K_{l2}, K_{l3}, K_{l4} decays in and beyond the SM

Emilie Passemar
Indiana University/Jefferson Lab.

NA62 Kaon Physics Handbook
MITP, University of Mainz
Mainz, January 19, 2015

Outline

1. Introduction and Motivation
2. K_{l2} decays: Lepton Universality and R_K
3. K_{l3} decays: CKM Unitarity and Callan-Treiman
4. K_{l3} & K_{l4} decays: T violation
5. Conclusion and outlook

See the excellent review by Cirigliano, Ecker, Neufeld, Pich, Portoles'12

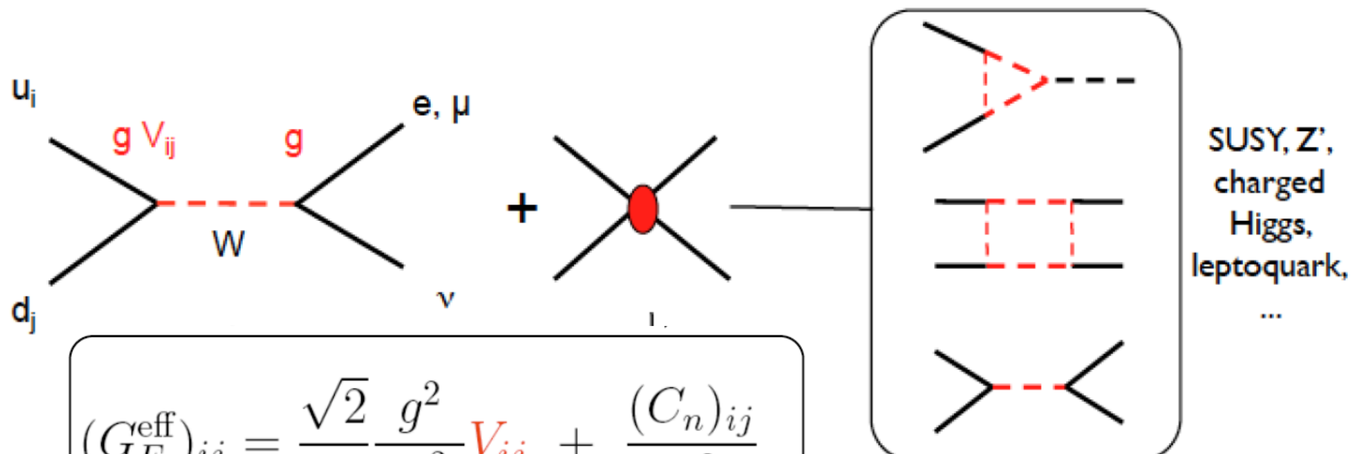
1. Introduction and Motivation

1.1 (Semi)-leptonic decays

- Studying semileptonic decays: Mediated by W exchange in the SM
  - Only V-A structure
  - *Lepton Universality*
  - *Cabibbo Universality:* $\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}}$

Negligible (B decays)

- Indirect searches of new physics, several possible high-precision tests:



$$\left(G_F^{\rm eff}\right)_{ij} = \frac{\sqrt{2}}{8}\frac{g^2}{M_W^2} V_{ij} + \frac{(C_n)_{ij}}{\Lambda^2}$$

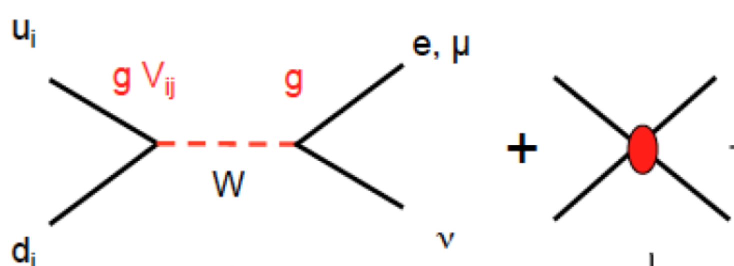
- BSM effects : $\boxed{\Delta \sim \frac{c_n}{g^2}\frac{M_W^2}{\Lambda^2} \leq 10^{-2} - 10^{-3} \longleftrightarrow \Lambda\text{~1-10 TeV}}$

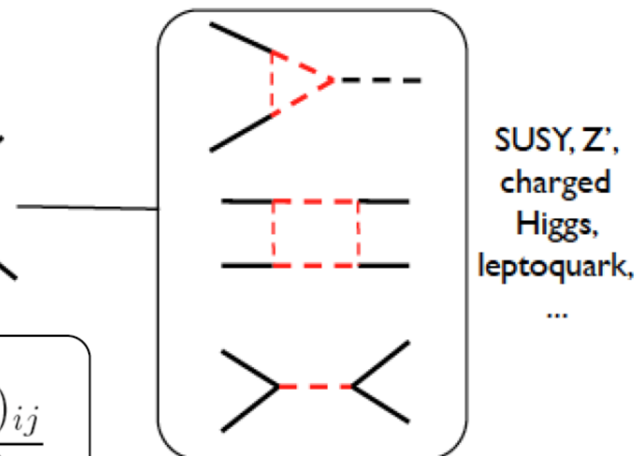
1.1 (Semi)-leptonic decays

- Studying semileptonic decays: Mediated by W exchange in the SM
  - Only V-A structure
  - *Lepton Universality*
  - *Cabibbo Universality:* $\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}}$

Negligible (B decays)

- Indirect searches of new physics, several possible high-precision tests:





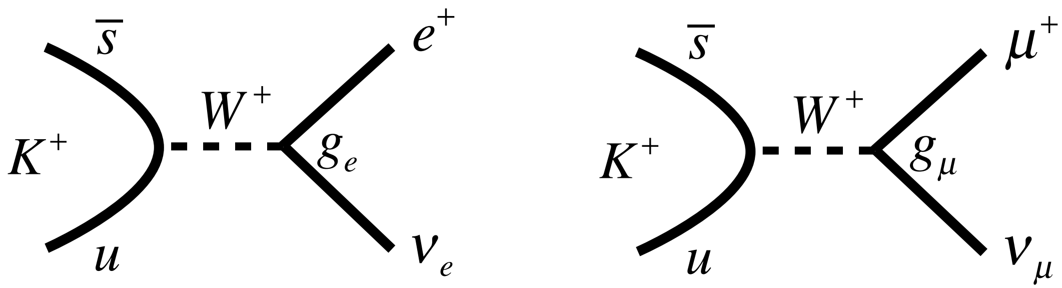
$$(G_F^{\text{eff}})_{ij} = \frac{\sqrt{2}}{8}\frac{g^2}{M_W^2}V_{ij} + \frac{(C_n)_{ij}}{\Lambda^2}$$

- Look for new physics by comparing different processes: helicity suppressed K_{l2}, helicity allowed K_{l3} etc..

2. Lepton Universality: R_K

2.1 K_{l2} decays

- K_{l2} decays



Only the *axial current* contributes in the SM

- The branching ratio in the SM: see *S. Descotes-Genon's* talk

$$B(K \to \ell\nu) = \frac{G_F^2 |V_{us}|^2}{8\pi} f_K^2 m_K m_\ell^2 \left(1 - \frac{m_\ell^2}{M_K^2}\right)^2 \left(1 + 2\frac{\alpha}{\pi} \log \frac{M_Z}{M_\rho}\right)$$
$$\left(1 + \frac{\alpha}{\pi} F(m_\ell / m_K)\right)(1 + O(\alpha))$$

Marciano & Sirlin'93, Finkemeier'96, Cirigliano & Rosell'07

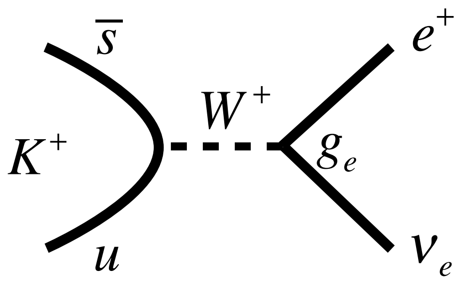
– *Short distance* effects (universal)

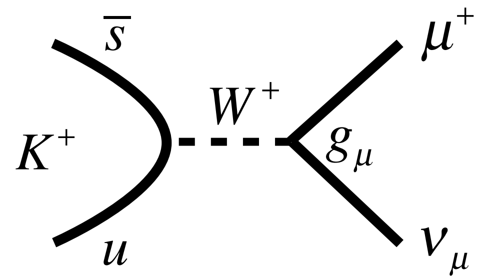
– Long distance effects (universal)

– Structure dependent effects (process dependent)

2.2 R_K

- K_{l2} decays





- Define the RK ratio to reduce the theoretical uncertainties: most of the hadronic and radiative contributions cancel

Cirigliano, Rosell'07

$$R_K^{SM} = \frac{\Gamma(K^+ \to e^+\nu_e[\gamma])}{\Gamma(K^+ \to \mu^+\nu_\mu[\gamma])} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_K^2 - m_e^2}{m_K^2 - m_\mu^2}\right)^2 (1+\delta R_{QED}) = 2.477(1)\times 10^{-5}$$

$g_e / g_\mu = 1$

in the standard model

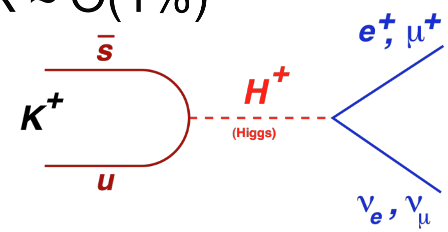
Experimental result:

NA62-R_K:
R_K = (2.488 ± 0.007$_{stat}$ ± 0.007$_{syst}$) × 10^{-5}
R_K = (2.488 ± 0.010) × 10^{-5}

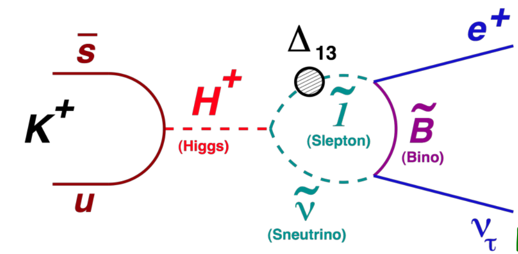
- Compatible with SM but experimental uncertainty one order of magnitude higher than theory ⇨ *NA62*

2.3 Test of New Physics in R_K

- R_K sensitive to *lepton flavour violating effects*, $\Delta R/R \approx O(1\%)$



- 2HDM – tree level: ⇨ additional contribution due to charged Higgs, does not contribute to R_K



- Possibility to constrain LFV at one loop in MSSM

  Masiero, Paradisi, Petronzio'06,'08

- Update and extension by *Girrbach & Nierste'12*

  – *LFV:* $R_K^{LFV} \approx R_K^{SM}(1+0.013)$

  – Can become negative if interference with LFC effects:

  $$R_K^{LFV} \approx R_K^{SM}(1-0.032)$$

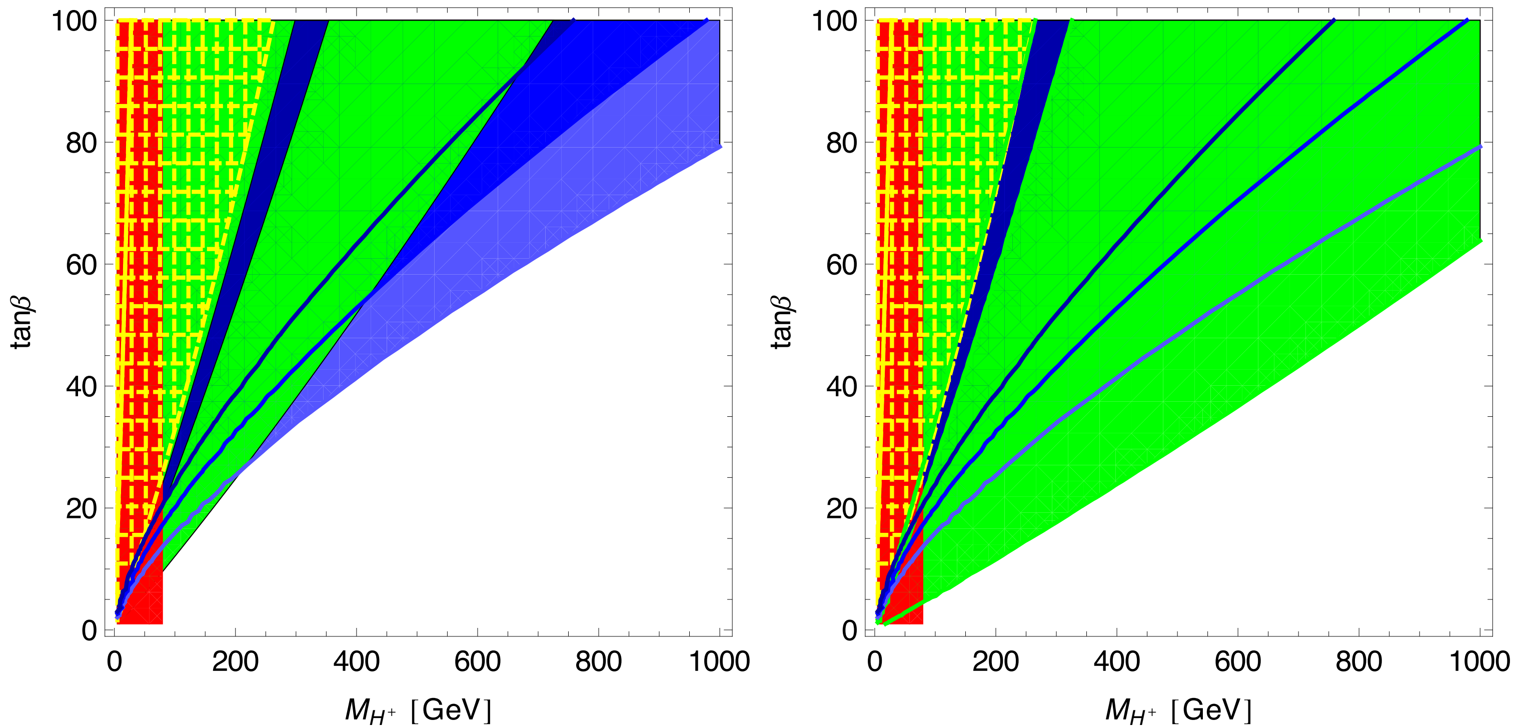
  Ex : $\tan\beta = 40$, $M_H = 500$ GeV, $\Delta^{31}_R = 5\times10^{-4}$.

