Solenoid Spectrometers for Measuring Neutron Distributions

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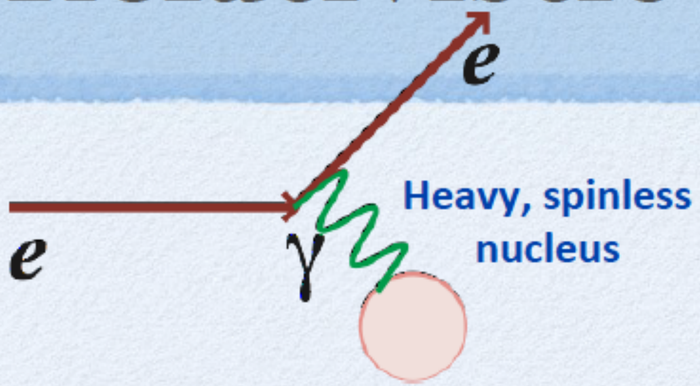
Thanks to Kumar, Michaels, Gertz, Thiel, …

Outline

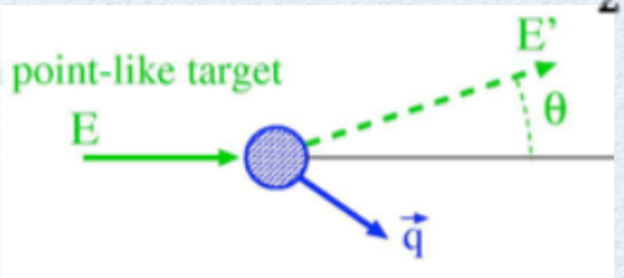
- Nuclear PVES and neutron distributions
- Chuck's ^{48}Ca program
- "Focussing" solenoids
- Application to ^{48}Ca

Relativistic Electron Scattering

and nuclear size



4-momentum transfer $q^2 = -4EE'\sin^2\frac{\theta}{2}$



Q2: -(4-momentum)2 of the virtual photon

$$Q \approx \frac{hc}{\lambda}$$

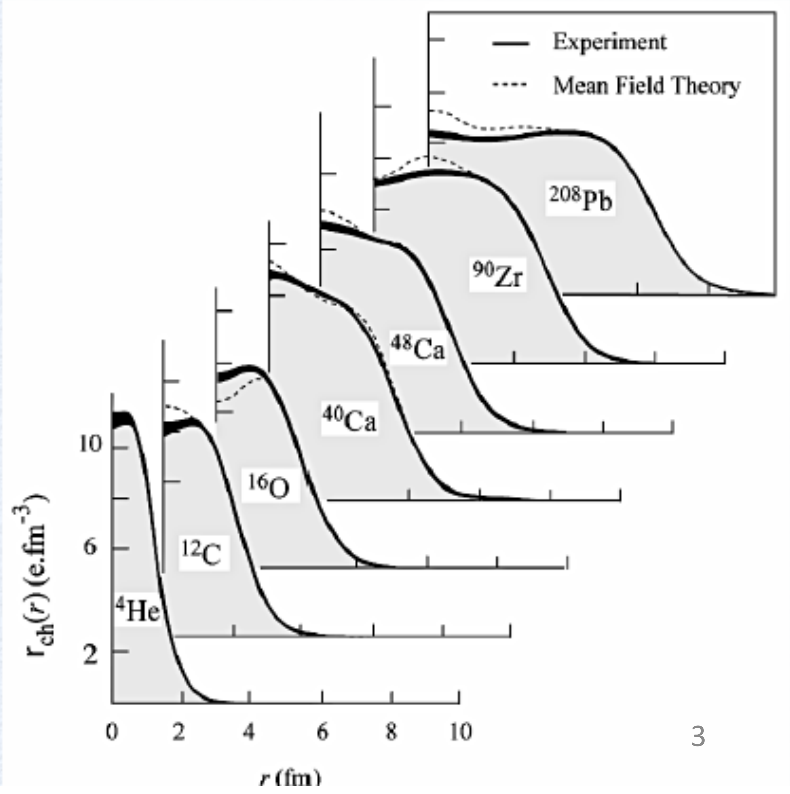
Differential Cross Section

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \frac{4Z^2\alpha^2E^2}{q^4}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} |F(q)|^2$$

As Q increases, nuclear size modifies formula

Neglecting recoil, form factor *F(q)* is the Fourier transform of charge distribution



