Beam Normal Spin Asymmetry: Theory Trivia

Misha Gorshteyn - Institut für Kernphysik, Mainz U.

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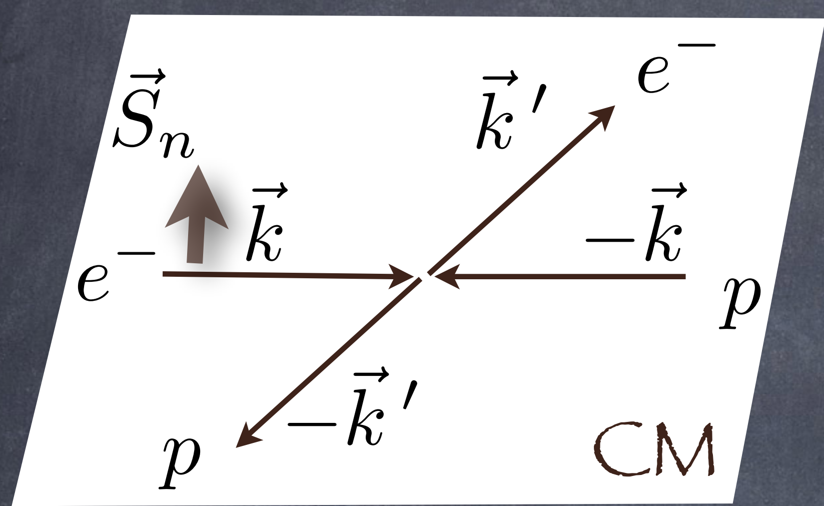
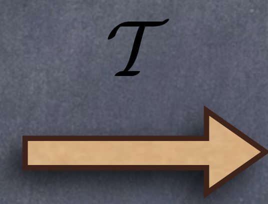
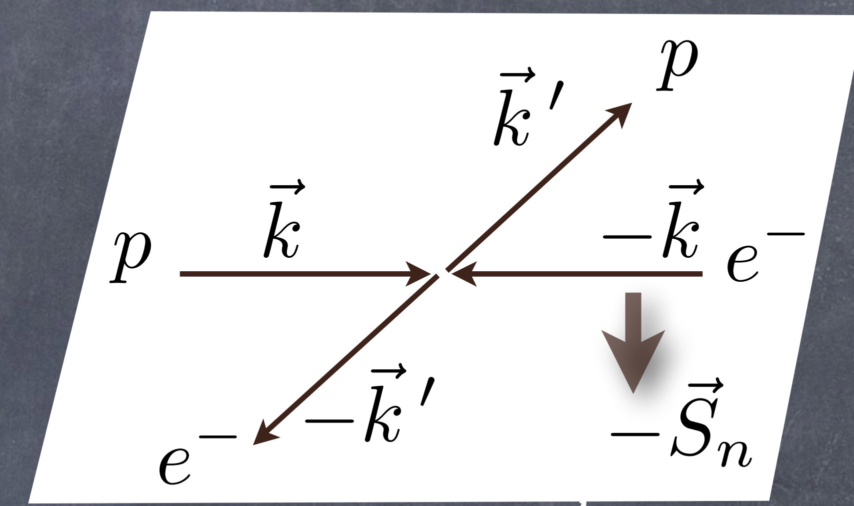
Elastic e-p scattering with e polarized normal to the reaction plane

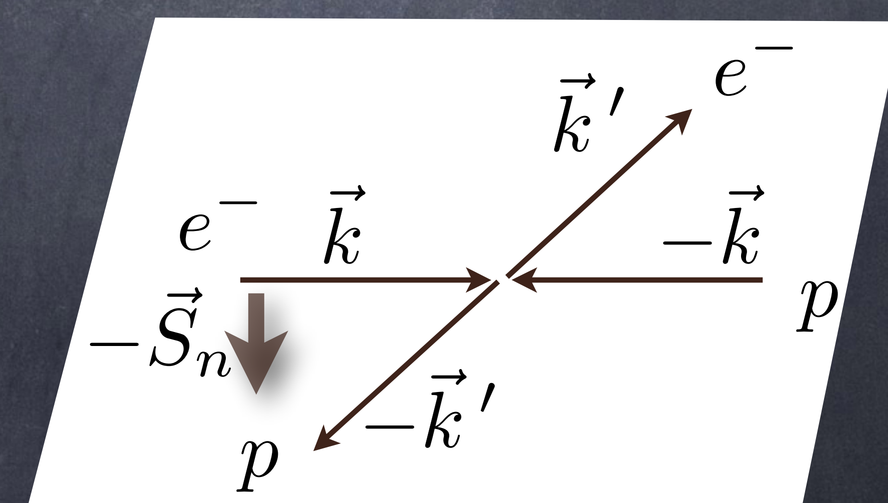






180° rotation around y-axis

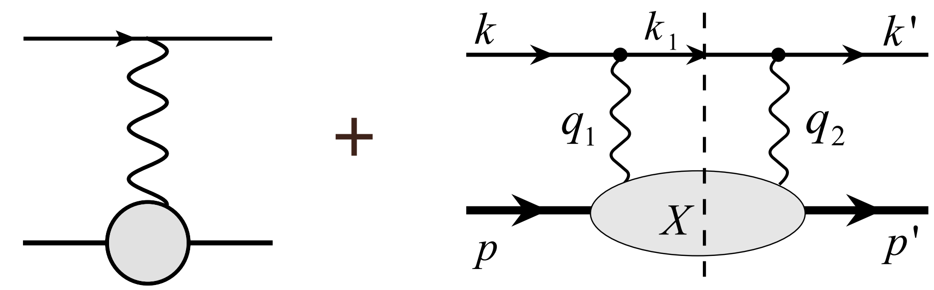
$$T(S_n, \vec{k}, \vec{k}') \to \eta_1 T^*(-S_n, -\vec{k}, -\vec{k}') \to \eta_1\eta_2 T^*(-S_n, \vec{k}, \vec{k}')$$



Mismatch between time-reversed states is due to imaginary part of the amplitude (in absence of CP- and CPT-violation)

Elastic e-p scattering in presence of two-photon exchange

$$T_{ep} = T_{1\gamma} + T_{2\gamma} + \mathcal{O}(\alpha^2)$$



Purely real

Has imaginary part

$$\begin{aligned} T_{ep}(S_n = +1/2) &= T_{1\gamma} + T_{2\gamma} \\ T_{ep}(S_n = -1/2) &= T_{1\gamma} + T_{2\gamma}^* \end{aligned}$$

$$B_n = \frac{\sigma(S_n = +1/2) - \sigma(S_n = -1/2)}{\sigma(S_n = +1/2) + \sigma(S_n = -1/2)} = \frac{2\mathrm{Im}\,(T_{1\gamma}^* T_{2\gamma})}{|T_{1\gamma}|^2}$$

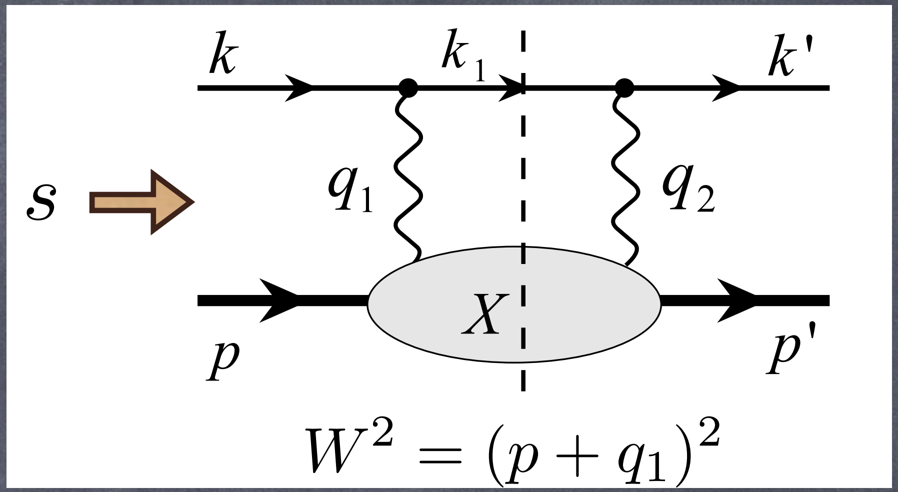
De Rújula et al., NP B35 (1971) 365

$$B_n \sim \alpha \frac{m}{E} \sim 10\, p.p.m.\, \frac{500 MeV}{E}$$

Important background to PVES measurements
(sub- to few p.p.m. asymmetries)