2.3 Test of New Physics in R_K

- R_K sensitive to *lepton flavour violating effects*, $\Delta R/R \approx O(1\%)$

- If 0.05% effect on R_K found at NA62 (blue constraint):

Girrbach & Nierste'12



2.3 Test of New Physics in R_K

- R_K sensitive to neutrino mixing parameters within SM extensions involving a *fourth generation of quarks and leptons*

$$\frac{1-|U_{e4}|^2}{1-|U_{\mu 4}|^2}$$

Lacker & Menzel'10

- R_K sensitive to neutrino mixing parameters within SM extensions involving sterile neutrinos. Depends on on masses, hierarchy, and mixings of new neutrino states

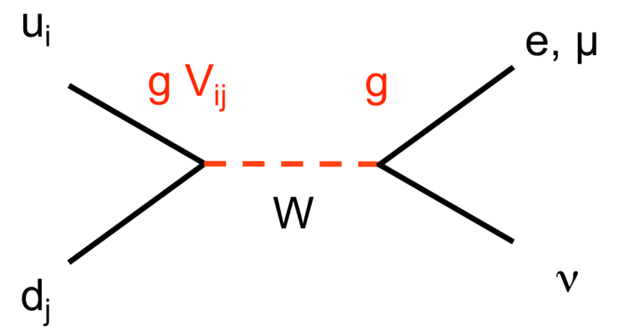
Abada et al.'12

3. CKM Unitarity and Callan-Treiman relation

3.1 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

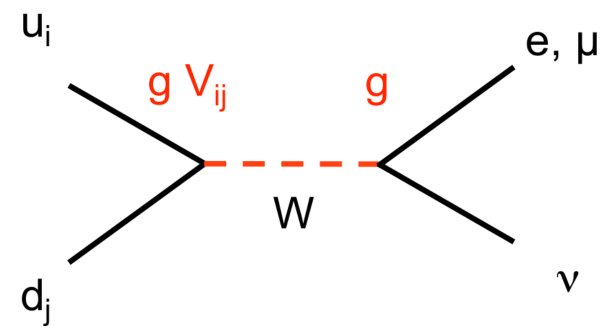
V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to \ell\, \nu_\ell$
V_{us}	$K \to \pi \ell \nu_\ell$	$\Lambda \to p e \nu_e$	$K \to \ell\, \nu_\ell$



3.1 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to \ell \nu_\ell$
V_{us}	$K \to \pi \ell \nu_\ell$	$\Lambda \to p e \nu_e$	$K \to \ell \nu_\ell$



- These are the *golden modes* to extract V_{ud} and V_{us}
  - Only the *vector current* contributes
  - Normalization known in SU(2) [SU(3)] symmetry limit
  - Corrections start at 2nd order in SU(2) [SU(3)] breaking

Ademollo & Gato, Berhands & Sirlin

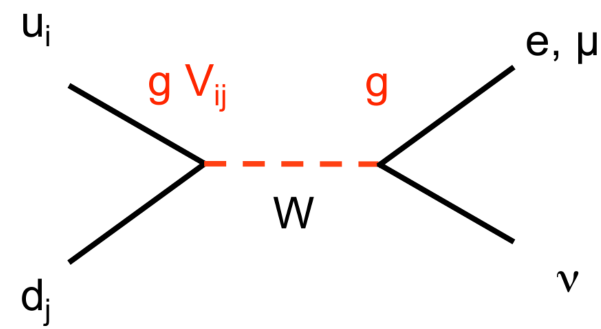
- Currently the most precise determination of V_{ud} and V_{us}

⇨ V_{ud} (*0.02 %*) and V_{us} (*0.5 %*)

3.1 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to \ell \nu_\ell$
V_{us}	$K \to \pi \ell \nu_\ell$	$\Lambda \to p e \nu_e$	$K \to \ell \nu_\ell$



- K_{l2}/π_{l2}
  - Only the *axial current* contributes
  - Need to know the decay constants F_K, F_π
    ⇒ *Lattice QCD*
  - Probe different BSM operators than from the vector case
- Input on F_K/F_π ⇒ V_{us}/V_{ud} very precisely

3.2 V_{us}/V_{ud} from K_{l2}/π_{l2}

- From K_{l2}/π_{l2}:

$$\boxed{\frac{\Gamma\left(K\to\mu\nu[\gamma]\right)}{\Gamma\left(\pi\to\mu\nu[\gamma]\right)}=\frac{m_{K^\pm}}{m_{\pi^\pm}}\frac{\left(1-m_\mu^2/m_{K^\pm}^2\right)}{\left(1-m_\mu^2/m_{\pi^\pm}^2\right)}\frac{f_K^2}{f_\pi^2}\frac{|V_{us}|^2}{|V_{ud}|^2}\left(1+\delta_{\mathrm{EM}}\right)}$$

Inputs needed :

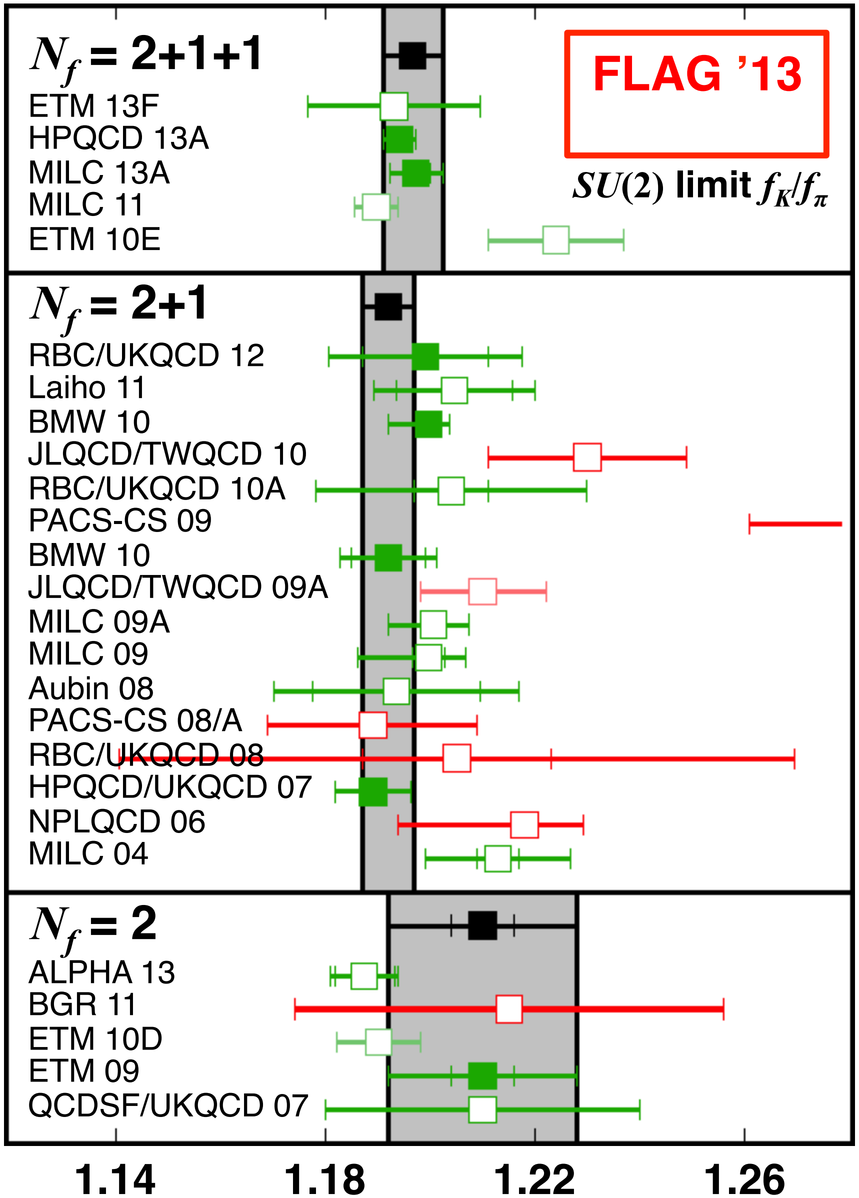
→ Experimental BRs from FlaviaNet kaon WG review *Antonelli et al.'10*

→ F_K/F_π *Lattice calculations*

→ Electromagnetic and isospin breaking corrections

Marciano'04, Knecht et al.'99

F_K/F_π from lattice QCD



- Corrections for IB taken into account in FLAG averages

FLAG'13

- N_f=2+1

$$\frac{F_K}{F_\pi} = 1.192 \pm 0.005$$

- N_f=2+1+1

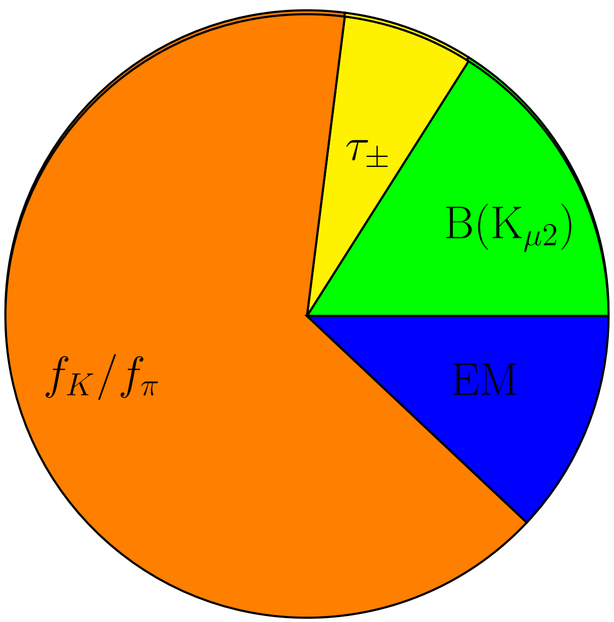
$$\frac{F_K}{F_\pi} = 1.194 \pm 0.005$$

3.2 V_{us}/V_{ud} from K_{l2}/π_{l2}

- From K_{l2}/π_{l2}:

$$\frac{\Gamma\left(K\to\mu\nu[\gamma]\right)}{\Gamma\left(\pi\to\mu\nu[\gamma]\right)} = \frac{m_{K^\pm}}{m_{\pi^\pm}}\frac{\left(1-m_\mu^2/m_{K^\pm}^2\right)}{\left(1-m_\mu^2/m_{\pi^\pm}^2\right)}\frac{f_K^2}{f_\pi^2}\frac{|V_{us}|^2}{|V_{ud}|^2}\left(1+\delta_{\rm EM}\right)$$

Moulson@CKM2014



Choice of $f_{K\pm}/f_{\pi\pm}$		V_{us}/V_{ud}
N_f = 2+1	1.192(5)	**0.2315(10)**
N_f = 2+1+1	1.1960(25)	**0.2308(6)**

What role for *NA62*?

