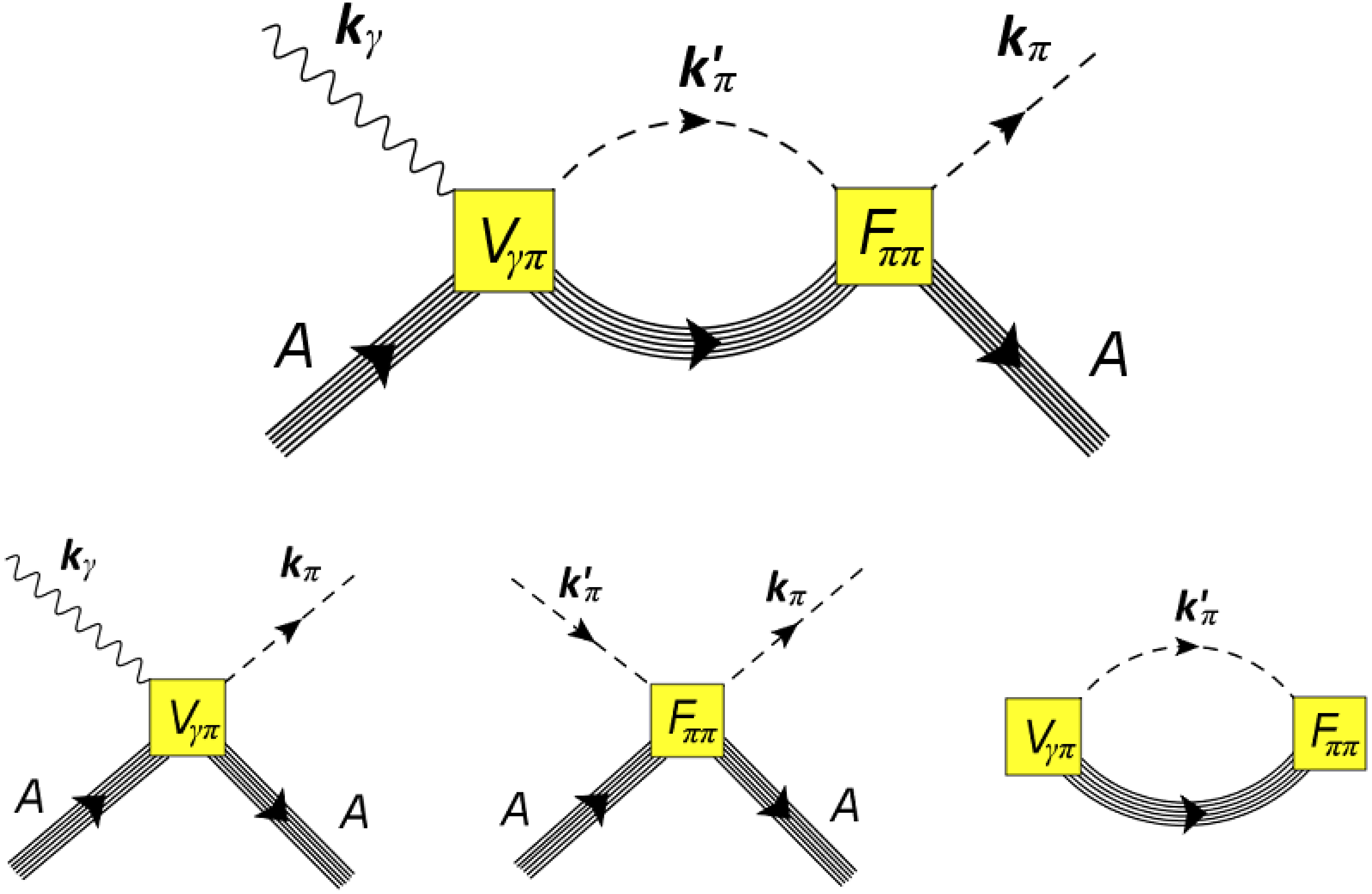
Coherent photoproduction of pions on spin-zero nuclei

Slava Tsaran

Pion photoproduction: DWA



Pion-nucleus scattering: Lippmann-Schwinger equation

$$\Psi^{\pm}_{\bar{k}}(\bar{k}') = (2\pi)^3 \delta^3(\bar{k} - \bar{k}') + \frac{T^{\pm}(\bar{k}', \bar{k})}{\frac{k^2}{2m} - \frac{k'^2}{2m} \pm i\varepsilon} \quad (1)$$

$$\boxed{T(\bar{k}', \bar{k}) = U(\bar{k}', \bar{k}) + \int \frac{d^3\bar{k}''}{(2\pi)^3} \frac{U(\bar{k}', \bar{k}'')T(\bar{k}'', \bar{k})}{k^2 - k''^2 + i\varepsilon}} \quad (2)$$

$$T = U + U(G_{\mathrm{ON}} + G_{\mathrm{OFF}})T \quad (3)$$

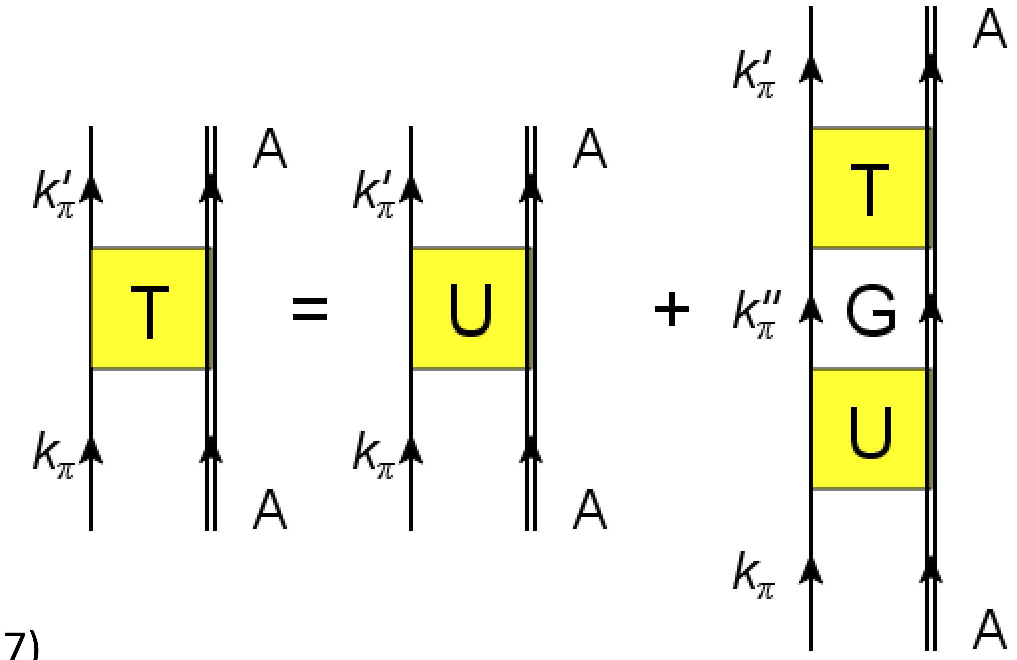
$$G_{\mathrm{ON}}(k, k'') = -i\frac{\pi}{2}\frac{1}{k}\delta(k - k'') \quad (4)$$

