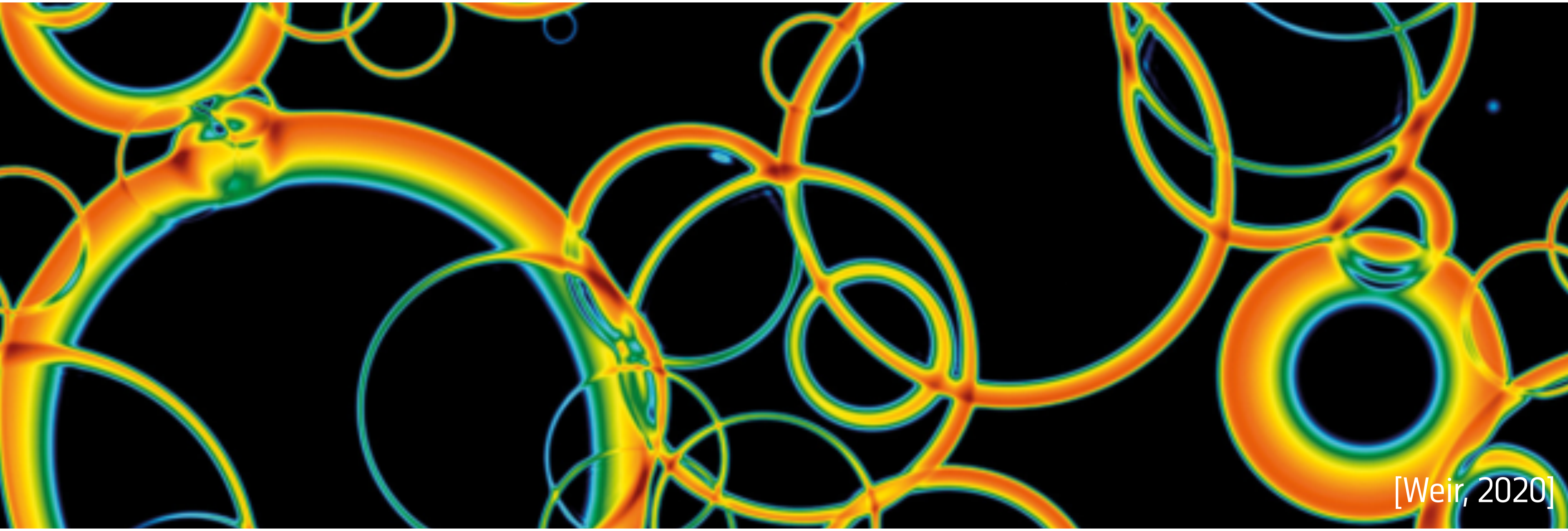
Perturbative aspects of the electroweak phase transitions with the complex singlet



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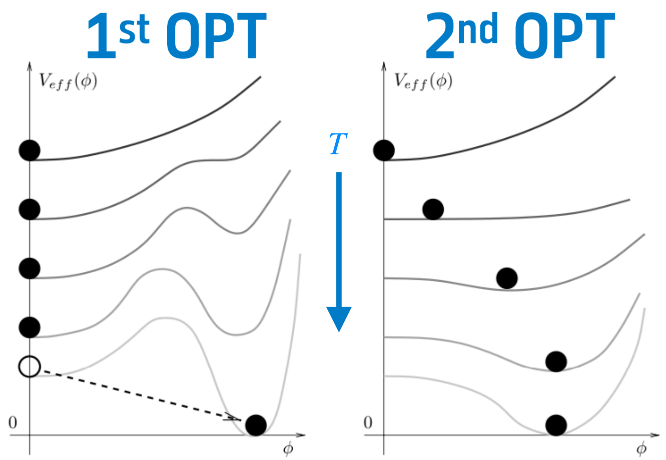
Based on: **2511.14831**

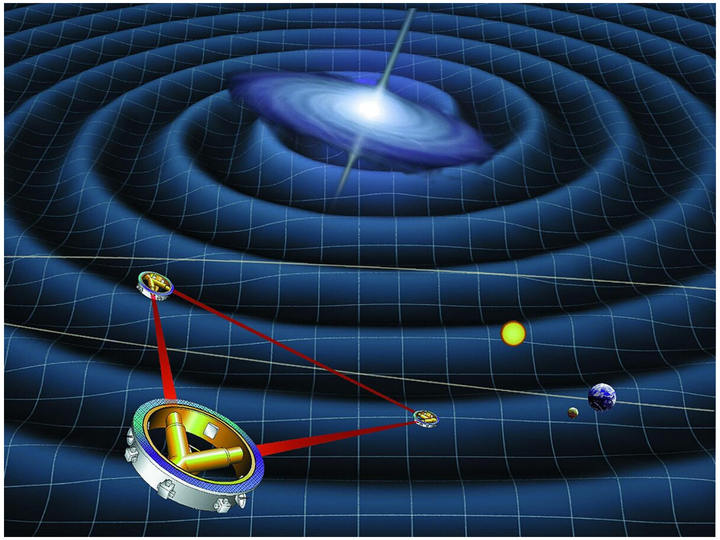
YOUNGST@RS:Shaping the Universe, 27/01/2026

High Energy Theory

Motivation: First Order Phase Transition (FOPT) in the Early Universe

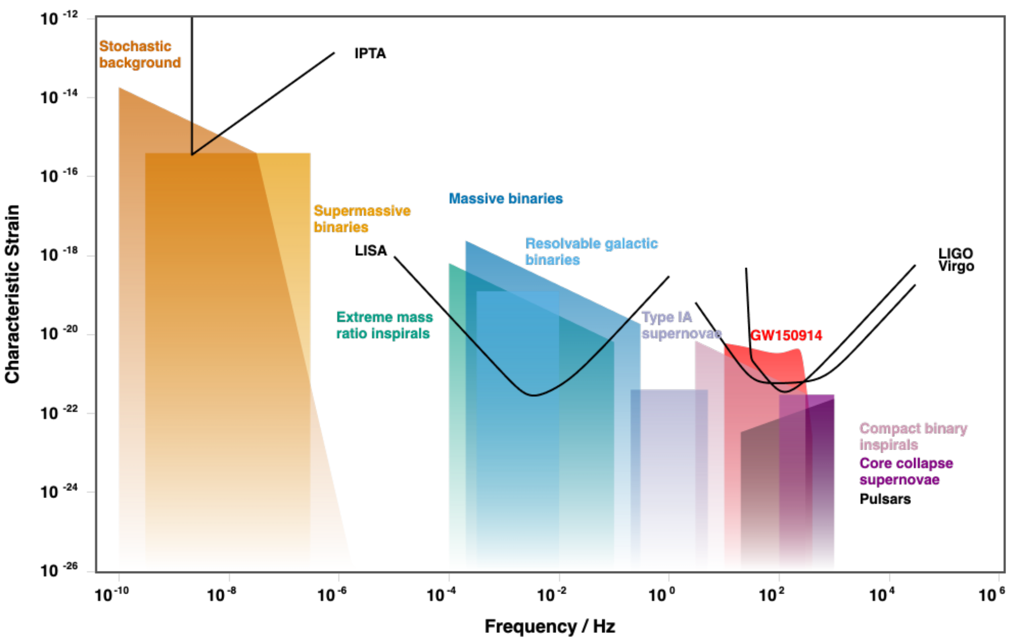
- Mechanism to satisfy Sakharov's conditions for generation matter-antimatter asymmetry
- First order phase transitions = hope for perturbative description



