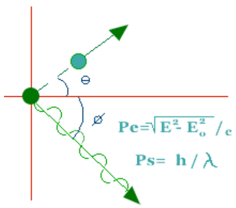
Complete experimental picture of proton and neutron properties in ^{40}Ca and ^{48}Ca and its theoretical interpretation

NSkin Mainz
5/23/2016

Washington University in St. Louis



Wim Dickhoff
Bob Charity
Helber Dussan
Hossein Mahzoon
Mack Atkinson

- Motivation
- Green's functions method
  - ab initio
  - as a framework to analyze experimental data (and extrapolate and predict properties of exotic nuclei)

--> dispersive optical model (DOM)

- Focus on recent DOM —> DSM developments
- Can do more than expected!
- Conclusions

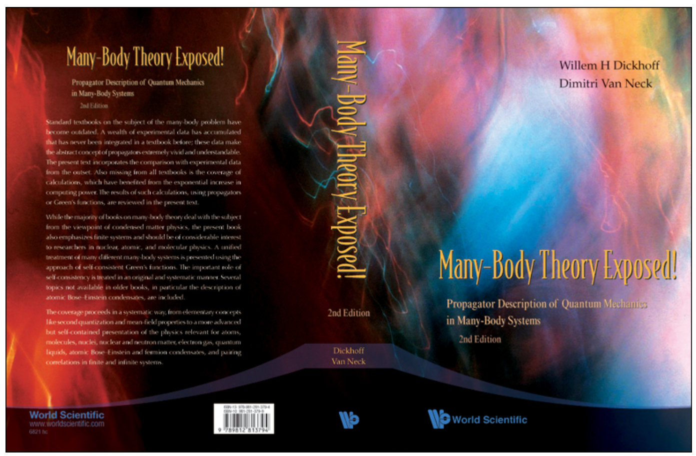
reactions and structure

Motivation

- Rare isotope physics requires a **much** stronger link between nuclear reactions and nuclear structure descriptions
- We need an ab initio approach for optical potential —> optical potentials must therefore become **nonlocal** and **dispersive**
- Current status to extract structure information from nuclear reactions involving strongly interacting probes **unsatisfactory**
- Intermediate step: dispersive optical model as originally proposed by Claude Mahaux —> in need of **extensions** some discussed here

Remarks

- Given a Hamiltonian, a perturbation expansion can be generated for the single-particle propagator or Green's function
- Dyson equation determines propagator in terms of nucleon self-energy <—> also referred to as optical potential at positive energy
- Self-energy is causal and obeys dispersion relations relating its real and imaginary part and must also be nonlocal (even in HF)
- See e.g. —>



- Data constrained self-energy acts as ideal interface between ab initio theory and experiment and allows unexpected predictions!

Propagator / Green's function

- Lehmann representation

$$G_{\ell j}(k,k';E) = \sum_m \frac{\langle\Psi_0^A|\, a_{k\ell j}\,|\Psi_m^{A+1}\rangle\,\langle\Psi_m^{A+1}|\, a^\dagger_{k'\ell j}\,|\Psi_0^A\rangle}{E-(E_m^{A+1}-E_0^A)+i\eta} + \sum_n \frac{\langle\Psi_0^A|\, a^\dagger_{k'\ell j}\,|\Psi_n^{A-1}\rangle\,\langle\Psi_n^{A-1}|\, a_{k\ell j}\,|\Psi_0^A\rangle}{E-(E_0^A-E_n^{A-1})-i\eta}$$

- Any other single-particle basis can be used
- Overlap functions --> numerator
- Corresponding eigenvalues --> denominator
- Spectral function

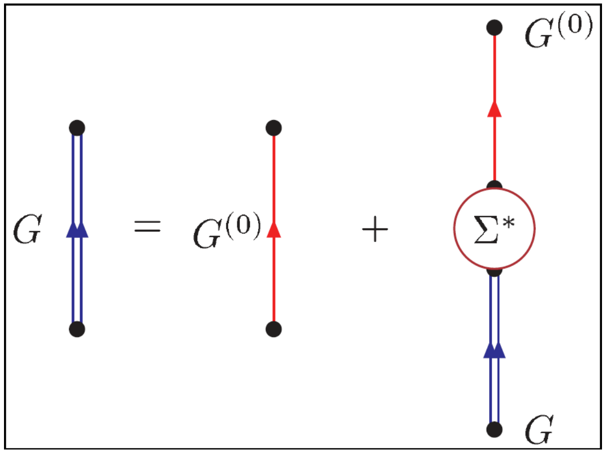
$$S_{\ell j}(k;E) = \frac{1}{\pi}\,\mathrm{Im}\,G_{\ell j}(k,k;E) = \sum_n \left|\langle\Psi_n^{A-1}|\, a_{k\ell j}\,|\Psi_0^A\rangle\right|^2 \delta(E-(E_0^A-E_n^{A-1})) \qquad E \le \varepsilon_F^-$$

- Spectral strength in the continuum

$$S_{\ell j}(E) = \int_0^\infty dk\; k^2\; S_{\ell j}(k;E)$$

- Discrete transitions $\sqrt{S^n_{\ell j}}\;\phi^n_{\ell j}(k) = \langle\Psi_n^{A-1}|\, a_{k\ell j}\,|\Psi_0^A\rangle$
- Positive energy —> see later

Propagator from Dyson Equation and "experiment"



Equivalent to ...

Schrödinger-like equation with: $E_n^- = E_0^A - E_n^{A-1}$

Self-energy: non-local, energy-dependent potential

With energy dependence: spectroscopic factors < 1

$\Rightarrow$ as extracted from (e,e′p) reaction

$$\frac{k^2}{2m}\phi_{\ell j}^n(k) + \int dq\; q^2\; \Sigma_{\ell j}^*(k,q;E_n^-)\;\phi_{\ell j}^n(q) = E_n^-\;\phi_{\ell j}^n(k)$$

Spectroscopic factor $\quad S_{\ell j}^n = \int dk\; k^2\; \left|\langle \Psi_n^{A-1} | \, a_{k\ell j} \, | \Psi_0^A \rangle\right|^2 < 1$

Dyson equation also yields $\left[\chi_{\ell j}^{elE}(r)\right]^* = \langle \Psi_{elE}^{A+1} | \, a_{r\ell j}^\dagger \, | \Psi_0^A \rangle$ for positive energies

Elastic scattering wave function for protons or neutrons

Dyson equation therefore provides:

Link between scattering and structure data from **dispersion relations**

reactions and structure

Propagator in principle generates

- Elastic scattering cross sections for p and n
- Including all polarization observables
- Total cross sections for n
- Reaction cross sections for p and n
- Overlap functions for adding p or n to bound states in Z+1 or N+1
- Plus normalization --> spectroscopic factor
- Overlap function for removing p or n with normalization
- Hole spectral function including high-momentum description
- One-body density matrix; occupation numbers; natural orbits
- Charge density
- Neutron distribution
- p and n distorted waves
- Contribution to the energy of the ground state from V_{NN}

reactions and structure

Dispersive Optical Model

- Claude Mahaux 1980s
  - connect traditional optical potential to bound-state potential
  - crucial idea: use the dispersion relation for the nucleon self-energy
  - smart implementation: use it in its subtracted form
  - applied successfully e.g. to ^{40}Ca and ^{208}Pb in a limited energy window
  - employed traditional volume and surface absorption potentials and a local energy-dependent Hartree-Fock-like potential
  - Reviewed in Adv. Nucl. Phys. **20**, 1 (1991)
- Radiochemistry group at Washington University in St. Louis: Charity and Sobotka propose to use the DOM for a sequence of Ca isotopes —> data-driven extrapolations to the drip line
  - First results PRL 97, 162503 (2006)
  - Subsequently —> attention to data **below** the Fermi energy related to ground-state properties —> Dispersive Self-energy Method (**DSM**)

Optical potential <--> nucleon self-energy

- e.g. Bell and Squires --> elastic T-matrix = reducible self-energy
- e.g. Mahaux and Sartor Adv. Nucl. Phys. **20**, 1 (1991)
  - relate dynamic (energy-dependent) real part to imaginary part
  - employ subtracted dispersion relation

General dispersion relation for self-energy:

$$\text{Re }\Sigma(E) = \Sigma^{HF} - \frac{1}{\pi}\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\text{Im }\Sigma(E')}{E-E'} + \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\text{Im }\Sigma(E')}{E-E'}$$

Calculated at the Fermi energy $\varepsilon_F = \frac{1}{2}\left\{(E_0^{A+1} - E_0^A) + (E_0^A - E_0^{A-1})\right\}$

$$\text{Re }\Sigma(\varepsilon_F) = \Sigma^{HF} - \frac{1}{\pi}\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\text{Im }\Sigma(E')}{\varepsilon_F-E'} + \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\text{Im }\Sigma(E')}{\varepsilon_F-E'}$$

