NATIONAL SCIENCE CENTRE
POLAND

Work in progress

Bubble nucleation rate with out-of-eq effects

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Advisor: Bogumiła Świeżewska

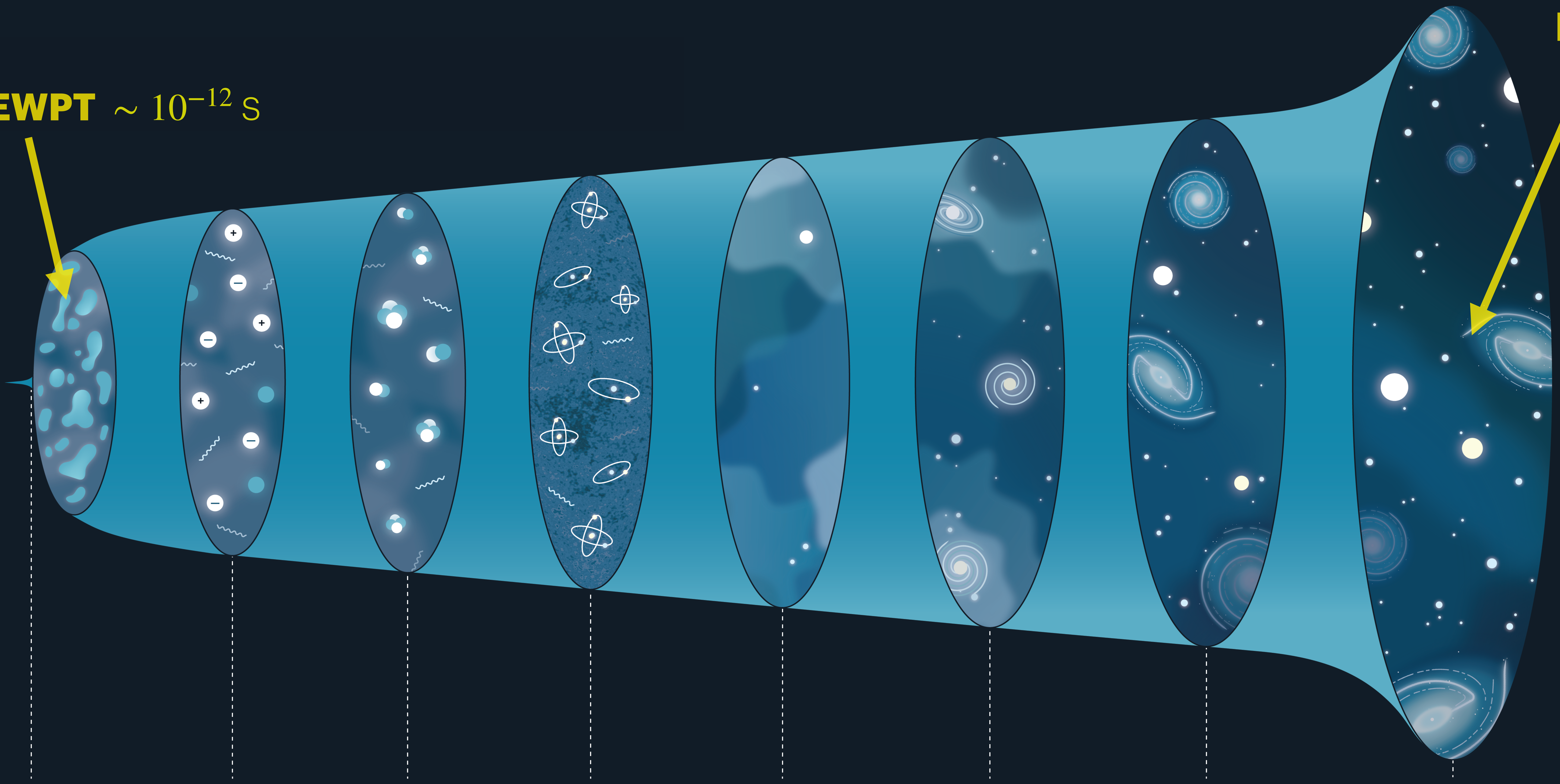
Collaborator: Wenyuan Ai

Youngst@rs 2026 - Shaping the universe:
framework and footprints of cosmological PTs

UNIWERSYTET
WARSZAWSKI

EWPT $\sim 10^{-12}$ s

We are here!



10^{-32} seconds	**Inflation**
1 microsecond	**First Particles**
3 minutes	**First Nuclei**
380,000 years	**CMB**
200 million years	**First Stars**
400 million years	**Galaxies & Dark Matter**
10 billion years	**Dark Energy**
13.8 billion years	**Today**

[“History of the universe”, NASA infographic]

New exploration window

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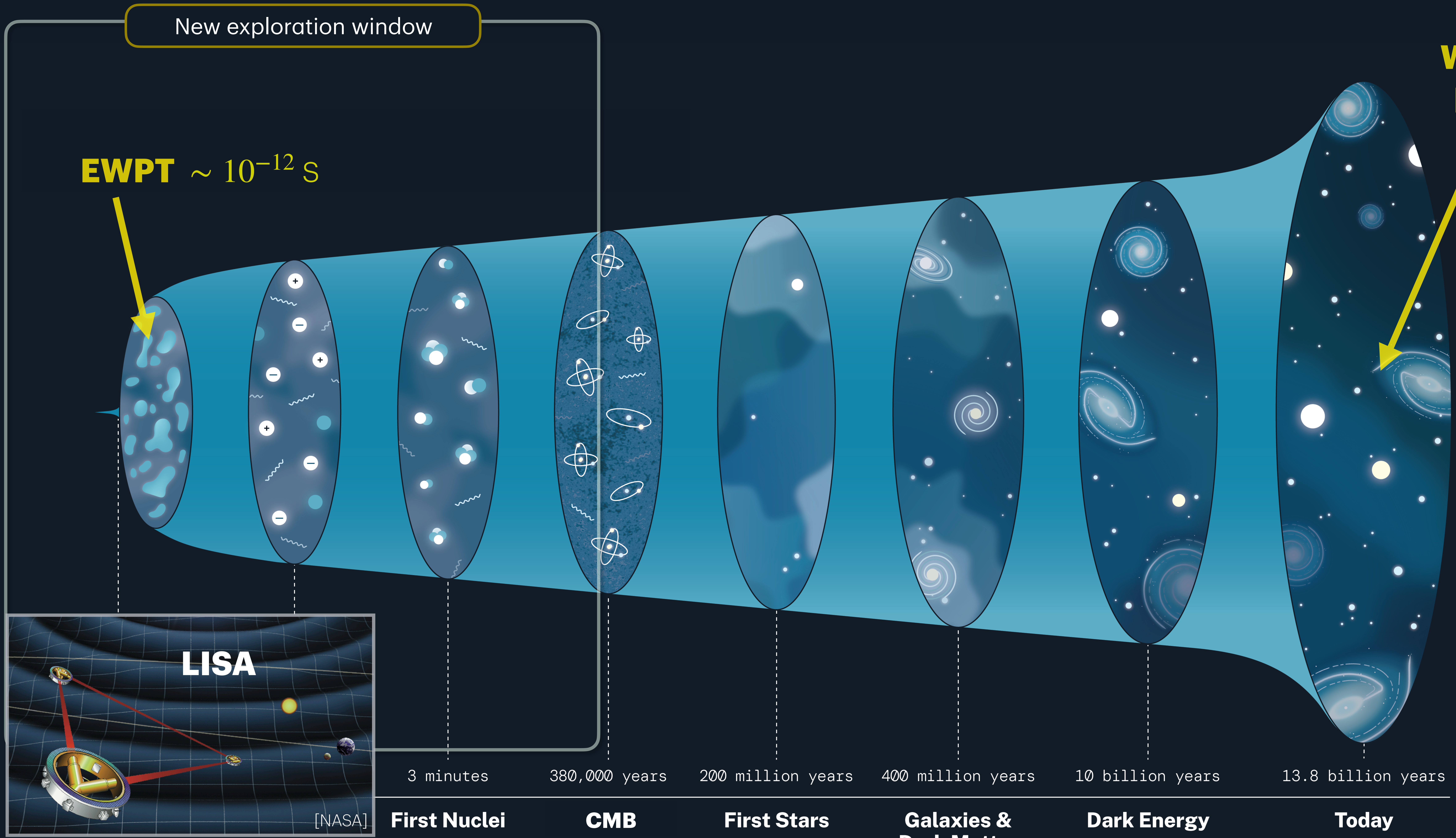