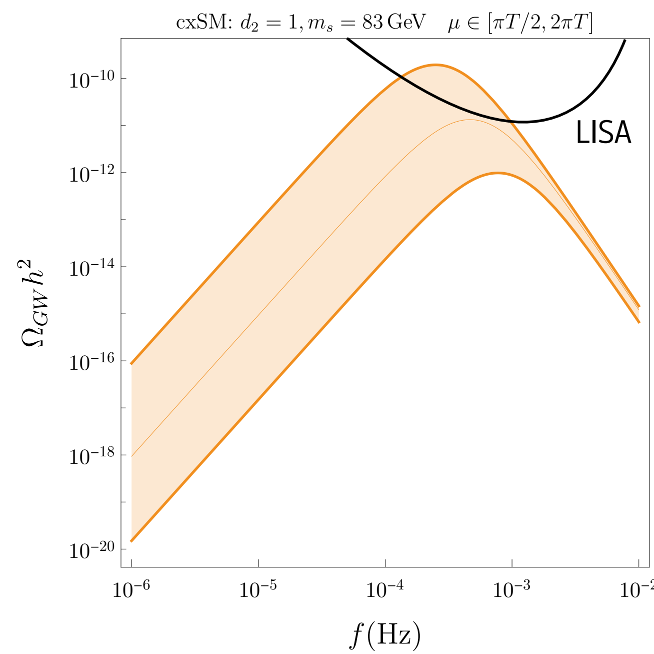


Motivation: First Order Phase Transition (FOPT) in the Early Universe

- Perfect candidate for gravitation wave detection in close future experiments (LISA, etc)
- Large uncertainties in theoretical predictions

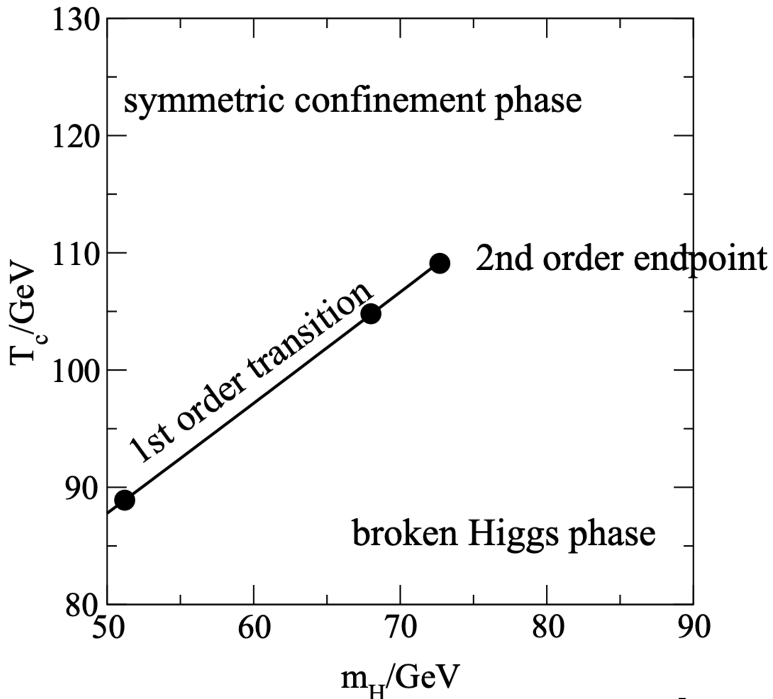


[gwplotter]



Motivation: Extended scalar sector

- No electroweak FOPT in the SM: Higgs is too light
- Naturally occurring in most of extensions of Higgs sectors (singlets, doublets, SUSY, SMEFT etc)



[Laine, 2000]

Gravitational wave signal: working pipeline

Physical inputs

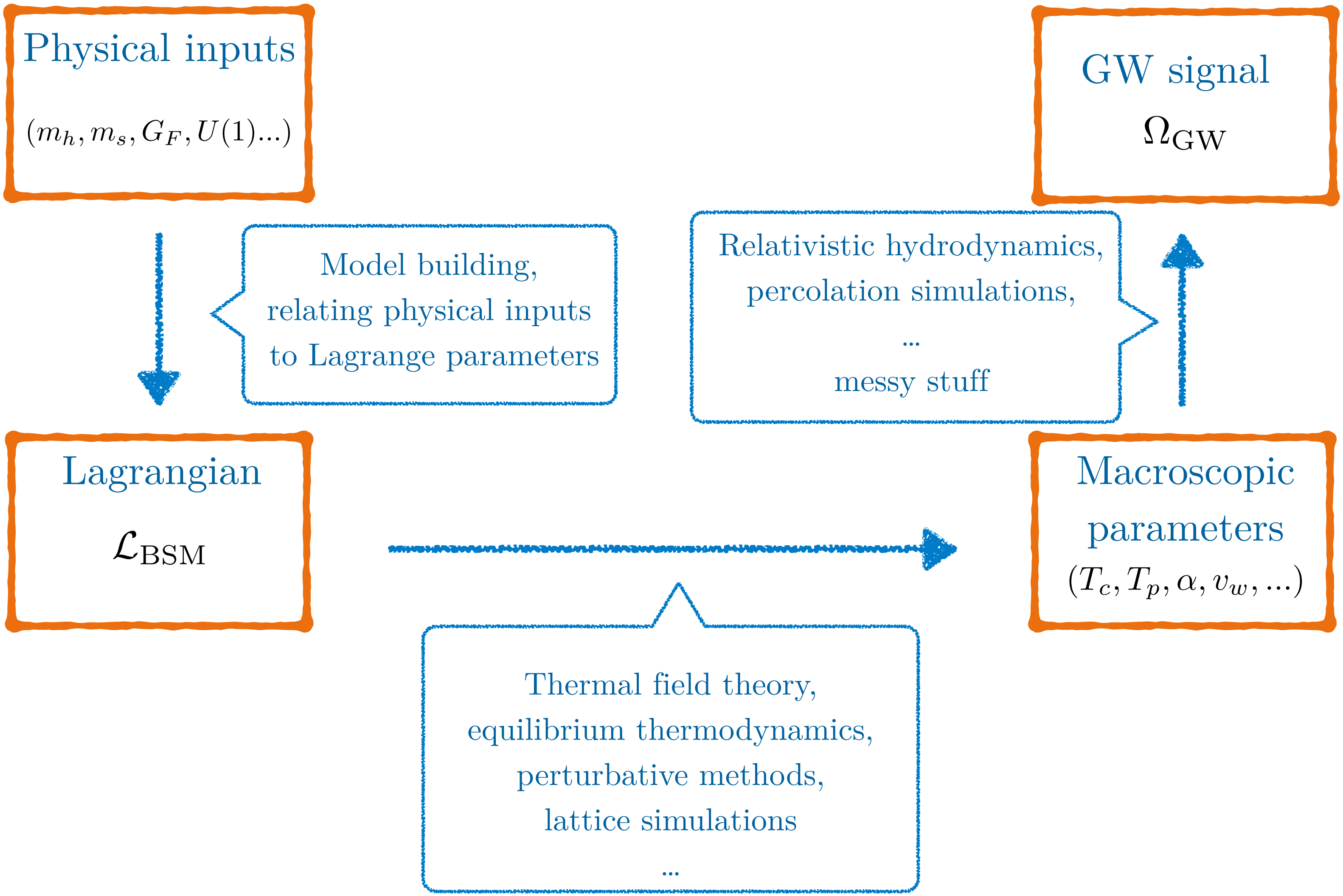
$(m_h, m_s, G_F, U(1)...)$

→

GW signal

$\Omega_{\rm GW}$

Gravitational wave signal: working pipeline

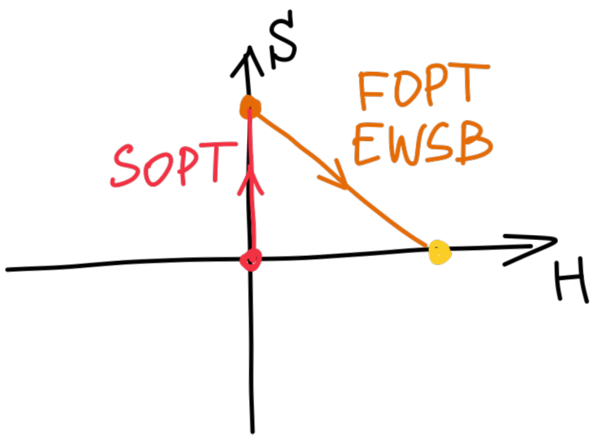


Benchmark model: cxSM

- SM + complex singlet — a realistic toy model

$$\mathcal{L}_{cxSM} \supset D_\mu \Phi^\dagger D^\mu \Phi - \mu_h^2 \Phi^\dagger \Phi - \lambda_h (\Phi^\dagger \Phi)^2 + \frac{1}{2}|\partial^\mu S|^2 - \frac{1}{2}\mu_s^2 |S|^2 - \frac{\lambda_s}{4}|S|^4 - \frac{1}{2}\lambda_{hs}|S|^2 \Phi^\dagger \Phi.$$

