Symmetry energy and pure neutron systems

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Neutron Skins of Nuclei
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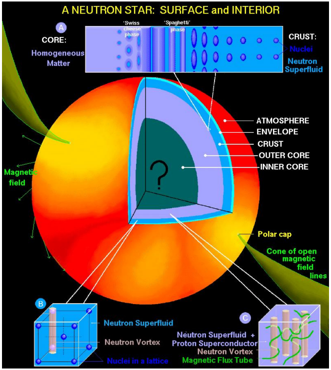
Neutron stars

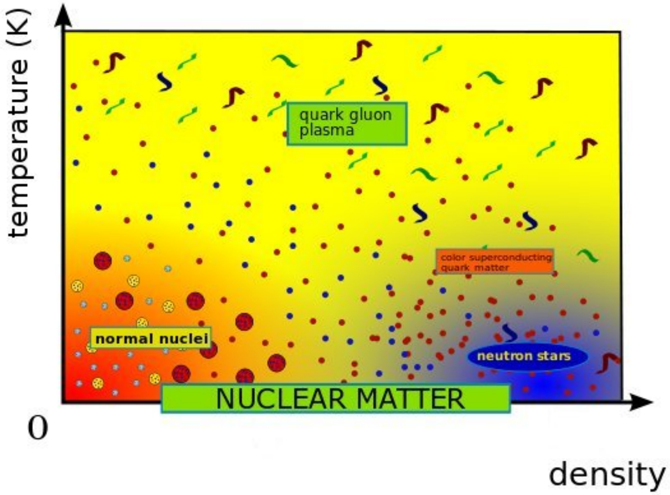
Neutron star is a wonderful natural laboratory



D. Page

- Atmosphere: atomic and plasma physics
- Crust: physics of superfluids (neutrons, vortex), solid state physics (nuclei)
- Inner crust: deformed nuclei, pasta phase
- Outer core: nuclear matter
- Inner core: hyperons? quark matter? π or K condensates?

Homogeneous neutron matter



Outline

- The model and the method
- Neutron drops: energies and radii, correlations radii-skin and radii-L
- Equation of state of neutron matter, three-neutron force and symmetry energy
- Neutron star structure and symmetry energy
- Conclusions

Nuclear Hamiltonian

Model: non-relativistic nucleons interacting with an effective nucleon-nucleon force (NN) and three-nucleon interaction (TNI).

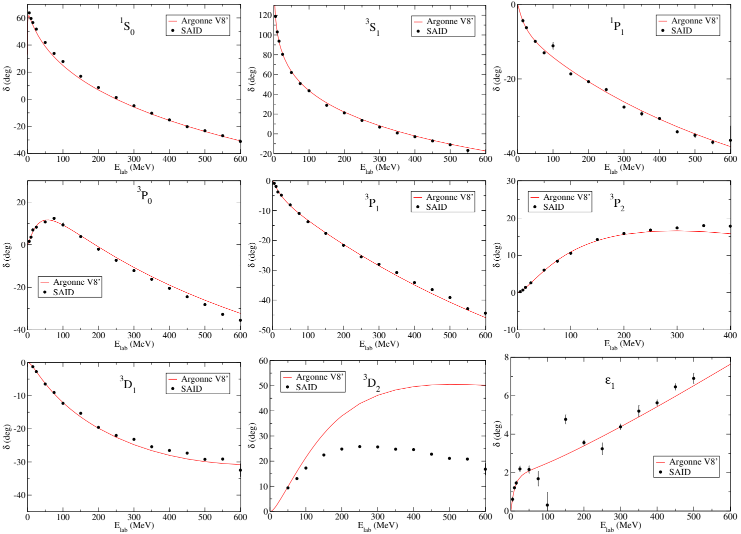
$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

v_{ij} NN fitted on scattering data. Sum of operators:

$$v_{ij} = \sum O_{ij}^{p=1,8} v^p(r_{ij}) \,, \quad O_{ij}^p = (1, \vec{\sigma}_i \cdot \vec{\sigma}_j, S_{ij}, \vec{L}_{ij} \cdot \vec{S}_{ij}) \times (1, \vec{\tau}_i \cdot \vec{\tau}_j)$$

NN interaction - Argonne AV8'.

