Neutron Stars and the Symmetry Energy

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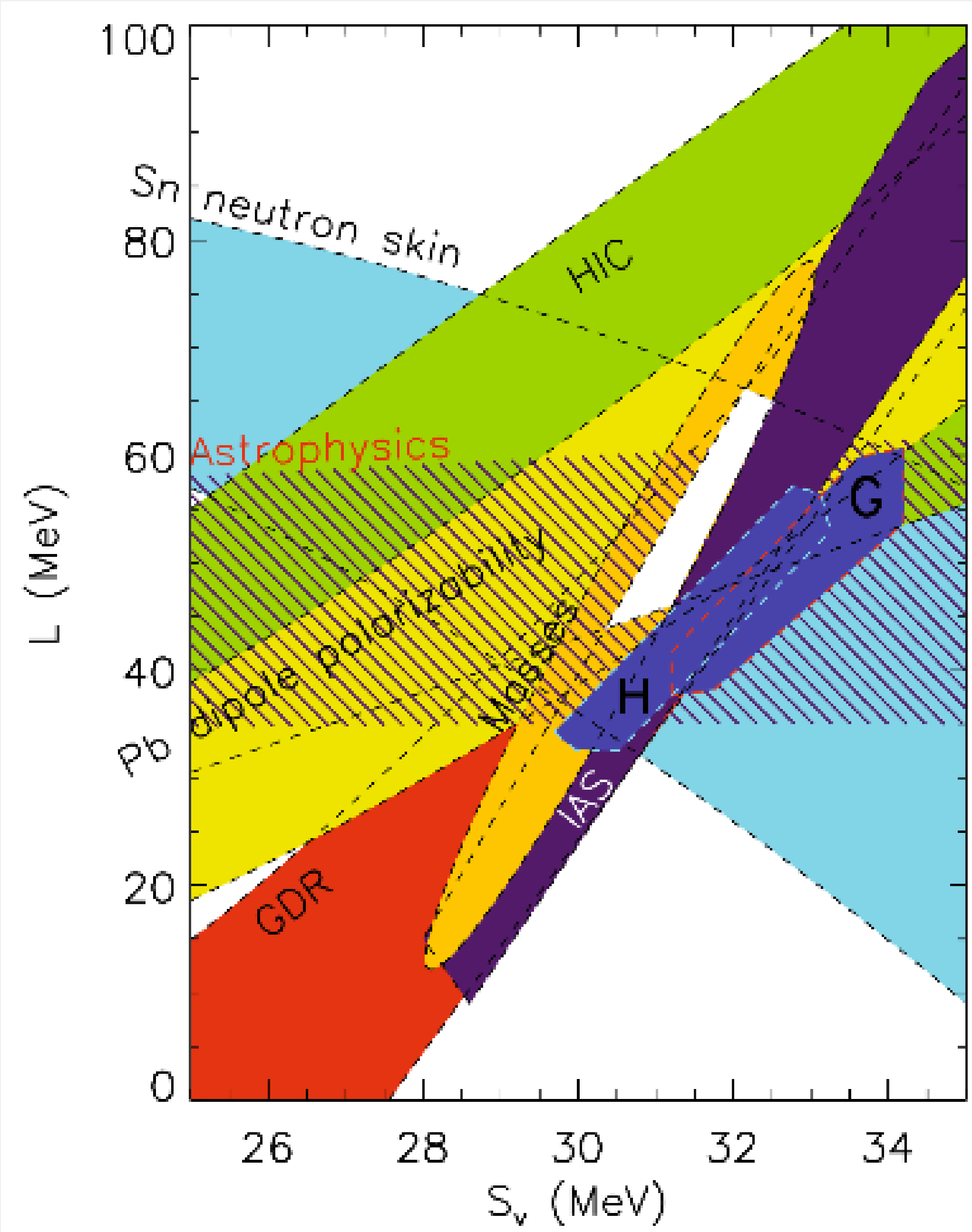
Outline

- Symmetry energy and correlations
- Bayesian inference
- Neutron Stars
- Masses and Radii
- Some (not all) other related observations

Nuclear Symmetry Energy

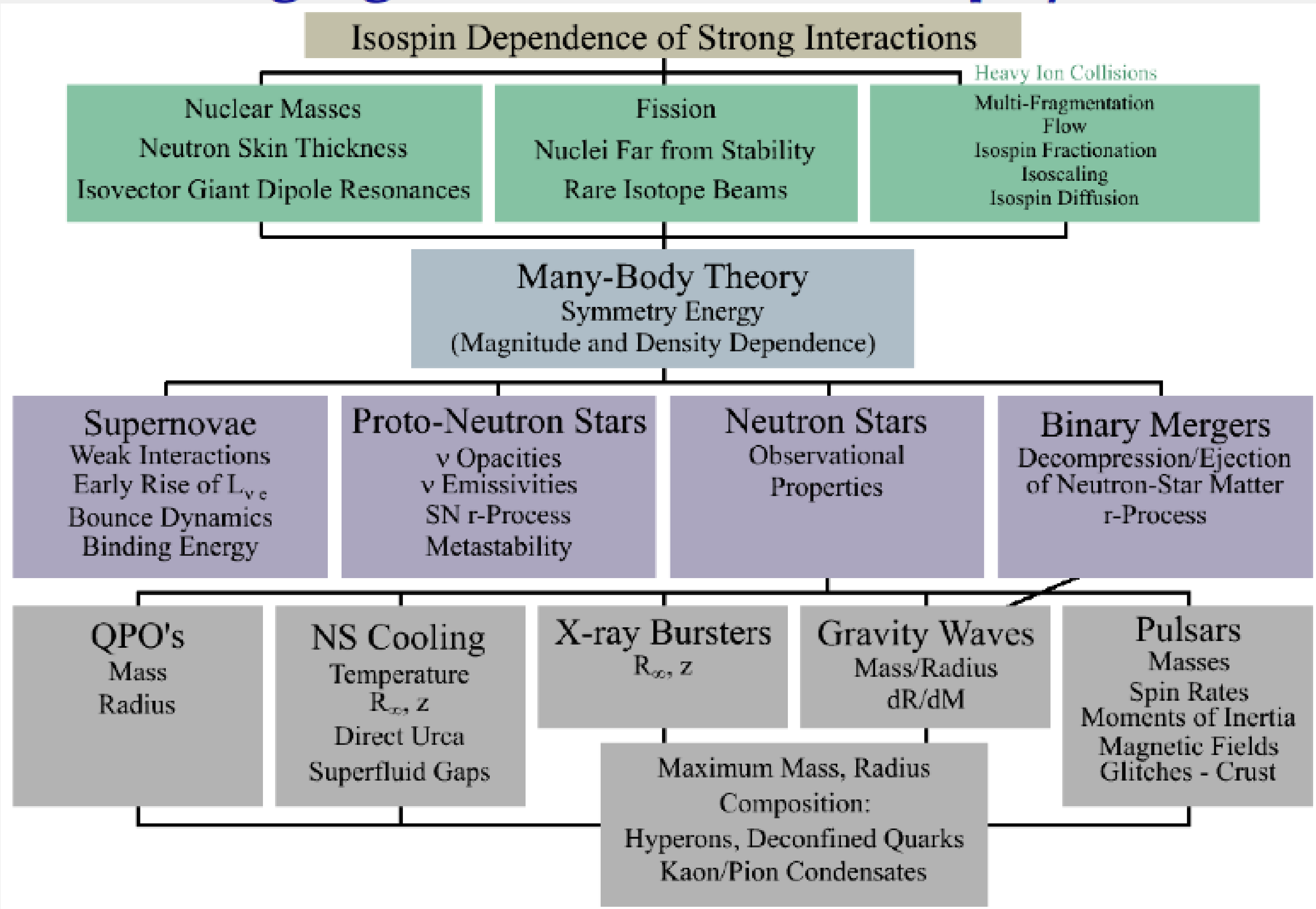


From Lattimer and Steiner (2014)

- Most uncertain part of n-n interaction near saturation
- Combined progress in experiment, theory, and observation
- All of these constraints have uncontrolled systematics
- Need to go beyond the saturation density
- Wide astrophysical impact

[based on Shetty et al. (2007), Trippa et al. (2008), Tsang et al. (2009), Chen et al. (2010), Kortelainen et al. (2010), Tamii et al. (2011), Gandolfi et al. (2012), Hebeler et al. (2012), Steiner et al. (2012), Roca-Maza et al. (2013), Danielewicz et al. (2014)]

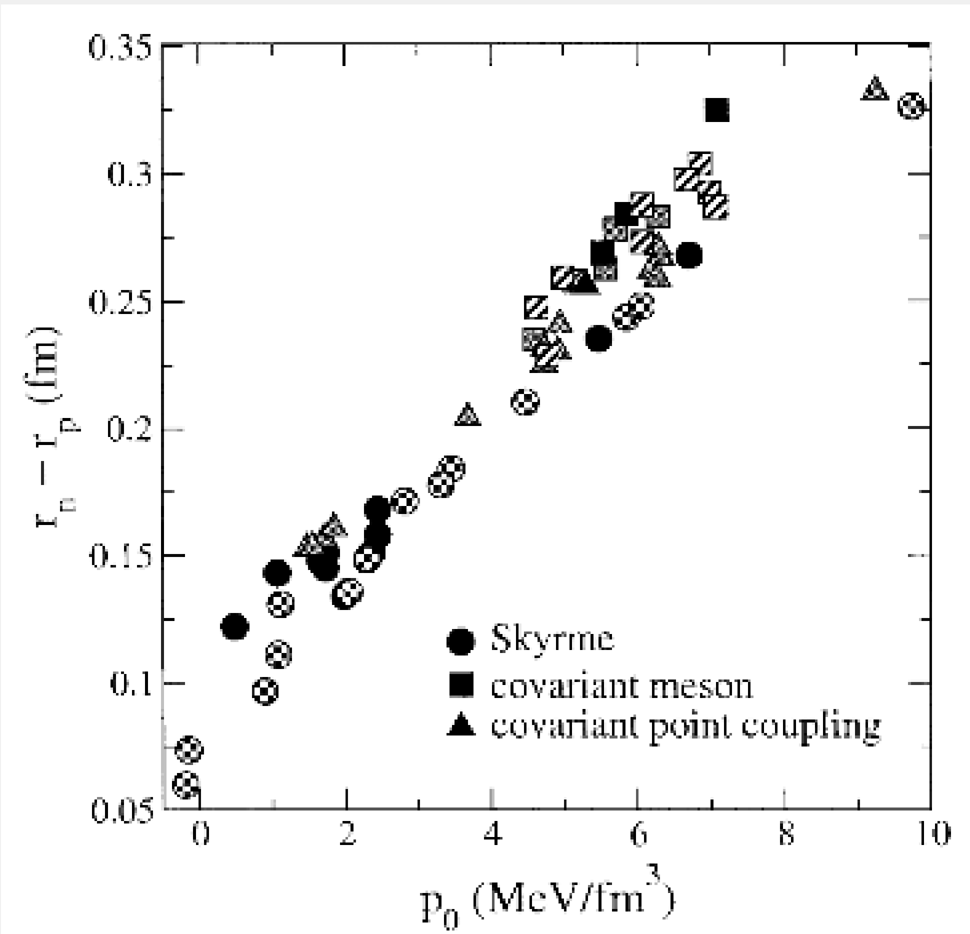
Bridging Nuclear and Astrophysics



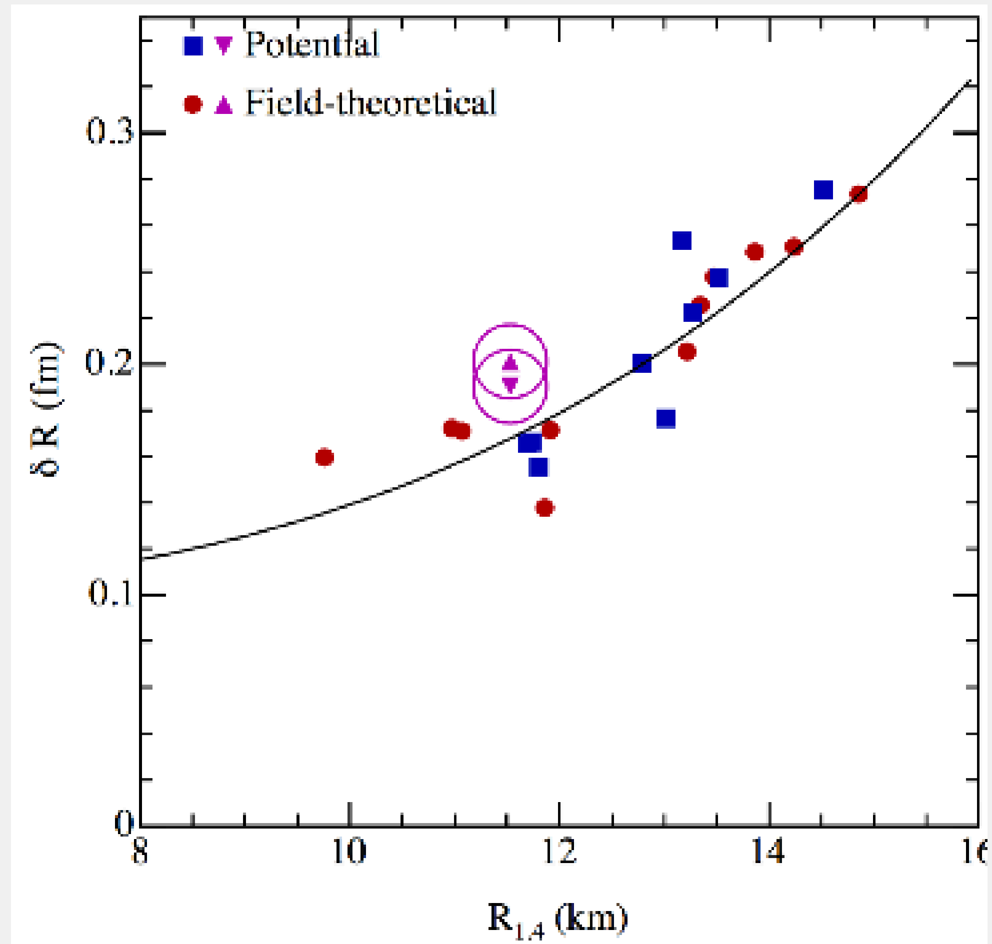
Steiner, et al. (2005)

Correlations and/or "Universal Relations"

- Take advantage of relationships between observables



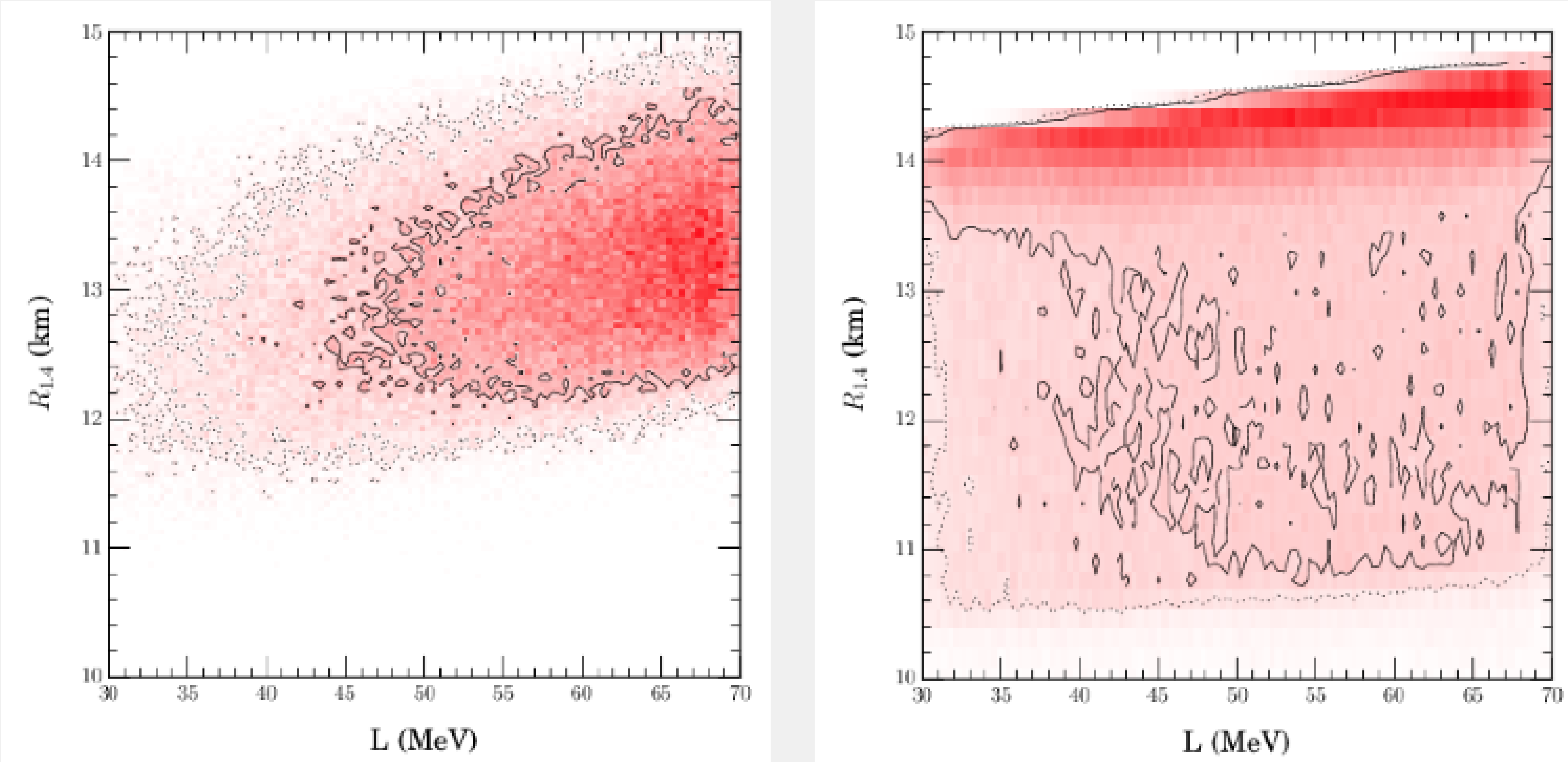
Furnstahl (2002)
(p_0 is related to L)



Steiner et al. (2005)
based on Horowitz and Piekarewicz (2001)

- 2-3 models and a dozen parameterizations (not optimally chosen) for each model
- Plots like this can be dangerous!

How Correlations Fail



line-segments in $\log P - \log \varepsilon$ line-segments in $P - \varepsilon$

Based on Steiner et al. (2016)

- The correlation between L and $R_{1.4}$ is strongly dependent on model assumptions

Bayesian Inference vs. χ^2 fitting

$$\chi^2 = \sum_i \left[\frac{(\text{data})_i - (\text{model})_i}{(\text{err})_i} \right]^2$$

- Maximize the likelihood: $\mathcal{L} = \exp(-\chi^2/2)$ with respect to model parameters
- Not unique when uncertainties in independent variable are large
- Bayes theorem: $P[\mathcal{M}_i|D] \propto P[D|\mathcal{M}_i]P[\mathcal{M}_i] = \mathcal{L} \times \text{prior}$
- Determine parameters through marginalization, i.e.

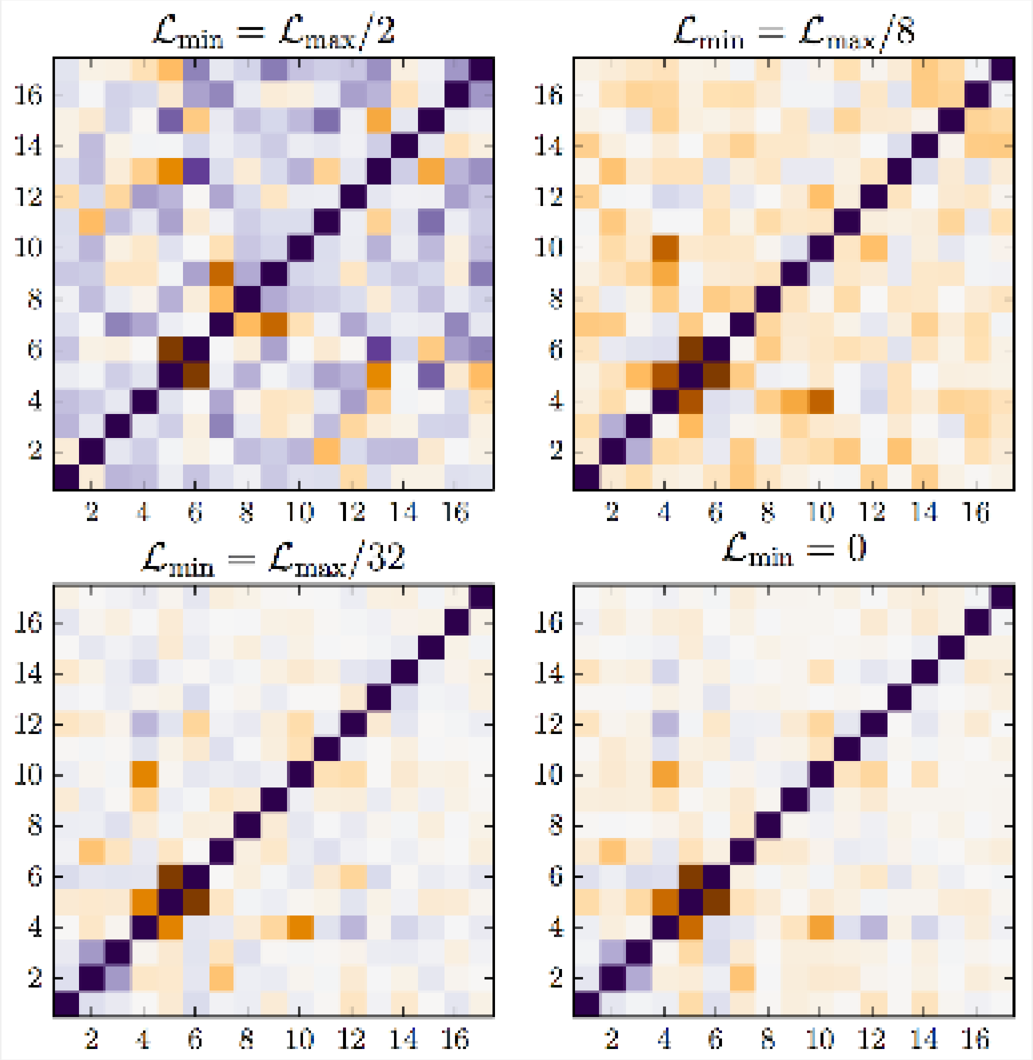
$$P(\mathcal{M}_i^0) = \int \delta(\mathcal{M}_i - \mathcal{M}_i^0) P[D|\mathcal{M}_i] P[\mathcal{M}_i] \, d\mathcal{M}$$

$$= \int \delta(\mathcal{M}_i - \mathcal{M}_i^0) \exp\left(-\chi^2/2\right) P[\mathcal{M}_i] \, d\mathcal{M}$$