- Zero singlet vev at zero temperature (“flip-flop” transition)
- Collider “nightmare scenario”
- New scalars: possible Dark Matter candidates
- Extra input parameters: $m_S = m_A, \lambda_s, \lambda_{hs}$



Effective potential: vacuum structure

- Field-theoretic definition:

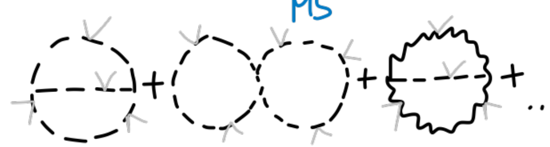
$$V_{eff}[\phi] = \frac{\Gamma[\phi]}{(vol)}$$

where $\Gamma[\phi] = W[J] - \int d^4x\, J(x)\phi(x)$ and $W[J]$ is the generating functional of connected Green's functions

$$V^{eff} = V_{tree} + V_{1\text{-}loop} + V_{2\text{-}loop} + \dots$$

$$V^{T=0}_{1\text{-}loop} = -\frac{i}{2}\int \frac{d^4k}{(2\pi)^4} \ln\left(k^2 - m_i^2\right) \overset{\overline{MS}}{=} \sum_P \frac{n_i \cdot m_i^4}{64\pi^2}\left(\log\left(\frac{m_i^2}{\mu^2}\right) - c_i\right)$$

$V^{T=0}_{2\text{-}loop} \supset$

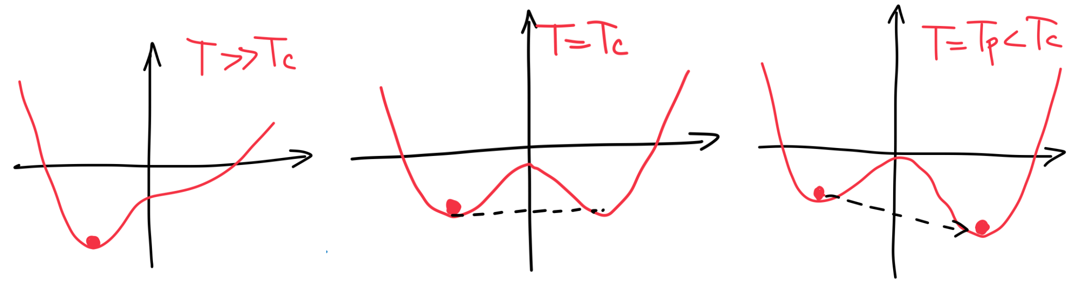


- <u>Note:</u> discussion for effective action follows same logic as presented here, just more lengthy

Phase transition thermodynamics.

Free energy density = pressure = effective potential in the minimum

Critical temperature T_c



Nucleation temperature

$$\int_{t_c}^{t_n} dt \frac{\Gamma(t)}{H(t)^3} = \int_{T_n}^{T_c} \frac{dT}{T} \frac{\Gamma(T)}{H(T)^4} \sim 1,$$

Latent heat

$$L(T) = \Delta\rho = \Delta p + T\frac{d\Delta p}{dT}$$

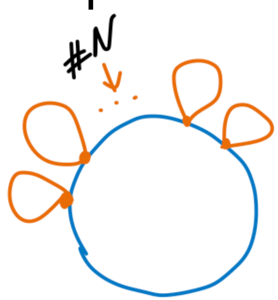
$$L_c \equiv L(T_c) = \Delta\rho\big|_{T=T_c} = T_c \frac{d\Delta p}{dT}\bigg|_{T=T_c} = -T_c \frac{d\Delta V_{eff}}{dT}\bigg|_{T=T_c}$$

Renormalization scale dependence & Daisy resummation

- Thermal loop corrections, which are dominant during the phase transition, introduce large renormalization scale dependence.

$$V_{therm} = \underbrace{V_{tree} + V^{T=0}_{1\text{-}loop}}_{\mu\text{-inv}} + \underbrace{V^{T\neq 0}_{1\text{-}loop}}_{\mu\text{-non-inv!}}$$

- The presence of hierarchy between hard ($\sim T$) and soft ($\sim gT$) scales requires resummation of the hard modes, which messes up with the loop order



$$\sim m^3 T \left(\frac{gT}{m}\right)^{2N}$$

* a daisy

[Arnold, 1992]

Gauge dependence

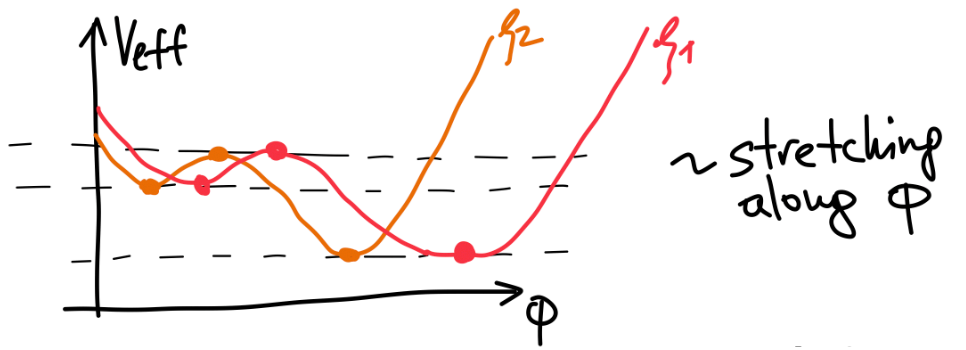
- The effective action itself is an intrinsically gauge dependent quantity, as it's defined for the non-zero source term.

- But, it's gauge dependent according to Nielsen identity:

$$\frac{\partial V_{eff}}{\partial \xi} = C_i(\varphi_i, \xi)\frac{\partial V_{eff}}{\partial \varphi_i}$$

[Nielsen, 1975]

- V_{eff} is gauge invariant at stationary point (extremums)



- Gauge invariant results can be obtained by systematic $\hbar$-expansion

[Patel, et.al., 2011]

Separating logarithms

- Large μ- dependence and need for resummations indicate large separation of scales

- Relevant scales:

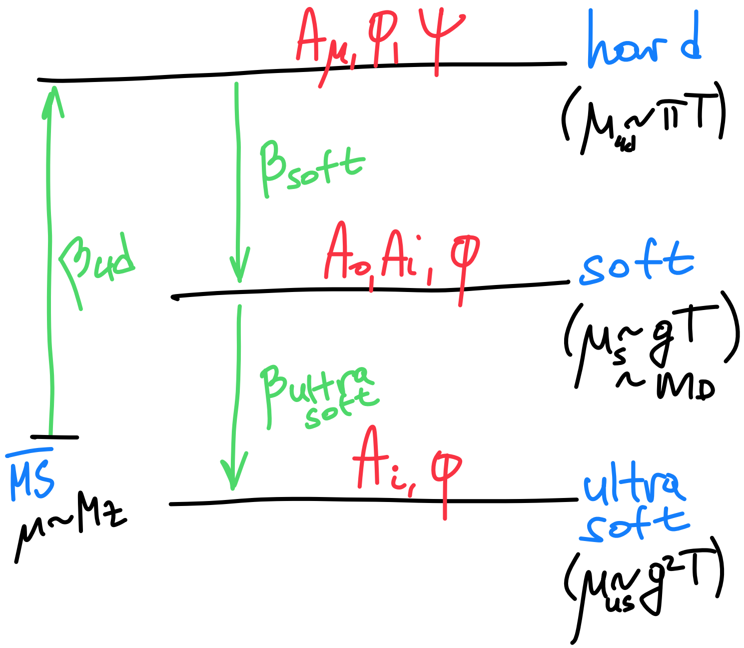
  - hard mode $\sim \pi T$
  - soft $\sim gT \sim m_D$
  - ultrasoft $\sim g^2 T$

- Basically, we have too many logarithms:

$$\log\left(\frac{\pi T}{\mu}\right), \log\left(\frac{gT}{\mu}\right), \log\left(\frac{g^2 T}{\mu}\right)$$