Theoretical understanding of the Bn

Im part of the TPE diagram:
on-shell hadronic states only
- integral over data (kinda)



$$\mathrm{Im}T_{2\gamma} = e^4 \int \frac{d^3\vec{k}_1}{2E_1(2\pi)^3} \frac{\bar{u}(k')\gamma_\nu(\not{k}_1 + m_e)\gamma_\mu u(k)}{Q_1^2 Q_2^2} \mathrm{Im}W^{\mu\nu}(W^2, Q_1^2, Q_2^2, t)$$

$W^{\mu\nu}$ - spin-independent doubly virtual Compton tensor in the arbitrary kinematics;
In general unknown (depends on inclusive excited states);
Study the integrals to see where the input is necessary.

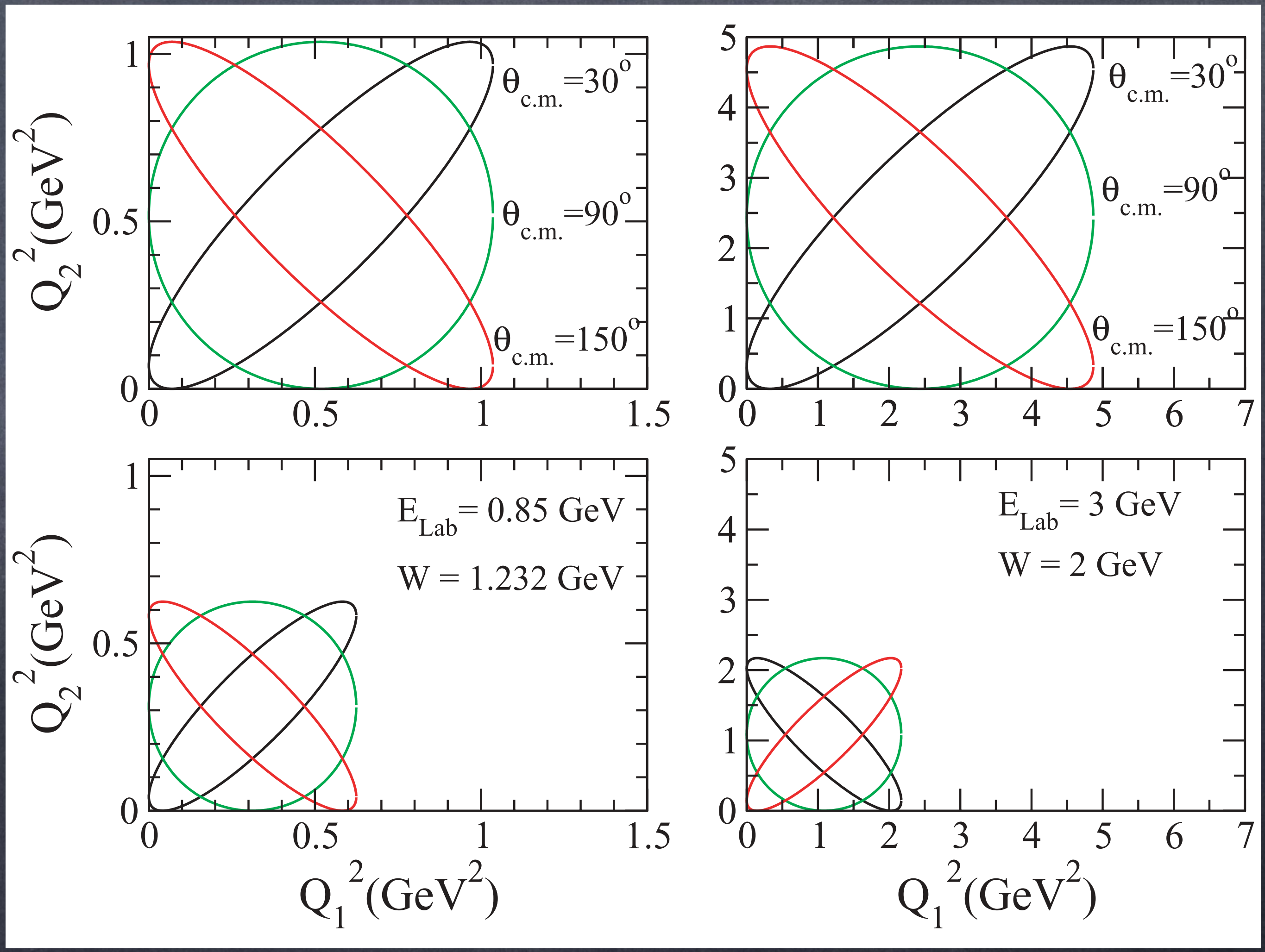
Where is the input needed?

$$d^3\vec{k}_1 \rightarrow \int_{M^2}^{s} dW^2 \int_0^{(Q_1^2)^{max}} dQ_1^2 \int_{(Q_2^2)^{min}}^{(Q_2^2)^{max}} dQ_2^2$$

Elastic (on-shell nucleon) - W=M

Inelastic - $(M+m_\pi)^2 < W^2 < s$

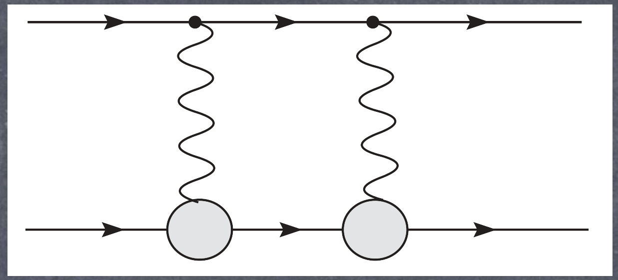
$Q^2{}_{min} < Q_{1,2}{}^2 < Q^2{}_{max}$



For $s = W^2$: $Q_{1,2}{}^2 = 0$

Bn with elastic intermediate states

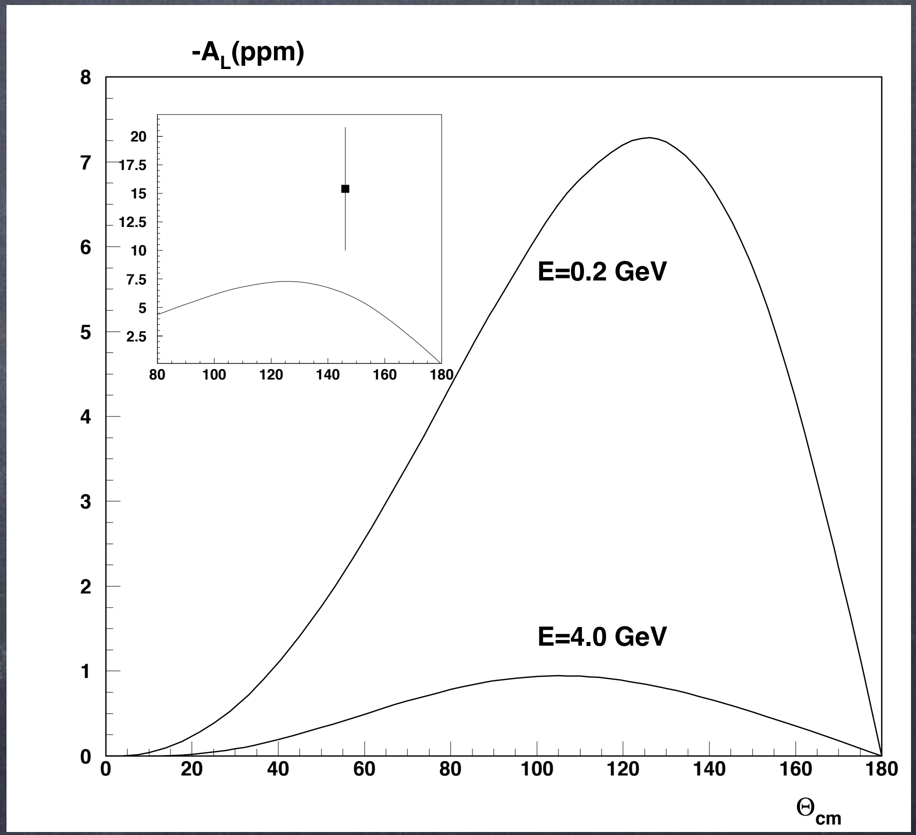
On-shell ground state inside the box



Im $W^{\mu\nu}$ - exactly calculable in terms of measured form factors G_E, G_M, F_C, ...