C-REX (& PREX) : Z^0 of weak interaction : sees the neutrons

	proton	neutron
Electric charge	1	0
Weak charge	0.08	1

T.W. Donnelly, J. Dubach, I. Sick
Nucl. Phys. A 503, 589, 1989

C. J. Horowitz, S. J. Pollock, P. A. Souder, R. Michaels
Phys. Rev. C 63, 025501, 2001

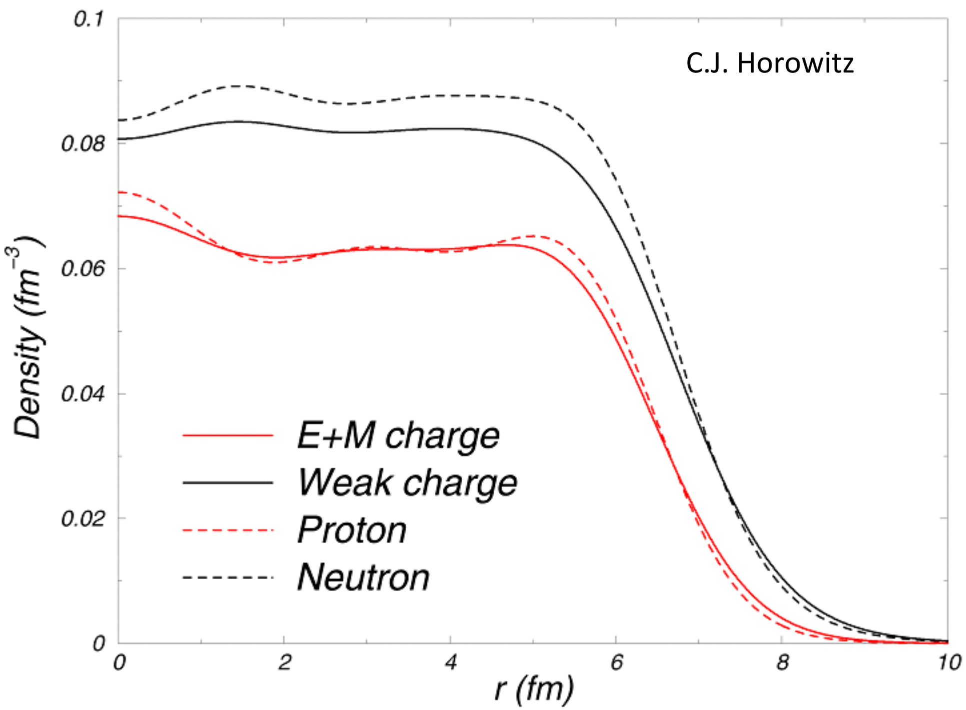
^{208}Pb

Neutron form factor

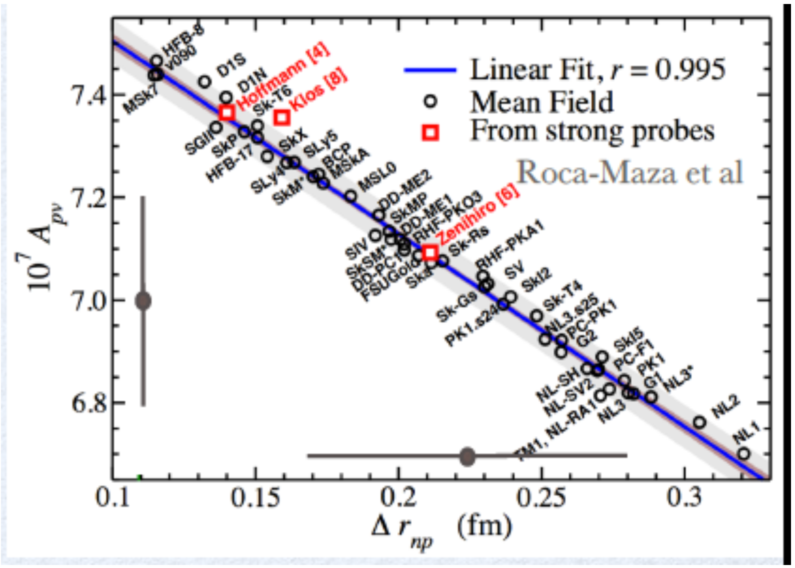
$$F_N(Q^2) = \frac{1}{4\pi} \int d^3r \; j_0(qr) \; \rho_N(r)$$

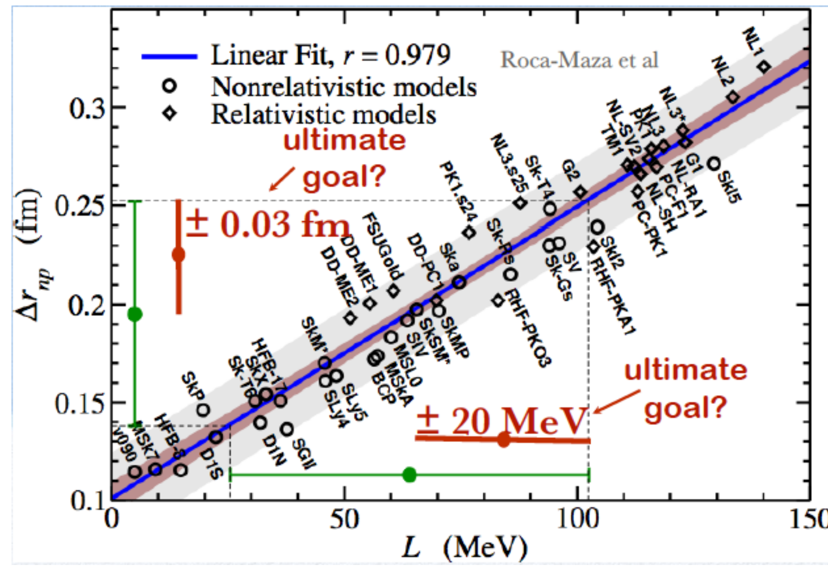
Parity Violating Asymmetry

$$A = \frac{G_F Q^2}{2\pi\alpha\sqrt{2}} \left[\underbrace{1 - 4\sin^2\theta_W}_{\approx 0} - \frac{F_N(Q^2)}{F_P(Q^2)} \right]$$



Prex and Crex: A_{PV} and R_n and L

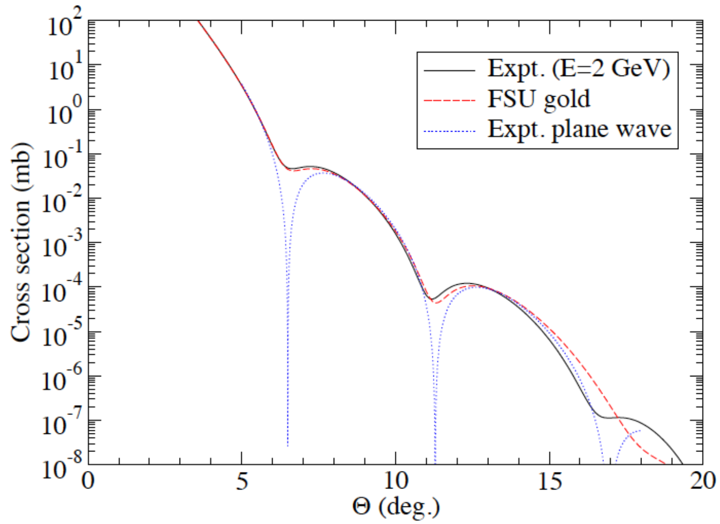




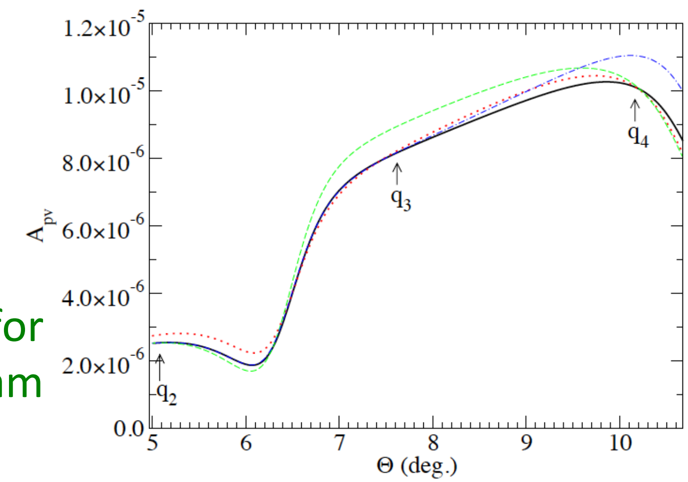
L is the density dependence of the symmetry Energy, a critical parameter for neutron stars, etc.

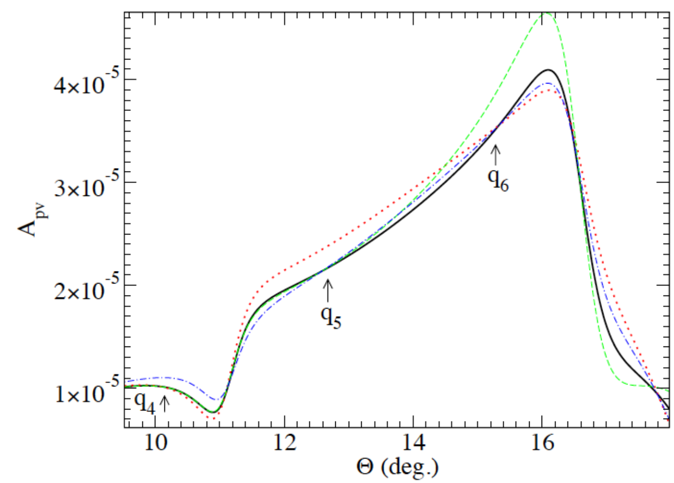
Other parameters, such as central density, require additional Q^2 points

Program for ^{48}Ca (Horowitz)