$$K \equiv (1 - G_{\mathrm{OFF}})^{-1} U \quad (5)$$

$$K = U + U G_{\mathrm{OFF}} K \qquad T = K + K G_{\mathrm{ON}} T \quad (6)$$

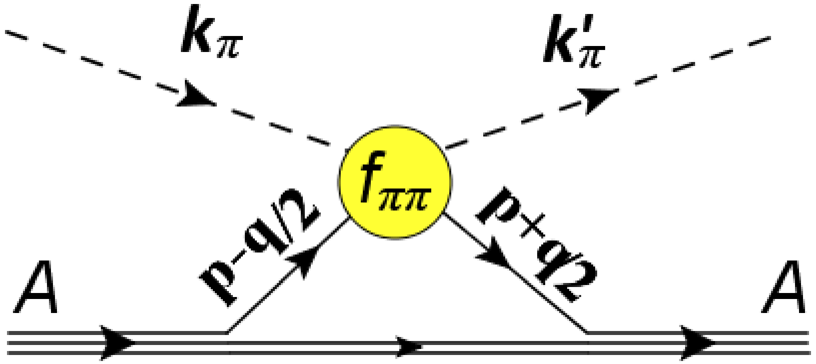
$$K(\bar{k}', \bar{k}) = U(\bar{k}', \bar{k}) + \mathrm{p.v.}\!\int \frac{d^3\bar{k}''}{(2\pi)^3} \frac{U(\bar{k}', \bar{k}'')K(\bar{k}'', \bar{k})}{k^2 - k''^2} \quad (7)$$

Static limit: $$F_{\pi\pi} = -\frac{1}{4\pi} T \quad (8)$$



Pion-nucleus scattering: tρ potential

Impulse approximation:



$$T(\bar{k}',\bar{k}) = \langle \bar{k}'|T^{\pi A}_{IA}|\bar{k}\rangle = \sum_{\alpha\beta}\langle \bar{k}',\alpha|t^{\pi A \mathrm{c.m.}}_{IA}|\bar{k},\beta\rangle\langle\Phi_A|\hat{c}^\dagger_\alpha\hat{c}_\beta\Phi_B\rangle = \tag{1}$$

$$\sum_{\alpha<F}\int\frac{d^3\bar{p}}{(2\pi)^3}\phi^*_\alpha(\bar{p}+\bar{q}/2)\phi_\alpha(\bar{p}-\bar{q}/2)t^{\pi A \mathrm{c.m.}}_{\pi N}(\bar{k}',\bar{p}+\bar{q}/2;\bar{k},\bar{p}-\bar{q}/2)$$

Factorization approximation:

$$\langle \bar{k}'|T^{\pi A}_{IA}|\bar{k}\rangle = \rho(\bar{q})t^{\pi A \mathrm{c.m.}}_{\pi N}(\bar{k}',\bar{k};\bar{q}) \tag{2}$$

$$\rho(\bar{q}) = \int\frac{d^3\bar{p}}{(2\pi)^3}\phi^*_\alpha(\bar{p}+\bar{q}/2)\phi_\alpha(\bar{p}-\bar{q}/2), \quad \rho(0)=A \tag{3}$$

$$t^{\pi N \mathrm{c.m.}}_{\pi N}(\bar{k}'_{\mathrm{c.m.}},\bar{k}_{\mathrm{c.m.}};\bar{q}_{\mathrm{c.m.}}) = -4\pi(b_o + c_o\bar{k}'_{\mathrm{c.m.}}\cdot\bar{k}_{\mathrm{c.m.}}) \tag{4}$$

$$\bar{k}'_{\mathrm{c.m.}}\cdot\bar{k}_{\mathrm{c.m.}} \approx \frac{1}{(1+\varepsilon)^2}\left[\bar{k}'\cdot\bar{k} - \frac{\varepsilon}{2}\bar{q}^2\right], \quad \varepsilon = \omega/m_N \tag{5}$$

$$\boxed{\langle \bar{k}'|T^{\pi A}_{IA}|\bar{k}\rangle = -4\pi\rho(\bar{q})\left((1+\varepsilon)b_o + \frac{c_o}{1+\varepsilon}\left[\bar{k}'_{\mathrm{c.m.}}\cdot\bar{k}_{\mathrm{c.m.}} - \frac{\varepsilon}{2}\bar{q}^2\right]\right)} \tag{6}$$

Pion-nucleus scattering: optical potential

$$\left(-\nabla^2 + m_\pi^2\right)\Phi_{\bar{k}}(\bar{r}) + \tilde{U}(\omega,\bar{r})\Phi_{\bar{k}}(\bar{r}) = \left(\omega - V^{\text{Coul}}(\bar{r})\right)^2 \Phi_{\bar{k}}(\bar{r}) \quad (1)$$

$$\tilde{U}(\omega,\mathbf{r})\Phi_{\mathbf{k}}(\mathbf{r}) = \int d^3\mathbf{r}\, U(\mathbf{r},\mathbf{r}')\Phi_{\mathbf{k}}(\mathbf{r}') \quad (2)$$

$$U_{t\rho}(\bar{k}',\bar{k}) = -4\pi\rho(q)\left((1+\varepsilon)b_o + \frac{c_o}{1+\varepsilon}\left[\bar{k}'_{\text{c.m.}}\cdot\bar{k}_{\text{c.m.}} - \frac{\varepsilon}{2}\bar{q}^2\right]\right) \quad (3)$$

$$\tilde{U}(\omega,\mathbf{r})| = -4\pi\left[(1+\varepsilon)b_o\rho(r) - \frac{c_o}{1+\varepsilon}\nabla\rho(r)\nabla + \frac{c_o}{(1+\varepsilon)}\frac{\varepsilon}{2}\left(\nabla\rho(r)\right)^2\right]; \quad (4)$$