Bn @ SAMPLE: H-target, E = 200 MeV, θ = 150 deg.

Exp. point: S. Wells et al, PRC 63 (2001) 064001
Curves (proton): A. Afanasev et al, hep-ph/0208260



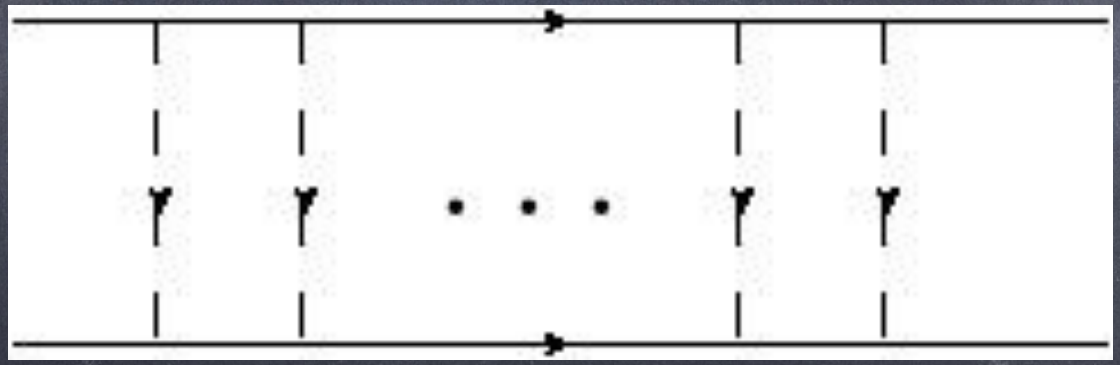
- IR-finite
- 1/E behavior
- larger at backward angles

Bn with elastic intermediate states

For spin=0 target - overlap of two Coulomb densities separated by a distance fixed by the external kinematics

$$B_n \approx -\frac{m_e}{E}\frac{Z\alpha}{\pi}\tan^3\frac{\theta_{cm}}{2}\int\int\frac{dQ_1^2 dQ_2^2}{\sqrt{(Q_+ - Q_2^2)(Q_2^2 - Q_-)}}\frac{Q_1^2 + Q_2^2 - Q^2}{2Q_1^2 Q_2^2}\frac{F_C(Q_1^2)F_C(Q_2^2)}{F_C(Q^2)}$$

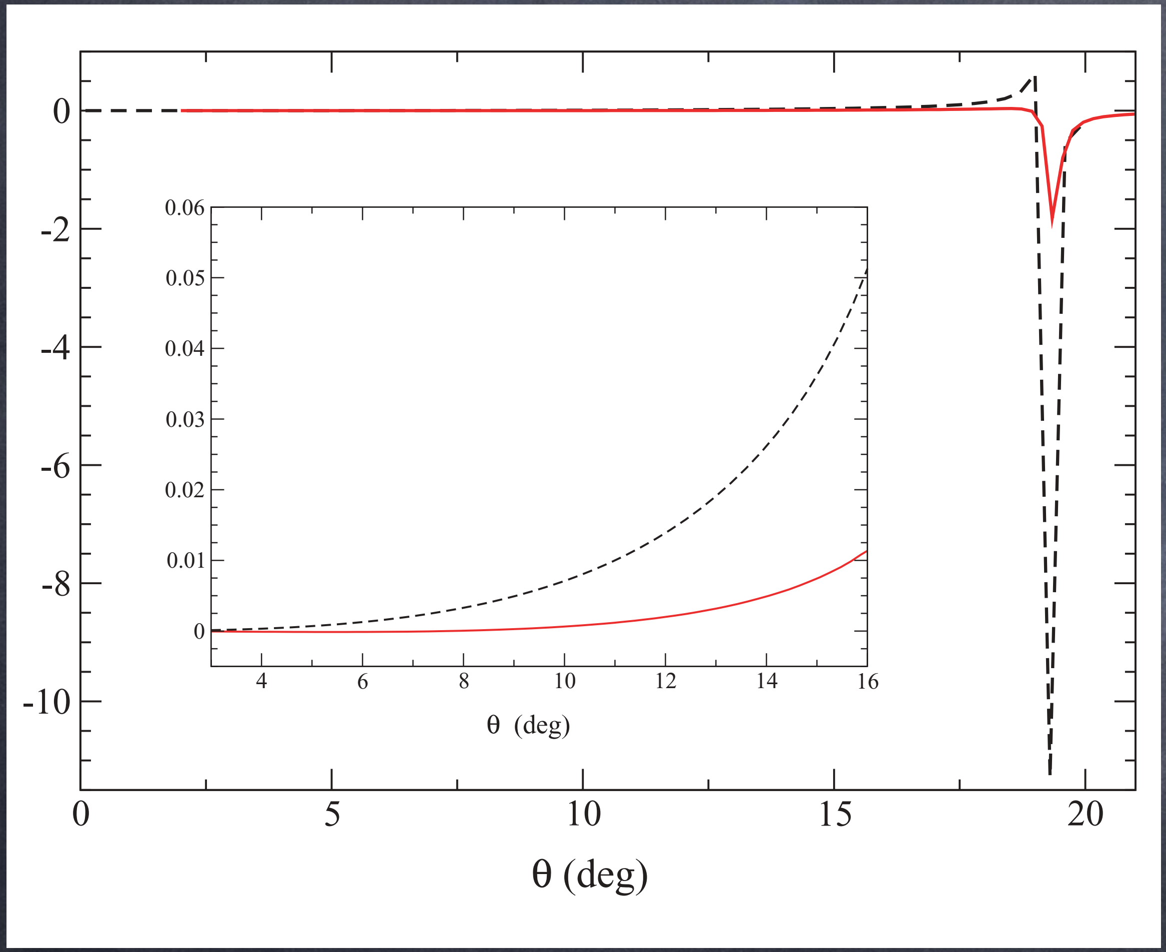
For highly-charged nuclei higher orders $O(Z\ \alpha)^n$ need to be resumed
- Coulomb distortions