Plots are for 2 GeV beam

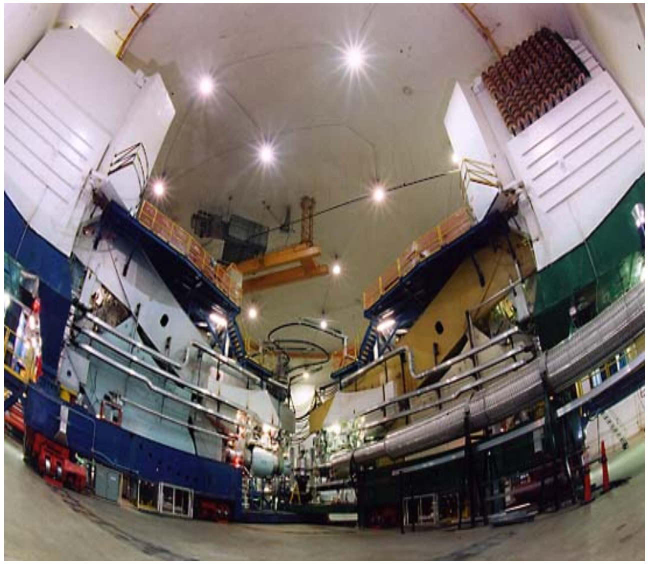




q_i fm^{-1}	E GeV	$\frac{d\sigma}{d\Omega}$ mb	A_{pv} ppm	T days	a_i fm^{-3}	$\Delta a_i/a_i$ %
0.45					0.0752	1.1
0.90	2.06	2.44	2.54	5	0.0468	5.9
1.35	3.09	1.07×10^{-1}	8.31	7	-0.0438	7.6
1.80	4	2.9×10^{-3}	9.92	10	-0.0147	27
2.24	4	4.05×10^{-4}	22.5	15	0.0161	29
2.69	4	9.7×10^{-6}	36.5	23	0.0066	90

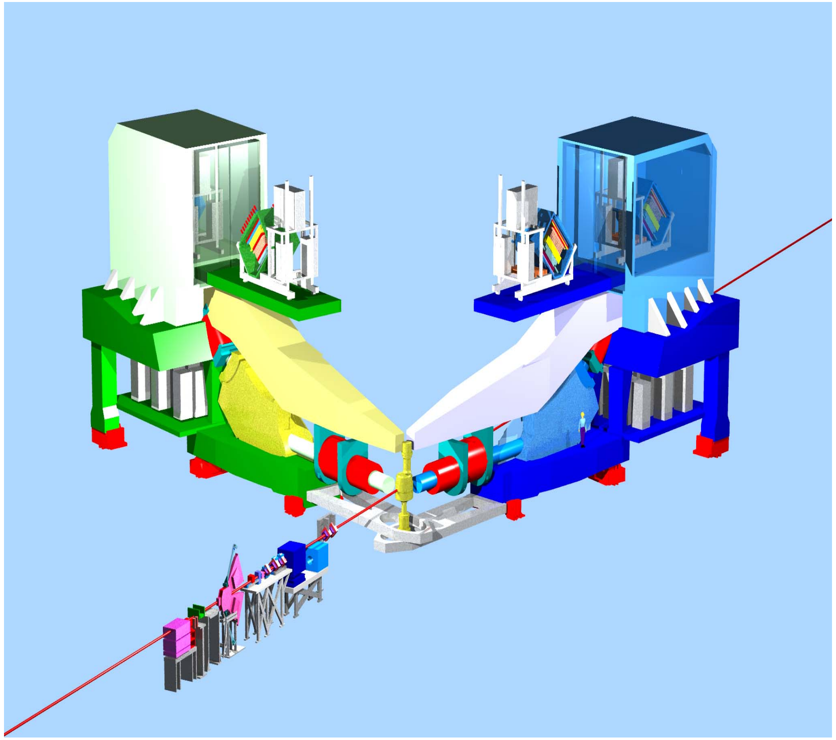
Part I -- Considering the HRS in Hall A

The high-resolution spectrometers are well suited to suppress background and discriminate inelastic states.



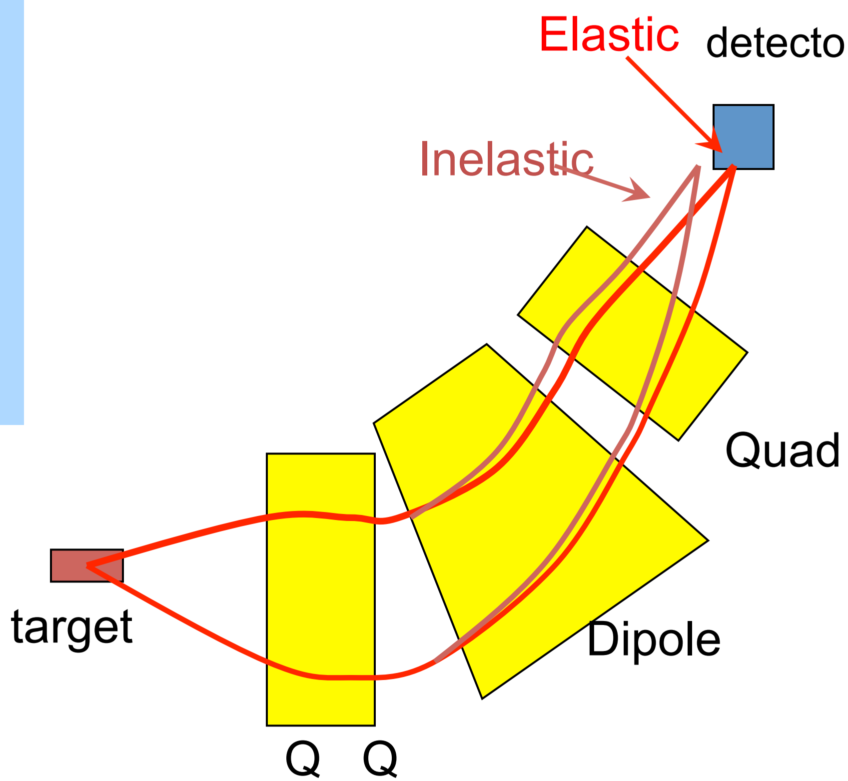
- Septum Magnet
- "Trombone" target, variable Z position to reach

High Resolution Spectrometers -- HRS



Spectrometer Concept:
Resolve Elastic

1^{st} excited state ^{48}Ca 3.8 MeV



Left-Right symmetry to control transverse polarization systematic

^{48}Ca

Nicholas's Kinematics

uses already-approved CREX-1 kinematics as one point.