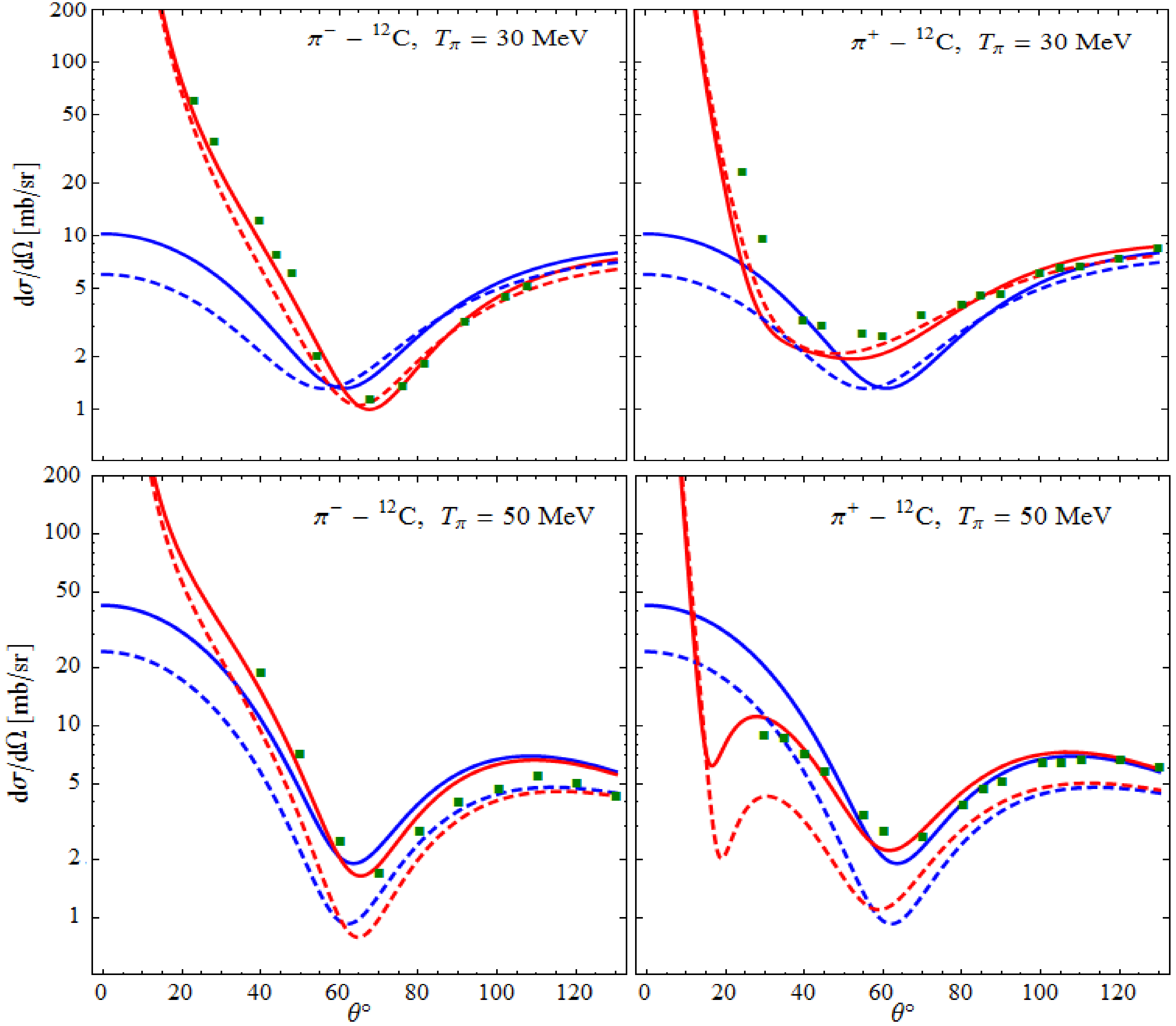
$$\boxed{\begin{aligned}\tilde{U}(\omega,\bar{r}) = -4\pi\Bigg[&p_1 b_o\rho(r) + p_2 B_o\rho^2(r) - \bar{\nabla}\frac{\frac{c_o}{p_1}\rho(r) + \frac{C_o}{p_2}\rho^2(r)}{1}\bar{\nabla} + \\ &\frac{1}{2}\left(1-\frac{1}{p_1}\right)c_o\left(\nabla^2\rho(r)\right) + \frac{1}{2}\left(1-\frac{1}{p_2}\right)C_o\left(\nabla^2\rho^2(r)\right)\Bigg]\end{aligned}} \quad (5)$$

K. Stricker et al., Phys. Rev. C 25 (1982) 952

$$p_1 = \frac{1+\varepsilon}{1+\varepsilon/A} \qquad p_2 = \frac{1+\varepsilon/2}{1+\varepsilon/(2A)}$$

$$K(\bar{k}',\bar{k}) = U(\bar{k}',\bar{k}) + \text{p.v.}\int\frac{d^3\bar{k}''}{(2\pi)^3}\frac{U(\bar{k}',\bar{k}'')K(\bar{k}'',\bar{k})}{k^2-k''^2} \quad (6)$$

Pion-nucleus elastic scattering: testing of potential

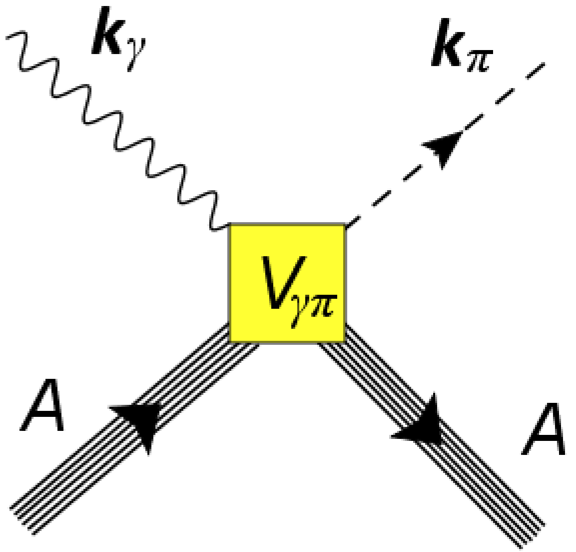


Red curves are for $\pi^\pm$
Blue curves are for π^0

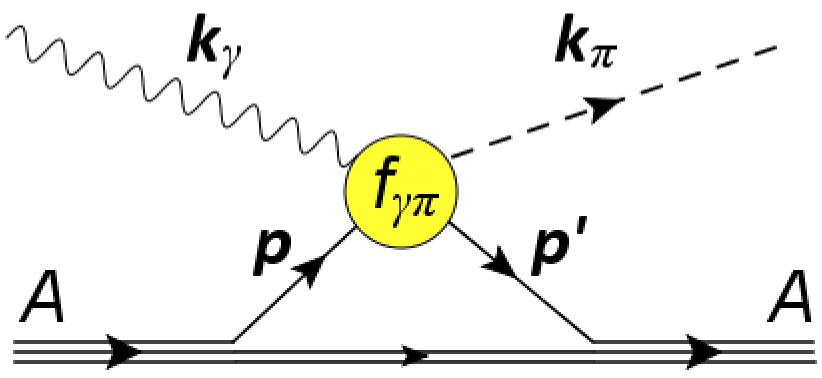
Solid curves are solutions of L.-SH.
Dashed curves are for Born approx.

Potential parameters was taken from the $0 \le T_\pi \le 50$ MeV fit by *J.A.Carr et al., PRC 25 (1982) 952*

Pion photoproduction: PWIA



Pion photoproduction: PWIA



$$V^{\lambda}_{\gamma\pi}(\mathbf{k}_\gamma,\mathbf{k}_\pi) = W_A \langle 0| \sum_{0}^{A} e^{i(\mathbf{k}-\mathbf{q})\mathbf{r}_j} f_{\gamma\pi}(\mathbf{k}_\gamma,\mathbf{k}_\pi,\mathbf{p}_j)|0\rangle \qquad (1)$$

$$f_{\gamma\pi} = \frac{1}{2}\left(f^{p}_{\gamma\pi} + f^{n}_{\gamma\pi}\right) \qquad (2)$$

$$\mathrm{p} = -\frac{1}{A}\mathrm{k}_\gamma - \frac{A-1}{2A}\mathrm{q}, \quad \mathrm{p}' = -\frac{1}{A}\mathrm{k}_\pi + \frac{A-1}{2A}\mathrm{q} \qquad (3)$$

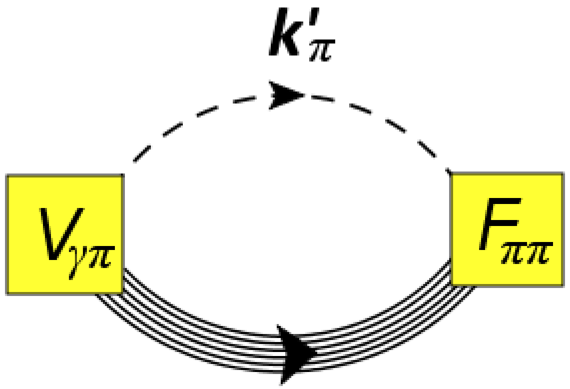
MAID2007: $\mathcal{F} = i\tilde{\boldsymbol{\sigma}}\cdot\epsilon\, F_1 + \boldsymbol{\sigma}\cdot\hat{\mathrm{k}}\,\boldsymbol{\sigma}\cdot(\hat{\mathrm{q}}\times\epsilon)\,F_2 + i\boldsymbol{\sigma}\cdot\hat{\mathrm{q}}\,\tilde{\mathrm{k}}\cdot\epsilon\,F_3 + i\boldsymbol{\sigma}\cdot\hat{\mathrm{k}}\,\tilde{\mathrm{k}}\cdot\epsilon\,F_4$ (4)

$$\tilde{f}^{\lambda}_{\gamma\pi} = F_2(\tilde{\mathbf{k}}_\pi,\tilde{\mathbf{k}}_\gamma,W)\left[\hat{\tilde{\mathbf{k}}}_\gamma \times \hat{\tilde{\mathbf{k}}}_\pi\right]\cdot\mathbf{e}_\lambda \qquad (5)$$