- Integrals can be computationally demanding
- So long as the priors are weakly varying, the Gaussian dominates

Gaussian Approximation

- $\mathcal{L} = \exp(-\chi^2/2)$, so if if χ^2 is nearly quadratic near the minimum then the likelihood is Gaussian in the model parameters
- If this approximately holds, the covariance matrix is related to the second derivatives
- This approximation can fail badly
- Important for MC simulation of uncertainties; covariance matrix doesn't contain all the information



Covariance matrices from a fit to neutron star mass and radius data. Upper-left is near best-fit, lower-right is full likelihood.
Steiner (2015)

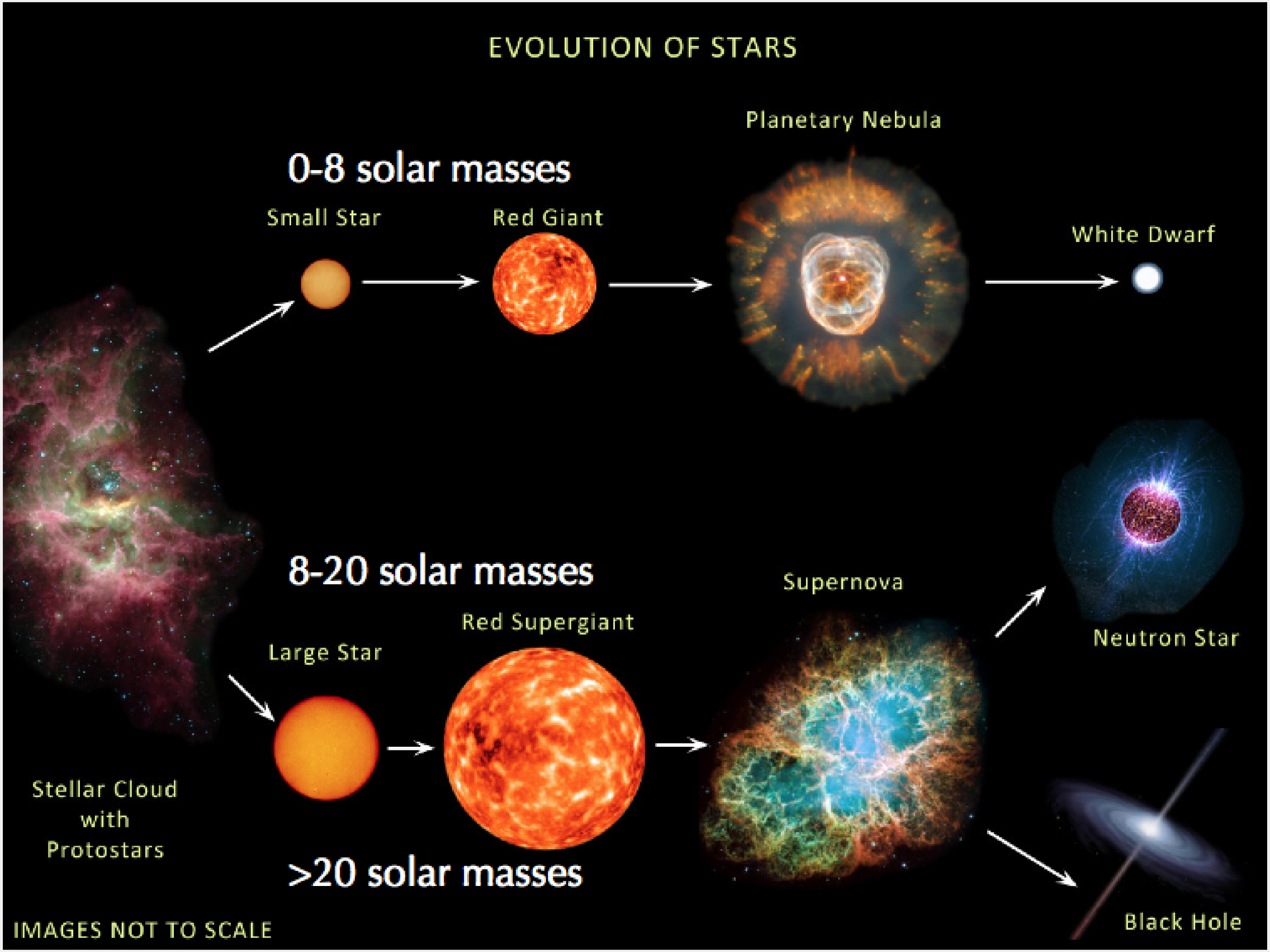
Model Comparison and Bayes Factors

- Frequentist: smallest χ^2 wins (likelihood ratio test)
- Only value of $\mathcal{L}$ at the best fit parameter set is relevant
- Bayesian: compute evidence

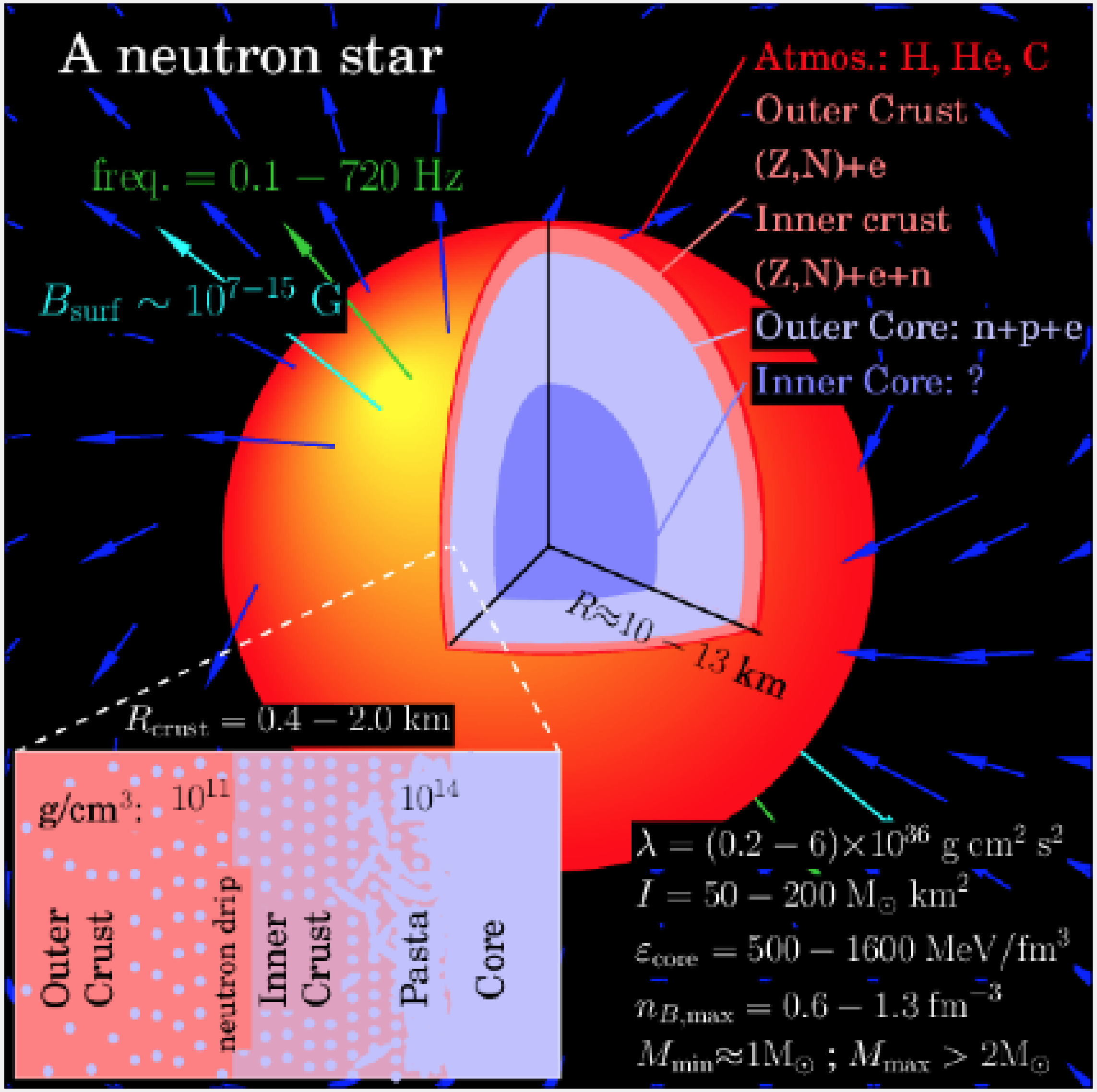
$$E[\mathcal{M}] = \int P[D|\mathcal{M}_i]P[\mathcal{M}_i]\, d\mathcal{M}$$

- Bayes factor = ratio of two evidence integrals
- B > 10 indicates a strong preference for the model in the numerator
  Bayes factors work just like gambling odds
- Bayes factors can automatically disfavor models with too many parameters

Stellar Evolution



Neutron Star Composition

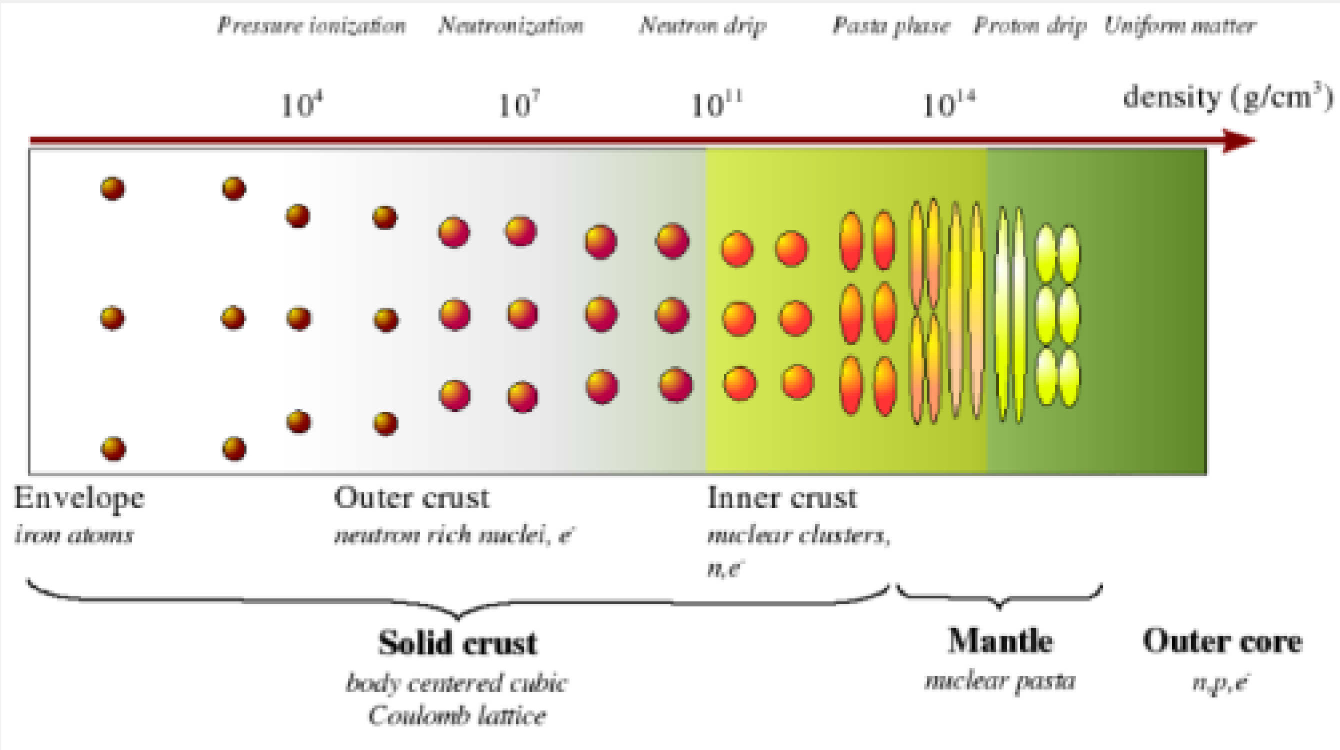


Inspired by D. Page; open source (python)

- Outer crust: of neutron-rich nuclei
  $\mu_n = \mu_p + \mu_e$
- Inner crust: neutron-rich nuclei embedded in a sea of quasi-free superfluid neutrons
- Outer core: fluid of neutrons, protons, and electrons
- Inner core: hyperons? Bose condensates? deconfined quark matter?

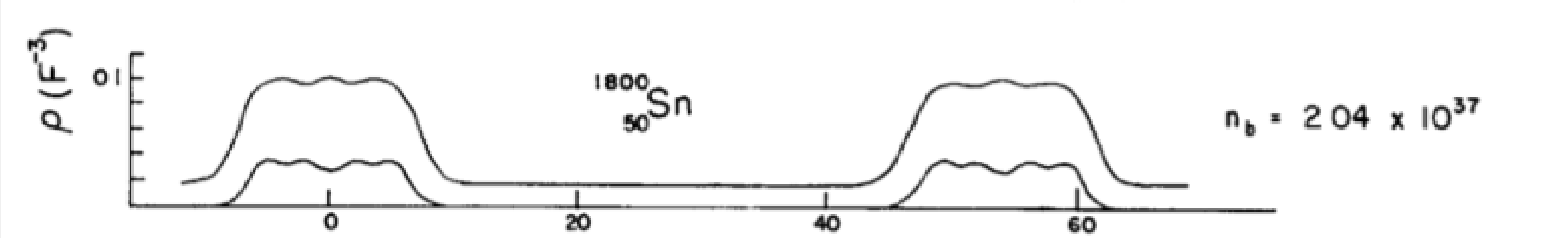
What are the correct degrees of freedom for the effective field theory which describes dense matter?

Structure of Matter in the Neutron Star Crust



Picture from N. Chamel

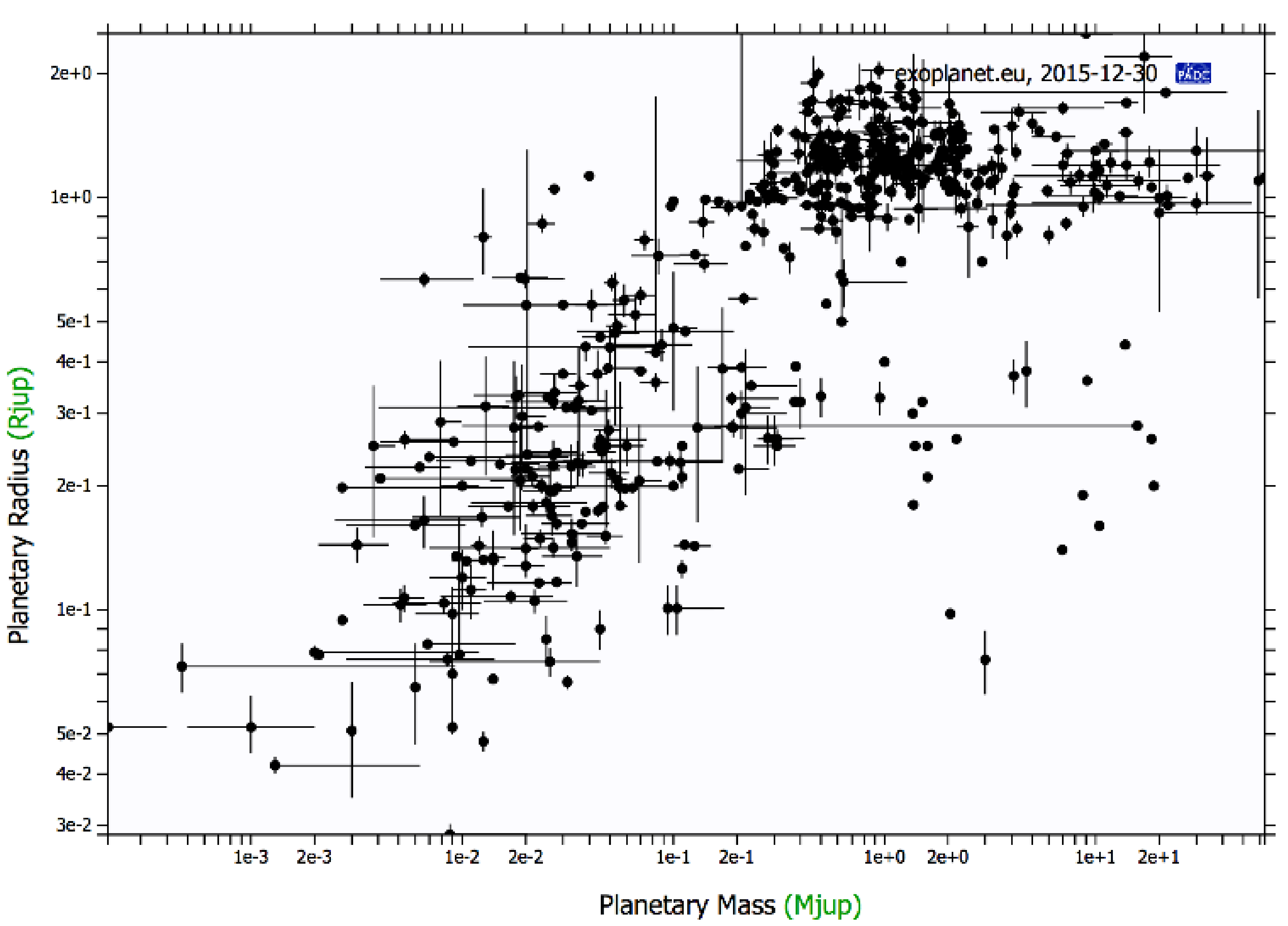
- Neutron-rich nuclei
- Sea of superfluid neutrons
- Coulomb lattice
- Strong interactions
- Pasta



Negele and Vautherin (1973!)

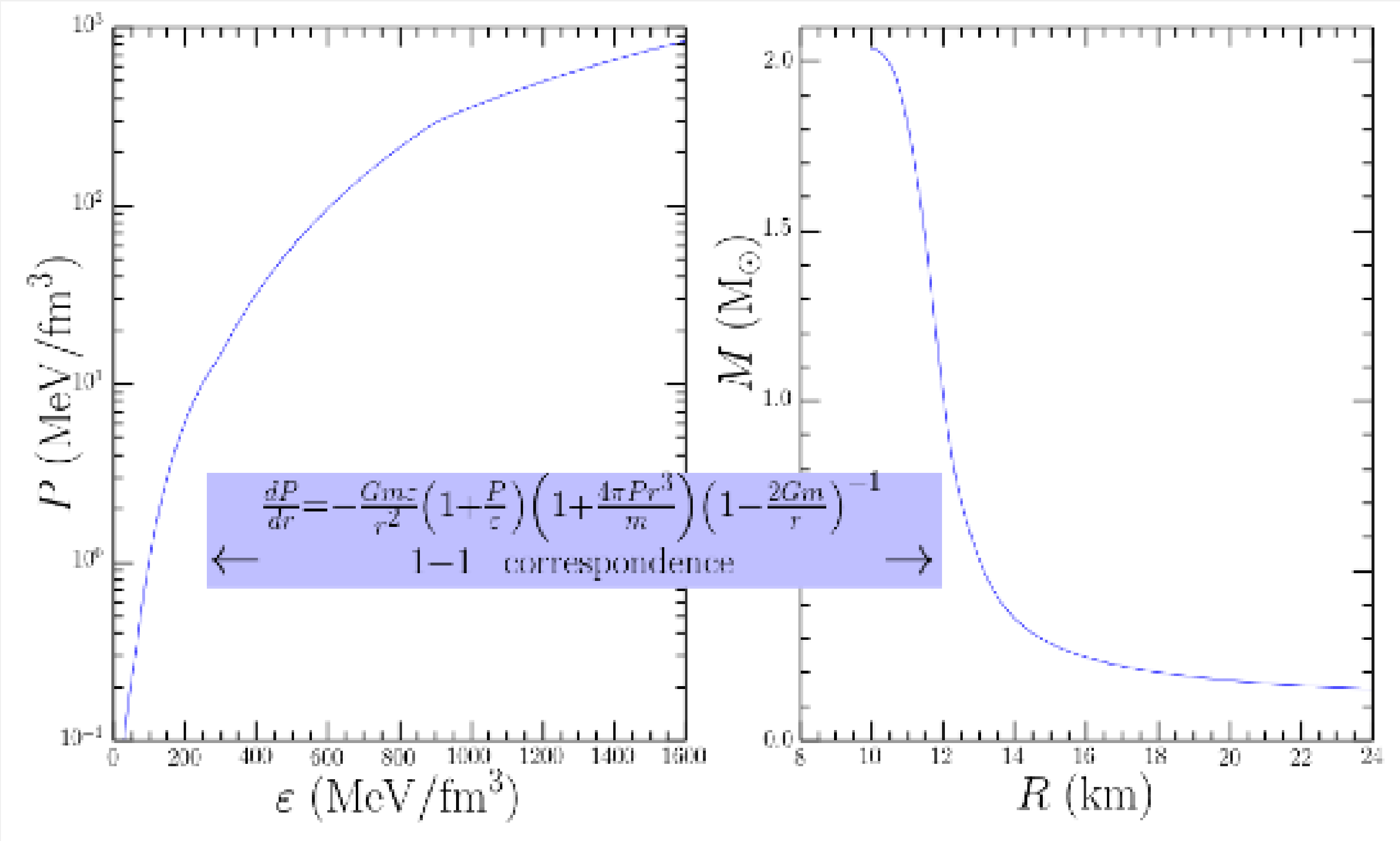
Pretty much the most complicated system you can think of

Planetary masses and radii



Neutron Star Masses and Radii and the EOS

- Neutron stars (to better than 10%) all lie on one universal mass-radius curve
  (Largest correction is rotation)



- Two 2 $M_\odot$ neutron stars
  Demorest et al. (2010), Antoniadis et al. (2013)
- As of 2007, neutron star radii ranged from 8-16 km
  Lattimer and Prakash (2007)
- Now 10-13 km is more likely
  Steiner et al. (2013)