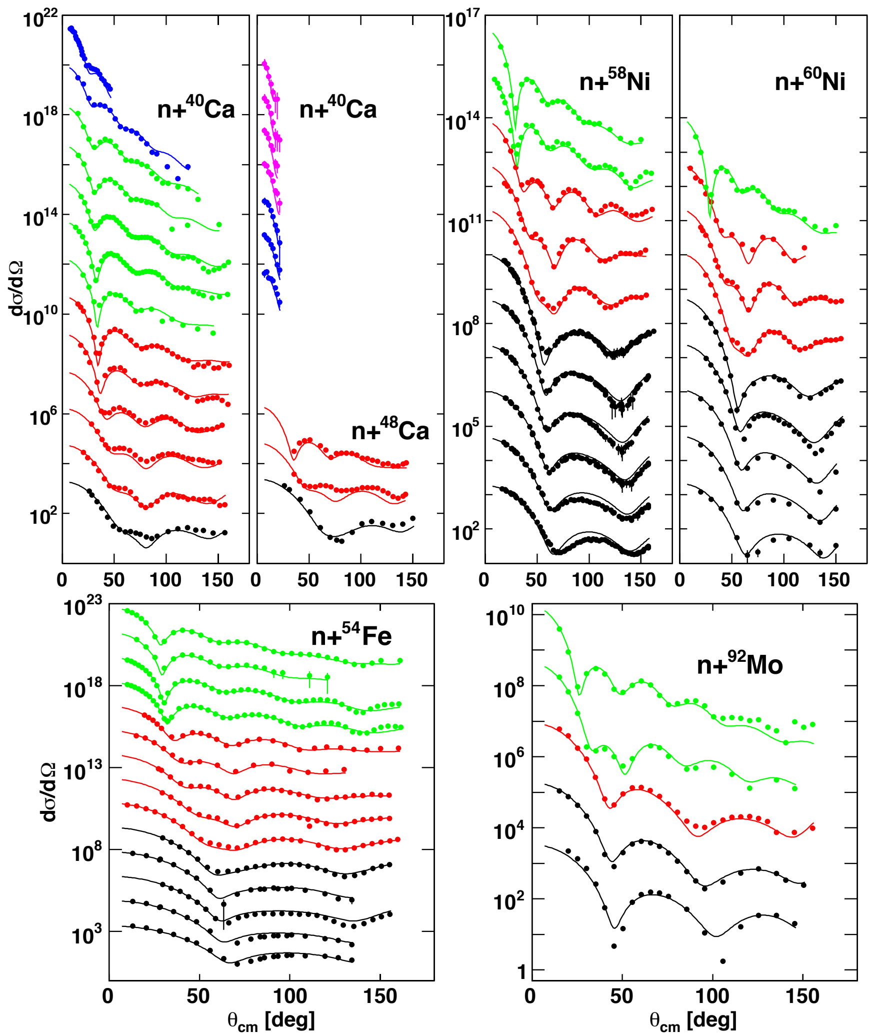
Subtract

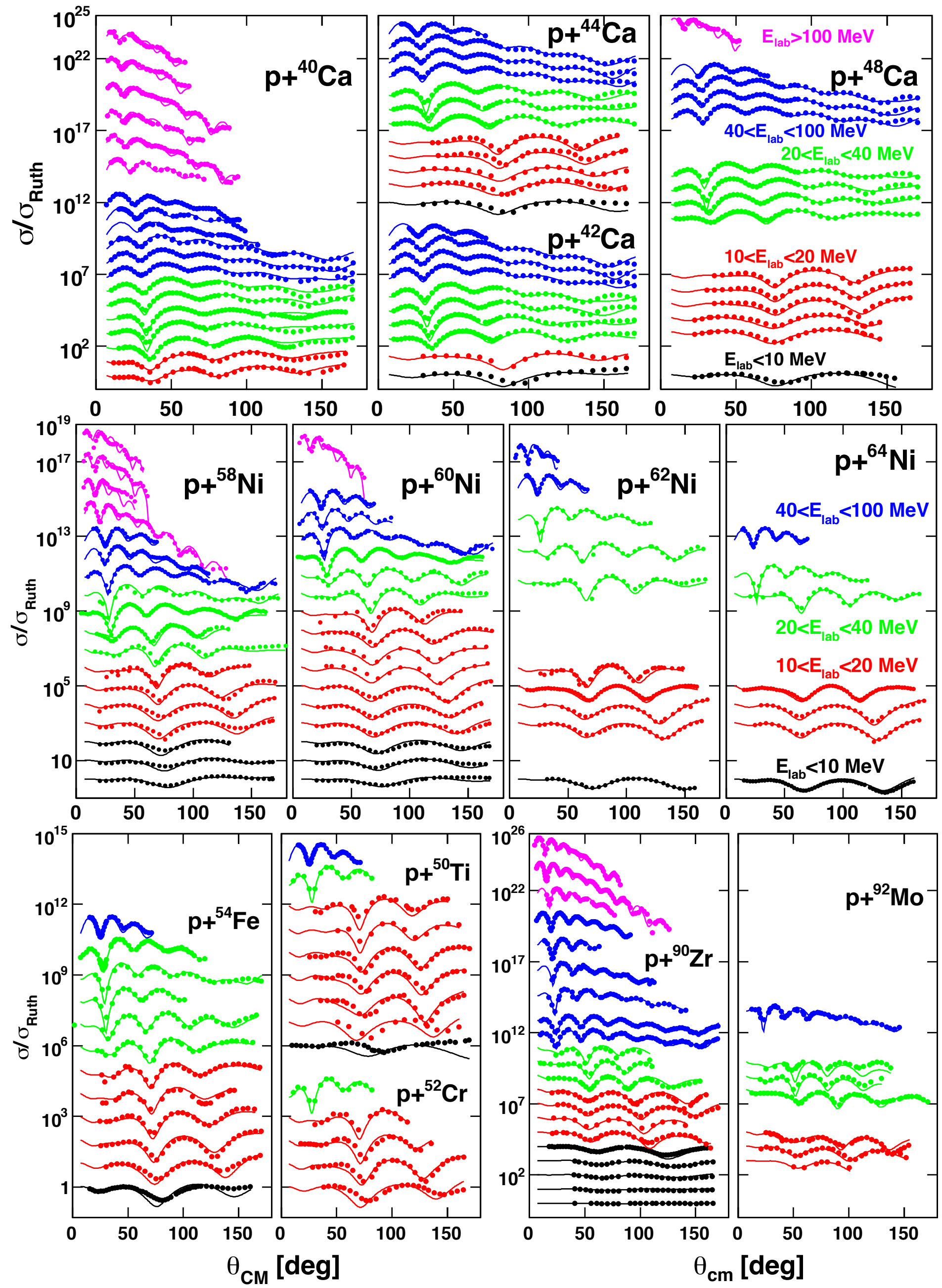
$$\text{Re }\Sigma(E) = \text{Re }\Sigma^{\widetilde{HF}}(\varepsilon_F)$$

$$- \quad \frac{1}{\pi}(\varepsilon_F - E)\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\text{Im }\Sigma(E')}{(E-E')(\varepsilon_F-E')} + \frac{1}{\pi}(\varepsilon_F - E)\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\text{Im }\Sigma(E')}{(E-E')(\varepsilon_F-E')}$$

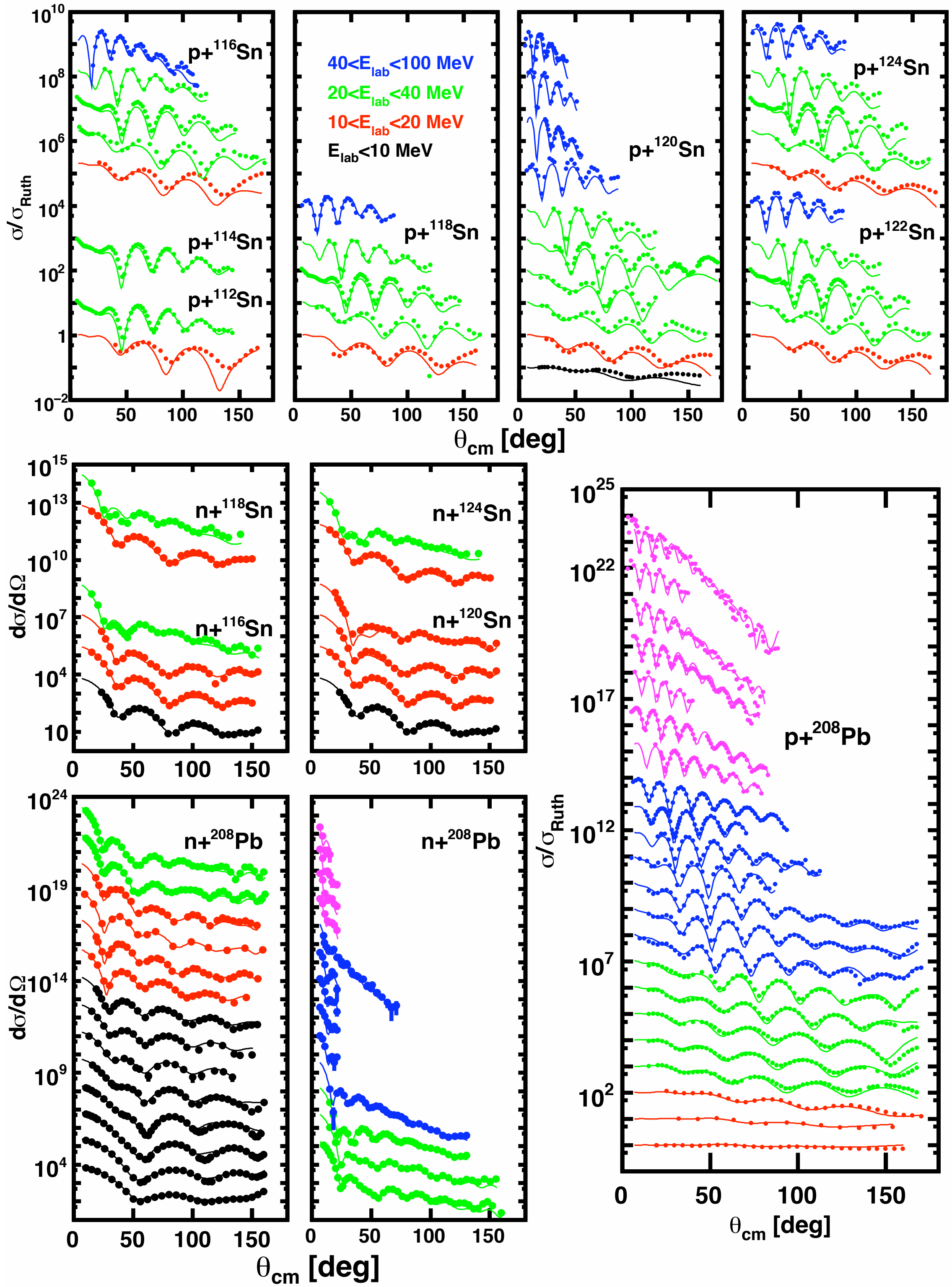
Elastic scattering data for protons and neutrons

- Local DOM implementation





J. Mueller et al.
PRC83,064605 (2011), 1-32



Recent local DOM analysis --> towards global

J. Mueller et al.
PRC83,064605 (2011), 1-32

Nonlocal DOM implementation PRL112,162503(2014)