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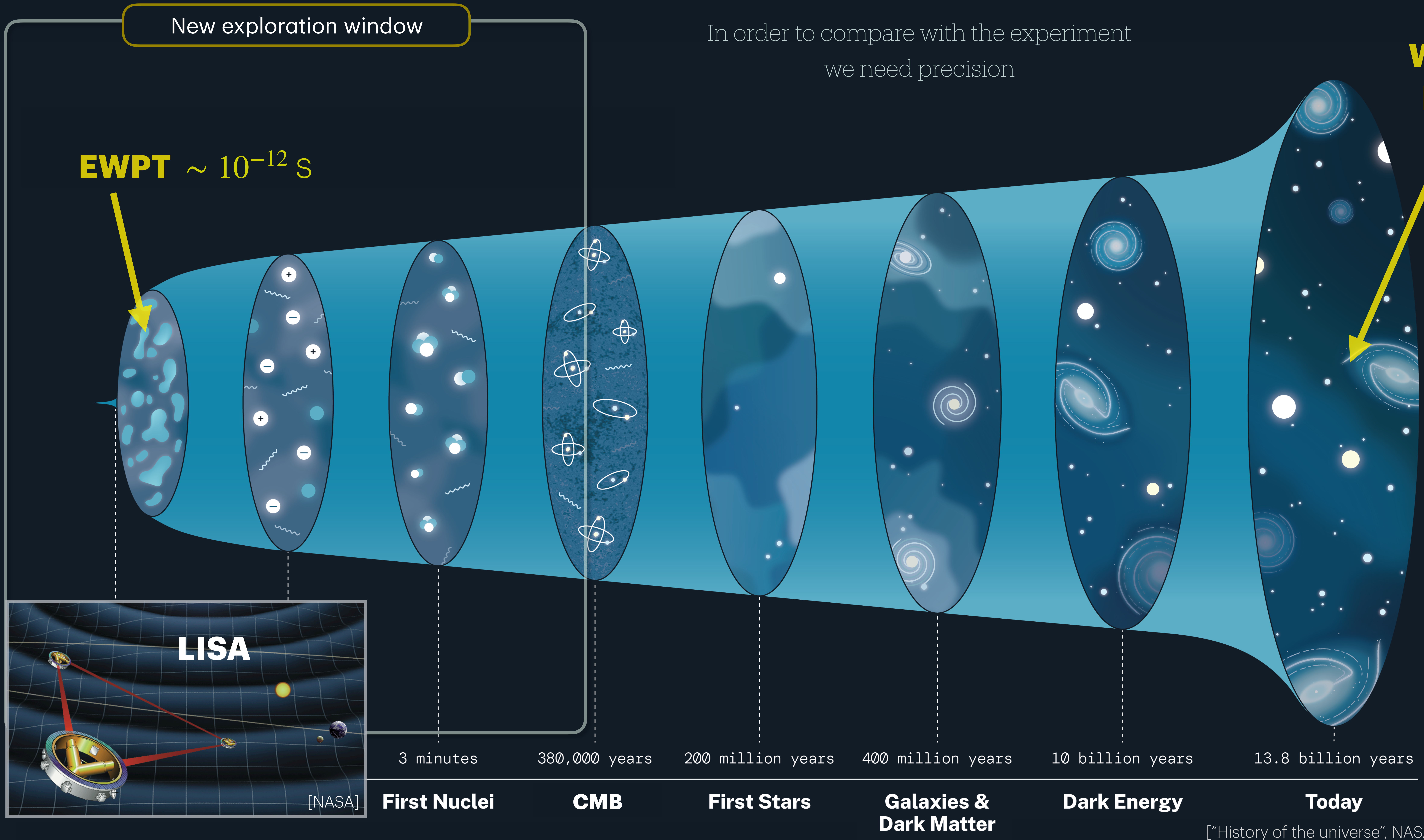

New exploration window

In order to compare with the experiment we need precision

We are here!

EWPT $\sim 10^{-12}\,\mathrm{s}$

LISA

[NASA]

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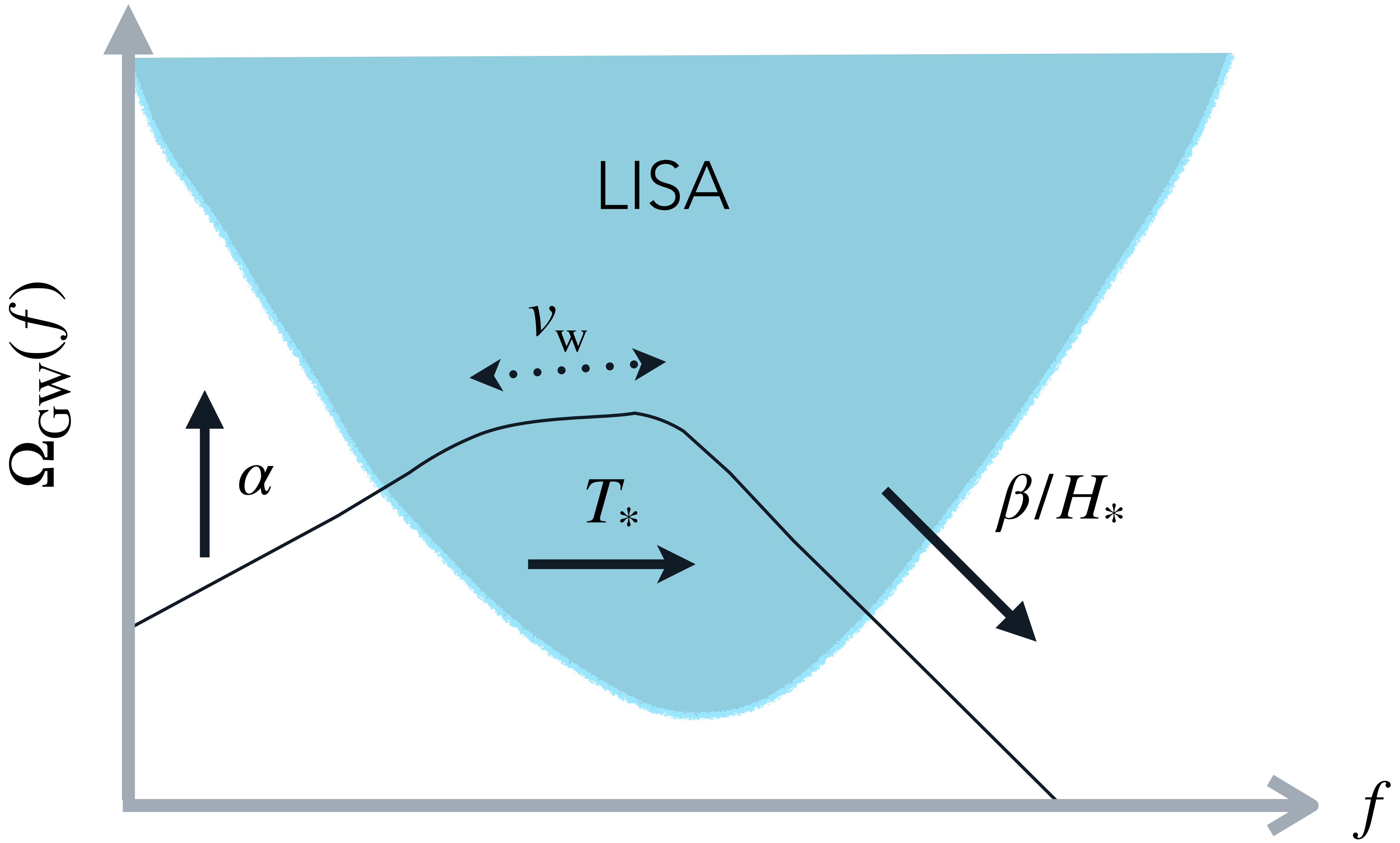
Gravitational waves and the nucleation rate

$$\Omega_{\mathrm{GW}} = \Omega_{\phi} + \Omega_{\mathrm{sw}} + \Omega_{\mathrm{turb}}$$



[Adapted from B. Świeżewska's slides, Scalars 2025]

$$\Omega_i \sim \# \left(\frac{\kappa_i(\alpha)\,\alpha}{1+\alpha}\right)^{a_i} \left(\frac{H_*}{\beta}\right)^{b_i} \Delta_i(v_w)\, S_i(f)$$

[Caprini et al., JCAP04(2016)001]
[Recent templates: JHEP07(2025)217, JCAP10(2024)020]