Phase shifts, AV8'



Difference AV8′-AV18 less than 0.2 MeV per nucleon up to A=12.

Scattering data and neutron matter

Two neutrons have

$$k \approx \sqrt{E_{lab}\, m/2}\,, \qquad \to k_F$$

that correspond to

$$k_F \to \rho \approx (E_{lab}\, m/2)^{3/2}/2\pi^2\,.$$

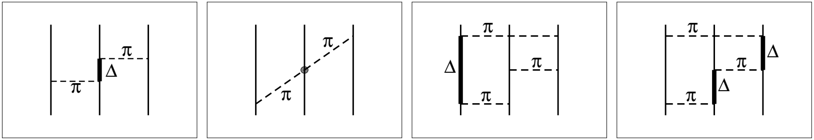
E_{lab}=150 MeV corresponds to about 0.12 fm^{-3}.

E_{lab}=350 MeV to 0.44 fm^{-3}.

Argonne potentials useful to study dense matter above ρ_0=0.16 fm^{-3}

Three-body forces

Urbana–Illinois V_{ijk} models processes like



$+$ short-range correlations (spin/isospin independent).

Urbana UIX: Fujita-Miyazawa plus short-range.

Quantum Monte Carlo

Projection in imaginary-time t:

$$H\,\psi(\vec{r}_1 \dots \vec{r}_N) = E\,\psi(\vec{r}_1 \dots \vec{r}_N) \qquad \psi(t) = e^{-(H-E_T)t}\psi(0)$$

Ground-state extracted in the limit of $t \to \infty$.

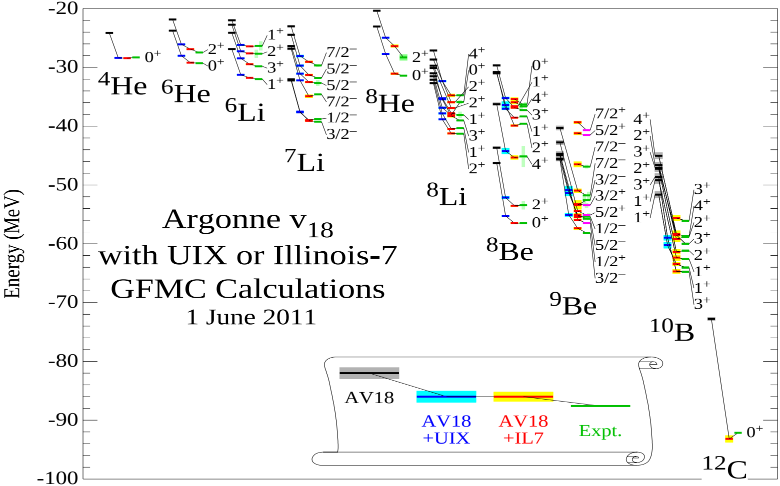
Propagation performed by

$$\psi(R,t) = \langle R|\psi(t)\rangle = \int dR'\,G(R,R',t)\psi(R',0)$$

- $dR' \to dR_1\,dR_2 \dots,\; G(R,R',t) \to G(R_1,R_2,\delta t)\,G(R_2,R_3,\delta t)\dots$
- Importance sampling: $G(R,R',\delta t) \to G(R,R',\delta t)\,\Psi_I(R')/\Psi_I(R)$
- Constrained-path approximation to control the sign problem. Unconstrained calculation possible in several cases (exact).

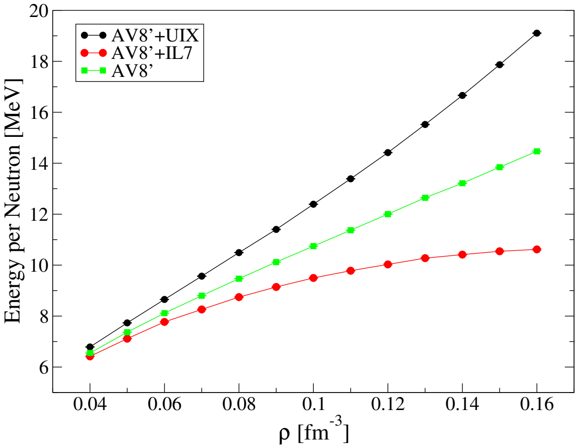
Ground–state obtained in a **non-perturbative way.** Systematic uncertainties within 1-2 %.