Bayesian Inference for Masses and Radii

- Two-dimensional fitting problem

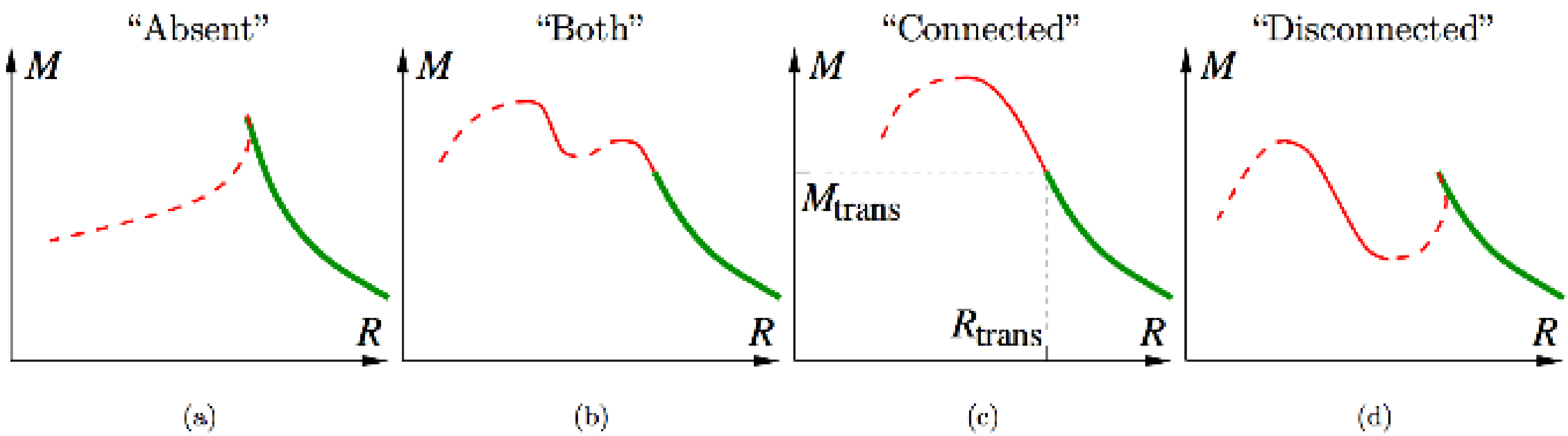


FIG. 2: Four possible topologies of the mass-radius relation for hybrid stars. The thick (green) line is the hadronic branch. Thin solid (red) lines are stable hybrid stars; thin dashed (red) lines are unstable hybrid stars. In (a) the hybrid branch is absent. In (c) there is a connected branch. In (d) there is a disconnected branch. In (b) there are both types of branch. In realistic neutron star $M(R)$ curves, the cusp that occurs in cases (a) and (d) is much smaller and harder to see [13, 14]

Alford et al. (2013)

- Data is poor, so the models are not fully constrained
- Choice of prior distribution introduces additional model dependence
- Typically simulate several models with $\sim 10^5$ points each

Thus we're forced to do more work

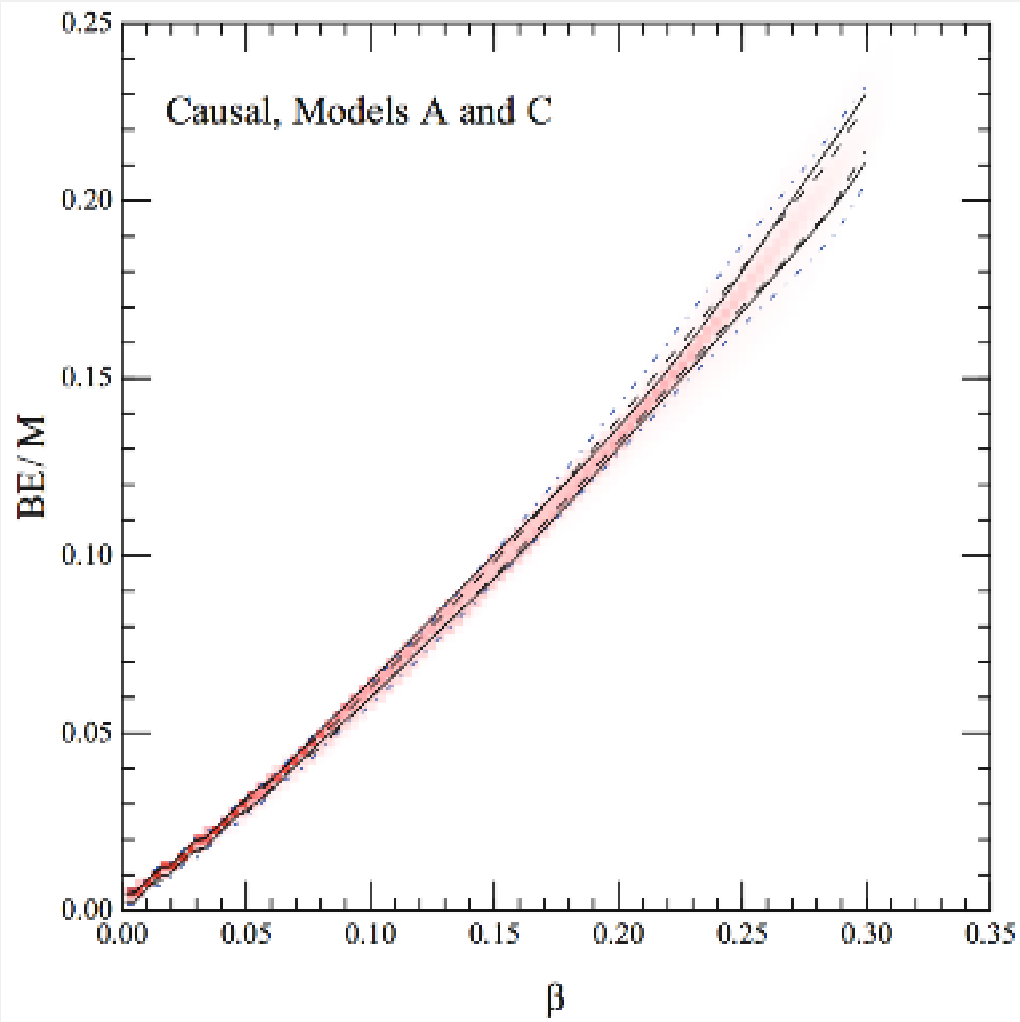
Posterior Confidence Ranges and Evidence Integrals

Model	N_H	Dist.	Comp.	$R_{1.4}$ (km)	I
Base	G13	G13	H	11.11–11.88	$(1.77 \pm 0.09) \times 10^{-8}$
Base	G13	G13	H+He	11.36–12.84	$(4.50 \pm 0.21) \times 10^{-3}$
Base	G13	Alt	H	10.73–11.65	$(1.86 \pm 0.18) \times 10^{-6}$
Base	G13	Alt	H+He	11.45–13.32	$(3.71 \pm 0.21) \times 10^{-1}$
Base	G13	H10	H	10.77–11.71	$(1.23 \pm 0.09) \times 10^{-7}$
Base	G13	H10	H+He	11.36–13.44	$(4.28 \pm 0.35) \times 10^{-3}$
Base	D90	G13	H	10.67–11.51	$(4.65 \pm 0.48) \times 10^{-3}$
Base	D90	G13	H+He	11.31–12.64	$(2.14 \pm 0.19) \times 10^{+2}$
Base	D90	Alt	H	10.85–11.79	$(9.40 \pm 1.22) \times 10^{-3}$
Base	D90	Alt	H+He	11.37–12.61	$(4.06 \pm 0.36) \times 10^{+2}$
Base	D90	H10	H	10.78–11.70	$(4.78 \pm 0.73) \times 10^{-3}$
Base	D90	H10	H+He	11.23–12.62	$(1.57 \pm 0.07) \times 10^{+3}$
Base	H10	G13	H	10.87–11.82	$(1.04 \pm 0.08) \times 10^{+0}$
Base	H10	G13	H+He	11.15–12.38	$(1.84 \pm 0.12) \times 10^{+2}$
Base	H10	Alt	H	11.03–12.07	$(1.39 \pm 0.20) \times 10^{+2}$
Base	H10	Alt	H+He	11.04–12.31	$(1.44 \pm 0.10) \times 10^{+2}$
Base	H10	H10	H	10.78–11.95	$(7.52 \pm 0.65) \times 10^{+1}$
Base	H10	H10	H+He	11.31–12.66	$(5.30 \pm 0.22) \times 10^{+2}$
Exo	G13	G13	H	9.15–10.81	$(7.32 \pm 0.63) \times 10^{-6}$
Exo	G13	G13	H+He	10.52–11.77	$(4.46 \pm 0.38) \times 10^{-2}$
Exo	G13	Alt	H	10.42–11.39	$(1.21 \pm 0.19) \times 10^{-3}$
Exo	G13	Alt	H+He	10.88–12.59	$(7.33 \pm 0.78) \times 10^{-1}$
Exo	G13	H10	H	10.61–11.41	$(2.23 \pm 0.48) \times 10^{-5}$
Exo	G13	H10	H+He	10.76–12.38	$(1.67 \pm 0.16) \times 10^{-2}$
Exo	D90	G13	H	9.39–10.97	$(5.46 \pm 1.74) \times 10^{-1}$
Exo	D90	G13	H+He	10.53–12.45	$(2.29 \pm 0.13) \times 10^{+1}$
Exo	D90	Alt	H	9.86–11.44	$(3.04 \pm 0.42) \times 10^{-1}$
Exo	D90	Alt	H+He	10.90–12.31	$(4.46 \pm 0.22) \times 10^{+1}$
Exo	D90	H10	H	9.60–11.38	$(2.27 \pm 0.50) \times 10^{-1}$
Exo	D90	H10	H+He	10.61–12.28	$(2.59 \pm 0.15) \times 10^{+1}$
Exo	H10	G13	H	9.87–11.49	$(5.15 \pm 0.51) \times 10^{+0}$
Exo	H10	G13	H+He	10.60–11.99	$(4.67 \pm 0.46) \times 10^{+1}$
Exo	H10	Alt	H	10.45–11.74	$(5.17 \pm 0.64) \times 10^{+1}$
Exo	H10	Alt	H+He	10.53–11.81	$(7.49 \pm 0.75) \times 10^{+1}$
Exo	H10	H10	H	10.42–11.72	$(2.83 \pm 0.21) \times 10^{+1}$
Exo	H10	H10	H+He	10.74–12.39	$(8.93 \pm 0.47) \times 10^{+1}$

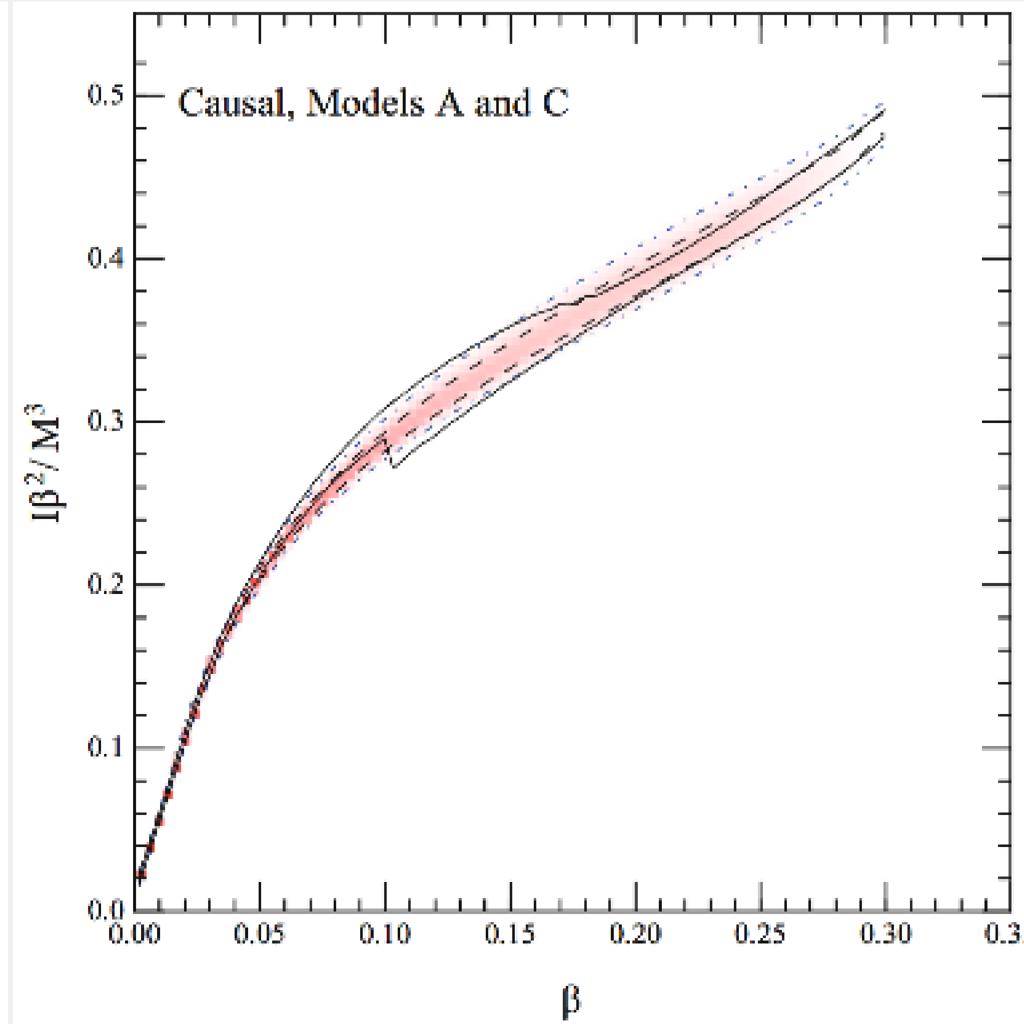
Lattimer and Steiner (2014)

- We try all sorts of combinations, and compare them
- Model-to-model variation provides estimates of systematics
- Not all systematic uncertainties are easily handled this way

Mass Measurements Alone



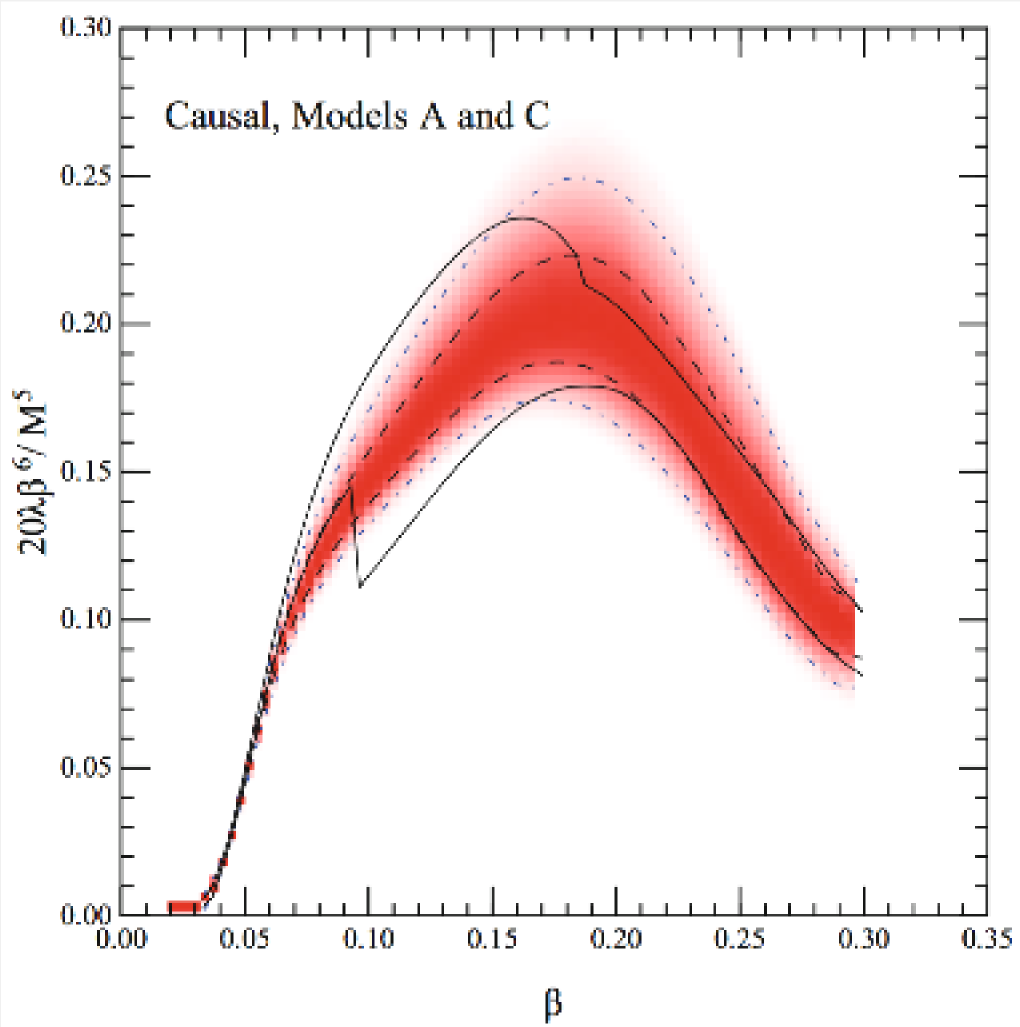
Steiner et al. (2016)



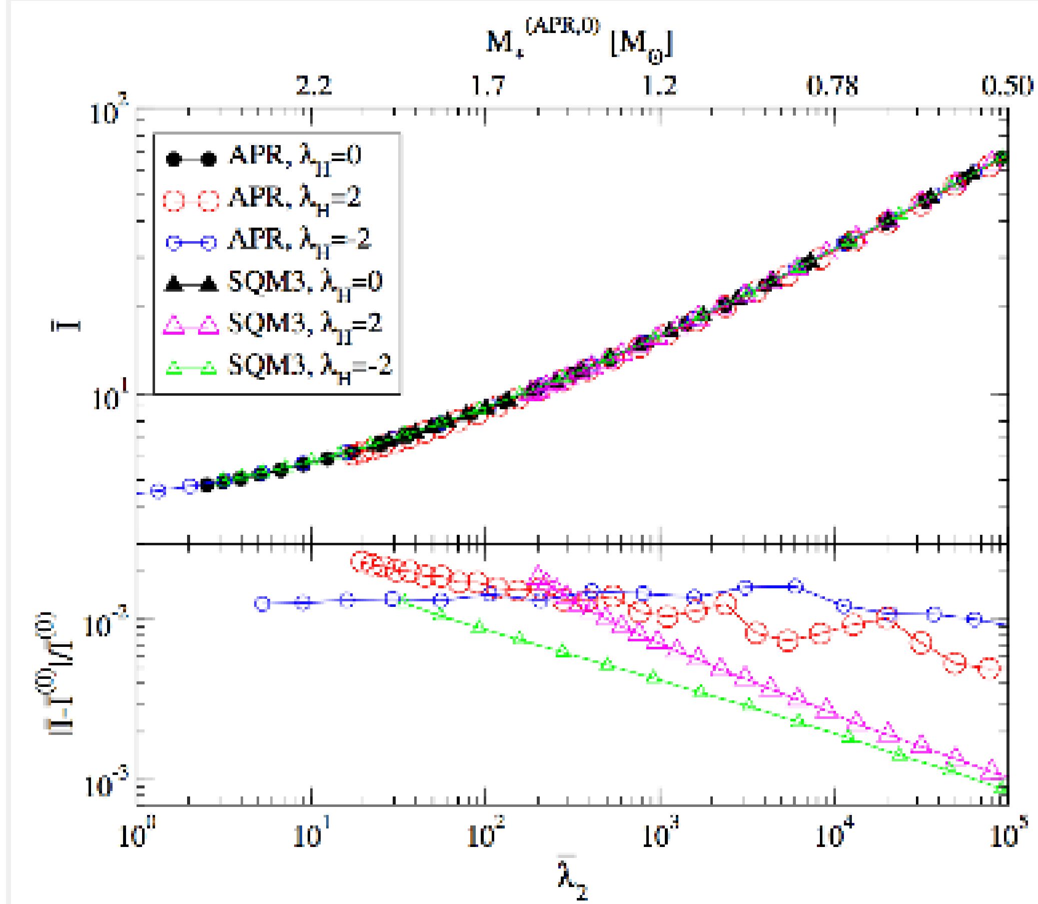
Steiner et al. (2016)

- Compactness strongly correlated with binding energy and moment of inertia

Mass Measurements Alone - II



Steiner et al. (2016)



Yagi and Yunes (2015)

- Compactness weakly correlated with tidal deformability
- Tidal deformability very strongly correlated with moment of inertia

Speed of sound

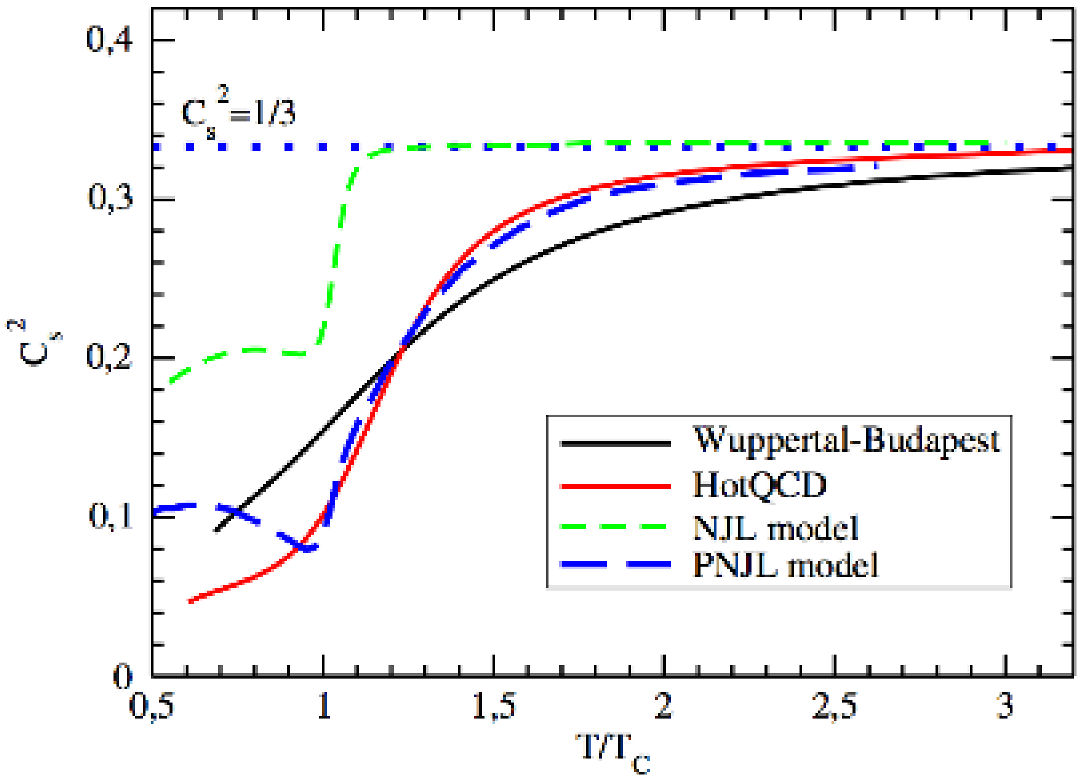


FIG. 4 (color online). Speed of sound squared as a function of T/T_C. The red solid curve corresponds to the HotQCD data, the black solid one to the Wuppertal-Budapest data, the short-dashed one to the NJL model, and the long-dashed one to the Polyakov loop extended NJL model fit to the HotQCD lattice data.