- Particle number --> **nonlocal** imaginary part
- Microscopic FRPA & SRC --> different nonlocal properties above and below the Fermi energy Phys. Rev. C84, 034616 (2011) & Phys. Rev.C84, 044319 (2011)
- Include charge density in fit
- Describe high-momentum nucleons <--> (e,e′p) data from JLab

Implications

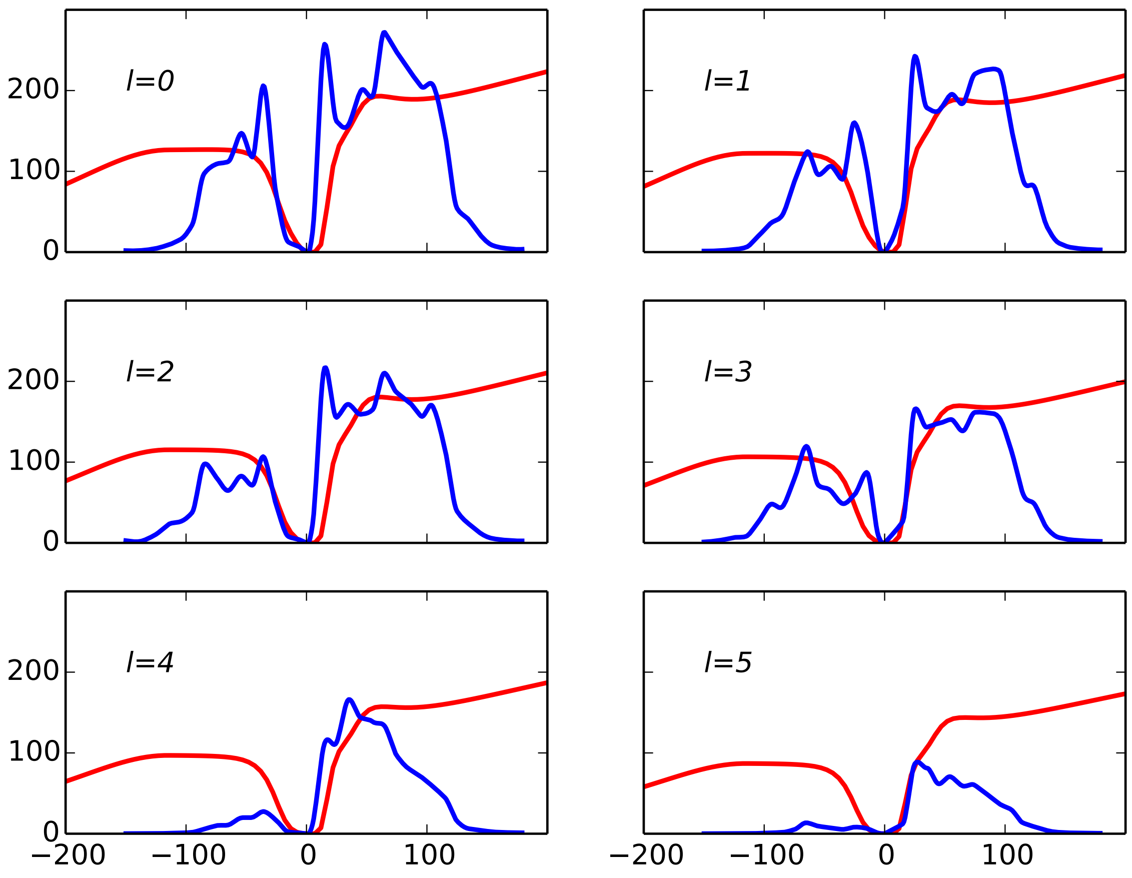
- Changes the description of hadronic reactions because interior nucleon wave functions depend on non-locality
- Consistency test of the interpretation of (e,e′p) possible
- Independent "experimental" statement on size of three-body contribution to the energy of the ground state--> two-body only:

$$E/A = \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^\infty dk k^2 \frac{k^2}{2m} n_{\ell j}(k) + \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^\infty dk k^2 \int_{-\infty}^{\varepsilon_F} dE\ E S_{\ell j}(k;E)$$

reactions and structure

Comparison with ab initio FRPA calculation

- Volume integrals of imaginary part of nonlocal ab initio (FRPA) self-energy compared with DOM result for ^{40}Ca

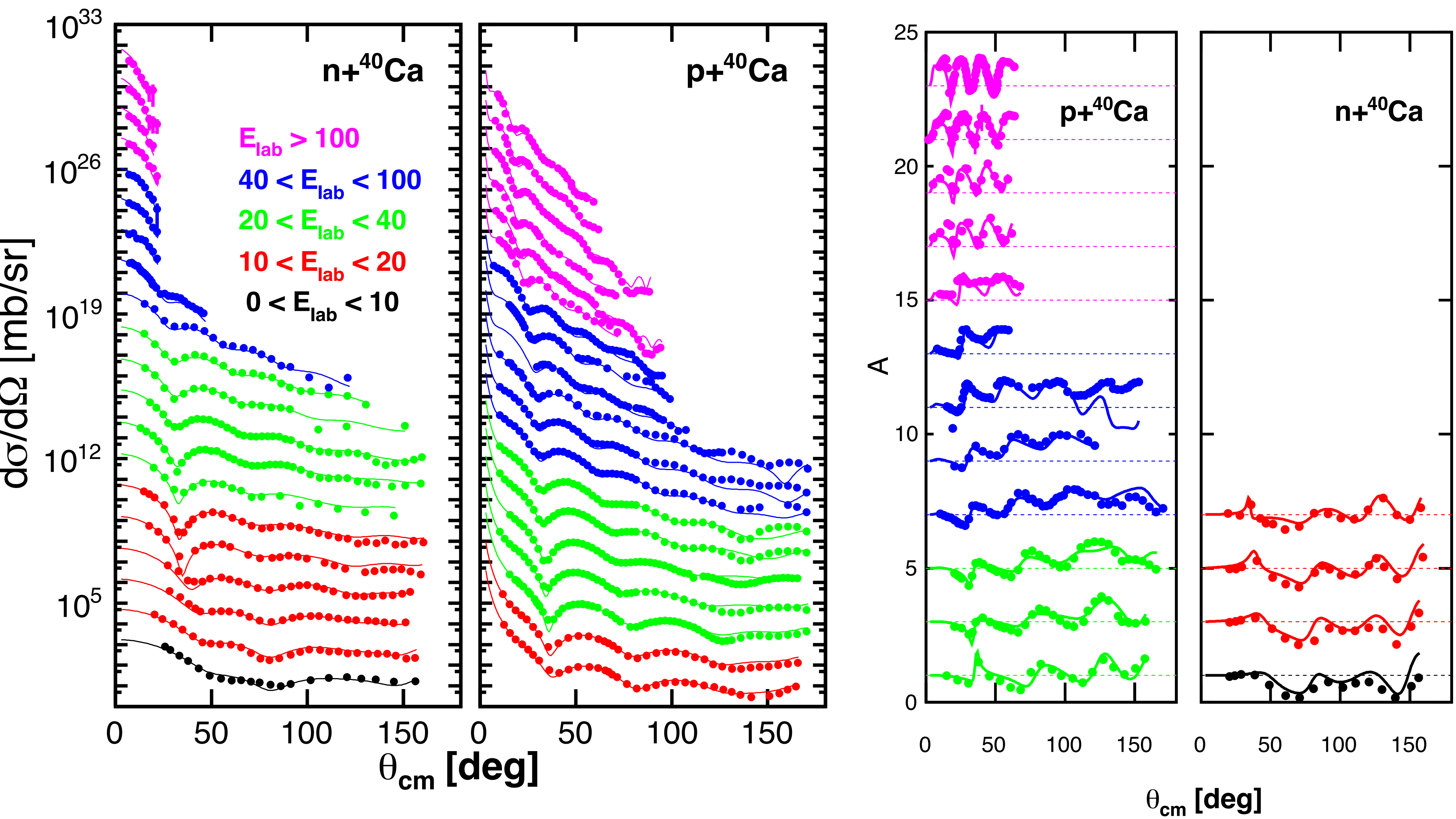


- Ab initio

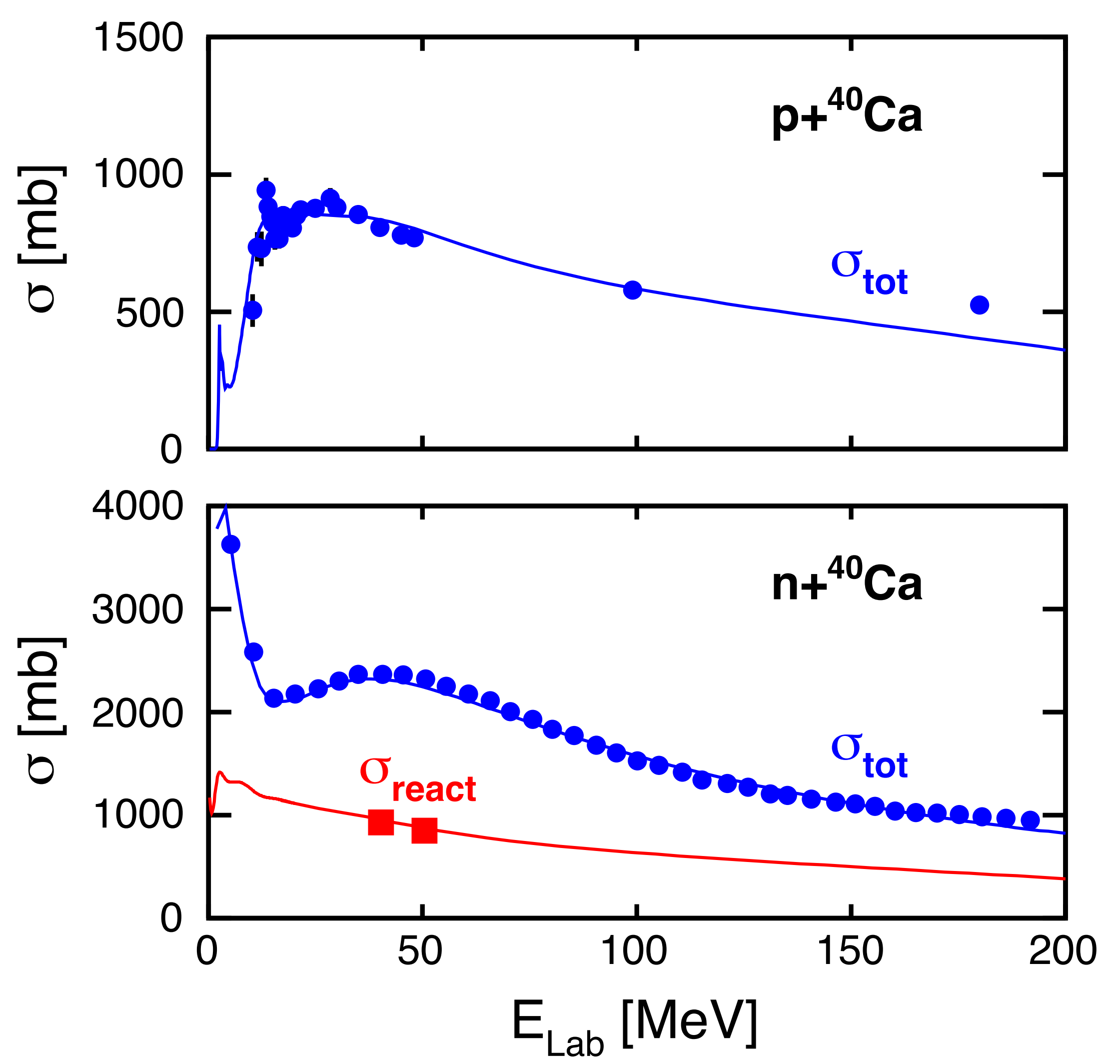
S. J. Waldecker, C. Barbieri and W. H. Dickhoff
Microscopic self-energy calculations and dispersive-optical-model potentials.
Phys. Rev. C84, 034616 (2011), 1-11.