E (GeV)	θ	q fm^{-1}	A (ppm)	T (days)	a_i	$\Delta a_i/a_i$
1.1	4	0.39	0.67	5 ?	0.0551	1.3 %
2.2 CREX-1	4 approved.	0.78	2.22	45	0.0646	0.77 %
2.2	6	1.17	4.82	7 ?	-0.0194	17 %
3.9	4.5	1.56		10 ?	-0.0328	
4.0	5.5	1.95		15 ?	-0.0018	
4.0	6.5	2.34		23 ?	0.0200	

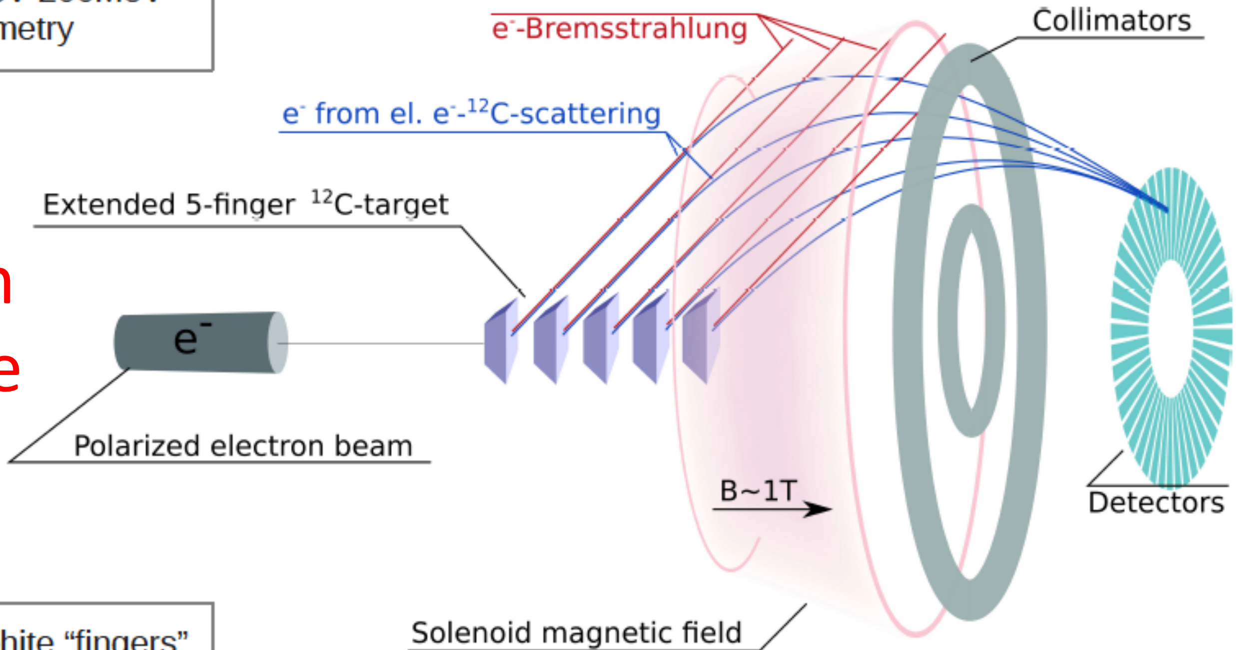
Work in progress …

Question: What criteria do we use to optimize the experiment run time ?

Solenoidal Spectrometer

MESA:
- 150µA
- 150MeV-200MeV
- Polarimetry

Provides maximum possible solid angle



Target:
- 5 graphite "fingers"
- 5 g/cm² total
- 36mm spacing

Gertz

Mathematics of Focusing Solenoids

Unusually simple!

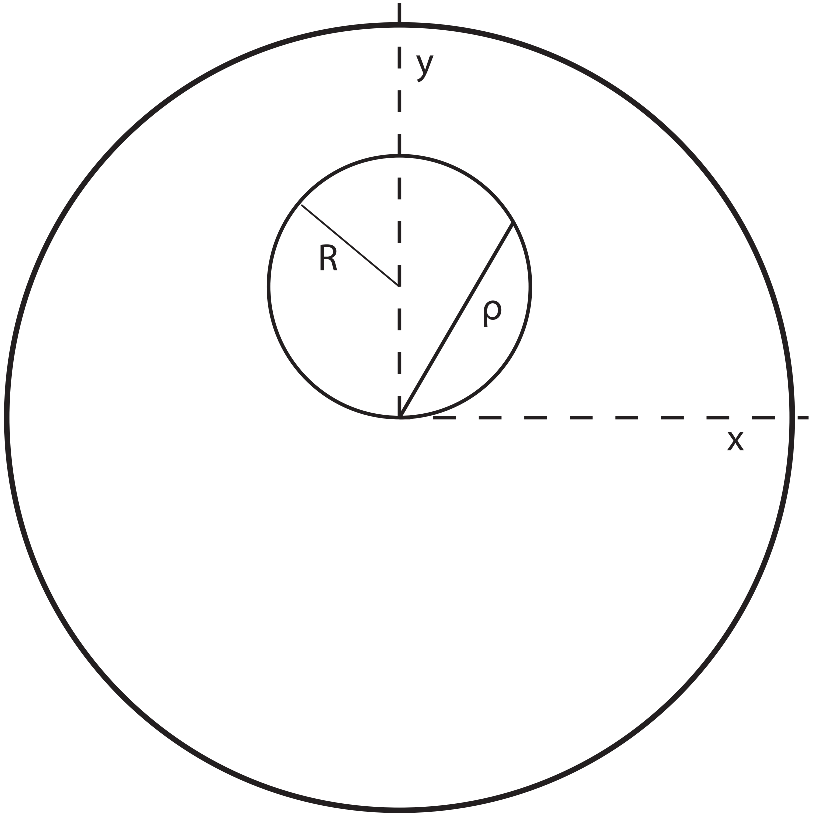
Consider a uniform magnetic field in the z direction. A particle has momentum p and velocity v. Initially, the particle makes an angle of θ with the magnetic field lines. Then

$$p_\perp = p\sin\theta; \quad p_\parallel = p\cos\theta; \quad v_\perp = p\sin\theta; \quad v_\parallel = p\cos\theta$$

The radius of curvature is given by

$$R = \frac{p\sin\theta}{0.3B}.$$

Geometry and Variables



Trajectory in a Solenoid

If the particle starts at the origin with $\mathbf{v}$ in the $x-z$ plane, the motion is given by

$$z = vt\cos\theta; \quad x = R\sin(\Omega t); \quad y = R[1-\cos(\Omega t)],$$

where

$$\Omega = v\sin\theta/R.$$

Let $\rho(z)$ be the distance of the particle from the z axis. Then

$$\rho^2 = x^2 + y^2 = R^2\{\sin^2(\Omega t) + [1+\cos(\Omega t)]^2\} = 2R^2[1-2\cos(\Omega t)] =$$

$$4R^2\sin^2\left(\frac{\Omega}{2}t\right).$$

Therefore

$$\rho = 2R\left|\sin\left(\frac{\Omega}{2}t\right)\right|.$$

We want $z = v\cos\theta t$ as the independent variable, so

$$\frac{\Omega}{2}t = \frac{1}{2}\frac{v\sin\theta}{R}\frac{z}{v\cos\theta} = \frac{z}{2R}\tan\theta$$

and finally

$$\rho(z) = 2R\left|\sin\left[\frac{z}{2R}\tan\theta\right]\right|.$$

This equation holds regardless of the plane of the initial velocity.

Interpretation

Each trajectory returns to the z axis at a distance

$$z_0 = 2\pi R \cot\theta.$$

Also

$$R = \frac{p \sin\theta}{0.3b}.$$

If θ increases a bit, R increases a bit, but z_0 decreases a bit. Therefore, there is a point on the trajectories where ρ is the same. If p is slightly different, the crossing point is at lower ρ.