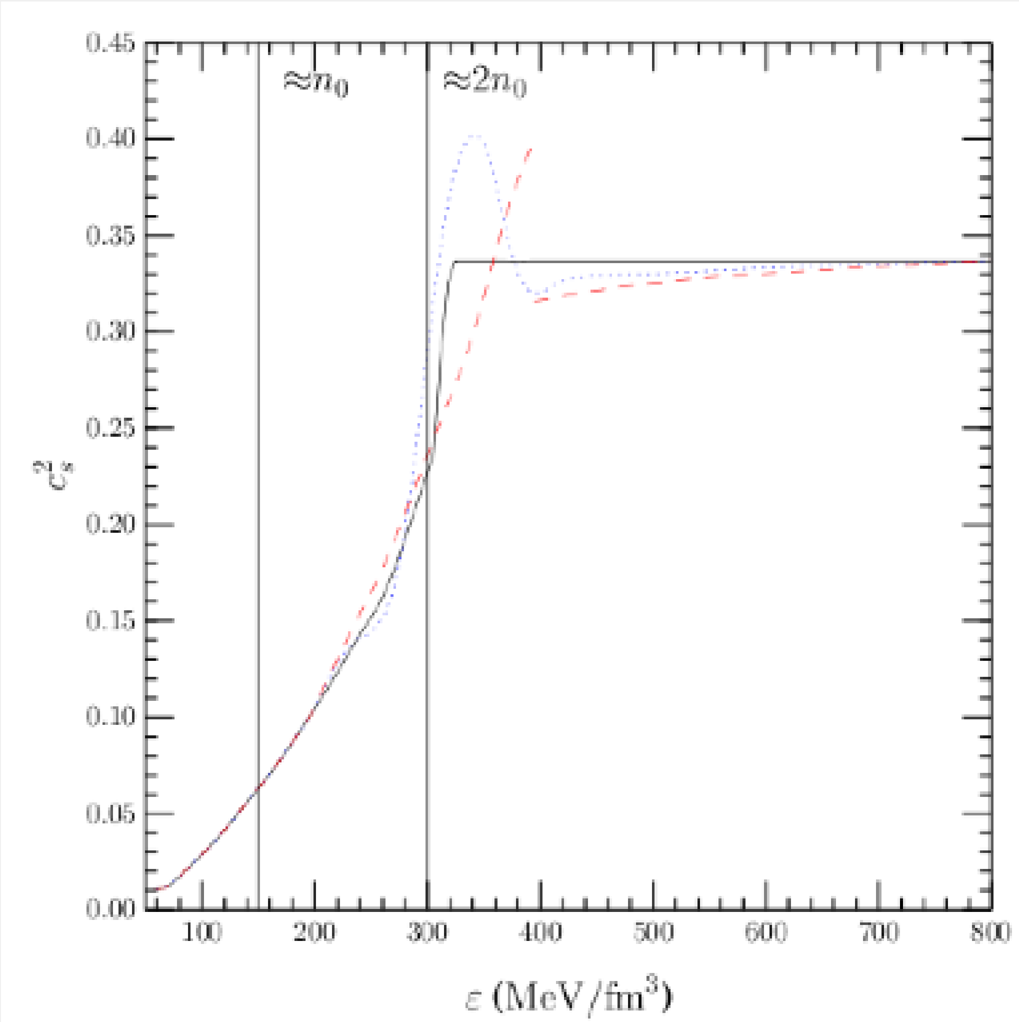
Plumari et al. (2011)

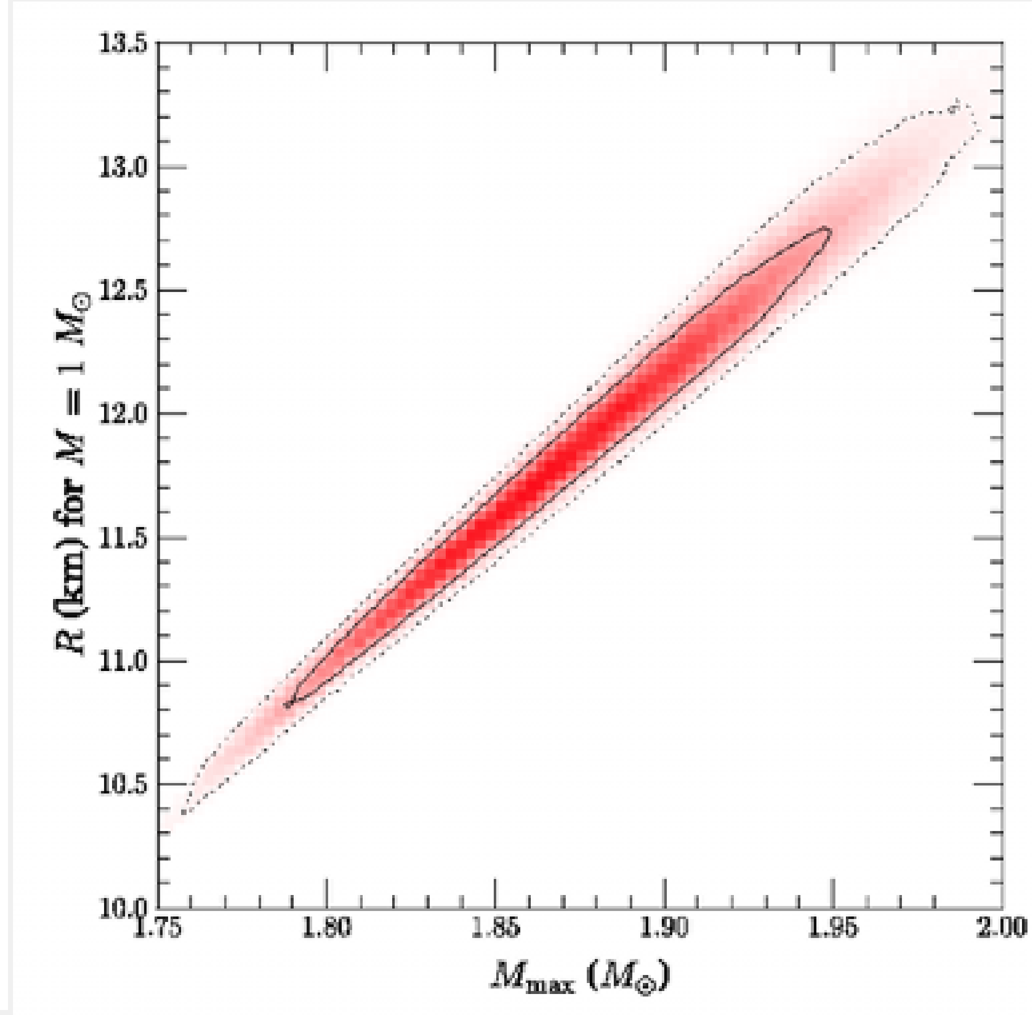
- The speed of sound at zero density and finite temperature: $c_s^2 \to 1/3$ as $T \to \infty$
- What happens at high density and zero temperature?
- Perturbation theory suggests c_s^2 increases to 1/3 from below
  Kurkela et al. (2010)
- $c_s^2 \approx 1/12$ in neutron matter at the saturation density
- Is $c_s^2 > 1/3$ anywhere in the universe?

A Hypothesis

- Remember $c_s^2 = dP/d\epsilon$
- Ok, let's assume that $c_s^2 < 1/3$
- This limits the pressure, thus also limits the maximum mass
- Can we produce a 2 $M_\odot$ neutron star?

Assume $c_s^2 < 1/3$ everywhere

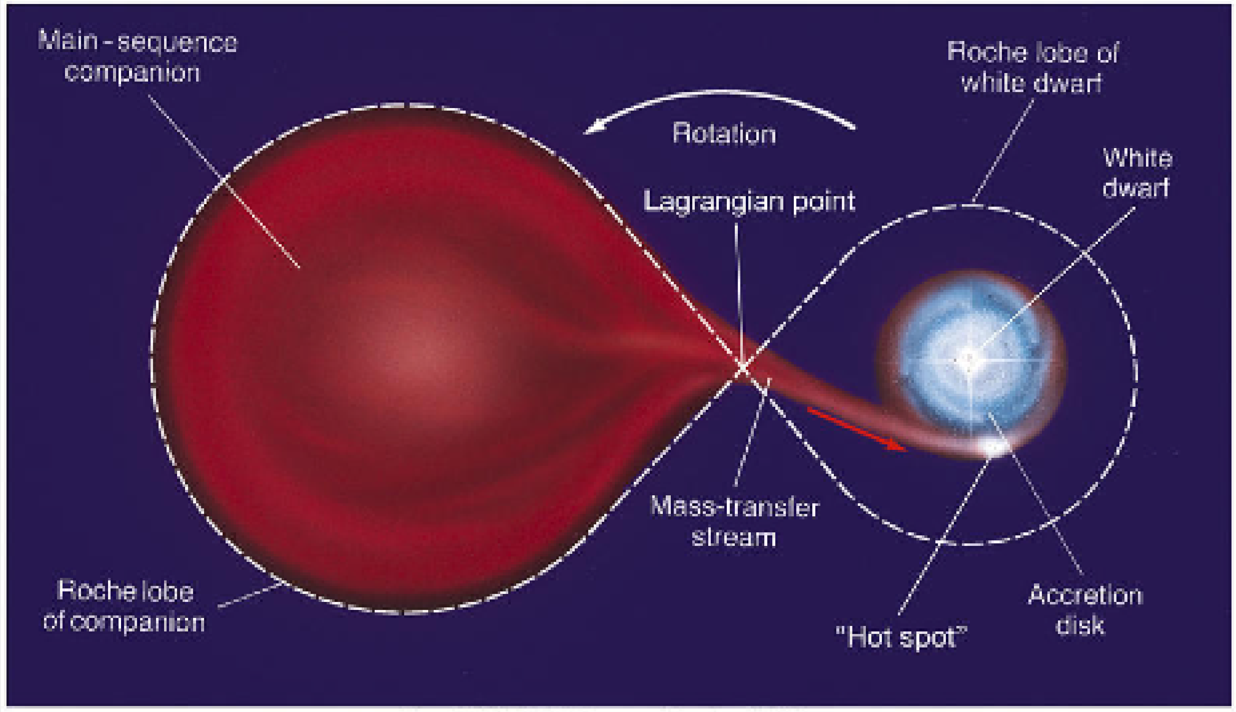




Bedaque and Steiner (2015)

- Make the speed of sound as large as possible (black curve)
- Failure! c_s^2 must be non-trivial at high densities. Why?
- May imply a phase transition at high-density, or the introduction of some new length scale

Low-mass X-ray binaries



- H and He accreted is unstable
- Accretion is unstable and sporadic
- X-ray burst, burns H and He to heavier elements

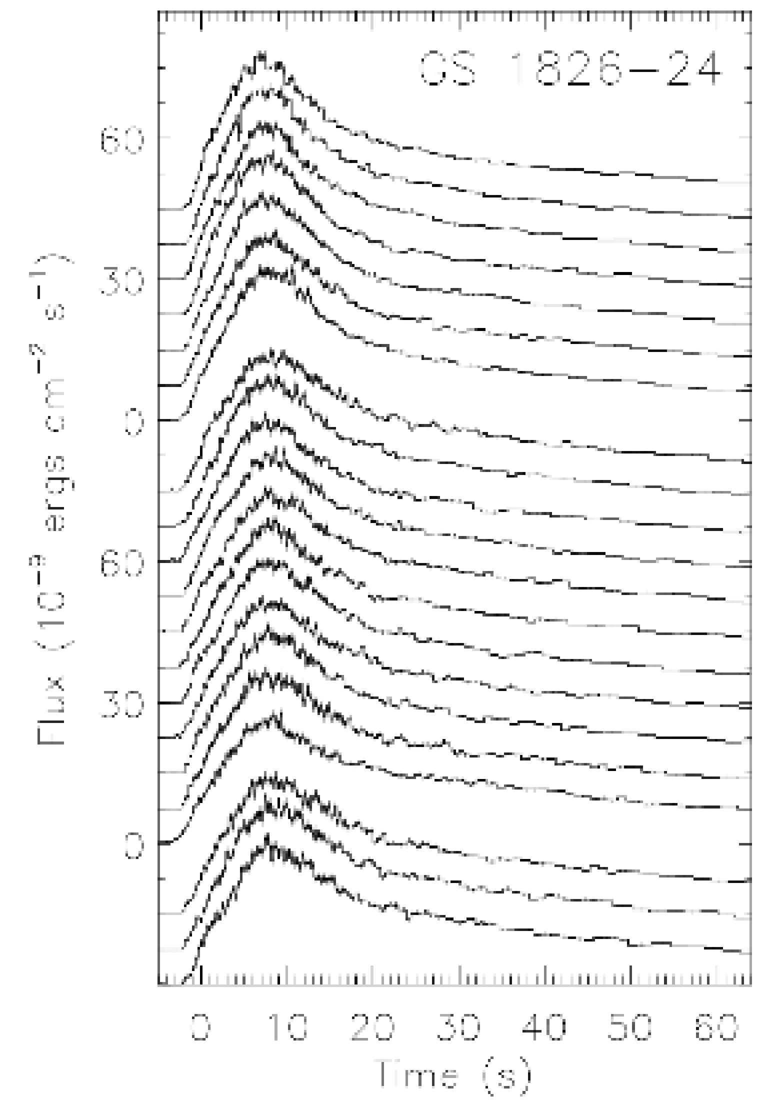


FIG. 1.— Profiles of 20 X-ray bursts from GS 1826−24 observed by *RXTE* between 1997–2002, plotted with varying vertical offsets for clarity. The upper group of 7 bursts were observed in 1997–98, the middle group of 10 bursts in 2000, while the lower group of 3 were observed in 2002. The bursts from each epoch have been time-aligned by cross-correlating the first 8 seconds of the burst. Error bars indicate the 1σ uncertainties.

X-ray bursts from GS 1826-24 from Galloway et al. (2004)

Radius Measurements in qLMXBs

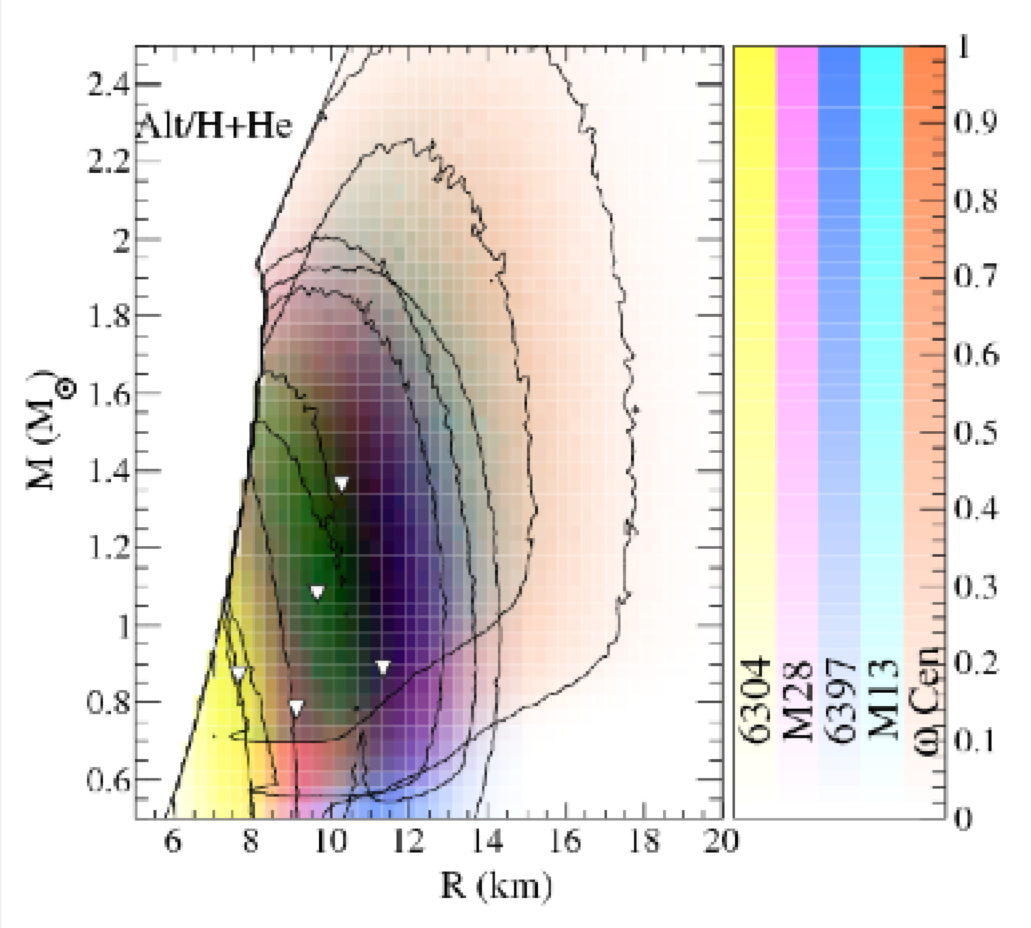
Quiescent LMXBs

- Blackbody-like spectrum of X-rays

$$F \propto T_{\rm eff}^4 \left(\frac{R_\infty}{D} \right)^2$$

i.e. Rutledge et al. (1999)

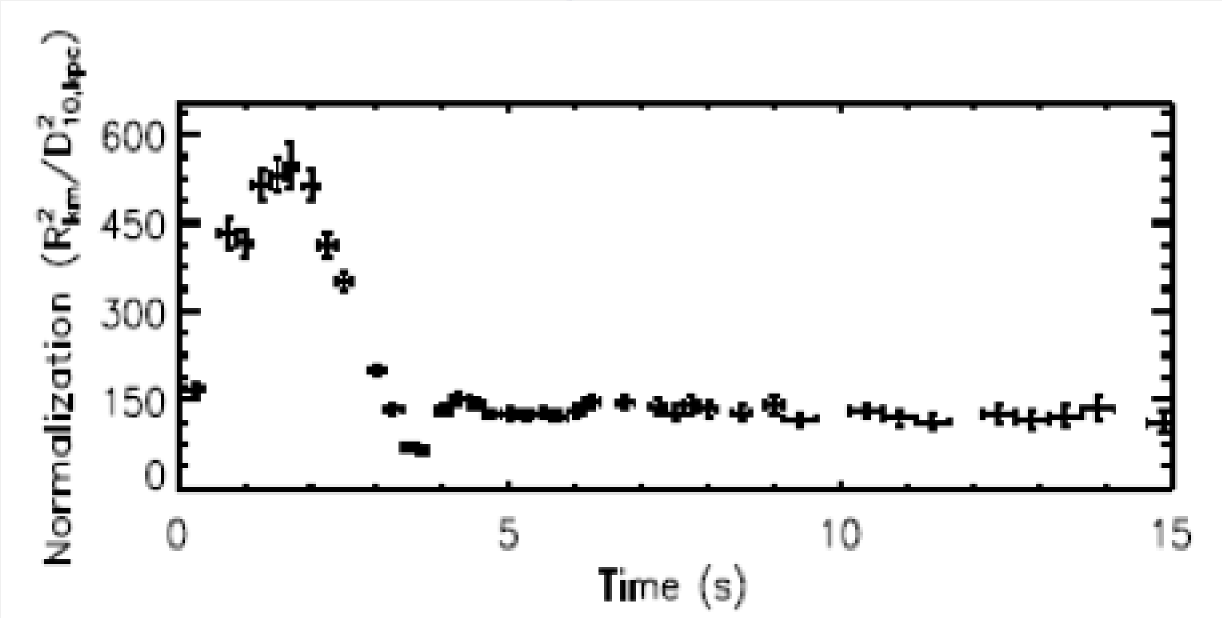
- Measure flux of photons and their energy distribution
- Know distance if in a globular cluster
- Neutron star atmosphere calculation needed to determine $T/T_{\rm eff}$



Lattimer and Steiner (2013) - Probability distributions for five neutron stars, colors added together

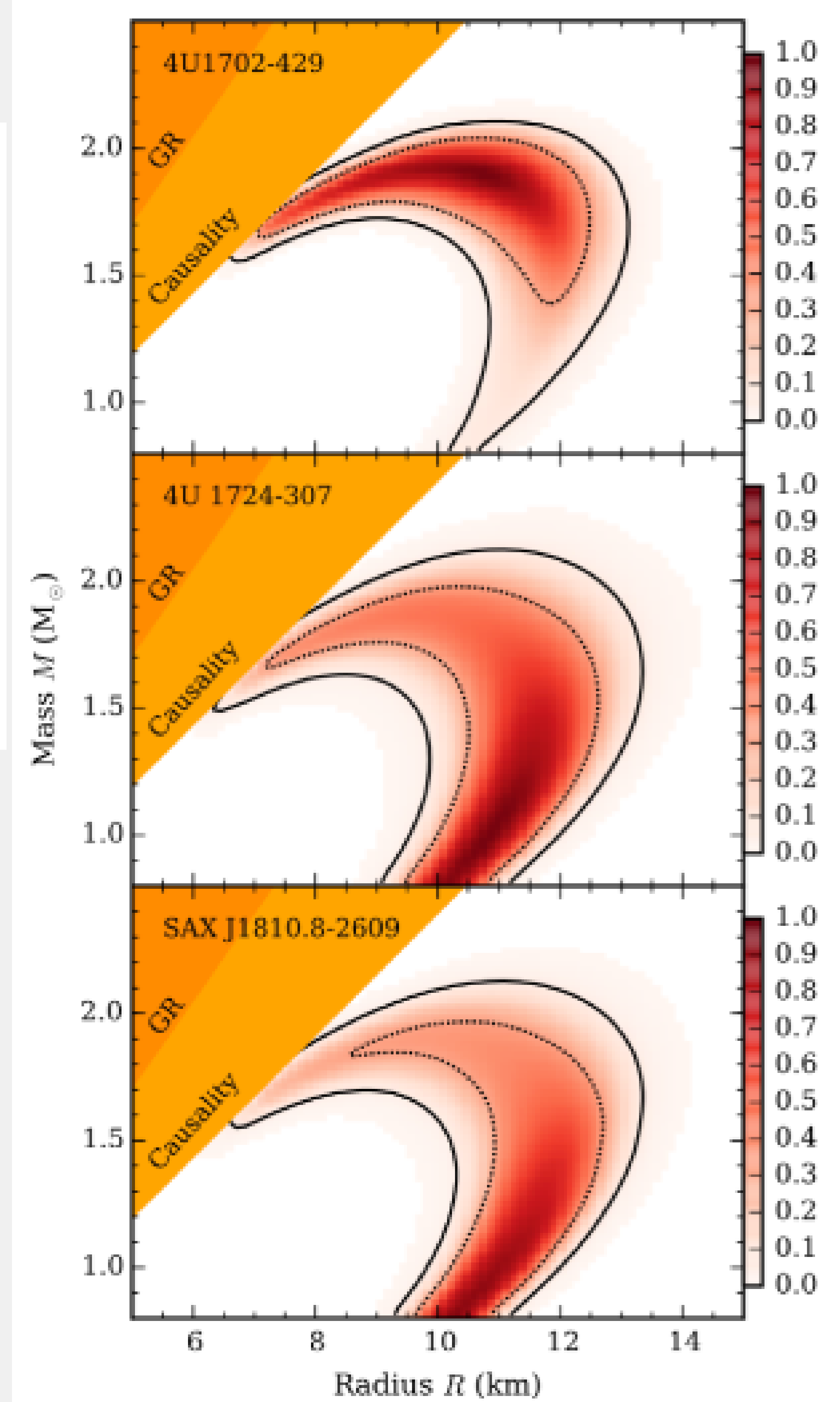
- Between 8 and 12 radius data points all together