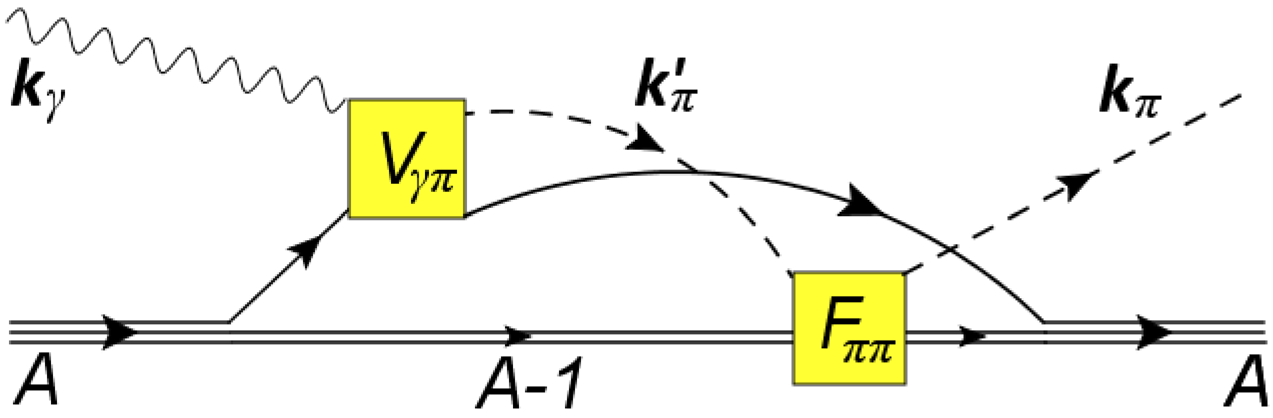
$$W = \sqrt{(k_\gamma + E_N(p))^2 - (\mathbf{k}_\gamma + \mathbf{p})^2} \qquad (6)$$

$$V^{\lambda}_{\gamma\pi}(\mathbf{k}_\gamma,\mathbf{k}_\pi) = W_A F_A(q) f_2(\mathbf{k}_\gamma,\mathbf{k}_\pi) \qquad (7)$$

$$F^{\mathrm{ch}}_A(q) = F_A(q) F^{\mathrm{ch}}_p(q) \qquad (8)$$


$\boldsymbol{k}'_{\pi}$
$V_{\gamma\pi}$
$F_{\pi\pi}$

Pion photoproduction: DWIA



$$F^{\lambda}_{\gamma\pi}(\mathbf{k}_\gamma,\mathbf{k}_\pi) = V^{\lambda}_{\gamma\pi}(\mathbf{k}_\gamma,\mathbf{k}_\pi) - \frac{\alpha}{(2\pi)^2}\int \frac{d\mathbf{k}'_\pi}{\mathcal{M}(k'_\pi)}\frac{F_{\pi\pi}(\mathbf{k}_\gamma,\mathbf{k}'_\pi)V^{\lambda}_{\gamma\pi}(\mathbf{k}'_\pi,\mathbf{k}_\gamma)}{E(k_\pi)-E(k'_\pi)+i\varepsilon} \qquad (1)$$

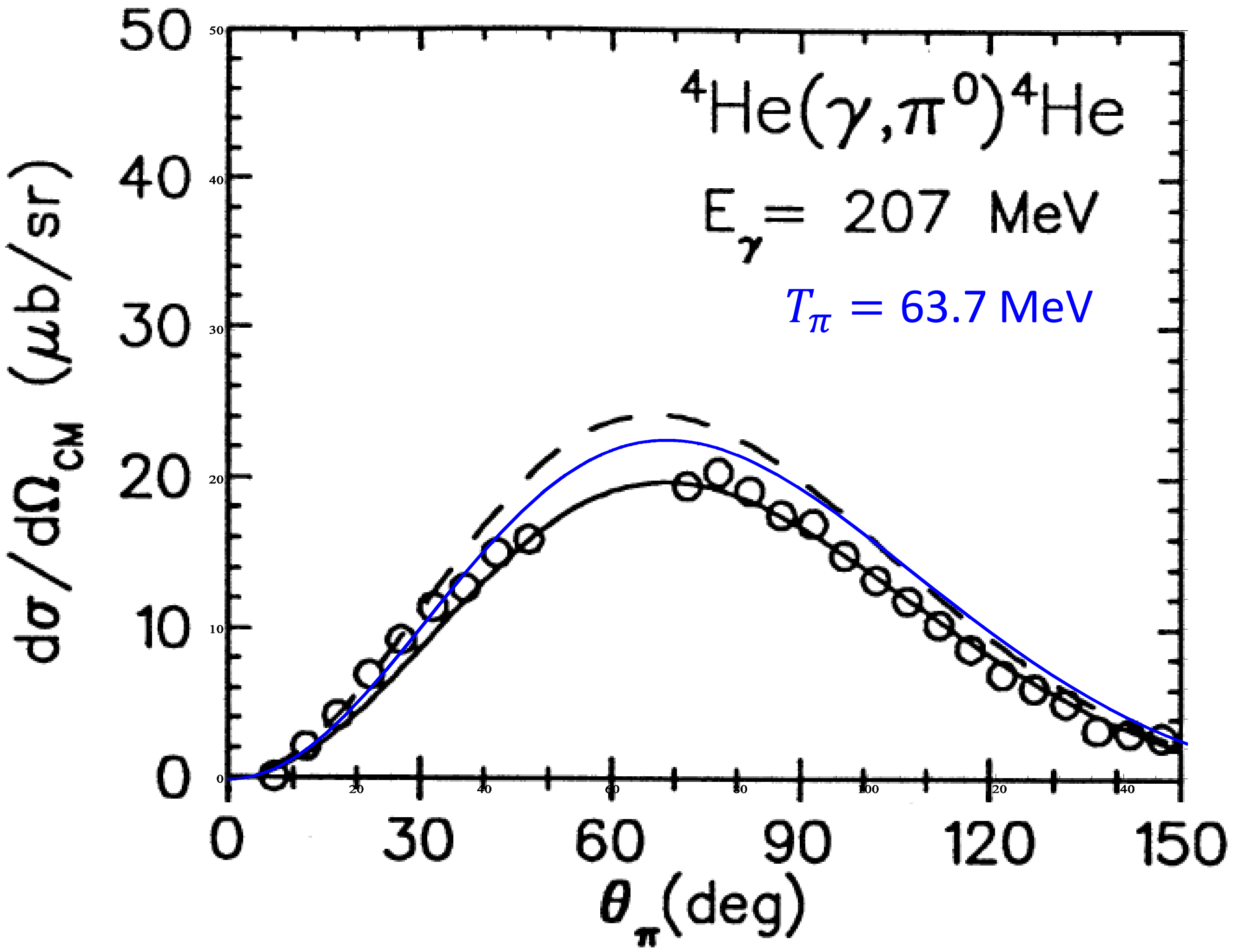
$$\alpha = \frac{A-1}{A} \qquad M(k_\pi) = \omega(k_\pi)E_A(k_\pi)/E(k_\pi)$$