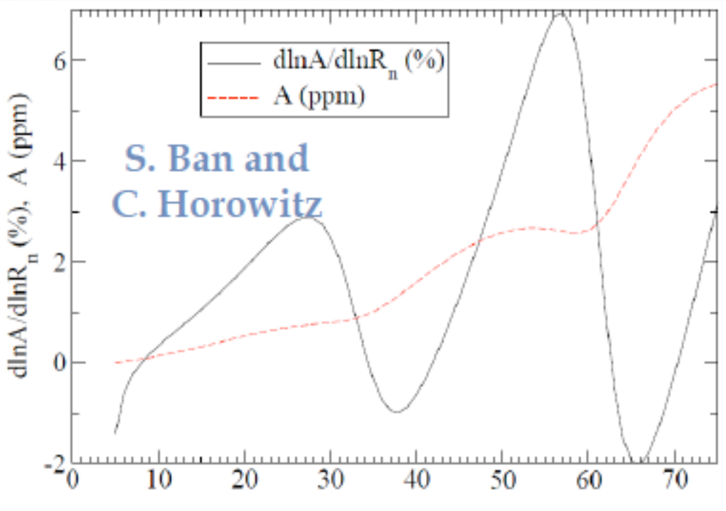
The effect is easy to see in a realistic magnetic field with azimuthal symmetry. Simply generate tracks starting at the origin and plot $\rho(z)$. This should approximately look like

$$\rho(z) = |\sin(kz)|.$$

By slightly varying θ and p, the above features can be seen.

"Pseudo" energy focus

Solenoid Spectrometer for Nuclei: (Obsolete version)

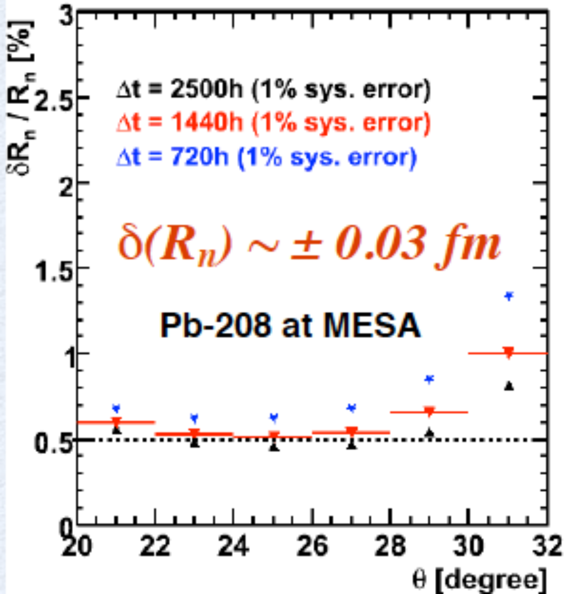


200 MeV: FOM peaks around 25 degrees

Not surprising: same Q^2 as PREX

In elastic scattering, the only parameter is Q^2

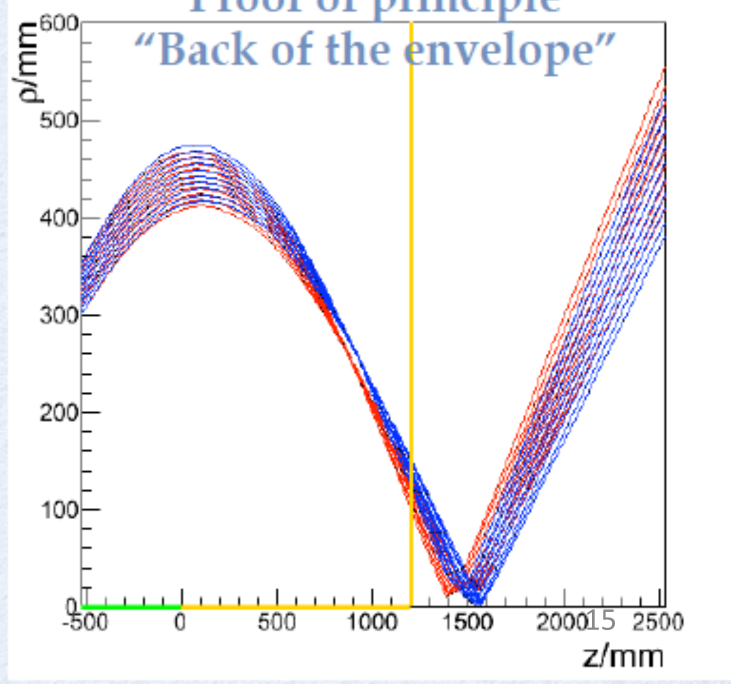
Why might one do better than PREX-II? Very simple: HRS picks up about 25% of the azimuth



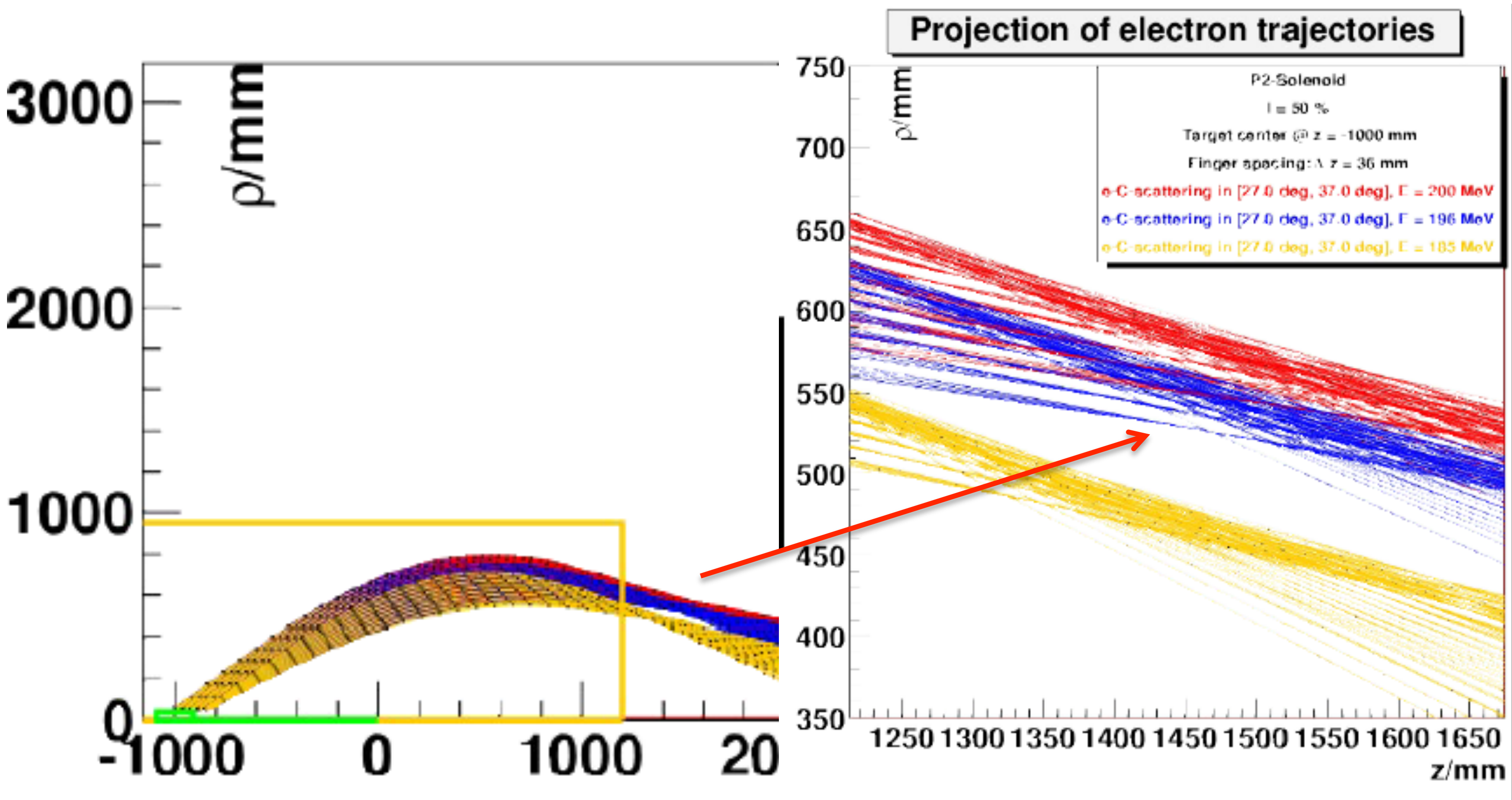
solenoidal spectrometer will separate inelastics over the full range of the azimuth