Photospheric Radius Expansion X-ray Bursts



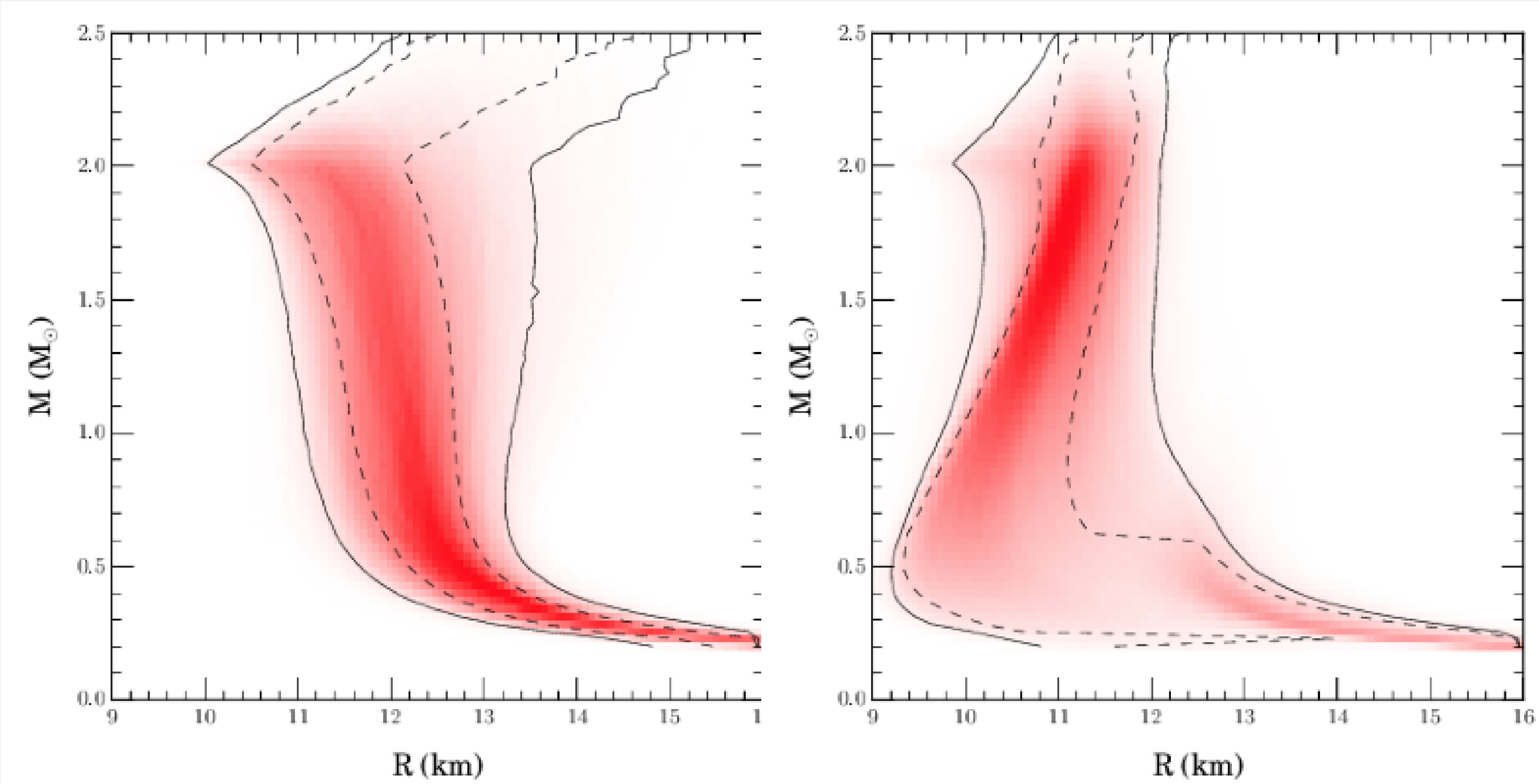
Ozel et al.

- Central idea: balance between radiation pressure and gravity provides a new calibration point
- Use "hard-state" bursts to ensure accretion doesn't poison radius measurement
- Previously gave 14 km radii, difficult to reconcile with qLMXB radii, now in agreement
- Still quite a bit of debate...



Nättila et al. (2016)

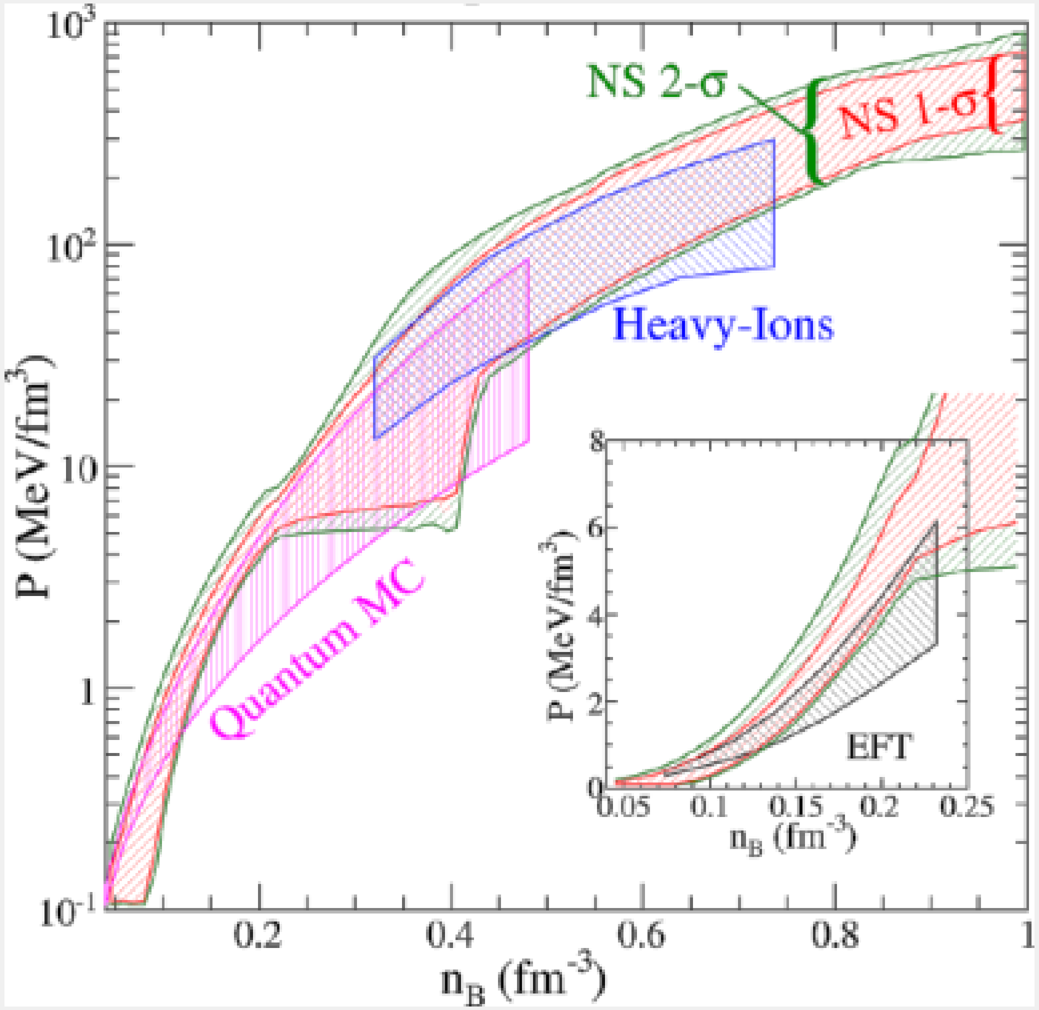
Presence of Phase Transitions Above Saturation



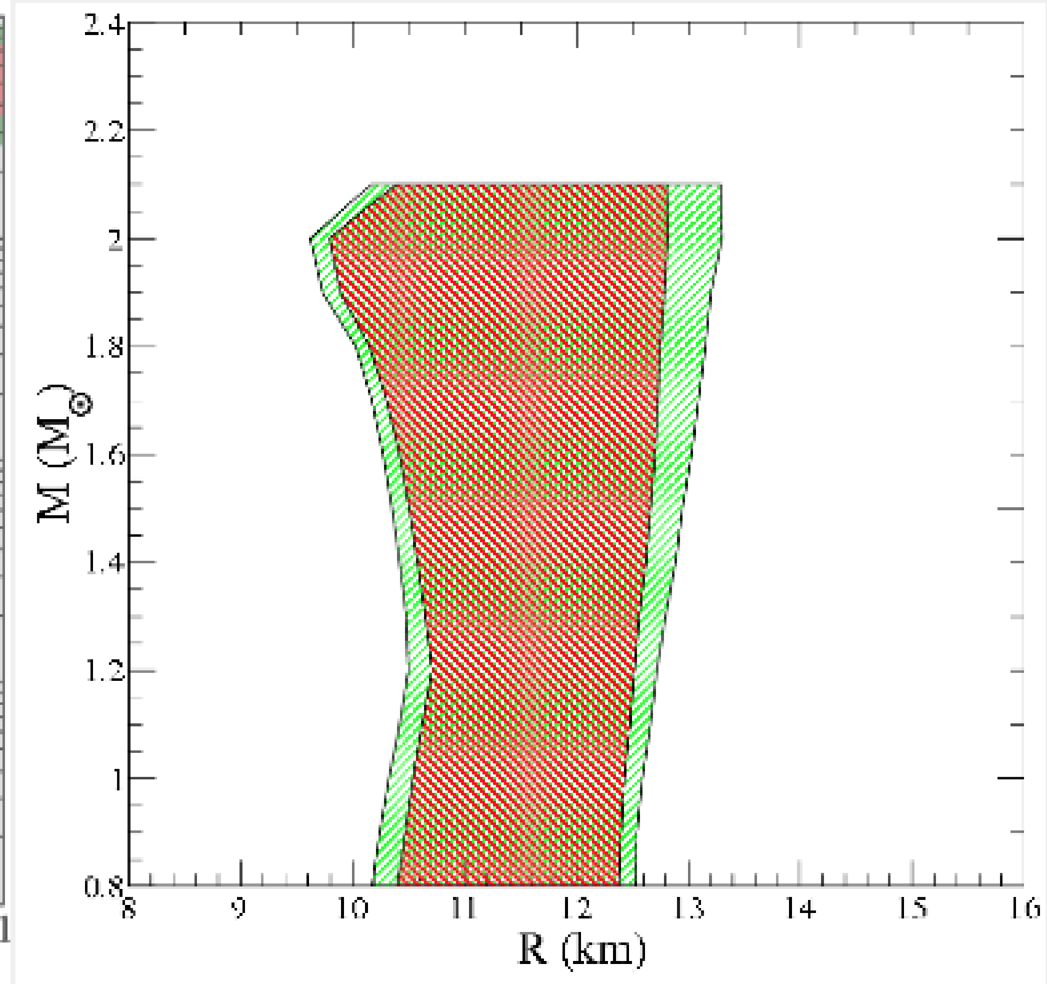
Heinke et al. (in prep.)

- Choice of prior distribution has a significant affect on radius, even after fixing data
- The principal difference here is the possible presence of a phase transition just above saturation
- Phase transition probably ruled out by heavy-ion collision data, but to how far?

M-R and EOS results



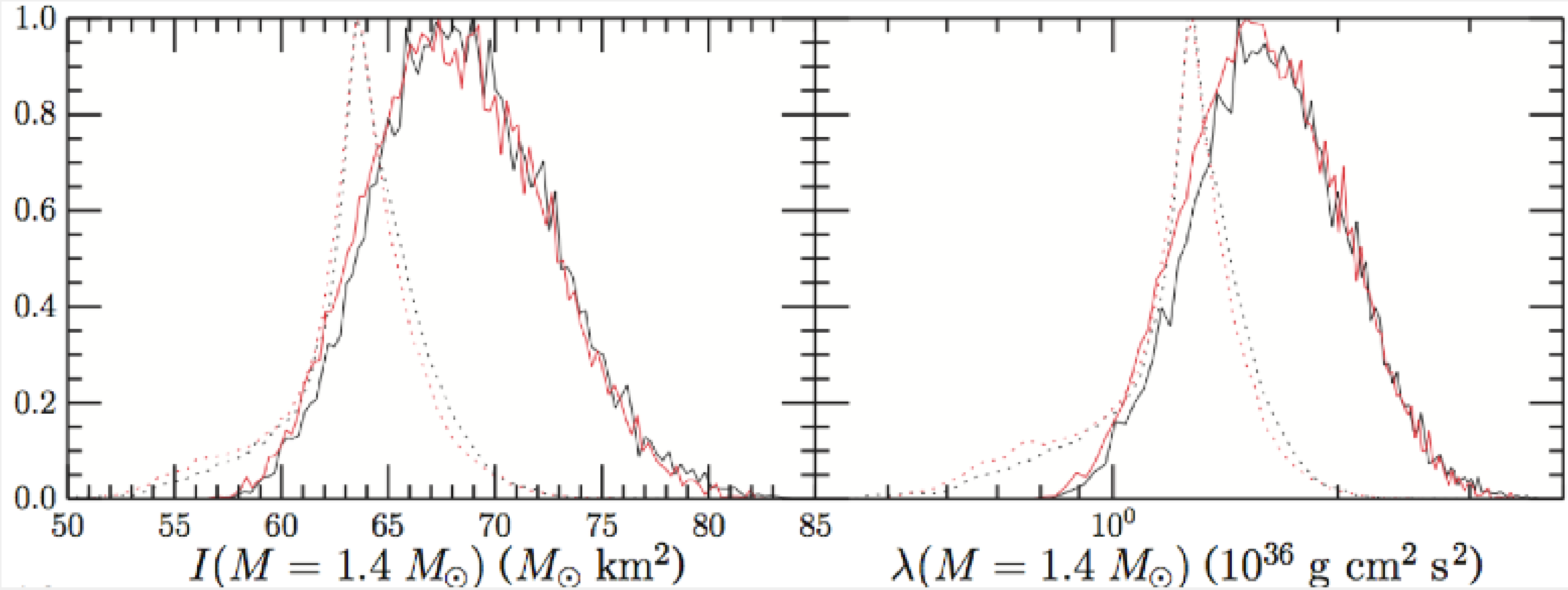
Steiner et al. (2013)



Steiner et al. (2013)

- Ensure bounds enclose results from different priors
- 10-13 km radii
- EOS consistent with nuclear physics constraints
- From 2013, but this won't change quickly

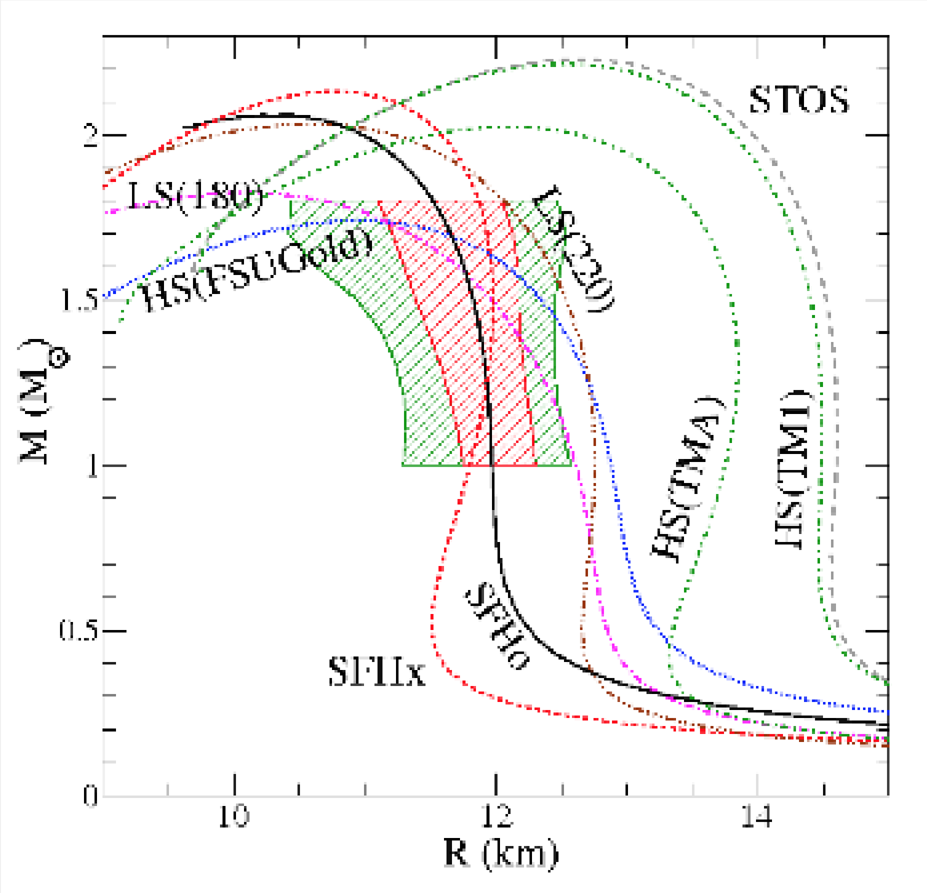
I and λ results



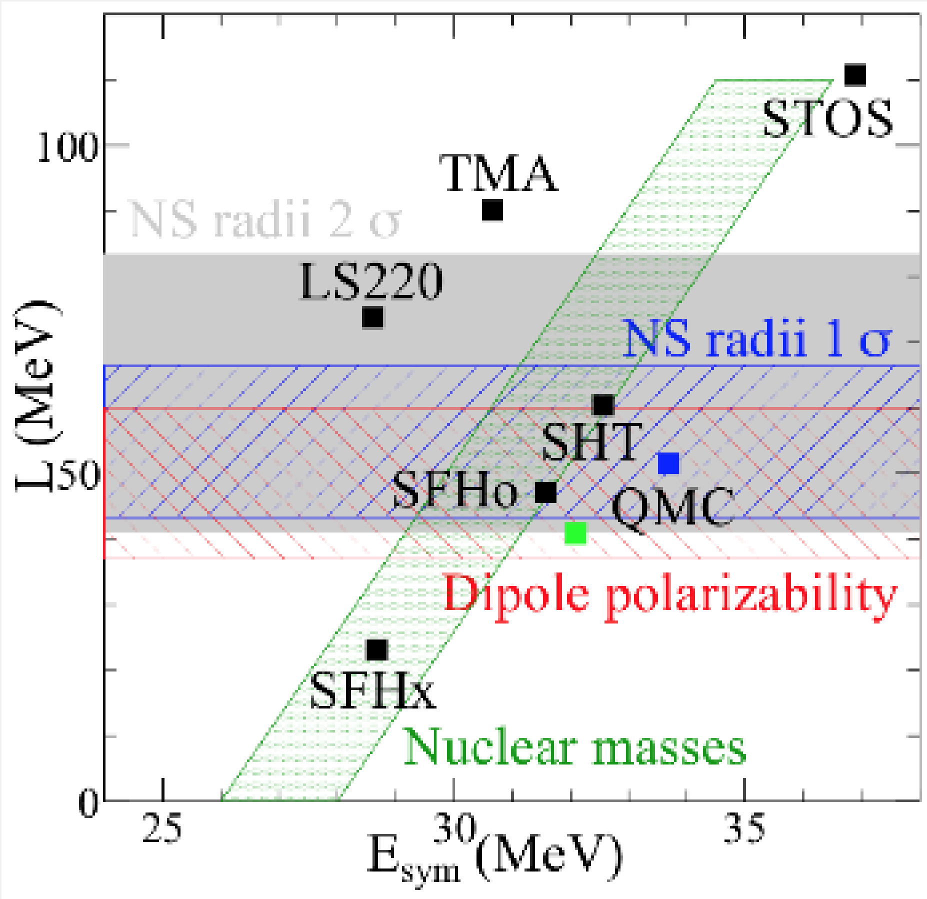
Steiner, Gandolfi, Fattoyev, and Newton (2015)

- Predict moments of inertia and tidal deformabilities
- Tidal deformability is like a tidal polarizability for gravitational fields
- Important counterpoint to LIGO

EOS for astrophysical simulations



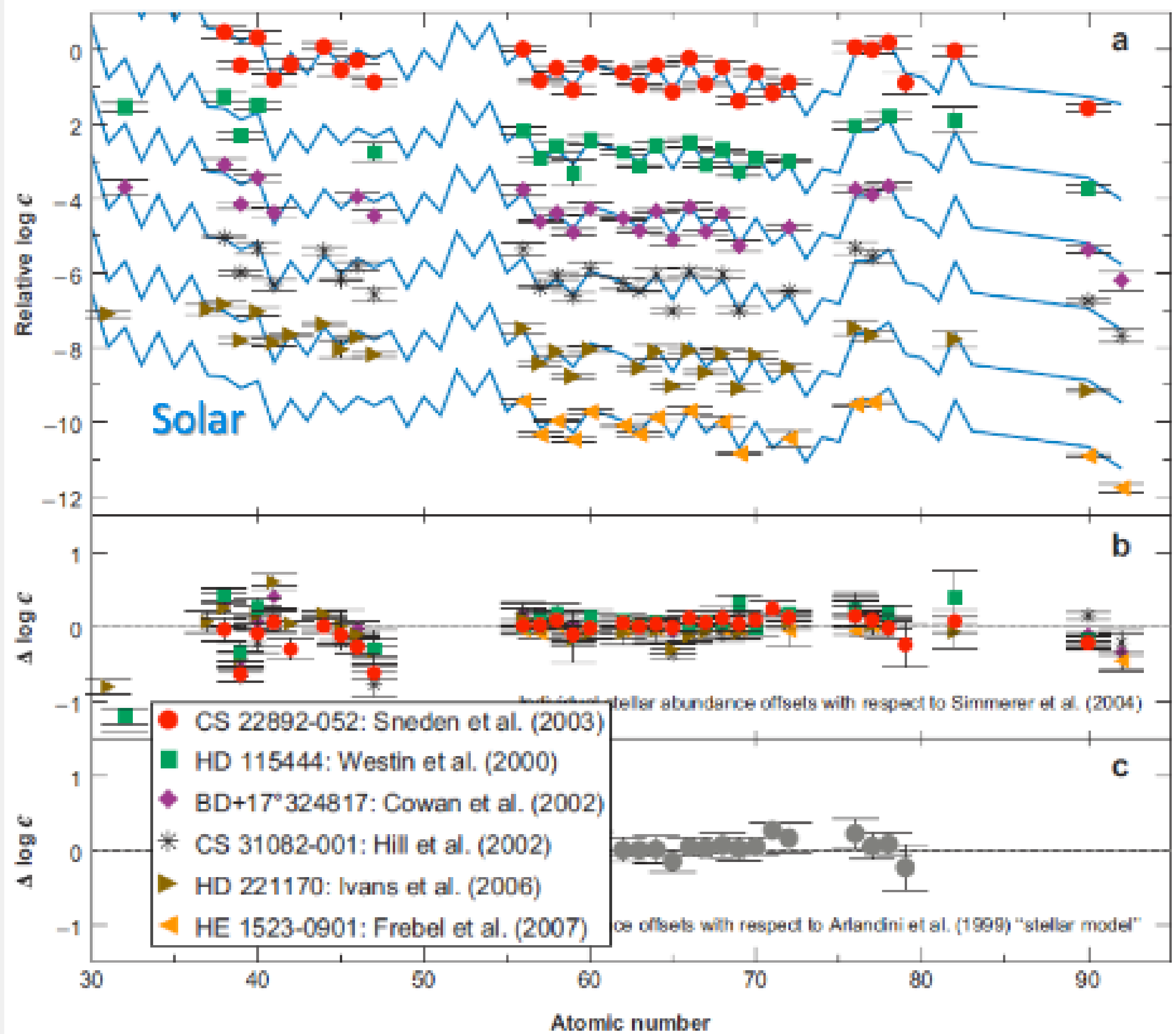
Steiner, Hempel, and Fischer (2013)