$$(\tilde{K},\, \beta/H_*,\, T_*)$$

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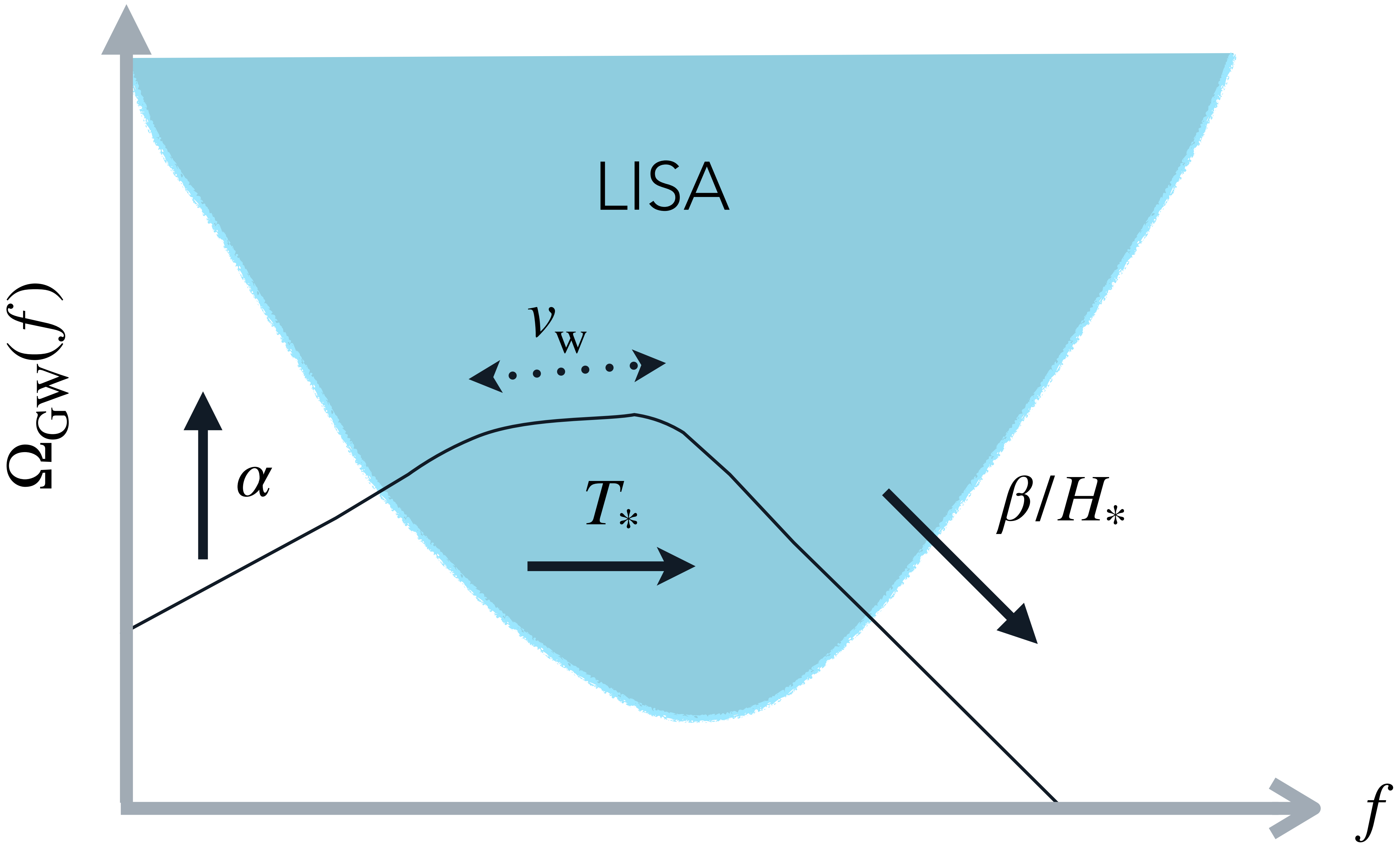

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$(\tilde{K},\ \beta/H_*,\ T_*)$

$$\Gamma(T) = A(T)\, e^{-S_{\rm eff}[\phi_{\rm b}, T]}$$

$$S_{\rm eff}[\phi_{\rm b}, T] = \begin{cases} S_{\rm d+1}[\phi_{\rm b}] & \text{low-T} \\ S_{\rm d}[\phi_{\rm b}]/T & \text{high-T} \end{cases}$$

(In equilibrium)

Improvements in the calculation

- **Naive estimates** (d=spatial dimensions)

$$\Gamma(T) \sim \underbrace{[A_{\mathrm{dyn}}]}_{\sim T} \underbrace{[A_{\mathrm{stat}}]}_{\sim T^d} \mathcal{J}_\phi \, e^{-S_{\mathrm{eff}}[\phi_{\mathrm{b}},T]}$$

[Linde, Nuclear Physics B216 (1983) 421]

Improvements in the calculation

BubbleDet
Ekstedt, Gould, Hirvonen.
[arXiv:2308.15652v2]

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$$A_{\rm dyn} = \frac{\omega_-}{2\pi}\,,$$

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Dissipative effects are neglected!

[Hänggi, Talkner, Borkovec. Rev.Mod.Phys.62 (1990), 251]

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- **Langer's dissipative model**

$$A_{\mathrm{dyn}} = \frac{1}{2\pi}\left[\sqrt{\tilde{\omega}_-^2 + \frac{\tilde{\eta}}{4}} - \frac{\tilde{\eta}}{2}\right]$$

[Langer, Annals of Physics 54 (1969) 258-275]
[Hänggi, Mojatabai. Phys.Rev.A.26 (1982), 1168]

[Assumes: Langevin dynamics]

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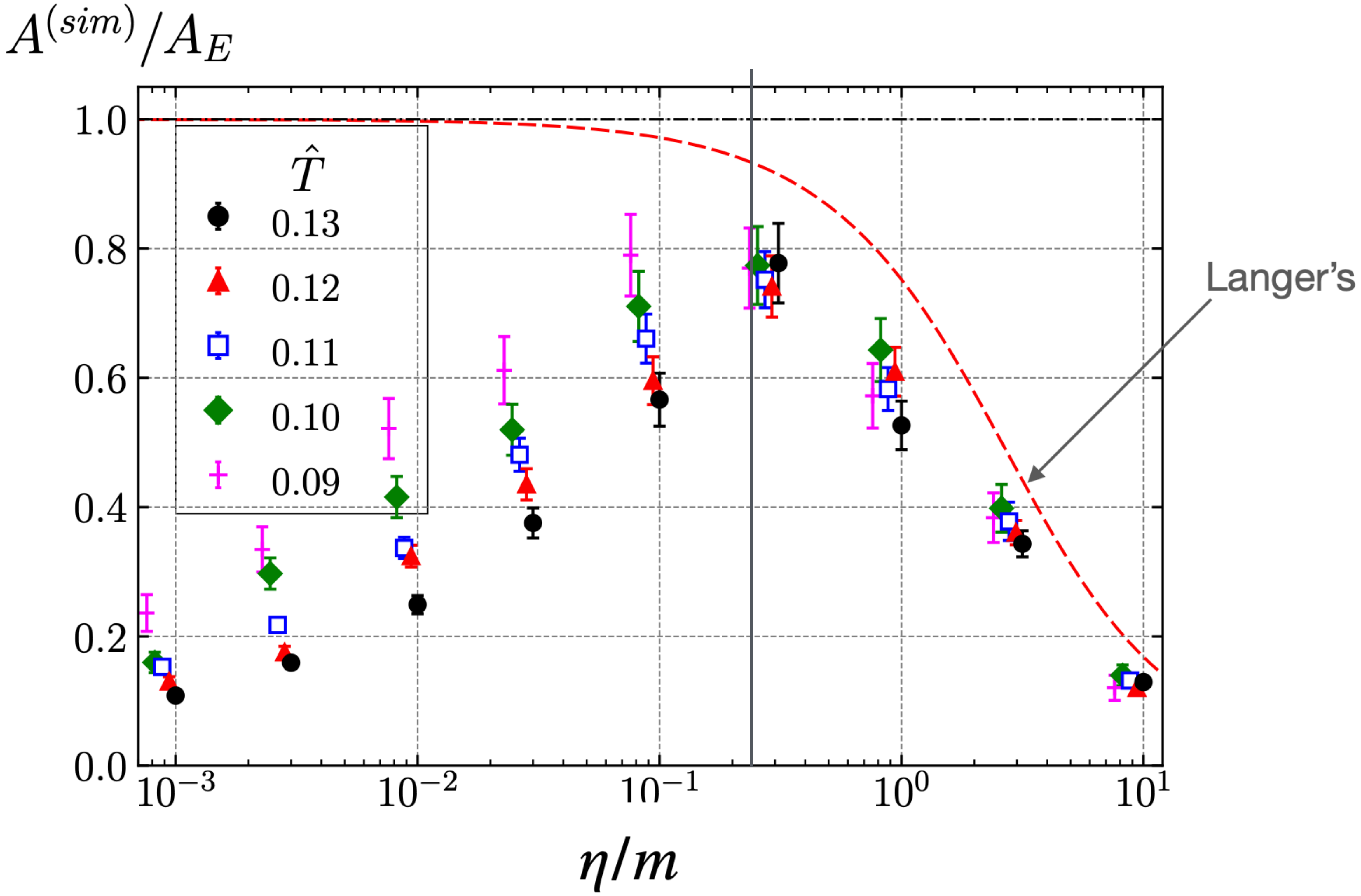
Langer’s rate breaks down for weak dissipation