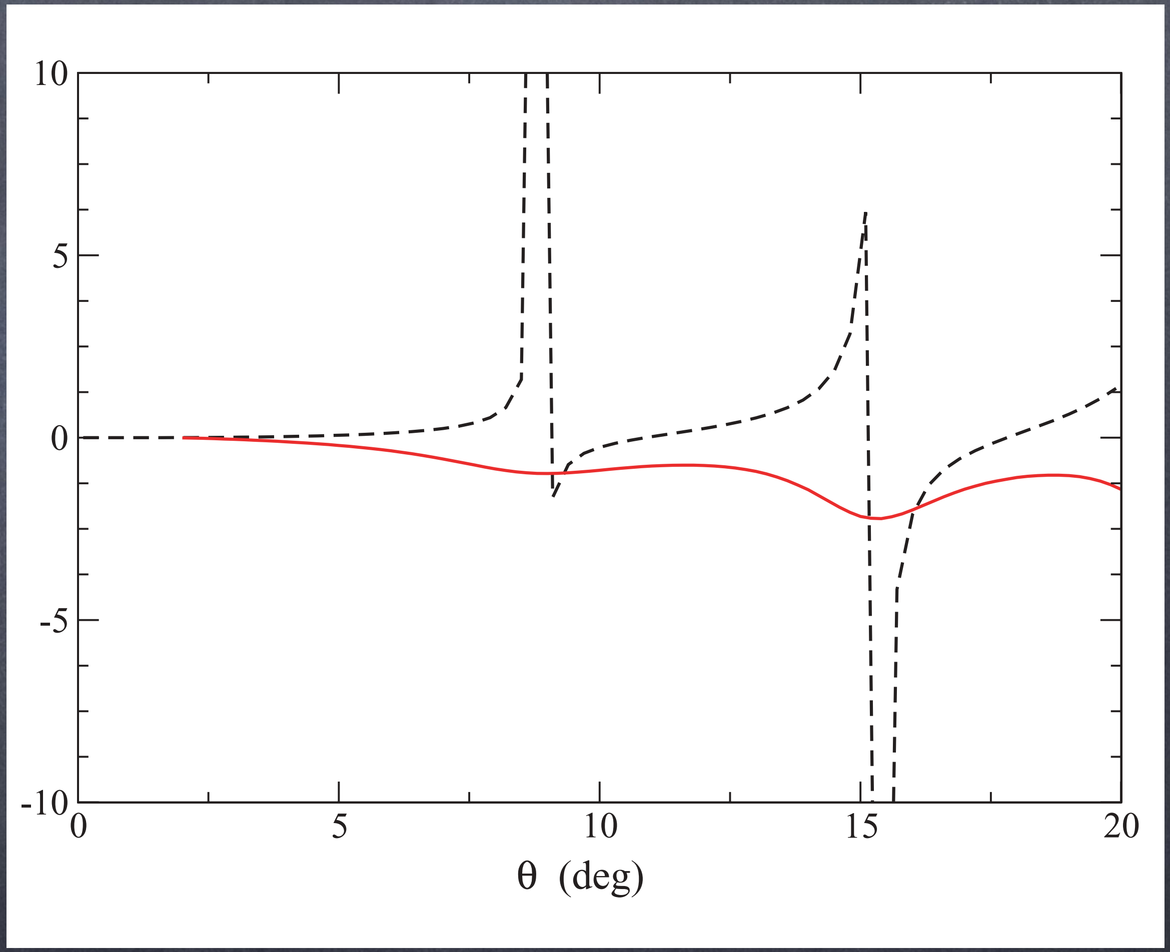
Was done for Bn on He-4, Pb-208, … by Cooper, Horowitz, PRC 72 (2005)
- solve Schrödinger eq. with phenomenological charge densities

Leading order in α (two photon exchange) vs. Coulomb distortion effects (resummed infinite photon exchange)

Bn (p.p.m.) for He-4, E = 3 GeV



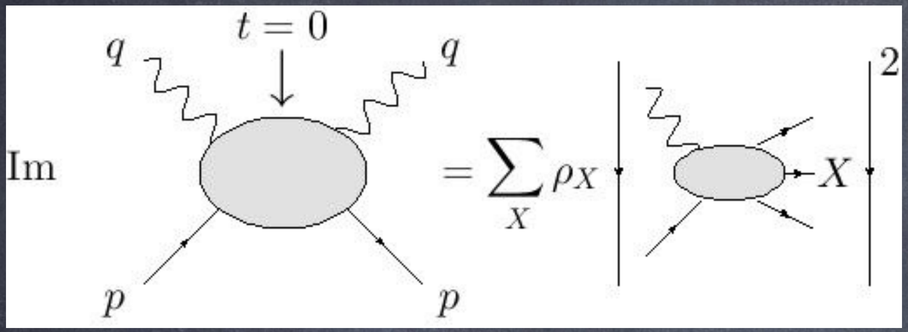
Bn (p.p.m.) for Lead, E = 855 MeV



Curves: Cooper, CJH, PRC 72 (2005) MG, CJH, PRC 77 (2008)

Inelastic states: forward angles

Forward spin-independent Compton tensor - from Optical Theorem: can use inelastic data as direct input



$$W^{\mu\nu} = 2\pi\left[-g^{\mu\nu}F_1^{\gamma\gamma} + \frac{P^\mu P^\nu}{(P\cdot q_1)}F_2^{\gamma\gamma}\right]$$

Bn features a large $\log(Q^2/m^2)$ - comes from small Q^2 but all W's

$$I = \int \frac{d\Omega_1}{Q_1^2 Q_2^2} = \frac{2\pi}{Qk_1\sqrt{Q^2k_1^2 + 4m_e^2(E-E_1)^2}} \ln \frac{\sqrt{Q^2k_1^2 + 4m_e^2(E-E_1)^2} + Qk_1}{\sqrt{Q^2k_1^2 + 4m_e^2(E-E_1)^2} - Qk_1}$$

Finite result protected by m_e and $E - E_1 >$ inelastic threshold

$$I(Q^2 \gg m_e^2) = \frac{2\pi}{Q^2k_1^2}\ln\frac{Q^2k_1^2}{m_e^2(E-E_1)^2}$$

$$I(Q^2 \ll m_e^2) = \frac{\pi}{m_e^2(E-E_1)^2} \qquad I(k_1 \to 0) = \frac{\pi}{m_e^2E^2}$$

Inelastic states: forward angles

Correct the input for (slightly) off-forward kinematics

Phenomenological input: Compton differential cross section

$$\frac{d\sigma^{\gamma p\to\gamma p}}{dt} \approx \left.\frac{d\sigma^{\gamma p\to\gamma p}}{dt}\right|_{t=0} \times exp[Bt]$$

$$t = -Q^2$$

$$\left.\frac{d\sigma^{\gamma p\to\gamma p}}{dt}\right|_{t=0} \sim |T_{\gamma p\to\gamma p}(t=0)|^2$$

$$\sigma_{\gamma p\to X} \sim \mathrm{Im}T_{\gamma p\to\gamma p}(t=0)$$

$$\Longrightarrow \quad \sigma_{\gamma p\to X} \to \sigma_{\gamma p\to X} \times exp[Bt/2]$$

Afanasev, Merenkov, PRD 70 (2004); PL B599 (2004);
MG, PRC 73 (2006); PL B644 (2007);
MG, Horowitz PRC 77 (2008)