Light nuclei spectrum computed with GFMC



Carlson, *et al.*, RMP (2015)

Neutron matter and the "puzzle" of the three-body force



Maris, *et al.*, PRC (2013)

Note: AV8'+UIX and (almost) AV8' are stiff enough to support observed neutron stars, but AV8'+IL7 too soft. → How to reconcile with nuclei???

Neutron matter and the "puzzle" of the three-body force

Extended Data Table 2 | Key observables from chiral interactions. Predictions for ^{48}Ca (based on the interactions used in this work): binding energy BE, neutron separation energy S_n, three-point-mass difference Δ, electric-charge radius R_{ch}, and the weak-charge radius R_W. The last two columns show the symmetry energy of the nuclear equation of state and its slope L at saturation density. Energies are in MeV and radii in fm. Theoretical uncertainty estimates are about 1% for radii and energies.

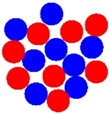
Interaction	BE	S_n	Δ	R_{ch}	R_W	S_v	L
$NNLO_{sat}$	404	9.5	2.69	3.48	3.65	26.9	40.8
1.8/2.0 (EM)	420	10.1	2.69	3.30	3.47	33.3	48.6
2.0/2.0 (EM)	396	9.3	2.66	3.34	3.52	31.4	46.7
2.2/2.0 (EM)	379	8.8	2.61	3.37	3.55	30.2	45.5
2.8/2.0 (EM)	351	8.0	2.41	3.44	3.62	28.5	43.8
2.0/2.0 (PWA)	346	7.8	2.82	3.55	3.72	27.4	44.0
Experiment	415.99	9.995	2.399	3.477			

Hagen, *et al.*, Nature Physics (2016)

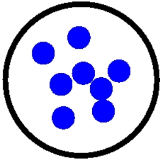
Similar trend: very low simmetry energies, soft EOS, probably leading too small radii in neutron stars.

Neutron drops

What are, and why to study neutron drops?



NP self−bound



N confined

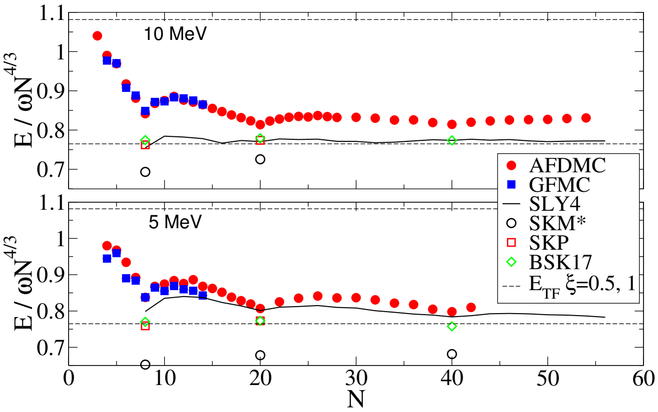
They model **inhomogeneous neutron matter**. Ab-initio→DFT

Neutrons are confined by an external potential:

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \sum_i V_{ext}(r_i)$$

V_{ext} tuned to change boundary conditions and densities.

External well: harmonic oscillator with $\hbar\omega$=5, 10 MeV. Interaction: AV8'+UIX

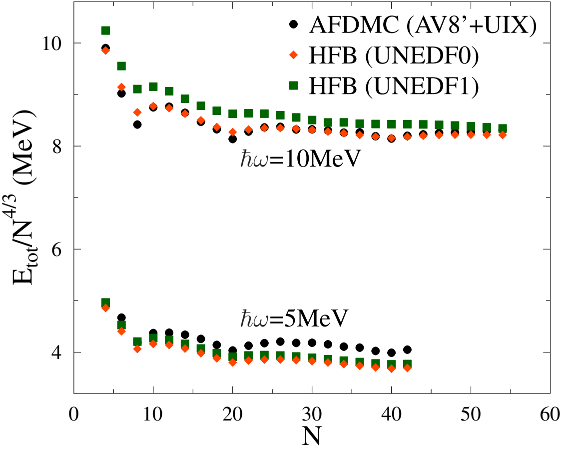


Gandolfi, Carlson, Pieper, PRL (2011)

Skyrme systematically overbind neutron drops.

