Theoretical predictions of low energy parameters in $K^+ \to \pi^+ \bar{\ell}\ell$ and $K_S \to \pi^0 \bar{\ell}\ell$

– A phenomenological approach –

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Based on a work in preparation

Amplitude analysis

G. D'Ambrosio, G. Isidori and J. Portoles, Phys. Lett. B **423**, 385 (1998)

G. Buchalla, G. D'Ambrosio and G. Isidori, Nucl. Phys. B **672**, 387 (2003)

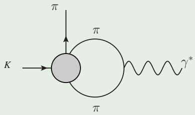
The amplitude $K^+ \to \pi^+ \bar{\ell}\ell$ is dominated by the exchange of a virtual photon

$$\mathcal{A}\left[K^+(k) \to \pi^+(p)\gamma^*(q)\right] = \frac{W_+(z)}{(4\pi)^2}\left[z(k+p)_\mu - (1-r_\pi^2)q_\mu\right]$$

with $r_x \doteq \frac{M_x}{M_K}$ and $z = \frac{q^2}{M_K^2}$, such that

$$\mathcal{A}\left[K(k) \to \pi(p)\ell^+(p_+)\ell^-(p_-)\right] = -\frac{\alpha}{4\pi M_K^2}\, W_+(z)\, (k+p)_\mu\, \bar{u}_\ell(p_-)\gamma^\mu v_\ell(p_+)$$

Dominance of the pion loop



$$W_+(z) = \underbrace{G_F M_K^2(a_+ + b_+ z)}_{\text{polyn. part}} + \underbrace{W_+^{\pi\pi}(z)}_{\pi-\text{loop}}$$

Can we predict a_+ and b_+ ?

Experimentally: $a_+ = -0.587 \pm 0.01$ $b_+ = -0.655 \pm 0.04$

G. D'Ambrosio, G. Ecker, G. Isidori and J. Portoles, JHEP **9808** (1998)
G. D'Ambrosio, G. Isidori and J. Portoles, Phys. Lett. B **423**, 385 (1998)
G. Buchalla, G. D'Ambrosio and G. Isidori, Nucl. Phys. B **672**, 387 (2003)

Reconstruction approach

- Need for an interpolation of form factor W_+ between SD and LD.
- a_+ and b_+ are sufficient to encode the interplay

$$W_+(z) \underset{z\to 0}{\sim} G_F M_K^2 a_+ + \left(G_F M_K^2 b_+ + \frac{3r_\pi^2(\alpha_+ - \beta_+) - \beta_+}{180 r_\pi^6} \right) z$$

$\alpha_+ = -20.6 \cdot 10^{-8}$ and $\beta_+ = -2.6 \cdot 10^{-8}$ obtained from exp. $K \to 3\pi$

Our proposal

- Make the interpolation using the Bardeen-Buras-Gérard framework
- Extract the values of a_+ and b_+ by identification at $z \to 0$

The key quantity here is the scale where SD and LD match

Short Distance information - An estimation of the matching scale

$$\mathcal{H}^{\Delta S=1}_{\text{eff.}} = -\frac{G_F V^*_{us} V_{ud}}{\sqrt{2}} \left[C_-(\mu^2) Q_-(\mu^2) + C_7(\mu^2) Q_7 \right]$$

with

$$\begin{aligned} Q_- &= 4(\bar{s}_L \gamma^\nu u_L)(\bar{u} \gamma_\nu d_L) - 4(\bar{s}_L \gamma^\nu d_L)(\bar{u}_L \gamma_\nu u_L) \\ Q_7 &= 2\alpha(\bar{s}_L \gamma^\nu d_L)(\bar{e} \gamma_\nu e) \end{aligned}$$

- Q_- is dominant in $K^0 \to \pi^+\pi^-$. Since $C_-(\mu^2) \left\langle \pi^+\pi^- \;\middle|\; Q_-(\mu^2) \;\middle|\; K^0 \right\rangle$ is μ^2-independent implies that (RGE)

$$C_-(\mu^2) \sim \left(\ln \mu^2 \right)^{-\frac{4}{9}} \quad ,$$

- RGE also predicts $C_7(\mu^2) \sim \left(\ln \mu^2 \right)^{\frac{5}{9}}$. It shows explicitly the 1 loop correction

$$C_7(\mu^2) \sim C_-(\mu^2) \ln \mu^2 \sim \left(\ln \mu^2 \right)^{1-\frac{4}{9}} \quad .$$

- Then $\left\langle \pi^+ \gamma^* \;\middle|\; Q_-(\mu^2) \;\middle|\; K^+ \right\rangle \sim \frac{1}{3} \ln \frac{\mu^2}{M_\pi M_K}$

  For $a_+|_{\text{exp.}} \approx -0.6$, one has $\mu \approx 0.6$ GeV.