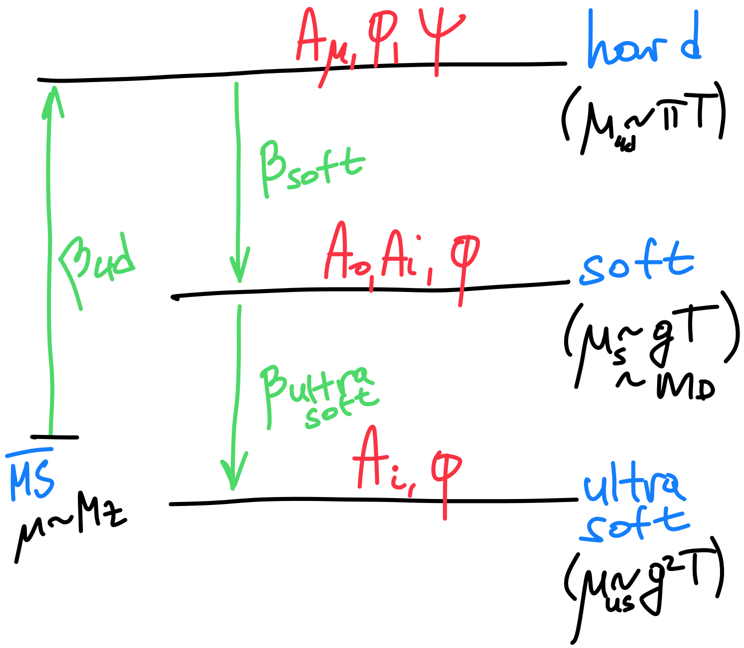
[Kajantie et al, 1994]

High-T EFT



A_μ, φ, Ψ
hard
$(\mu_{4d} \sim \pi T)$
β_{soft}
β_{4d}
A_0, A_i, φ
soft
$(\mu_s \sim gT \sim m_D)$
$\beta_{ultrasoft}$
A_i, φ
ultra soft
$(\mu_{us} \sim g^2 T)$
$\overline{MS}$
$\mu \sim M_Z$

High-T EFT



A_μ, φ, ψ
hard
$(\mu_{4d} \sim \pi T)$
β_{soft}
β_{4d}
A_0, A_i, φ
soft
$(\mu_s \sim gT \sim m_D)$
$\beta_{ultrasoft}$
A_i, φ
ultra soft
$(\mu_{us} \sim g^2 T)$
$\overline{MS}$
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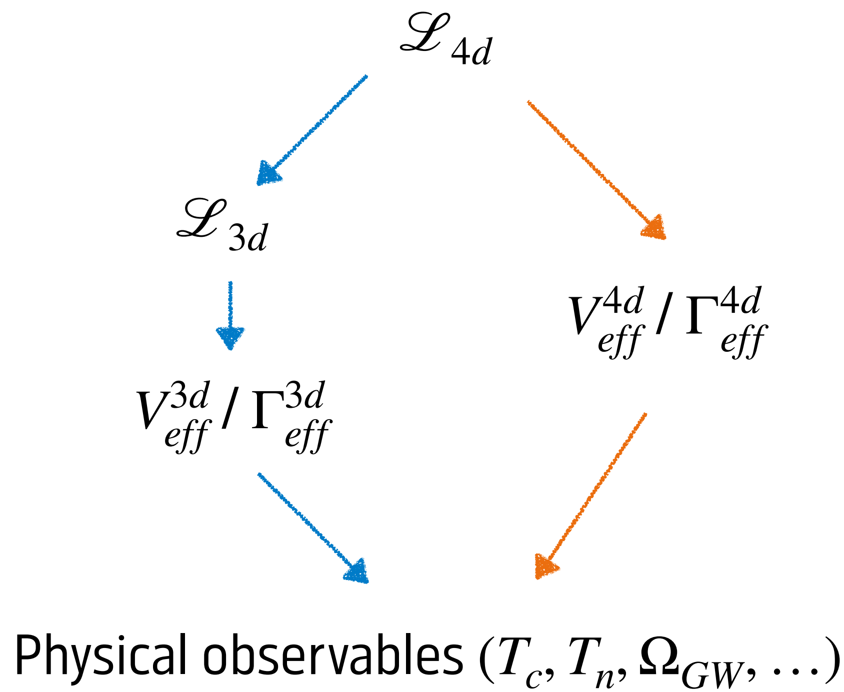
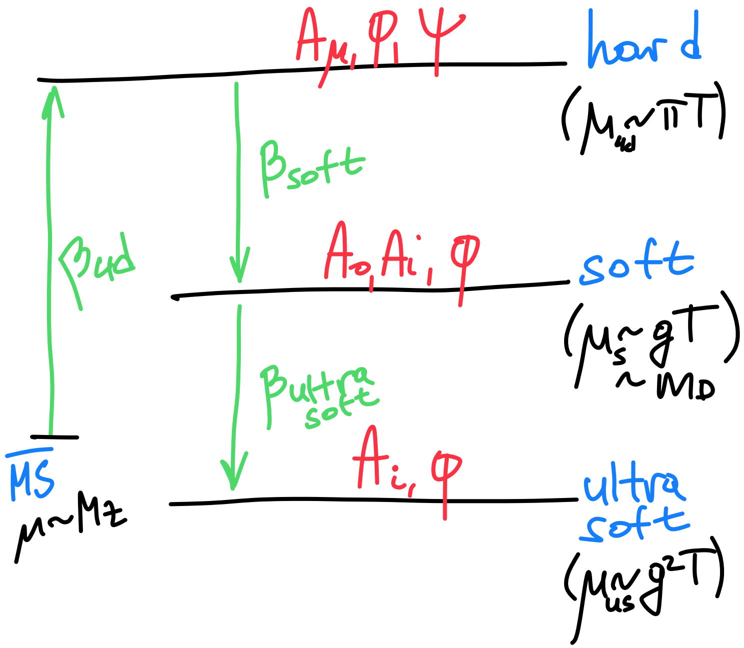
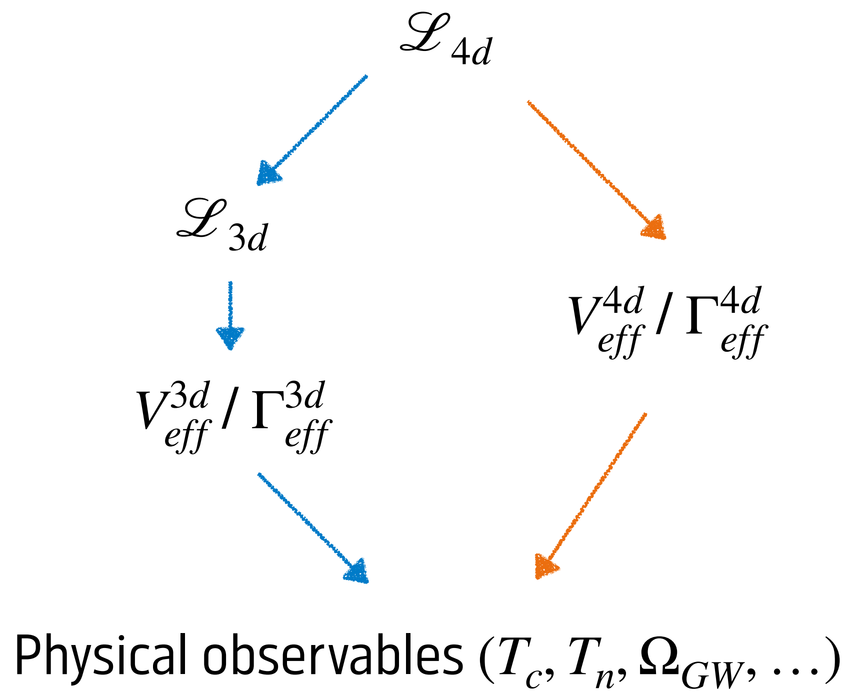


$\mathscr{L}_{4d}$
$\mathscr{L}_{3d}$
V_{eff}^{4d} / Γ_{eff}^{4d}
V_{eff}^{3d} / Γ_{eff}^{3d}
Physical observables $(T_c, T_n, \Omega_{GW}, \ldots)$