reactions and structure

Differential cross sections and analyzing powers



Reaction (p&n) and total (n) cross sections

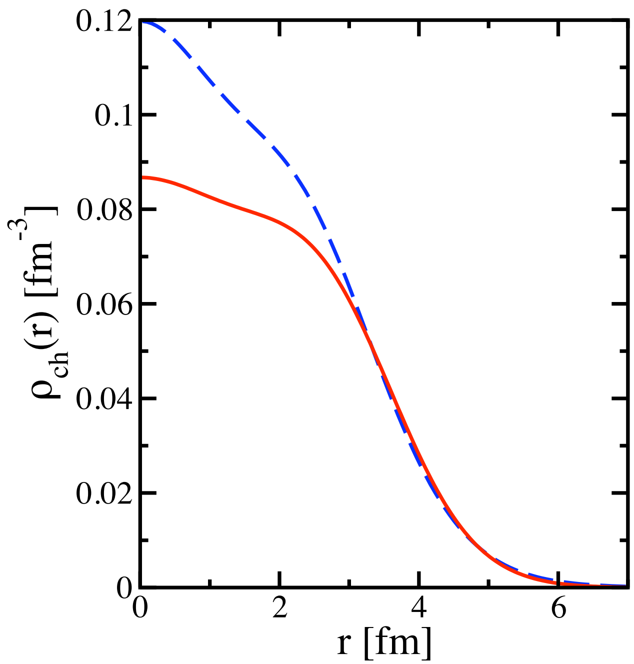


Critical experimental data—> charge density

Local version

radius correct…

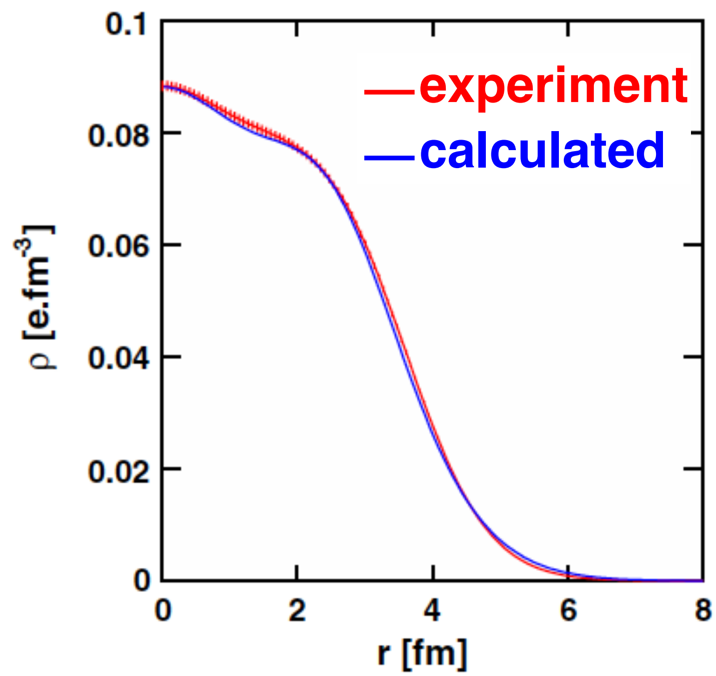
PRC82, 054306 (2010)



Charge density ^{40}Ca

Non-locality essential

PRL 112,162503(2014)



High-momentum nucleons —> JLab can also be described —> E/A

Do elastic scattering data tell us about correlations?

- Scattering T-matrix (neutrons)

$$\Sigma_{\ell j}(k,k';E) = \Sigma^*_{\ell j}(k,k';E) + \int dq q^2 \Sigma^*_{\ell j}(k,q;E) G^{(0)}(q;E) \Sigma_{\ell j}(q,k';E)$$

- Free propagator $G^{(0)}(q;E) = \dfrac{1}{E - \hbar^2 q^2/2m + i\eta}$
- Propagator

$$G_{\ell j}(k,k';E) = \frac{\delta(k-k')}{k^2} G^{(0)}(k;E) + G^{(0)}(k;E)\Sigma_{\ell j}(k,k';E) G^{(0)}(k;E)$$

- Spectral representation

$$G^p_{\ell j}(k,k';E) = \sum_n \frac{\phi^{n+}_{\ell j}(k)\left[\phi^{n+}_{\ell j}(k')\right]^*}{E - E^{*A+1}_n + i\eta} + \sum_c \int_{T_c}^{\infty} dE' \, \frac{\chi^{cE'}_{\ell j}(k)\left[\chi^{cE'}_{\ell j}(k')\right]^*}{E - E' + i\eta}$$

- Spectral density for $E > 0$

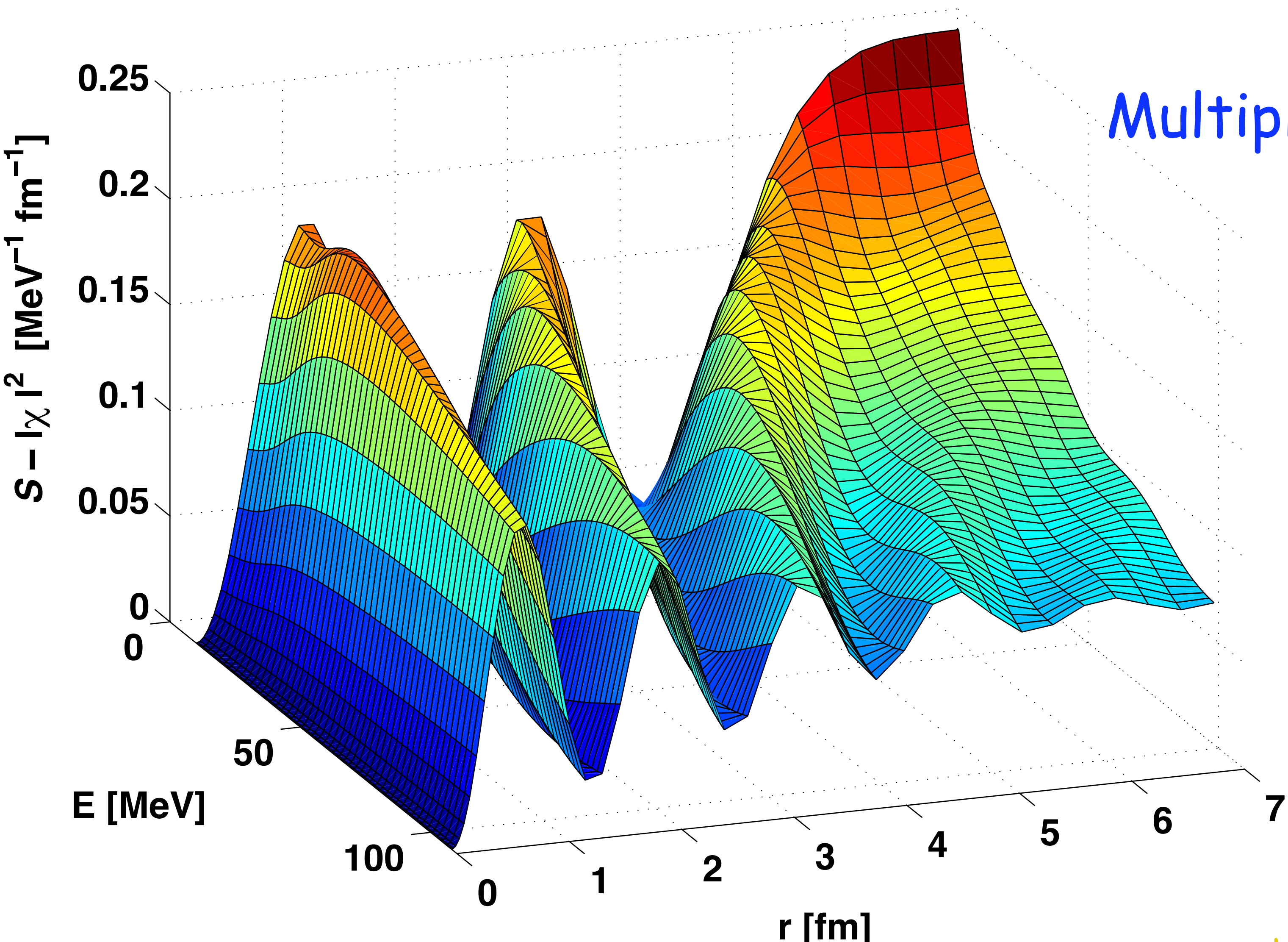
$$S^p_{\ell j}(k,k';E) = \frac{i}{2\pi}\left[G^p_{\ell j}(k,k';E^+) - G^p_{\ell j}(k,k';E^-)\right] = \sum_c \chi^{cE}_{\ell j}(k)\left[\chi^{cE}_{\ell j}(k')\right]^*$$