The Bardeen–Buras–Bardeen-Buras-Gérard framework

A. J. Buras, J. -M. Gerard and W. A. Bardeen, Eur. Phys. J. C **74**, 2871 (2014)
J. M. Gerard, Acta Phys. Polon. B **21**, 257 (1990)

Long Distance Prescriptions

- The chiral Lagrangian $\mathcal{O}(p^2)$ is valid up to a scale M^2 Typically: $6 < M < 1$ GeV).
- M^2 is identified to a *cut-off regulator*:

no more dimensional regularisation

- All higher order corrections –including chiral $\mathcal{O}\left(p^4\right)$– are encoded in the quadratic divergences in M^2.

Short Distance Prescriptions

- RGE are valid up to the scale $\mu^2 = M^2$
- The matching appears between the $\ln M^2$ and the quadratic divergences

Application of the prescriptions in a pure χPT context

A. J. Buras, J. -M. Gerard and W. A. Bardeen, Eur. Phys. J. C 74, 2871 (2014)

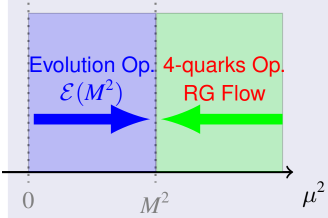
J. M. Gerard, Acta Phys. Polon. B 21, 257 (1990)

- The amplitude is now a function of M^2

$$\mathcal{A} = -\frac{G_F V_{us}^* V_{ud}}{\sqrt{2}} \left\langle \pi^+ \ell^+ \ell^- \;\middle|\; C_-(M^2) Q_-(M^2) + C_7(M^2) Q_7(M^2) \;\middle|\; K^* \right\rangle$$

$$W_+(z) \longmapsto W_+(z, M^2)\ .$$

- χPT is valid as an approximation at $M^2 = 0$.
  Need for an evolution operator in LD (below M^2) equivalent to the RG Flow in SD (above M^2)



$$\boldsymbol{Q_-(M^2) = \mathcal{E}(M^2) Q_-(0)}$$

The *evolution operator* is:

$$\mathcal{E}(M^2) \doteq 1 + \frac{3}{16\pi^2}\left[\frac{M^2}{f_\pi^2} + \frac{M_K^2}{4 f_\pi^2} \ln\left(1 + \frac{M^2}{\tilde{m}}\right)\right]$$

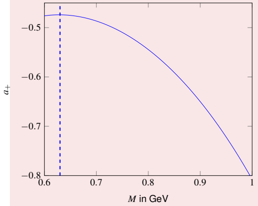
with $\tilde{m} \approx 0.3$ GeV.

The expression of the form factor is now

$$W_+(z, M^2) = \frac{M_K^2 G_F V_{us}^* V_{ud}}{\sqrt{2}} \sqrt{Z_\pi Z_K} \times \left[C_-(M^2)\mathcal{E}(M^2) \left\langle \, \pi^+ \gamma^*(q) \, \mid Q_-(0) \mid K^+ \, \right\rangle + 4\pi C_7(M^2) \right] \, ,$$

and using the cut-off regulator

$$\left\langle \, \pi^+ \gamma^*(q) \, \mid Q_-(0) \mid K^+ \, \right\rangle = \chi\left(\frac{z}{r_\pi^2}\right) + \chi\,(z) - \frac{5}{9} + \frac{1}{3} \ln \frac{M^2}{M_\pi M_K} \, .$$



$$a_+(M^2) = -\frac{V_{us}^* V_{ud}}{\sqrt{2}} \sqrt{Z_\pi Z_K} \left\{ -4\pi C_7(M^2) + C_-(M^2) \left[-\frac{5}{9} + \frac{1}{3} \ln \frac{M^2}{M_\pi M_K} \right] \mathcal{E}(M^2) \right\}$$