Simulations

(Pîrvu, Shkerin, Sibiryakov. 2024)

$$V(\phi) = \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$$

$$\phi_{\text{f.v.}} = 0$$
$$\phi_{\text{t.v.}} = \pm\infty$$



[Pîrvu et al., IntJModPhysA39(2024)34, 2445007, arXiv:2407.06263v1]

$$\Gamma_{\text{Langer}} = \frac{1}{\pi V}\left[\sqrt{\omega_- + \frac{\eta}{2}} - \frac{\eta}{2}\right]\beta\,\text{Im}F$$

$$\Gamma_{\text{E}} = \frac{\omega_-}{\pi V}\ \beta\,\text{Im}F$$

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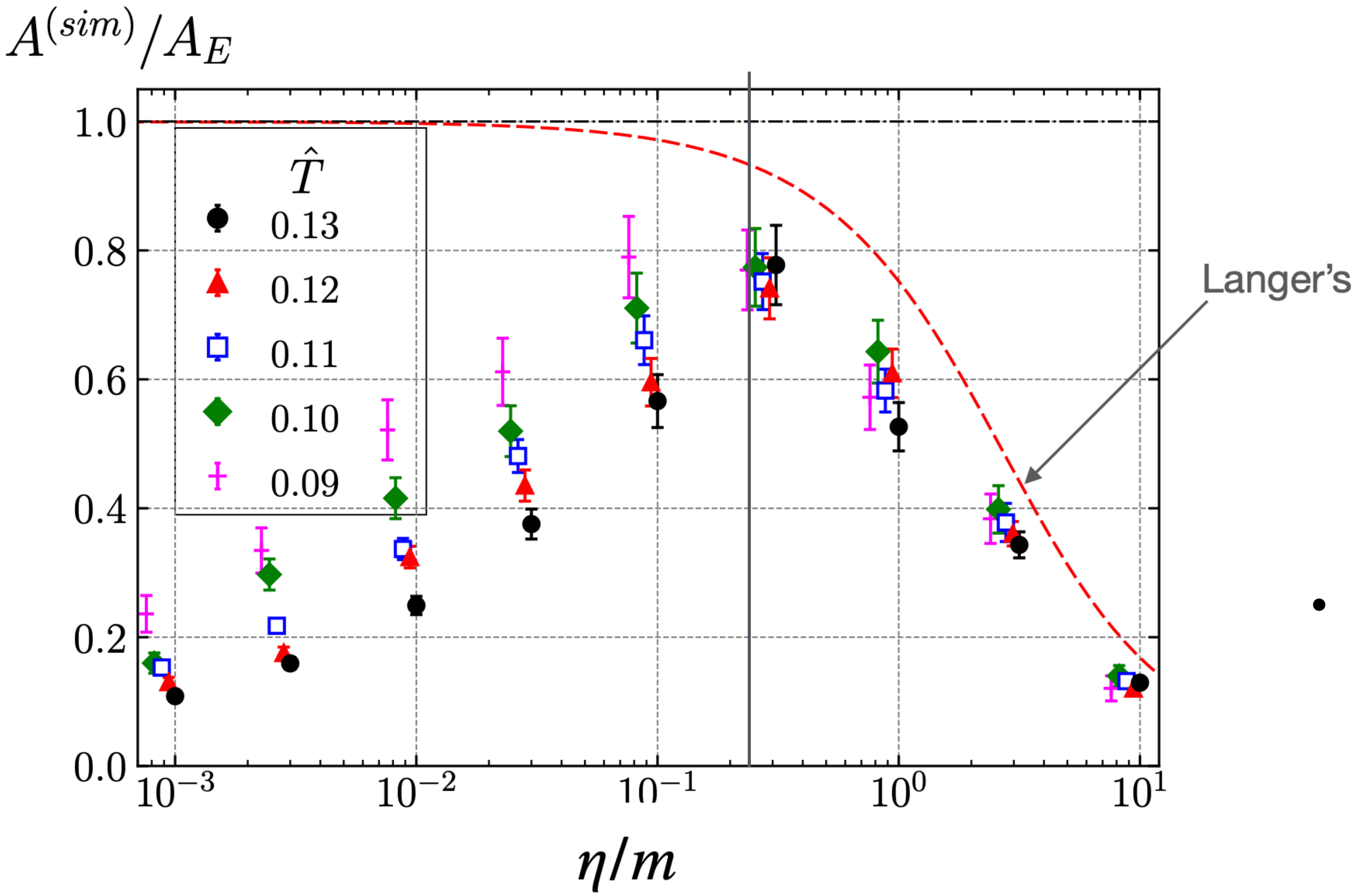

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- **By direct comparison with the instanton rate, it is clear that >>> Langer's rate formula breaks down at low η**

$$\eta \ll \frac{\omega_-}{\beta F_{\text{b}}} \quad \Rightarrow \quad \eta^{-1} \sim \tau_{\text{thermal}} \gg \tau_{\text{decay}}\,\beta F_{\text{b}}$$

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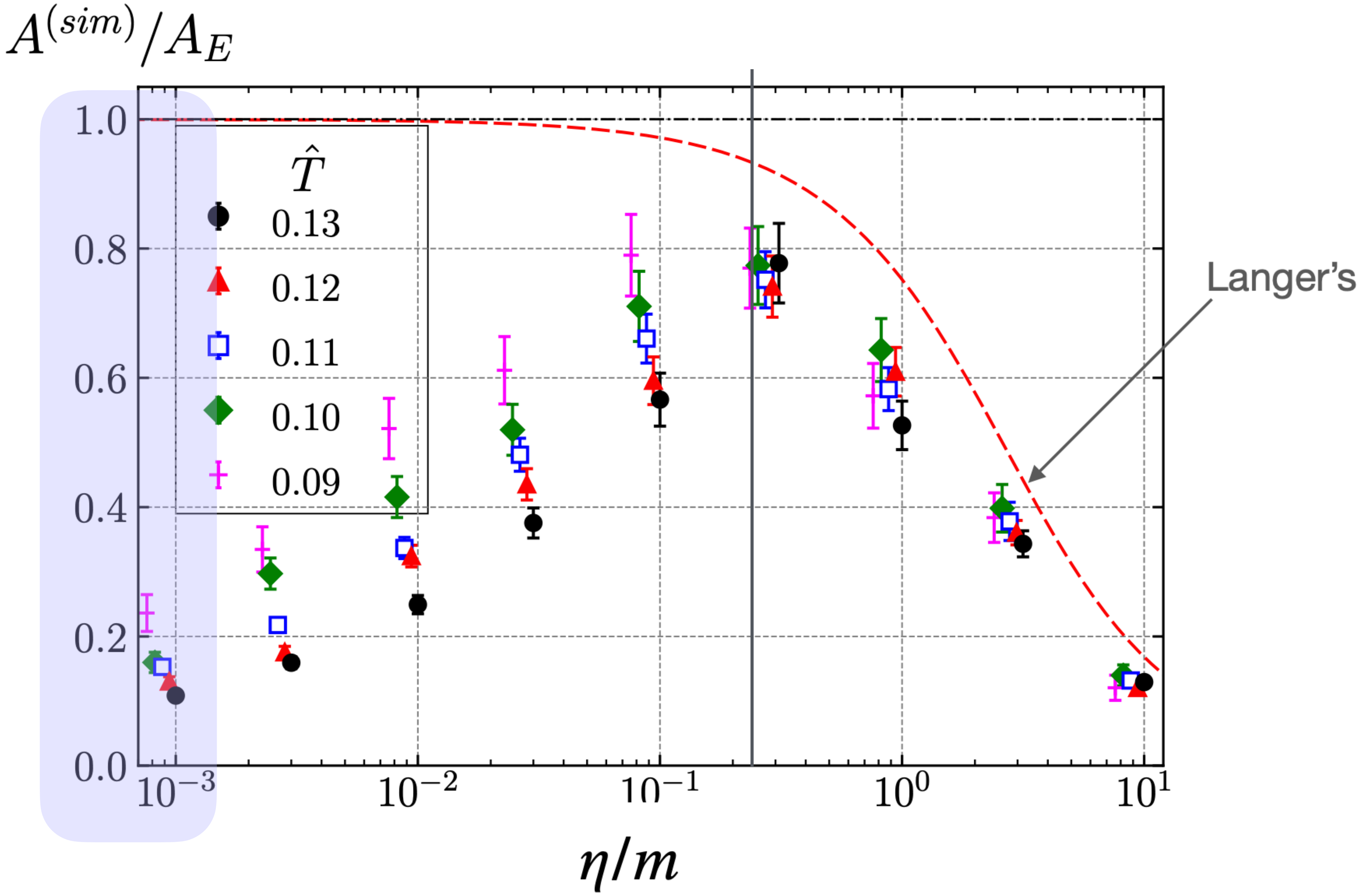