High-T EFT



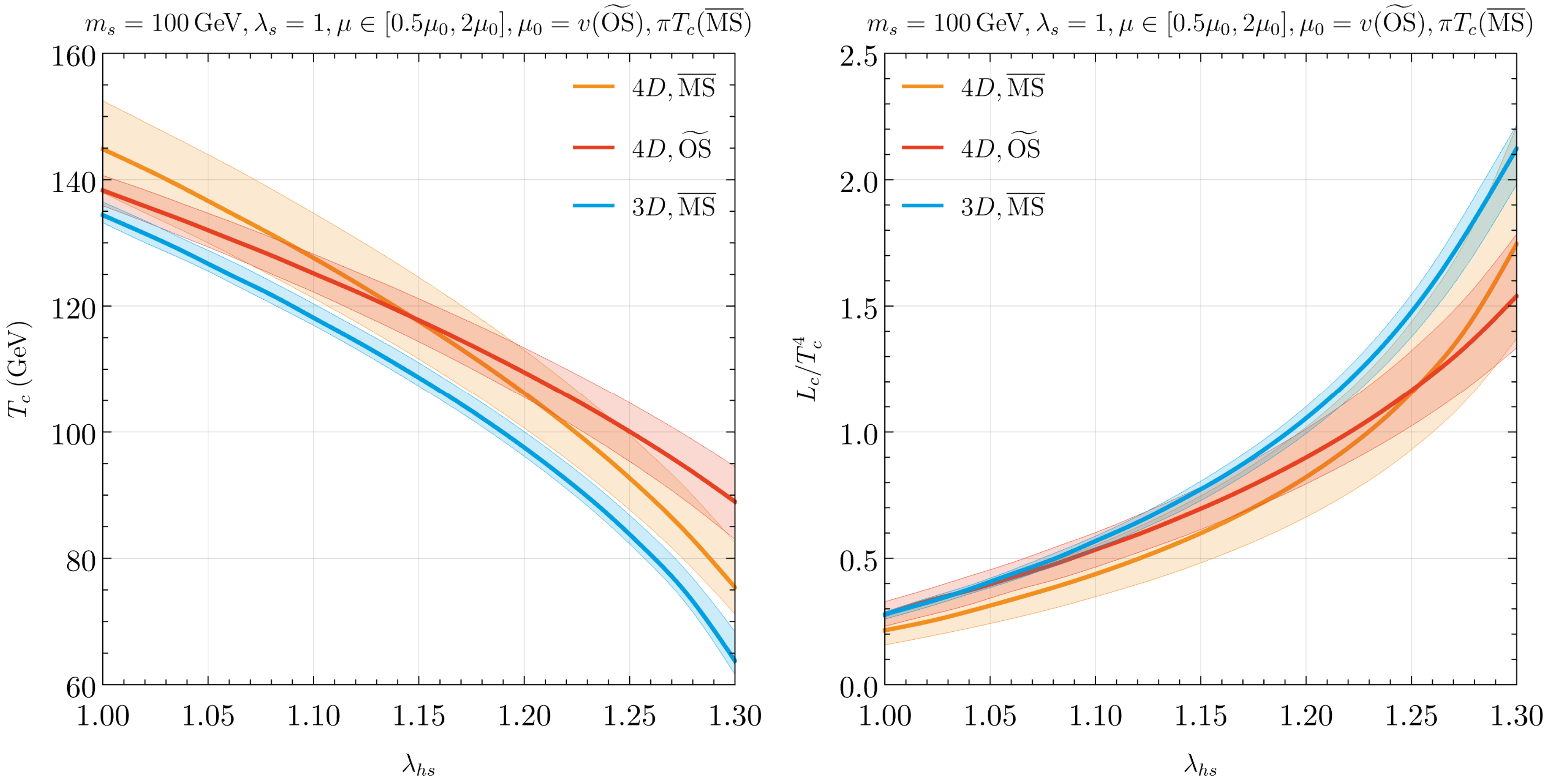


Temperature is “integrated out”

- Use $T = 0$ QFT framework
- Resummations (daisies, etc) are included systematically
- Gauge invariance is straightforward

- Validity of EFT truncation can be studied by higher-dimensional operators in the EFT