$$F_2^{\Delta}\left(\tilde{k}'_\pi,\tilde{k}_\pi\right) \Rightarrow \frac{\tilde{k}'_\pi}{\tilde{k}_\pi}g(\tilde{k}'_\pi,\tilde{k}_\pi)F_2^{\Delta}\left(W(\tilde{\mathbf{k}}_\gamma,\tilde{\mathbf{k}}_\pi),\tilde{\theta}\right) \quad (2) \qquad g(\tilde{k}'_\pi,\tilde{k}_\pi) = \left(\frac{\Lambda^2+\tilde{k}_\pi^2}{\Lambda^2+\tilde{k}_\pi'^2}\right), \quad \Lambda = 450\mathrm{MeV} \quad (3)$$

$$F_2\left(\tilde{k}'_\pi,\tilde{k}_\pi\right) - F_2^{\Delta}\left(\tilde{k}'_\pi,\tilde{k}_\pi\right) \Rightarrow g(\tilde{k}'_\pi,\tilde{k}_\pi)\left[F_2\left(\tilde{k}'_\pi,\tilde{k}_\pi\right) - F_2^{\Delta}\left(\tilde{k}'_\pi,\tilde{k}_\pi\right)\right] \quad (4)$$

Testing of DWIA result

D. Drechsel et al., Nuclear Physics A 660 (1999) 423-438

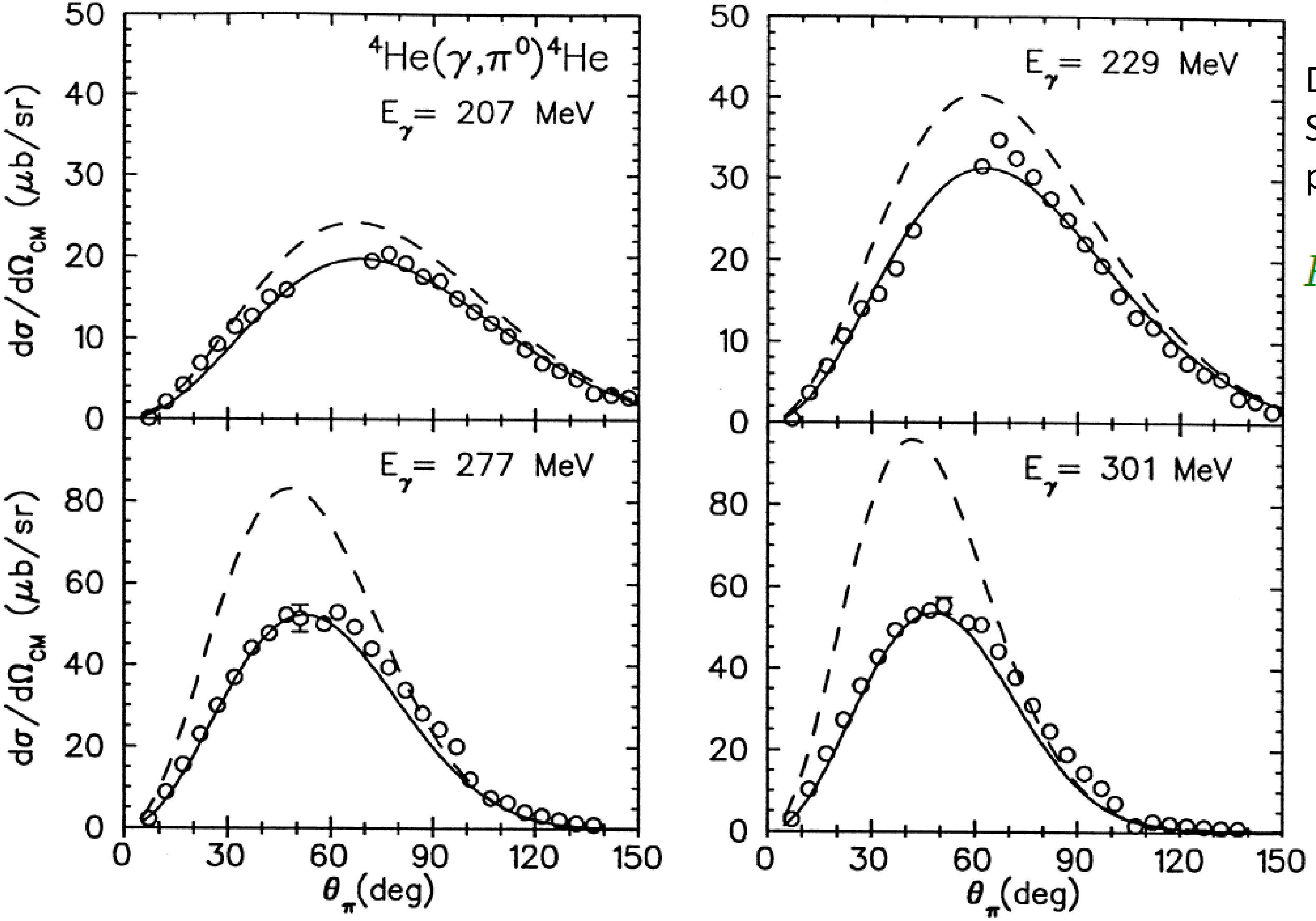


Dashed curves are the DWIA result.
Solid curves are obtained with parameterization of Δ self-energy. Experimental data are from *F. Rambo et al., Nucl. Phys. A 660 (1999) 69*

Blue curve is our DWIA result.