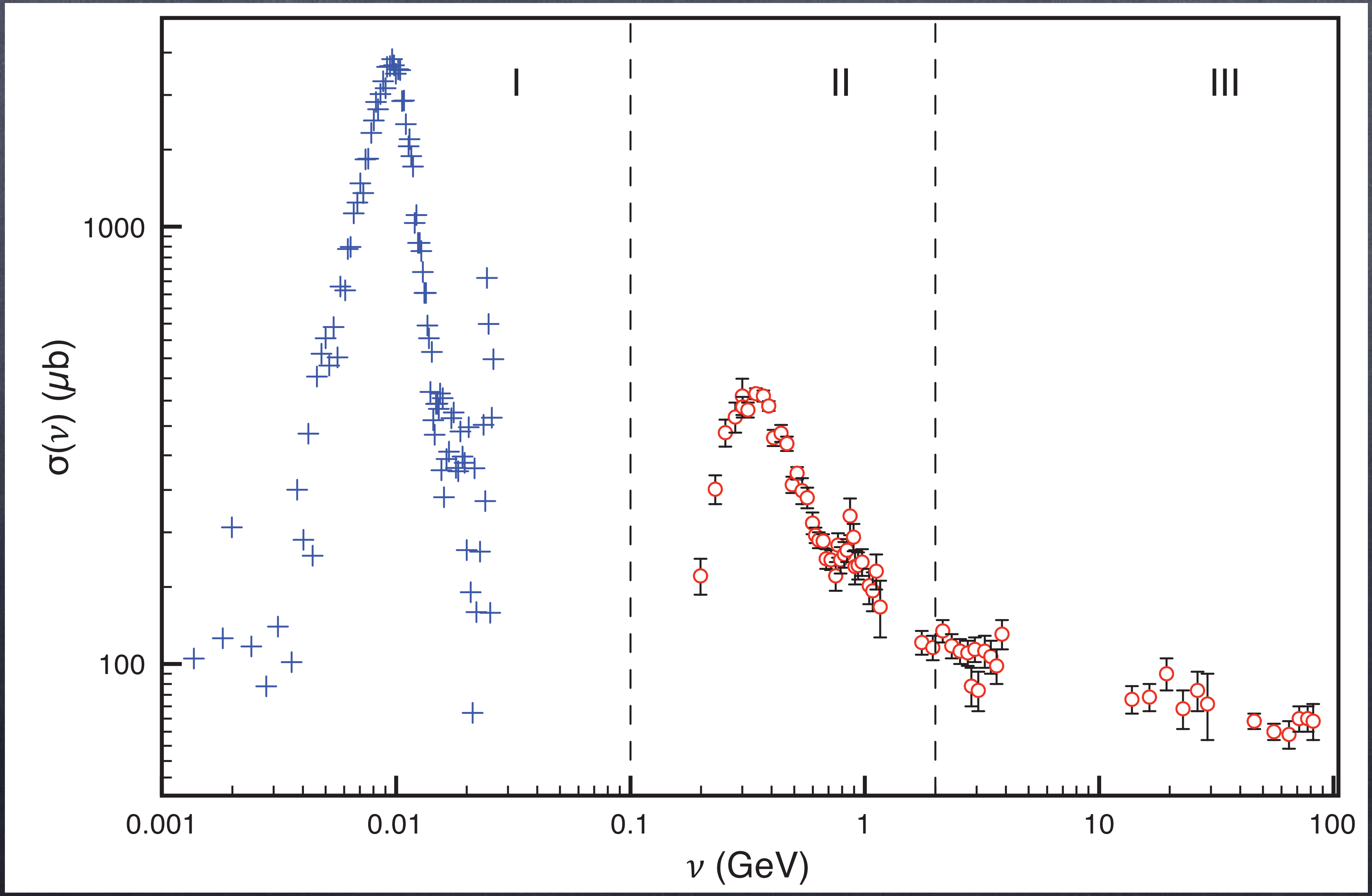
$$B_n^{inel} \approx -\frac{1}{4\pi^2}\frac{m_e Q}{E^2}\frac{A}{Z}\int_{\omega_\pi}^{E} d\omega\,\omega\,\sigma_{\gamma N}(\omega)\ln\left(\frac{Q^2}{m_e^2}\frac{(E-\omega)^2}{\omega^2}\right)\frac{e^{-BQ^2/2}}{F_C(Q^2)}$$

Real photoabsorption cross section per nucleon

Dominance of $\log(Q^2/m^2)$ is assumed - only real photoabsorption!
Finite virtualities in the loop - suppressed by an extra Q^2

If hadronic contributions dominate - good for nuclei, too;
If low-lying nuclear states are important - inadequate

Total photoabsorption by Pb-208 (per nucleon in μbarn)



If hadronic contributions dominate - good for nuclei, too;
If low-lying nuclear states are important - inadequate

$$B_n^{inel} \approx -\frac{1}{4\pi^2}\frac{m_e Q}{E^2}\frac{A}{Z}\boxed{\int_{\omega_\pi}^{E} d\omega\omega\sigma_{\gamma N}(\omega)}\ln\left(\frac{Q^2}{m_e^2}\frac{(E-\omega)^2}{\omega^2}\right)\frac{e^{-BQ^2/2}}{F_C(Q^2)}$$

Hadronic contributions enhanced due to energy weighting

Check for PB-208

Tables from Harvey et al., PR 136 (1964)

TABLE I. Integrated cross sections in MeV-b, up to 28 MeV, for Pb isotopes and Bi.

Isotope	$\int_0^{28}\sigma(\gamma,n)dE$	$\int_0^{28}\sigma(\gamma,2n)dE$	$\int_0^{28}\sigma dE$	$\int_0^{28}\sigma dE+W$	$0.06NZ/A$
Pb^{206}	2.22	0.56	2.78±0.28	3.07±0.36	2.96
Pb^{207}	2.05	0.60	2.65±0.27	2.95±0.30	2.97
Pb^{208}	1.96	0.95	2.91±0.29	3.21±0.32	2.98
Bi^{209}	2.17	0.76	2.93±0.29	3.25±0.33	3.00

TABLE II. Lorentz line parameters and σ_{-2} values for Pb isotopes and Bi.

Isotope	Peak σ_0 (mb)	Width Γ (MeV)	E_0 (MeV)	σ_{-2} (mb/MeV)	$0.00225A^{5/3}$ (mb/MeV)
Pb^{206}	525	3.75	13.7	15.6±1.6	16.2
Pb^{207}	485	3.87	13.6	14.5±1.5	16.3
Pb^{208}	495	3.78	13.6	14.1±1.4	16.4
Bi^{209}	520	3.83	13.5	16.6±1.7	16.6

$$\alpha_D = \frac{1}{2\pi^2}\int\frac{d\omega}{\omega^2}\sigma(\omega) \approx 20 fm^3$$

$$\frac{1}{2\pi^2}\int d\omega\sigma(\omega) \approx 0.15\text{MeV barn}$$

$$\longrightarrow \quad \frac{1}{2\pi^2 A}\int d\omega\omega\sigma(\omega) \approx 3\times 10^{-5}$$

Nuclear photoabsorption in the hadronic range

Energies up to 1 GeV:
integrated cross section scales as A;