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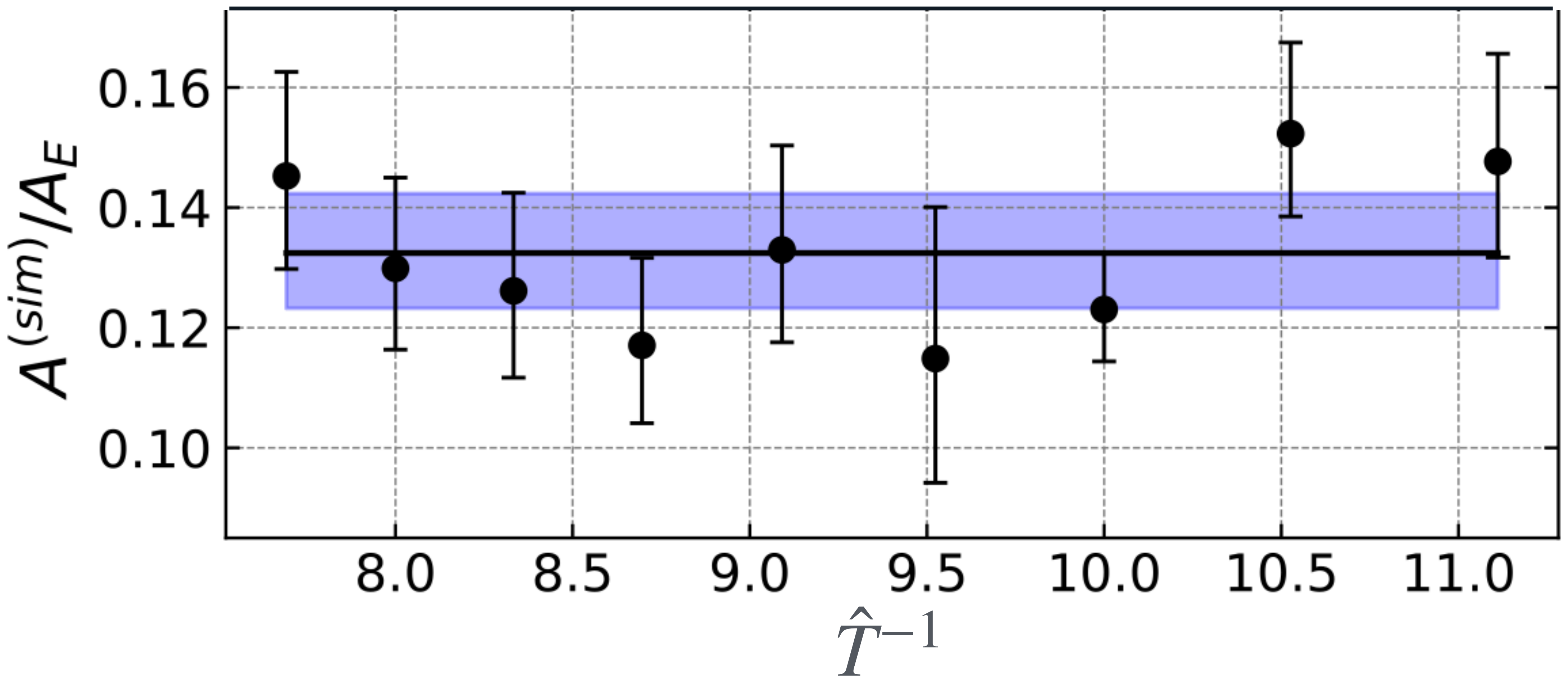
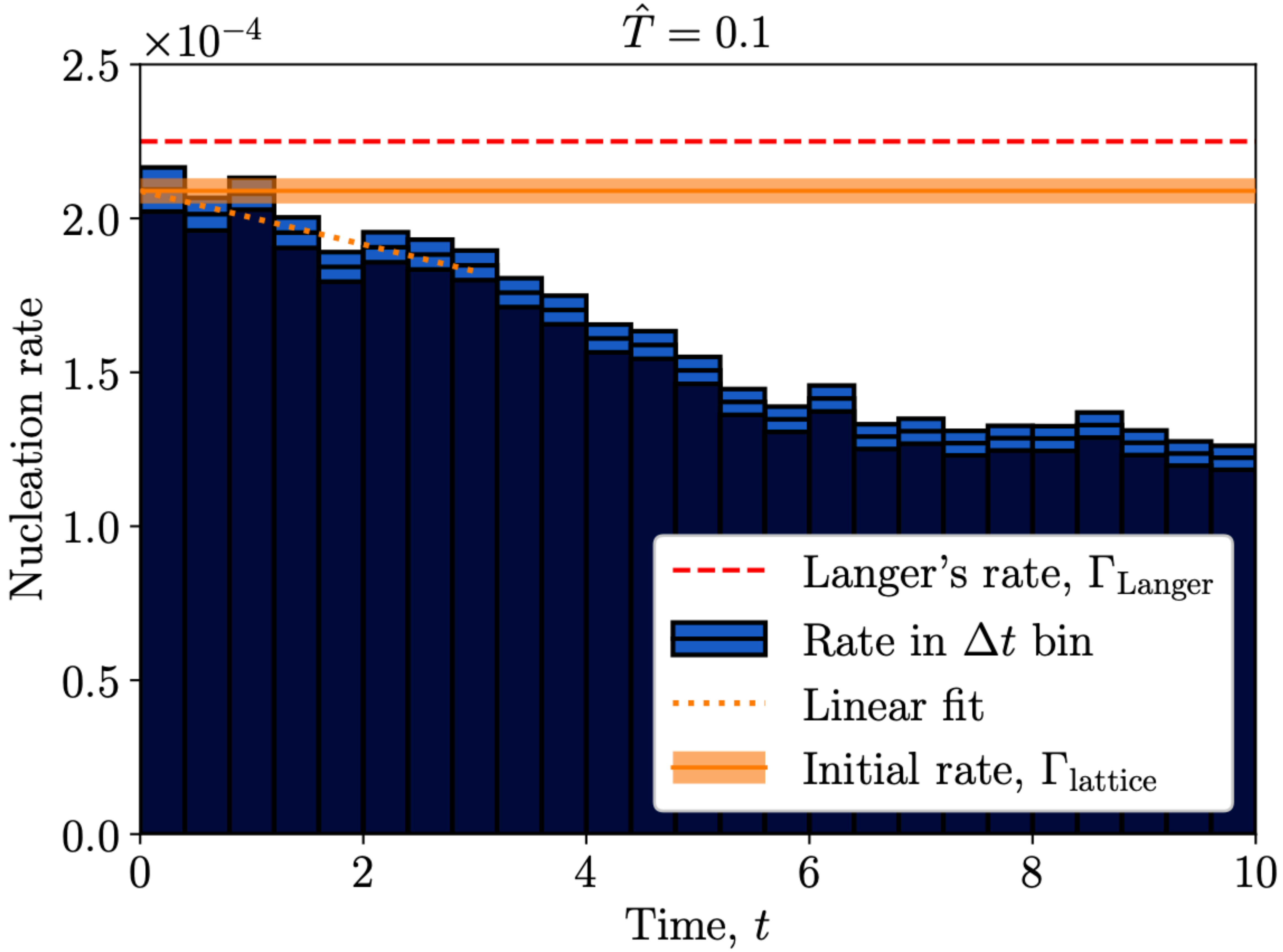


$$m_{\text{th}}^2 = m^2 - \frac{3\lambda T}{2m} \equiv m^2\left(1 - \frac{3}{2}\hat{T}\right)$$

Simulations

(Gould, Hirvonen. 2024)



[Gould, Hirvonen, arXiv:2505.22732]

- **Initial thermalization procedure**
- **Non-perturbative extraction of the rate**
- **Hamiltonian evolution post-thermalization**

Conservative systems follow Langer's rate without friction

- **In out-of-eq settings, the system is not initially thermalized… thus, the validity of Langer's rate depends on the values of η.**

Theory (Langer)

Assumptions:

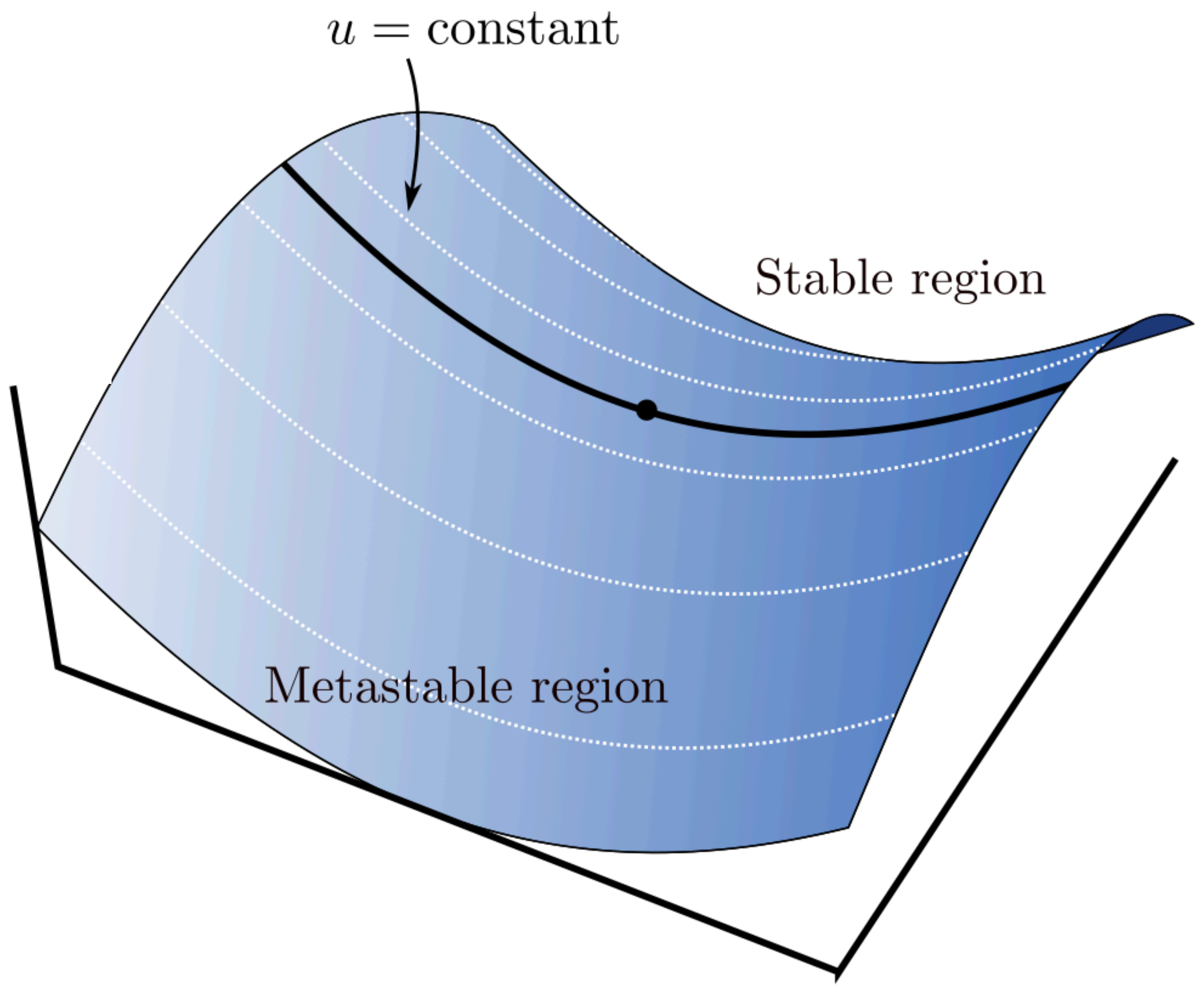
[Additive Gaussian noise]