Influence of Δ self-energy

D. Drechsel et al., Nuclear Physics A 660 (1999) 423-438



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Influence of Δ self-energy

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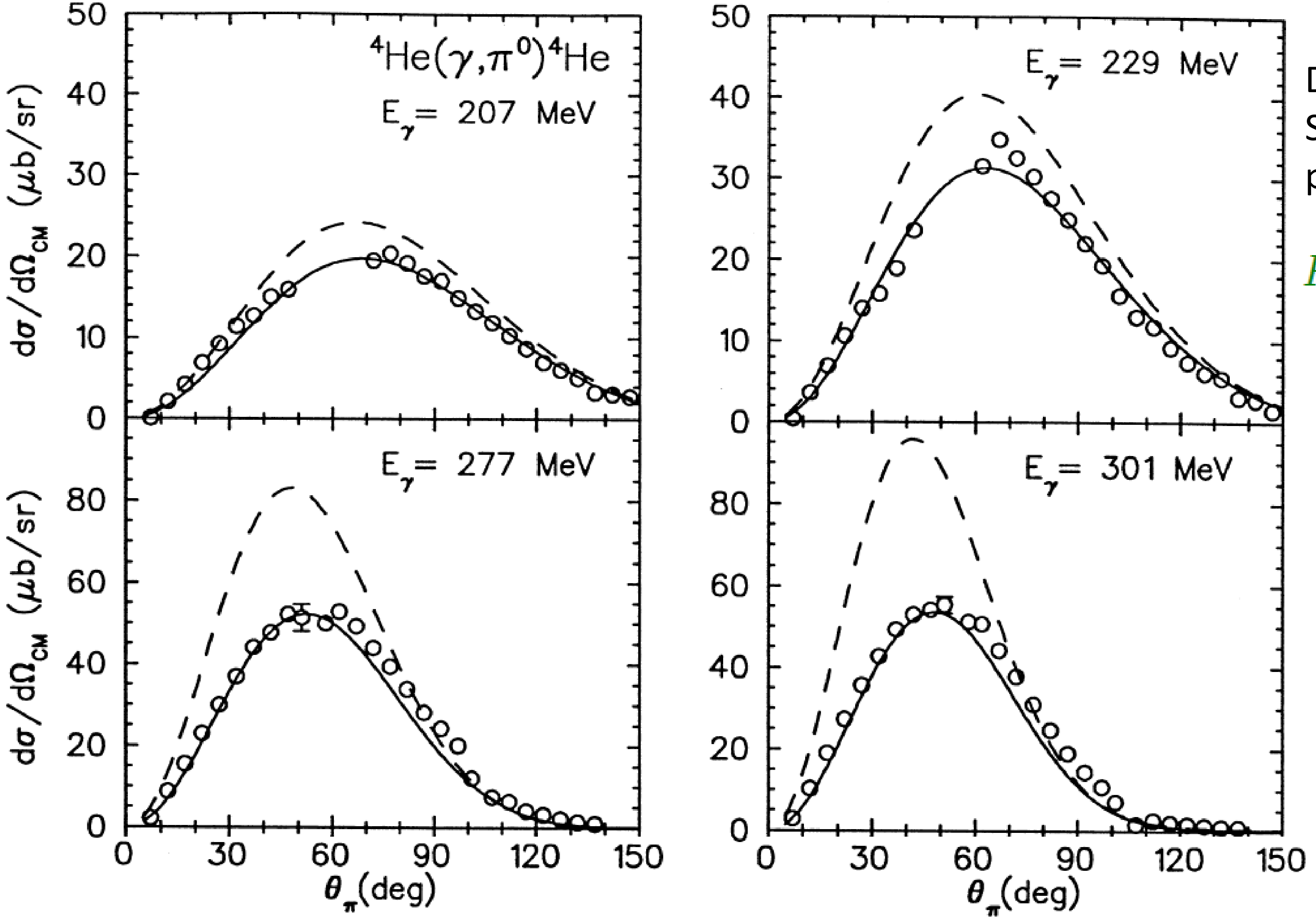

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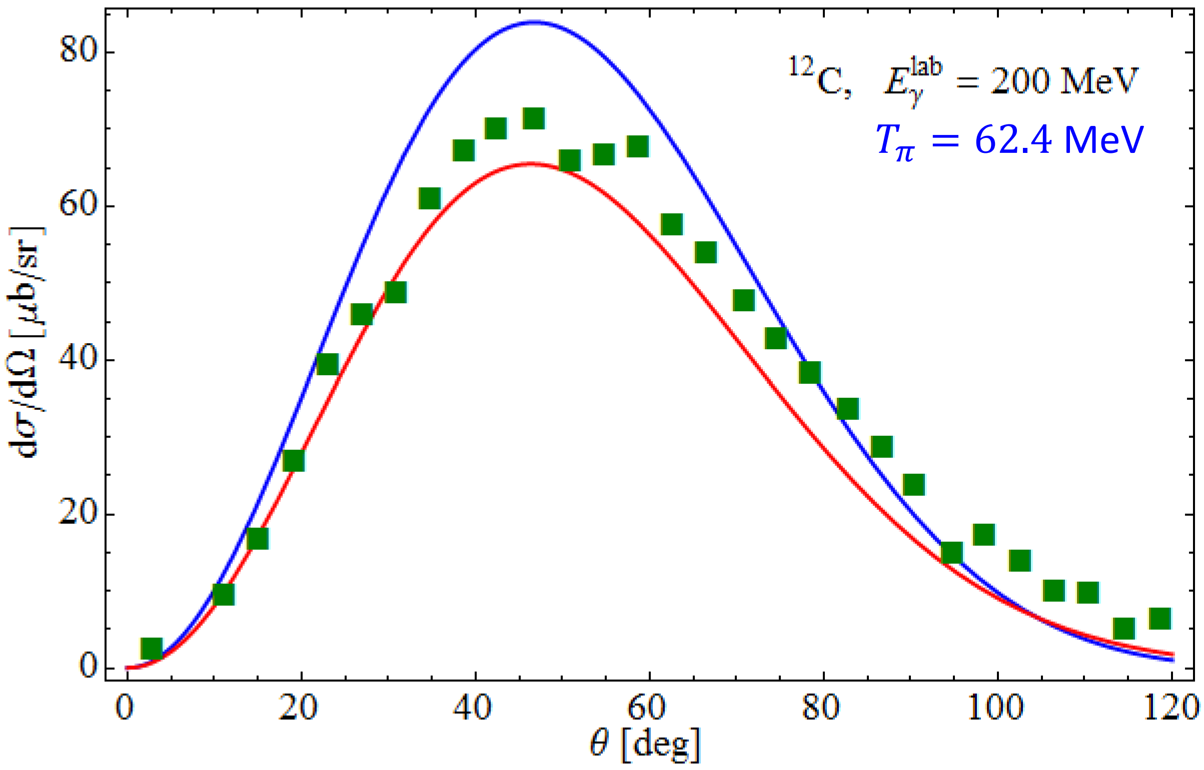
Experimental data are from
F. Rambo et al., NPA 660 (1999) 69

$$\frac{1}{W-\overline{M}_\Delta+i\overline{\Gamma}_\Delta/2} \Rightarrow \frac{1}{W-\overline{M}_\Delta+i\overline{\Gamma}_\Delta/2-\Sigma_\Delta}$$