- Coordinate space $S^p_{\ell j}(r,r';E) = \sum_c \chi^{cE}_{\ell j}(r)\left[\chi^{cE}_{\ell j}(r')\right]^*$
- Elastic scattering also explicitly available

$$\chi^{elE}_{\ell j}(r) = \left[\frac{2mk_0}{\pi\hbar^2}\right]^{1/2} \left\{ j_\ell(k_0 r) + \int dk k^2 j_\ell(kr) G^{(0)}(k;E)\Sigma_{\ell j}(k,k_0;E) \right\}$$

Adding an $s_{1/2}$ neutron to ^{40}Ca

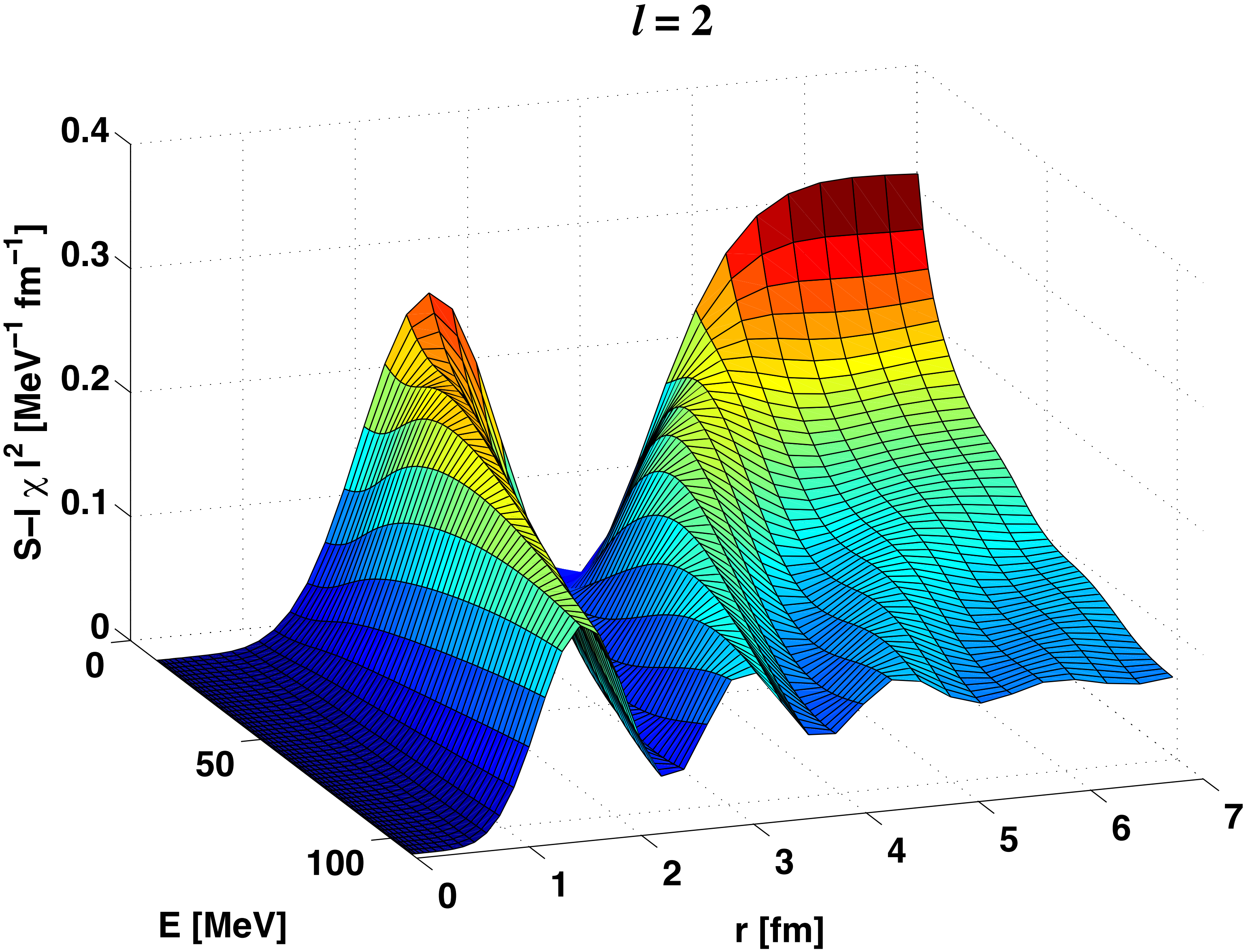
- Inelastically!
- Zero when there is no absorption!



Multiplied by r^2

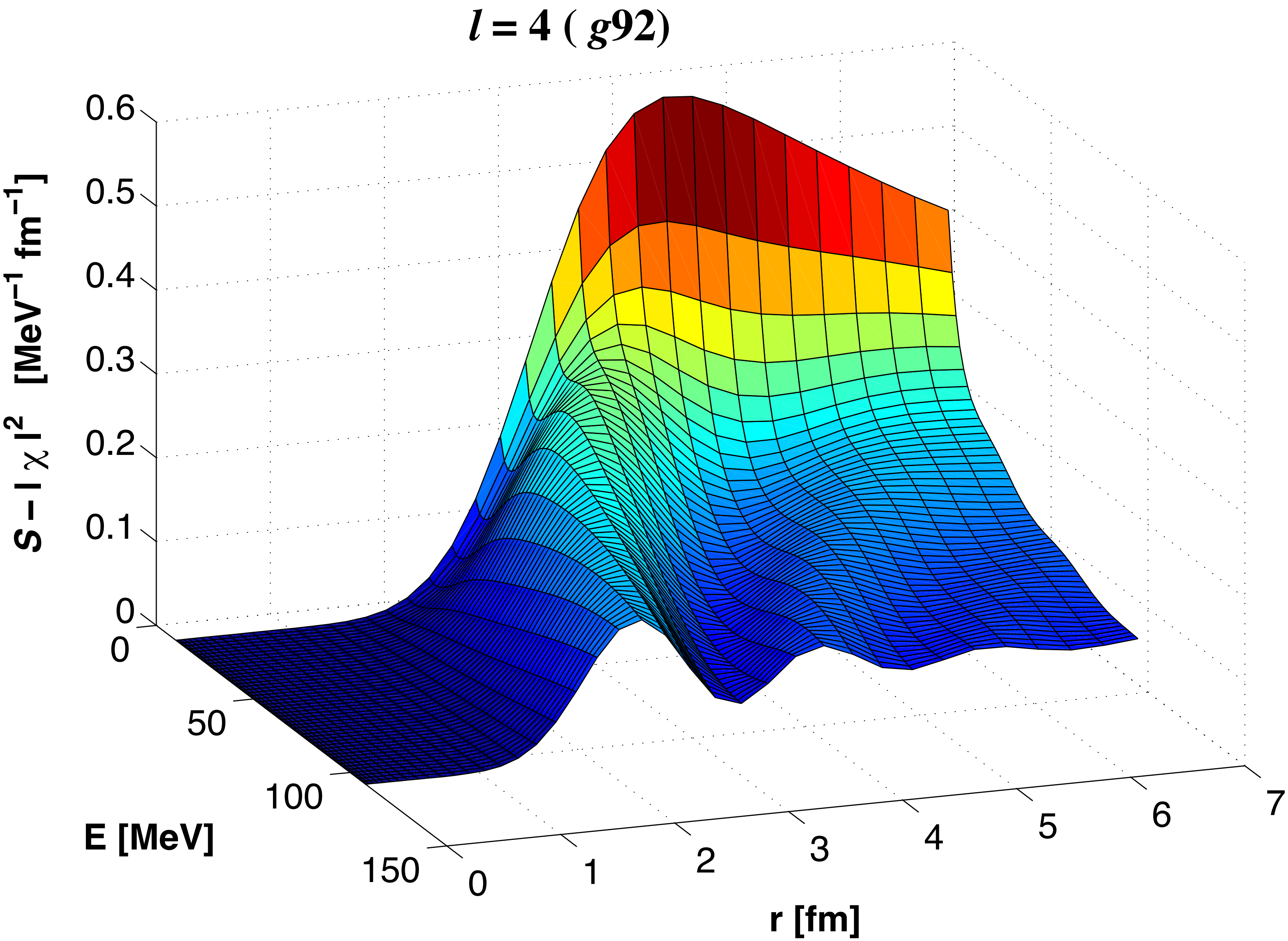
$d_{3/2}$

- One node now



No nodes

- Asymptotically determined by inelasticity



Determine location of bound-state strength

- Fold spectral function with bound state wave function

$$S_{\ell j}^{n+}(E) = \int dr\ r^2 \int dr'\ r'^2 \phi_{\ell j}^{n-}(r) S_{\ell j}^{p}(r, r'; E) \phi_{\ell j}^{n-}(r')$$