Nuclear shadowing stronger above that energy

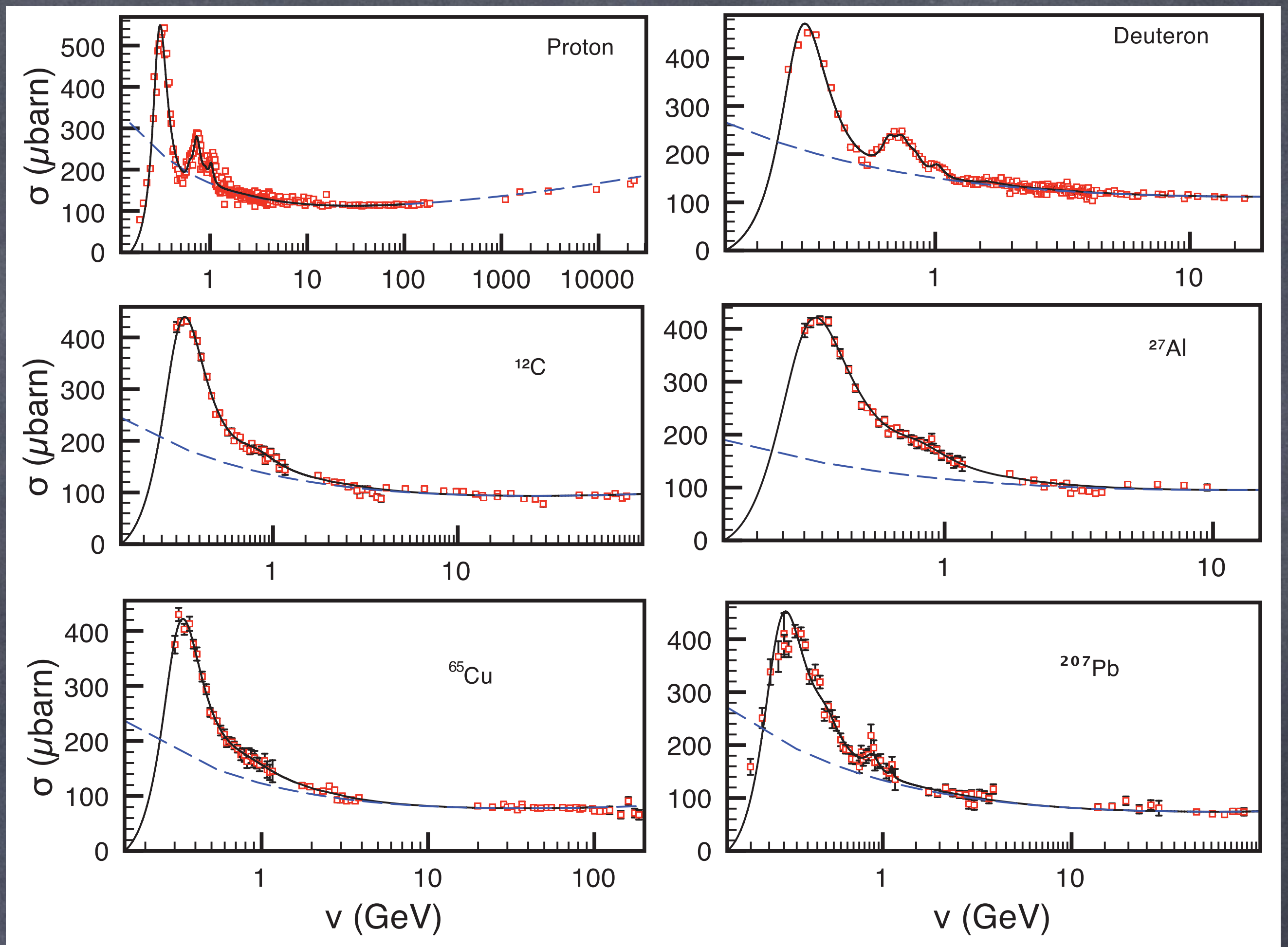


TABLE II. Contributions to the finite energy sum rule for selected targets in units of GeV·μb. The entries in the second row are taken from a review on nuclear data in Ref. [24].

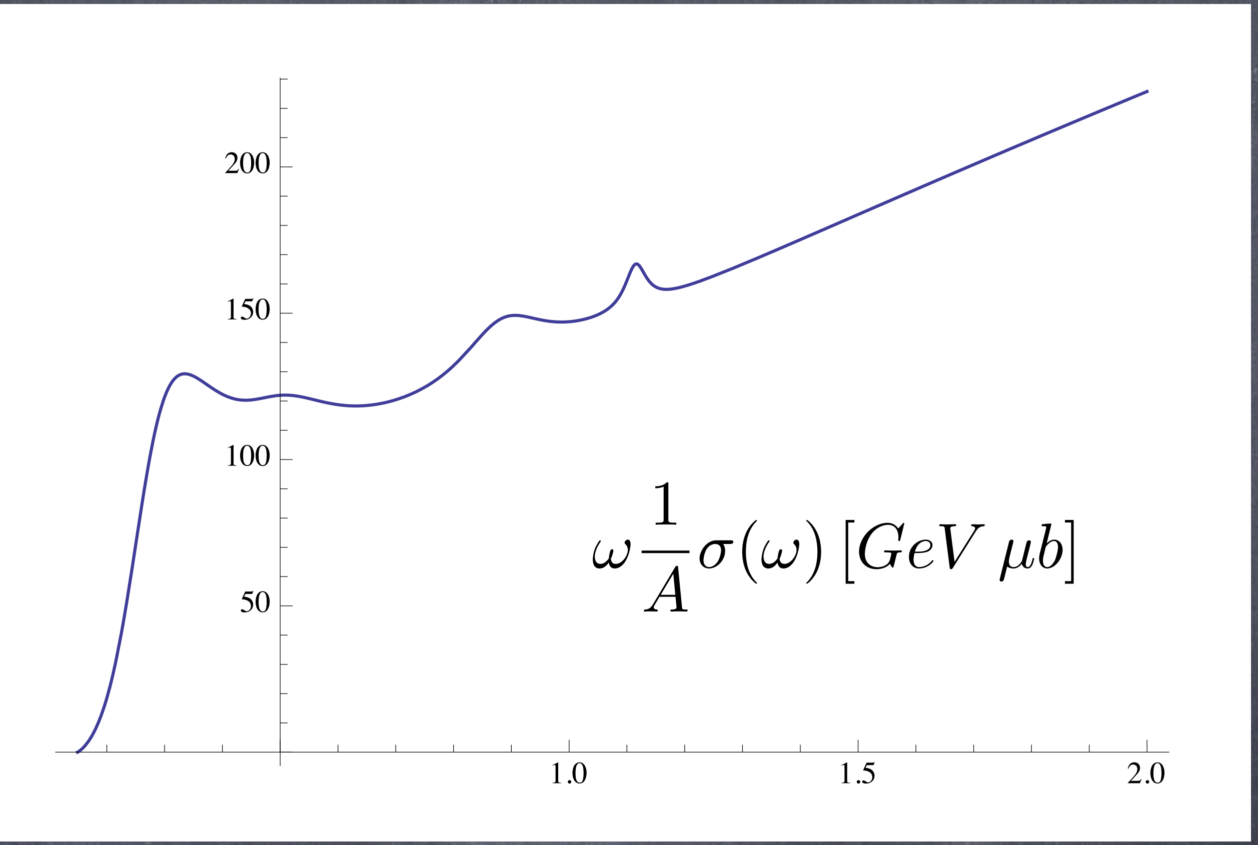
	Proton	Deuteron	$^{12}_{6}$C	$^{27}_{13}$Al	$^{65}_{29}$Cu	$^{207}_{82}$Pb
$\frac{1}{2\pi^2 A}\sigma_{\mathrm{int}}^{\mathrm{had}}$	18.60 ± 0.31	17.46 ± 0.51	16.80 ± 0.62	16.54 ± 1.50	16.16 ± 0.57	16.57 ± 1.02
$\frac{1}{2\pi^2 A}\sigma_{\mathrm{int}}^{\mathrm{nucl}}$	–	–	0.197	0.30	0.480	0.69
$\frac{1}{2\pi^2} c_R \frac{(E/GeV)^{1/2}}{1/2}$	14.19 ± 0.16	11.54 ± 0.39	10.96 ± 0.63	7.67 ± 1.66	11.03 ± 0.52	13.10 ± 1.31
r.h.s. of Eq. (17)	-4.21 ± 0.35	-5.92 ± 0.65	-6.04 ± 0.88	-9.17 ± 2.24	-5.61 ± 0.77	-4.16 ± 1.66
$-\left(2 + \frac{ZN}{A^2}\right)\frac{\alpha}{M}$	-6.06	-6.82	-6.82	-6.82	-6.81	-6.78
$\frac{1}{2\pi^2} c_P (E/GeV)$	6.72 ± 0.02	6.92 ± 0.12	5.65 ± 0.11	6.19 ± 0.59	4.53 ± 0.06	4.16 ± 0.25
$-\frac{Z^2}{A^2}\frac{\alpha}{M}$	-3.03	-0.76	-0.76	-0.70	-0.60	-0.48
$\mathrm{Re}T^{\alpha=0}$	-0.72 ± 0.35	0.25 ± 0.65	-1.14 ± 0.89	-3.68 ± 2.31	-1.71 ± 0.77	-0.48 ± 1.68