Pion photoproduction on ^{12}C: DWIA

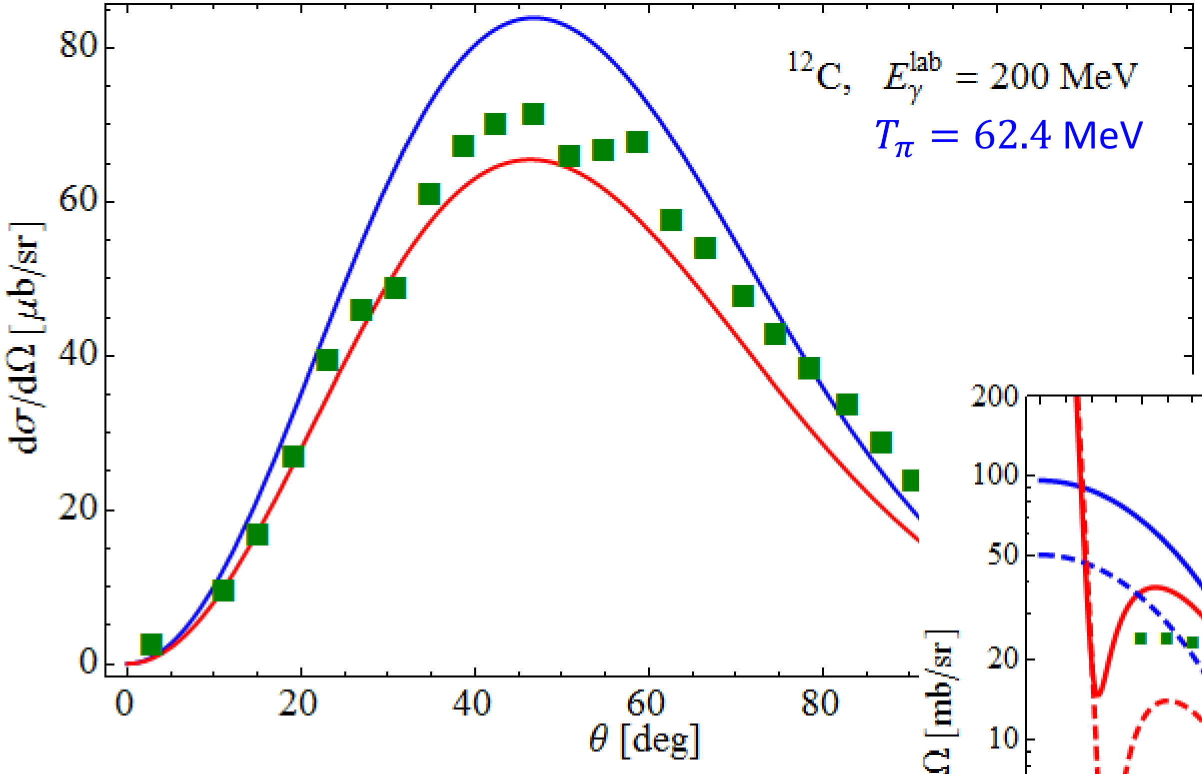


Blue curve is differential cross section in DWIA approx.
Red curves is the same, but without Δ contribution.

Data from

B. Krusche et al., Physics Letters B 526 (2002) 287-294

Pion photoproduction on ^{12}C: DWIA



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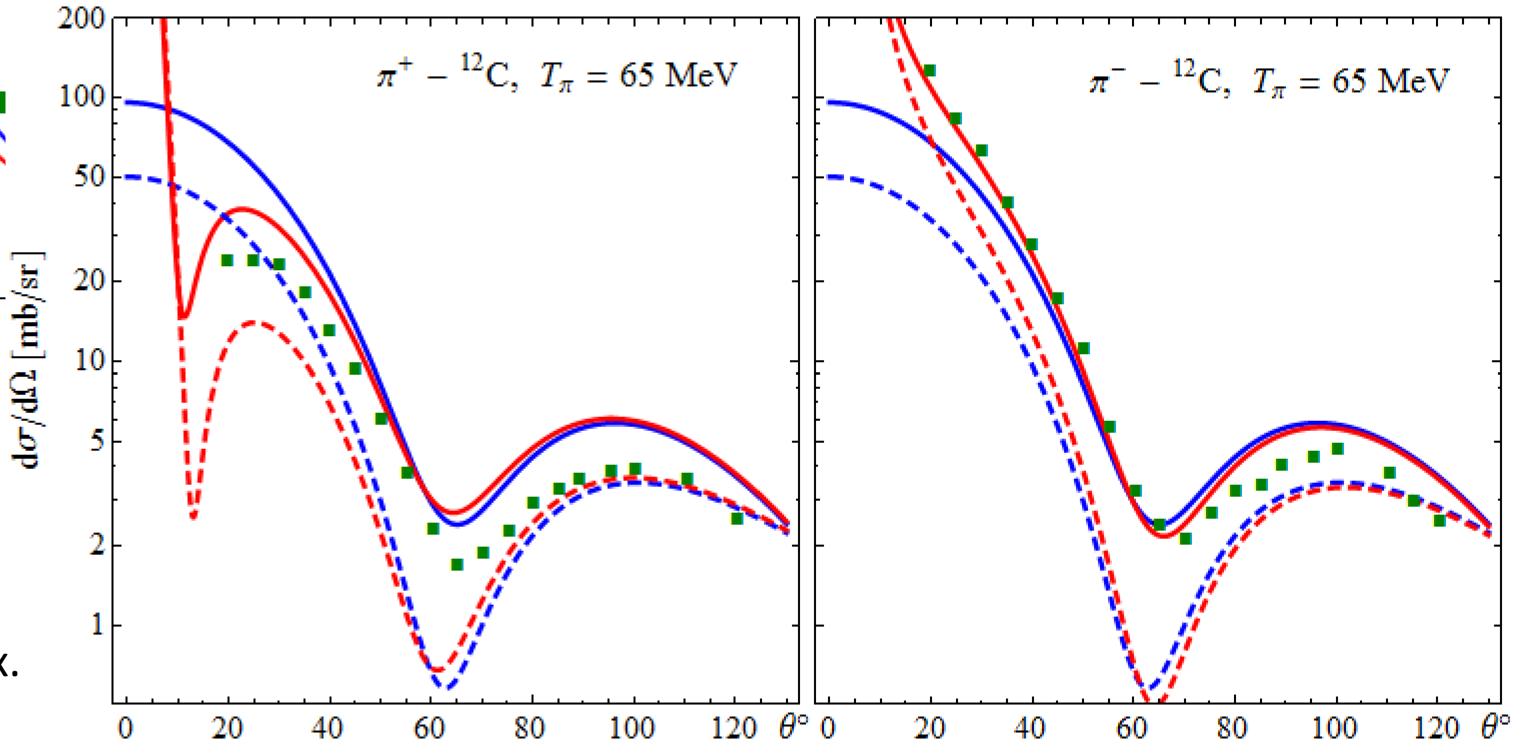

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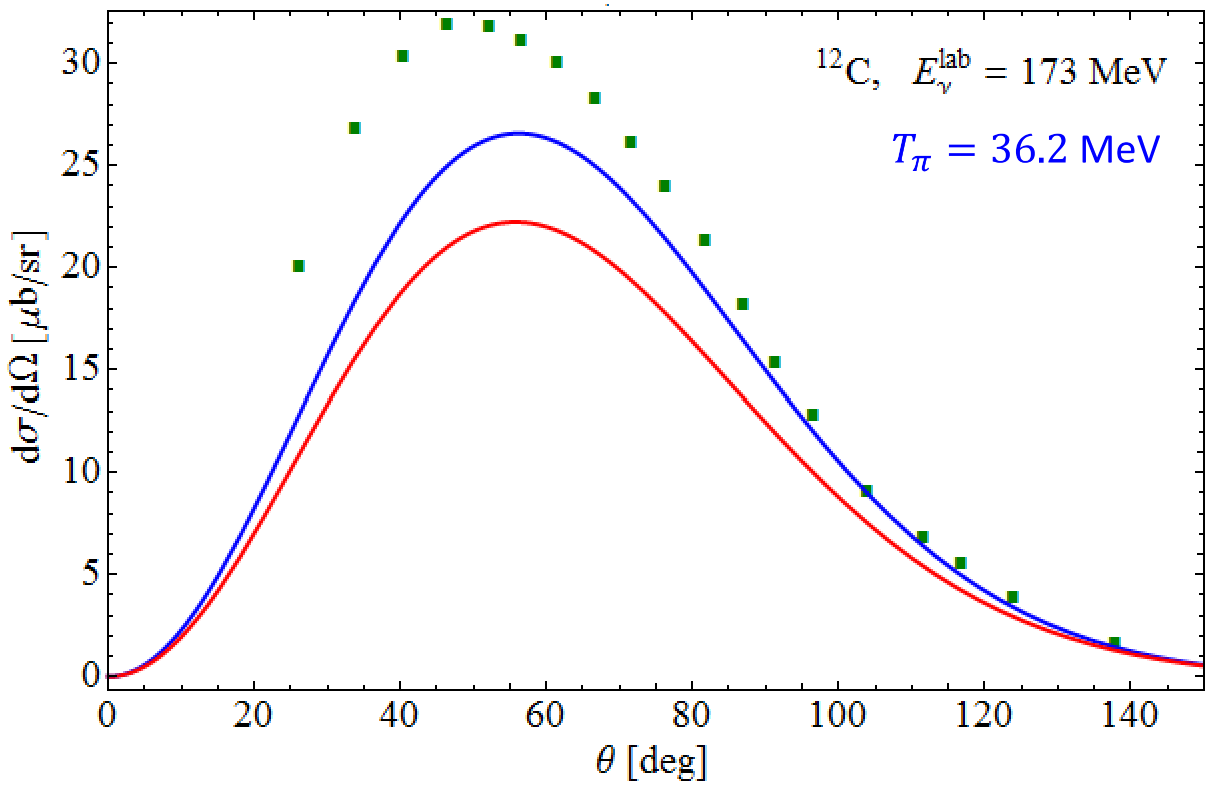
Pion photoproduction on ^{12}C: DWIA