- —> Addition probability of bound orbit
- Also removal probability

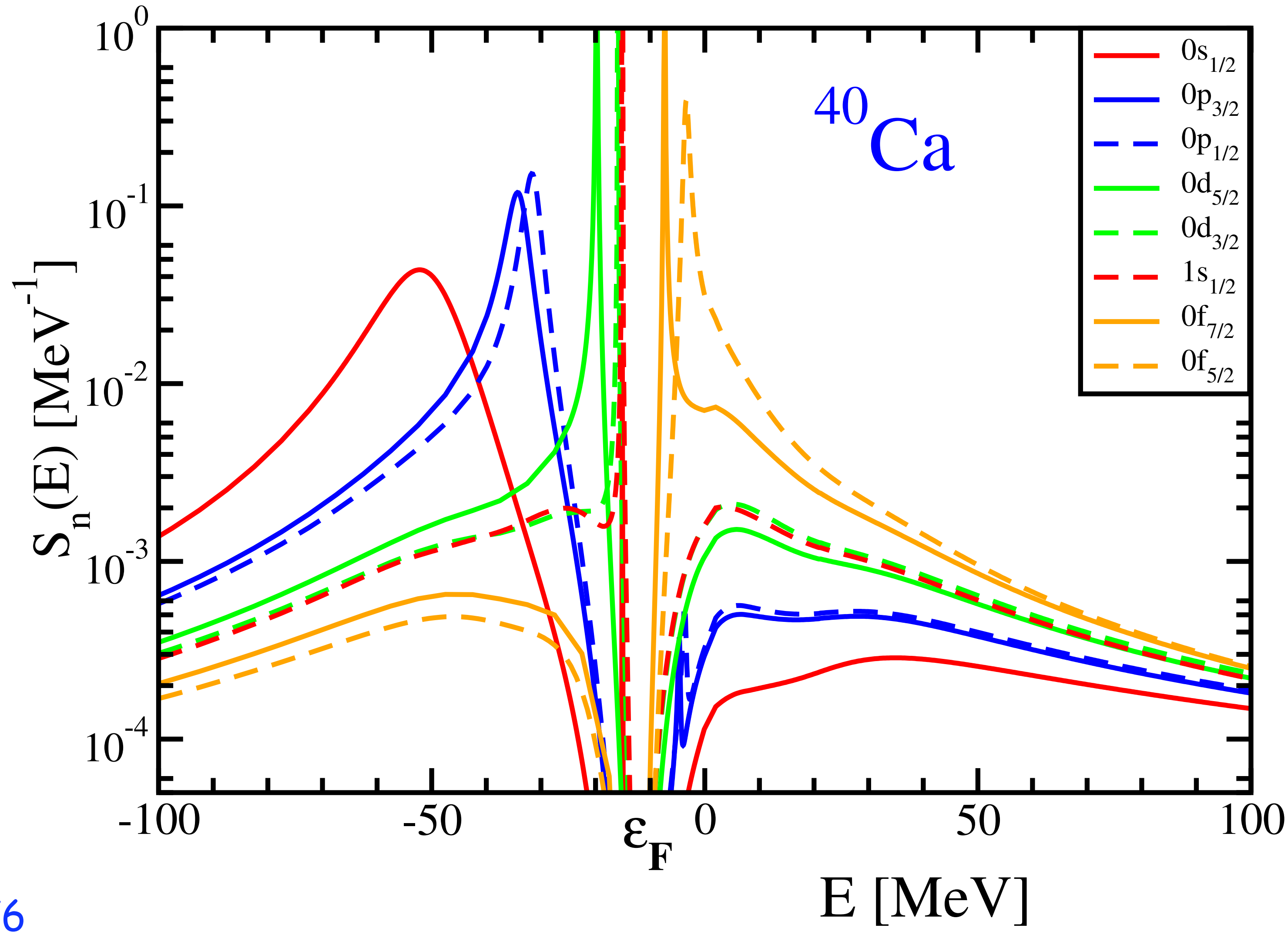
$$S_{\ell j}^{n-}(E) = \int dr r^2 \int dr' r'^2 \phi_{\ell j}^{n-}(r) S_{\ell j}^{h}(r, r'; E) \phi_{\ell j}^{n-}(r')$$

- Overlap function $\sqrt{S_{\ell j}^{n}} \phi_{\ell j}^{n-}(r) = \langle \Psi_n^{A-1} | \, a_{r\ell j} \, | \Psi_0^A \rangle$

- Sum rule $1 = n_{n\ell j} + d_{n\ell j} = \int_{-\infty}^{\varepsilon_F} dE\ S_{\ell j}^{n-}(E) + \int_{\varepsilon_F}^{\infty} dE\ S_{\ell j}^{n-}(E)$

Spectral function for bound states

- [0,200] MeV —> constrained by elastic scattering data



$S_{0d3/2}$= 0.76

$S_{1s1/2}$ = 0.78

0.15 larger than NIKHEF analysis!

PRC90, 061603(R) (2014)

Quantitatively

- Orbit closer to the continuum —> more strength in the continuum
- Note "particle" orbits
- Drip-line nuclei have valence orbits very near the continuum

Table 1: Occupation and depletion numbers for bound orbits in ^{40}Ca. $d_{nlj}[0,200]$ depletion numbers have been integrated from 0 to 200 MeV. The fraction of the sum rule that is exhausted, is illustrated by $n_{n\ell j} + d_{n\ell j}[\varepsilon_F, 200]$. Last column $d_{nlj}[0,200]$ depletion numbers for the CDBonn calculation.

orbit	$n_{n\ell j}$ DOM	$d_{n\ell j}[0,200]$ DOM	$n_{n\ell j} + d_{n\ell j}[\varepsilon_F, 200]$ DOM	$d_{n\ell j}[0,200]$ CDBonn
$0s_{1/2}$	0.926	0.032	0.958	0.035
$0p_{3/2}$	0.914	0.047	0.961	0.036
$1p_{1/2}$	0.906	0.051	0.957	0.038
$0d_{5/2}$	0.883	0.081	0.964	0.040
$1s_{1/2}$	0.871	0.091	0.962	0.038
$0d_{3/2}$	0.859	0.097	0.966	0.041
$0f_{7/2}$	0.046	0.202	0.970	0.034
$0f_{5/2}$	0.036	0.320	0.947	0.036

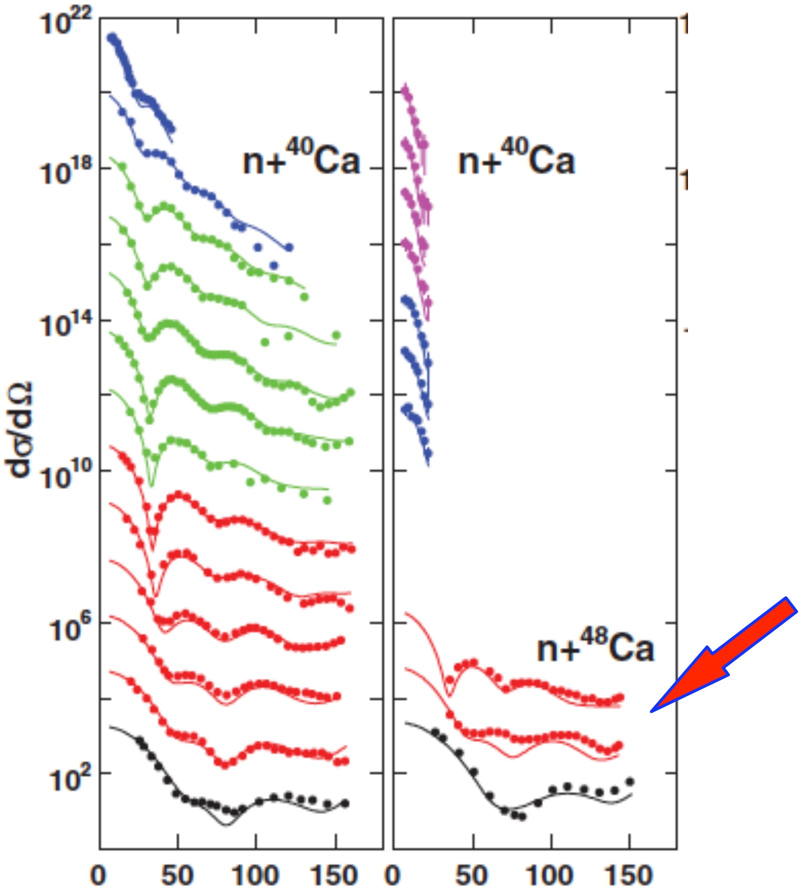
PRC90, 061603(R) (2014)

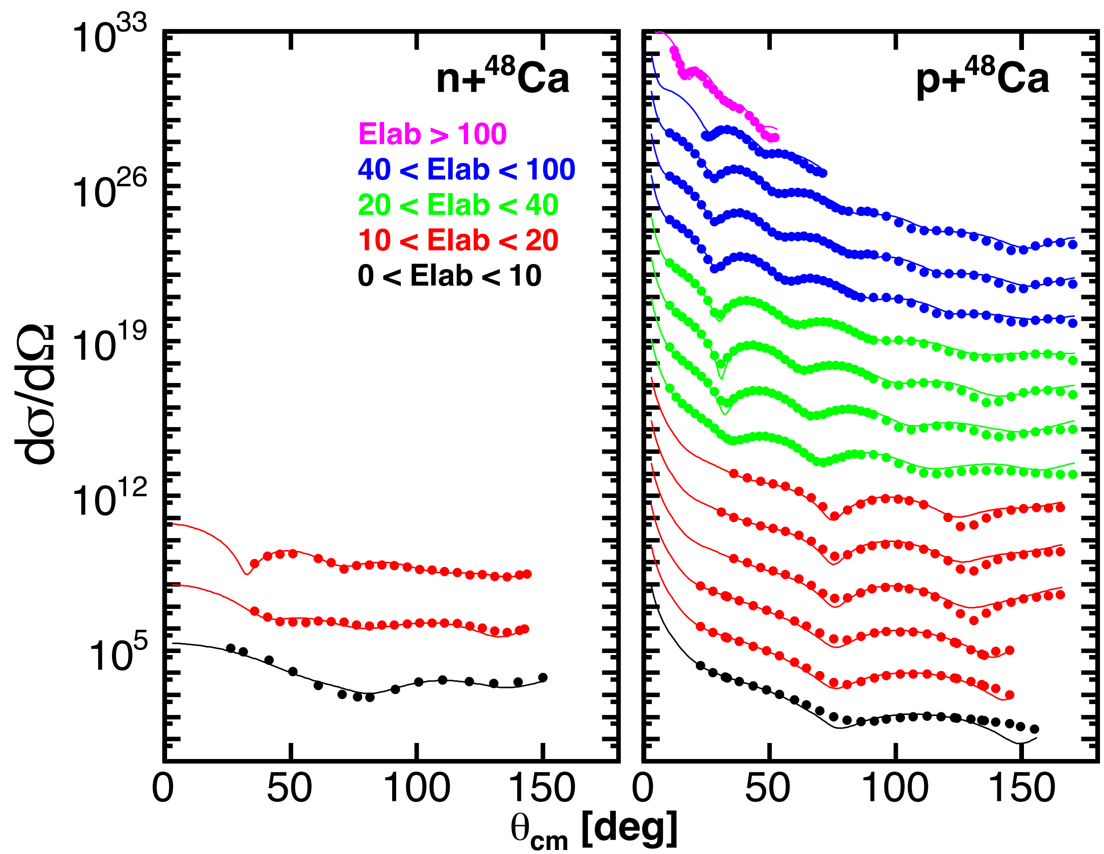
New DOM results for ^{48}Ca

- Change of proton properties when 8 neutrons are added to ^{40}Ca?
- Change of neutron properties?
- Can hard to measure quantities be indirectly constrained?