3.2 V_{us} from K_{l3}

- Master formula for $K \to \pi l \nu_l$:

$$\boxed{\Gamma\left(K \to \pi l \nu [\gamma]\right) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{EW}^K \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^l \left(1 + \delta_{\mathrm{EM}}^{Kl} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)^2}$$

- Experimental inputs from FlaviaNet review *Antonelli et al.'10,*

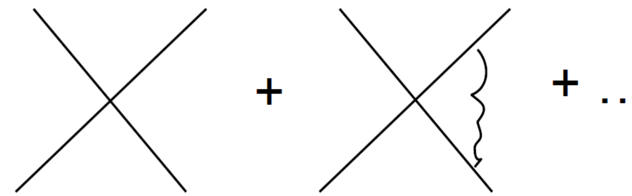
  ⇒ Update by *M. Moulson at CKM2014*

3.2 V_{us} from K_{l3}

- Master formula for $K \to \pi l \nu_l$:

$$\Gamma(K \to \pi l \nu[\gamma]) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 \boxed{S_{EW}^K} |V_{us}|^2 \left| f_+^{K^0\pi^-}(0) \right|^2 I_K^l \left(1 + \delta_{\mathrm{EM}}^{Kl} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)^2$$

- Theoretical inputs :
  - S_{ew} : Short distance electroweak correction



$$S_{\mathrm{ew}} = 1 + \frac{2\alpha}{\pi}\left(1 + \frac{\alpha_S}{4\pi}\right)\log\frac{m_Z}{m_\rho} + O\left(\frac{\alpha\alpha_S}{\pi^2}\right)$$

$$\Rightarrow \quad \boxed{S_{\mathrm{ew}} = 1.0232}$$

Sirlin'82

3.2 V_{us} from K_{l3}

- Master formula for $K \to \pi l \nu_l$:

$$\Gamma(K \to \pi l \nu [\gamma]) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{EW}^K |V_{us}|^2 \left| \boxed{f_+^{K^0\pi^-}(0)} \right|^2 I_K^l \left(1 + \delta_{\rm EM}^{Kl} + \delta_{\rm SU(2)}^{K\pi}\right)^2$$

- Theoretical inputs :
  - S_{ew} : Short distance electroweak correction $\Rightarrow$ $\boxed{S_{\rm ew} = 1.0232}$
  - $f_+(0)$: vector form factor at zero momentum transfer:
    Hadronic matrix element:

$$\langle \pi(p_\pi) | \bar{\rm s}\gamma_\mu {\rm u} | {\rm K}(p_{\rm K}) \rangle = \left[(p_K + p_\pi)_\mu - \frac{\Delta_{K\pi}}{t}(p_K - p_\pi)_\mu \right] \underbrace{f_+(t)}_{\text{vector}} + \frac{\Delta_{K\pi}}{t}(p_K - p_\pi)_\mu \underbrace{f_0(t)}_{\text{scalar}}$$

$$t = q^2 = (p_\mu + p_{\nu_\mu})^2 = (p_K - p_\pi)^2, \quad \bar{f}_{0,+}(t) = \frac{f_{0,+}(t)}{f_+(0)}$$

In chiral limit $f_+(0) = 1$, calculation of SU(3) breaking crucial

$\Rightarrow$ *ChPT* with resonances or *lattice*

3.2 V_{us} from K_{l3}

- Master formula for $K \to \pi l \nu_l$:

$$\Gamma(K \to \pi l \nu [\gamma]) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{EW}^K |V_{us}|^2 \left| f_+^{K^0\pi^-}(0) \right|^2 \boxed{I_K^l} \left(1 + \delta_{\mathrm{EM}}^{Kl} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)^2$$

- Theoretical inputs :
  - S_{ew} : Short distance electroweak correction ⇨ $S_{\mathrm{ew}} = 1.0232$
  - $f_+(0)$: vector form factor at zero momentum transfer

    ⇨ *ChPT* with resonances or *lattice*
  - I_K : Phase space integral ⇨ need a *parametrization* for the normalized form factors to fit the experimental distributions

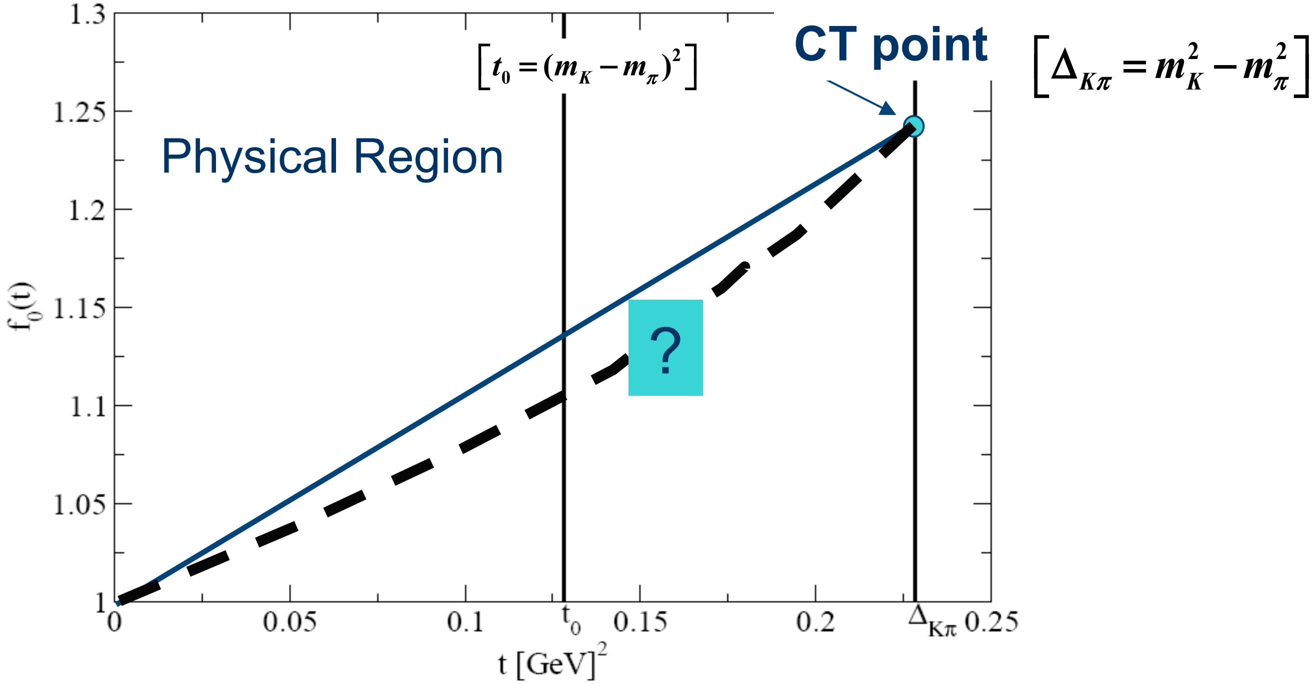
    Taylor expansion :

$$\bar{f}_{+,0}(s) = 1 + \lambda'_{+,0} \frac{s}{m_\pi^2} + \frac{1}{2} \lambda''_{+,0} \left(\frac{s}{m_\pi^2}\right)^2 + \ldots$$

⇨ *Dispersive parametrization*

Dispersive representation for the form factors

- Take the Kπ rescattering into account *Bernard, Oertel, E.P., Stern'06, '09*
- Allow to determine the slope and *curvature* of the scalar form factor

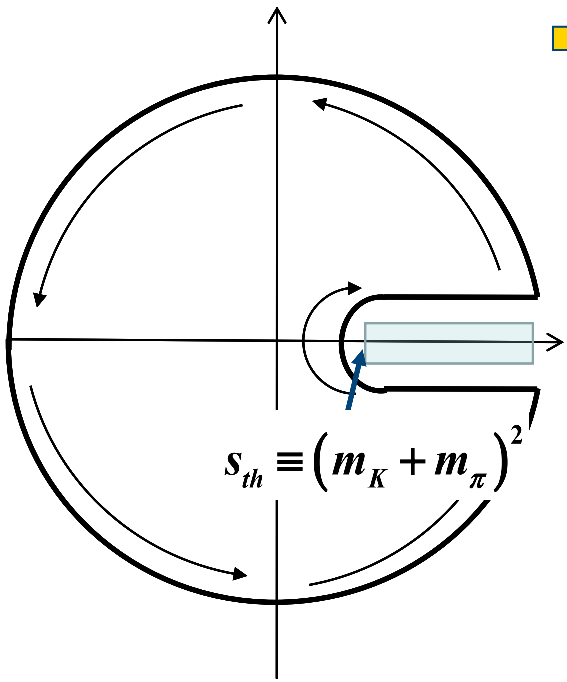


- Use the CT theorem for the scalar FF ⇒ Write a twice substracted dispersion relation for ln f(t) at t=0 and at the CT point for the scalar FF

Dispersive representation for the form factors

Bernard, Oertel, E.P., Stern'06, '09

- Omnès representation:



$$\bar{f}_{+,0}(s) = \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\phi_{+,0}(s')}{s'-s-i\varepsilon}\right]$$

$\phi_{+,0}$ (s): phase of the form factor

$$s < s_{in} \;:\; \phi_{+,0}(s) = \delta_{K\pi}(s)$$

Kπ scattering phase

$$s \geq s_{in} \;:\; \phi_{+,0}(s)$$

$$\phi_{+,0}(s) = \phi_{+,0as}(s) = \pi \pm \pi \;\left(\bar{f}_{+,0}(s) \to 1/s\right)$$

[Brodsky&Lepage]

- Subtract dispersion relation to weaken the high energy contribution of the phase. Improve the convergence but sum rules to be satisfied.

Dispersive representation for the form factors

Bernard, Oertel, E.P., Stern'06, '09

- Scalar form factor:

$$\bar{f}_0(t) = \exp\left[\frac{t}{\Delta_{K\pi}}\left(\ln C - G(t)\right)\right] \quad \text{with} \quad G(t) = \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi}\int_{t_{K\pi}}^{\infty}\frac{ds}{s}\frac{\phi_0(s)}{(s-\Delta_{K\pi})(s-t)}$$

- Vector form factor:

$$\bar{f}_+(t) = \exp\left[\frac{t}{m_\pi^2}\left(\Lambda_+ + H(t)\right)\right] \quad \text{with} \quad H(t) = \frac{m_\pi^2 t}{\pi}\int_{t_{\pi K}}^{\infty}\frac{ds}{s^2}\frac{\phi_+(s)}{(s-t)}$$

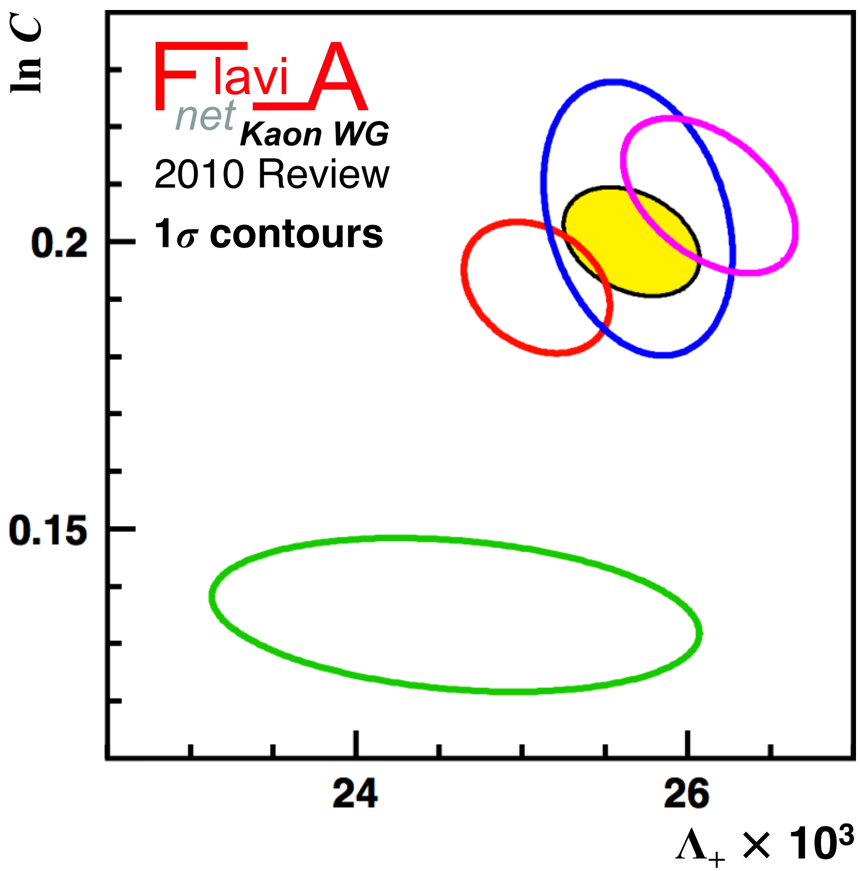
$$I_{K\ell} = \frac{2}{3}\int_{m_\ell^2}^{t_0}\frac{dt}{M_K^8}\bar{\lambda}^{3/2}\left(1+\frac{m_\ell^2}{2t}\right)\left(1-\frac{m_\ell^2}{2t}\right)^2 \times \left(\bar{f}_+^2(t) + \frac{3m_\ell^2\Delta_{K\pi}^2}{(2t+m_\ell^2)\bar{\lambda}}\bar{f}_0^2(t)\right),$$

Moulson@CKM2014

Dispersive parameters for $K_{\ell 3}$ form-factors

$K_{e3} + K_{\mu 3}$ averages from KTeV KLOE ISTRA+ NA48 2010 fit

For NA48, only K_{e3} data included in 2010 fit



$$\Lambda_+ \times 10^3 = 25.61 \pm 0.41$$
$$\ln C = 0.2004(91)$$
$$\rho(\Lambda_+, \ln C) = -0.328$$
$$\chi^2/\text{ndf} = 5.6/5\ (34\%)$$