[Classical nucleation theory]

$$\Box \phi_\xi(x) = -V'(\phi_\xi) - \eta\dot{\phi}_\xi(x) + \xi(x),$$

$$\langle \xi(x)\xi(x') \rangle_\xi = 2\eta T\delta(x - x')$$

$$\langle \xi(x) \rangle_\xi = 0$$



[Gould, Hirvonen, PRD 104 (2021) 9, 096015]

[Berera et al.., PRDD 100 (2019), 076005]

Theory (Langer)

Assumptions:

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[Classical nucleation theory]

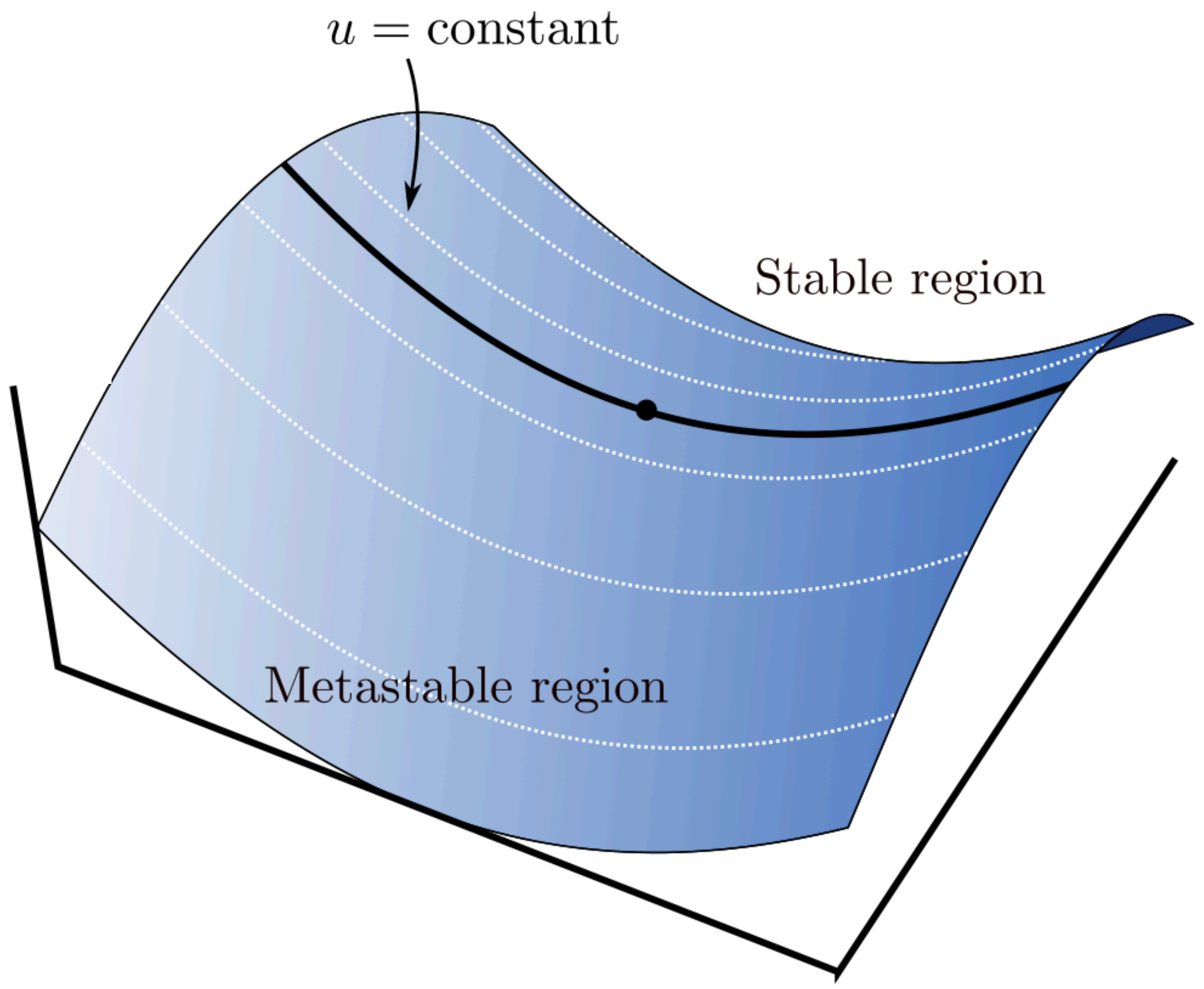
$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\phi_\xi \\ \pi_\xi\end{pmatrix} = \begin{pmatrix}\pi_\xi \\ \nabla^2\phi_\xi - V'(\phi_\xi) - \eta\pi\end{pmatrix} + \begin{pmatrix}0 \\ \xi\end{pmatrix}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\phi_\xi \\ \pi_\xi\end{pmatrix} = -\begin{pmatrix}0 & -1 \\ 1 & \eta\end{pmatrix}\begin{pmatrix}\partial E/\partial\phi \\ \partial E/\partial\pi\end{pmatrix} + \begin{pmatrix}0 \\ \xi\end{pmatrix}$$

$$\frac{d}{dt}P(\phi,\pi;t) = -\overrightarrow{\nabla}\cdot\vec{J}$$

$$P_{\text{steady}}(\phi,\pi) = P_0(\phi_{\text{fv}},\pi_{\text{fv}})\,\zeta(u)$$

$$\zeta(\phi_{\text{fv}},\pi_{\text{fv}}) \to 1\,, \quad \zeta(\phi > \phi_{\text{b}}\,,\pi) \to 0$$



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$$E[\phi,\pi] = \int_x\left[\frac{\pi^2}{2} + \left(\overrightarrow{\nabla}\phi\right)^2 + V(\phi)\right]$$