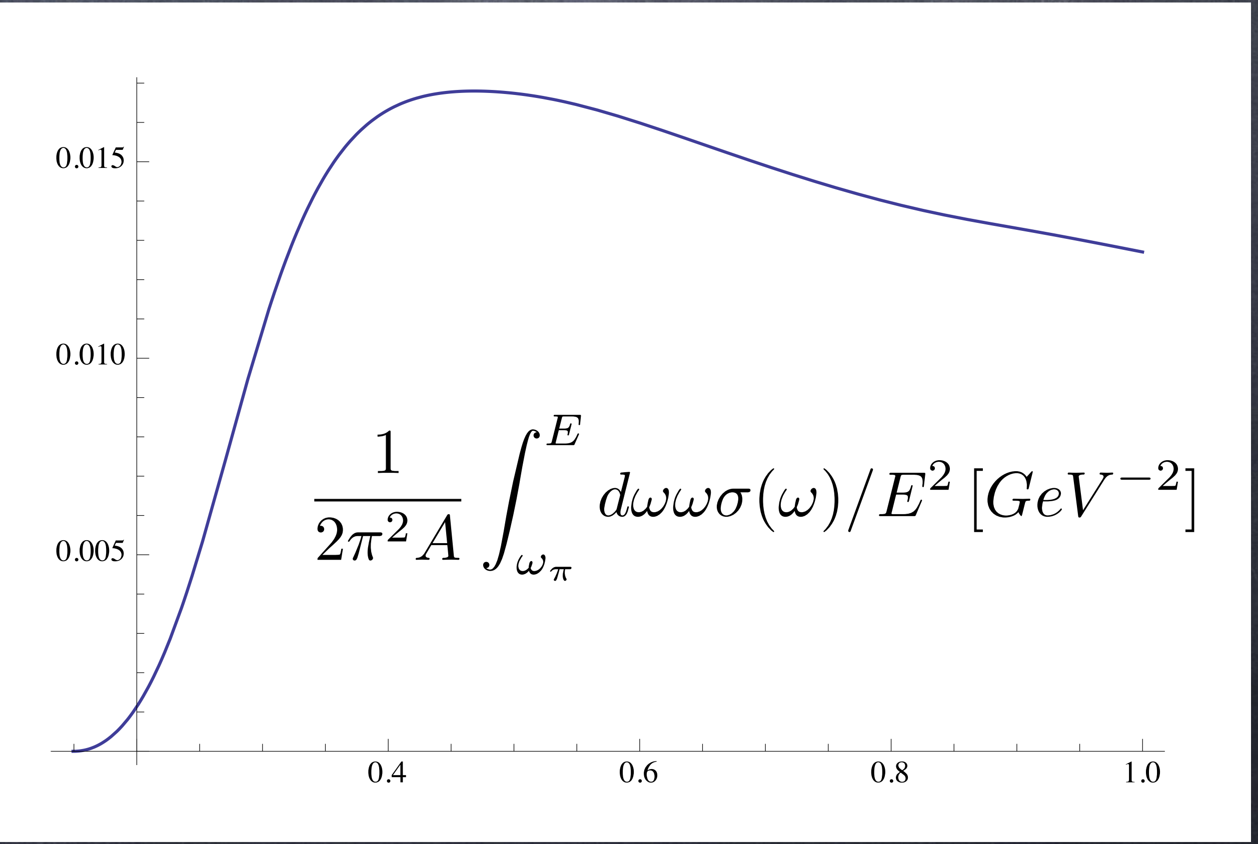
MG, Hobbs, Londergan, Szczepaniak, PRC 84 (2011)

Nuclear photoabsorption in the hadronic range

$$\frac{1}{2\pi^2 A}\int_{\omega_\pi}^{1\,GeV} d\omega\omega\sigma(\omega) \approx 0.012$$

$$\frac{1}{2\pi^2 A}\int_{0}^{28\,MeV} d\omega\omega\sigma(\omega) \approx 3\times 10^{-5}$$





Assumptions seem to be OK

Compton slope parameter B

$$\frac{d\sigma^{\gamma p \to \gamma p}}{dt} \approx \left.\frac{d\sigma^{\gamma p \to \gamma p}}{dt}\right|_{t=0} \times exp[Bt]$$

Known for proton and He-4

$$B(A=1) = (7 \pm 1)\, GeV^{-2}$$ Bauer et al, Rev.Mod.Phys. 50 (1978)

$$B(A=4) = (32 \pm 2)\, GeV^{-2}$$ Alexanian et al, Sov. J. Nucl.Phys. 45 (1987)

What matters is Compton to elastic form factor ratio!

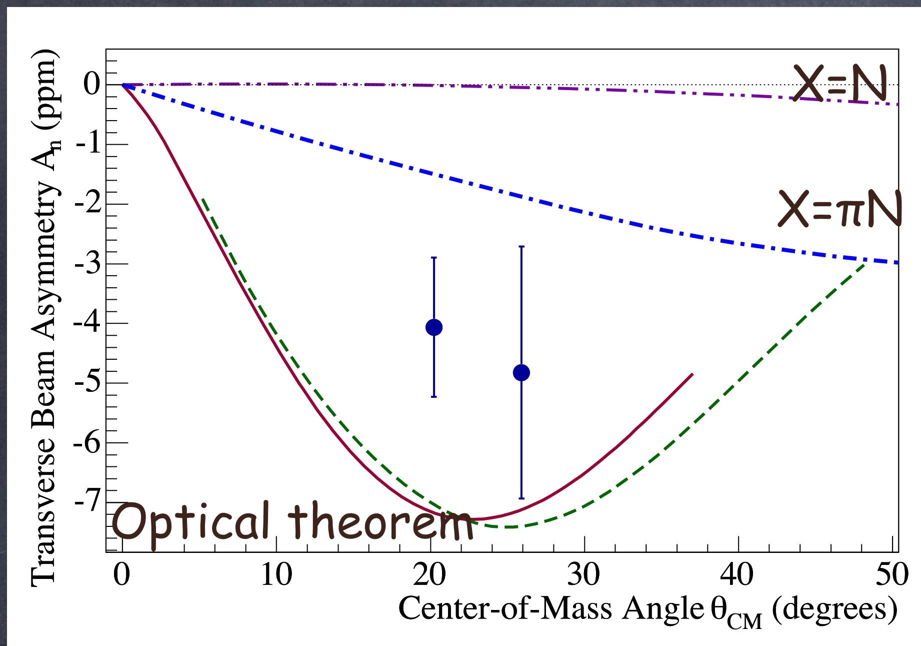
Proton: $$\frac{Exp[-BQ^2/2]}{F(Q^2)} = Exp[-(3.5 \pm 0.5)(Q^2/GeV^2)]\,(1 + (Q^2/0.71\, GeV^2))^2$$

Helium-4: $$\frac{Exp[-BQ^2/2]}{F(Q^2)} = \frac{Exp[-(16 \pm 1)(Q^2/GeV^2)]}{Exp[-11.74(Q^2/GeV^2)](1 - [2.573Q^2/GeV^2]^6)}$$

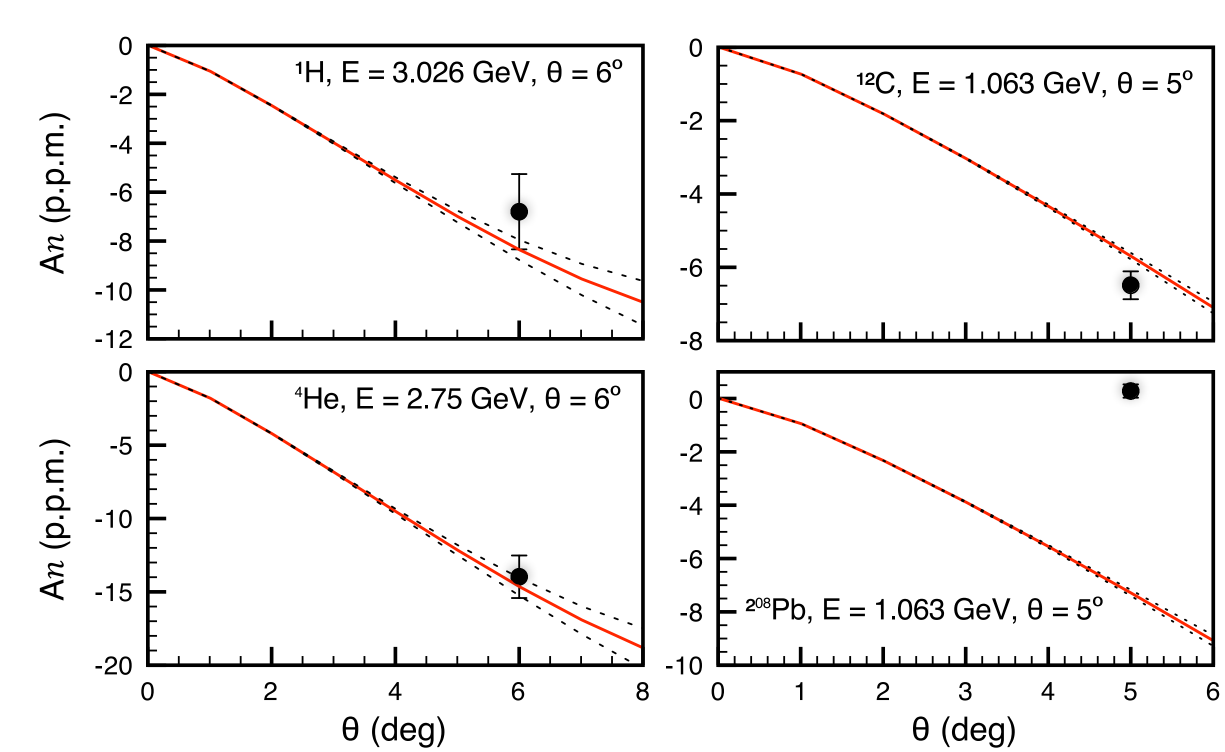
The ratio was taken roughly the same for all nuclei;
Numerically is irrelevant at low Q^2