Integrals

Mode	Quad-lin	Disp
$K^0{}_{e3}$	0.15457(20)	**0.15476(18)**
$K^+{}_{e3}$	0.15894(21)	**0.15922(18)**
$K^0{}_{\mu 3}$	0.10266(20)	**0.10253(16)**
$K^+{}_{\mu 3}$	0.10564(20)	**0.10559(17)**

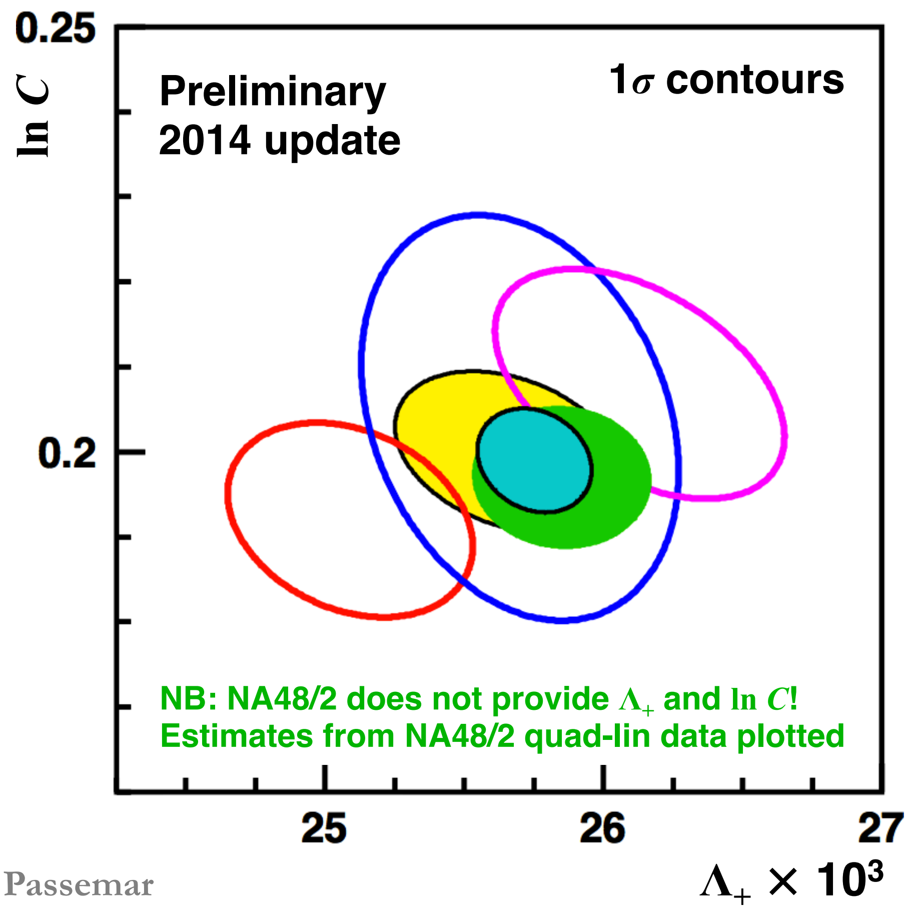
Maximum change 0.2% if same data used as for quad-lin fits

Moulson@CKM2014

Dispersive parameters for $K_{\ell 3}$ form-factors

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2 ’12 prel** **2010 fit** **Update**

For NA48, only K_{e3} data included in fits



$$\Lambda_+ \times 10^3 = 25.75 \pm 0.36$$
$$\ln C = 0.1985(70)$$
$$\rho(\Lambda_+, \ln C) = -0.202$$
$$\chi^2/\text{ndf} = 5.9/7\ (55\%)$$

Integrals

Mode	Update	2010
$K^0{}_{e3}$	**0.15481(14)**	0.15476(18)
$K^+{}_{e3}$	**0.15927(14)**	0.15922(18)
$K^0{}_{\mu 3}$	**0.10253(13)**	0.10253(16)
$K^+{}_{\mu 3}$	**0.10558(14)**	0.10559(17)

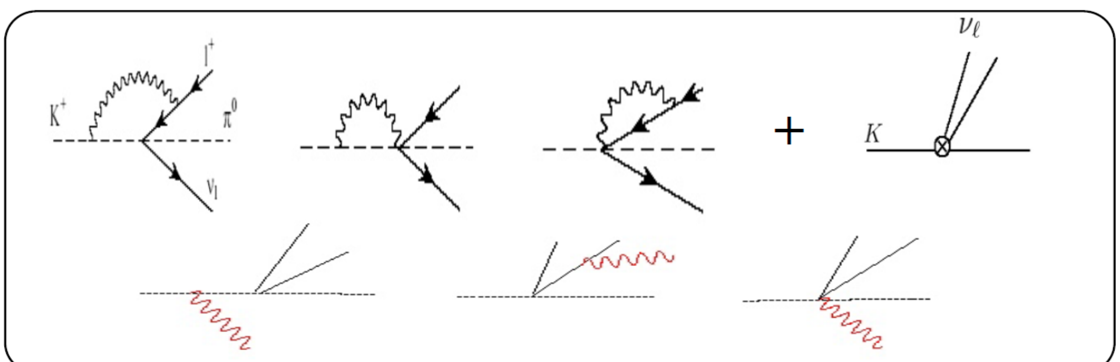
Only tiny changes in central values

3.2 V_{us} from K_{l3}

- Master formula for $K \to \pi l \nu_l$:

$$\Gamma(K \to \pi l \nu[\gamma]) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{EW}^K |V_{us}|^2 \left| f_+^{K^0\pi^-}(0) \right|^2 I_K^l \left(1 + \boxed{\delta_{\rm EM}^{Kl}} + \delta_{\rm SU(2)}^{K\pi}\right)^2$$

- Theoretical inputs :
  - S_{ew} : Short distance electroweak correction ⇒ $\boxed{S_{\rm ew} = 1.0232}$
  - $f_+(0)$: vector form factor at zero momentum transfer
    ⇒ *ChPT* with resonances or *lattice*
  - I_K *:* Phase space integral ⇒ *Dispersive parametrization* for the FFs
  - $\delta_{\rm EM}^{Kl}$: Long-distance electromagnetic corrections



→ ChPT to O(p²e²)

→ Fully inclusive prescription for real photons

→ Uncertainties: LECs (100%)

3.2 V_{us} from K_{l3}

- Master formula for $K \to \pi l \nu_l$:

$$\Gamma(K \to \pi l \nu [\gamma]) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{EW}^K |V_{us}|^2 \left| f_+^{K^0\pi^-}(0) \right|^2 I_K^l \left(1 + \boxed{\delta_{\mathrm{EM}}^{Kl}} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)^2$$

- Theoretical inputs :
  - S_{ew} : Short distance electroweak correction ⇨ $\boxed{S_{\mathrm{ew}} = 1.0232}$
  - $f_+(0)$: vector form factor at zero momentum transfer
    ⇨ *ChPT* with resonances or *lattice*
  - I_K : Phase space integral ⇨ *Dispersive parametrization* for the FFs
  - $\delta_{\mathrm{EM}}^{Kl}$: Long-distance electromagnetic corrections

⇨

Mode	$\delta_{\mathrm{EM}}^{K\ell}$ (%)
K_{e3}^0	0.495 ± 0.110
$K_{e3}^\pm$	0.050 ± 0.125
$K_{\mu3}^0$	0.700 ± 0.110
$K_{\mu3}^\pm$	0.008 ± 0.125

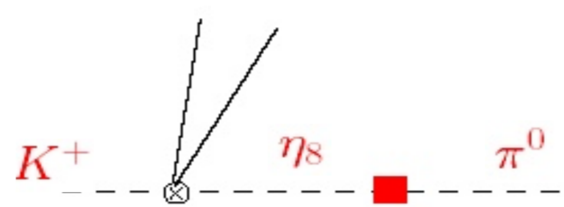
Cirigliano, Giannotti, Neufeld'08

3.2 V_{us} from K_{l3}

- Master formula for $K \to \pi l \nu_l$:

$$\Gamma(K \to \pi l \nu [\gamma]) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{EW}^K |V_{us}|^2 \left| f_+^{K^0\pi^-}(0) \right|^2 I_K^l \left(1 + \delta_{\mathrm{EM}}^{Kl} + \boxed{\delta_{\mathrm{SU(2)}}^{K\pi}}\right)^2$$

- Theoretical inputs :
  - S_{ew} : Short distance electroweak correction ⇨ $\boxed{S_{\mathrm{ew}} = 1.0232}$
  - $f_+(0)$: vector form factor at zero momentum transfer
    ⇨ *ChPT* with resonances or *lattice*
  - I_K : Phase space integral ⇨ *Dispersive parametrization* for the FFs
  - $\delta_{\mathrm{EM}}^{Kl}$: Long-distance electromagnetic corrections
  - $\delta_{\mathrm{SU(2)}}^{K\pi}$:isospin breaking corrections $\boxed{\delta_{\mathrm{SU(2)}}^{K\pi} = \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} - 1}$



+ IB in one loop graphs + CT

Isospin breaking corrections

Gasser&Leutwyler'85

- $\delta^{K\pi}_{SU(2)} = \dfrac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} - 1$ In ChPT at O(p⁴) : $\delta^{K\pi}_{SU(2)} = \dfrac{3}{4}\dfrac{1}{Q^2}\left[\dfrac{m_K^2}{m_\pi^2} + \dfrac{\chi_{p^4}}{2}\left(1+\dfrac{m_s}{\hat{m}}\right)\right]$

$$Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} \qquad \left[\hat{m} \equiv \frac{m_u + m_d}{2}\right]$$

- Or equivalently using R

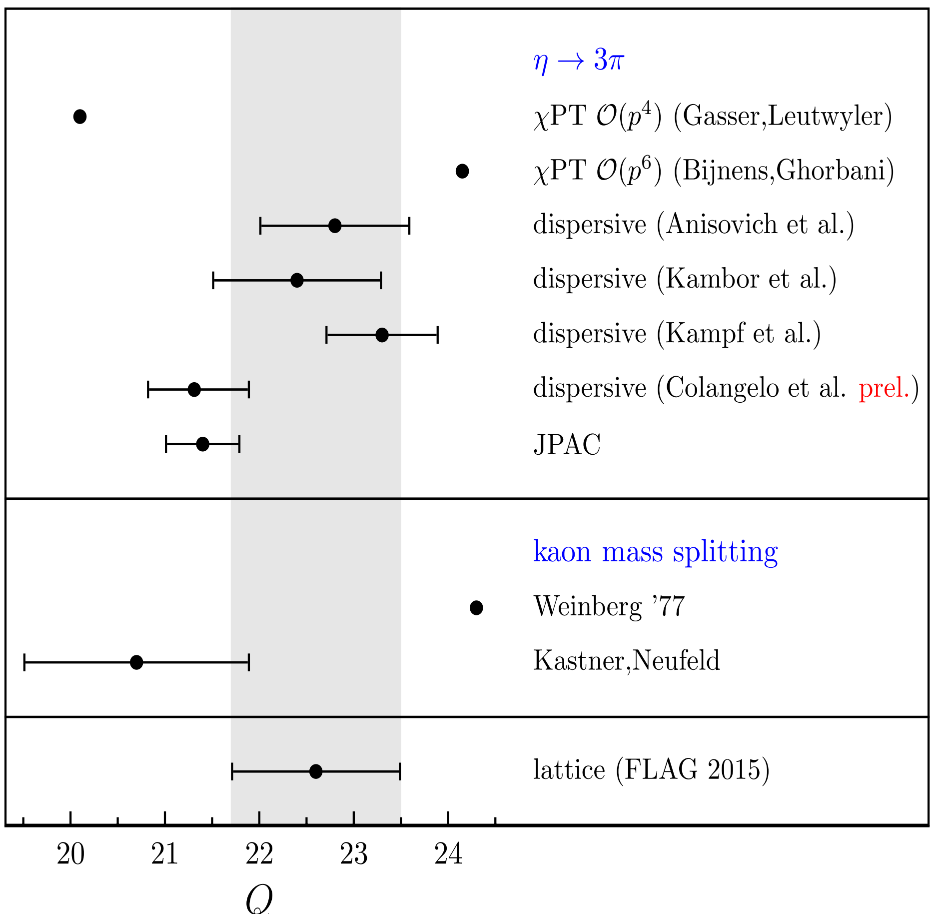
$$\delta^{K^+\pi^0}_{\mathrm{SU}(2)} = \frac{3}{4}\frac{1}{R}\left(1 + \chi_{p^4} + \Delta_M + \mathcal{O}(m_q^2)\right) \qquad R \equiv \frac{m_s - \hat{m}}{m_d - m_u}$$