A real-time EFT perspective

k

Quantum
UV modes

UV

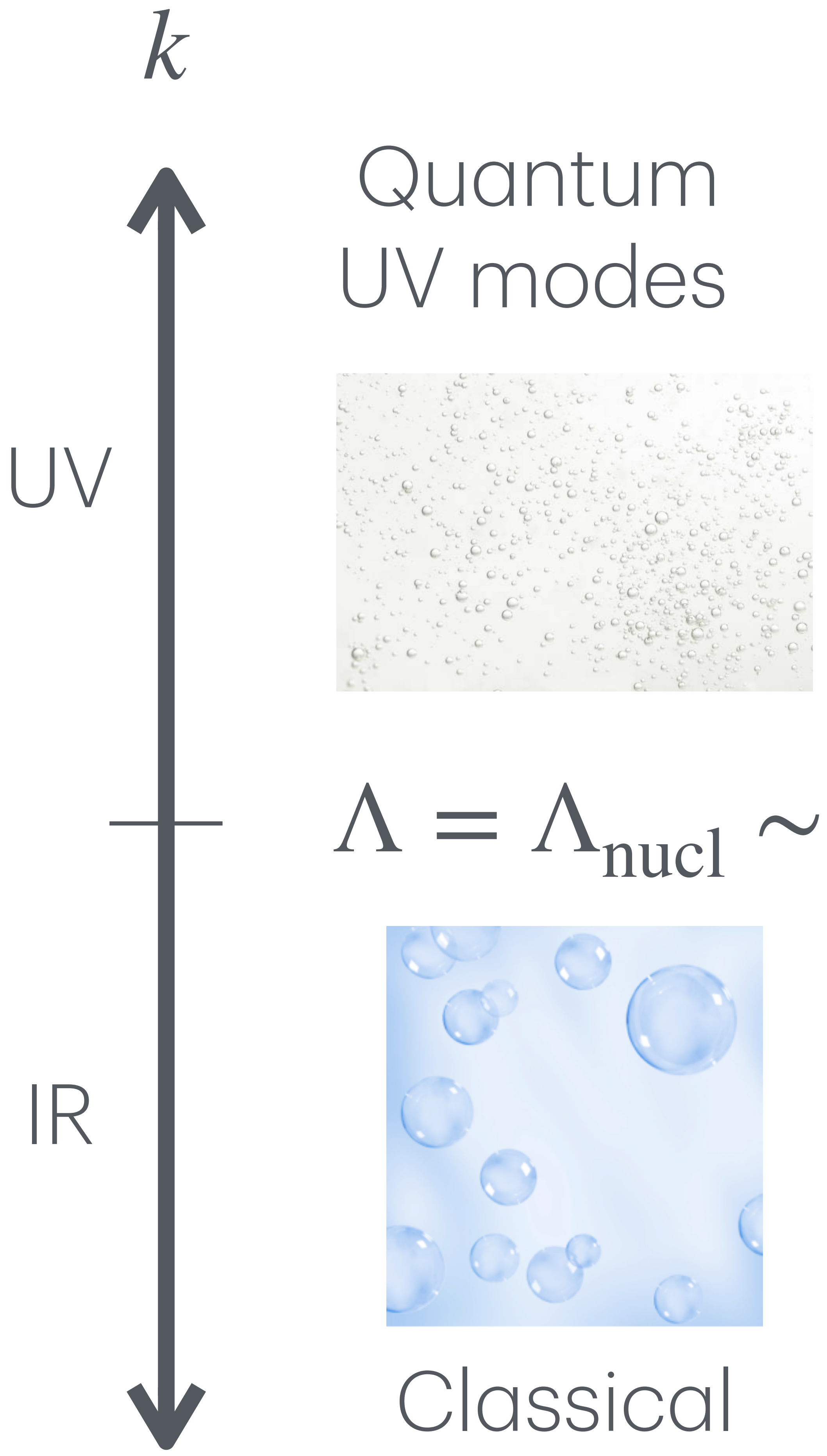
$\Lambda = \Lambda_{\text{nucl}} \sim R_c^{-1}$ (inverse critical bubble radius)

IR

Classical
IR modes

highly-occupied @ high-T

A real-time EFT perspective



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1. There is a hierarchy of scales

- EFT for the IR modes. (UV: integrated out)
- $\phi(x) = \phi_{\text{IR}}(x) + \phi_{\text{UV}}(x)$

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2. A precise real-time description requires

Closed-time-path formalism
Density matrices

A real-time EFT perspective

$$\phi(x) = \phi_{\mathrm{IR}}(x) + \phi_{\mathrm{UV}}(x)$$

A real-time EFT perspective

$$\phi(x) = \phi_{\rm IR}(x) + \phi_{\rm UV}(x)$$

$$\hat{\rho}_{\rm IR}(t_f) = {\rm Tr}_{\rm UV}\{\hat{U}(t_f\,, t_i)\,\hat{\rho}\,(t_i)\hat{U}^\dagger(t_f\,, t_i)\}$$

A real-time EFT perspective

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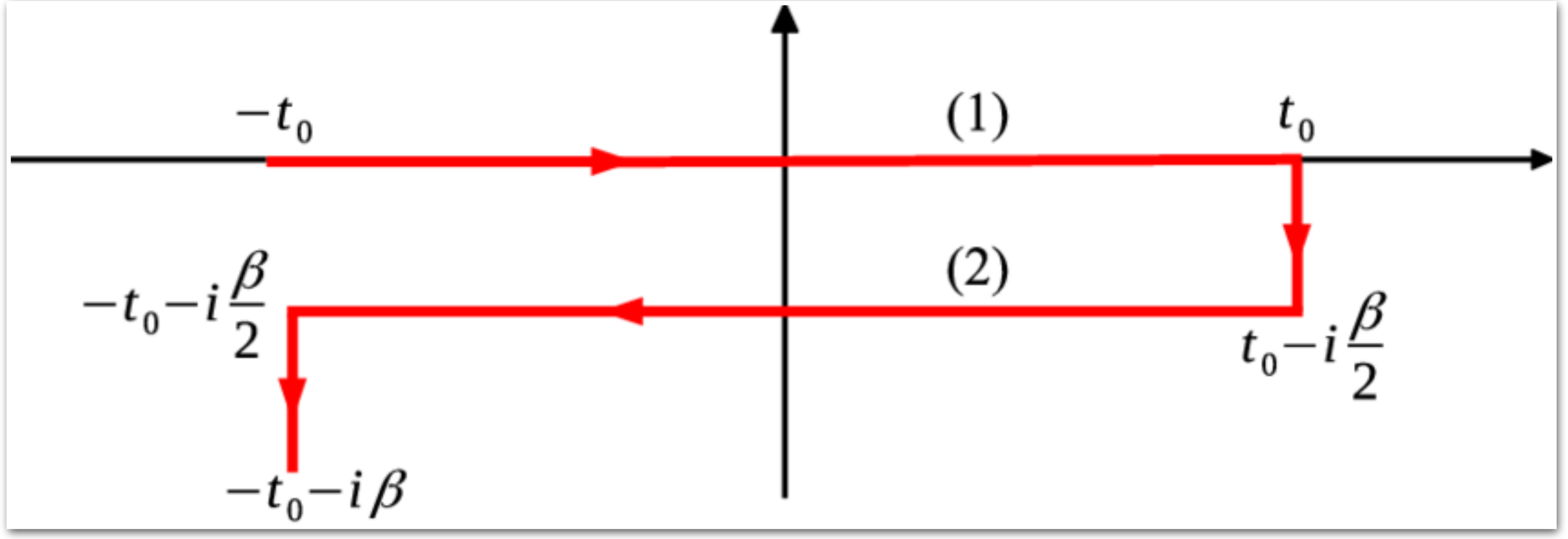

Partition function:

$$Z = \int_{\rm B.C.} D\phi^+_{\rm IR} D\phi^-_{\rm IR}\ \rho_{\rm IR,ini}[\phi^+_{\rm IR}, \phi^-_{\rm IR}]\ e^{\frac{i}{\hbar}S_{\rm eff,\Lambda}[\phi^+_{\rm IR},\phi^-_{\rm IR}]}$$

A real-time EFT perspective

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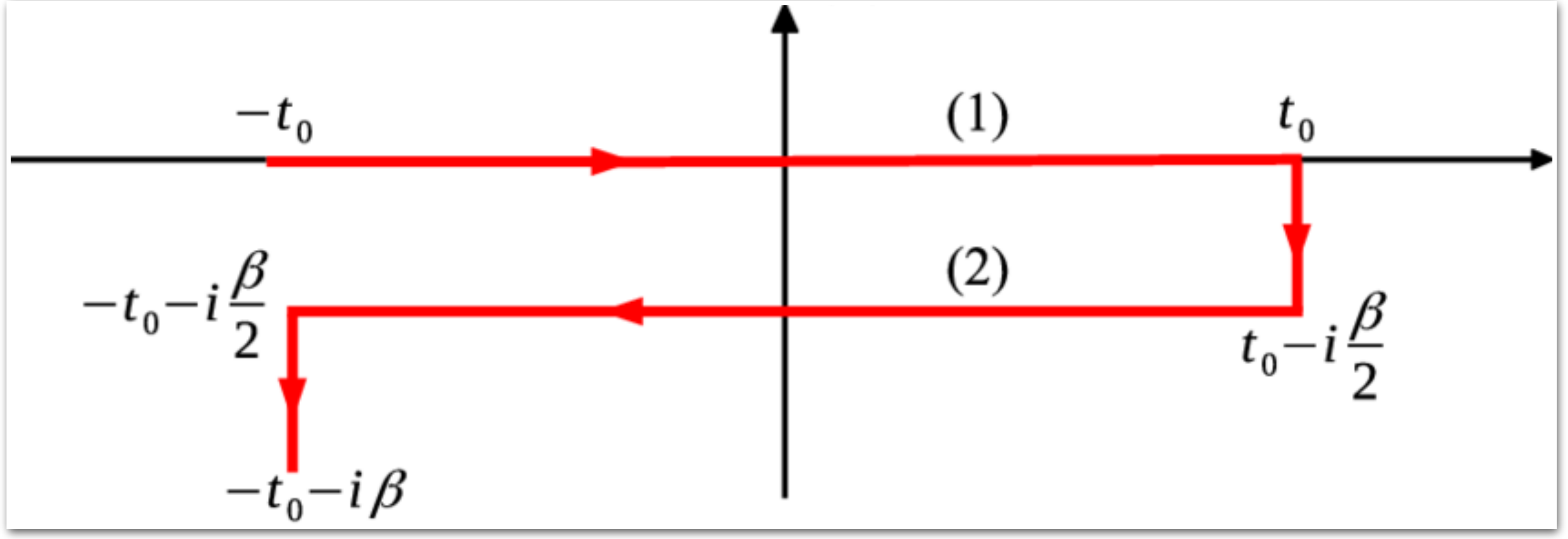