0.5% Rn in 1500 hours of running; same luminosity as PREX

M. Thiel



Example: Existence of High-resolution Focus for ^{12}C



Elastic 4.2 MeV level

K. Gertz

15 MeV Isovector level

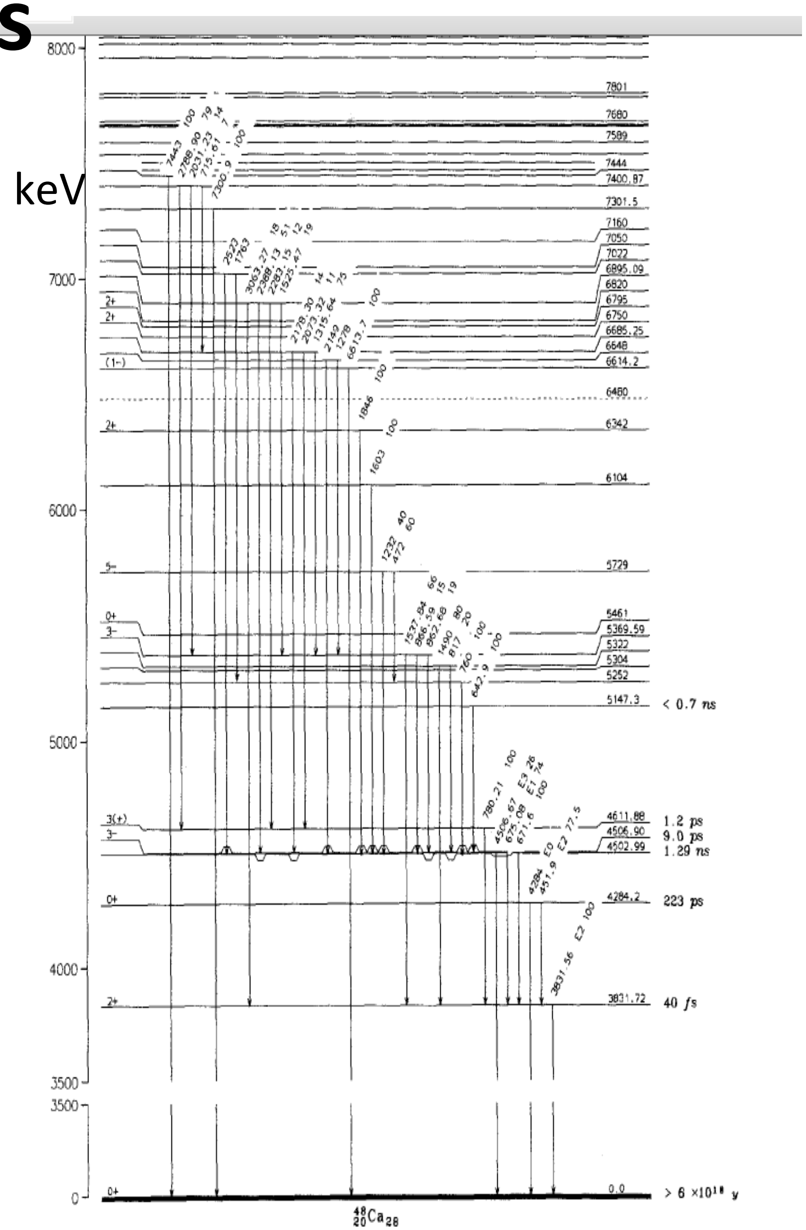
Solenoid and ^{48}Ca Program

- How high in Q^2 can we go?
- Do we have the necessary resolution?
- Is there an ideal accelerator?

Nuclear Levels

^{48}Ca

E (MeV)	J^P
Gnd	0^+
3.83	2^+
4.28	0^+
4.51	
4.61	



1824 EISENSTEIN, MADSEN, THEISSEN, CARDMAN, AND BOCKELMAN 188

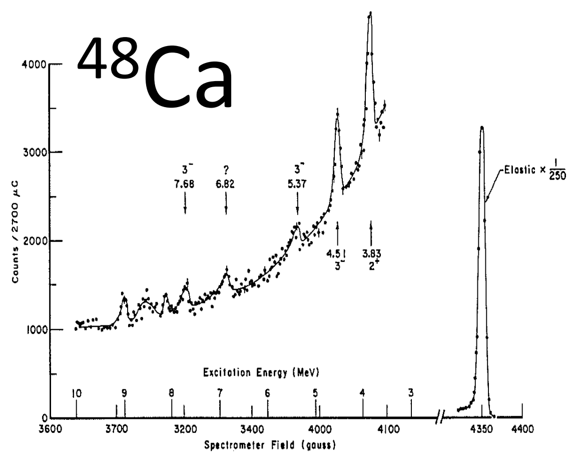


FIG. 7. Spectrum of electrons scattered from Ca^{48}. The incident energy is 60.1 MeV and $\theta=130°$. The spectrum has been corrected for background.

1826 EISENSTEIN, MADSEN, THEISSEN, CARDMAN, AND BOCK

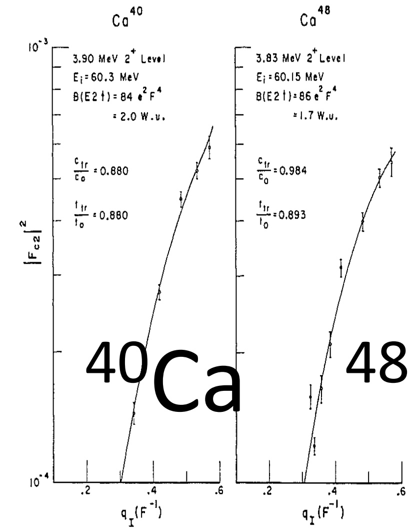


FIG. 8. Experiment and best theoretical first 2+ states in experimental points given in Table V.