Neutron drops

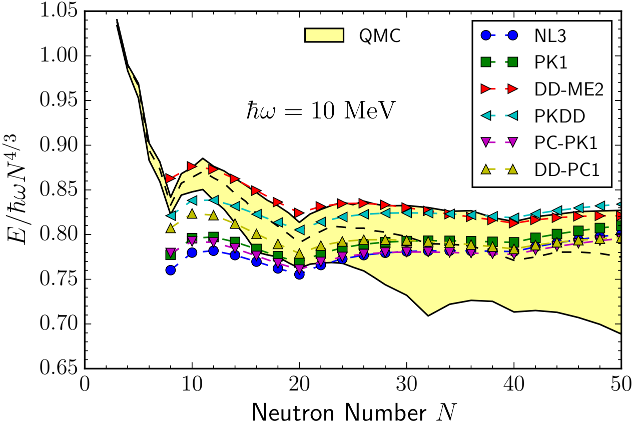
Ab-initio calculations meant as "experimental data". Interaction: AV8'+UIX



M. Kortelainen, *et al.*, PRC (2012).

Neutron drops

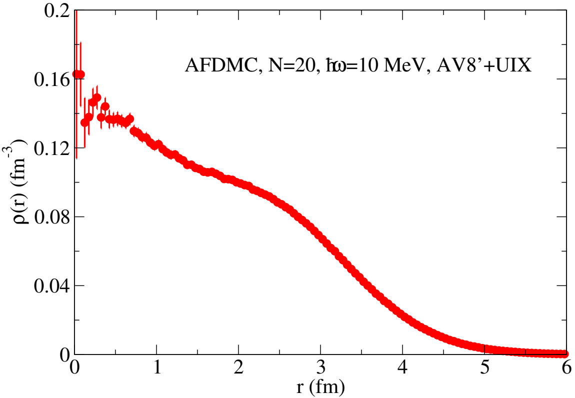
Comparison with other functionals. QMC: AV8'+UIX, AV8', AV8'+IL7 (from Maris, *et al.*, PRC 2013)



Zhao, Gandolfi, arXiv:1604.01490

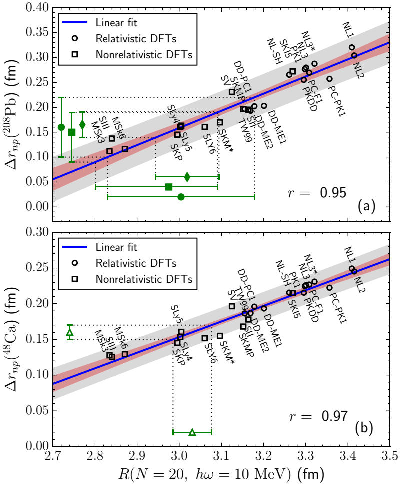
Neutron drops - radii

Note: tune V_{ext} or N to have central density about 0.16(2) fm^{-3}. For example, this is given by putting 20 neutrons in an external trap with $\hbar\omega$=10 MeV:



Rms radius: $\sqrt{< r^2 >} = 3.14(1)$ fm

Correlation between neutron drops radii and the skin thickness of ^{208}Pb and ^{48}Ca:

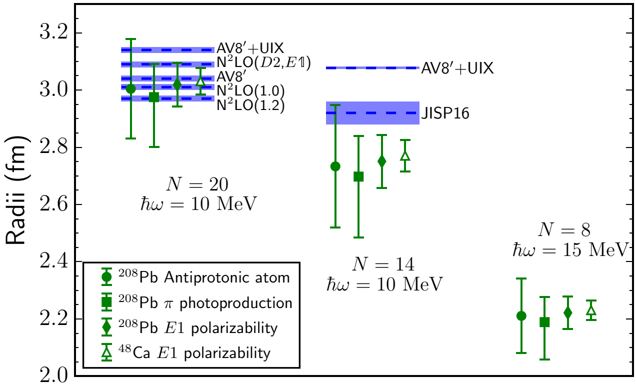


Zhao, Gandolfi, arXiv:1604.01490

Green points: antiprotonic atoms, pion photoproduction, electric dipole polarizability