What about neutrons?

- ^{48}Ca —> charge density has been measured
- Recent neutron elastic scattering **data** —> PRC83,064605(2011)
- Local DOM **OLD** Nonlocal DOM **NEW**

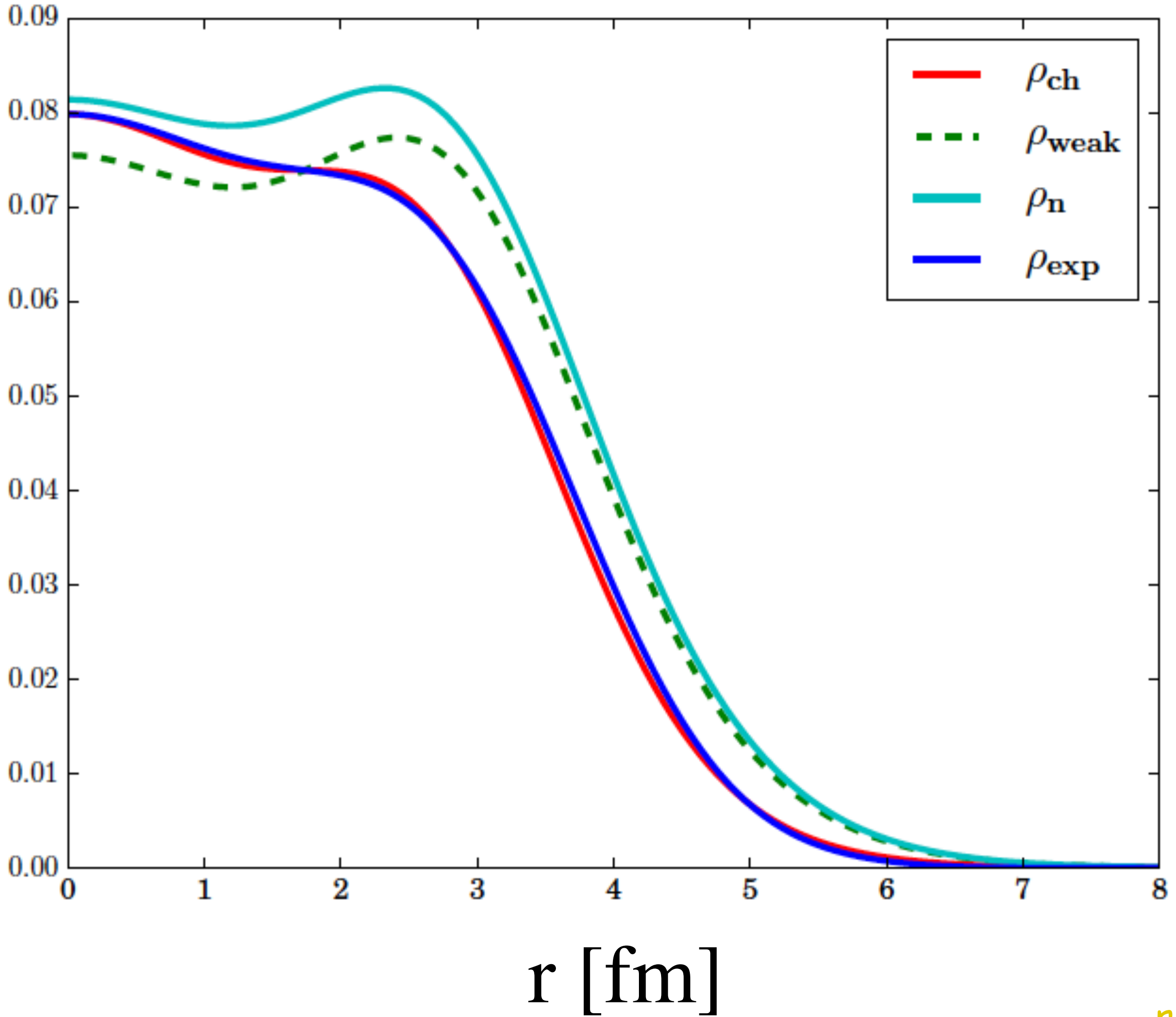




reactions and structure

Results ^{48}Ca

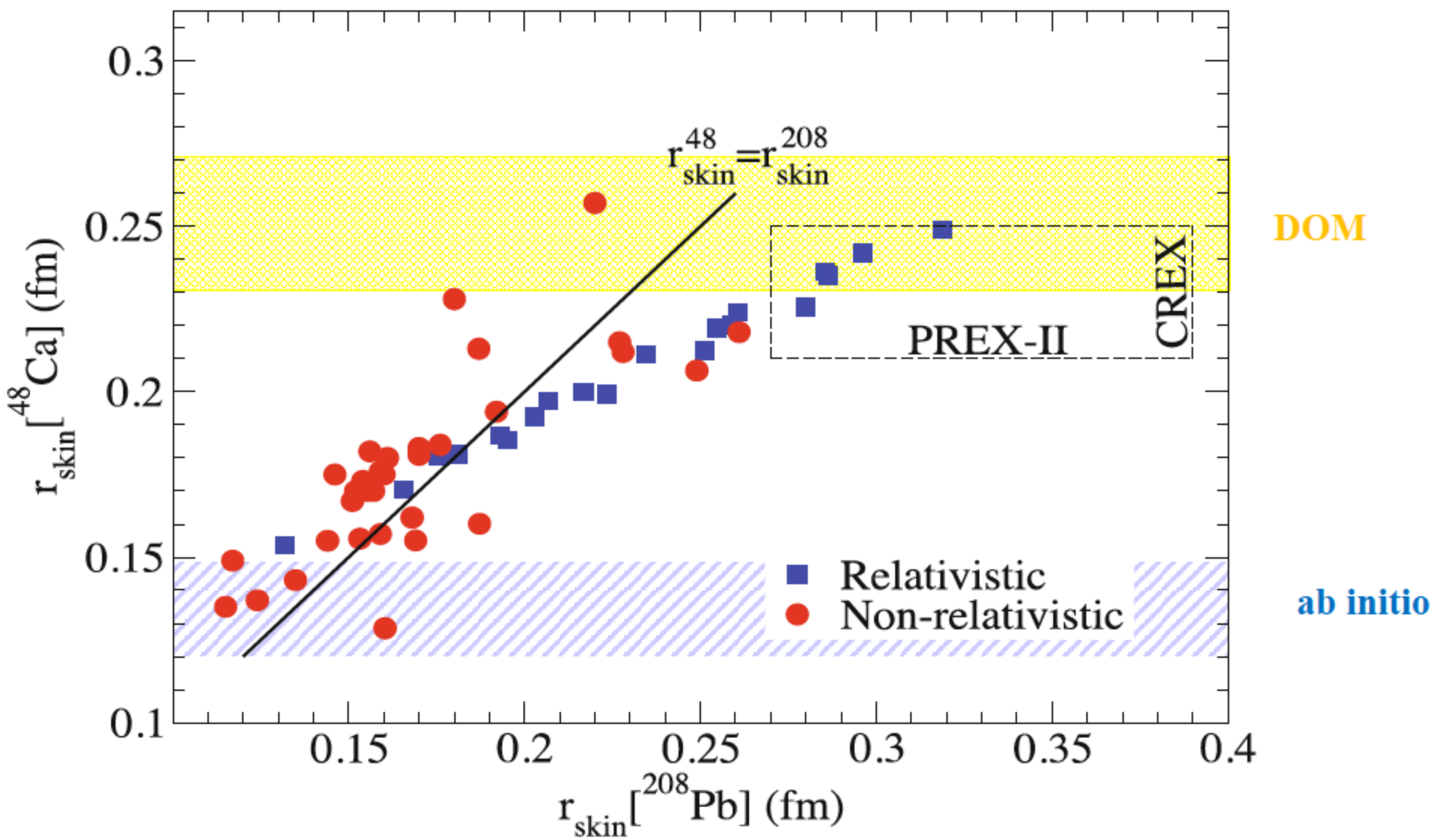
- Density distributions
- DOM —> neutron distribution —> R_n-R_p



Comparison of neutron skin with other calculations and future experiments…

- Figure adapted from

C.J. Horowitz, K.S. Kumar, and R. Michaels, Eur. Phys. J. A (2014)



