\beta_\ell - 1}{\beta_\ell + 1} \right) + \frac{\pi^2}{12\beta_\ell} + 3 \log \frac{m_\ell}{\mu} + \chi(\mu)$$

LECs strike again!

$$\chi(\mu) = \chi_{\gamma\gamma}(\mu) + \chi_{\text{SD}}$$

## 5 | LFUV and $K_L \rightarrow \ell^+ \ell^-$

- Can we disentangle long- and short-distance effects?

$$G_F V_{ud} V_{us}^* C_{7A}^{\text{NP}} = -\frac{\alpha}{F_K} \left( \frac{2\Gamma_{\gamma\gamma}}{\pi m_K^3} \right)^{1/2} \chi_{\text{NP}}$$

- Look for difference between lepton modes

$$C_{7A}^{\mu\mu} - C_{7A}^{ee} = -\frac{\alpha}{F_K G_F V_{ud} V_{us}^*} \left( \frac{2\Gamma_{\gamma\gamma}}{\pi m_K^3} \right)^{1/2} (\chi^{\mu\mu} - \chi^{ee})$$

- Correlate with B-sector using MFV

$$C_{10}^{B,\mu\mu} - C_{10}^{B,ee} = \frac{2\pi}{F_K G_F V_{td} V_{ts}^*} \left( \frac{2\Gamma_{\gamma\gamma}}{\pi m_K^3} \right)^{1/2} (\chi^{\mu\mu} - \chi^{ee})$$

## 5 | LFUV and $K_L \rightarrow \ell^+ \ell^-$

- Naive fit to measured rates yields two solutions per channel

Channel	$\chi$ (Solution 1)	$\chi$ (Solution 2)
$ee$	$5.1^{+15.4}_{-10.3}$	$-(57.5^{+15.4}_{-10.3})$
$\mu\mu$	$3.75 \pm 0.20$	$1.52 \pm 0.20$

- Suppose uncertainty can reduced by factor of  $\sim 10$ :

$$\chi^{\mu\mu} - \chi^{ee} \sim 1.3 \pm 1.3$$

$$C_{10}^{B,\mu\mu} - C_{10}^{B,ee} \sim 3.5 \pm 3.5$$

- **Caveat:** Fit assumes  $\chi^{\mu\mu} = \chi^{ee}$  to all orders
- Corrections of  $O(m_\ell/m_{\pi,\eta,\eta'})$  likely to be important  
[Masjuan, Sanchez-Puertas (2015)]

## 6 | LFV decays

- Apart from tiny effects due to neutrino oscillations, LFV not present in SM
- Amplitude factorises, so no problems with LECs
- Key modes

$$\text{Br}[K_L \rightarrow \mu^\pm e^\mp] \propto \{|C_{7V}^{\mu e}|^2 + |C_{7A}^{\mu e}|^2\}$$
$$\text{Br}[K^+ \rightarrow \pi^+ \mu^\pm e^\mp] \propto \{|C_{7V}^{\mu e}|^2 + |C_{7A}^{\mu e}|^2\}$$

- Bounds on amplitude correlated with B-sector coefficients

	$K_L \rightarrow \mu^\pm e^\mp$	$K^+ \rightarrow \pi^+ \mu^\pm e^\mp$	$K_L \rightarrow \pi^0 \mu^\pm e^\mp$	$K^+ \rightarrow \pi^+ \mu^\pm e^\mp$ (NA62 projection)
$( C_{7V}^{\mu e} ^2 +  C_{7A}^{\mu e} ^2)^{1/2}$	$< 1.3 \times 10^{-6}$	$< 2.2 \times 10^{-5}$		$< 5.1 \times 10^{-6}$
$( y_{7V}^{\mu e} ^2 +  y_{7A}^{\mu e} ^2)^{1/2}$			$< 0.040$	
$( C_9^{B,\mu e} ^2 +  C_{10}^{B,\mu e} ^2)^{1/2}$	$< 0.71$	$< 12$	$< 35$	$< 2.7$

- Strongest bound from  $K_L \rightarrow \mu^\pm e^\mp$  but...remove GIGATRACKER?

## 7 | Summary

- Several anomalies in B-decays hint at New Physics of LFUV origin
- The MFV hypothesis implies correlations between Wilson coefficients and B-sectors
- These correlations can be tested at NA62! ( $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  key)
- Complementary searches for LFV decays also correlated

$$K_L \rightarrow \mu^\pm e^\mp \quad K^+ \rightarrow \pi^+ \mu^\pm e^\mp$$

- Three logical possibilities:
  1. New Physics explanations for B-anomalies + MFV implies signal at NA62 sensitivities
  2. If searches negative at NA62 can rule out MFV solutions
  3. If signal seen near current sensitivities can also rule out MFV

One final prediction...