Neutron drops

Radii: AFDMC calculations with various Hamiltonians, compared to what might be extracted from experiments:

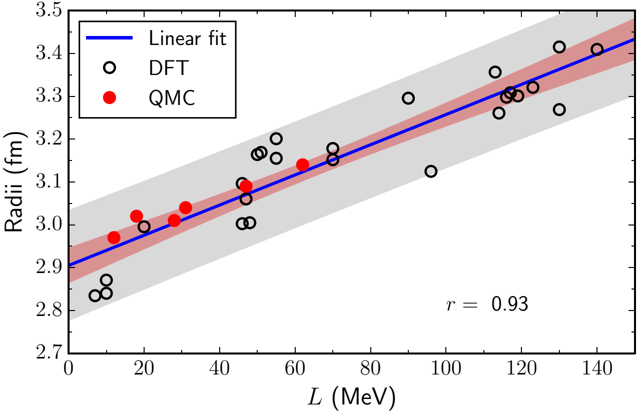


Zhao, Gandolfi, arXiv:1604.01490

Note, $N = 14$ has central density of about 0.12 fm^{-3}, N=8 we don't know yet...

Neutron drops

Radius of 20 neutrons in a $\hbar\omega = 10$ MeV harmonic trap as a function of L (from the EOS)



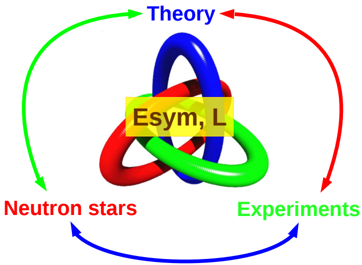
Zhao, Gandolfi, arXiv:1604.01490

Neutron matter equation of state

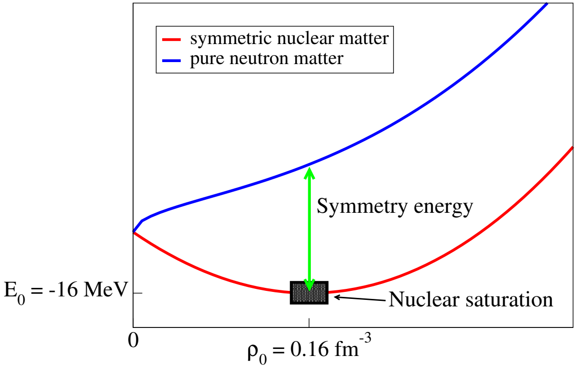
Why to study neutron matter at nuclear densities?

- EOS of neutron matter gives the symmetry energy and its slope.
- Assume that NN is very good - fit scattering data with very high precision.
  Three-neutron force ($T = 3/2$) very weak in light nuclei, while $T = 1/2$ is the dominant part. No direct $T = 3/2$ experiments.

Why to study symmetry energy?



What is the Symmetry energy?



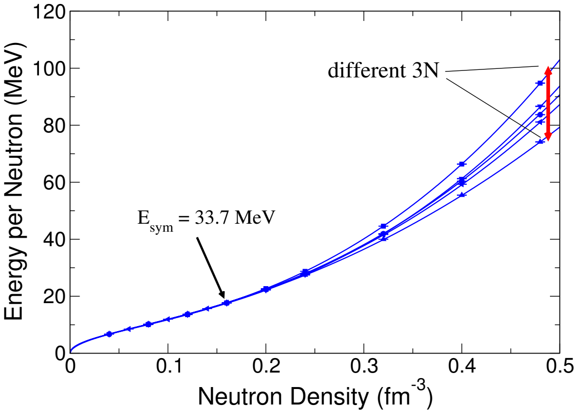
Assumption from experiments:

$$E_{SNM}(\rho_0) = -16 MeV\,, \quad \rho_0 = 0.16 fm^{-3}\,, \quad E_{sym} = E_{PNM}(\rho_0) + 16$$

At ρ_0 we access E_{sym} by studying PNM.

Neutron matter

We consider different forms of three-neutron interaction by only requiring a particular value of E_{sym} at saturation.