Bn at forward angles

H, E = 3 GeV

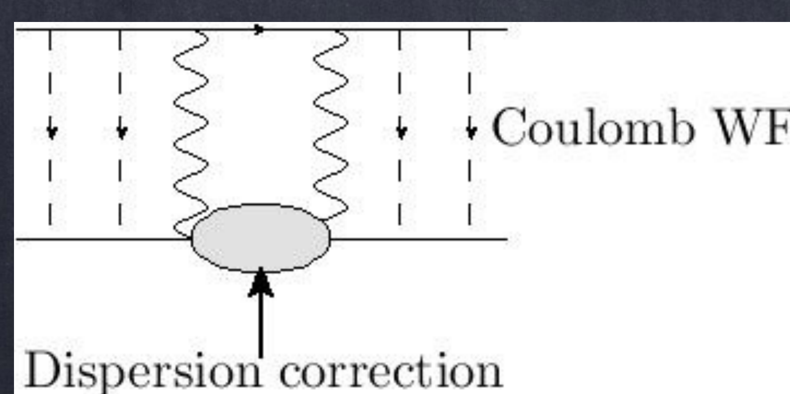


Armstrong et al [G0], '05



Abrahamyan et al. [HAPPEX and PREx], 12

Excellent description for light nuclei and very forward angles;
Somewhat worse for larger angles (G0) - Q^2 not small anymore, include corrections
Fails completely for lead - realistic nuclear calculation needed

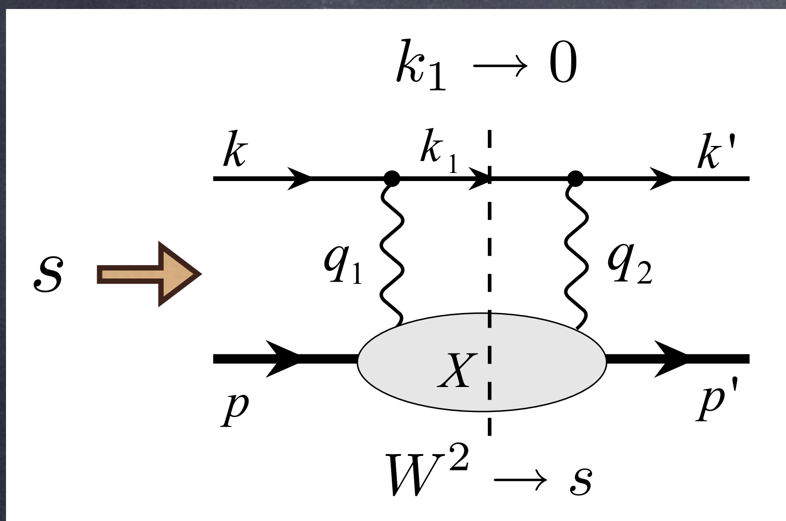


Bn Summary

- Bn measures the Im part of TPE amplitude;
  - few to few 10's p.p.m. effect - background for PVES;
  - forward: unitarity model - great job except Pb;
  - Compton slope - the only free parameter;
  - Carbon news: theory below data at 570 MeV, 20 deg.
  - Needed: beyond two-photon exchange and beyond the standard (ground state) Coulomb distortion

Bn at backward angles

Equivalent photon kinematics



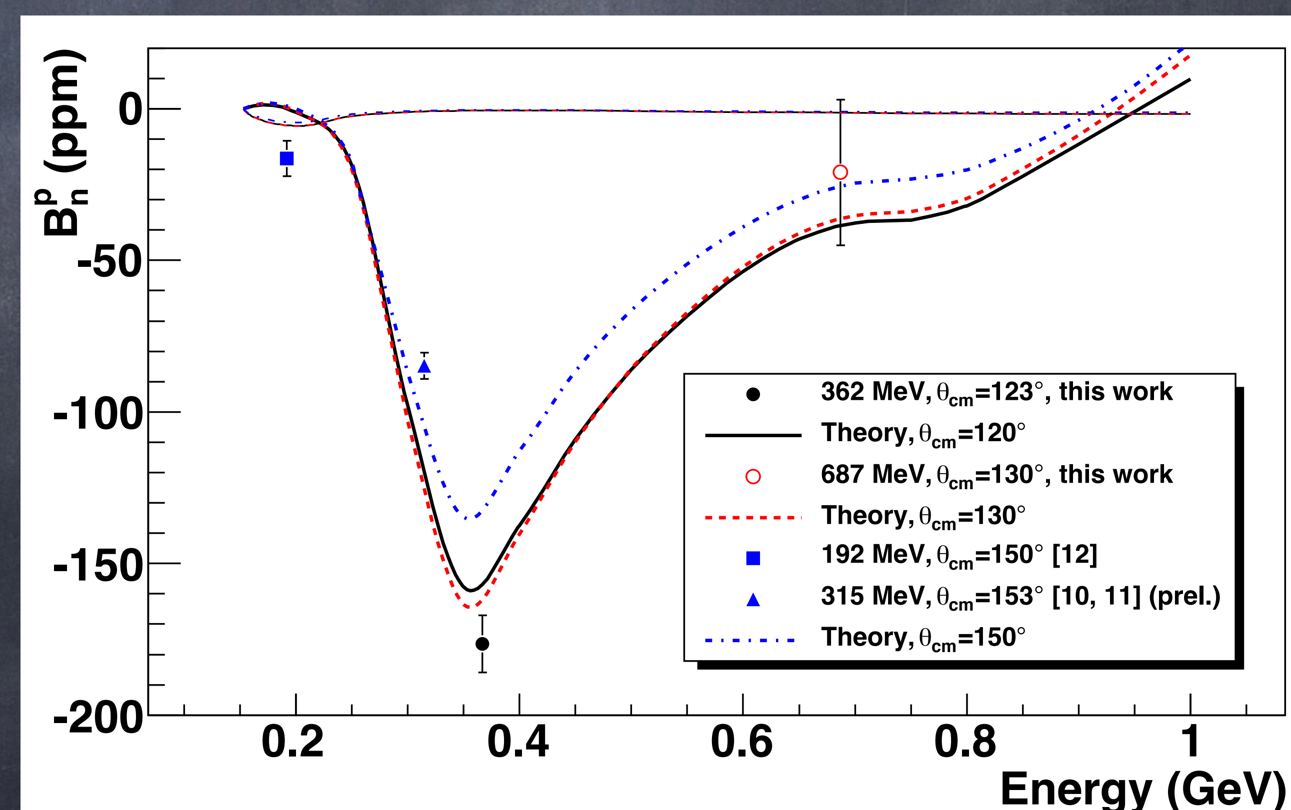
$$q_1 \to k \qquad q_2 \to k'$$
$$Q^2_{1,2} \quad \to \quad 0$$
$$W^{\mu\nu}(W^2, Q_1^2, Q_2^2) \to W^{\mu\nu}(s, 0, 0)$$

$$B_n^{inel} \sim -\frac{1}{32\pi^2}\frac{m}{E}\tan(\theta/2)\frac{G_E g_{\mu\nu} W^{\mu\nu}_{RCS}(s,t)}{\epsilon G_E^2 + \tau G_M^2}\ln^2\frac{-t}{m^2}$$

Double log enhancement

Data from: Maas et al, '05 +
Wells et al, '00

Reasonable description
except for SAMPLE point



* Theory: Pasquini, Vanderhaeghen '04; MG '06