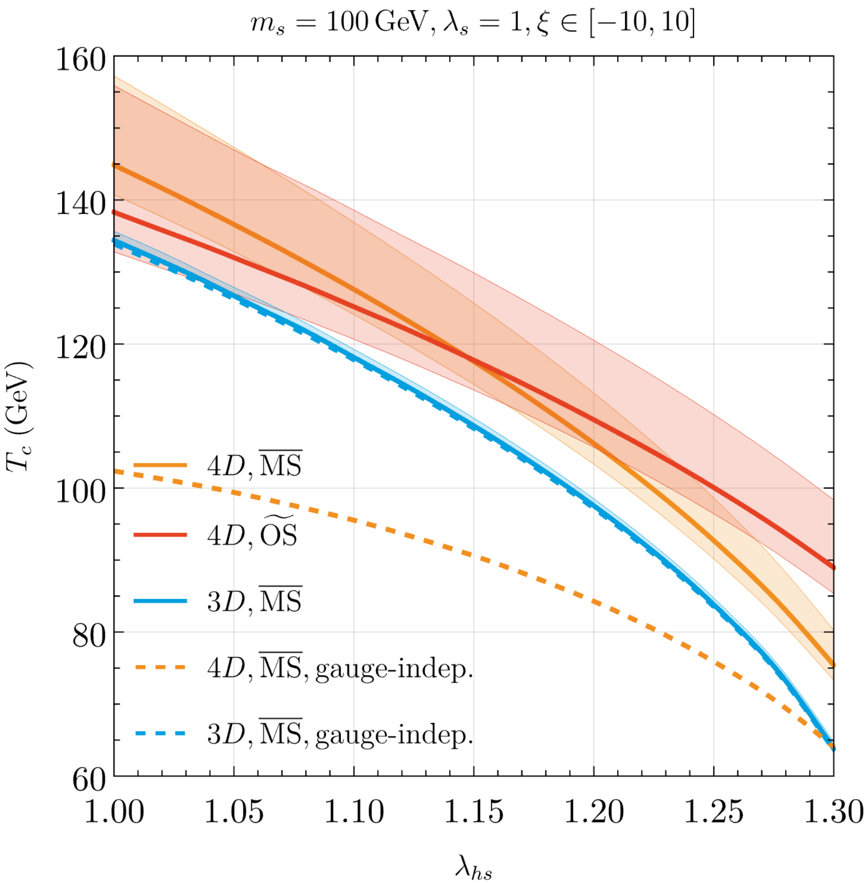
Renormalization scale dependence

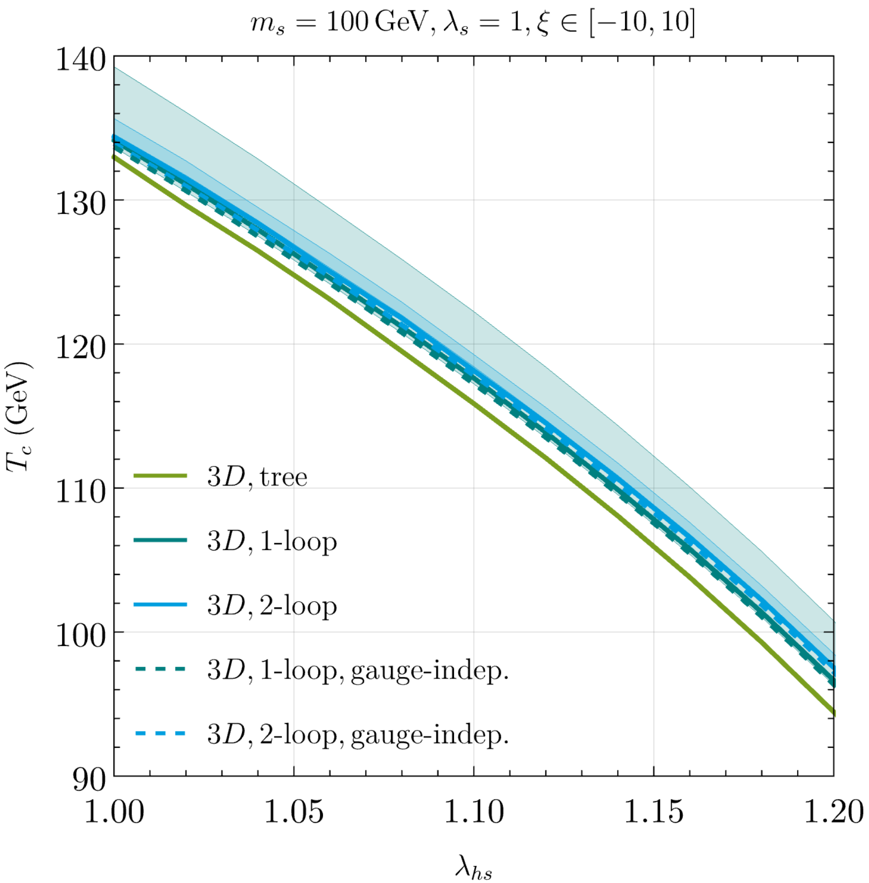


1-loop potential
+ AE-daisy resummation

2-loop potential
+ $\mathcal{O}(g^4)$-matching

Gauge dependence





Nucleation: bounce revisited

$$\Gamma(T) \approx A e^{-S_B/T}$$

$$S_B \equiv S\left[\phi_i^B\right] = 4\pi \int_0^\infty d\rho \rho^2 \left[\frac{1}{2}\left(\frac{d\phi_i^B(\rho)}{d\rho}\right)^2 + V_0(\phi_i^B(\rho))\right]$$

$$V_0 \to V_{eff}$$

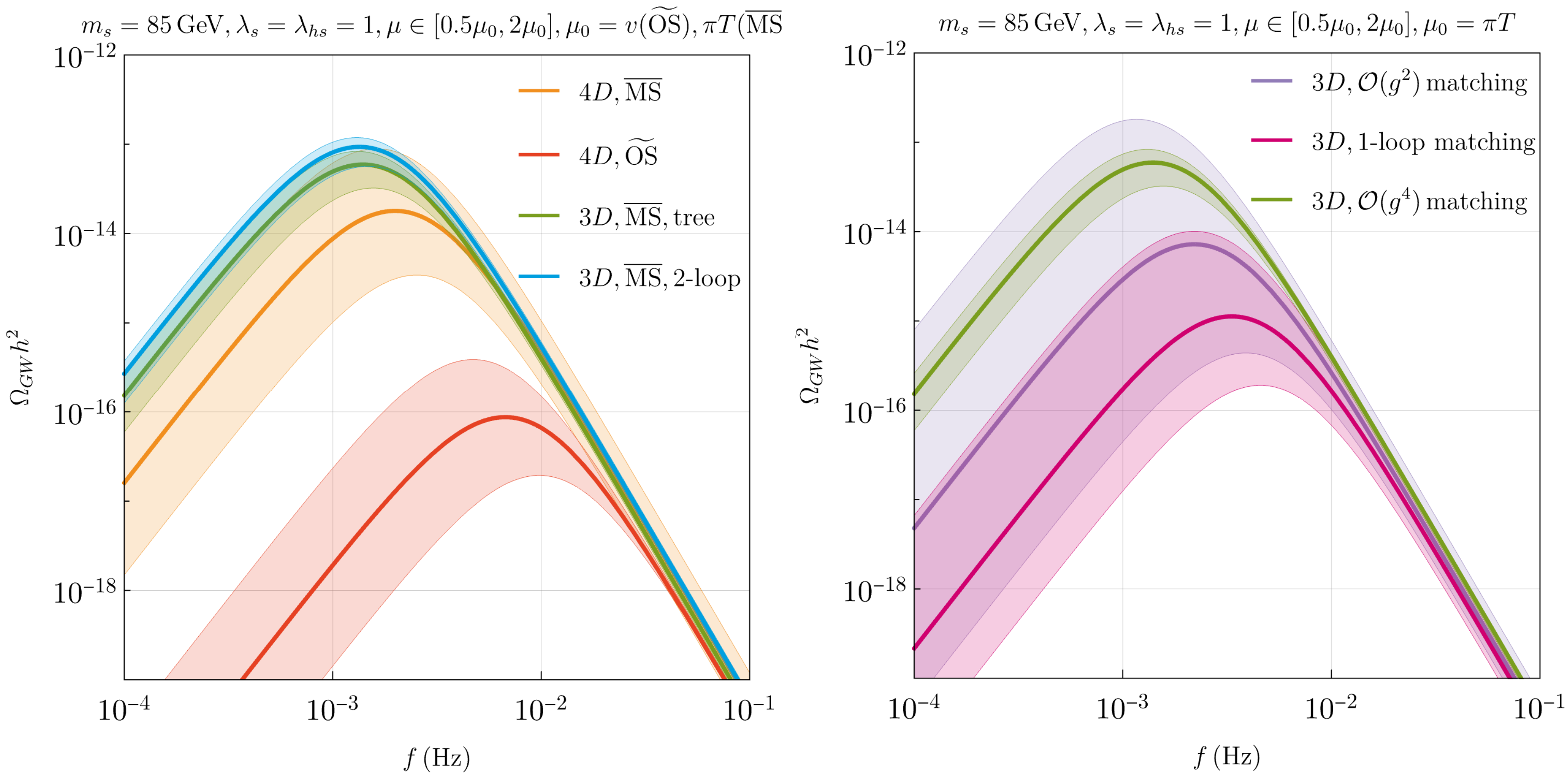
- In 4D, bounce may not even exist (again, because quantum/ thermal corrections significantly change the vacuum structure)

- Double counting of fluctuations
- Not a strict perturbative expansion in any parameter
- Gauge dependent

Separate **UV/IR** physics!