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Influence functional:

$$e^{\frac{i}{\hbar}S_{\mathrm{eff},\Lambda}[\phi^{+}_{\mathrm{IR}},\phi^{-}_{\mathrm{IR}}]} = e^{\frac{i}{\hbar}S_{C}[\phi_{\mathrm{IR}}]} \int_{\mathrm{B.C.}} D\phi^{+}_{\mathrm{UV}} D\phi^{-}_{\mathrm{UV}}\ \rho_{\mathrm{UV,ini}}[\phi^{+}_{\mathrm{UV}}, \phi^{-}_{\mathrm{UV}}]\ e^{\frac{i}{\hbar}\,(S_{C}[\phi_{\mathrm{UV}}]+S_{\mathrm{mix,C}}[\phi_{\mathrm{UV}};\phi_{\mathrm{IR}}])}$$

A real-time EFT perspective

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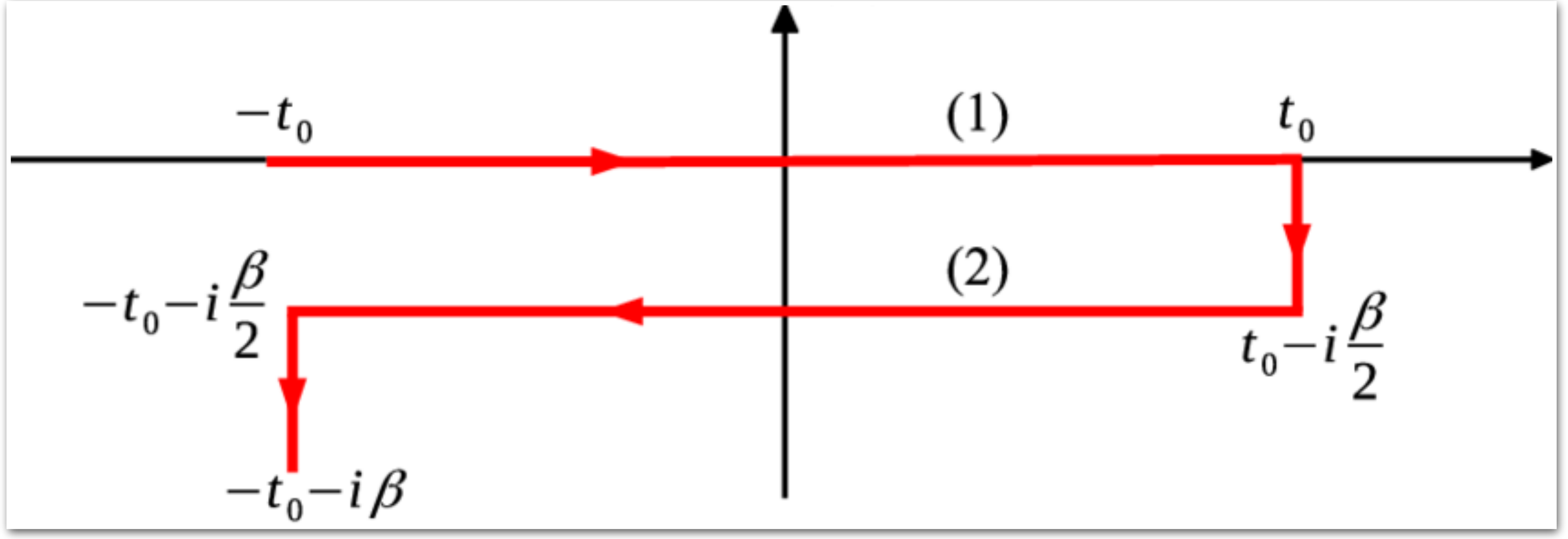

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$$e^{\frac{i}{\hbar} S_{\rm eff,\Lambda}[\phi^{+}_{\rm IR},\phi^{-}_{\rm IR}]} = e^{\frac{i}{\hbar} S_C[\phi_{\rm IR}]} \int_{\rm B.C.} D\phi^{+}_{\rm UV} D\phi^{-}_{\rm UV}\; \rho_{\rm UV,ini}[\phi^{+}_{\rm UV}, \phi^{-}_{\rm UV}]\; e^{\frac{i}{\hbar}\left(S_C[\phi_{\rm UV}] + S_{{\rm mix},C}[\phi_{\rm UV};\phi_{\rm IR}]\right)}$$

Reminiscent of:

> Background-field method

> Wilsonian effective action

A real-time EFT perspective

$$S_{\mathrm{eff},\Lambda}[\phi^+_{\mathrm{IR}},\phi^-_{\mathrm{IR}}] = S_C[\phi_{\mathrm{IR}}] - i\hbar \ln \left\langle e^{\frac{i}{\hbar} S_{\mathrm{mix},C}[\phi_{\mathrm{UV}};\phi_{\mathrm{IR}}]} \right\rangle_{\mathrm{UV,ini}} = S_C[\phi_{\mathrm{IR}}] + S_{\mathrm{IF}}[\phi^+_{\mathrm{IR}},\phi^-_{\mathrm{IR}}]$$

Feynman-Vernon influence functional

A real-time EFT perspective

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Feynman-Vernon influence functional

For a single scalar field with self-coupling λ, the mixing action $S_{\mathrm{mix,C}}$ can be parametrized via

$$-\lambda\, c_\ell \int_x f_\ell(x)\, \phi^\ell_{\mathrm{UV}}(x)$$

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In the semiclassical limit, up to $\mathcal{O}(\lambda^3)$ corrections:

$$\left.\frac{\delta S_{\mathcal{C}}[\phi_{\text{IR}}]}{\delta \phi^\Delta_{\text{IR}}(x)}\right|_{\phi^\Delta_{\text{IR}}=0} + \left.\frac{\delta \operatorname{Re} S_{\text{IF}}}{\delta \phi^\Delta_{\text{IR}}(x)}\right|_{\phi^\Delta_{\text{IR}}=0} = -\int_y \xi_{4-\ell}(y) \left.\frac{\delta f^\Delta_\ell(y)}{\delta \phi^\Delta_{\text{IR}}(x)}\right|_{\phi^\Delta_{\text{IR}}=0}$$

A real-time EFT perspective

$$\Box\,\bar{\phi}_{\mathrm{IR}}(x) + V'\big(\bar{\phi}_{\mathrm{IR}}(x)\big) - \sum_{\ell} \frac{\partial f_\ell\big(\bar{\phi}_{\mathrm{IR}}(x)\big)}{\partial \bar{\phi}_{\mathrm{IR}}(x)} \int_y G^R_{\ell\ell}(x, y; \lambda)\, f_\ell\big(\bar{\phi}_{\mathrm{IR}}(y)\big) = \sum_{\ell} \xi_\ell(x)\, \frac{\partial f_\ell\big(\bar{\phi}_{\mathrm{IR}}(x)\big)}{\partial \bar{\phi}_{\mathrm{IR}}(x)}\,.$$

Dissipation

Noise

A real-time EFT perspective

$$\Box\,\bar{\phi}_{\rm IR}(x) + V'\big(\bar{\phi}_{\rm IR}(x)\big) - \sum_{\ell} \frac{\partial f_\ell\big(\bar{\phi}_{\rm IR}(x)\big)}{\partial \bar{\phi}_{\rm IR}(x)} \underbrace{\int_y G^R_{\ell\ell}(x,y;\lambda)\, f_\ell\big(\bar{\phi}_{\rm IR}(y)\big)}_{\text{Dissipation}} = \underbrace{\sum_{\ell} \xi_\ell(x)\, \frac{\partial f_\ell\big(\bar{\phi}_{\rm IR}(x)\big)}{\partial \bar{\phi}_{\rm IR}(x)}}_{\text{Noise}}\,.$$

[Memory-dependent friction]

[Multiplicative Gaussian noise]

A real-time EFT perspective

$$\Box\,\bar{\phi}_{\mathrm{IR}}(x) + V'\big(\bar{\phi}_{\mathrm{IR}}(x)\big) - \sum_{\ell} \frac{\partial f_{\ell}\big(\bar{\phi}_{\mathrm{IR}}(x)\big)}{\partial \bar{\phi}_{\mathrm{IR}}(x)} \int_{y} G^{R}_{\ell\ell}(x, y; \lambda)\, f_{\ell}\big(\bar{\phi}_{\mathrm{IR}}(y)\big) = \sum_{\ell} \xi_{\ell}(x)\, \frac{\partial f_{\ell}\big(\bar{\phi}_{\mathrm{IR}}(x)\big)}{\partial \bar{\phi}_{\mathrm{IR}}(x)}\,.$$

Dissipation

Noise

[Memory-dependent friction]

[Multiplicative Gaussian noise]

> Out-of-eq Fluctuation-dissipation relations exist.
> In the High-T limit, the kernels become local.
> Langer’s rate is reproduced only if we neglect multiplicative noises.

Summary

- The nucleation rate depends on microphysics.
- There are theoretical uncertainties in the GW spectrum due to dissipation.
- The nucleation rate requires due care at low friction.
- Effective (generalized) Langevin equations can be derived from first principles (*Influence action)*.
- Realistic physics goes beyond additive white noise.
- Memory effects can affect the nucleation rate (deviations from the traditional result).
- There might be theoretical uncertainties in the GW spectrum due to dissipation.

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