Isospin breaking corrections

Gasser & Leutwyler'85

- $$\boxed{\delta^{K\pi}_{\mathrm{SU(2)}} = \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} - 1}$$ In ChPT at O(p⁴) : $$\boxed{\delta^{K\pi}_{\mathrm{SU(2)}} = \frac{3}{4}\frac{1}{Q^2}\left[\frac{m_K^2}{m_\pi^2} + \frac{\chi_{p^4}}{2}\left(1+\frac{m_s}{\hat{m}}\right)\right]}$$

$$Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} \qquad \left[\hat{m} \equiv \frac{m_u + m_d}{2}\right]$$



Q on the higher side

$$\boxed{\delta^{K\pi}_{\mathrm{SU(2)}} = (2.36 \pm 0.22)\%}$$

Inputs from *H. Leutwyler'96*
M. Walker

Q on the lower side

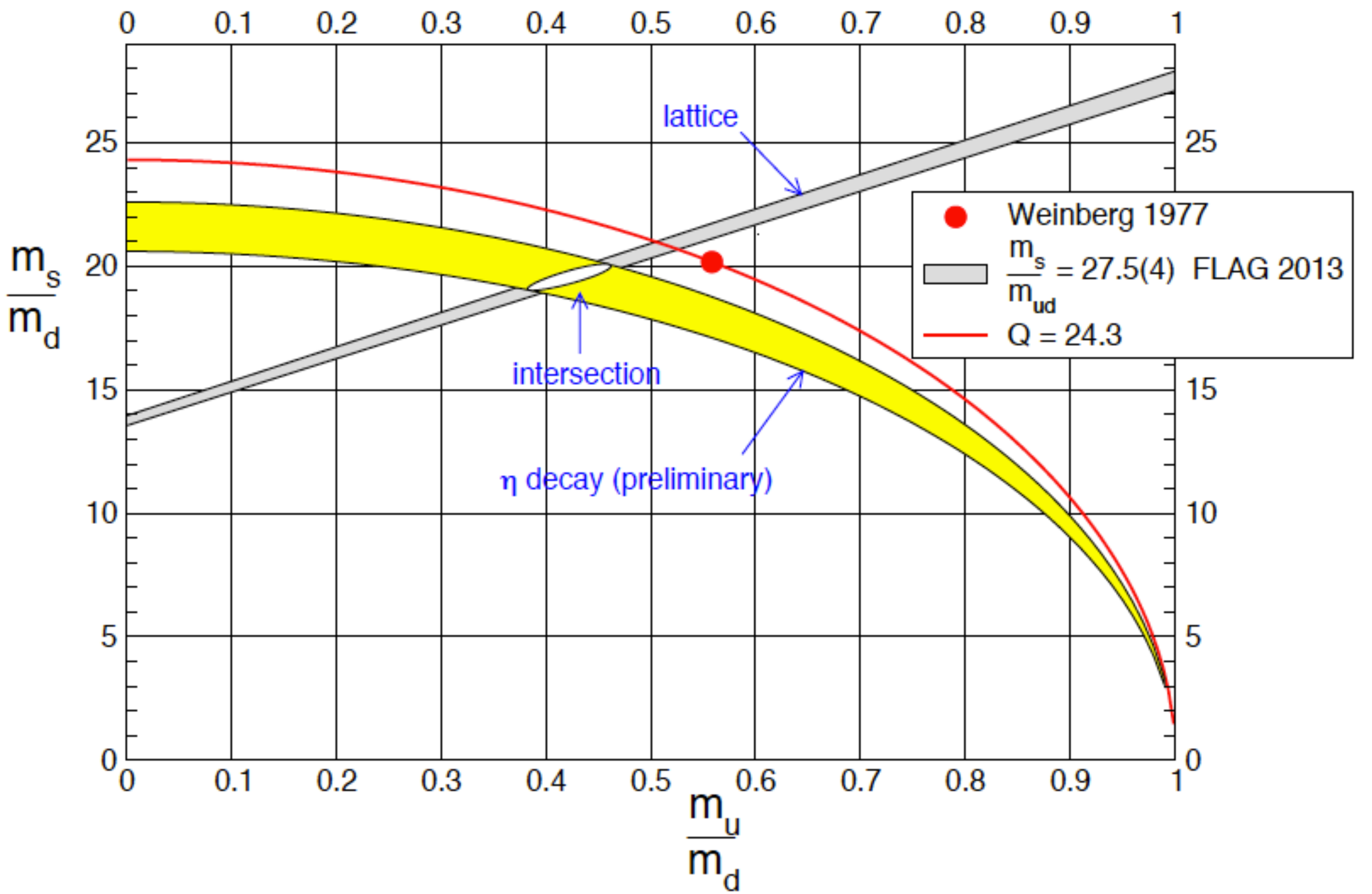
$$\boxed{\delta^{K\pi}_{\mathrm{SU(2)}} = (2.9 \pm 0.4)\%}$$

Knecht et al.'99, Kastner&Neufeld'08

Inputs from Dashen's corrections
Anantharayan&Moussalam'04

Q and Leutwyler's Ellipse

H. Leutwyler



Isospin breaking corrections

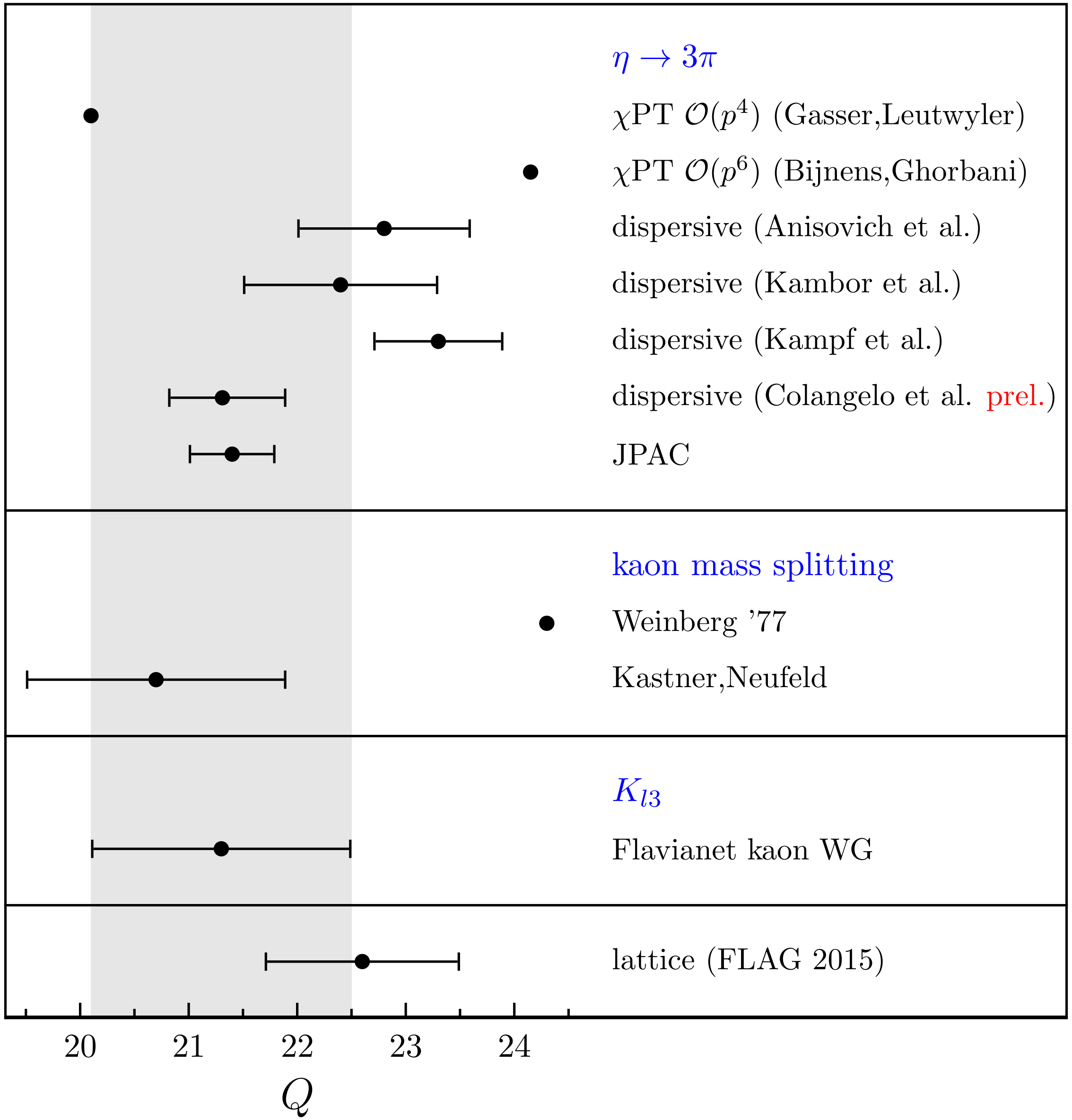
- Other way round: use the difference between K^+ and K^0 ⇨ Q

$$\Delta^{K}_{SU(2)} = \frac{\Gamma_{K^+_{\ell 3}}}{\Gamma_{K^0_{\ell 3}}} \cdot \frac{I^{K^0\ell}}{I^{K^+\ell}} \left(\frac{M_{K^0}}{M_{K^+}}\right)^5 - \frac{1}{2} - \left[\Delta^{K^+\ell}_{\mathrm{EM}} - \Delta^{K^0\ell}_{\mathrm{EM}}\right] \longrightarrow (2.7 \pm 0.4)\ \%$$

~ 0.15% from TH

- $\delta^{K\pi}_{\mathrm{SU(2)}}$ driven by the value of Q

Isospin breaking corrections



Isospin breaking corrections

- Other way round: use the difference between K^+ and K^0 ⇨ Q

$$\Delta^K_{SU(2)} = \frac{\Gamma_{K^+_{\ell 3}}}{\Gamma_{K^0_{\ell 3}}} \cdot \frac{I^{K^0\ell}}{I^{K^+\ell}} \left(\frac{M_{K^0}}{M_{K^+}}\right)^5 - \frac{1}{2} - \left[\Delta^{K^+\ell}_{\mathrm{EM}} - \Delta^{K^0\ell}_{\mathrm{EM}}\right] \longrightarrow (2.7 \pm 0.4)\ \%$$

~ 0.15% from TH

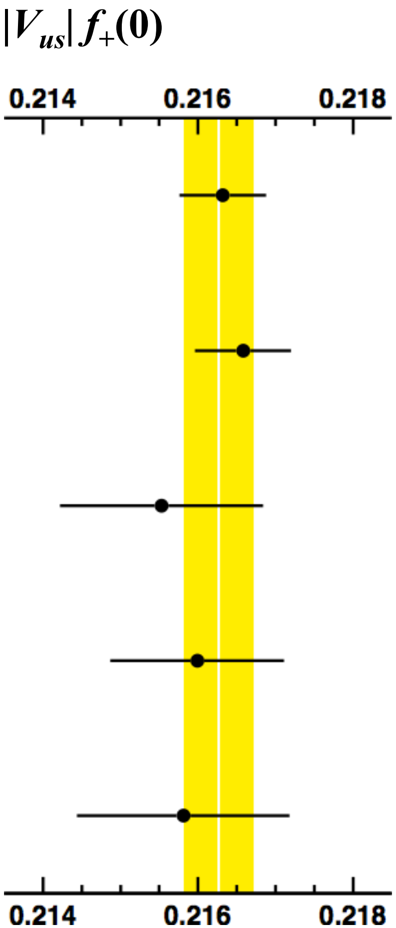
- $\delta^{K\pi}_{\mathrm{SU(2)}}$ driven by the value of Q

- At the moment the uncertainties are large ⇨ need improvement
  - On the theory side: we use only one loop ChPT in *FlaviaNet Kaon WG'10*
    ⇨ need to assess chiral higher order corrections, see talk by *J. Bijnens*

    Bijnens&Ghorbani'07

  - On the experimental side: reduce the uncertainties ⇨ *NA62*

$|V_{us}|\, f_+(0)$ from world data: 2010

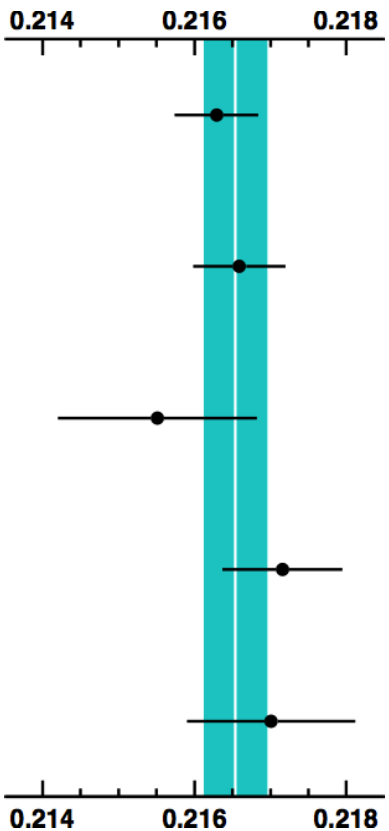


		% err	Approx. contrib. to % err from: BR	τ	Δ	Int
$K_L e3$	**0.2163(6)**	**0.26**	0.09	0.20	0.11	0.06
$K_L \mu 3$	**0.2166(6)**	**0.29**	0.15	0.18	0.11	0.08
$K_S e3$	**0.2155(13)**	**0.61**	0.60	0.03	0.11	0.06
$K^\pm e3$	**0.2160(11)**	**0.52**	0.31	0.09	0.40	0.06
$K^\pm \mu 3$	**0.2158(14)**	**0.63**	0.47	0.08	0.39	0.08