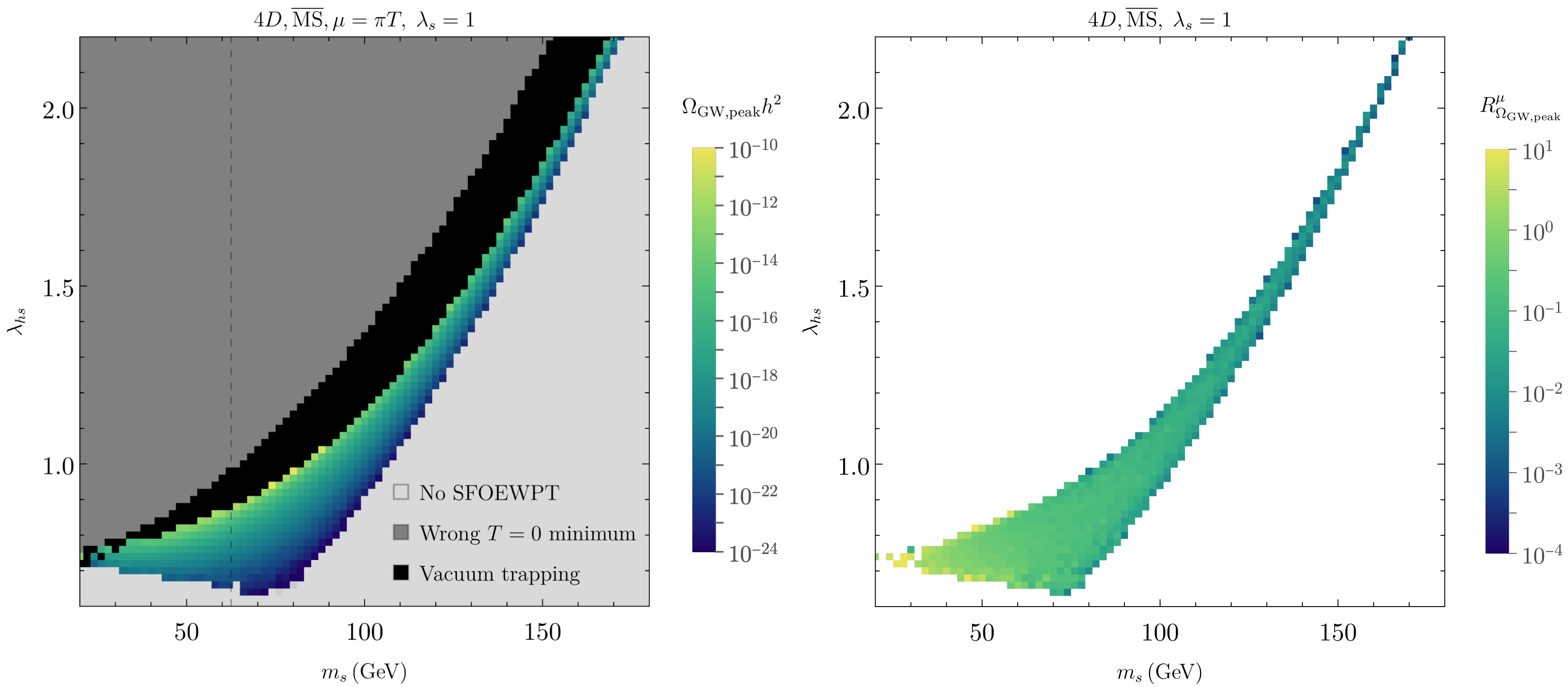
EFTs

Predicted gravitational wave signal



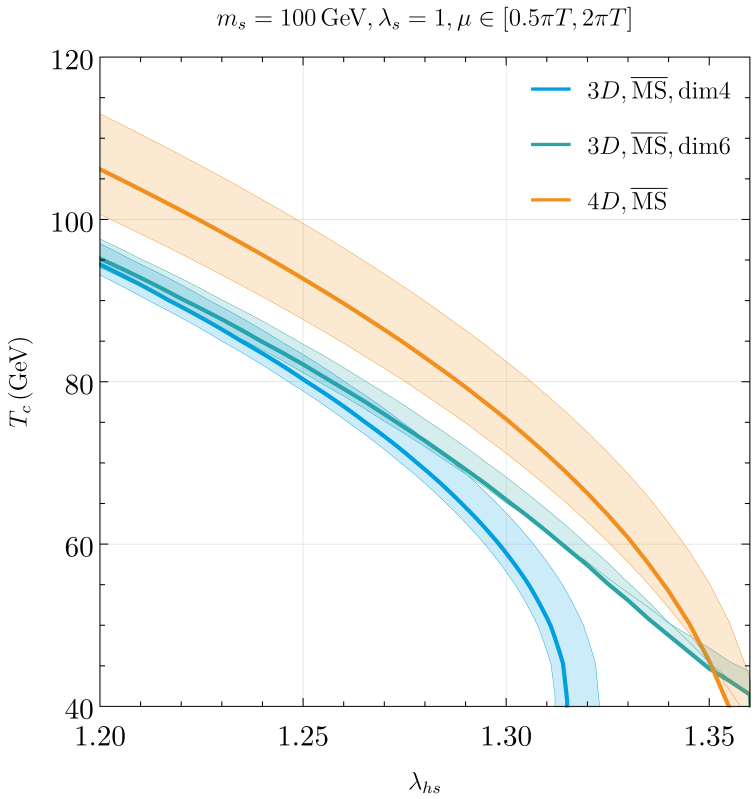
$$\Omega_{\mathrm{GW}} = \Omega_{\mathrm{GW}}^{\text{sound waves}}$$

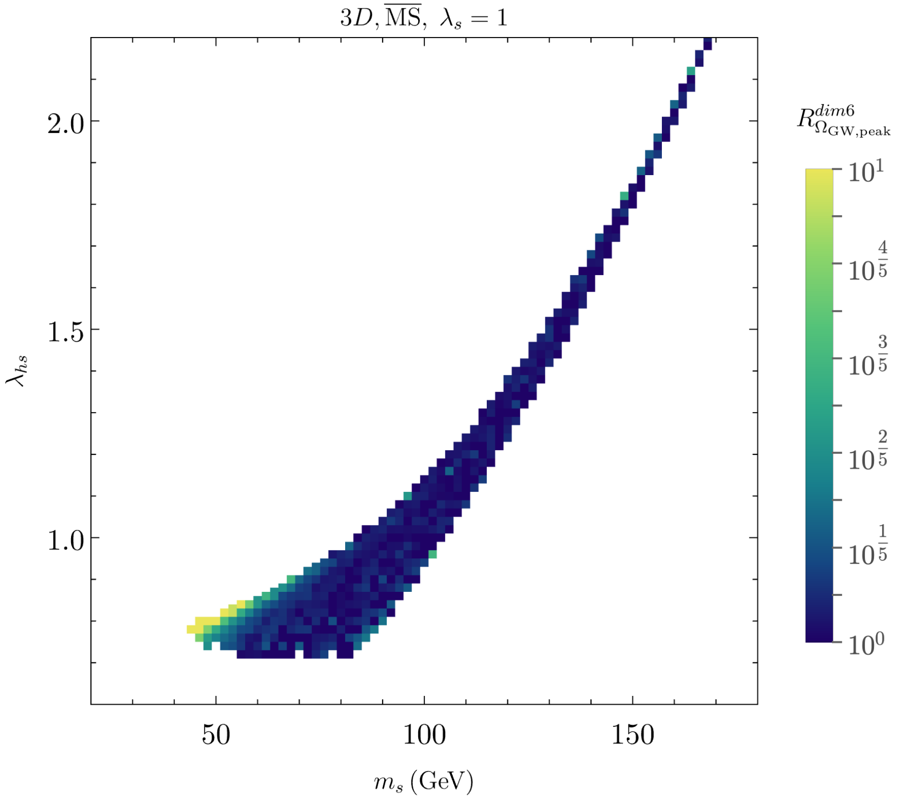
Model parameter scans



- Quiet a lot of fine-tuning for strong FOPT: usually the case for realistic BSM models

Impact of higher-dimensional operators

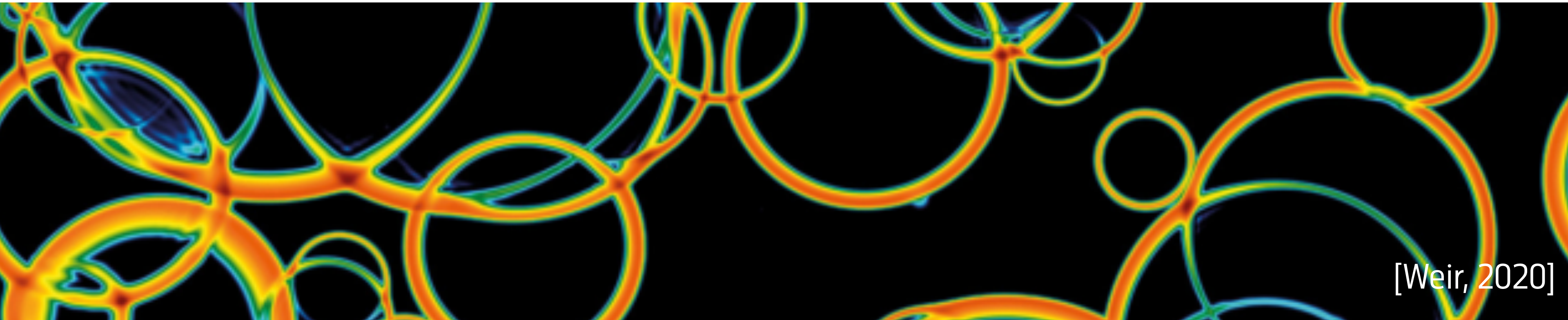




- Higher-dimensional operators in 3D-EFT approach are relevant in region of strong PT.