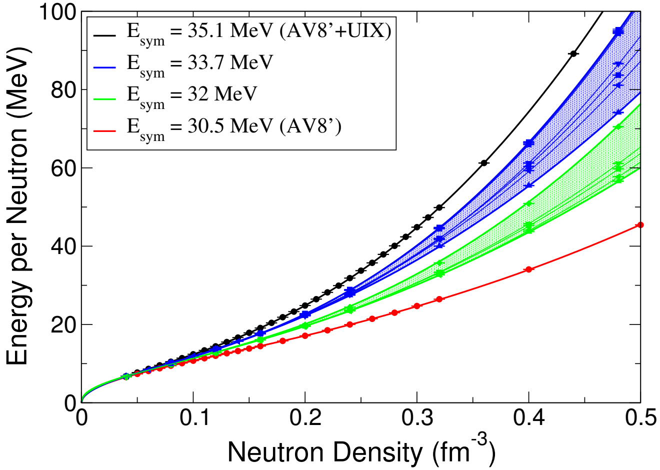


different 3N:

- $V_{2\pi} + \alpha V_R$
- $V_{2\pi} + \alpha V_R^{\mu}$ (several μ)
- $V_{2\pi} + \alpha \tilde{V}_R$
- $V_{3\pi} + \alpha V_R$

Neutron matter

Equation of state of neutron matter using Argonne forces:

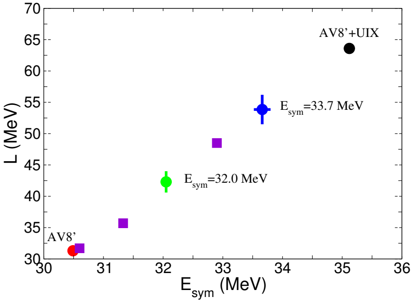


Gandolfi, Carlson, Reddy, PRC (2012)

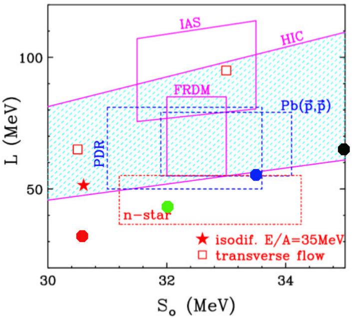
Neutron matter and symmetry energy

From the EOS, we can fit the symmetry energy around ρ_0 using

$$E_{sym}(\rho) = E_{sym} + \frac{L}{3}\frac{\rho - 0.16}{0.16} + \cdots$$



Gandolfi *et al.*, EPJ (2014)



Tsang *et al.*, PRC (2012)

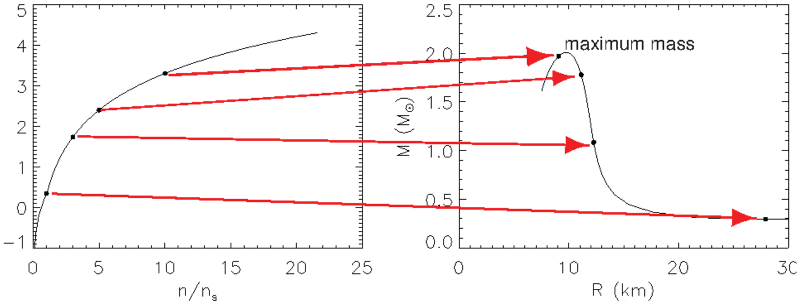
Very weak dependence to the model of 3N force for a given E_{sym}.
Knowing E_{sym} or L useful to constrain 3N! (within this model...)

Neutron matter and neutron star structure

TOV equations:

$$\frac{dP}{dr} = -\frac{G[m(r) + 4\pi r^3 P/c^2][\epsilon + P/c^2]}{r[r - 2Gm(r)/c^2]},$$

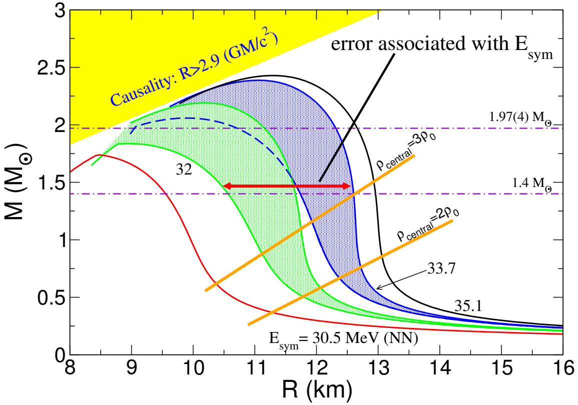
$$\frac{dm(r)}{dr} = 4\pi\epsilon r^2,$$