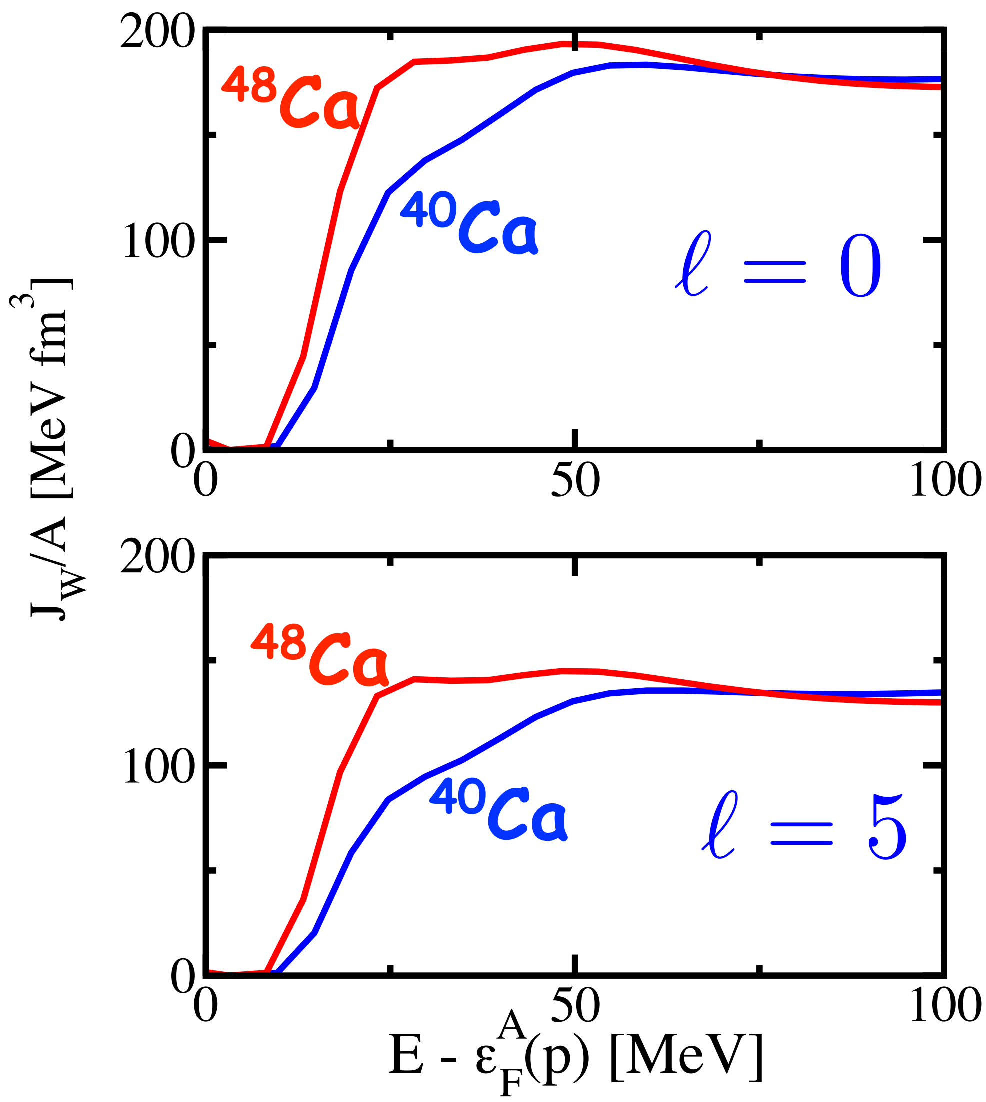
- "Ab initio":

G. Hagen et al., Nature Phys. 12, 186 (2016)

--> drip line

Volume integrals for $^{40\text{-}48}Ca$

- Protons see the same interior but a different surface!



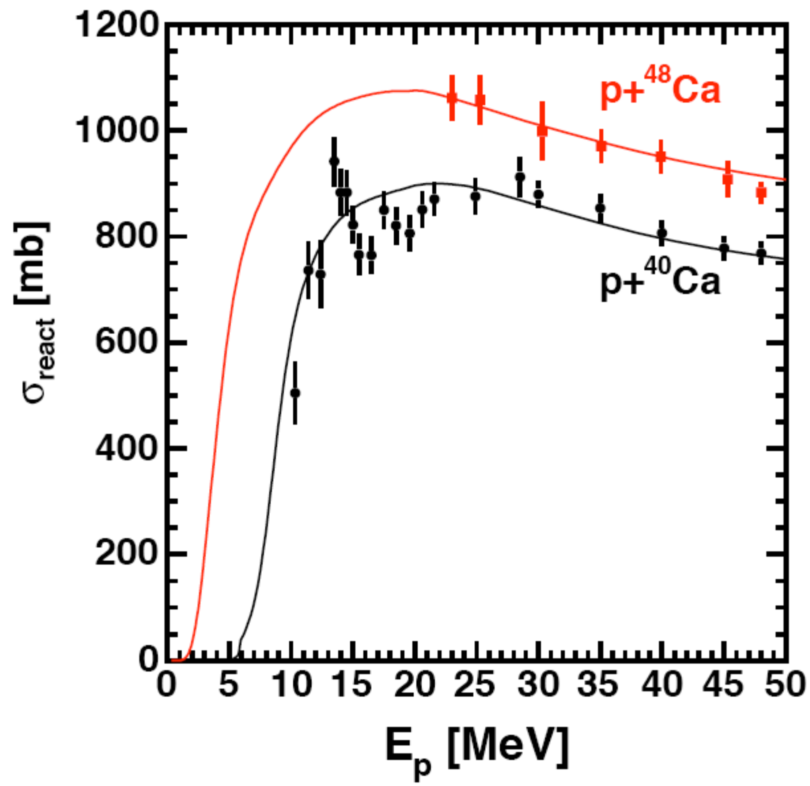
--> drip line

Quantitative comparison of ^{40}Ca and ^{48}Ca

Spectroscopic factors	^{40}Ca	p ^{48}Ca	n ^{48}Ca
$0d_{3/2}$	0.76	0.65 ↓	0.80 ↑
$1s_{1/2}$	0.78	0.71 ↓	0.83 ↑
$0f_{7/2}$	0.73	0.59 ↓	0.84 ↑

reactions and structure

Why are protons in ^{48}Ca more correlated than in ^{40}Ca?

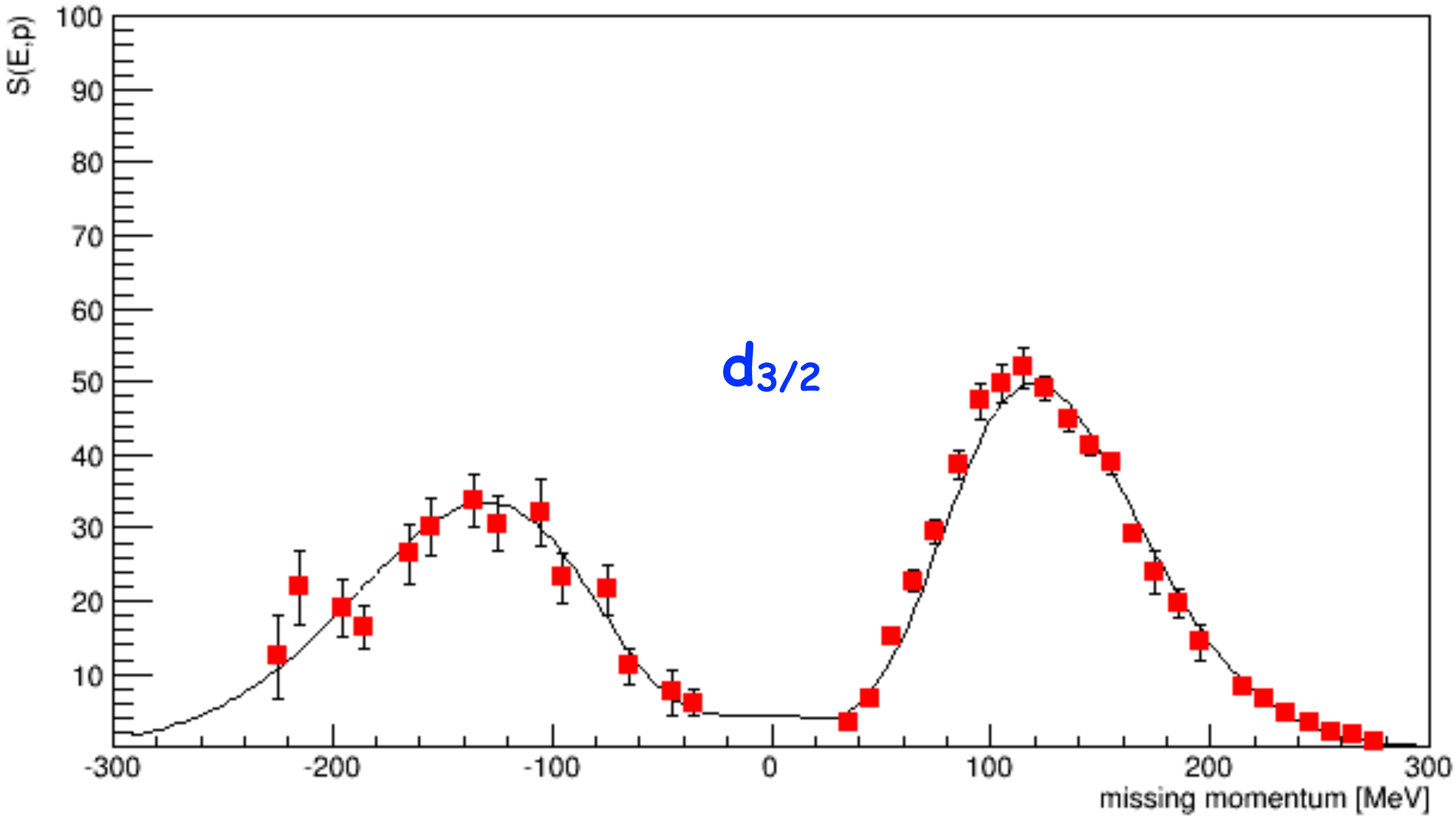


Loss of flux in the elastic channel

Answer: data require more surface absorption in ^{48}Ca than in ^{40}Ca

Very recent analysis (preliminary)

- NIKHEF (e,e'p) data with only DOM input (Atkinson in progress)



Conclusions

- It is possible to link nuclear reactions and nuclear structure
- Vehicle: **nonlocal** version of **Dispersive Optical Model** (Green's function method) as developed by Mahaux —> DSM
- Can be used as input for analyzing nuclear reactions
- Can predict properties of exotic nuclei
- "Benchmark" for ab initio calculations: e.g. V_{NNN} —> binding
- Can describe ground-state properties
  - charge density & momentum distribution
  - spectral properties including high-momentum Jefferson Lab data
- **Elastic scattering determines depletion of bound orbitals**
- Outlook: reanalyze many reactions with nonlocal potentials...
- For N ≳ Z sensitive to properties of neutrons —> weak charge prediction, large neutron skin, perhaps more...

reactions and structure