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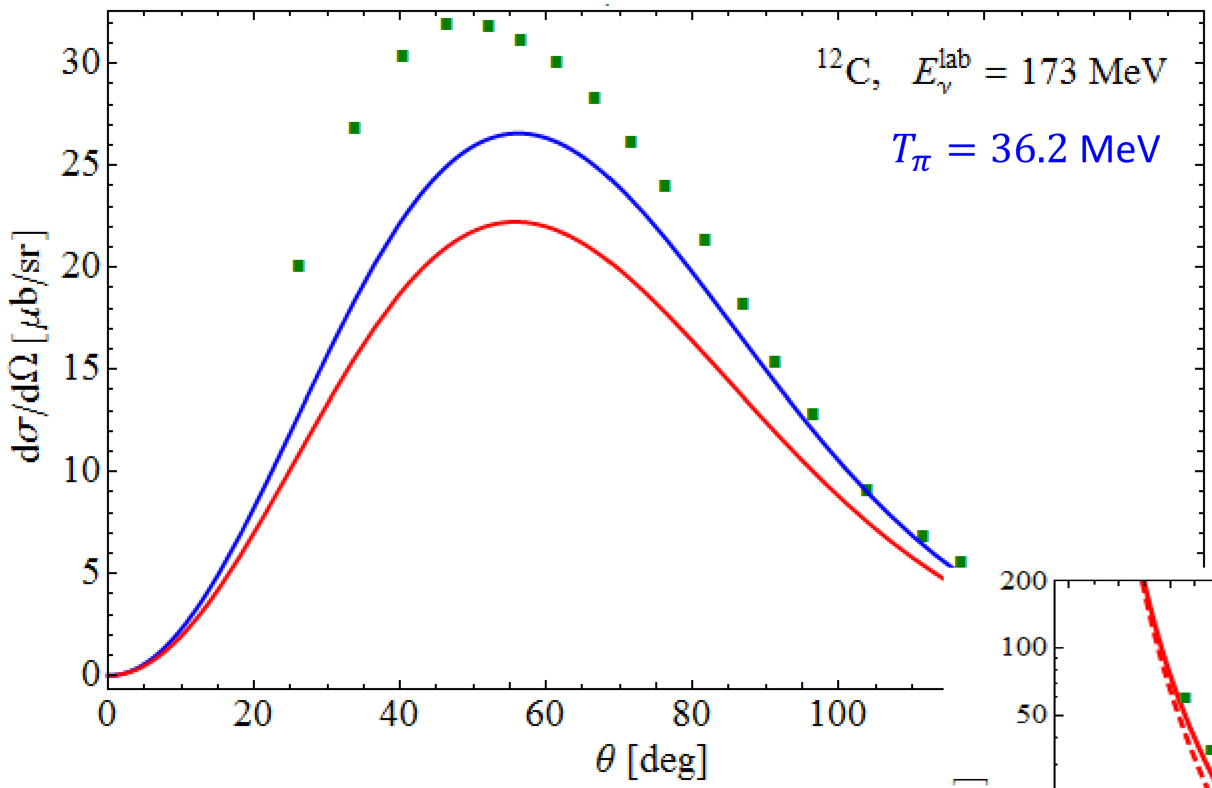
R. W. Gothe et al., Physics Letters B 355 (1995) 59-64

Pion photoproduction on ^{12}C: DWIA



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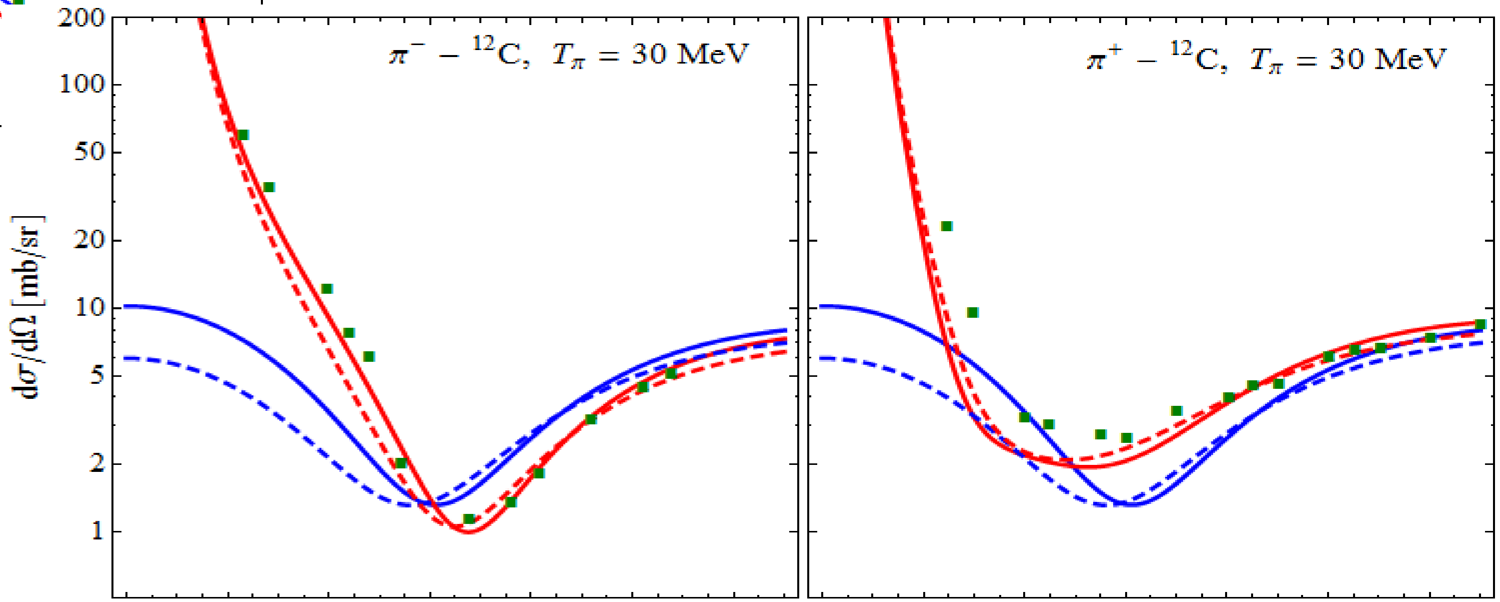

Red curves are for $\pi^\pm$
Blue curves are for π^0
Solid curves are solutions of L.-SH.
Dashed curves are for Born approx.

Work to be done

- Perform fits of optical potential for ^{4}He and ^{12}C
- Extend fit to other nuclei i.e. ^{40}Ca,^{208}Pb …
- Study of model sensitivities:
  - Δ resonance suppression
  - deviations from tρ
  - investigate size of effects beyond the impulse approximation
- Theoretical error estimate