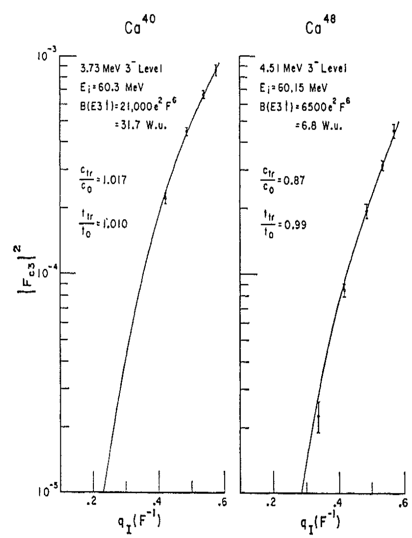


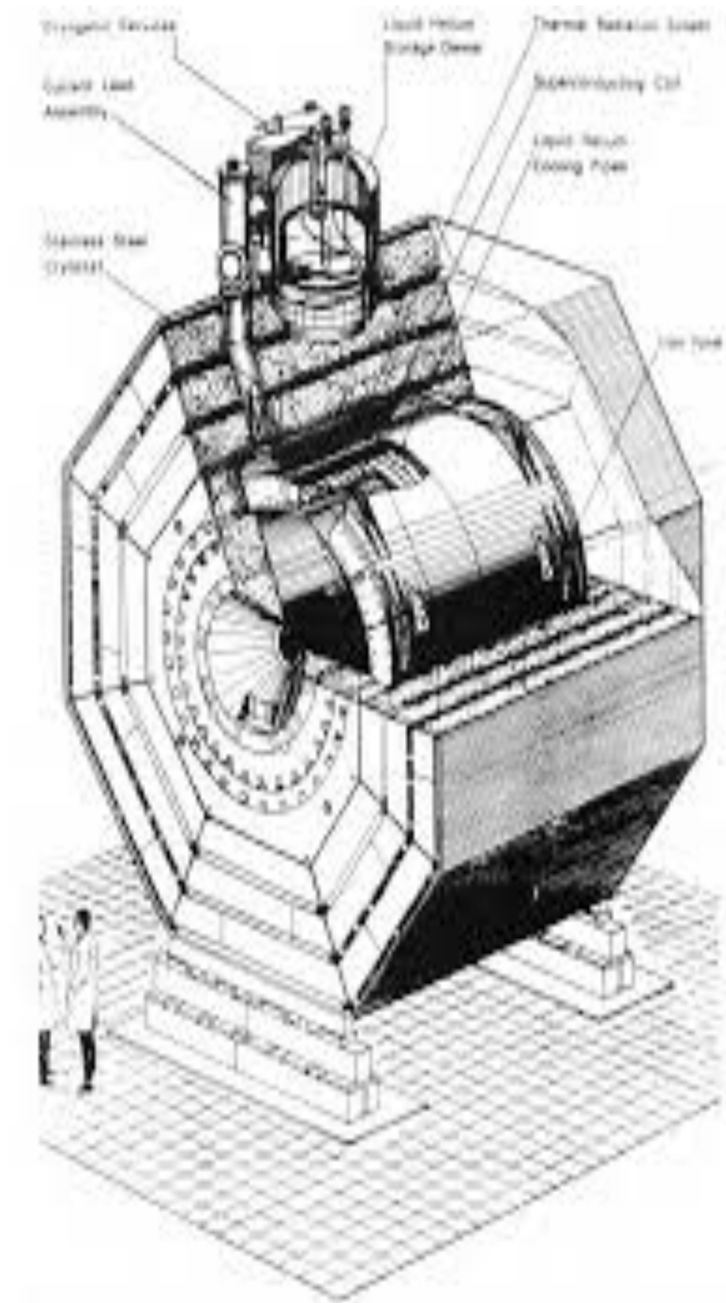
FIG. 9. Experiment and best theoretical first 3− states in experimental values

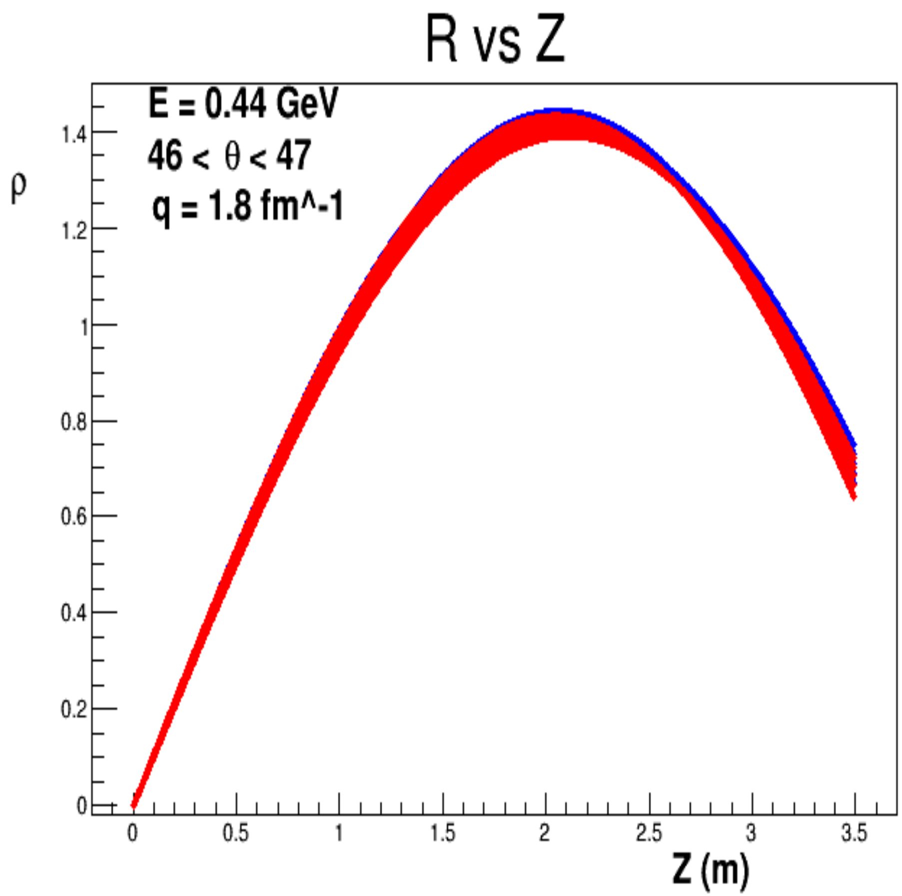
Electron scattering data and **form factors** for low-lying states of ^{48}Ca

SOLENOID SPECTROMETER

The CLEO-II magnet is being moved to Hall A for the SOLID spectrometer. Here, we try using it for superCREX.