The condition $\partial_{M^2} a_+ = 0$ provides $M^2 = 0.63$ GeV

$$a_+\left((0.63 \text{ GeV})^2\right) = -0.5$$

$$a_+\big|_{\text{exp.}} = -0.587 \pm 0.01$$

Inclusion of vector contributions

A. J. Buras, J. -M. Gerard and W. A. Bardeen, Eur. Phys. J. C **74**, 2871 (2014)

J. M. Gerard, Acta Phys. Polon. B **21**, 257 (1990)

- Resonances play a crucial role in the intermediate region for the interpolation.
- The prescription says that $\mathcal{E}$ is now also a function of M_V^2

$$\mathcal{E}(M^2) \longmapsto \mathcal{E}(M^2, M_V^2) = \mathcal{E}(M^2) + \Delta\mathcal{E}(M^2, M_V^2)$$

with

$$\Delta\mathcal{E}(M^2, M_V^2) = \frac{3}{16\pi^2}\left[-\frac{9}{16}\frac{M^2}{f_\pi^2} + \frac{3}{8}\frac{M^2}{f_\pi^2}\frac{M_V^2}{M^2+M_V^2} + \frac{3}{16}\frac{M_V^2}{f_\pi^2}\ln\left(1+\frac{M^2}{M_V^2}\right)\right]$$

Replacing the evolution operator by this new expression the vector contribution is implemented

- Modifying the EM form factor too: $1 \longmapsto 1 + z\frac{M_K^2}{M_V^2}$

This leads to new expressions for W_+, a_+, b_+.

Inclusion of vector contributions

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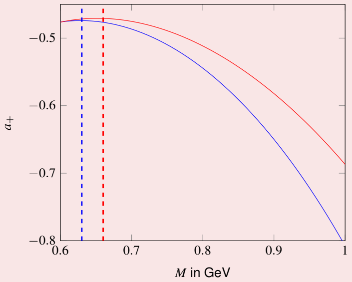

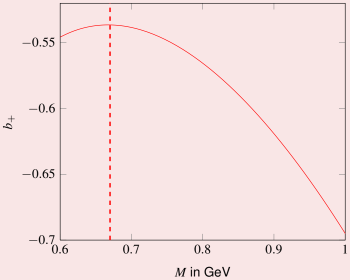

$$a_+ = -0.5$$

$$a_+\big|_{\text{exp.}} = -0.587 \pm 0.01$$

$$b_+ = -0.6$$

$$b_+\big|_{\text{exp.}} = -0.655 \pm 0.04$$

Application to $K_S \to \pi^0 \bar{\ell}\ell$

A. J. Buras, J. -M. Gerard and W. A. Bardeen, Eur. Phys. J. C **74**, 2871 (2014)

J. M. Gerard, Acta Phys. Polon. B **21**, 257 (1990)

G. Buchalla, G. D'Ambrosio and G. Isidori, Nucl. Phys. B **672**, 387 (2003)

Application of the prescriptions

The form factor is given by

$$\begin{aligned} W_S(z, M^2) = &\frac{M_K^2 G_F V_{us}^* V_{ud}}{\sqrt{2}} \sqrt{Z_\pi Z_K} \\ &\times \left[C_-(M^2) \mathcal{E}(M^2, M_V^2) \left\langle \pi^0 \gamma^*(q) \mid Q_-(0) \mid K_S \right\rangle - 4\pi C_7(M^2) \right] \end{aligned}$$

with

$$\left\langle \pi^0 \gamma^*(q) \mid Q_-(0) \mid K_S \right\rangle = 2\chi(z) - \frac{5}{9} + \frac{1}{3} \ln \frac{M^2}{M_K^2}$$

$$a_S = 1.1 \qquad\qquad b_S = 0.5$$

Experimentally: $|a_S| = 1.08^{+0.26}_{-0.21}$

One should have $b_S/a_S = M_K^2/M_V^2 \approx 0.4$, in our case one has 0.5.

Conclusions

- We have shown that applying the Bardeen–Buras–Gérard prescriptions to interpolate SD and LD contributions to the form factors of $K^+ \to \pi^+\bar{\ell}\ell$ and $K_S \to \pi^0\bar{\ell}\ell$ gives reasonable prediction to the characteristic low energy constants a_i, b_i involved in the form factor.
- Our work lies on the fact that we assume the evolution operator coming from the $K \to 2\pi$ and $K \to 3\pi$ description.
- The BBG framework could be completed by a more "explicit" interpolation method for the relevant Green functions, but the result should be the same.
- We expect that the inclusion of higher order resonances could lead to more precise determinations.