Average: $|V_{us}|\, f_+(0)$ = 0.2163(5) **χ^2/ndf = 0.77/4 (94%)**

Moulson@CKM2014

$|V_{us}| f_+(0)$ from world data: Update



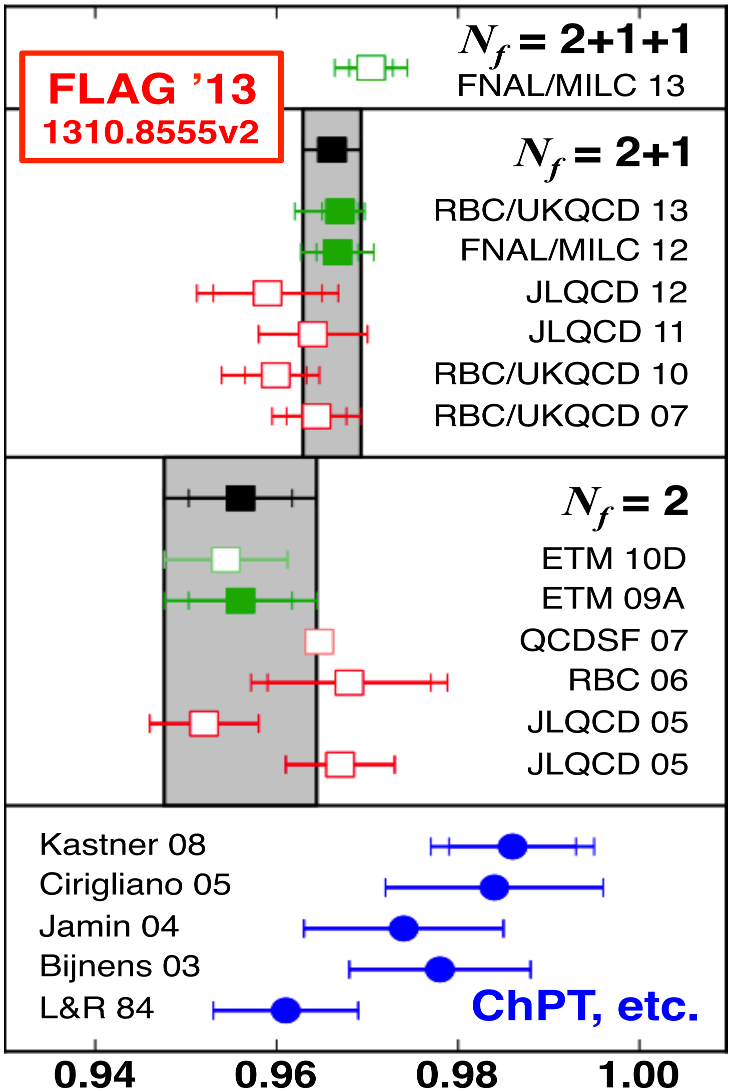
Approx. contrib. to % err from:

		% err	BR	τ	Δ	Int
$K_L e3$	**0.2163(6)**	**0.26**	0.09	0.20	0.11	0.05
$K_L \mu 3$	**0.2166(6)**	**0.28**	0.15	0.18	0.11	0.06
$K_S e3$	**0.2155(13)**	**0.61**	0.60	0.02	0.11	0.05
$K^{\pm} e3$	**0.2172(8)**	**0.36**	0.27	0.06	0.23	0.05
$K^{\pm} \mu 3$	**0.2170(11)**	**0.51**	0.45	0.06	0.23	0.06

Average: $|V_{us}| f_+(0)$ = 0.2165(4) **χ^2/ndf = 1.61/4 (81%)**

3.2 V_{us} from K_{l3}

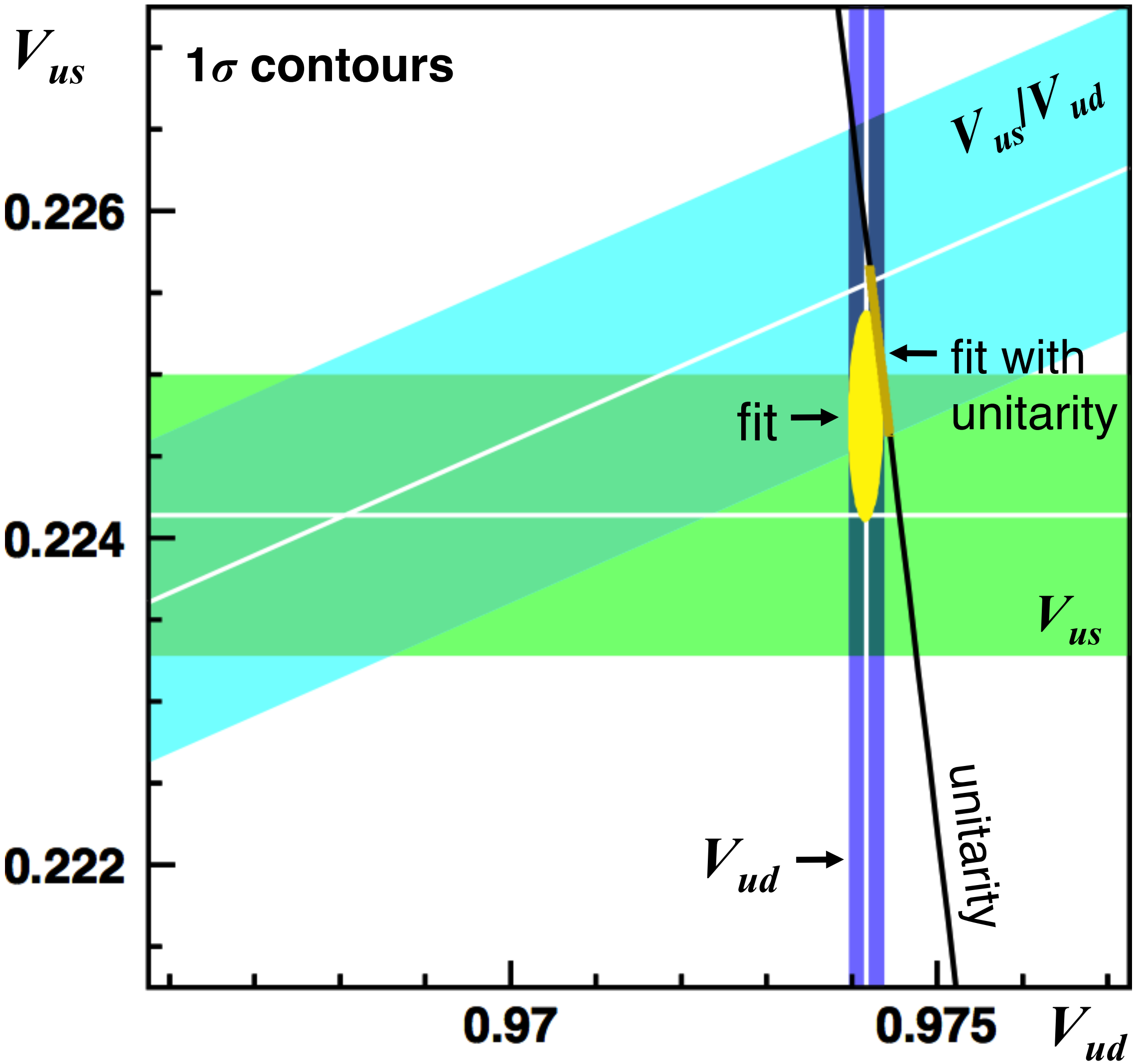
Moulson@CKM2014



Choice of $f_+(0)$		V_{us}
N_f = 2+1	0.9661(32)	**0.2241(9)**
N_f = 2+1+1	0.9704(32)	**0.2232(9)**

3.3 Global fit to V_{us} & V_{ud}

Moulson@CKM2014



V_{ud} = **0.97416(21)**

V_{us} = **0.2248(7)**

χ^2/ndf = 1.16/1 (28.1%)

Δ_{CKM} = **–0.0005(5)**

–1.0σ

See also talk *by S. Descotes-Genon*

3.4 Looking for New Physics with K_{l2} and K_{l3}

- Effective Theory approach: ⇨ see talk by *J. Camelich*

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \dots$$

- Δ_{CKM} a constraining quantity: ⇨ *S. Jaeger*'s talk

3.4 Looking for New Physics with K_{l2} and K_{l3}

	Operator	$K^+ \to \pi^+ \nu\bar{\nu}$	$K_L \to \pi^0 \nu\bar{\nu}$	$K_L \to \pi^0 \ell^+\ell^-$	$K_L \to \ell^+\ell^-$	$K^+ \to \ell^+\nu$	$P_T(K^+ \to \pi^0\mu^+\nu)$	$\Delta_{\rm CKM}$	ϵ'/ϵ	ϵ_K	in MSSM?
$O_{lq}^{(1)}$	$(\bar{D}_L\gamma^\mu S_L)(\bar{L}_L\gamma_\mu L_L)$	✓	✓	✓	hs	–	–	–	–	–	✓
$O_{lq}^{(3)}$	$(\bar{D}_L\gamma^\mu\sigma^i S_L)(\bar{L}_L\gamma_\mu\sigma^i L_L)$	✓	✓	✓	hs	hs	✓	✓	–	–	✓
O_{qe}	$(\bar{D}_L\gamma^\mu S_L)(\bar{l}_R\gamma_\mu l_R)$	–	–	✓	hs	–	–	–	–	–	small
O_{ld}	$(\bar{d}_R\gamma^\mu s_R)(\bar{L}_L\gamma_\mu L_L)$	✓	✓	✓	hs	–	–	–	–	–	small
O_{ed}	$(\bar{d}_R\gamma^\mu s_R)(\bar{l}_R\gamma_\mu l_R)$	–	–	✓	hs	–	–	–	–	–	small
$O_{lq}^\dagger$	$(\bar{u}_R S_L)\cdot(\bar{l}_R L_L)$	–	–	–	–	✓	✓	✓	–	–	tiny
$(O_{lq}^t)^\dagger$	$(\bar{u}_R\sigma_{\mu\nu} S_L)\cdot(\bar{l}_R\sigma^{\mu\nu} L_L)$	–	–	–	–	–	?	?	–	–	tiny
O_{qde}	$(\bar{d}_R S_L)(\bar{L}_L l_R)$	–	–	✓	✓	–	–	–	–	–	tiny
$O_{qde}^\dagger$	$(\bar{D}_L s_R)(\bar{l}_R L_L)$	–	–	✓	✓	✓	✓	✓	–	–	large $\tan\beta$
$O_{\varphi q}^{(1)}$	$(\bar{D}_L\gamma^\mu S_L)(H^\dagger D_\mu H)$	✓	✓	✓	hs	–	–	–	✓	(✓)	✓
$O_{\varphi q}^{(3)}$	$(\bar{D}_L\gamma^\mu\sigma^i S_L)(H^\dagger D_\mu\sigma^i H)$	✓	✓	✓	hs	hs	✓	✓	✓	(✓)	✓
$O_{\varphi d}$	$(\bar{d}_R\gamma^\mu s_R)(H^\dagger D_\mu H)$	✓	✓	✓	hs	–	–	–	✓	(✓)	large $\tan\beta$ (non-MFV)

from: SJ, talk at NA62 Handbook workshop 2009

3.4 Looking for New Physics with K_{l2} and K_{l3}

- Callan-Treiman theorem:

Bernard, Oertel, E.P., Stern'06, '08

$$C = \bar{f}_0(\underset{m_K^2 - m_\pi^2}{\Delta_{K\pi}}) = \frac{F_K}{F_\pi f_+(0)} + \Delta_{CT} = \underbrace{\frac{F_K |V^{us}|}{F_\pi |V^{ud}|} \frac{1}{f_+(0)|V^{us}|} |V^{ud}|}_{\text{Very precisely known from Br(Kl2/}\pi\text{l2), }\Gamma\text{(Ke3) and }|V_{ud}|} r + \Delta_{CT}$$

$$\boxed{B_{\exp} = 1.2446(41)}$$

- In the Standard Model : $\boxed{r = 1}$ $\left(\ln C_{SM} = 0.2141(73)\right)$ $\Delta_{CT} = (-3.5 \pm 8).10^{-3}$

- In presence of new physics, new couplings : $\boxed{r \neq 1}$

NLO value + large error bars in agreement with

Bijnens&Ghorbani'07
Kastner & Neufeld'08

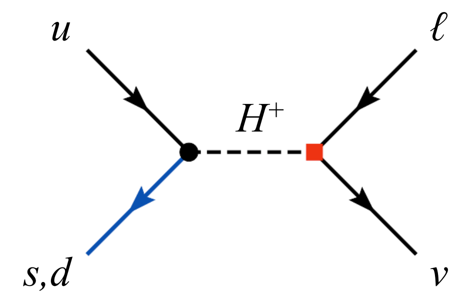
Experiment K_{e3}+$K_{\mu3}$	ln C
NA48'07 ($K_{\mu3}$ alone)	0.144(14)
KLOE'08	0.204(25)
KTeV'10	0.192(12)
NA48 (preliminary)	?