Based on Steiner, Hempel, and Fischer (2013)

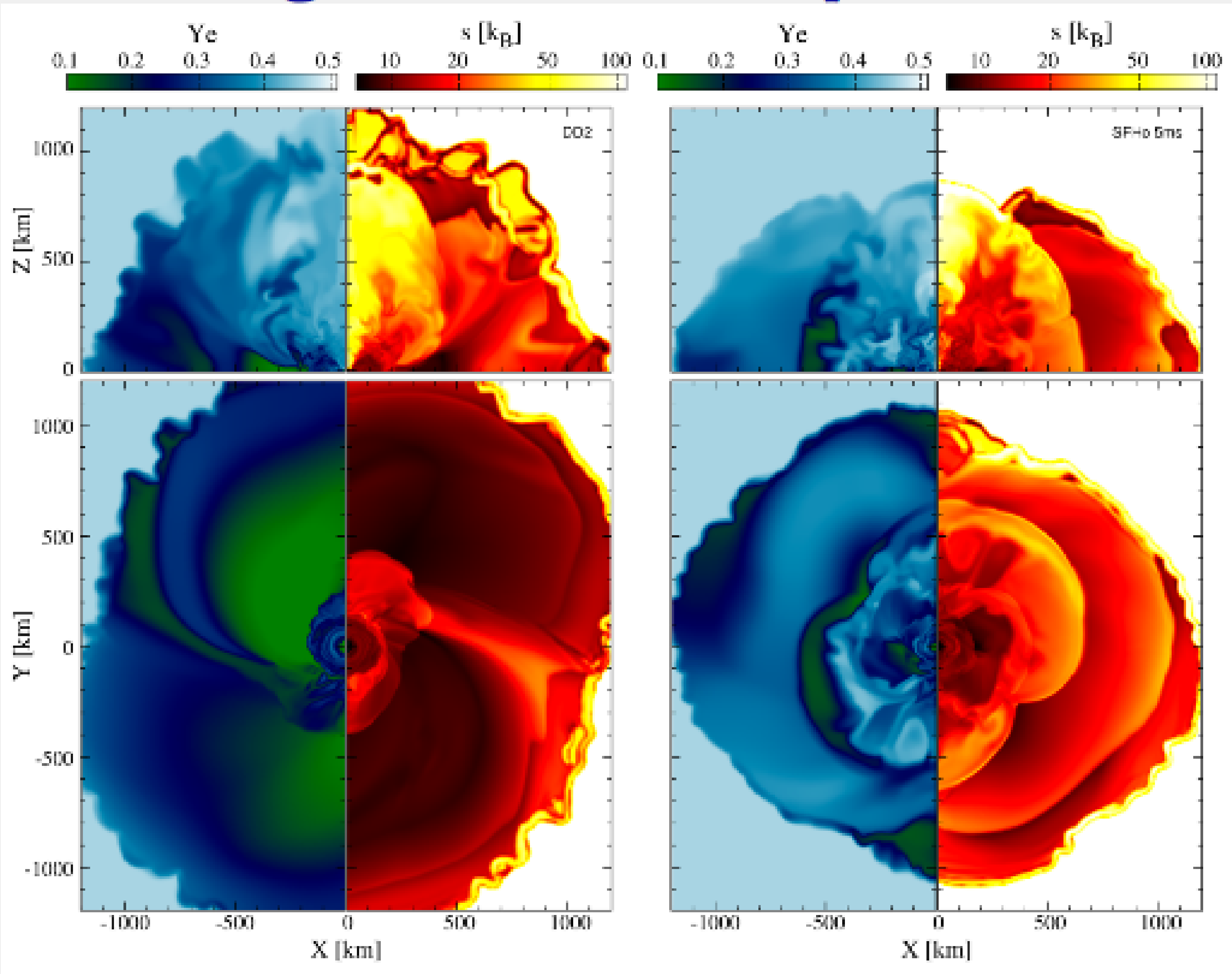
- Limited number of EOS tables (Y_e, n_B, T) which satisfy $M - R$ constraints and the $S - L$ correlation
- Current EOS uncertainties too small to explain explosion
- Many simulation properties are weakly correlated with the symmetry energy

Mergers and the r-process



- r-process nuclei observed in stars is universal: same pattern from event to event
- May occur in neutron star mergers, but is it universal?

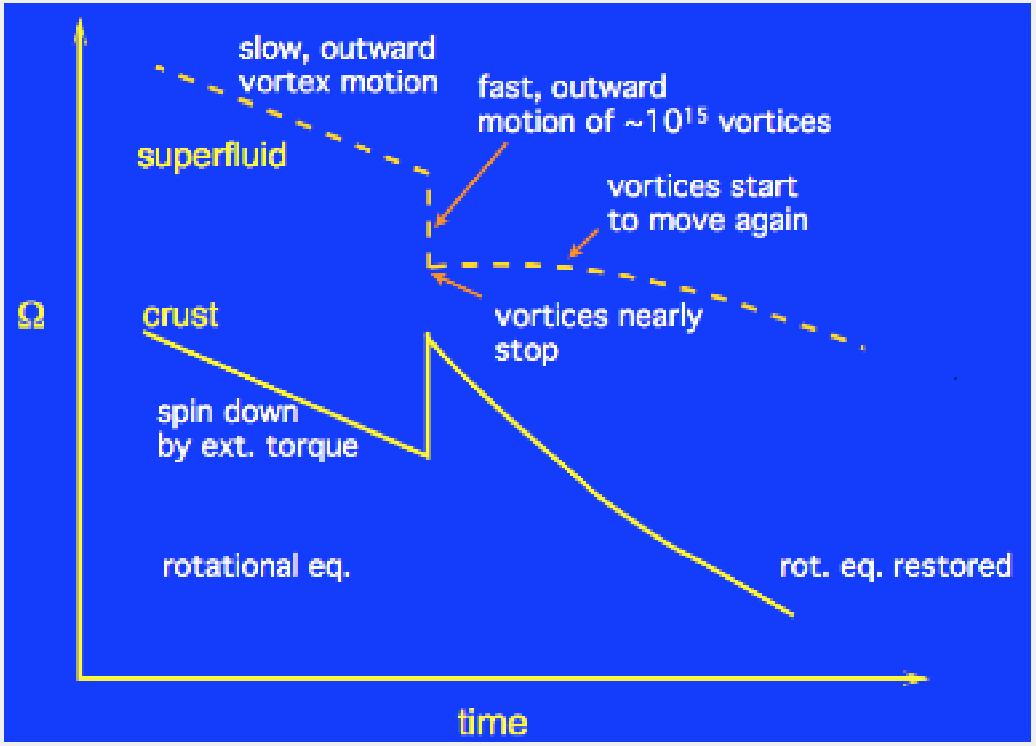
Mergers and the r-process



Sekguchi et al. (2015), DD2 (large radii) on left and SFHo (small radii) on right

- Small radii lead to higher Y_e and more universal r-process production
- Smaller radii also lead to larger amounts of ejected r-process material

Pulsar Glitch Mechanism



Picture from B. Link

- Superfluid component, decoupled from rotation at the surface
- Natural to associate the superfluid component with the superfluid neutrons in the crust
- What is the mechanism for the sudden change?

- Superfluid vortices pinned to the lattice
- Neutron star spins down, vortices bend creating tension, eventually they must shift lattice sites
- Quasi-free neutrons are entrained with the lattice
  Chamel 2012, Chamel et al. 2013

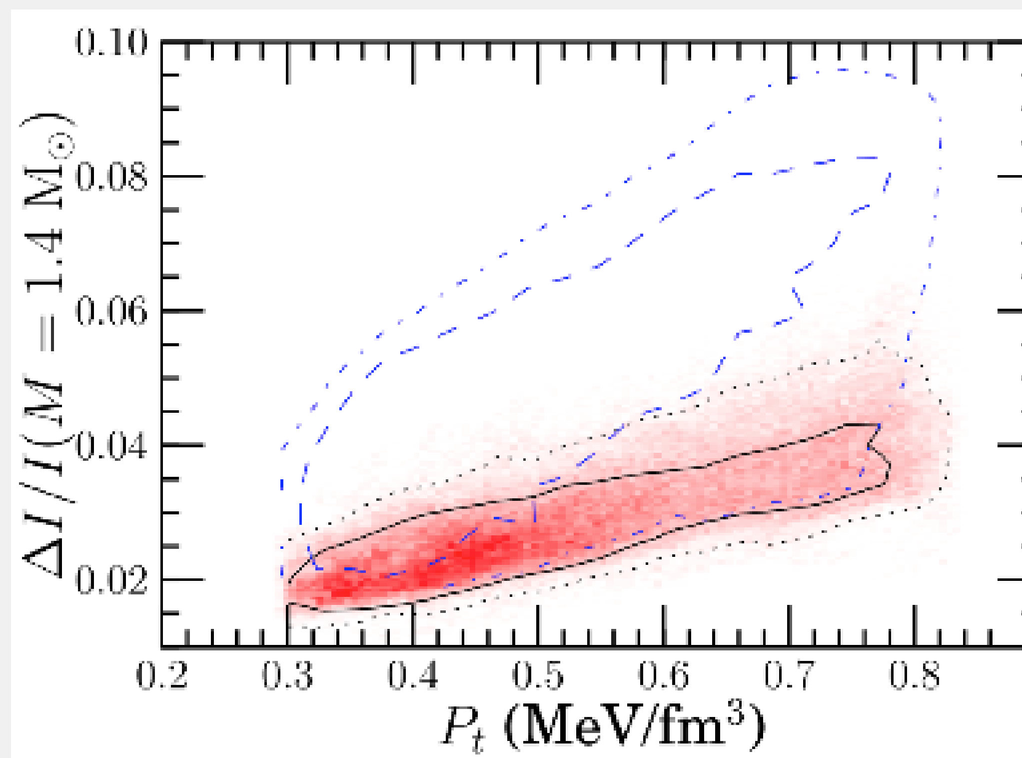
Is There Enough Superfluid in the Crust?

- We require 1.6% of I to explain glitches in Vela
  Link, Epstein, and Lattimer (1999)

- Entrainment: 75-85% of otherwise superfluid neutrons 'connected' to the lattice
  N. Chamel (2012)

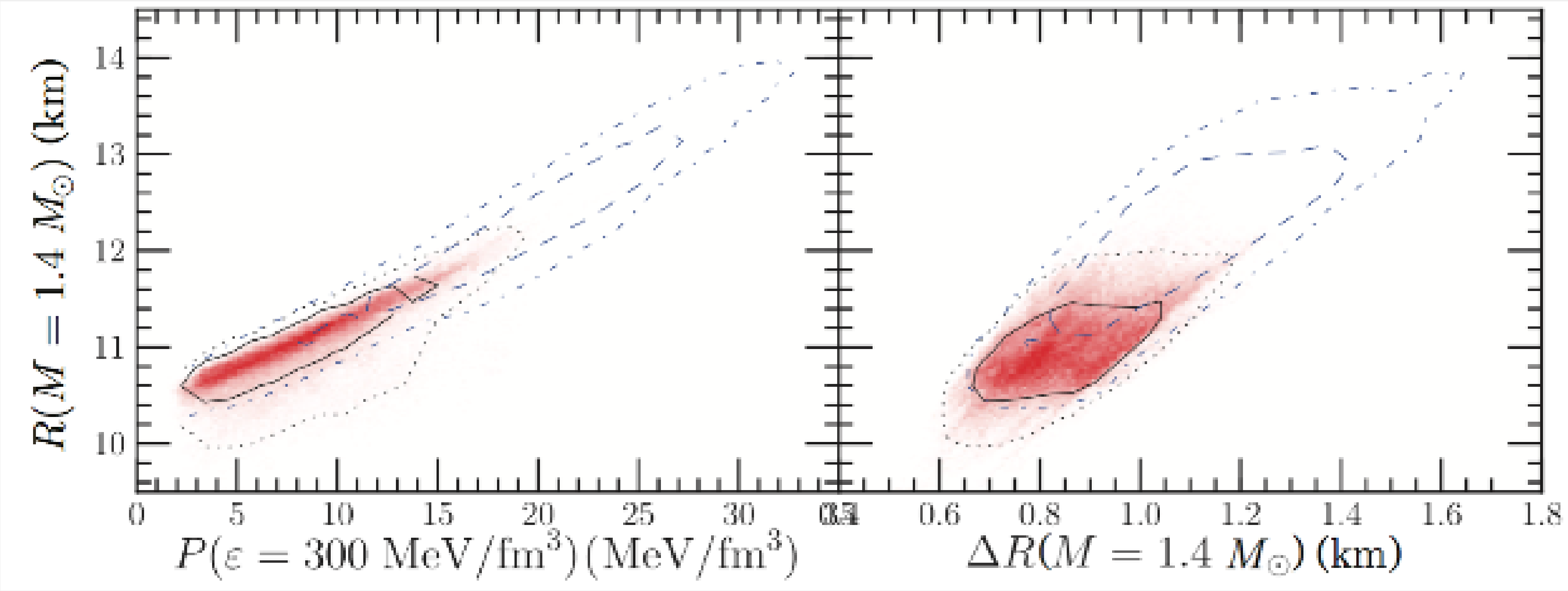
- Current M and R observations suggest there is not enough I in the crust
  See Andersson et al. (2012)

- Unless the systematics force much larger neutron star radii and P_t and L are large

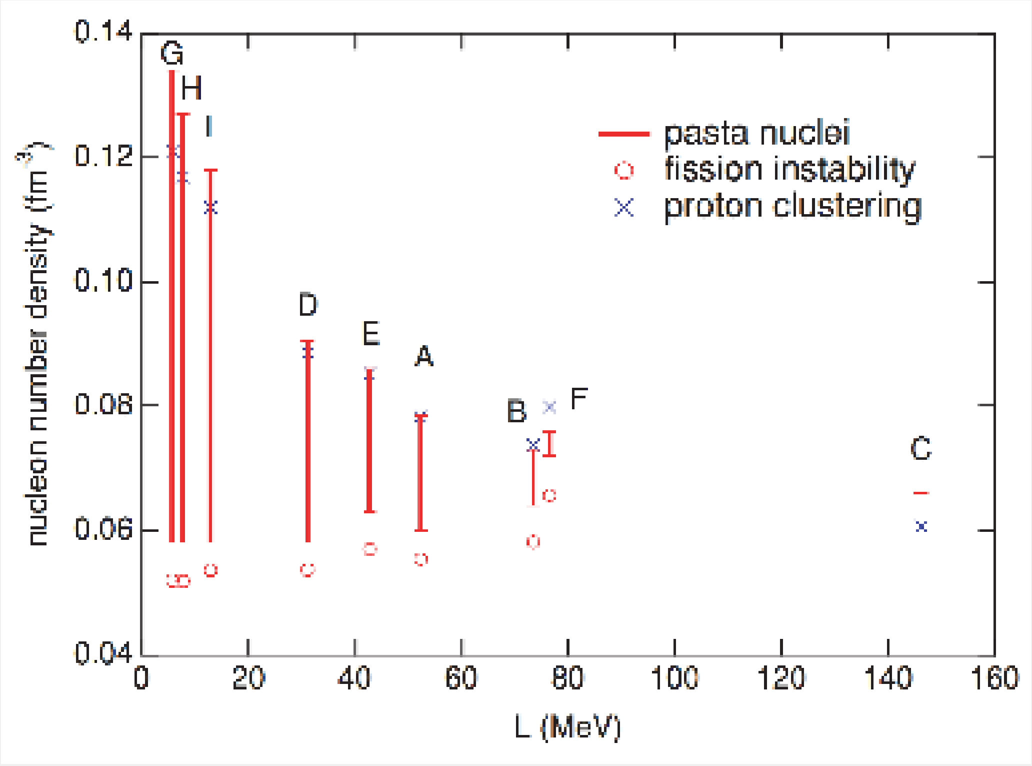


Steiner et al. (2014); black and red are with M & R observations, blue contours are with $I = 70\ M_\odot\ km^2$

Crust Thickness and Pasta

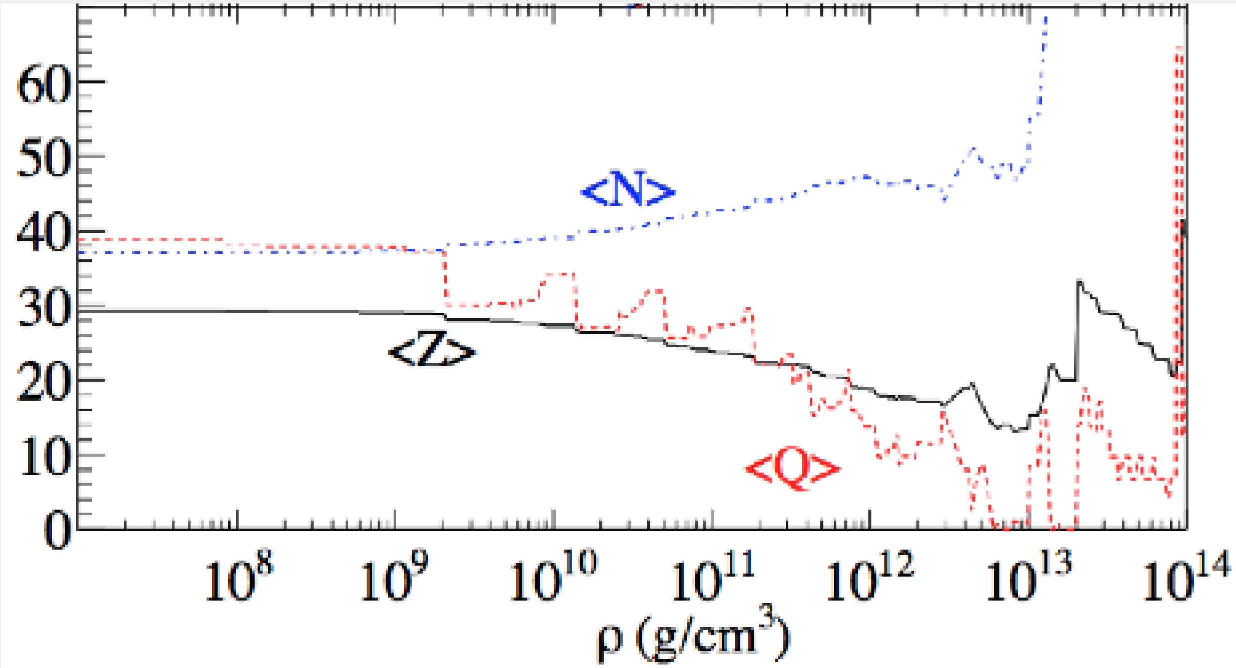


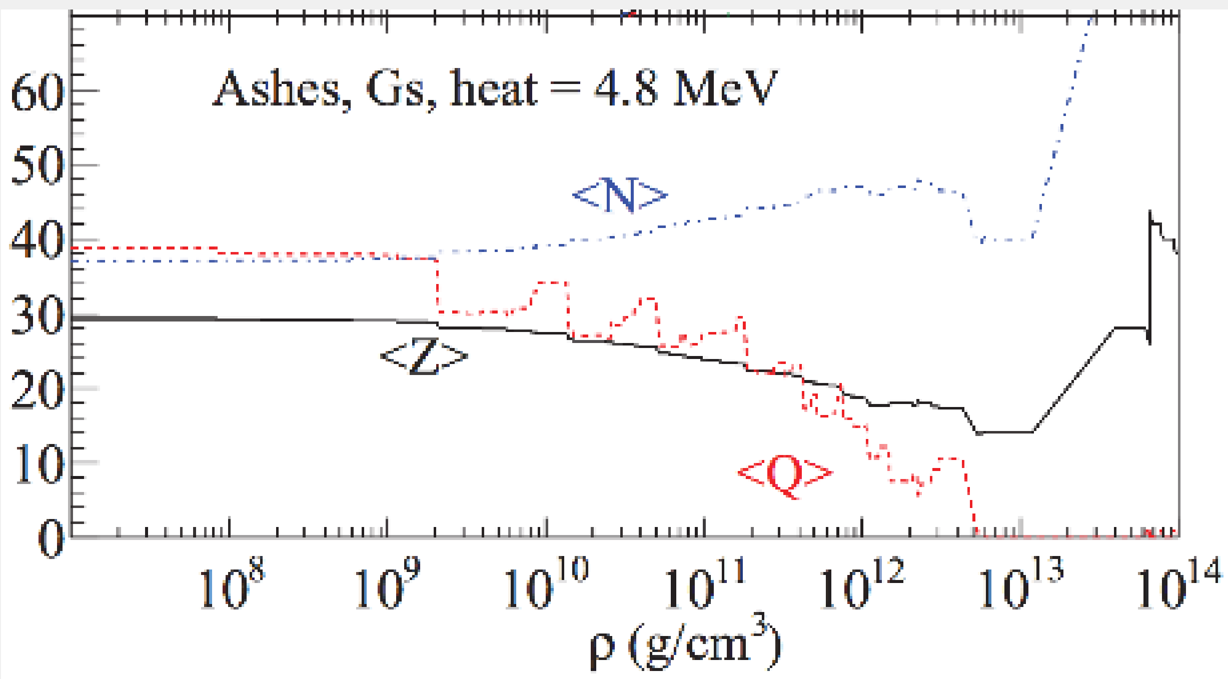
Steiner, Gandolfi, Fattoyev, and Newton (2015)



Oyamatsu and Iida (2007)

- L connected to crust thickness and the extent of the nuclear pasta





Steiner (2012)

Deep Crustal Heating

- L connected to the amount of heating in an accreting neutron star crust

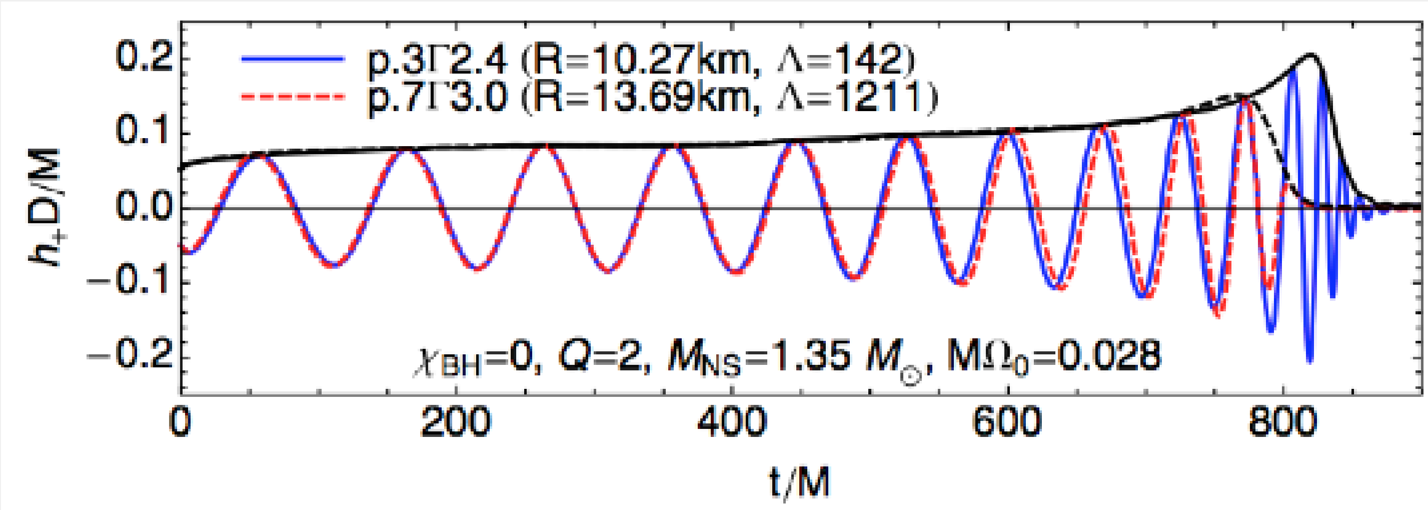
Summary

- Do we want a measurement of L to within 20 MeV? Yes please! L is important for a wide range of observables and we want to rule out models which are unphysical.
- Is L the only remaining uncertainty for the description of neutron star structure and evolution? No, almost all neutron star observables have additional systematics which we do not yet understand.
- Will we ask for better constraints on L afterwards? Most certainly yes. Please sign us up for the ± 1 MeV experiment. Thank you.
- The motivation for PREX (from my perspective) is not that it will provide the last word on neutron star radii, but rather that L is a critical parameter with such a broad impact.
- Do radii require exotic nuclear physics? Not at the moment no.