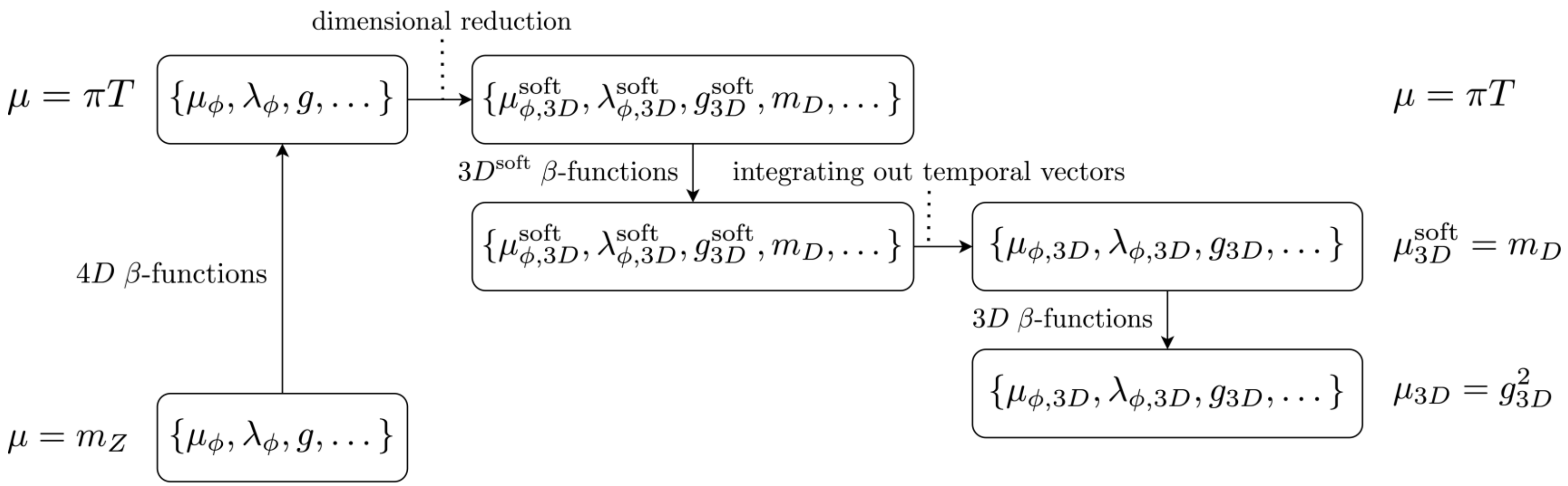
Conclusions: take-home messages.

- Phase transitions come with a lot of theoretical uncertainties: there's hope for perturbative treatment → lot's of things to do for physicists.
- Thermally driven phase transitions introduce new scaling relations, which require modification of the usual perturbation theory.
- Large scale separations can be rigorously taken into account with the help of EFT techniques (dimensional reduction) — within which, higher dimensional operators are relevant for strong PTs.
- Gauge dependence is also a good indicator of applicability of perturbative expansion for phase transitions: gauge invariance rely on perturbative expansion.



Back-up:

Dimensional reduction pipeline



Lagrange parameters determination

Another possible source of uncertainties

Physical inputs (m_h, m_s, m_z, G_F)

n-loop $\overline{MS}$ relations

$$m_h = M_h + \Pi_h(p^2 = m_h^2)$$

$$V_{eff}^{4d,T=0}$$

OS-like renormalization

$$\partial_h V_{tree} = \partial_h V_{eff} + \partial_h V_{c.t.}$$

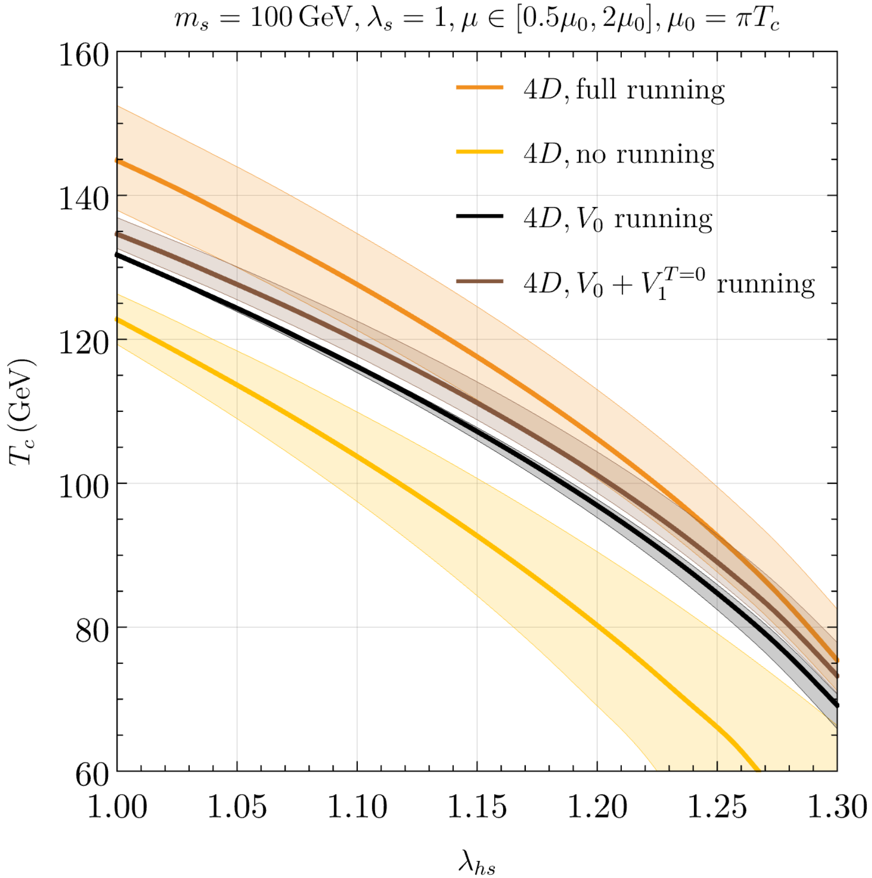
$$\partial_h^2 V_{tree} = \partial_h^2 V_{eff} + \partial_h^2 V_{c.t.}$$

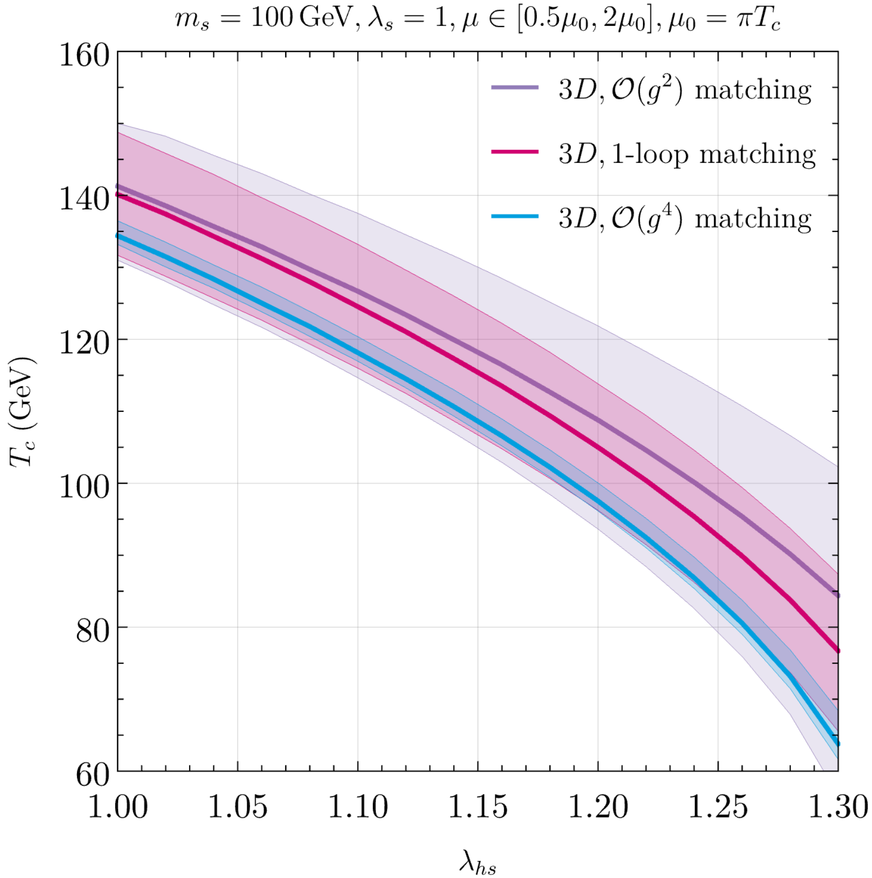
$$(\sim \Pi_h(p^2 = 0))$$

Lagrange parameters $(\mu_h, \lambda, g, \ldots)$

- Missing momentum contribution
- Gauge dependent
- Taking derivatives must be handled carefully*
- Goldtone catastrophe in Landau gauge
- Only scalar potential couplings are renormalised

Renormalization scale dependence





3d EFT loop convergence