3.4 Looking for New Physics with K_{l2} and K_{l3}

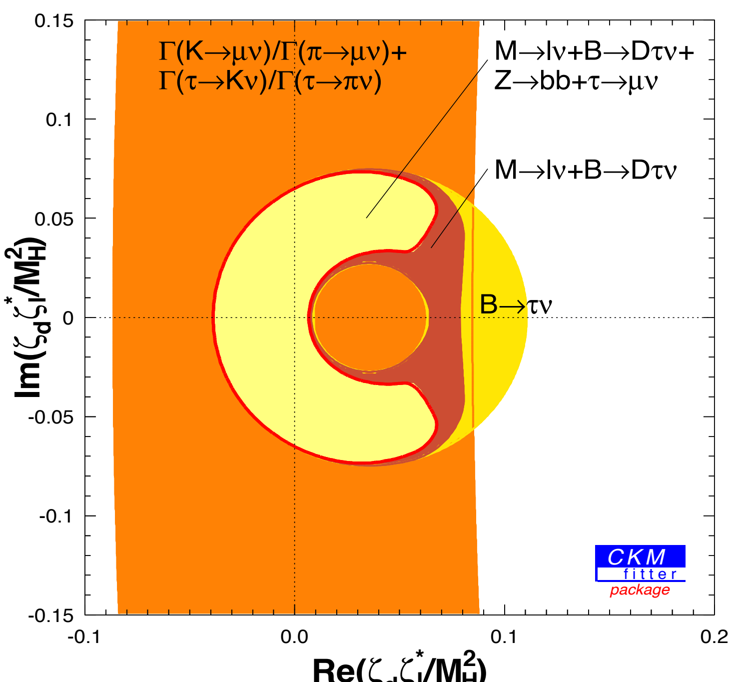
- Ex: Constraints on the aligned 2-Higgs-doublet model:

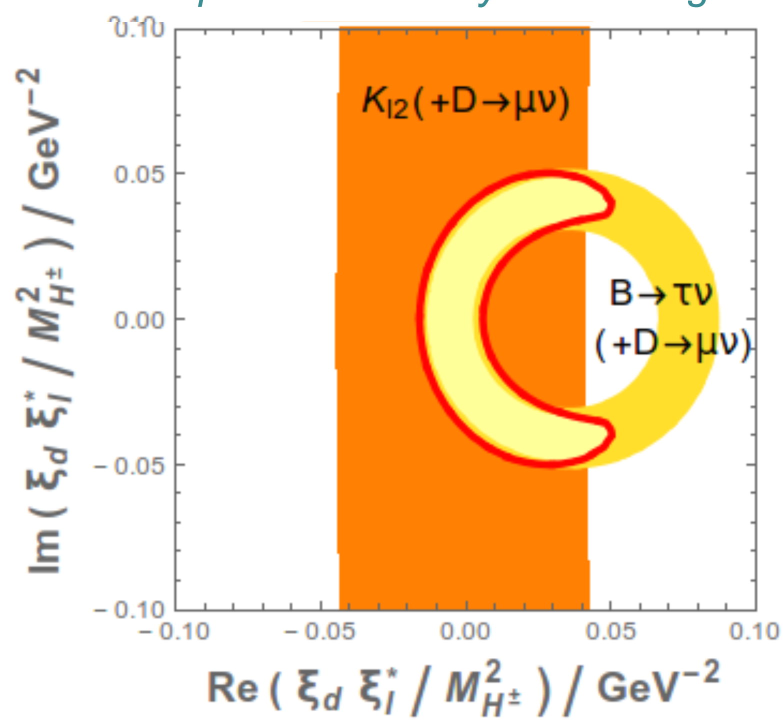
Jung, Pich, Tuzon'10
Pich@HQL'12

$$\mathcal{L}_Y = -\frac{\sqrt{2}}{v} H^+ \left\{ \bar{u} \left[\varsigma_d V_{\rm CKM} M_d \mathcal{P}_R - \varsigma_u M_u^\dagger V_{\rm CKM} \mathcal{P}_L \right] d + \varsigma_l \left(\bar{\nu} M_l \mathcal{P}_R l \right) \right\} + \text{h.c.}$$



Update: Courtesy of M. Jung



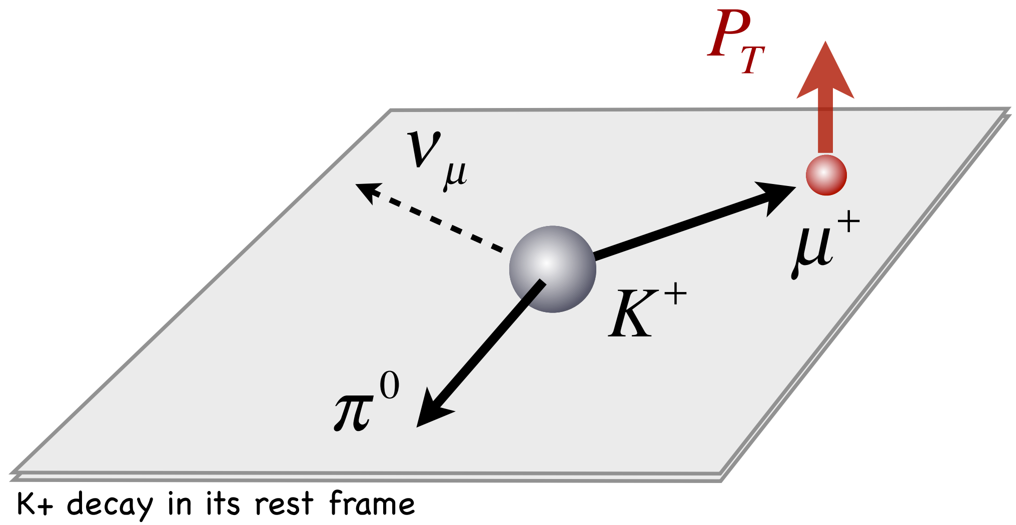


4. T violation in semileptonic kaon decays

4.1 T-odd polarization asymmetry in Kmu3 decays

- Study of direct CP violation possibly due to non-standard mechanisms, with the help of T-odd correlation variables

$$P_T = \frac{\vec{\sigma}_\mu \cdot (\vec{p}_\pi \times \vec{p}_\mu)}{\left|\vec{p}_\pi \times \vec{p}_\mu\right|}$$

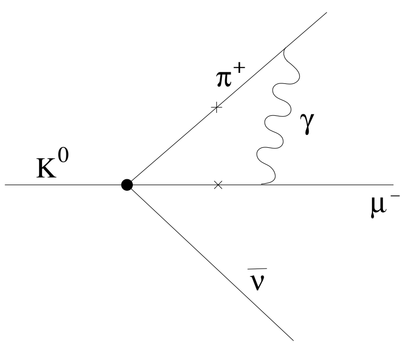


Strauch@Kaon2013

- Violates T in the absence of FSI *D'Ambrosio & Isidori'98*

- Case of $K_L \to \pi^+ \mu^- \nu_l$: FSI *large* $\langle P_T \rangle_{\rm FSI} \sim 10^{-3}$

Okun & Khriplovich'68



- Case of $K^+ \to \pi^0 \mu^+ \nu_l$: FSI does not exceed 10^{-5}

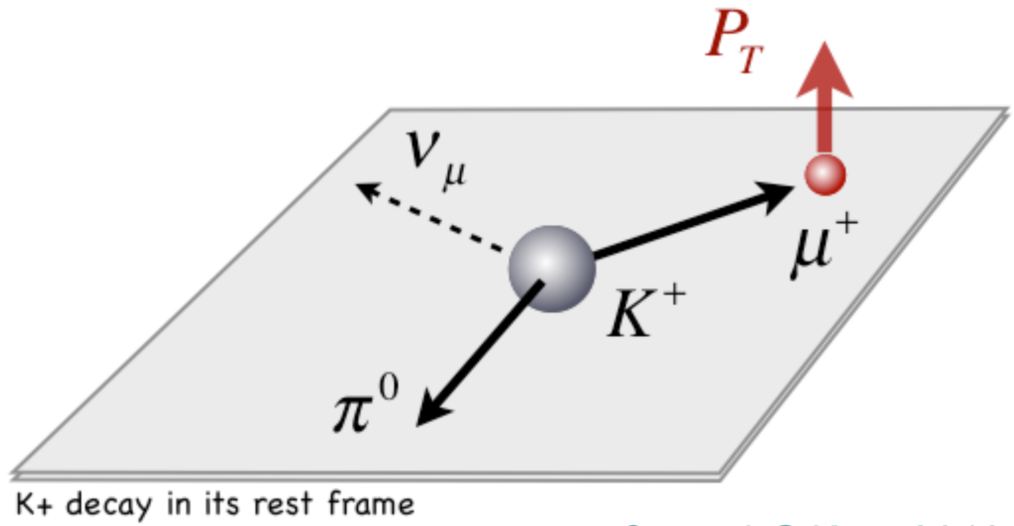
Zhitnitsky'80, Efrosinin et al.'00

⇨ good probe of T violating effect

4.1 T-odd polarization asymmetry in Kmu3 decays

- Study of direct CP violation possibly due to non-standard mechanisms, with the help of T-odd correlation variables

$$\boxed{P_T = \frac{\vec{\sigma}_\mu \cdot (\vec{p}_\pi \times \vec{p}_\mu)}{\left|\vec{p}_\pi \times \vec{p}_\mu\right|}}$$



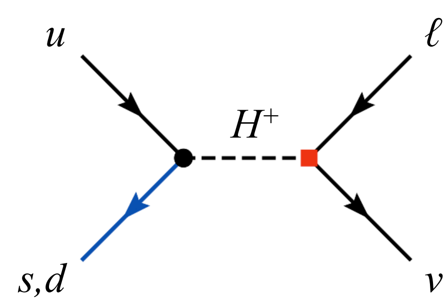
Strauch@Kaon2013

- In the SM: CP-violating contribution to P_T very small : ~10^{-7}

Cheng'83
Bigi & Sanda'00

- Sensitive probe for physics beyond the SM :
  - Multi-Higgs
  - SUSY with squarks mixing
  - SUSY with R-parity breaking
  - Leptoquark model etc…

Kohl'10; Paton et al.'06



López Castro et al.'09

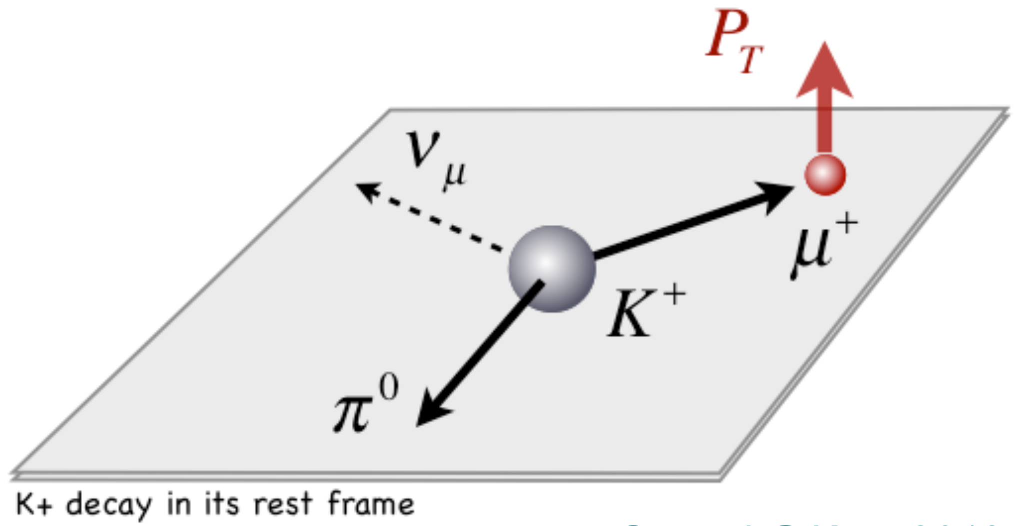
4.1 T-odd polarization asymmetry in Kmu3 decays