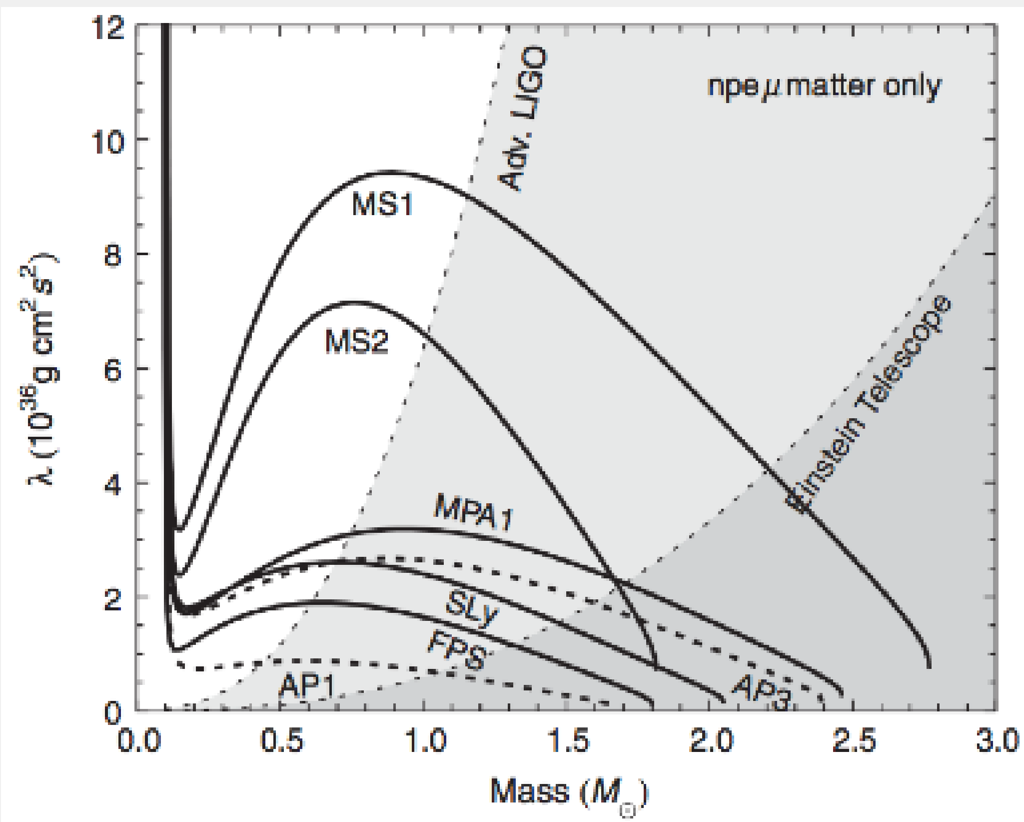
Future

- Data rules all: more experiment (PREX, FRIB, FAIR, RIKEN, etc.) and observation (NICER, LIGO, LOFT!)
  Watts et al. RMP (2016)
- Uncertainty quantification from microphysics ⇒ macrophysics
  Mumpower et al. (2016)
- Optimize interactions to minimize prediction systematics
- Shrinking uncertainties for heavier nuclei:
  - Pushing ab-initio to higher A
  - Squeezing DFT systematics
- Going beyond S and L:
  - Nuclei near and beyond the drip line
  - Magic numbers in neutron-rich nuclei
  - More astrophysical simulations with sensitivity studies

Neutron Star Tidal Deformabilities



Lackey et al. (2014)



Hinderer et al. (2010)

- Gravitational wave signal from an NS merger measures tidal deformability λ
- Point masses early on; deformation near 400 Hz
- Easier to detect larger tidal deformations

New EOSs better for the r-process

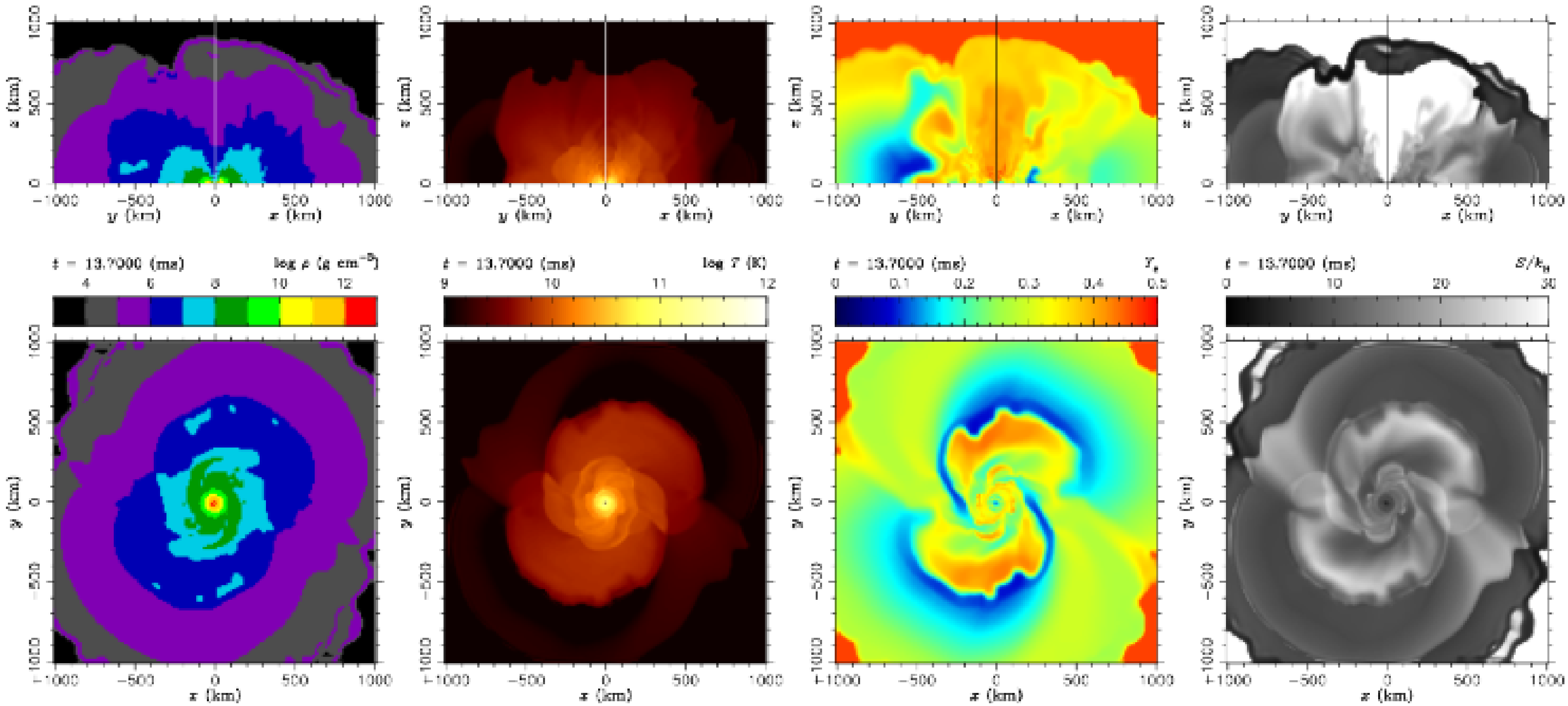


Figure 2. Color-coded distributions for density, temperature, Y_e, and S/k_B (from left to right) on the x–y (lower panels), x–z (positive sides of top panels), and y–z (negative sides of top panels) planes at the end of simulation.

- Wanajo et al. (2014) find that mergers with typical EOSs have small $Y_e \sim 0.1$, which gives an overabundance of nuclei with $A > 130$
- the SFHo EOS gives larger Y_e, and thus a r-process distribution that more closely matches observations
- L also impacts Y_e in neutrino driven winds

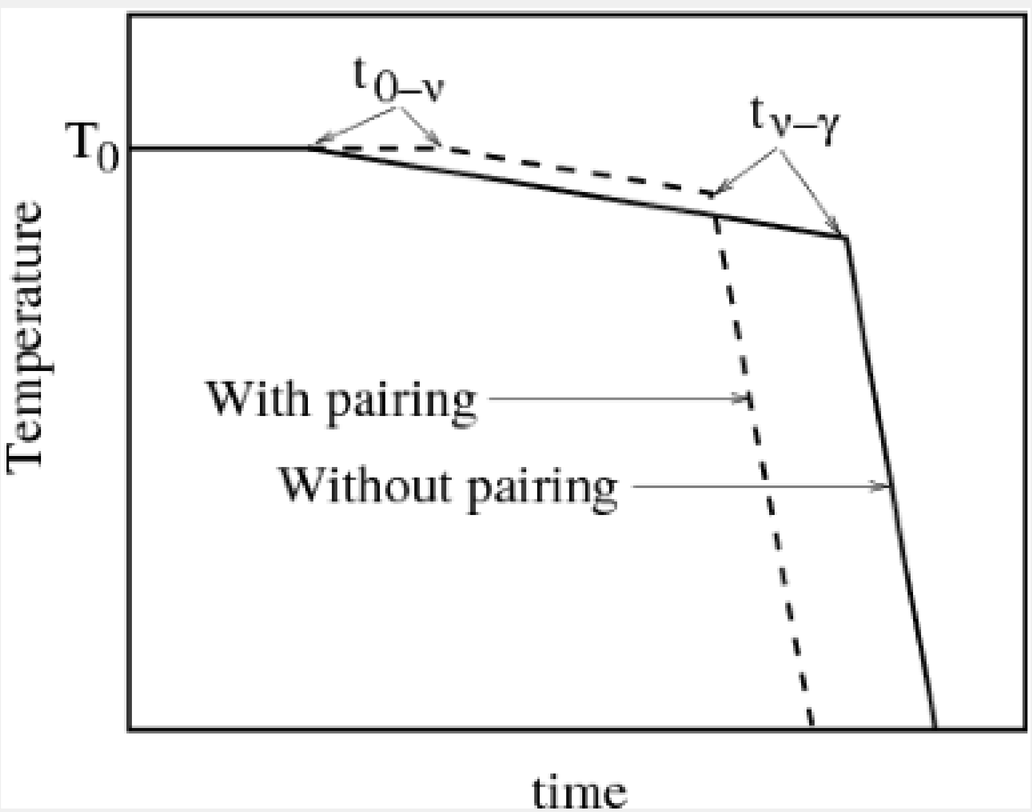
  Roberts et al. (2012)

Thermal Emission from Isolated Neutron Stars

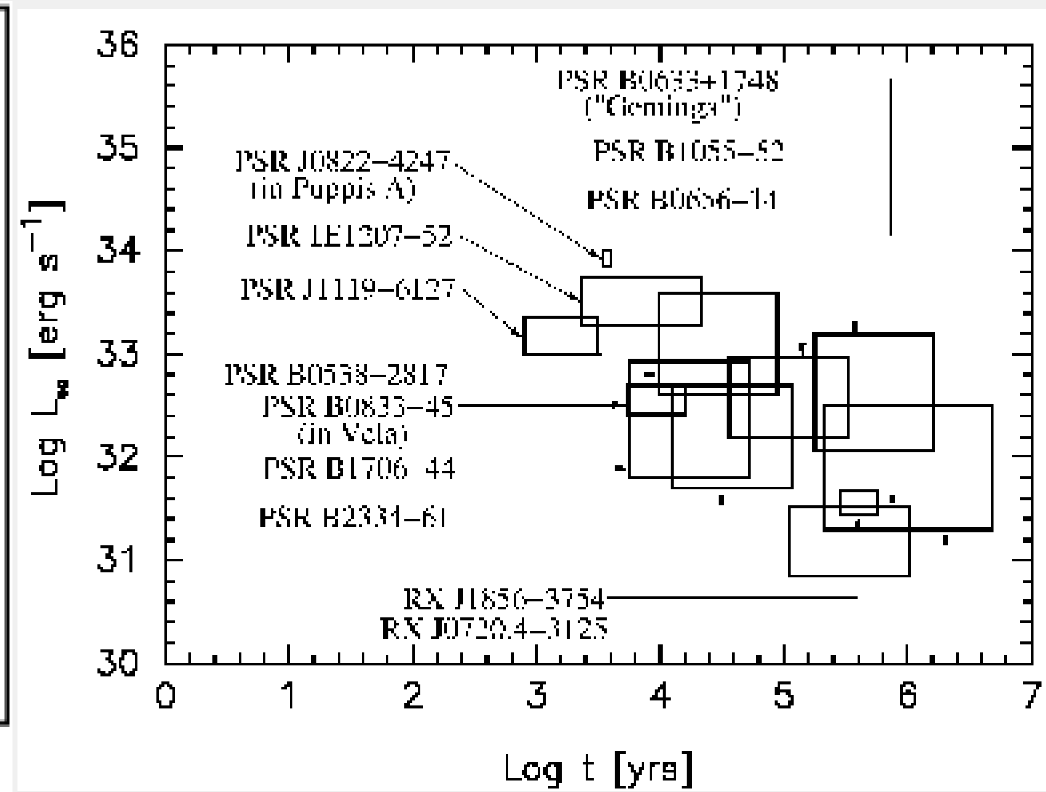
- No distance measurement required
- Requires a model of the NS atmosphere to associate the observed spectrum with a luminosity or temperature

$$C_V \frac{dT}{dt} = L_\nu + L_\gamma\,, \quad L_\gamma \sim T^{2+4\alpha}\,, \quad L_\nu \sim T^8 \text{ (Modified Urca)}\,, \quad C_V \sim CT$$

- Age assumed from spin-down age or associated with a supernova remnant



Page, et al (2004)



Page, et al (2009)

Fitting two-dimensional data

- One-dimensional fit:

$$\chi^2 = \sum_i \left(\frac{M_i - D_i}{U_i}\right)^2; \quad \mathcal{L} \propto \exp\left(-\chi^2/2\right)$$

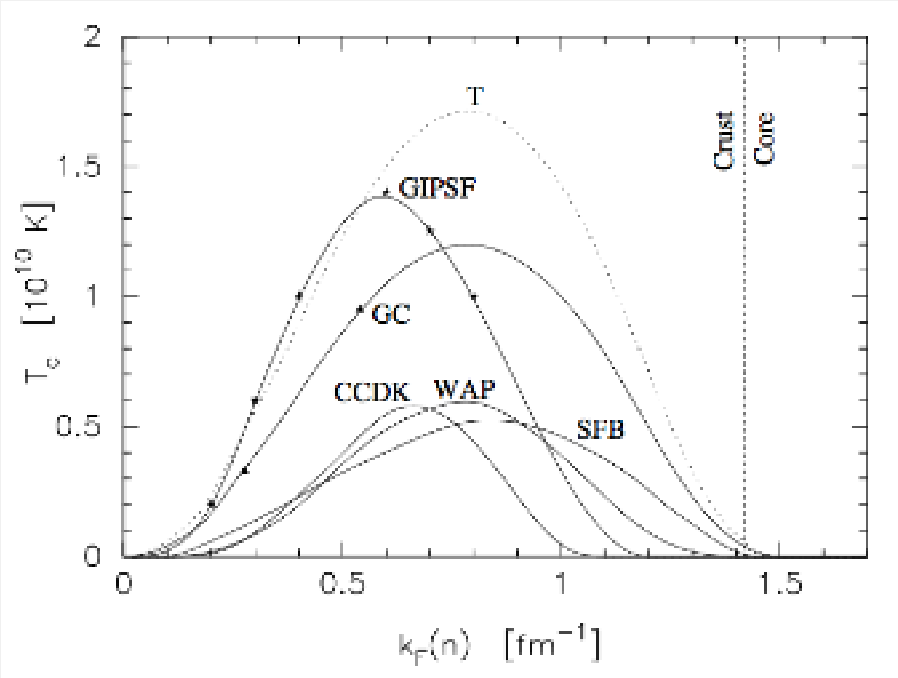
- Fit a data set $(t_i \pm \delta t_i, T_i \pm \delta T_i)$ to a function $T(t)$
- No unique method, typical frequentist approaches are:
  - orthogonal least-squares
  - geometric mean regression
  - Deming regression
- Bayesian generalization of several frequentist methods

$$\mathcal{L} \propto \prod_i \int dt \sqrt{\left[\left(\frac{dT}{dt}\right)^2 + 1\right]} \exp\left\{\frac{-[t - t_i]^2}{2\delta t_i^2}\right\} \exp\left\{\frac{-[T(t) - T_i]^2}{2\delta T_i^2}\right\}$$

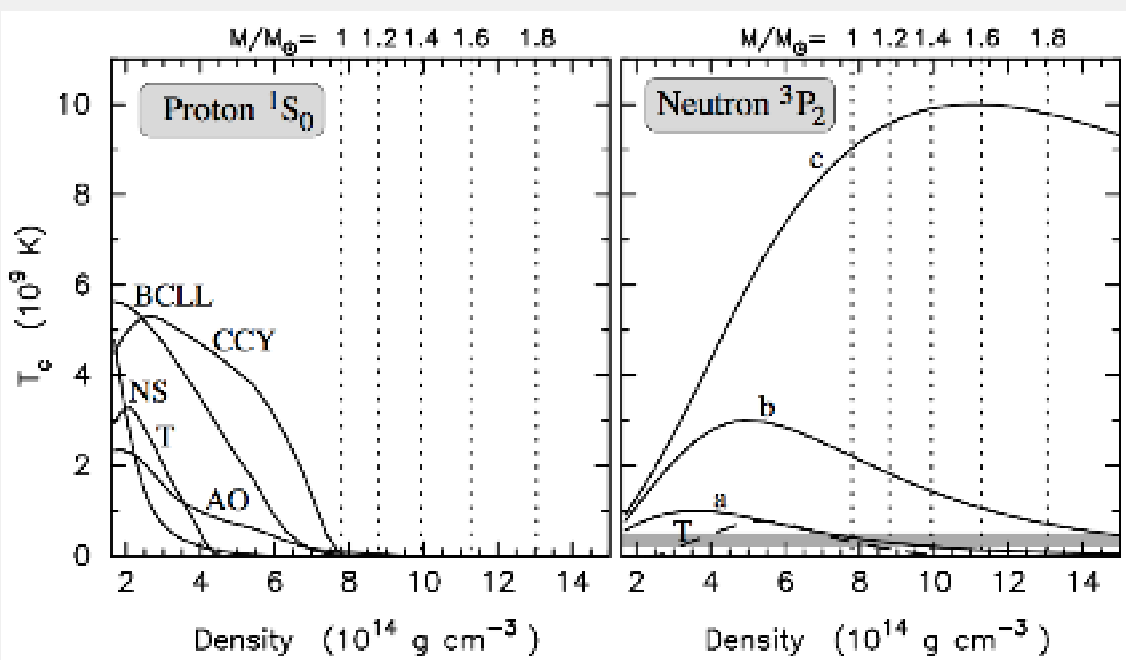
- Prior distribution manifest in the line element
- Limiting forms imply traditional χ^2 fit

Neutron Star Superfluidity

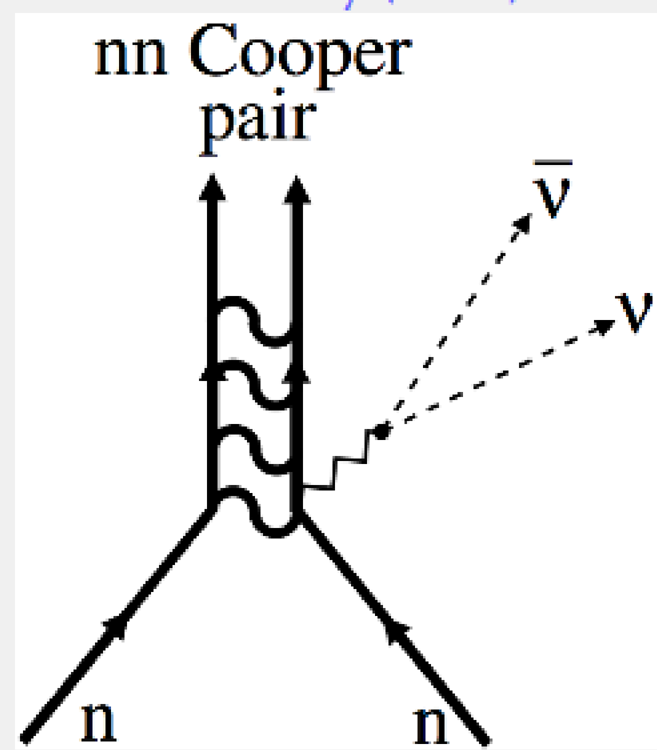
(See our review at 1302.6626)