J. Lattimer

Neutron star structure

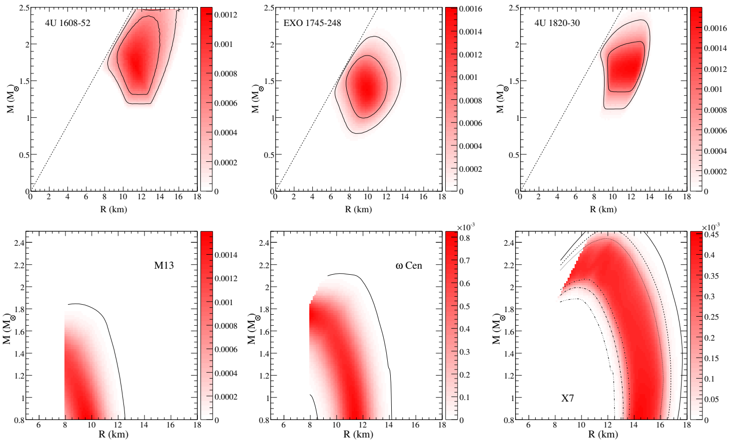
EOS used to solve the TOV equations.



Gandolfi, Carlson, Reddy, PRC (2012).

Accurate measurement of E_{sym} put a constraint to the radius of neutron stars, **OR** observation of M and R would constrain E_{sym}!

Neutron stars



Steiner, Lattimer, Brown, ApJ (2010)

Neutron star observations can be used to 'measure' the EOS and constrain E_{sym} and L. (Systematic uncertainties still under debate...)

Neutron star matter

Neutron star matter model:

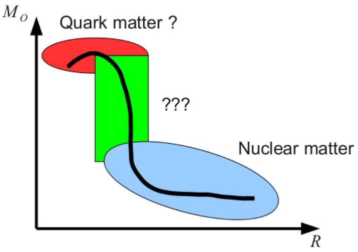
$$E_{NSM} = a\left(\frac{\rho}{\rho_0}\right)^{\alpha} + b\left(\frac{\rho}{\rho_0}\right)^{\beta}, \quad \rho < \rho_t$$

(form suggested by QMC simulations),

and a high density model for $\rho > \rho_t$

i) two polytropes

ii) polytrope+quark matter model

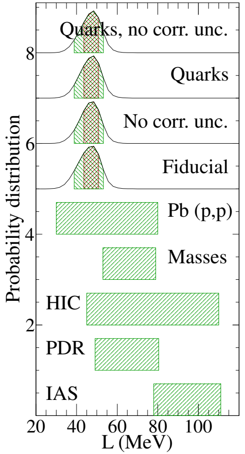


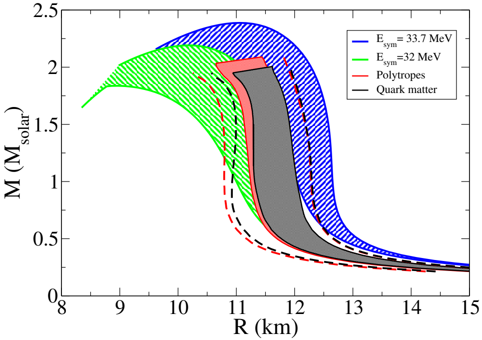
Neutron star radius sensitive to the EOS at nuclear densities!

Direct way to extract E_{sym} and L from neutron stars observations:

$$E_{sym} = a + b + 16, \quad L = 3(a\alpha + b\beta)$$

Neutron star matter really matters!





$32 < E_{sym} < 34\,MeV, \; 43 < L < 52\,MeV$

Steiner, Gandolfi, PRL (2012).

Uncertainties on observations not included!

Summary

- Neutron drops provide an "artificial" useful system to calibrate density functionals
- Radii of neutron drops correlated to the skin thickness of nuclei and to L
- Three-neutron force is the bridge between E_{sym} and neutron star structure.
- Neutron star observations becoming competitive with experiments.

For the future: use more dense neutron drops to predict L at different densities?

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- Andrew Steiner (UT/ORNL)