Operator	Observable	$K^+ \to \pi^+ \nu\bar{\nu}$	$K_L \to \pi^0 \nu\bar{\nu}$	$K_L \to \pi^0 \ell^+\ell^-$	$K_L \to \ell^+\ell^-$	$K^+ \to \ell^+\nu$	$P_T(K^+ \to \pi^0\mu^+\nu)$	Δ_{CKM}	ϵ'/ϵ	ϵ_K	from: SJ, talk at NA62 Handbook workshop 2009 in MSSM?
$O_{lq}^{(1)}$	$(\bar{D}_L\gamma^\mu S_L)(\bar{L}_L\gamma_\mu L_L)$	✓	✓	✓	hs	–	–	–	–	–	✓
$O_{lq}^{(3)}$	$(\bar{D}_L\gamma^\mu\sigma^i S_L)(\bar{L}_L\gamma_\mu\sigma^i L_L)$	✓	✓	✓	hs	hs	✓	✓	–	–	✓
O_{qe}	$(\bar{D}_L\gamma^\mu S_L)(\bar{l}_R\gamma_\mu l_R)$	–	–	✓	hs	–	–	–	–	–	small
O_{ld}	$(\bar{d}_R\gamma^\mu s_R)(\bar{L}_L\gamma_\mu L_L)$	✓	✓	✓	hs	–	–	–	–	–	small
O_{ed}	$(\bar{d}_R\gamma^\mu s_R)(\bar{l}_R\gamma_\mu l_R)$	–	–	✓	hs	–	–	–	–	–	small
$O_{lq}^{\dagger}$	$(\bar{u}_R S_L)\cdot(\bar{l}_R L_L)$	–	–	–	–	✓	✓	✓	–	–	tiny
$(O_{lq}^{t})^{\dagger}$	$(\bar{u}_R\sigma_{\mu\nu} S_L)\cdot(\bar{l}_R\sigma^{\mu\nu} L_L)$	–	–	–	–	–	?	?	–	–	tiny
O_{qde}	$(\bar{d}_R S_L)(\bar{L}_L l_R)$	–	–	✓	✓	–	–	–	–	–	tiny
$O_{qde}^{\dagger}$	$(\bar{D}_L s_R)(\bar{l}_R L_L)$	–	–	✓	✓	✓	✓	✓	–	–	large $\tan\beta$
$O_{\varphi q}^{(1)}$	$(\bar{D}_L\gamma^\mu S_L)(H^\dagger D_\mu H)$	✓	✓	✓	hs	–	–	–	✓	(✓)	✓
$O_{\varphi q}^{(3)}$	$(\bar{D}_L\gamma^\mu\sigma^i S_L)(H^\dagger D_\mu\sigma^i H)$	✓	✓	✓	hs	hs	✓	✓	✓	(✓)	✓
$O_{\varphi d}$	$(\bar{d}_R\gamma^\mu s_R)(H^\dagger D_\mu H)$	✓	✓	✓	hs	–	–	–	✓	(✓)	large $\tan\beta$ (non-MFV)

4.1 T-odd polarization asymmetry in Kmu3 decays

- Study of direct CP violation possibly due to non-standard mechanisms, with the help of T-odd correlation variables

$$\boxed{P_T = \frac{\vec{\sigma}_\mu \cdot (\vec{p}_\pi \times \vec{p}_\mu)}{\left|\vec{p}_\pi \times \vec{p}_\mu\right|}}$$



Strauch@Kaon2013

- Current experimental bound:

  $|P_T| < 0.0050$ at 90% C.L. *KEK E-246 Abe et al.'06*

- Aim of *TREK* experiment: improvement by a factor of 20
  NA62?

4.2 T-odd asymmetry using K14

Retico'02

- The K_{l4} decay rate:

$$\boxed{d\Gamma = G_F^2 |V_{us}|^2 N(s_\pi, s_l)\, J_5(s_\pi, s_l, \theta_\pi, \theta_l, \phi) ds_\pi ds_l d(\cos\theta_\pi) d(\cos\theta_l) d\phi}$$

$$\begin{aligned} J_5 &= 2(1-z_l)[I_1 + I_2\cos 2\theta_l + I_3 \sin^2\theta_l \cos 2\phi + I_4 \sin 2\theta_l \cos\phi + I_5 \sin\theta_l \cos\phi + \\ &+ I_6 \cos\theta_l + I_7 \sin\theta_l \sin\phi + I_8 \sin 2\theta_l \sin\phi + I_9 \sin^2\theta_l \sin 2\phi]\ , \end{aligned}$$

- Measuring K^+ and K^- :

$$\boxed{\begin{aligned} I_7(K^+_{\ell 4}) + I_7(K^-_{\ell 4}) &= \frac{m_\ell}{m_K} \frac{\lambda^{\frac{1}{2}}(m_K^2, s_\pi, s_\ell)}{(1-z_l)} \gamma \sqrt{s_\ell} \sqrt{s_\pi - 4m_\pi^2} \sin\theta_\pi \\ &\times \mathrm{Re}(HS^*)\ |C_+^V|^2\ \mathrm{Im}\left(\frac{C_-^S}{C_+^V}\right) \end{aligned}}$$

$$\langle \pi^+(p_+)\pi^-(p_-)|\bar{s}\gamma_5 u|K^+(p_K)\rangle = iS \qquad [\gamma = (1-\lambda(m_K^2, s_\pi, s_\ell)/(m_K^2 - s_\ell + s_\pi)^2)^{-\frac{1}{2}}].$$

$$\langle \pi^+(p_+)\pi^-(p_-)|\bar{s}\gamma^\mu u|K^+(p_K)\rangle = -\frac{H}{m_K^3}\epsilon_{\mu\nu\rho\sigma} L^\nu P^\rho Q^\sigma$$

4.2 T-odd asymmetry using Kl4

Retico'02

- The K_{l4} decay rate:

$$\boxed{d\Gamma = G_F^2 |V_{us}|^2 N(s_\pi, s_l)\, J_5(s_\pi, s_l, \theta_\pi, \theta_l, \phi) ds_\pi ds_l d(\cos\theta_\pi) d(\cos\theta_l) d\phi}$$

$$\begin{aligned} J_5 &= 2(1-z_l)[I_1 + I_2 \cos 2\theta_l + I_3 \sin^2\theta_l \cos 2\phi + I_4 \sin 2\theta_l \cos\phi + I_5 \sin\theta_l \cos\phi + \\ &+ I_6 \cos\theta_l + I_7 \sin\theta_l \sin\phi + I_8 \sin 2\theta_l \sin\phi + I_9 \sin^2\theta_l \sin 2\phi]\ , \end{aligned}$$

- Measuring K^+ and K^- :

$$\begin{aligned} I_7(K^+_{\ell 4}) + I_7(K^-_{\ell 4}) &= \frac{m_\ell}{m_K} \frac{\lambda^{\frac{1}{2}}(m_K^2, s_\pi, s_\ell)}{(1-z_l)} \gamma \sqrt{s_\ell} \sqrt{s_\pi - 4m_\pi^2} \sin\theta_\pi \\ &\times \mathrm{Re}(HS^*)\ |C_+^V|^2\ \mathrm{Im}\left(\frac{C_-^S}{C_+^V}\right) \end{aligned}$$

- Suppressed by lepton mass $\Rightarrow$ only possible to study in muon case
- New form factors should be determined from Lattice

5. Conclusion and Outlook

Conclusion and Outlook

- K_{l2}, K_{l3} and K_{l4} decays offer excellent probes of the SM and its extensions
  - On the theory side: the progress in Chiral EFT & Lattice QCD have allowed to reach a very good precision for the SM prediction:

    ⇒ EM and isospin breaking are included

  - On the experimental side: a lot of new measurements in the years 2000 by ISTRA (K+), KLOE, KTeV and NA48 (KL)

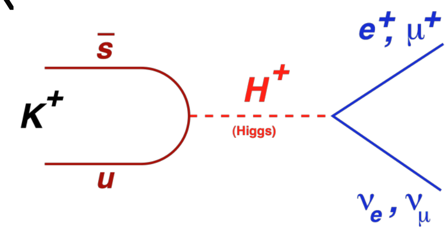
- Tests of lepton and Cabibbo universality at a level that competes with / complements collider physics

- NA62 could allow:
  - Push measurement of Ke2 to 0.1% ⇒ improve R_K
  - Improve the $K^{\pm}_{l3}$ measurements, ⇒ our knowledge of isospin breaking
  - Give a better constrain on T-violating new physics

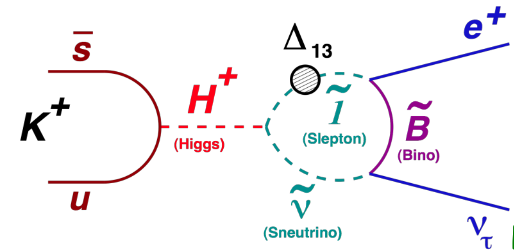
5. Back-up

2.3 Test of New Physics in R_K

- R_K sensitive to *lepton flavour violating effects*, $\Delta R/R \approx O(1\%)$



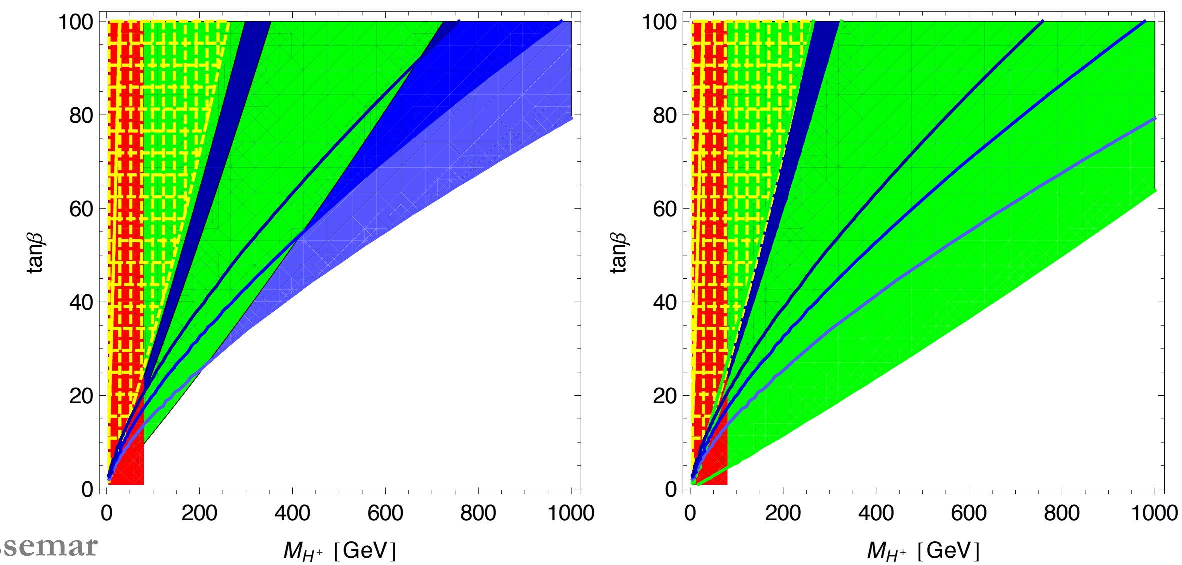
- 2HDM – tree level: ➡ additional contribution due to charged Higgs, does not contribute to R_K



- Possibility to constrain LFV at one loop in MSSM

  Masiero, Paradisi, Petronzio'06,'08

- Update and extension by *Girrbach & Nierste'12* ➡ consider other constraints



3.3 Different recent analyses

1. *Schneider, Kubis, Ditsche 2011*: 2-loop NREFT approach
   - allows investigation of isospin-violating corrections
   - relations between charged and neutral Dalitz plots

2. *Kampf, Knecht, Novotny, Zdrahal 2011*: Analytic dispersive approach
   - Amplitudes involve 6 parameters (subtraction constants)
   - Fit to Dalitz plot distribution (KLOE 2008: $\eta \to \pi^+\pi^-\pi^0$)
   - Predict Dalitz plot parameter α (neutral decay mode)
   - Match to absorptive part of NNLO chiral amplitude where differences between NLO and NNLO are small ⇨ R (Q)

   Problem: do not reproduce the Adler's zero

3.3 Different recent analyses

3. *Guo et al. 2015: JPAC* analysis, Khuri Treiman equations solved numerically using Pasquier inversion techniques
   - Madrid/Cracow ππ phase shifts, 3 subtraction constants
   - Fit experimental Dalitz plot (*WASA/COSY* 2014: $\eta \to \pi^+\pi^-\pi^0$) $\Rightarrow$ predict Dalitz plot parameter α
   - Match to NLO ChPT near Adler zero $\Rightarrow$ Q

4. *Colangelo, Lanz, Leutwyler, E.P. in progress:* dispersive approach following *Anisovich, Leutwyler*
   - Electromagnetic effects to NLO fully taken into account (*Ditsche, Kubis, Meißner'09*)
   - Dispersive amplitudes: Bern ππ phase shifts, 6 subtraction constants
   - Fit similtanously Charged (*WASA*, *KLOE*) and Neutral Dalitz plots (*MAMI*)
   - Matching to one loop ChPT : Taylor expand the partial wave around s=0