- Gap is energy gained from making a Cooper pair
- New ways of cooling

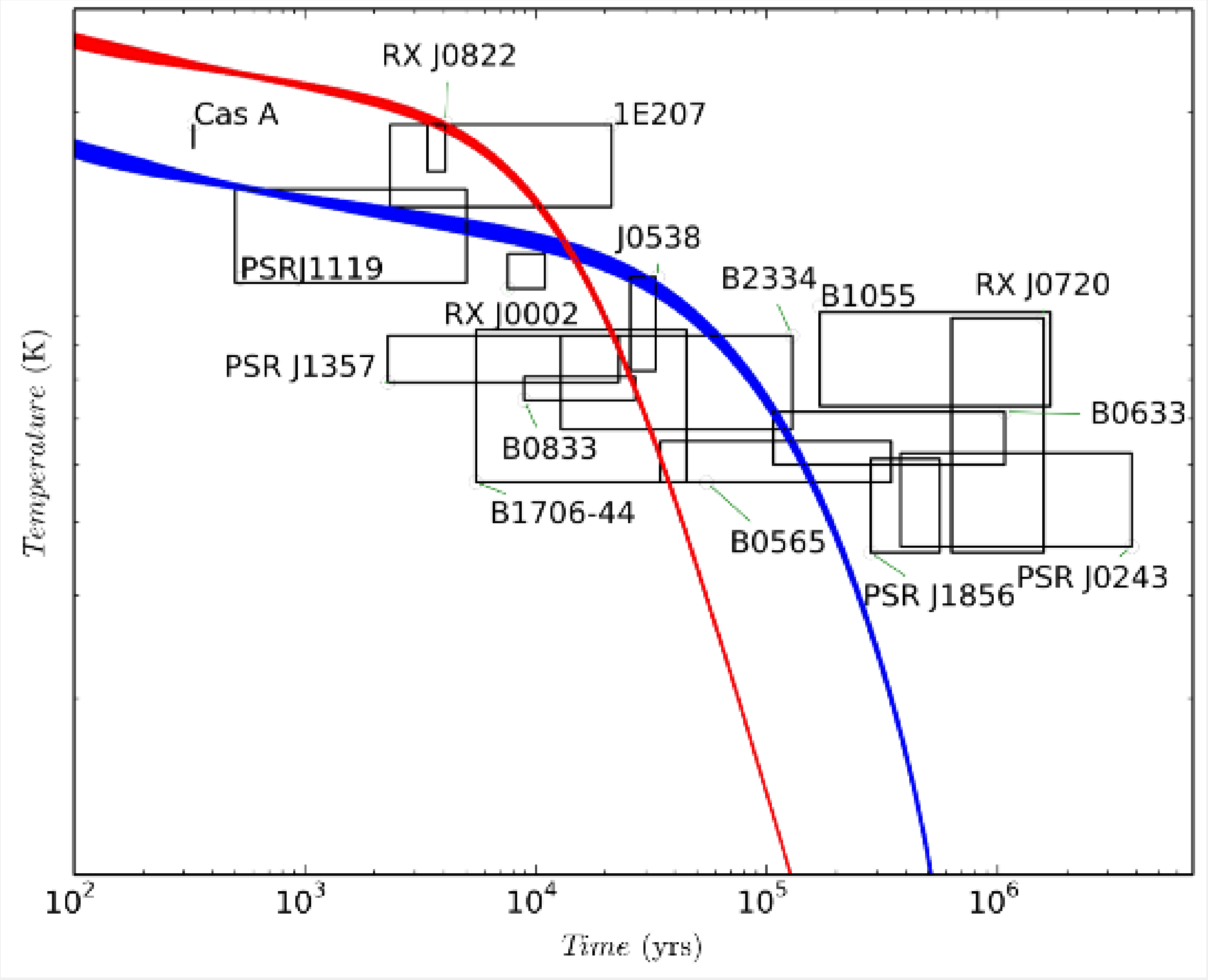


e.g. Steiner and Reddy (2009)



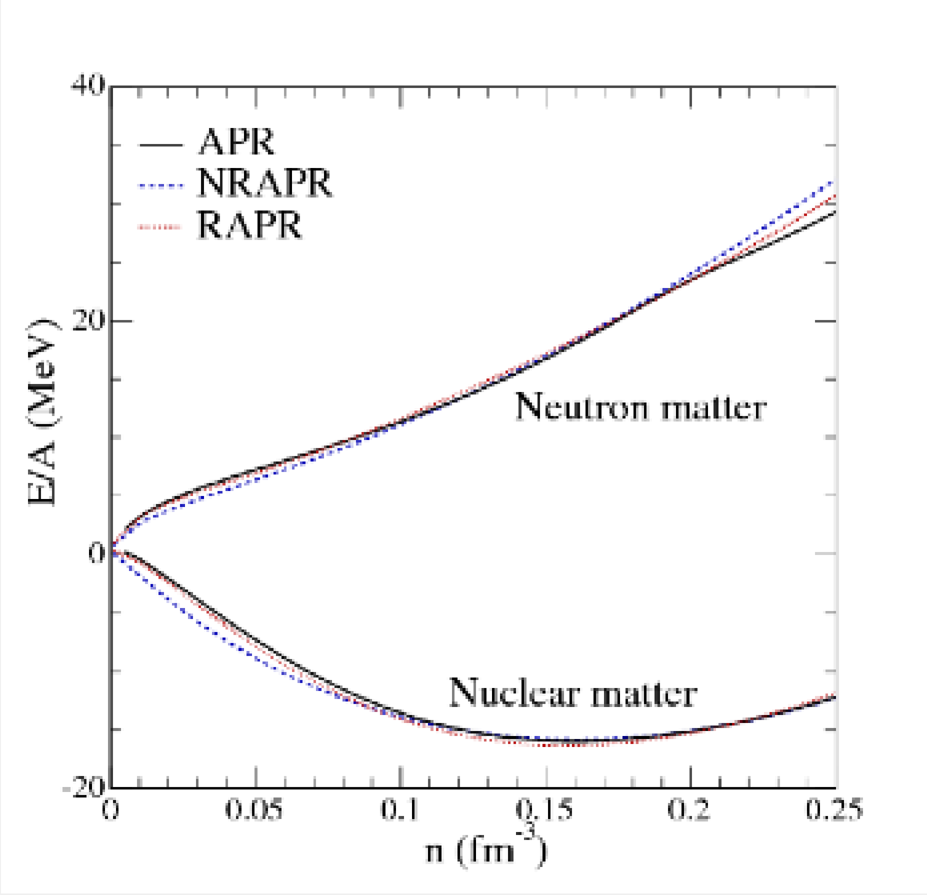
Detecting Neutron Star Superfluidity

- Data is reproduced only by a particular set of superfluid gaps, $T_{C,n} \approx 10^8$ K $T_{C,p} \approx 10^9$ K



Beloin, et al.

The Nuclear Symmetry Energy, Revisited



Steiner et al. (2005)

- Define $\alpha \equiv (n_n - n_p)/n$

$$S(n) = \frac{1}{2n}\left(\frac{\partial(nE/A)}{\partial\alpha}\right)_{\alpha=0}$$

- $\tilde{S}(n) = (E/A)_{\rm neut}(n) - (E/A)_{\rm nuc}(n)$
- If E/A is quadratic in α, $S(n) = \tilde{S}(n)$
- $S \equiv S(n_0)$
- L is the derivative, $L \equiv 3n_0 S'(n_0)$

- $4S(n) = \partial_\alpha\left[\mu_n(n,\alpha) - \mu_p(n,\alpha)\right]$
- $\tilde{S}(n)$ probed in neutron matter,
  $S(n)$ probed in nuclei, e.g. isovector giant dipole

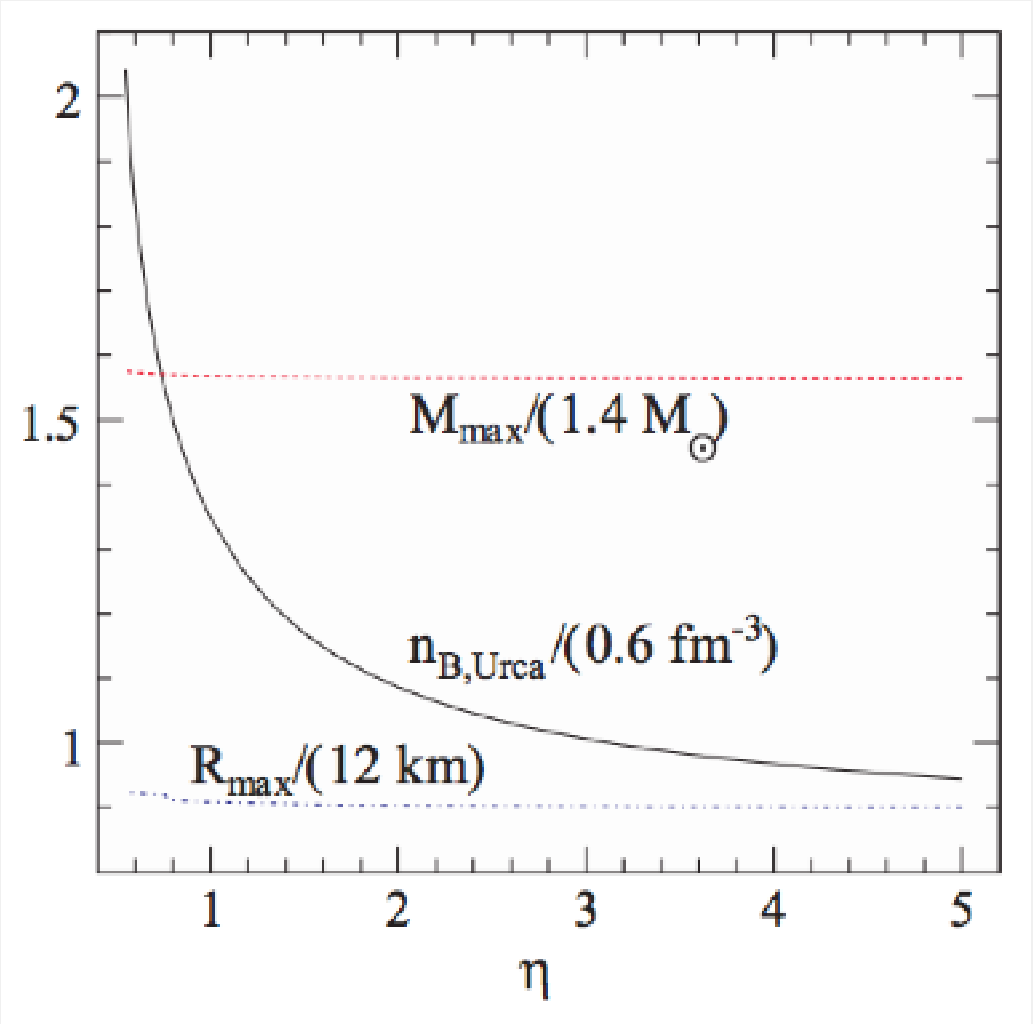
Quartic Terms and the Direct Urca Process

$$(E/A)(n,\alpha) = (E/A)_{\rm nuc}(n,\alpha) + \alpha^2 S(n) + \alpha^4 Q(n)$$

- Below saturation, quartic terms are likely "small", above saturation densities, they may be important

$$\eta(n) \equiv \frac{4S(n) + 5Q(n)}{4S(n) + Q(n)}$$

- $3/7 < \eta(n) < 5$
- Complicates connection between symmetry energy and direct Urca
- Superfluidity also very important, and depends on L



Steiner (2006)

Extrapolation: The Tail Wags the Dog

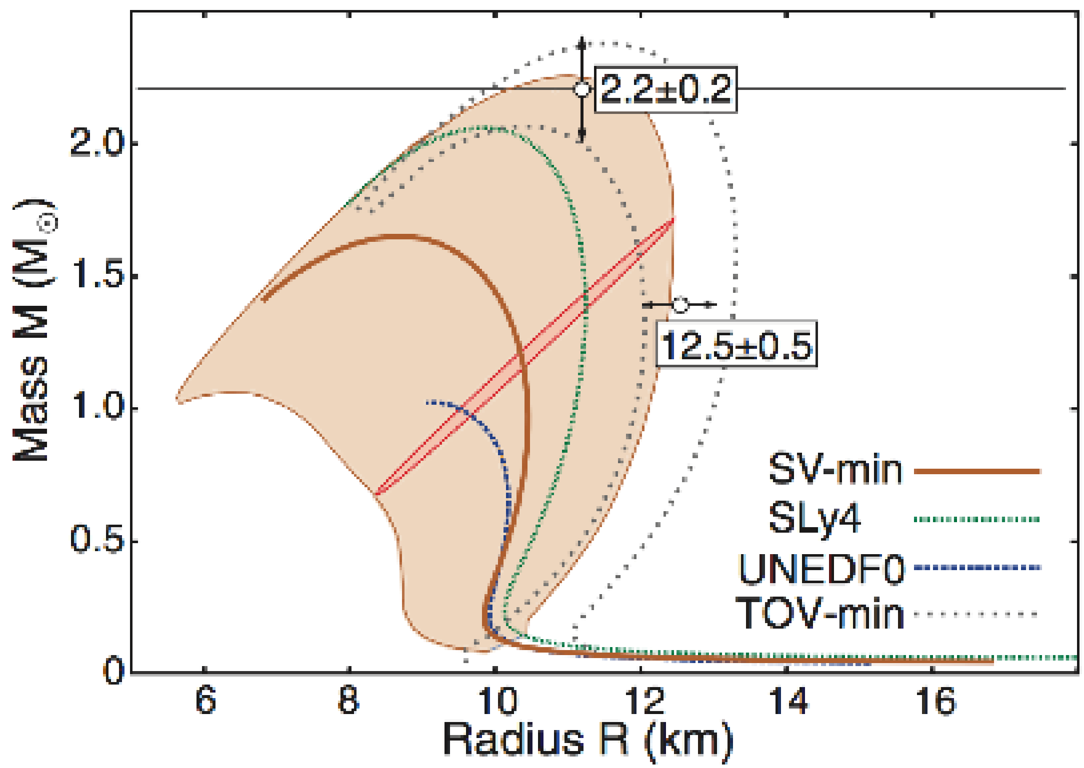


FIG. 2. (Color online) Mass-radius relation of SLy4 [1], UNEDF0 [22] and SV-min [24]. The uncertainty band for SV-min is shown. This band is estimated by calculating the covariance ellipsoid for the mass M and the radius R at each point of the SV-min curve as indicated by the ellipsoid. Also depicted (dotted lines) are uncertainty limits for TOV-min.

Erler et al. (2013)

- Many works extrapolate models of the nucleon-nucleon interaction up to very large densities
- Bayesian inference can alleviate this problem
- Arrange prior distribution to properly reweigh the dog and its tail
- Important issue when fitting disparate data sets to one model
- Related to "overfitting"

Implications for L and radii

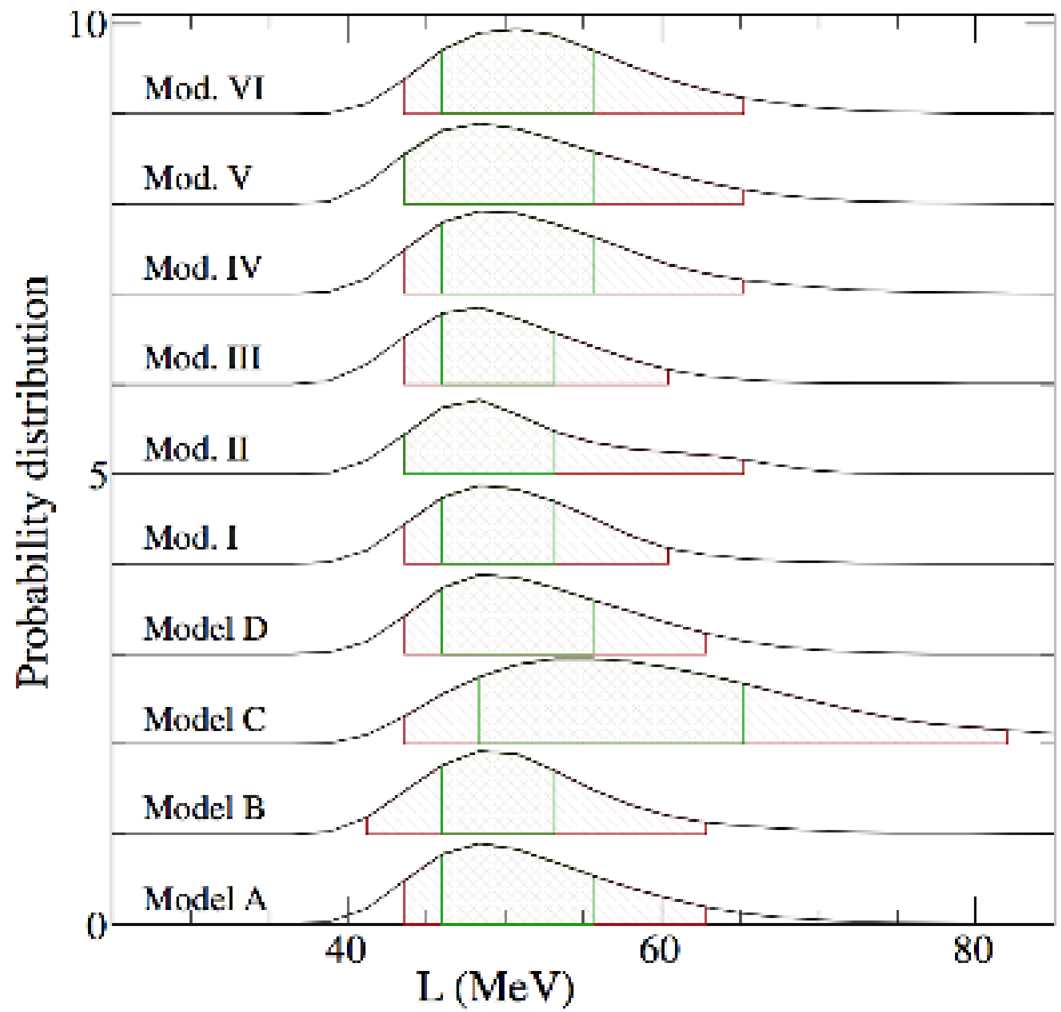


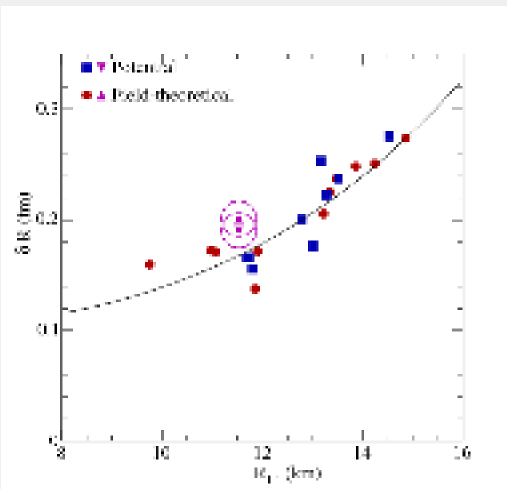
Figure 4. Limits on the density derivative of the symmetry energy, L. The single-hatched (red) regions show the 95% confidence limits and the double-hatched (green) regions show the 68% confidence limits.

Steiner et al. (2013)

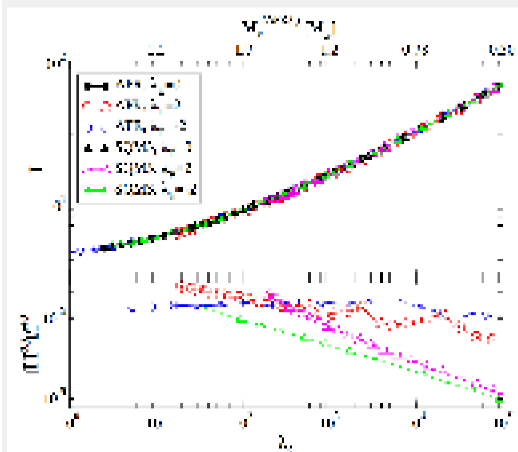
- Trying several different prior distributions
- Model C prefers strong phase transitions
- Less certain constraints on the skin thickness of lead
- Radius of a 1.4 $M_\odot$ neutron star is about 10.5 - 13 km

Representative Models

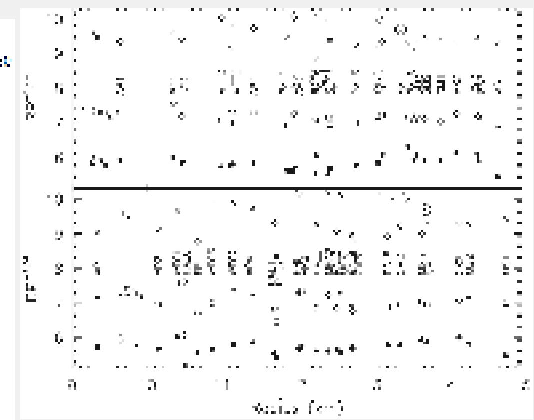
- It has been common to use a small set of models to represent a large parameter space



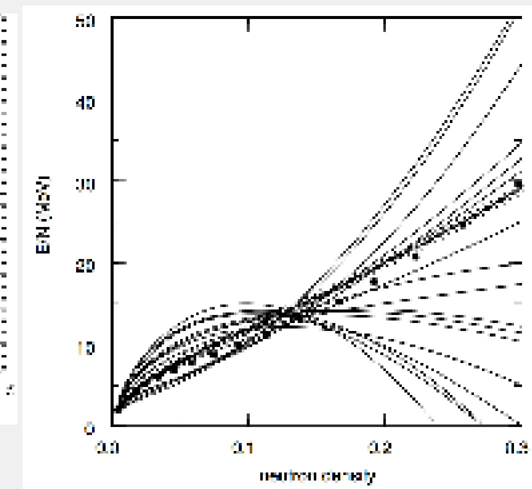
Steiner et al. (2005)



Yagi and Yunes (2015)



Lattimer and Prakash (2001)



Brown (2000)

- This is like doing a many-dimensional Monte Carlo integral with only a handful of haphazard points

- New computational power has allowed for more complete explorations of parameter space