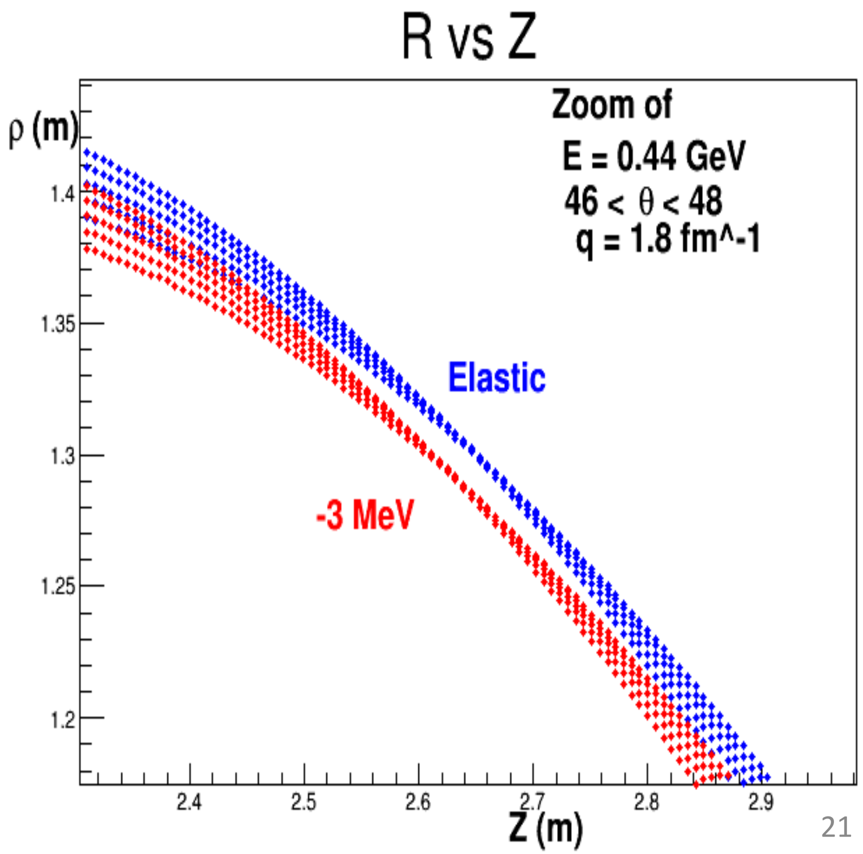
Credit: Paul Souder



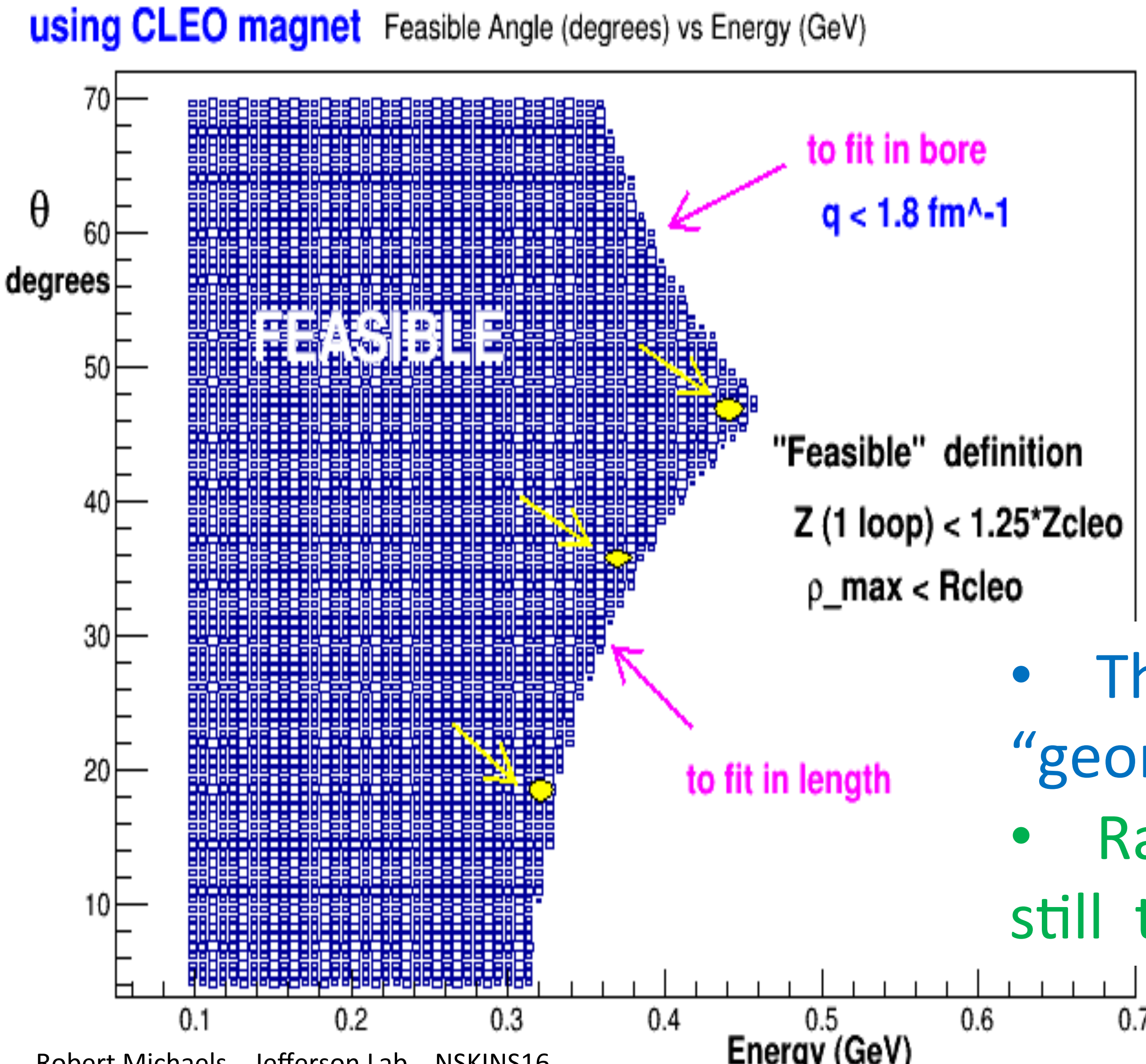


Example **1** of Trajectories

in CLEO Solenoid



Kinematic Limits of CLEO Magnet

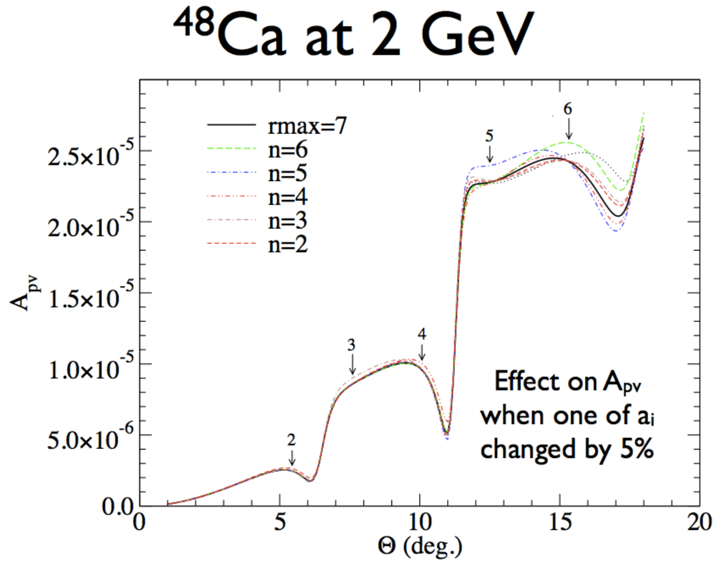


Beam energies are high for Mainz, low for JLab

- This addresses "geometric" feasibility.
- Rate estimates still to come.

Possible Program

- Use solenoid at Mainz for super-precise low Q^2 measurements.
- Use Jlab HRS for high Q^2 points.
- Use SoLID for medium-high points?



Conclusions for Solenoids

- Positive
  - Maximum solid angle.
  - Ample energy resolution for isolating elastic scattering.
- Negative
  - Inconvenient beam energies.
  - Maximum Q^2 limited.
  - Need BIG solenoid for large Q^2.